Tue, Feb 27

* Particles:- have mass but no shape or size
  ↳ motion: translational motion
  no rotation.

* Rigid body:- (no deformation) have shape, mass, size
  motion: translational motion
  and rotation motion.

STATICS :-

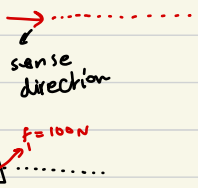
① Foundation of Dynamics
② structural analysis
③ machine design.
④ safety and reliability.
⑤ problem solving skills
⑥ Preparation for advance courses

ro

2.1 Scalar and vector quantities:-

we deal with Forces

Force :- vectors ↘ magnitude
↘ direction



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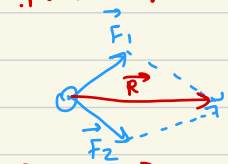
* Vector addition:-

⇒ Resultant Force:
↳ parallelogram method.
↳ Triangle method.

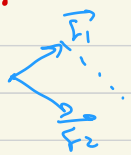
Parallel gram.

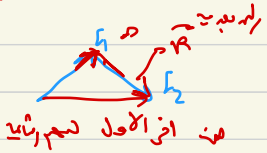


$\vec{R} = \vec{F_1} + \vec{F_2}$

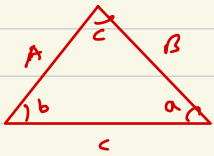


Triangle method





$\vec{R} = \vec{F_1} + \vec{F_2}$



Cosine Law:-

$$C = \sqrt{A^2 + B^2 - 2AB\cos(C)}$$

Sine Law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

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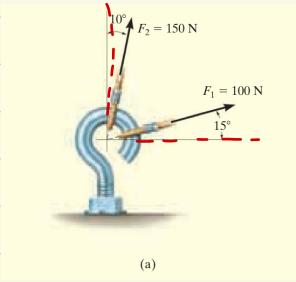
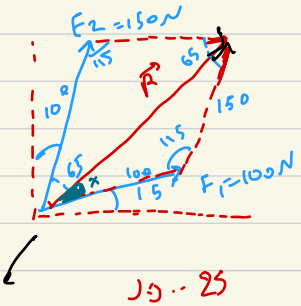
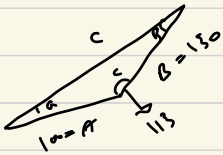
example 2.1 :-



$$\frac{360-(65)2}{2} = 115$$



$\therefore 25$



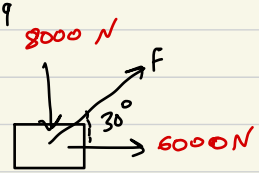
$$C = F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115}$$

$$F_R = 213 \text{ N}$$

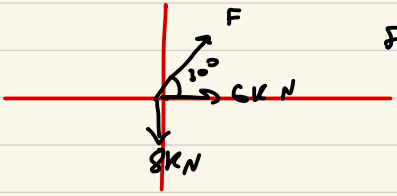
$$\frac{F_2}{\sin x} = \frac{F_R}{\sin 115} \rightarrow \frac{150}{\sin x} = \frac{213}{\sin 115}$$

$x = 54.8°$ from F x axis

Thu, Feb 29



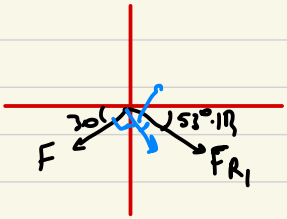
f_R as small as possible → minimum (F)



$F_{R_1} = 10\ kN$

$\theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ$

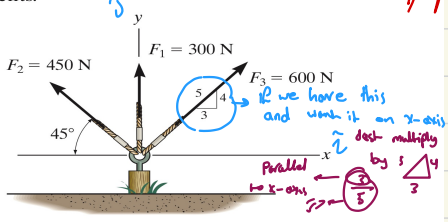
f_R as minimum → perpindicular on F



Sun, Mar 3

THE UNITS IS IMPORTANT

F2–7. Resolve each force acting on the post into its x and y components.



Prob. F2–7

786

$+\rightarrow F_x = F_{R(x)}$

$+\rightarrow F_{1(x)} = 0$ N

$+\uparrow F_{1(y)} = 300$ N

$+\rightarrow F_{2(x)} = -450 \cos 45°$ N

or $\leftarrow + = 450 \cos 45°$

$+\uparrow F_{2(y)} = 450 \sin 45°$ N

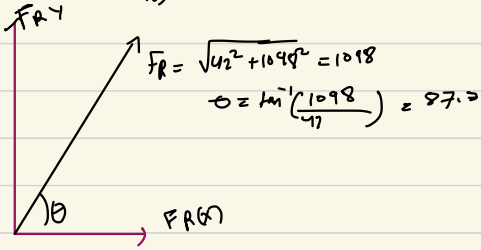
FIRST WAY

$+\rightarrow F_{3(x)} = 600\left(\frac{3}{5}\right)$ N

$+\uparrow F_{3(y)} = 600(4/5)$ N

$F_{R(x)} = 0 - 318 + 360 = 42$ N

$F_{R(y)} = 1098$



SECOND WAY:

$F_1 = (0\hat{i} + 300\hat{j})$ N

$F_2 = (-450 \cos 45° \hat{i} + 450 \sin 45° \hat{j})$ N

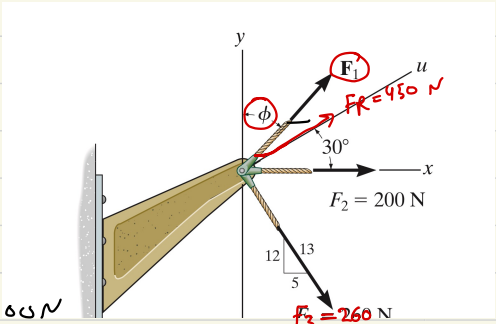
$F_3 = \left(600\left(\frac{3}{5}\right)\hat{i} + 600(4/5)\hat{j}\right)$ N

$F_R = \left(-450 \cos 45° + 600\left(\frac{3}{5}\right)\right)\hat{i} + \left(300 + 450 \sin 45° + 600(4/5)\right)\hat{j}$

vector هو اتجاه له مقدار واتجاه

$\hat{i}$ and $\hat{j}$ are vectors

***2–56.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



$F_R = 450\ N$

$\theta = 30^\circ$

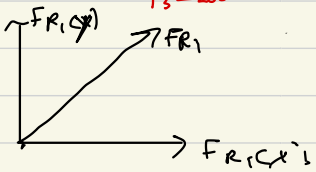
$F_2 = (200\,\hat{i} + 0\,\hat{j})\ N$

$F_3 = \left(260\left(\tfrac{5}{13}\right)\hat{i} + 260\left(\tfrac{12}{13}\right)\hat{j}\right)N$

$F_{R_1}(x) = 200 + 260\left(\tfrac{5}{13}\right) = 300\ N$

$F_{R_1}(y) = 0 + 260\left(\tfrac{12}{13}\right) = 281.6\ N$

↓ this way is so long another way:-



$450\cos 30^\circ = F_1 \sin\phi + 200 + 260\left(\tfrac{5}{13}\right)N \Rightarrow F\sin\phi = 89.41$ ...①

$450\sin 30^\circ = F_1\cos\phi + 0 + 260\left(\tfrac{12}{13}\right)N \rightarrow F\cos\phi = 465$ ...②

① ÷ ②

$\tan\phi = \dfrac{89.41}{465} \Rightarrow \phi = 109.8 \quad F_1 = 473.6\ N$

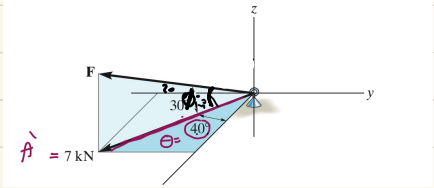
Tue, march 5

- RIGHT HAND RULE :-
  - * the tips of the fingers = x-axis
  - * the curl of the fingers = Y-axis
  - * the thumb = z-axis

$\vec{F} = F\hat{\imath}$ $\qquad \hat{\imath} = \frac{\vec{F}}{F}$

unit vector = $\frac{\text{vector}}{\text{magnitude}}$

2–62. Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the *x*–*y* plane is 7 kN.



my solution

$F' = 7\,KN$

$F_x = F' \cos\theta = 5.36\,KN$

$F_y = F' \sin\theta = 4.49\,KN$

$F_z =$

$7 = F \sin\phi$

$?? = F \cos\phi$

$F\cos\phi = F_z$

RIGHT SOLUTION:

$$\vec{u}_F = \frac{\vec{F}}{F} = \frac{F_x}{F}\tilde{\imath} + \frac{F_y}{F}\tilde{\jmath} + \frac{F_z}{F}\tilde{k}$$

$$\vec{u}_F = \frac{F\cos 30^\circ \cos 40^\circ}{F} - \frac{F\cos 30^\circ \sin 40^\circ}{F} + \frac{F\sin 30^\circ}{F}$$

$$0.6634\tilde{\imath} - 0.5567\tilde{\jmath} + 0.5\tilde{k}$$

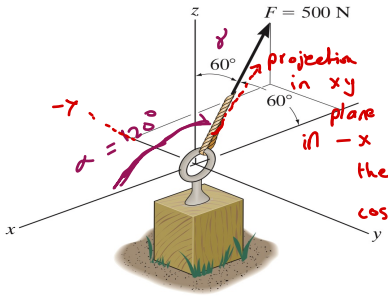
$\cos\alpha$ $\qquad$ $\cos\beta$ $\qquad$ $\cos\gamma$

$\alpha = 48.4^\circ$ $\qquad$ $\beta = 124^\circ$ $\qquad$ $\gamma = 60^\circ$

$7 = F\cos 30^\circ$

$F = 8.081\,KN$

F2–14. Express the force as a Cartesian vector.



in −x and +y then $\cos\beta$ = ⊖ neg

$1 = \cos^2 120^\circ + \cos^2 60^\circ + \cos^2 \gamma$

$\cos^2\beta = 0.5$

$\cos\beta = \pm 0.707$

$\cos\beta = -0.707$

$\beta = 135^\circ$

$\vec{F} = F\,\vec{u}_F$

$= 500\,(\cos 120^\circ \hat{i} + \cos 135^\circ \hat{j} + \cos 60^\circ \hat{k})$

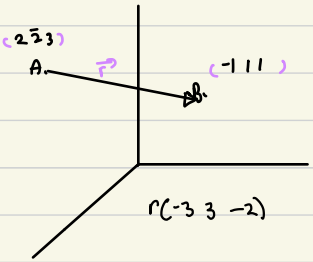
$500\,(-0.5 - 0.707 + 0.5)$

$500(-0.707) = -353.5$

$\vec{F} = -353.5\ N$

* little problem if the forces are not from the origin:-

POSITION VECTOR (r):

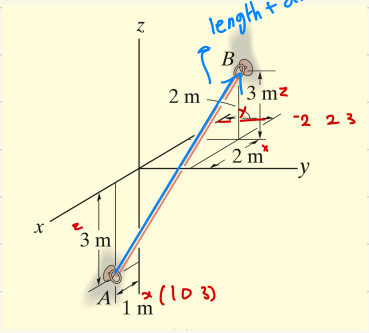


$\vec{r} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j} + (B_z - A_z)\hat{k}$ m

$= (-3\hat{i} + 3\hat{j} - 2\hat{k})$ m

$= \sqrt{-3^2 + 3^2 + -2^2}$

$= 4.7$



length + direction

Position vector → unit vector → vector

$\vec{r} = -3\hat{i} + 2\hat{j} + 6\hat{k}$

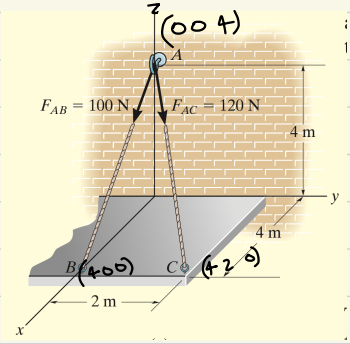
$|r| = \sqrt{3^2 + 2^2 + 6^2} = 7$ m length

