Dynamics

Ch12 Kinmatics of a particle
Ch16 Kinmatics of a rigid body
we will describe the motion: V(velocity)
: a (acceleration)
: S (position)

and then ↓
ch 13, ch 14, ch 15 Kinetics
↳ "what is causing the motion?" → "THE FORCE"

Ch 12

Kinematics of a particle

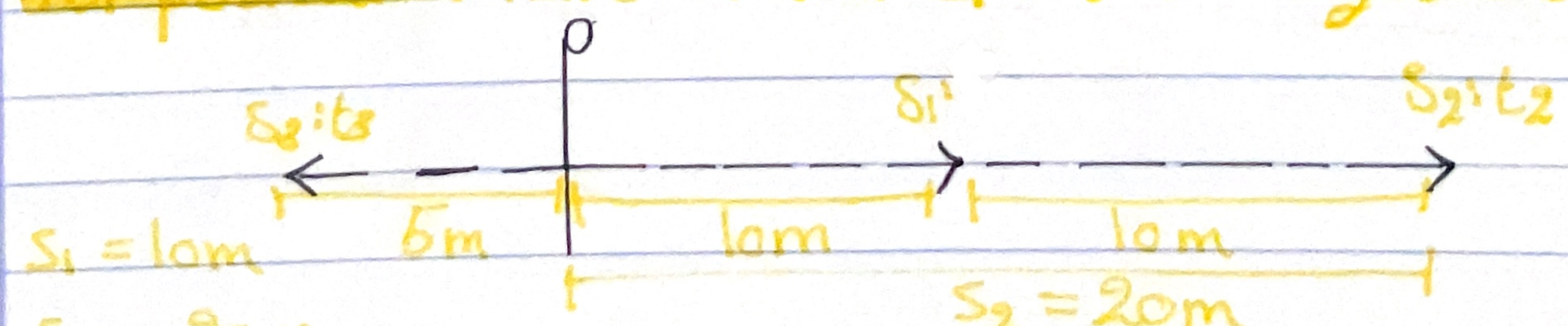
ch12.2

Rectilinear Kinematics (continous motion)

straight line motion.

① position "S":- a vector quantity
direction ← → distance

For a position I have a reference point that I set the distance from it



بدها اشارة لأنها متجهة

2. Displacement (ΔS):- Change in position

$$\Delta S = S_F - S_i$$

$\Delta S_{2-1} = S_2 - S_1 = 20 - 10 = 10m$, $\Delta S_{2-0} = 20 - 0 = 20m$, $\Delta S_{3-0} = -5 - 0 = -5$

The displacement is also a quantity

3. Distance (ST):- positive scalar

(we will add all the distance traveled all by together.)

$ST = S_1 + S_2 + S_3$

$ST = 10 + 10 + 20 + 5 = +45m$

(no negative sign bc it is scalar quan)

Always positive

4. Velocity:

$V_{avg} = \frac{\Delta S}{\Delta t}$ Vector quantity m/s

$V_{inst} = \frac{dS}{dt}$ ..... (1) m/s

$dv = \frac{dS}{dt}$, $dt = \frac{dS}{dv}$ } V, S, t

5. acceleration

$a_{avg} = \frac{V_{avg}}{\Delta t} = \frac{\Delta V}{\Delta t}$ m/s^2 Vectorquantity

$a_{inst} = \frac{dv}{dt}$ m/s^2 ..... (2)

$dt = \frac{dv}{da}$

$da = \frac{dv}{dt}$ (a, v, t)

$ads = vdv$ ..... (3) a, v, s

speed is scalar quantity

Velocity is a vector quantity

when the body stops initially
we abs have at that point V=(0)

page 10

Ex 12.1, The car on the left in the photo and in Fig. 12-2 moves in straight line such that for a short time its velocity is defined by $V = (0.6t^2 + t)$ m/s, where t is in seconds. Determine its position and acceleration when $t = 3s$, when $t = 0$, when $s = 0$

Solu: $s = ??$, $a = ??$, $t = 3s$, $V = (0.6t^2 + t)$ m/s

$V = \frac{ds}{dt}$ $\quad ds = V dt$

$\int_0^s ds = \int_0^t (0.6)t^2 + t \, dt$

* $s = \frac{0.6t^3}{3} + \frac{t^2}{2} = (0.2t^3 + \frac{t^2}{2})$ m

plug $t = 3$ → $s = 9.9$m

think in the que step by step to know the diffrance between if it is statics or dynamics

$a = \frac{dv}{dt}$ $\quad a = \frac{d(0.6t^2 + t)}{dt}$

and since it is in

$a = (1.2t + 1)$ m/s²

* at $t = 3s$ → $a = 4.6$ m/s²

Ex, 12.5, A particle moves along a horizontal path with a velocity of $V = (3t^2 - 6t)$ m/s, where t is the time in seconds. IF it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particles average velocity and average speed during the time ~~traste~~ interval.

$s = -4$m $\quad s = -2$m

Solu:- $V(3t^2 - 6t)$ m/s $\quad V = \frac{ds}{dt}$, $ds = Vdt$ ---------- $s = 0$ $\quad s = 6.125$ m

V: F(t)

① ST in $\Delta t = 3.5$ s $\quad \delta = \int_0^t 3t^2 - 6t \, dt$

② Vavg. (Vsp.)avg $\quad s = t^3 - 3t^2$ m

ST ?? Find all positions

m:	s	0	-2	-4	0	6.125
t:	t	0	1	2	3	3.5

at the turning point $V = 0$

$V = 3t^2 - 6t = 0$

$t = 0$ and $2s$

$ST = 4 + 4 + 6.125 = 14.125$m

$(V_{sp.})_{avg} = ST = \frac{14.125}{3.5} = 4.04$ m/s

$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s \text{ at } t=3.5 - s \text{ at } t=0}{\Delta t}$

$= \frac{6.125 - 0}{3.5} = 1.45$ m/s

Que :- "What if the "a" was constant?