$u_r = \frac{\vec{r}}{|r|} = \frac{-3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

$\cos\alpha = \frac{-3}{7} \rightarrow \alpha =$

$\cos\beta = \frac{2}{7} \rightarrow \beta =$

$\cos\gamma = \frac{6}{7} \rightarrow \gamma =$



$r_{AB} = 4\hat{i} - 4\hat{k}$ Position vector

$|r| = \sqrt{4^2 + -4^2} = 5.66$ m

$u_{AB} = \frac{\vec{r}}{r} = \frac{4}{5.66}\hat{i} - \frac{4}{5.66}\hat{k}$ unit vector

$\vec{F}_{AB} = |F_{AB}|\, u_{AB} = 100\,N\left(\frac{4}{5.66}\hat{i} - \frac{4}{5.66}\hat{k}\right)$

$= (70.7\hat{i} - 70.7\hat{k})$ vector

$r_{AC} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ position vector

$|r_{AC}| = \sqrt{4^2 + 2^2 + 4^2} = 6$ m

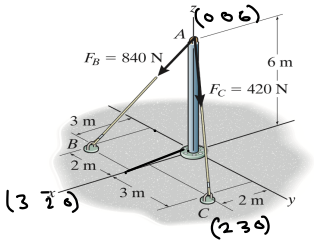
$u_{AC} = \frac{\vec{r}}{|r|} = \frac{4}{6}\hat{i} + \frac{2}{6}\hat{j} - \frac{4}{6}\hat{k}$

$\vec{F}_{AC} = |F_{AC}|\, u_{AC}$

$= 120$

⋮

F2–23. Determine the magnitude of the resultant forc
at A.



Prob. F2–23

$r_{AB} = 3\tilde{i} - 2\tilde{j} - 6\hat{k}$ position vector

$|r|_{AB} = \sqrt{3^2+2^2+6^2} = 7\ m$

$\vec{F}_{AB} = |F_{AB}|\, u_{r_{AB}}$

$= 840\left(\frac{3}{7}\tilde{i} - \frac{2}{7}\tilde{j} - \frac{6}{7}\hat{k}\right)$

$(360\,\tilde{i} - 240\hat{j} - 720\,\hat{k})\ N$

$r_{AC} = 2\hat{i} + 3\tilde{j} - 6\tilde{k}$

$|r_{AC}| = \sqrt{2^2+3^2+6^2} = 7\ m$

$\vec{F}_{AC} = |F_{AC}|\ u_{AC}$

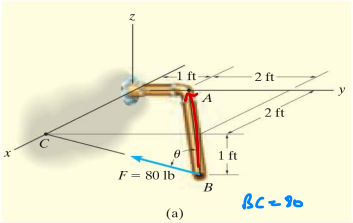
$= 420\left(\frac{2}{7}\tilde{i} + \frac{3}{7}\tilde{j} - \frac{6}{7}\hat{k}\right)$

$(120\,\hat{i} + 180\,\hat{j} - 360\,\hat{k})\ N$

$F_R = (360+120)\tilde{i} + (-240+180)\hat{j} + (-720-360)\hat{k}$

Sun, march 10

The pipe in Fig. 2–44*a* is subjected to the force of $F = 80$ lb. Determine the angle θ between **F** and the pipe segment BA and the projection of **F** along this segment.



(a)

BC = 90

BA (A → B)

A (0 1 0)

B (2 3 -1)

C (2 0 0)

$\vec{r}_{BA} = -2\vec{i} - 2\vec{j} + 1\vec{k}$ N

$|r|_{BA} = \sqrt{4+4+1} = \sqrt{9} = 3$ m magnitude

$\vec{r}_{BC} = 0\vec{i} - 3\vec{j} + 1\vec{k}$ N

$|r|_{BC} = \sqrt{9+1} = \sqrt{10}$ m magnitude

$\vec{A}.\vec{B} = |A|\,|B| \cos\theta$

$= 3\sqrt{10}\cos\theta$

6 + 1

$\cos\theta = \dfrac{\vec{r}_{BA} \cdot \vec{r}_{BC}}{|r_{BA}|\,|r_{BC}|} = \dfrac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = \dfrac{7}{3\sqrt{10}} = 0.73786$

$\theta = 42.5°$

$\theta = \cos^{-1} 0.73786$

$\vec{F}_{BC} \cdot \vec{U}_{BA} = |F_{BC}|\,(1)\cos\theta$

$\vec{F}_{BC} \cdot \vec{U}_{BA} = \vec{F}_{BA}$

الطريقة الصعبة

الطريقة الأسهل

$\vec{U}_{BC} = \dfrac{-3}{\sqrt{10}}\vec{i} + \dfrac{1}{\sqrt{10}}\vec{k}$

الذي يتبقى

$\vec{F}_{BA} = \vec{F}_{BC}\cos\theta$

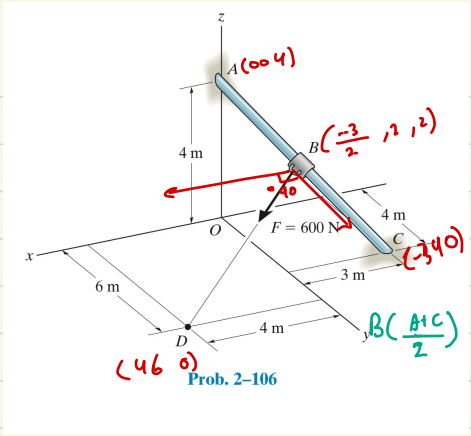
$\vec{F}_{BC} = |F_{BC}|\,\vec{U}_{BC} = 80\left(\dfrac{-3}{\sqrt{10}}\vec{i} + \dfrac{1}{\sqrt{10}}\vec{k}\right)$ N

$= (-75.89\vec{j} + 25.3\vec{k})$ N

$\vec{U}_{BA} = \dfrac{-2}{3}\vec{i} - \dfrac{2}{3}\vec{j} + \dfrac{1}{3}\vec{k}$

$+75.89\left(\dfrac{-2}{3}\right)\vec{j} + 25.3\left(\dfrac{1}{3}\right)\vec{k} = \vec{F}_{BA}$

59 N



Prob. 2–106

determine the components of F that act along rod AC and perpendicular to it, point B is located at the midpoint of the rod

$\vec{r}_{BC} = (-1.5\hat{i} + 2\hat{j} - 2\hat{k})$ N

$|r|_{BC} = \sqrt{1.5^2 + 2^2 + 2^2} = 3.2$ m

$\vec{u}_{BC} = \frac{-1.5}{3.2}\hat{i} + \frac{2}{3.2}\hat{j} - \frac{2}{3.2}\hat{k}$ N

$= -0.468\hat{i} + 0.625\hat{j} - 0.625\hat{k}$

$F_{BD} \cdot \vec{u}_{BC} = F_{BD} (\cos\theta\, \hat{i}) = F_{BC}$

$-462 \times 0.468 + (336)(0.625) + (-169.2 \times -0.625) = 600 \cos\theta = F_{BC}$

$F_{BC} = 99.53$ // AC

$\vec{r}_{BD} = 5.5\hat{i} + 4\hat{j} - 2\hat{k}$ N

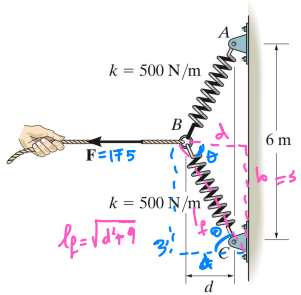
$|r|_{BD} = \sqrt{5.5^2 + 4^2 + 2^2} = 7.09$ m

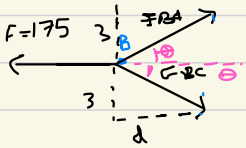
$\vec{u}_{BD} = \left(\frac{5.5}{7.09}\hat{i} + \frac{4}{7.09}\hat{j} - \frac{2}{7.09}\hat{k}\right)$ N

$\vec{F}_{BD} = F_{BD}\vec{u}_{BD} =$

$(402\hat{i} + 336\hat{j} - 169.2\hat{k})$ N

3–23. The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the displacement d of the cord from the wall when a force $F = 175$ N is applied to the cord.





$$\cos\theta = \frac{d}{\sqrt{d^2+9}}$$

$$-175 + 500s\cos\theta + 500s\cos\theta = 0$$

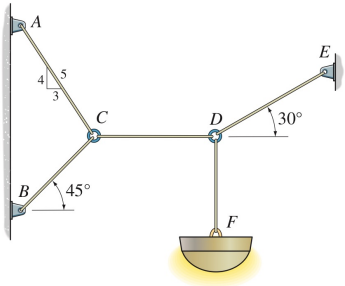
$$175 = 1000(l_f - l_0)\cos\theta$$

$$175 = 1000\left(\sqrt{d^2+9} - 3\right)\cos\theta$$

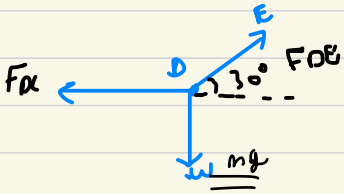
$$175 = 1000\left(\sqrt{d^2+9} - 3\right)\left(\frac{d}{\sqrt{d^2+9}}\right)$$

$$175 = 1000d - \frac{3000d}{\sqrt{d^2+9}}$$

3–30. Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.



m ?!



$\xrightarrow{+} \Sigma F_x = 0$

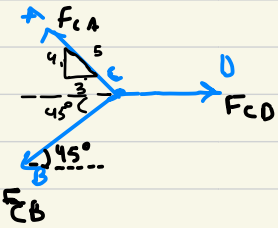
$F_{DE}\cos 30^\circ - F_{DC} = 0$

$+\uparrow \Sigma F_y = 0$

$F_{DE}\sin 30^\circ - mg = 0$

$F_{DE} = (19.62\,m)\ N$ ، $F_{DC} = (16.99\,m)\ N$

لازم نقلع كل القوى ! بدنا نشوف مين فيها أكبر شي (400)



① $16.99m - F_{CA}(3/5) - F_{CB}\cos 45^\circ$

② $F_{CA}(4/5) - F_{CB}\sin 45^\circ$

solve

$F_{CB} = (13.93\,m)\ N$

$F_{CA} = (13.14\,m)\ N$

we want maximum mass

$19.62m = 400 \rightarrow m = 20.4\ kg$

غوص لي محل اي force بدي تتحمل بس 400 N

Thu, Mar, 14

Chapter 4 :-

4.1 moment of force

Scalar formation :-

rigid body :- mass, size, shape → translational, rotational

moment :- tendacy for a body to rotate about a point which is not on the line of action of the force

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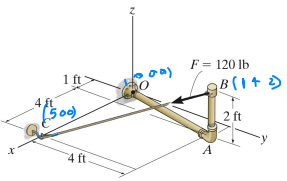
Sun, march 17

F4–11. Determine the moment of force **F** about point *O*. Express the result as a Cartesian vector.



Prob. F4–11

$\vec{r}_C = 5\hat{i} + 0\hat{j} + 0\hat{k}$ N

$\vec{r}_{BC} = -4\hat{i} - 4\hat{j} - 2\hat{k}$ N $\qquad |\vec{r}_{BC}| = \sqrt{4^2+4^2+2^2} = 6$ m

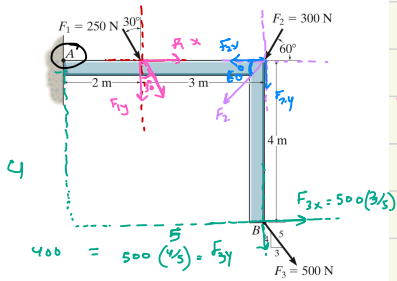
$\vec{U}_{BC} = \left(\frac{-4}{6}\hat{i} - \frac{4}{6}\hat{j} - \frac{2}{6}\hat{k}\right)$ N

$\vec{F} = F\,\vec{U}_{BC} = 120\,(0.6667\hat{i} - 0.6667\hat{j} - 0.3333\hat{k}) = 80\hat{i} - 80\hat{j} +$

Tue, march 19

4–7. Determine the moment of each of the three forces about point *A*.

***4–8.** Determine the moment of each of the three forces about point *B*.



4.7 :-

$r = -2\tilde{i}$

$M_{1y} = -2 \times (250 \cos 30)$

$M_{1x} = 0$

$M_{2y} = -300 \sin 60^\circ$ (-ve)

$M_{3x} = +500(3/5)(4)$

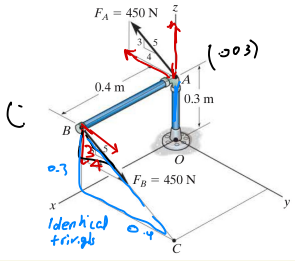
$M_{3y} = -500(4/5)(5)$

$F_{3x} = 500(3/5) = 300$

$400 = 500(4/5) = F_{3y}$

Thu, March 21

F4–24. Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.



$A(0\ 0\ 0.3)$ $B(0.4\ 0\ 0.3)$ $C(0.4\ 0.4\ 0)$

$\vec{r}_{AB} = 0.4\,\hat{i}$ m

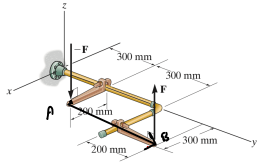
$F = 450\,(4/5)\,\hat{j} + 450\,(3/5)\,\hat{k}$ N

$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} 0.4 & 0 & 0 \\ 0 & 360 & -270 \end{vmatrix}$

$M = 108\,\hat{j} + 144\,\hat{k}$

or $\vec{F}_{BC} = 450\left(\dfrac{0.4\,\hat{j} - 0.3\,\hat{k}}{\sqrt{0.4^2 + 0.3^2}}\right) = 360\,\hat{j} - 270\,\hat{k}$

4–93. If the magnitude of the couple moment acting on the pipe assembly is 50 N · m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the *x*–*y* plane.



Probs. 4-92/93

$\vec{F} = a\,\vec{k}$

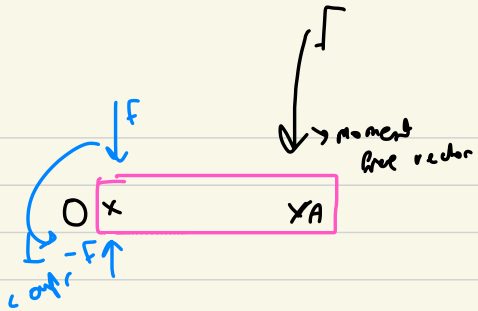
$A(0.2\ 0.3\ 0)$ m $B(0.3\ 0.8\ 0)$ m

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{r}_{AB} = (0.1\,\hat{i} + 0.5\,\hat{j})$ m

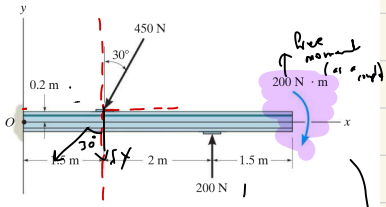
$50 = \begin{vmatrix} 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix}$

$(0.5F)\,\hat{i} + (0.1F)\,\hat{j}$



A translational
O " & rotational

4–101. Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.



Prob. 4–101

$\Sigma F_x = 450 \sin 30° = -225$ N ← neg

$\Sigma F_y = 450 \cos 30° = -389.7$

$F_R = \sqrt{225^2 + 190^2} = 294.5$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 40.2$

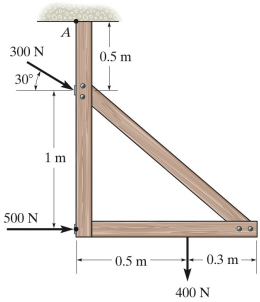
$(M)_O = \Sigma M_F + \Sigma M_C$

$225(0.2) - 389.7(0.5) + 200(1.5) - 200 = 111.5$

Solve (4.105) + (4.107)

Moment

4–105. Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.



Tue Mar 26

$F_x = 759.9$

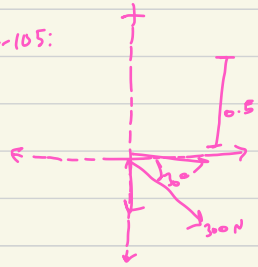
$F_y = -550$



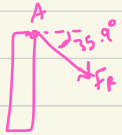
$\theta = 35.9°$

$F = 937.9$

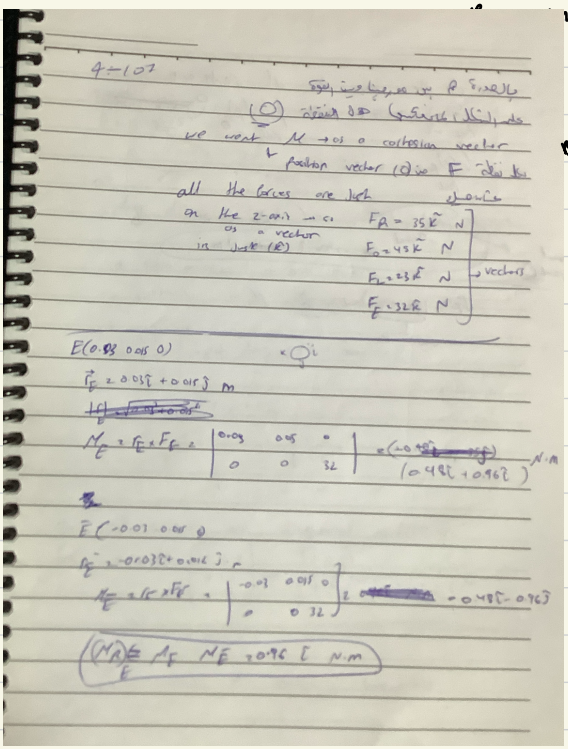
4-105:



$M_A = 300\cos 30°(0.5) - 300\sin 30°(1.5) + 50 = 650$ N.M



4-107:



4-107

(O) ... we want M → as a cartesian vector

↳ position vector (O) to F ...

all the forces are left ... on the z-axis → as a vector in unit (k)

$F_A = 35\vec{k}$ N

$F_B = 45\vec{k}$ N

$F_C = 23\vec{k}$ N

$F_E = 32\vec{k}$ N

→ vectors

E(0.8 0.6 0)

$\vec{r}_E = 0.8\vec{i} + 0.6\vec{j}$ m

$M_E = r_E \times F_E = \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & 32 \end{vmatrix}$ N.m

$\vec{E}(-0.02\ 0.04\ 0)$

$\vec{r}_E = -0.02\vec{i} + 0.04\vec{j}$ m

$M_E = r_E \times F_E = \begin{vmatrix} -0.02 & 0.04 & 0 \\ 0 & 0 & 32 \end{vmatrix}$

$(M_A)_E = M_E = 0.96\vec{i}$ N.m

(O) ... M as a cartesian vector

we do the same for all points and:

$(M_A)_z = 2.07\vec{i}$ N.m (Both)

$(M_A)_O = 0$

$(M_A)_R = -5.25\vec{i}$ N.m (Both)

$F_A = 135\times 2 = 270\vec{k}$ N

4.8 :

*example 4-41(a) all the forces pass through the point so it doesn't make a rotational

4-41(b) : the Forces don't pass through the point so it has trans- and r-

4-42 parallel force system

$F_R = \Sigma F$

$(M_R)_O = M_O + M_C$

ur Here

$F_R(d) = F_{R_1} + F_2 d_2 + F_3 d_3 \ldots\ldots$

obcluse they are area, mo

want this

d.i F_R ... (illegible)

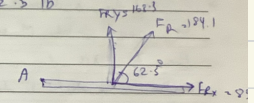
F 4-32

$F_{R(x)} = 50 \sin 30^\circ + 100(3/5) = 85$ lb

$F_{R(y)} = 50 \cos 30^\circ - 100(4/5) + 200 = 163.3$ lb

$F_R = \sqrt{85^2 + 163.3^2} = 184.1$

$\theta = \tan^{-1}\left(\frac{163.3}{85}\right) = 62.5$ lb



$(M_R)_A = M_R + M_C$

$(163.3)(d) = 50\cos 30^\circ(9) + 200(3) - 100\left(\frac{4}{5}\right)(6)$

$(163.3)(d) = 389.7 + 600 - 480$

~~509.7~~

$(163.3)(d) = 509.7$

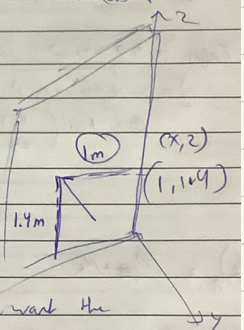
~~$d = 509.7$~~

$d = 3.12$ ft

4-124:

$F_R = 10\ kN$ but where ?? (d) ??

$= 2+3+5$



$(M_R) = \Sigma M$

* If we find the x-distance we need to take M about the z axis because x dist is ⊥ on zaxis

$(M_R)_z = \Sigma M_z$ we want the distance on the xz - plane

$-10(d) = -3(1.5) - 2(1.5) - 5(0.5)$

$\boxed{d = 1\ m}$ $x = 1$

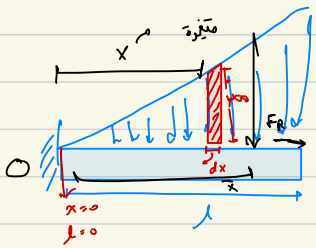
$(M_R)_x = \Sigma M$

$10(d) = 2(2.5) + 3(0.5) + 5(1.5)$

$(10)d = 14$ $z = 1.4$

$\boxed{d_z = 1.4\ m}$

Sun, Mar 31



distributed force مؤثرة على طول (infinite)

$w = w(x)$ N/m distributed

for example $w(x) = \frac{1}{2}x^2$ N/m

$x=0 \rightarrow w(0) = 0$ N

$x=1 \rightarrow w(1) = \frac{1}{2}$ N

$$\int dF_r = \int w(x)\,dx$$

$$F_r = \int w(x)\,dx \quad (dA)$$

$$F_r = \int dA = A \leftarrow \text{area under...}$$

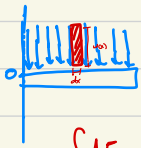
$$(M_R)_o = \Sigma M_o$$

$$(+F_R)(\bar{x}) = +\int_L x\,w(x)\,dx$$

Force

$$\bar{x} = \frac{\int_A x\,dA}{\int dA}$$

→ distribution of forces we need to replace with one F_R



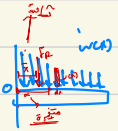
$$\int dF_R = \int w(x)\,dx$$

$$F_R = \int w(x)\,dx = \int dA = A$$

or so if we want F_R

F_R = Area under the curve

w(x) dF dx



$$(M_R)_o = \Sigma M_o$$

$$-F_R(\bar{x}) = -\int \bar{x}\,(dF)$$

$$-F_R(\bar{x}) = -\int x\,w(x)\,dx$$

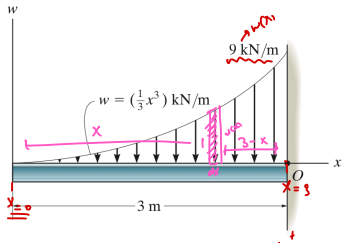
Forces

So $\bar{x}\int dA = \int x\,dA$

$$\bar{x} = \frac{\int x\,dA}{\int dA = F_R}$$

centroid

4–157. Determine the equivalent resultant force and couple moment at point O.