ac

1) $V : F(t)$

$$a_c = \frac{dv}{dt}$$

$$\int_{V_0}^{V} dv = \int_0^t a_c \, dt$$

$$V = V_0 + a_c t$$

2) $S : F(t)$

$$V = \frac{ds}{dt}$$

$$\int_{S_0}^{S} ds = \int_0^t (V_0 + a_c t)\, dt$$

$$S = S_0 + V_0 t + \tfrac{1}{2} a_c t^2$$

3) $V : F(S)$

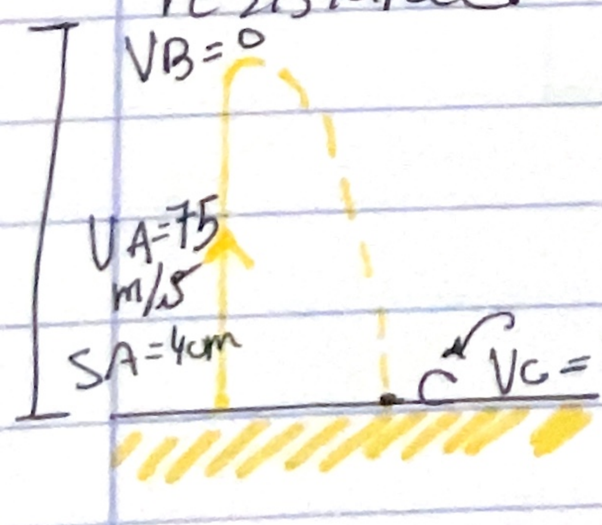
$$a_c \, ds = V \, dv$$

$$\int_{S_0}^{S} a_c \, ds = \int_{V_0}^{V} (V_0 + a_c t)\, dv$$

$$V^2 = V_0^2 + 2a_c(S - S_0)$$

① ② ③

Ex 12-3, During a test a rocket travels upward at 75 m/s, and when it is 40m from the ground its engine fails. Determine the maximum height S_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s due to gravity. Neglect the effect of the air resistance. Solu:-



$$V_B^2 = V_A^2 + 2a_c(S_B - S_A)$$

$$0 = 75^2 - 2(9.81)(S_B - 40)$$

$$\rightarrow S_B = 327\text{ m}$$

A and C

$$V_C^2 = V_A^2 + 2a_c(S_C - S_A)$$

$$V_C^2 = 75^2 - 2(9.81)(0 - 40)$$

$$V_C = -80.1\text{ m} = 80.1\downarrow \text{m}$$

12,22. A Sandbag is dropped from a ballon which is ascending vertically at a constant speed of 6m/s. IF the bag is released with the same upward velocity of 6m/s when $t=0$ and hits the ground when $t=8s$, determine the speed of the bag as it hits the ground and the altitude of the ballon at this instant.

Solu:-

*Sand bag

$V_i = 6m/s$, $a = -9.81 m/s^2$ *not zero

$V = V_0 + act$

$V = 6 - 9.81(8)$

$V = -72.4 m/s$

$S = S_0 + V_0 t + \frac{1}{2} a_c t^2$

$S = 0 + 6(8) - \frac{1}{2}(9.81)(8)$

$S = -265.9m$

$S = 265.9m$ *

*Balloon

~~$V = 6$~~ $V = 6m/s$, $a = 0$

$t = 8s$

$S = S_0 + V_0 t + \frac{1}{2} a_c t^2$

$S = 0 + 6(8) = 48m$

Altitude = S_{sand} + S_{Ballon}

$AL = 265.9 + 48 = 314m$

12.3 Rectilinear Kinematics:-

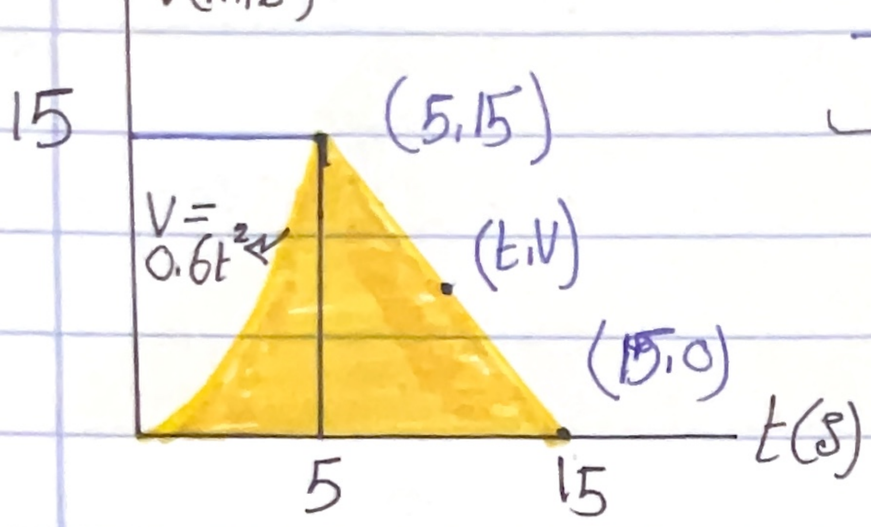


Erratic motion



Break up the system into intervals (time).

12-54, The v-t graph for the motion of a car as it moves along a straight road is shown. Draw the s-t and a-t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t=0$, $s=0$.



Solu:-

$$\frac{0-15}{15-5} = \frac{V-5}{t-5} \Rightarrow V = \frac{1}{2}(45-3t)\ m/s$$

↳ to find the 2nd shape equation.

① $0 < t \leq 5s$

$V = (0.6t^2)\ m/s$

$V = \frac{ds}{dt} \rightarrow ds = V\,dt \rightarrow \int_0^s ds = \int_0^t (0.6t^2)\,dt$

$s = 0.2t^3\ m$, at $t=0$ $s=0$, at $t=5$ $s=25m$

② $5 \leq t \leq 15$

$V = \frac{1}{2}(45-3t)$

$ds = V\,dt$

$\int_{25}^{s} ds = \int_5^t \frac{1}{2}(45-3t)\,dt \rightarrow s = 0.25(90t - 3t^2 - 275)m$