Prob. 4–157

Resultant force = area under the curve

$F_R = \int dA$

$F_R = \int_0^3 w\,dx = \int_0^3 \frac{1}{3}x^3 = \frac{1}{3}\frac{x^4}{4}\Big|_0^3 = 6.75$

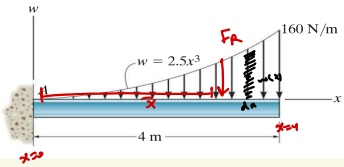
$\bar{x} = \frac{\int x\,dA}{\int dA} \rightarrow 6.75$

$\bar{X} = \frac{\int_0^3 x(\frac{1}{3}x^3)\,dx}{6.75}$

$\bar{X} = 0.6$

$M_R = 6.75(0.6) = 4.05$

F4–42. Determine the resultant force and specify where it acts on the beam measured from A.



$w(x) = 2.5x^3$ we take this, because it is the beam

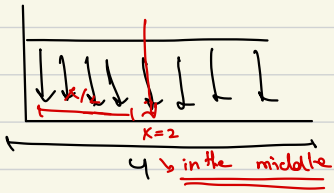
$F_R = \int dA$

$F_R = \int w(x)\,dx = \int_0^4 2.5x^3\,dx$

$F_R = \frac{2.5}{4}x^4\Big|_0^4 = 160$ N

$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^4 2.5x^4\,dx}{160} = \frac{\frac{2.5}{5}x^5\Big|_0^4}{160} = \frac{512}{160} = 3.2$ m

If the load is consistent

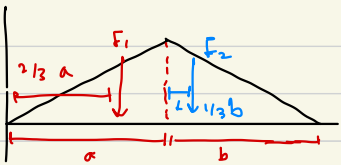


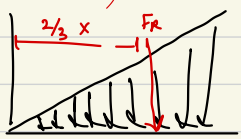
y is in the middle

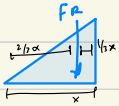
$\frac{1}{3}$ من القاعدة

or

من الزاوية القائمة

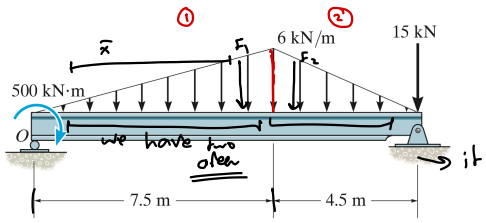






Tue, April 2

4–151. Replace the loading by a single resultant force, and specify the location of the force measured from point O.

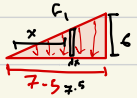


Prob. 4–151

$w(x) = 6$ KN/m

$F_R = \int w(x)\,dx = \int 6\,dx = 6x$

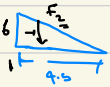
it doesn't move because it's supported even when there is F_R if stoped the translation and rotation



$+\downarrow$ $F_1 = \frac{1}{2} \times 7.5 \times 6 = A = 22.5$ KN $\downarrow$

$\bar{x}_1 = \frac{2}{3} \times 7.5 = 5$ m

$\downarrow^+$ $F_2 = \frac{1}{2} \times 6 \times 4.5 = A = 13.5$ KN $\downarrow$



$\bar{x} = \frac{1}{3} \times 4.5 + 7.5 = 9$

$F_R = \Sigma F = 22.5 + 13.5 = 51$ KN $\overset{+}{\downarrow}$

$(M_R)_o = \Sigma M_o + \Sigma M_c$

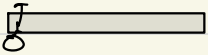
$-51d = -13.5(9) - 22.5(5) - 15 \times 2 - 500$

$d = 17.92 \rightarrow$ F_R ؛ Tab ؛ beam

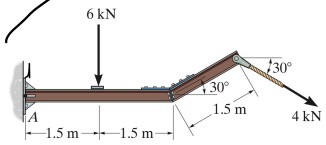
---

equilibrium of a rigid body Thu, April 4

* supports have forces ( ) the forces on the body to stop movement

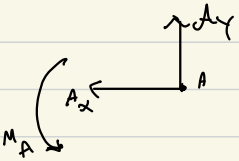


5–10. Determine the components of the support reactions at the fixed support A on the cantilevered beam.



Prob. 5–10

Pull sport → so no movement → reaction + moment (normal) in x and y



$\stackrel{+}{\rightarrow} \Sigma F(x) = 4\cos 30° - A_x = 0$

$A_x = 3.46\ kN$

$+\uparrow \Sigma F(y) = 0$

$F_y = A_y - 6 - 4\sin 30 = 0$

$A_y = 8\ kN \uparrow +$

$+\circlearrowleft M_A = 0$

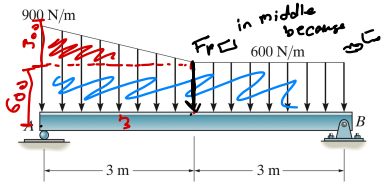
* اذا طلع عندي negative بمعنى انه وزت اتجاه غلط بس بعكس الاتجاه يصير

$MA - 6(1.5) - 4\sin 30(3 + 1.5\cos 30°) - 4\cos 30°(1.5\sin 30°)$

$+\ MA = 20.2\ kN/M$

* بحلل اتجاه اي forces عكس عامة اي forces اللي هنا

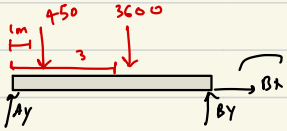
5–13. Determine the reactions at the supports.



$\downarrow F_{R\square} = 6 \times 600 = 3600\ N$

$\downarrow F_{R\triangle} = \frac{1}{2}(3)(900) = 450\ N$

distance = $1m = \frac{1}{3}\times 3$ from A



$\stackrel{+}{\rightarrow} \Sigma F_x = B_x = 0$

$+\uparrow \Sigma F_y = A_y + B_y - 4050$

$A_y + B_y = 4050$

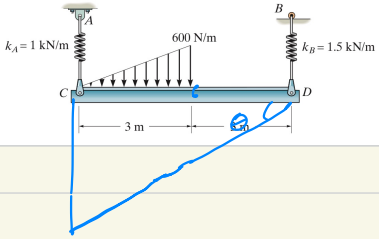
$\Sigma M_A = 0$

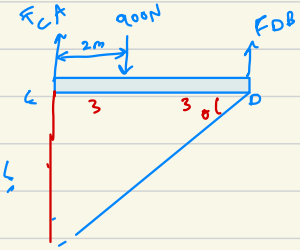
$-450(1) - 3600(3) + B_y(6) = 0$

$B_y = 1875\ N \uparrow$

plug into

*5–36. The beam of negligible weight is supported horizontally by two springs. If the beam is horizontal and the springs are unstretched when the load is removed, determine the angle of tilt of the beam when the load is applied.





$+\uparrow \Sigma F_y = -900(2) + F_{CA} + F_{DB} = 0 \quad \ldots (1)$

$\Sigma M_C = 0$

$F_{DB}(6) - 900(2) = 0$

$F_{DB} = 300\,N$

sub into 1

$F_{CA} = 600N$

$F_{DB} = K_{DB}\, S$

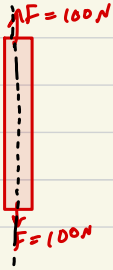
$300 = 1500\, S \rightarrow S_{DB} = 0.2$

$F_{CA} = K\, S_{CA}$

$600 = 1000\, S_{CA} \rightarrow S_{CA} = 0.6$

Sun, Apr 7

two-Force member



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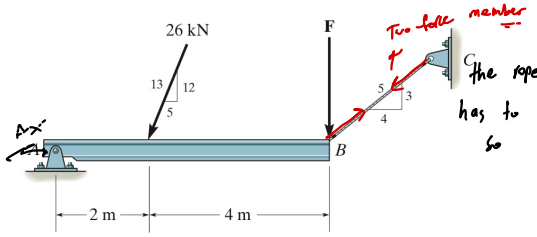
*5–48. If rope BC will fail when the tension becomes 50 kN, determine the greatest vertical load F that can be applied to the beam at B. What is the magnitude of the reaction at A for this loading? Neglect the thickness of the beam.

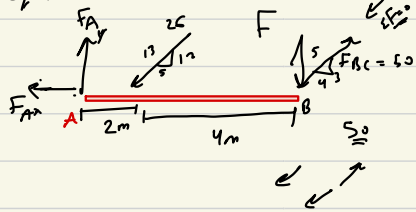


Probs. 5–47/48

Two-force member

the rope BC has to be in equilibrium so

$F_{BC} = 50\,kN$



$50(3/5) + F_{Ay} - 26(12/13) - F = 0$

$50(4/5) + F_{Ax} - 26(5/13) = 0$

$-40 + 10 = F_{Ax} \rightarrow F_{Ax} = 30$

$F_{AY} = 26(12/13) - 50(3/5)$

$F_{AY} = 74$

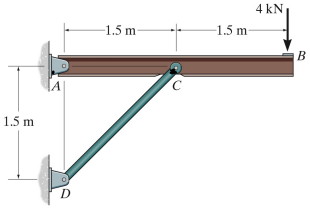
$\Sigma(M)_A = 0$

$-26(12/13)(2) - F(6) + 50(3/5)(6) = 0$

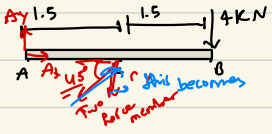
$-48 + 180 = F(6)$

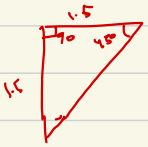
$F = 22\,kN$

F5–2. Determine the horizontal and vertical components of reaction at the pin A and the reaction on the beam at C.



Prob. F5–2





$\Sigma F_x = 0$ $\quad A_x + C_x = 0$ $\quad | \quad \Sigma F_y = 0$ $\quad A_y + C_y - 4 = 0$

$A_x = -8\ KN$

$-8KN \leftarrow$

$8 - 4 + A_y = 0$

$\Sigma (M)_A = -4(3) - C_y(1.5)$

$A_y = -4\ KN$

$= 4KN\downarrow$

$1.5\ C_y = -12$

$C \sin 45° = \frac{-12}{1.5} \times$

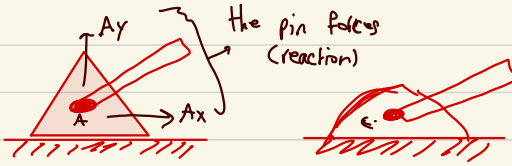
$F_C = -11.31 = 11.31$

Support:

we have equilibrium

1. pin/Hinge have two reaction/ one on x-axis and one on y-axis

ال support يعمل قوة عكس اتجاه القوة عليه عشان يعمل باتزان

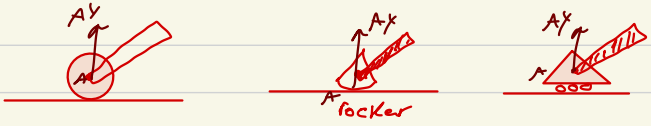


ممنوع ال moment

So it doesn't have a reaction moment

---

2. Roller/Rocker



I can move it along x-axis so it doesn't have a reaction on x-axis

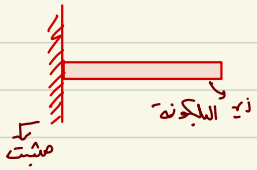
I can't move it along y-axis so it has a reaction

يقدر يدور

So no moment reaction

---

3. Fixed:



I can't move it along x-axis so we have a reaction on x-axis

I can't move it along y-axis " " " " " on Y-axis

I can't rotate it by moment " " " " " on moment

} so that we have a full equilibrium it doesn't move.

* خطوات الحل:

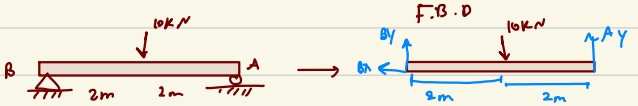
- F.B.D ← بدل الـ support بالـ Reaction تاعو
  - بحلل القوة الماثلة (إذا وجدت)
  - بحول الـ Distributed load ← Concentrated load

- المعادلات اللي بنستخدمها: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

* السؤال بيجي Find the reactions

---

①



$\Sigma F_x = 0 \quad -Bx = 0 \rightarrow Bx = 0 \rightarrow$

$\Sigma F_y = 0 \quad Ay + By = 10$

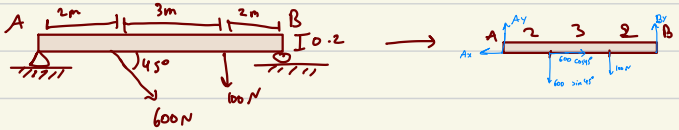
$\Sigma M = 0 \quad (M_B)_R = Ay(4) - 20 = 0 \rightarrow \boxed{Ay = 5\ KN} \uparrow$

عند كل نقطة $\Sigma M = 0$

$(M_A)_R = -By(4) + 20 = 0 \quad \boxed{By = 5KN} \uparrow$

---

②



$\Sigma F_x = 0 \rightarrow 600\cos 45° - Ax = 0 \quad \boxed{Ax = 424.3\ N} \leftarrow$

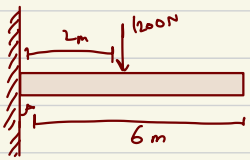
$\Sigma F_y = 0 \rightarrow Ay + By = 100 + 600\sin 45° \quad ---①$

$\Sigma M = 0 \quad (M_A)_R = By(7) - 100(5) - 600\sin 45°(2) - 600\cos 45°(0.2)$

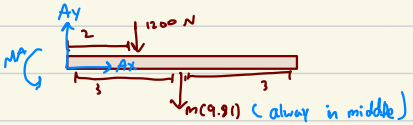
$7By = 1433.33 \rightarrow By = 204$

$Ay = 524.6 - 204 = 319.5$

③



mass of beam = 100 kg وزن الـ beam



$\Sigma F_x = 0 \rightarrow A_x = 0$

$\Sigma F_y = 0 \quad A_y = 1200 + 981 \Rightarrow \boxed{A_y = 2181\ N \uparrow}$

$+\circlearrowleft \Sigma M_A = 0 \quad M_A - 1200(2) - 981(3) = 0$

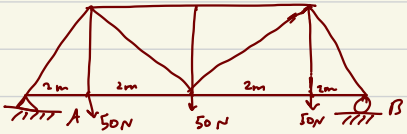
moment الـ unit هنا ← $M_A = 5343$ ↳ $\boxed{N.m}$

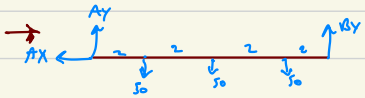
beam الـ support الـ reaction force

عكس ما بالرسم

---

④ this from chapter 6 (but let's see)





$A_X = 0$

$A_y + B_y = 150$

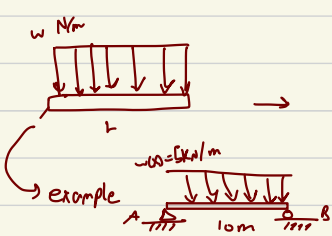
$+\circlearrowleft \Sigma M_A = 0 \quad -50(2) - 50(4) - 50(6) + B_y(8) = 0$

$B_y = 75_N \rightarrow 75 + A_y = 150$

$A_y = 75\ N \uparrow$

* دائماً عشان اوجد الـ Reaction لازم احول من dis load ← cons load.

* نفس الخطوات لإيجاد الـ Reaction



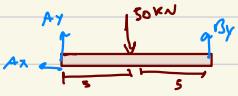
$F_R = \int w(x)\,dx = \int dA = A$ area under

$\bar{x} = \frac{1}{2}L$ Location

① example

$F_R = 5 \times 10 = 50\ KN$

location : in middle



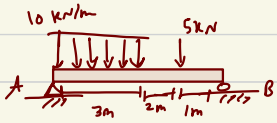
$A_x = 0$, $A_y + B_y = 50$

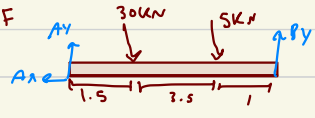
$\sum M_A = B_y(10) - 50(5)$

$B_y = \frac{250}{10} = 25$ ....

---

②

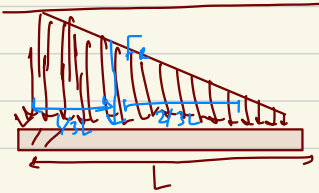




$A_x = 0$, $A_y + B_y = 35$, $\sum M_A = 0 \rightarrow 6B_y = 30(1.5) + 5(5)$

$B_y = 11.6$

$A_y = 23.4$

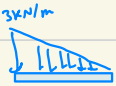


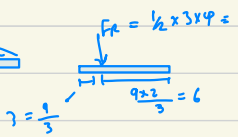
$F_R = \frac{1}{2} \times F \times L$

distance from الزاوية القائمة $= \frac{2}{3}d$

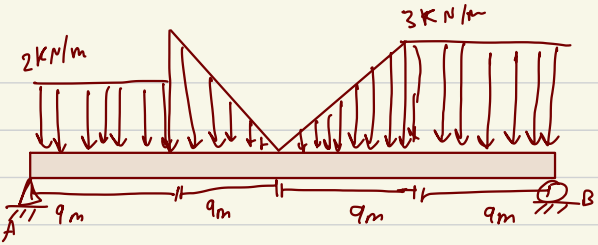
الزاوية المائلة $= \frac{1}{3}d$

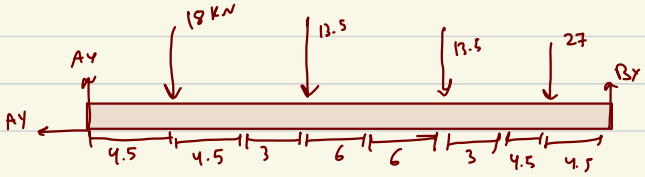
example





①





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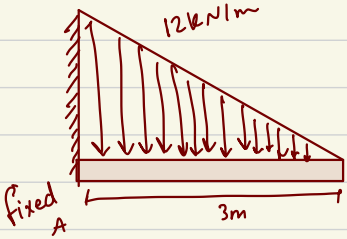
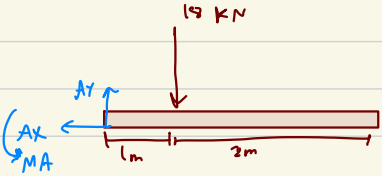
②



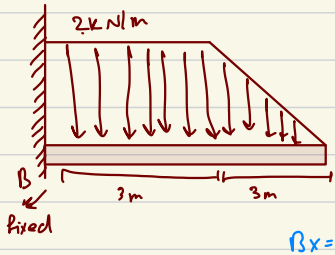


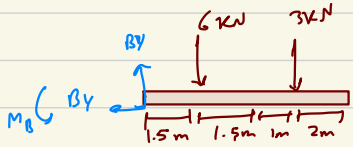
$A_X = 0$, $A_Y = 18$ KN ↑

$M_A = 18$ KN/m ↺

---

③

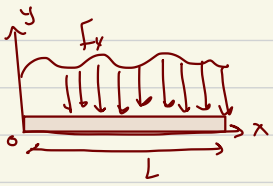




$B_X = 0$, $B_Y = 9$ KN ↑

$M_B = 6(1.5) + 3(4) \Rightarrow$ $M_B = 21$ KN·m ↺

Function load :

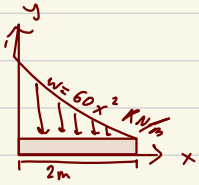


$$F_R = \int_0^L F(x)\,dx$$

location -

$$\bar{x} = \frac{\int_0^L x F(x)\,dx}{\int_0^L F(x)\,dx} = \frac{\int x w(x)\,dx}{\int w(x)\,dx}$$

example:-



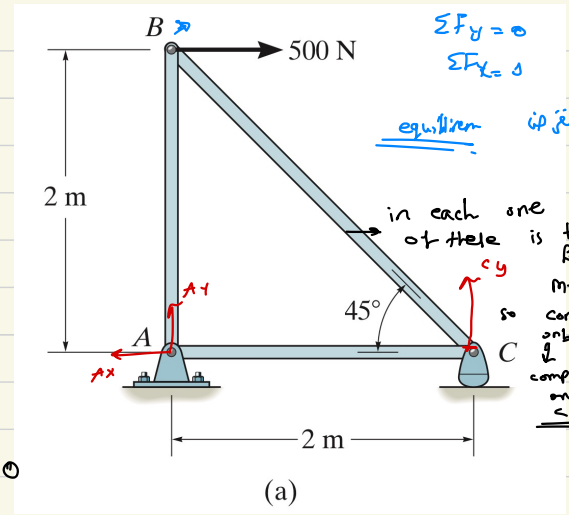
$$F = \int_0^2 60x^2 = \frac{60}{3}x^3\Big|_0^2 = 20 \times 8 = 160\ KN$$

$$\bar{x} = \frac{\int_0^2 60x^3}{160} = \frac{\frac{60}{4}x^4\Big|_0^2}{160} = \frac{15(2)^4}{160} = \frac{15 \times 16}{160} = \frac{240}{160} = 1.5$$

Sun, Apr 14

method of joints

example 6.1

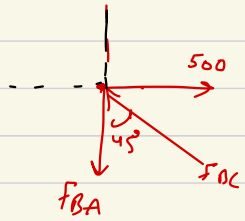


$\Sigma F_y = 0$

$\Sigma F_x = 0$

equilibrium in 2D

in each one of these is two force members so compression or tension



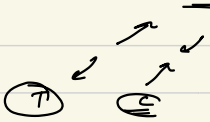
$\pm \Sigma F_x = 0 \quad 500 - F_{BC} \sin 45^\circ = 0$

$+\uparrow \Sigma F_y = 0 \quad F_{BC} \cos 45^\circ = F_{BA}$

$F_{BC} \sin 45^\circ = 500 \rightarrow F_{BC} = 707\ N$

$F_{BA} = 707 \cos 45 = 500\ N$

Two-force member





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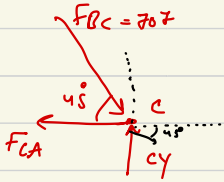
at Joint C

$\pm \Sigma F_x = 0 \quad F_{BC} \cos 45^\circ - F_{CA} = 0$

$+\uparrow \Sigma F_y = 0 \quad F_{CA} = 500\ N$

$C_y - F_{BC} \sin 45^\circ = 0$

$C_y = 500\ N$



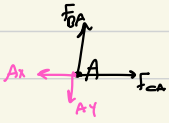
$F_{CA} = (-)$

---

at Joint A

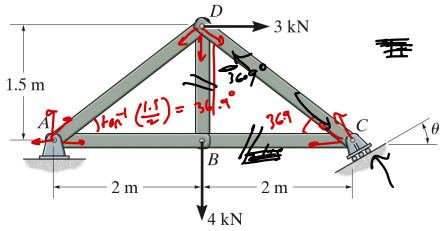
$\pm \Sigma F_x = 0 \quad + \uparrow \Sigma F_y = 0$

$A_y = 500\ N \quad A_x = 500\ N$



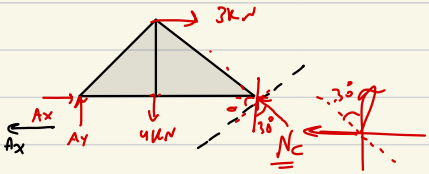
Tue April 16

6–6. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 30°$.