at $t = 5s \rightarrow S = 25m$

at $t = 15s \rightarrow s = 100m$

③ $0 < t \leq 5s$

$a = \frac{dv}{dt} = \frac{d(0.6t^2)}{dt}$

$a = 1.2t\ m/s^2$

at $t=0 \rightarrow a=0$

at $t = 5s \rightarrow a = 6\ m/s^2$

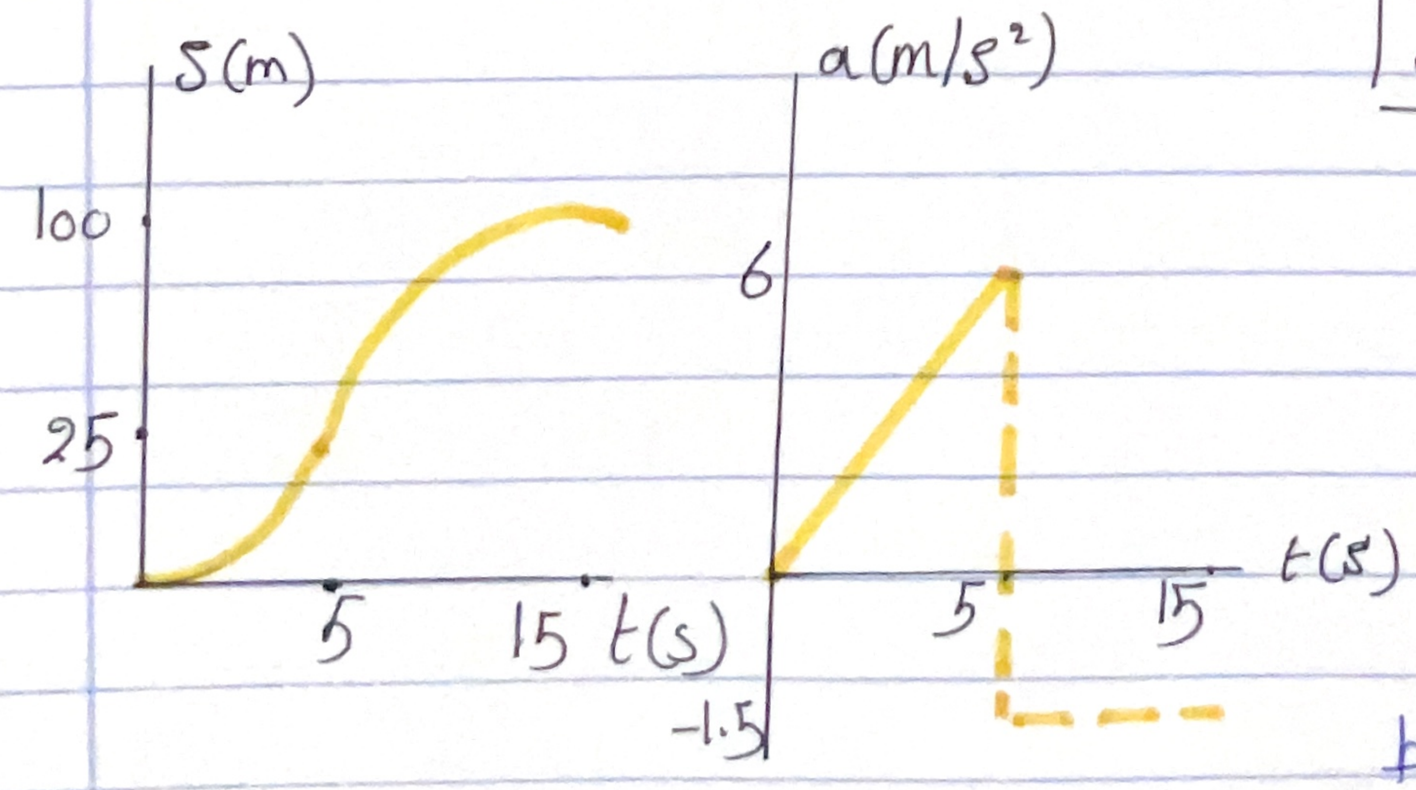
④ $5 \leq t \leq 15$

$a = \frac{dv}{dt} = \frac{d(\frac{1}{2}(45-3t))}{dt} = -1.5\ m/s^2$

$ST = 100\ m$

$V_{avg} = \frac{\Delta S T}{\Delta t}$

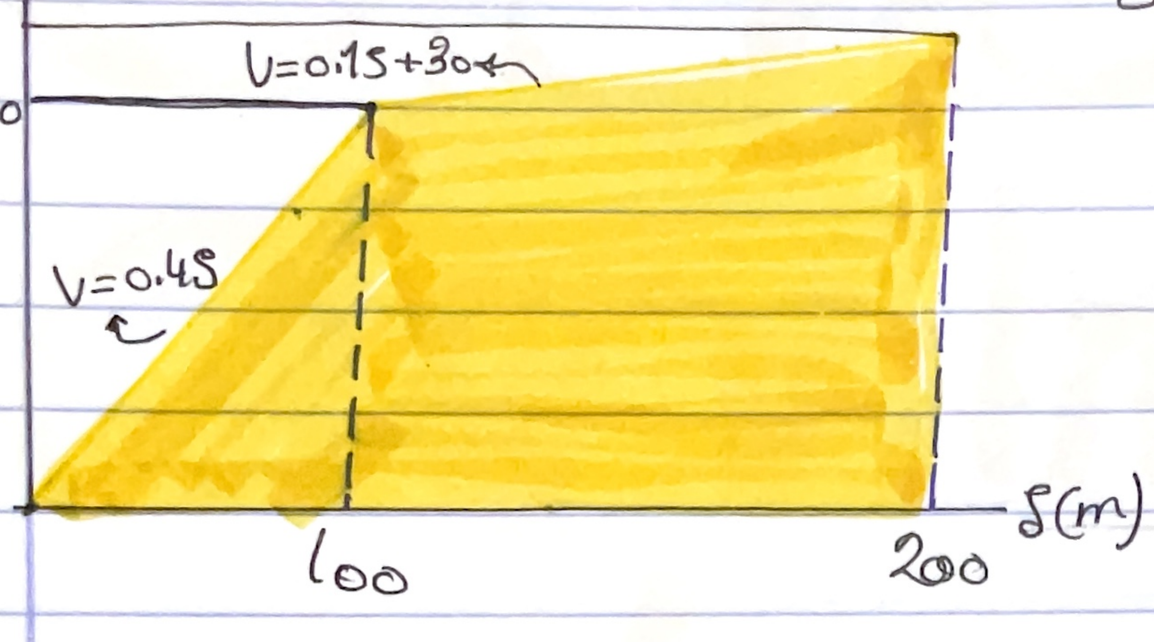
$V_{avg} = \frac{100}{15} = 6.67\ m/s$



لما بجي ارسم diagram
مش بس بدك تنتبهي للنقاط
لازم تنتبهي ل الاقتران الي حطتها بالأخير كمان لانه يا اما تكون const, linear, curved.

not existed in the book ig

Ex 12-53, The V-S graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100\,m$ and $s = 150\,m$. Draw the a-s graph



Solu:-

$0 < S < 100$ [a, V, S]

$a\,ds = V\,dv$

$a = V\frac{dv}{ds} = (0.4S)(0.4) = 0.16S$ m/s^2

at $s = 0$, $a = 0$

at $s = 100$, $a = 16\,m/s^2$

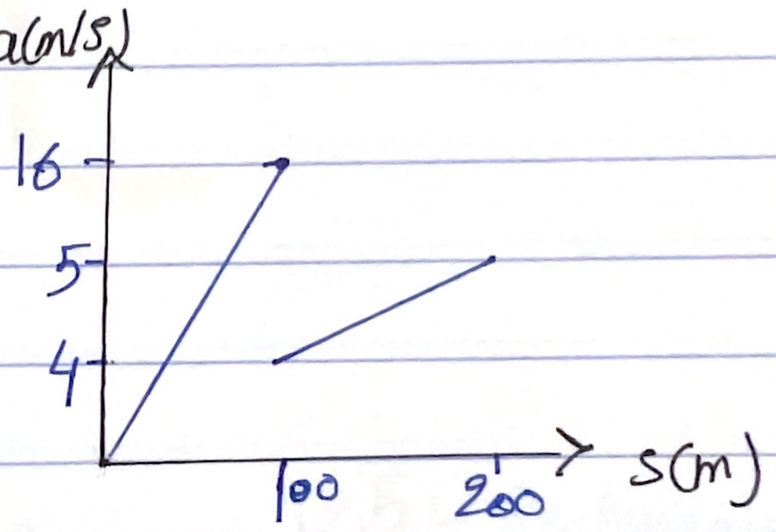
$100 \leq S \leq 200$

$a = (0.1S + 30)(0.1)$

$= (0.01S + 3)\,m/s^2$

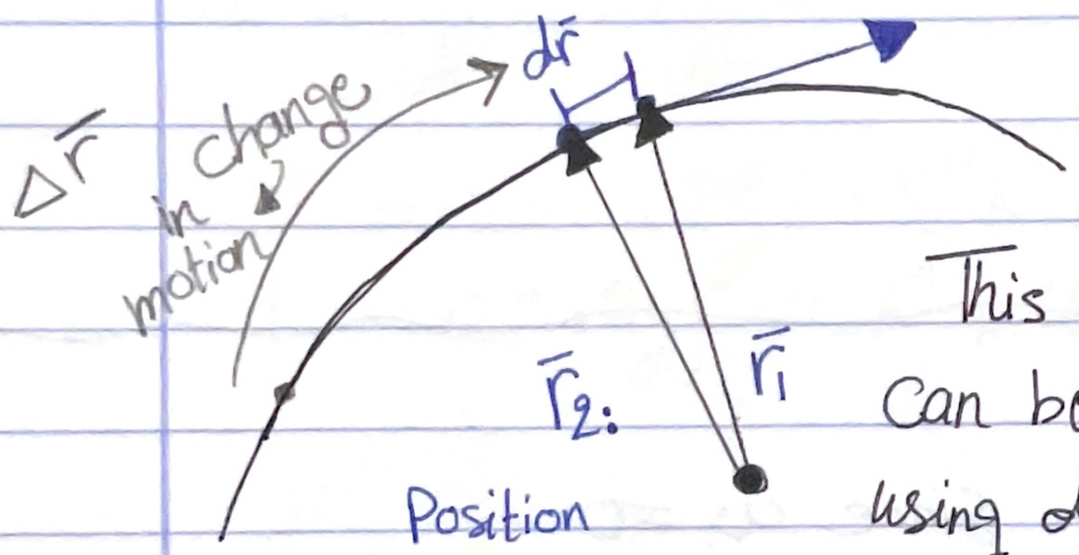
at $s = 100$, $a = 4\,m/s^2$

at $s = 200\,m$, $a = 5\,m/s^2$



* a-s graph means at the same exact a the plane have 2 diff values but not in the very same time tho*

General Curvilnear motion



Position Vector that defines the motion of a particle

This motion can be analyzed using different coordinat system:-

12.5:- Rectangular Coordinate System → 90°

12.7:- Normal Tangential Coordinate system

12.8:- Cylindrical components
↳ not included

لما تكون نقطة كتير على نقطة تانية بتاخد العربية كأنها Straigh line (فبتطل تمشي على القوس وتبقى كأنها خط مماس *)

$$\bar{V}_{avg} = \frac{\Delta\bar{r}}{\Delta t}$$

$$V_{inst} = \frac{d\bar{r}}{dt}$$



Tangent to the Curve

$\bar{a}$: inside of the curve
(الحركة تتعرف بالربط بأكبر زاوية اتجاه ال $\bar{a}$)

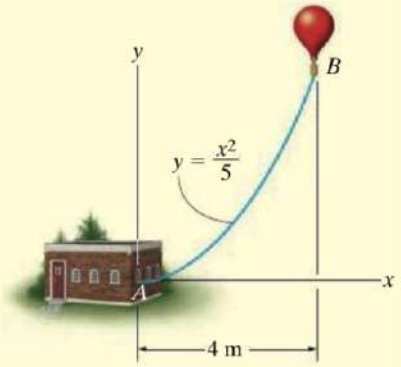
$$\bar{a}_{avg} = \frac{\Delta\bar{V}}{\Delta t}$$

$$a_{inst} = \frac{d\bar{V}}{dt} = \frac{V_{int}}{dt}$$

towards inside the curve
(عشان الحركة الدورانية تقل زي ما هي لازم اتجاه التسارع يكون للداخل)

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (2t)$ m, where t is in seconds. If the equation of the path is $y = x^2/5$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(2t) = 2 \text{ m/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. When $t = 2$ s, $x = 2(2) = 4$ m, Fig. 12–18*a*, and so

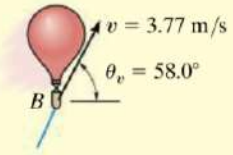
$$v_y = \dot{y} = \frac{d}{dt}(x^2/5) = 2x\dot{x}/5 = 2(4)(2)/5 = 3.20 \text{ m/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(2 \text{ m/s})^2 + (3.20 \text{ m/s})^2} = 3.77 \text{ m/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{3.20}{2} = 58.0° \qquad \textit{Ans.}$$



(b)

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(2) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/5) = 2(\dot{x})\dot{x}/5 + 2x(\ddot{x})/5$$

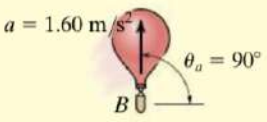
$$= 2(2)^2/5 + 2(4)(0)/5 = 1.60 \text{ m/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (1.60)^2} = 1.60 \text{ m/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{1.60}{0} = 90° \qquad \textit{Ans.}$$