Probs. 6–5/6

all joints have more than two unknowns, so we draw the free body diagram for the whole truss



$\Sigma F_y = 0$ $\quad N_C \cos 30° + A_y = 0$

$\Sigma F_x = 0$ $\quad A_x - N_C \sin 30° = 0$

$\Sigma M = 0$ $\quad \Sigma M_A = 0 \rightarrow N_C \cos 30° (4) - 3(1.5) - 4(2) = 0$

$N_C \, 4 \cos 30° = 8 + 4.5$ (12.5)

$N_C = 3.5 kN \rightarrow$ positive لهيك انا بما معناتو عزيزي الكتاب

$\Sigma M_C = 0$ (3.5, y=4.5)

$4(2) - 3(1.5) - A_y(4) = 0$

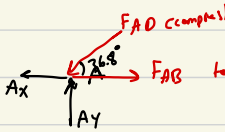
$A_y = 0.875 \, KN \uparrow$

$A_x - 3.5 \sin 30° = 0$

$A_x = -1.2 \, KN = 1.2 \, KN \leftarrow$

---

at joint A

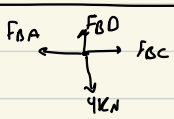


$A_y - F_{AD} \sin 36.8° = 0$

$F_{AB} - A_x - F_{AD} \cos 36.8° = 0$

$\frac{0.875}{\sin 36.8°} = F_{AD} = 1.46 \, KN$ compression

Joint B



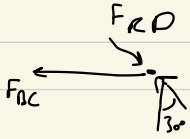
$F_{BD} = 4KN \uparrow$

$F_{BC} = 2.36 KN \rightarrow$

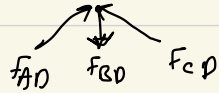
$F_{AB} = 1.2 + 1.46 \cos 36.8°$

$F_{AB} = 2.36 \, KN \rightarrow$

Joint C

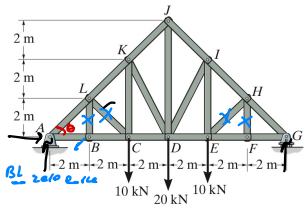


Joint D



Tuesday, April 16

6–11. Determine the force in each member of the *Pratt truss*, and state if the members are in tension or compression.



Prob. 6–11

→ the right hand side is the same as the left hand side and the same mesurments so if we do one side is the same on the other side

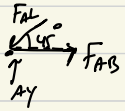
$\theta = \tan^{-1}\left(\frac{2}{2}\right) = 45°$

no x-axis force so $A_x = 0$ and $\sum F_x = -40$ so $C_y = A_y = 40_{kN}$

$C_y = A_y = 20\ kN\uparrow$

$F_{BL} = F_{LC} = F_{HE} = F_{HC} = 0$

Joint A:



$A_y - F_{AL}\sin 45° = 0$

$F_{AL}\sin 45° = A_y \rightarrow F_{AL} = 28.29$

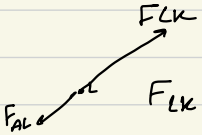
$F_{AB} - F_{AL}\cos 45° = 0$

$F_{AB} = 20k$

Joint B:

F_{BA} F_{BC} $F_{BC} = F_{BA} = 20kN$

Joint L:



$F_{LK} = F_{LA} = 28.28kN$

Joint K

$F_{KD} = 7.5\ kN(C)$

$F_{KJ} = 23.6\ kN(C)$

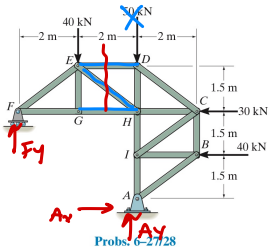
Joint J:

$F_{JL} = 23.6\ kN(C)$

$F_{JD} = 33.3kN(T)$

Thu, Apr 18

*6–28. Determine the force in members *ED*, *EH*, and *GH* of the truss, and state if the members are in tension or compression.



Probs. 6–27/28

using method of sections

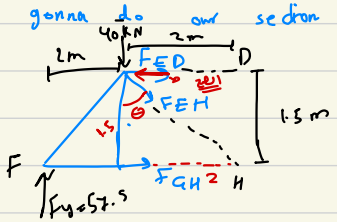
section على support reactions نطلع اول

$+\circlearrowleft \Sigma M_A = 0$

$40(2) + 30(3) + 40(1.5) - F_y(4)$

$F_y = \frac{230}{4} = 57.5\ KN \uparrow$

we're gonna do our section:



$\theta = \tan^{-1}\left(\frac{2}{1.5}\right) = 53.1°$

$\Sigma F_y = 57.5 - 40 - F_{EH}\cos(53.1°)$

$F_{EH} = \frac{17.5}{\cos(53.1)} = 29.16\ KN \searrow (T)$

① $+\circlearrowleft \Sigma M_H = 0$

$-57.5(4) + 40(2) + F_{ED}(1.5) = 0$

$F_{ED} = -100\ KN = 100\ N\ (C)$ ←

② $+\circlearrowleft M_E = 0$

$-57.5(2) + F_{GH}(1.5) = 0$

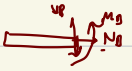
$F_{GH} = 76.7\ KN \rightarrow (T)$

$+\circlearrowleft \Sigma M_F = 0$ ← بعدين نزل صيغ

بـ ΣF ← استعلم بين نيم الاوية

Sunday, April 21

* Internal forces :



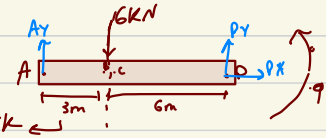
① normal force (N) : perpindicular on surface going out of it

② sheer force(V) : I assume the direction to make Clockwise moment

③ bending moment (M) : as if it : making the pot ∪ so . .

---

example p.344 :-

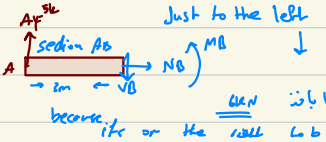


$\rightarrow \Sigma F_x = 0 \rightarrow D_x = 0$

$\circlearrowleft \Sigma M_D = 0$

$-A_y(9) + 6(6) + 9 = 0 \rightarrow A_y = 5KN$

I want to work on



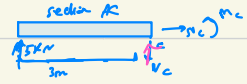
$\rightarrow \Sigma F_x = 0$

$N_B = 0$

$+\uparrow \Sigma F_y = 0 \quad 5 - V_B = 0$

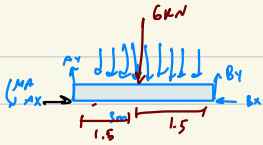
$V_B = 0 \quad M_B = 15KN.m$

Just to the left



$N_C = 0 \quad V_C - 1KN = V_C = 1K \uparrow$

$M_C = 15KN.m$



$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x = B_x = 0$

$\circlearrowleft^{+} \Sigma M_B = 0$

$M_A + 6(1.5) - A_y(3) = 0$

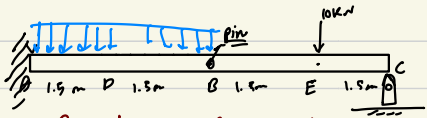
$+\circlearrowleft \Sigma M_A = 0$

$M_B - 6(1.5) + B_y(3) = 0$

$A_y = B_y$ because symmetry

$B_y = 3kN = A_y$

$M_A = 6kN \cdot m$



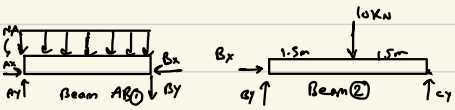
Compound beam (two beams)

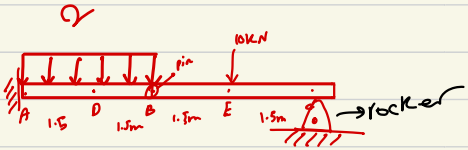
FBD to the two alone

find N, V, M at D & E

E is on the left of the 10kN Force

Tuesday, April 23





Compound beam (two beams)

- find N, V, and M at E & D

E is to the left of the 10KN force

Beam ②

$+\circlearrowleft \Sigma M_C = 0 = B_y(3) + 10(1.5)$

$\rightarrow B_y = -5KN = 5KN \uparrow$

$C_y = 5KN \uparrow$

Beam ①

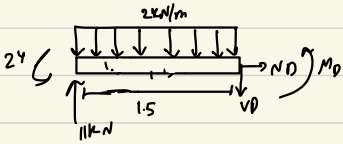
$+\uparrow \Sigma F_y = 0 \quad -5 - 2(3) + A_y = 0$

$A_y = 11\ KN \uparrow$

$+\circlearrowleft \Sigma M_B = 0$

$M_A - 11(3) + 6(1.5) = 0$

$M_A = 24\ KN.m$



$\Sigma F_x = 0 \rightarrow N_D = 0$

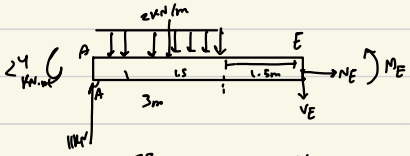
$\Sigma F_y = 0$

$11 - V_D - 2(1.5) = 0$

$V_D = 8KN \downarrow$

$\zeta^+ M_D = 0$

$M_D = -9.7\ KN.m$



$\Sigma F_x = 0 \quad \therefore N_E = 0$

$\Sigma F_y = 0$

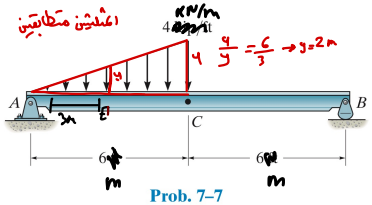
$11 - 2(3) - V_E = 0$

$V_E = 5\ KN \downarrow$

$+\circlearrowleft \Sigma M_E = 0 = 24 - 11(4.5) + 2(3)(1.5)$

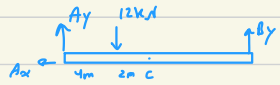
$M_E = 7.5$

7–7. Determine the internal shear force and moment acting at point C in the beam.



Prob. 7–7

N, V, M at $x = 3$ (E)

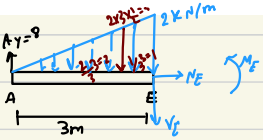


$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad A_y + B_y - 12 = 0$

$\Sigma M_B = 0 \rightarrow \quad -A_y(12) + 12(8) = 0$

$A_y = 8\,kN \uparrow \rightarrow B_y = 12 - 8 = 4$



$\Sigma F_x = 0 \quad N_E = 0$

$+\circlearrowleft \Sigma M_E = 0$

$-8(3) + \frac{1}{2}(3)(2)(1) + M_E = 0$

$M_E = 21\ kN.m$

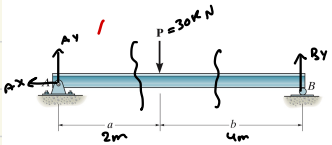
$\Sigma F_y = 0$

$8 - 3 - V_E = 0$

$V_E = 5k$

7-2 : sheer and moments (we're gonna use method of sections)

7–47. Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P =$ ~~6000~~ lb, $a =$ ~~5~~ ft, $b =$ ~~7~~ ft.