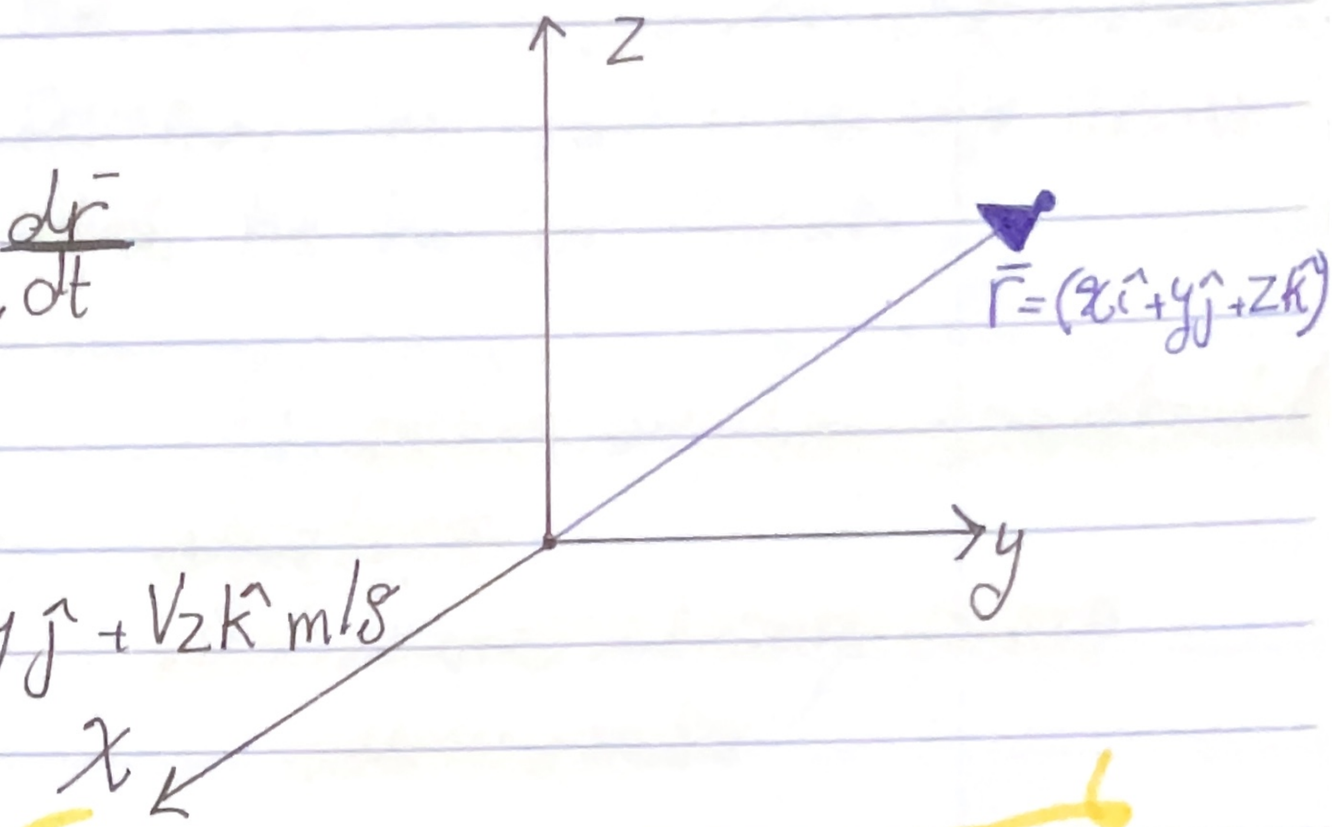


(c)

Fig. 12–18

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (2t)^2/5 = 0.8t^2$ and then taking successive time derivatives.

12.5 Rectangular coordinate system



$$\bar{V} = \frac{d\bar{r}}{dt}$$

$$\bar{V} = (\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k})\ m/s$$

$$\dot{x} = \frac{dx}{dt} = V_x$$

$$\dot{y} = \frac{dy}{dt} = V_y \qquad \bar{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\ m/s$$

$$\dot{z} = \frac{dz}{dt} = V_z$$

$$a = \frac{d^2r}{dt} = \frac{dv}{dt}$$

$$\bar{a} = \dot{V}_x\hat{i} + \dot{V}_y\hat{j} + \dot{V}_z\hat{k}\ m/s^2$$

$$\bar{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\ m/s^2$$

↳ called Fixed coordinate system

Check Example 12.9 page 39.

بالتوفيق
7. Dec 2025
9:52 Am

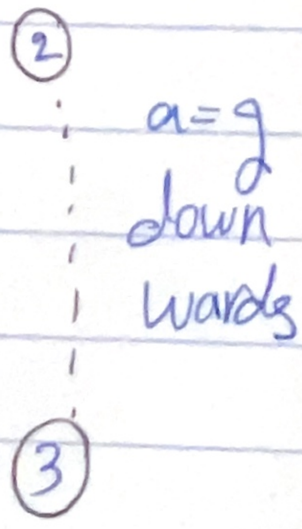
* and write it in the next page *

12.6 eqns

we have constant $a = g\downarrow$ we will use our 4 equations *

$V_{xf} = V_{oxi}$ always the same





Horizontal motion - Since $a_x = 0$

$\pm\rightarrow \quad v_x = (v_0)_x$

$+\rightarrow \quad x = x_0 + (v_0)_x t$

$+\rightarrow \quad v_x = (v_0)_x$

Vertical motion

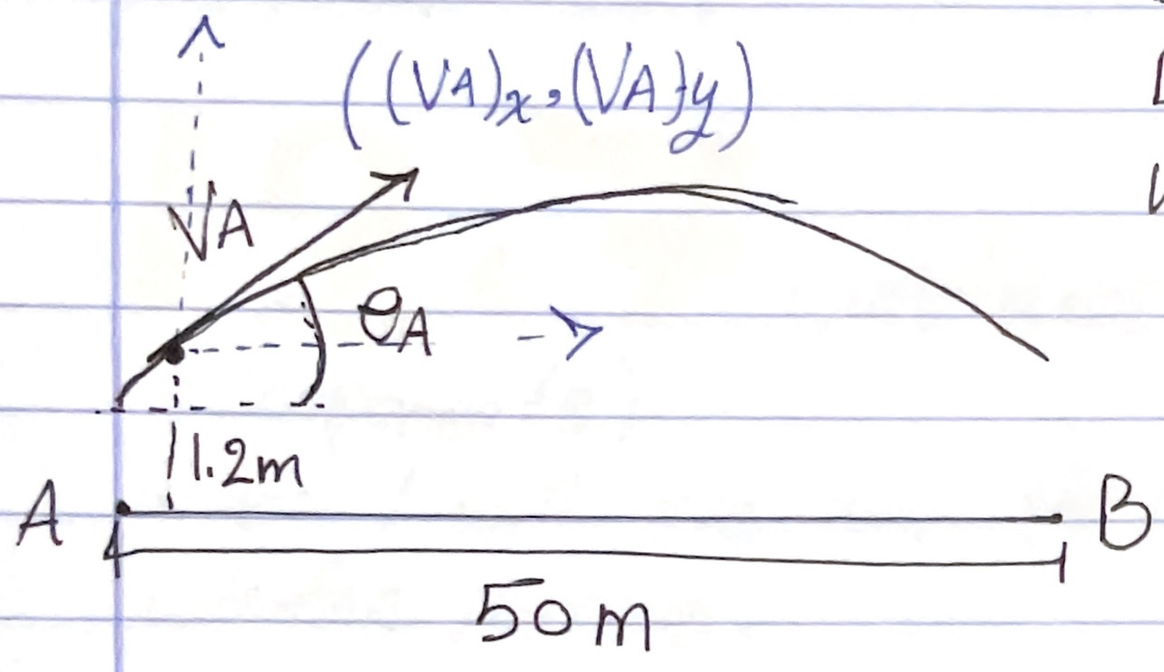
$+\uparrow \quad v_y = (v_0)_y - gt$

$+\uparrow \quad y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$

$+\uparrow \quad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$

12-85ex.

It is observed that the time for the ball to strike the ground at B is 2.5s. Determine the speed V_A and angle θ_A at which the ball was thrown.



(1) before anything you should adjust ur ref frame at any point u want.

Solu:—

$$y_B = y_A + (V_A)_y t - \frac{1}{2} g t^2$$

$t = 2.5s$

$x - x_0 = 50m$

$y_A = zero$

$y_B = -1.2m$

$x_A = zero$

$x_B = 50m$

$$-1.2 = 0 + (V_A)_y (2.5) - \frac{1}{2}(9.81)(2.5)^2$$

$$\rightarrow (V_A)_y = 11.78 m/s^2$$

$$(V_A) = \sqrt{V_{Ax}^2 + V_{Ay}^2} = 23.1 m/s$$

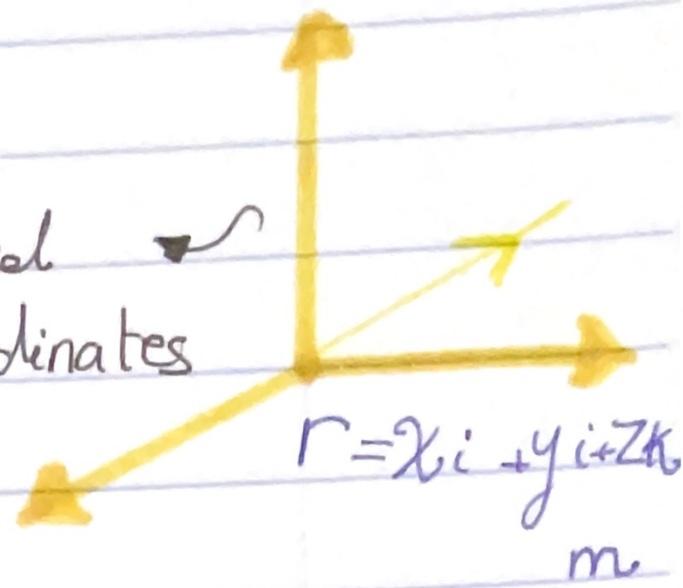
$$\theta = \tan^{-1}\frac{V_{Ay}}{V_{Ax}} = 30.5^\circ$$

$$x_B = x_A + (V_A)_x t$$

$$50 = 0 + (V_{Ax})(2.5)$$

$$\rightarrow (V_A)_x = 20 m/s$$

in 12.5 (Rectangular component)
to Find any point in the space I used to put an imaginary refrence frame just to find the point I want.



12.7 Curvilinear motion (Normal + Tangential Components).

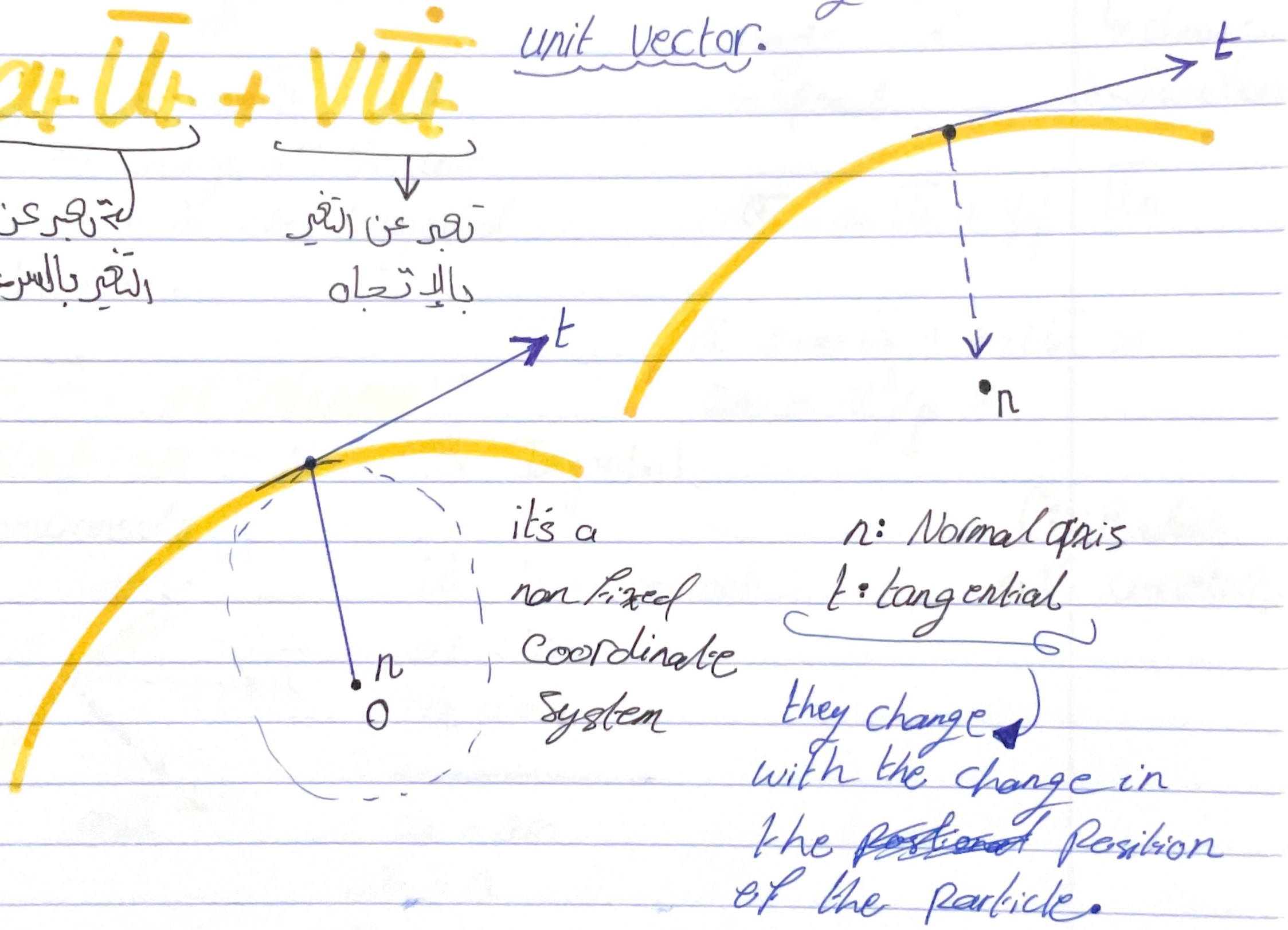
*now I will see how the coordinate system will move with the particle.