Prob. 7–47

we have diffrent x values before and after P

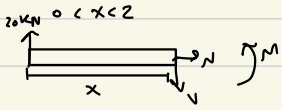
عشان لما بدي اعمل section بدي اعمل واحد من اليمين او من اليسار

FBD of AB $A_x = 0 \rightarrow$ $+\circlearrowleft \Sigma M_B = 0$

$A_y + B_y = 0$ $\quad 30(4) - A_y(6) = 0$

$A_y = \frac{120}{6} = 20\ kN \quad \therefore B_y = 10kN$

section 1:



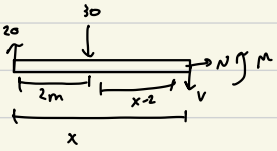
$\Sigma F_y = 0 \qquad \therefore V = 20\,KN$

$+\circlearrowleft \Sigma M = 0 \quad -20(x) + M = 0 \rightarrow M = 20x \ KN.m$

at $x=0 \rightarrow M=0$, at $x=2 \rightarrow M=40\,KN.m$

$2 < x < 6$

section 2:



$N=0$

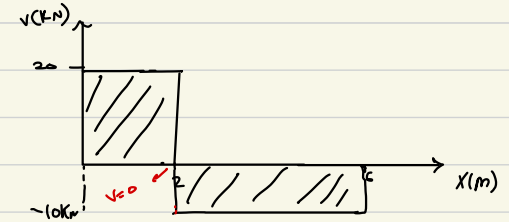
$\Sigma F_y = 0 \quad 20 - 30 - V = 0 \rightarrow V = -10\,KN \rightarrow V = 10\,KN \downarrow$

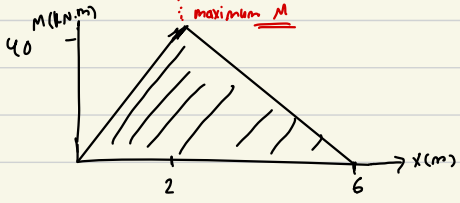
$+\circlearrowleft \Sigma M = 0 \qquad -20(x) + 30(x-2) + M = 0$

$M = 20x - 30x + 60 = -10x + 60$

at $x=2 \rightarrow M = -20 + 60 = 40\,KN.m$

at $x=6 \rightarrow M = -60 + 60 = 0\,KN.m$

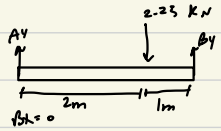


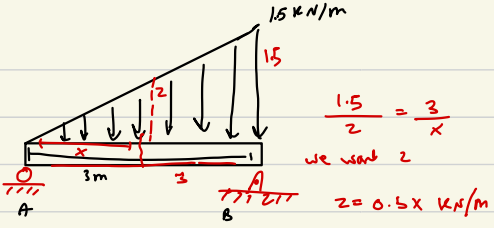


Sunday , April 28

* Question from the professor:- *

FBD on beam





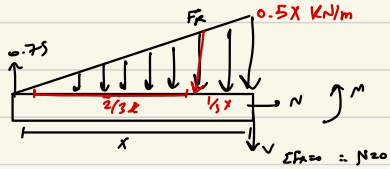
$+\uparrow \sum F_y = 0 \quad A_y + B_y = 2.25$

$+\circlearrowleft \sum M_B = 0 \quad 2.25(1) - A_y(3) = 0$

$A_y = \frac{2.25}{3} = 0.75 \text{ KN}$

$B_y = 2.25 - 0.75 = 1.5 \text{ KN}$

section ①: * when we do the sections we use the distributed load. not the F_R that we calculated.



$F_R = \left(\frac{1}{2}x\right)(0.5x) = 0.25x^2 \text{ KN}$

$0 < x < 3$

$\sum F_x = 0 \Rightarrow N = 0$

$+\uparrow \sum F_y = 0 \quad 0.75 - 0.25x^2 - V = 0$

$\boxed{V = -0.25x^2 + 0.75}$

: لازم نعمل جدول

① more than two points

② pay attention to (x^2) sign

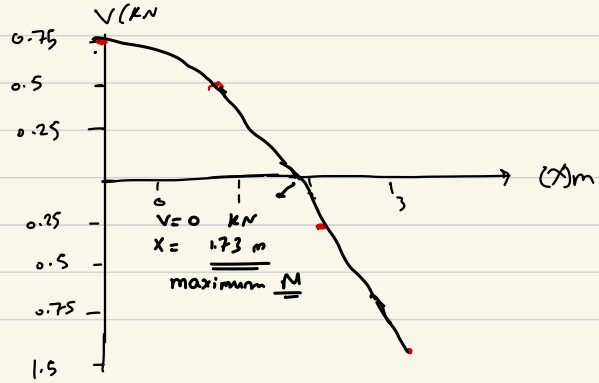
I WANT TO KNOW WHEN IT $\boxed{V=0}$ x-axis قطع

at $x=0 \rightarrow V = 0.75 \text{ KN}$

at $x=1 \rightarrow V = 0.5 \text{ KN}$

at $x=2 \rightarrow V = -0.25 \text{ KN}$

at $x=3 \rightarrow V = -1.5 \text{ KN}$



$+\circlearrowleft \Sigma M = 0$

$-0.75(x) + 0.2x^2\left(\frac{x}{3}\right) + M = 0$

$M = 0.75x - \frac{0.2}{3}x^3$

$x = 0 \rightarrow M = 0$ KN.m

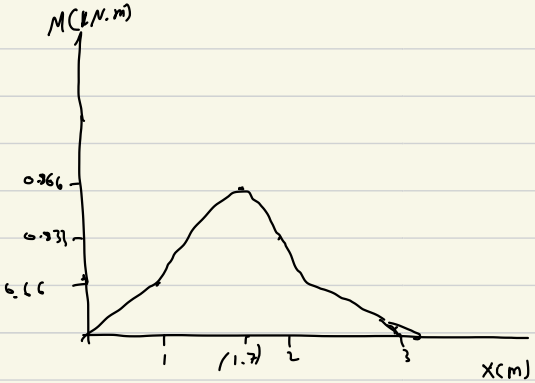
$x = 1 \rightarrow M = 0.55$ KN.m

$x = 2 \rightarrow M = 0.833$ KN.m

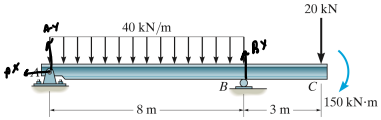
$x = 3 \rightarrow M = 0$ KN.m

$M_{maximum}$ at $x = 1.73$

$M_{max} = 0.866$



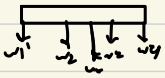
7–55. Draw the shear and moment diagrams for the beam.



Prob. 7–55

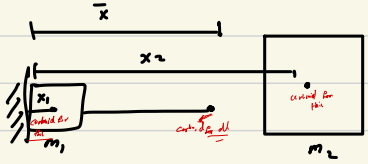
Tue, April 30

* Center of Gravity:-
the point through with the total weight of a body acts



* Centroid :-
the point of with the total area of a plane figure (rectangular, triangular, circle, regular) is assumed to be centred

Use the principle of moments, moments of total weight, mass, area, volume, etc..
= Σ moment of all weights, mass, volume, etc---



* 2 types of bodies:

① Discrete
↓ center of mass

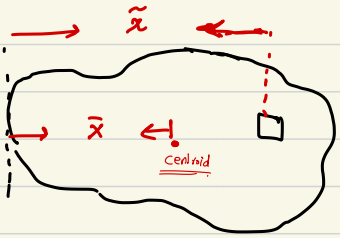
$$M_{total}\,\bar{x} = \sum m_i \tilde{x}_i$$

$$\bar{x} = \frac{m_1(\tilde{x}_1) + m_2(\tilde{x}_2)}{M_{total}\,(m_1+m_2)}$$

→ use length or mass (distributed mass) =

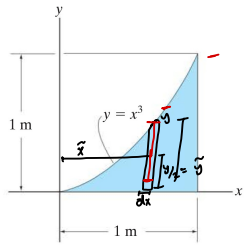
② Continuous :

$$W\bar{x} = \int \tilde{x}\, dw$$
$$\bar{x} = \int \tilde{x}\, dw / w$$
$$\bar{x} = \int \tilde{x}\, dw / \int dw$$



$\tilde{x}$ = centroid at the deltrientid element
$\bar{x}$ = centroid at the total area.

F9–2. Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area.



Prob. F9–2

$\bar{x} = X \qquad \bar{y} = y$

$dA = y\,dx$

$\tilde{y} = \frac{y}{2} = \frac{x^3}{2}$

$\bar{x} = \int \tilde{x}\,dA \,/\, \int dA$

$= \int x\,y\,dx \,/\, \int y\,dx$

$\int_0^1 x(x^3)\,dx \,/\, \int_0^1 x^3\,dx$

$\frac{x^5}{5}\Big|_0^1 \div \frac{x^4}{4}\Big|_0^1$

$X = \bar{X} = \frac{1}{5} \div \frac{1}{4} = 0.8$

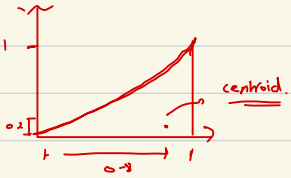
$\bar{y} = \int \tilde{y}\,dA \,/\, dA$

$= \int \frac{x^3}{2}\,y\,dx \,/ \int y\,dx$

$= \int_0^1 \frac{x^3}{2}(x^3)\,dx \,/\!/ \int x^3\,dx$

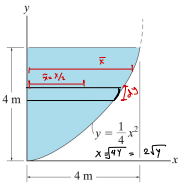
$= \frac{1}{2}\,\frac{x^7}{7}\Big|_0^1 \Big/ \frac{x^4}{4}\Big|_0^1$

$\bar{y} = 0.296$ m



9–9. Locate the centroid $\bar{x}$ of the shaded area.

9–10. Locate the centroid $\bar{y}$ of the shaded area.



Probs. 9–9/10

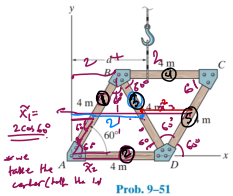
$dA = x\,dy$

$\bar{y} = \int y\,dA \,/ \int dA$

$= \frac{\int y\,x\,dy}{\int x\,dy} = \frac{\int y(2\sqrt{y})\,dy}{\int 2\sqrt{y}\,dy} = 2.4$ m

Sun, May 5

9–51. The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



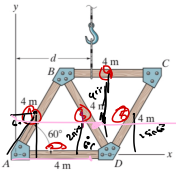
Prob. 9–51

member #	length	$\tilde{x}$	$\tilde{x}L$
1	4	$2\cos 60° = 1$	4
2	4	2	8
3	4	$2+2\cos 60° = 3$	12
4	4	$2+2 = 4$	16
5	4	$2+2+2\cos 60° = 5$	20
	$\Sigma L = 20$		$\Sigma \tilde{x}L = 60$

$$\bar{X} = \frac{\tilde{x}_1 L_1 + \tilde{x}_2 L_2 + \tilde{x}_3 L_3 + \cdots}{\Sigma L}$$

$$\bar{x} = \frac{\Sigma \tilde{x} L_i}{\Sigma L_i} = \frac{60}{20} = 3\text{ m}$$

9–51. The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



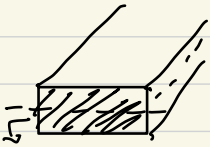
Prob. 9–51

member #	length	$\tilde{y}$	$\tilde{y}L$
1	4	$2\sin 60°$	
2	4	0	
3	4	$2\sin 60°$	
4	4	$4\sin 60°$	
5	4	$2\sin 60°$	
	$\Sigma L =$		$\Sigma \tilde{y}L =$

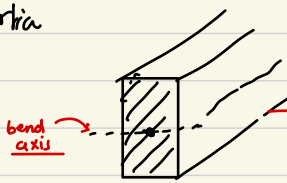
$\bar{y} = 1.73\text{ m}$

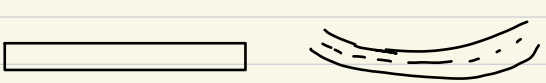
Thursday, May 9

Chapter 10: moment of inertia

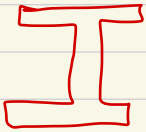


rotate 90°





the material here is far away from the bending axis ←



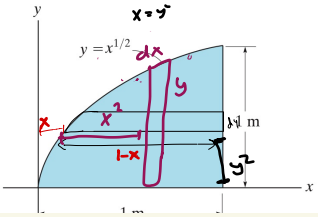
* we calculate the moment of inertia from an axis (x or y) and we choose distance perpendicular parallel to the axis (reference) we want

* when we calculate I in respect to the centroid. so (the origin of the x and y axis are on the centroid.) we call it $I_{centroid}$.
  ↳ centroidal axis

- if we want to calculate I with reference to an axis parallel to one of centroidal axis we just have to use the parallel axis theorem

10–5. Determine the moment of inertia for the shaded area about the x axis.