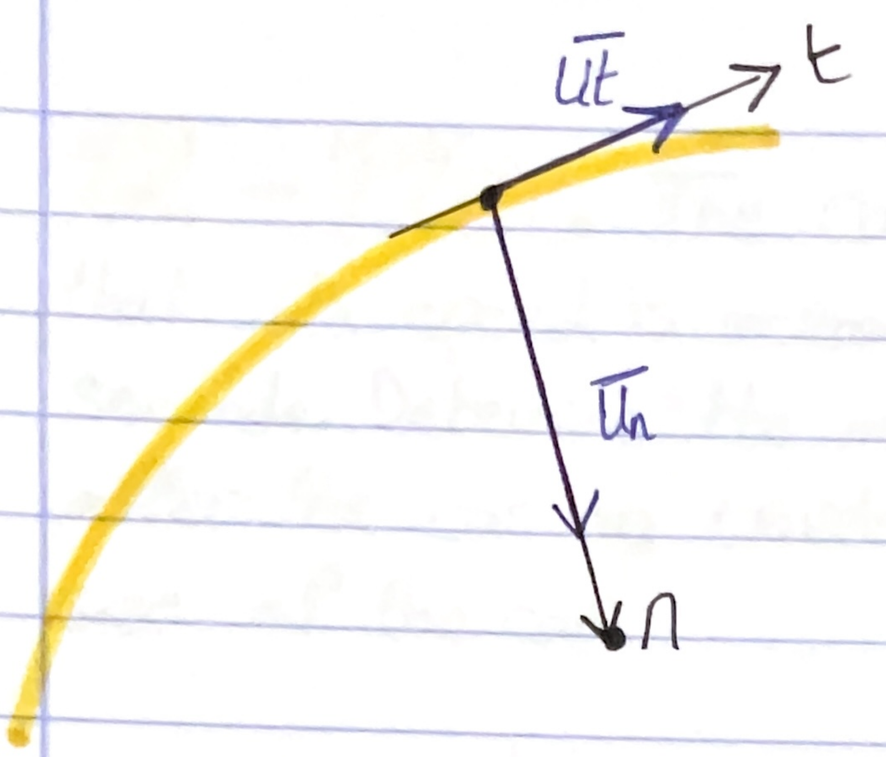
I need to create my own unit vector.

$a_t \vec{u}_t + v\dot{\vec{u}}_t$

$a_t \vec{u}_t$: تعبر عن التغير بالسرعة

$v\dot{\vec{u}}_t$: تعبر عن التغير بالإتجاه





$$\bar{V} = \dot{s}\,\bar{u}_t$$

$$= \frac{ds}{dt}\,\bar{u}_t$$

$$\bar{V} = V\,\bar{u}_t$$

↳ Velocity is ALWAYS tangent to the curve.

$$a = \frac{d\bar{V}}{dt} = \frac{d}{dt}(V\bar{u}_t)$$

use the product rule:-

$$= \frac{dv}{dt}\,\bar{u}_t + (V)\,\dot{\bar{u}}_t$$

$$= a_t\,\bar{u}_t + V\,\dot{\bar{u}}_t$$

$$\bar{a} = a_t\,\bar{u}_t + V\,\dot{\bar{u}}_t$$

change in speed ↔ $\bar{u}_t$; $V\,\dot{\bar{u}}_t$ ↳ change in direction

- When VT constant a_t = zero.

a_t, $\dot{\bar{u}}_t$ بتاخد تباطؤ لما تمشي عكس اتجاه Vt وتسارع لما تمشي مع اتجاه Vt.

$\dot{\bar{u}}_t \Rightarrow$ change of the unit vector in the tangential direction.

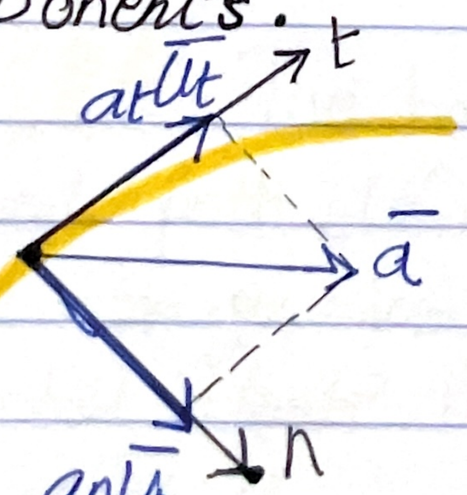
$$\bar{a} = a_t\,\bar{u}_t + \frac{V^2}{\rho}\,\bar{u}_n$$

$$\bar{a} = a_t\,\bar{u}_t + a_n\,\bar{u}_n \quad *$$

$$a_n = V^2/\rho \quad *$$

ρ :- Radius of curveture.

12.7 Curvilinear motion :- Normal & Tangential Components.



if V = constant

a_t = zero

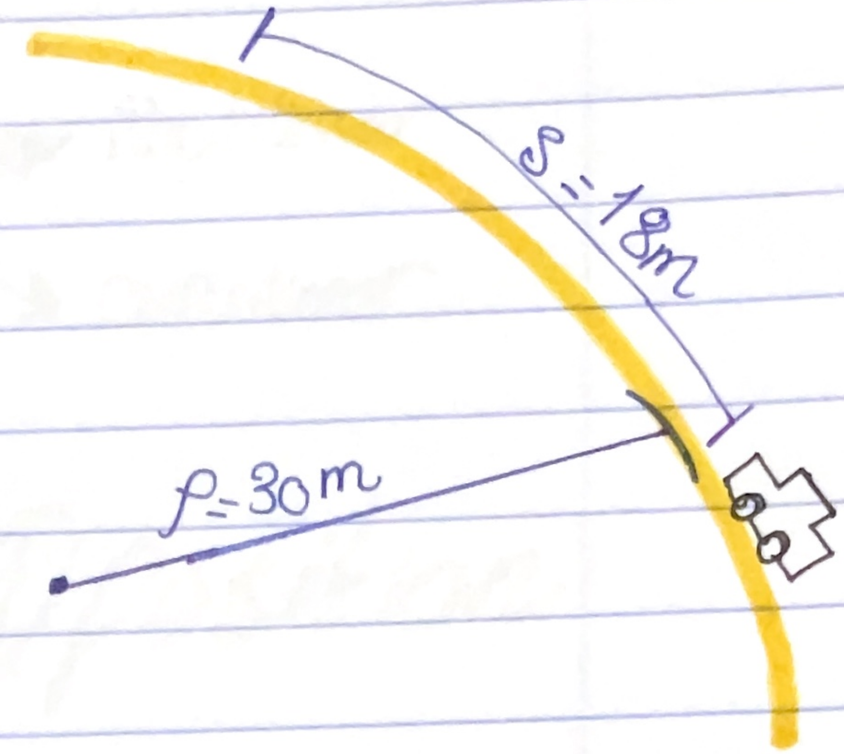
$a = a_n$

a_n = zero

$a_t = a$

Motion is Rectilinear

12-120. The car travels along the circular path such that its speed is increased by $a_t = (0.5)e^t \, m/s^2$. where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18m$ starting from rest. Neglect the size of the car.