10–6. Determine the moment of inertia for the shaded area about the y axis.



$$I_x = \int y^2\,dA = \int y^2 (1-x)\,dy$$

$$= \int_0^1 y^2 (1-y^2)\,dy = \int_0^1 y^2 - y^4\,dy = 0.1333$$

$$I_y = \int x^2 y\,dx = \int x^2 (x^{1/2})\,dx$$

$$= \int_0^1 x^{5/2}\,dx = 0.286\ m$$

Prob 10.26. p 544

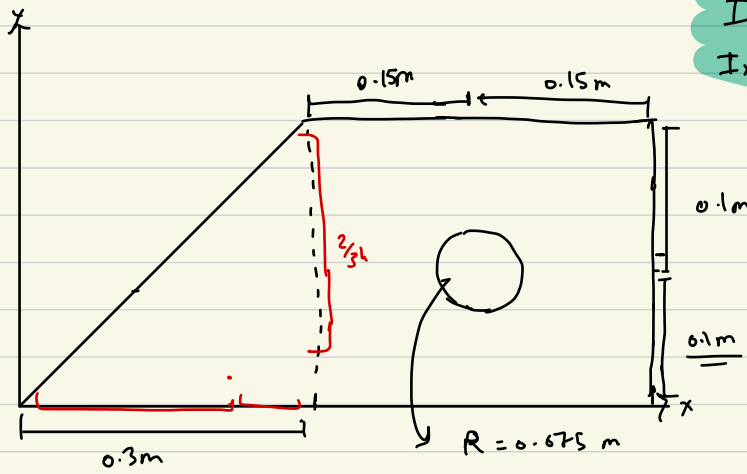
I_x ??

$I_x = I_{xc} + A d^2$ — d: distance between I_{xc} and axis

I_{xc} ▭ $= \frac{1}{12} bh^3$

I_x ○ $= \frac{\pi}{4} R^4$

I_x △ $= \frac{1}{36} bh^3$



$$I_{x\,▭} = \frac{1}{12}(300)(200)^3 + (300)(200)(100)^2 = 8(10^8)\ mm^4$$

$$I_{x\,△} = \frac{1}{36}(300)(200)^3 + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2 = 2(10^8)\ mm^4$$

$$I_{x\,○} = \frac{\pi}{4}(75)^4 + \pi(75)^2(100)^2 = 2.82(10^8)\ mm^4$$

$$I_{x\,total} = I_{x\,○} + I_{x\,△} - I_{x\,○} = 7.18(10^8)\ mm^4$$

* dynemics → there's acceliration

Dynamics :- accelerated motion of

- particles — translational motion
- Rigid body — translational and/or Rotational.

---

① Kinamatecs
↳ motion S, V, A
ch 12, 16

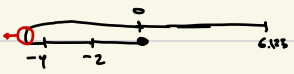
② Kinatecs
the reason of motion

Continue ex 12.5 , p. 14 :-

t	0	1	2	3	3.5
s	0	-2	-4	0	6.125

$V = 0$ عند نقطة رجوع

no distance here



(a) $S_T = 2+2+4+6.125 = 14.125$ m

b) $V_{avg} = \frac{\Delta S}{\Delta t} = \frac{(6.125 - 0)}{3.5} = 1.75$ m/s

c) $(V_{avg})_{sp} = \frac{S_T}{\Delta t} = \frac{14.125}{3.5} = 4.04$ m/s

---

When we see a question :- ① Dynamics or Statics
② linear Kinamatics or Kinatecs
↓
3 equations

12-54. $0 \le t \le 5$

$ds = v\,dt$

$s = \int_0^t 0.6t^2\,dt$

$s = 0.2t^3$

$t = 0 \rightarrow s = 0$

$t = 5 \rightarrow 25$

slope after t = 5

$\frac{0-15}{15-5} = \frac{v-15}{t-5}$

$v = \frac{1}{2}(45 - 3t)$ m/s

$5 \le t \le 15$

$s = \int_0^t \frac{1}{2}(45 - 3t)\,dt$

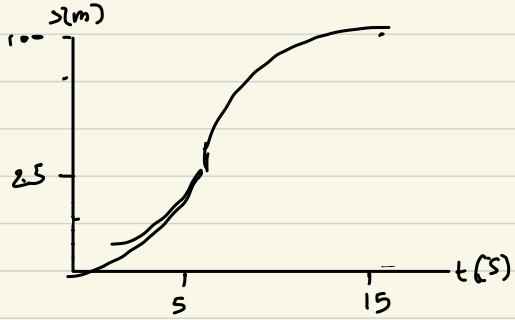
$s = \frac{1}{2}(45t - \frac{3}{2}t^2)$

at $t = 5 \rightarrow s = 25$ m

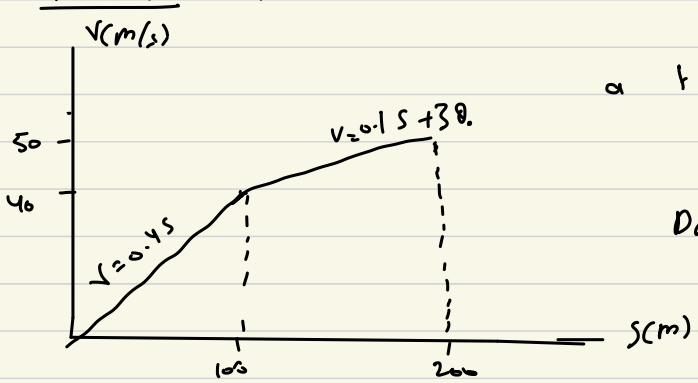
at $t = 15 \rightarrow s = 100$ m

$s_T = 100$ m

$\text{avg}(v_{sp}) = \frac{s_T}{15} = \frac{100}{15} = 6.67$ m/s



12-53: in the book



a t s = 400 m

s = 450

Draw the a-s graph

$a = v\frac{dv}{ds}$

$0 < s \leq 100$

$a = 0.4s / 0.4$

$a = 0.16s \ m/s^2$

$s = 0 \rightarrow a = 0$

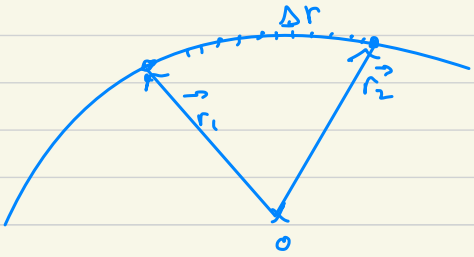
$s = 100 \rightarrow a = 16 \ m/s^2$

$100 \leq s \leq 200$

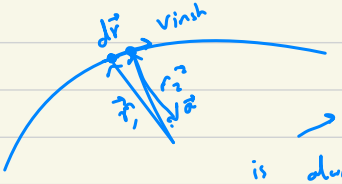
$a = 0.01s + 30$

General curvilinear motion:- (12.4)

Sun, May 19



$$\vec{v}_{avg} = \frac{\vec{\Delta r}}{\Delta t} \qquad \vec{a}_{avg} = \frac{\vec{v}_{avg}}{\Delta t}$$



$$\vec{v}_{inst} = \frac{d\vec{r}}{dt} \qquad a_{inst} = \frac{\vec{v}_{inst}}{dt}$$

→ is always tangent to the curve.

→ is always to the inside of the curve.

12-5:- rectangular components (x,y,z)

12-7: normal and tangentional components

12-8: cylindrical components

12-6:



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$\bar{V}_{inst} = dr/dt = (xi + yj + zk)$ m/s

$x = v_x \quad y = v_y \quad z = v_z$

$= \frac{dx}{dt} \quad = \frac{dy}{dt} \quad = \frac{dz}{dt}$

$v = (v_x i + v_y j + v_z k)$

$\bar{a} = \frac{d\vec{v}}{dt}$

$= (v_x i + v_y j + v_z k)$

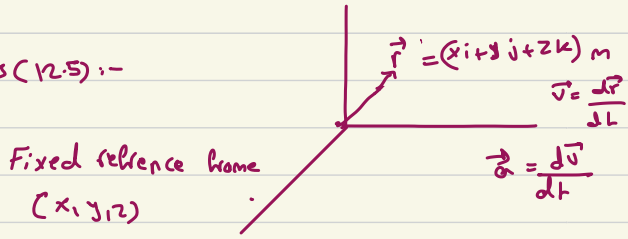
---

example: 12.9

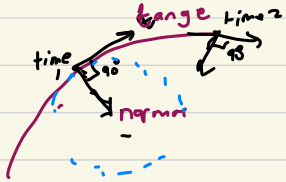
$\vec{r} = x i -$

Tuesday , May 22

- rectangular components (12.5) :-

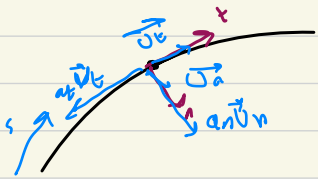


12.7 : Curvelinear motion : Normal and tangential components :-



- we define our axis according to the position

non-fixed refrence frame



- we will need two unit vectors on every axis

$$\vec{V} = s\vec{U}_t = \frac{ds}{dt}\vec{U}_t = v\vec{U}_t$$

always tangential to curve (path)

$$\vec{a} = \frac{d}{dt}\vec{V} = \frac{d}{dt}(v\vec{U}_t) = \frac{dv}{dt}\vec{U}_t + v\frac{d}{dt}\vec{U}_t$$

$$\vec{a} = \underbrace{\dot{v}\vec{U}_t}_{\text{change in speed.}} + \underbrace{v\frac{d\vec{U}_t}{dt}}_{\text{change of the unit vector in the tangential direction}}$$

change in direction

$$\vec{a} = a_t \vec{U}_t + v\rho \vec{U}_n$$

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

* when v = constant $\rightarrow$ $a = 0$

$a_n \neq 0$

$a = a_n$

a → total acceleration

a_n = normal acceleration

---

* $v \neq$ constant

$\Rightarrow$ straight line motion

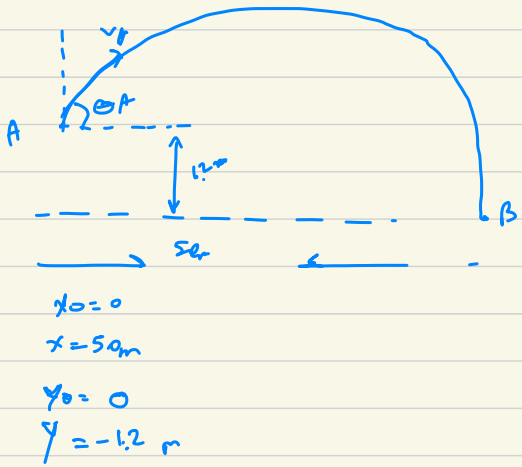
$a = a_t$, $a_n = 0$

---

* when (circular path) $\Rightarrow$ curvelinear motion

12-, 20 :-

$v = 5t$ '



$x_0 = 0$

$x = 50\ m$

$y_0 = 0$

$y = -1.2\ m$

* ah is observed that A the time for box to strike the ground. ah B = 2.5 s

v_A ?? → 22.1 m/s

θ_A ?? 30.5°

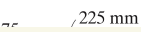
---

* chapter 12: Kinematics of a particle

chapter 16: Kinematics of rigid body

* chapter 16 :- Kinematics of a rigid body

F16–6. For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5\ \text{rad/s}^2$, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pulley D which is rigidly attached to gear B.



$V_C = \omega_D r_D$ (?)

$\omega_B r_D = 4.5(.125) = .$.

* mesh point: → نقطة تلامس

$V_A = V_B$

$(a_t)_A = (a_t)_B$

$\alpha_A r_A = \alpha_B r_B$

$4.5(.75) = \alpha_B(.225)$

α_B : اذا بدي اوجد ← بدي اوجد α_A

$\omega_B = (\omega_B)_0 + \alpha_B t$

$\omega_A = 4.5\ t$

$(\omega_A)_0 = (\omega_B)_0 = 0$

$\delta = \theta_B r_D$

$\theta_B = \cancel{(\theta_B)_0}^{\,0} + \cancel{\omega_B}^{\,0} + \frac{1}{2}\alpha_B (t)^2$

$\theta_B = 6.75$ rad.

$\delta = 6.75\, r_D = 6.75(125\text{mm}) = 844\text{ mm}$