$a_t = 0.5e^t$

$s = 18\,m$

$v = ?$, $a = ?$

$$a_t = \int_0^t 0.5e^t$$

$$V = 0.5(e^t - 1)$$

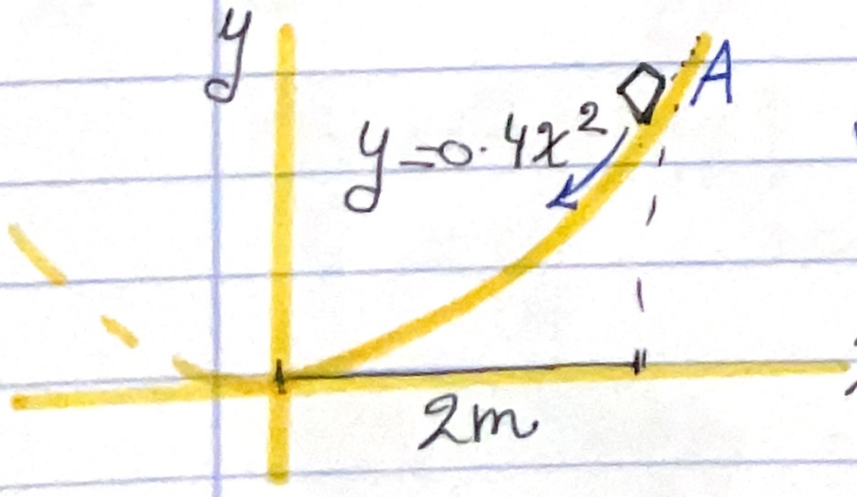
$$\int_0^v V = \int_0^t (0.5(e^t - 1)$$

$$a_n = \frac{V^2}{\rho} = \frac{19.85^2}{30} = 13.14$$

$t = 3.7$, $V = 19.18$

$$a_t = 0.5e^{3.70} = 20.39$$

12-124. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at $A(x_A = 2m, y_A = 1.6m)$, the speed is $v = 8\,m/s$ and the increase in speed is $dv/dt = 4\,m/s^2$. Determine the magnitude of the acceleration of the box at this instant.



increase in speed is tangential acceleration.

$y = 0.4x^2$, $y = 1.6$, $x = 2$

$a_t = 4\,m/s^2$

$y = 0.4x^2$, $\frac{dy}{dx} = 0.8x$

$$a_n = \frac{8^2}{8.39} = 7.628$$

$$a_t = \sqrt{(7.625)^2 + (4)^2}$$

$$a_t = 8.613\,m/s^2$$

$$\text{Curvature} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3}{\frac{d^2y}{dx^2}}$$

Ch.16

Planar Kinematics of a rigid Body.

Rigid body → Translation → Rectilinear
Translation → curvilinear
Rigid body → Rotation

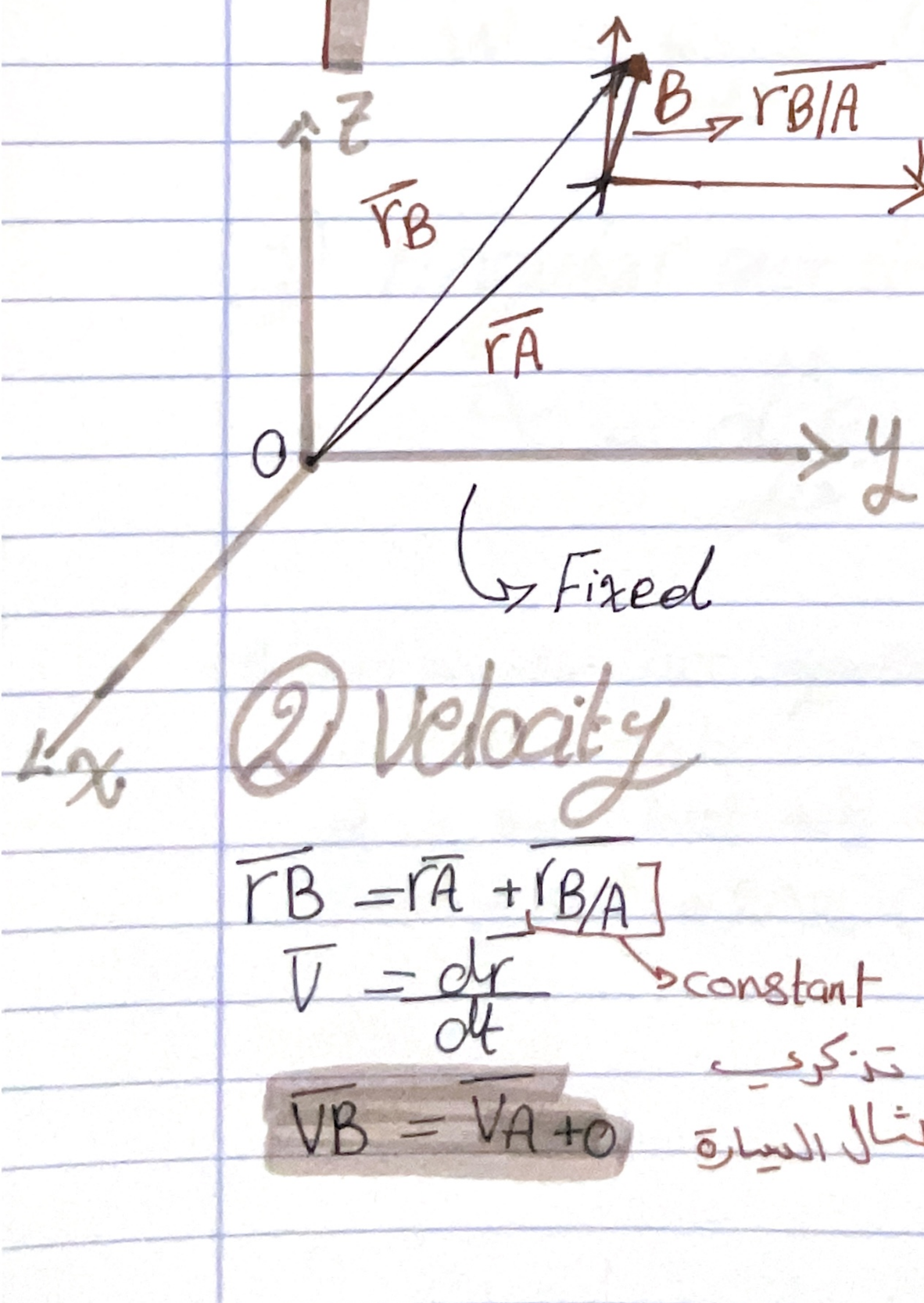
16.2: Opposition

Translation

By Vector addition

$\overline{rB} = \overline{rA} + \overline{r_{B/A}}$

* $\overline{rB}$ and $\overline{rA}$ is Variabels.
* $\overline{r_{B/A}}$ is Constant.



$\overline{r_{B/A}}$:-
position of B with respect to A.

② Velocity

$\overline{rB} = \overline{rA} + \overline{r_{B/A}}$

$\overline{V} = \frac{d\overline{r}}{dt}$ → constant

تذكري مثال السيارة

$\overline{VB} = \overline{VA} + 0$

③ acceleration

$\overline{a} = d^2\overline{r}/dt^2$

$\overline{a} = d\overline{v}/dt$

$\overline{aB} = \overline{a_A}$

16.3 Rotation about a fixed Axis

pls check sketch at page 322 these note are about it.

P: travels along a ciraular path
* But only a point so it cannot have angular motion. *

r: line that extends from point O to point P.

θ = angular displacement

dθ = change in angular displacement.
↳ rad / deg.

revolutions
1 rev = 2π rad.

② Angular Velocity :

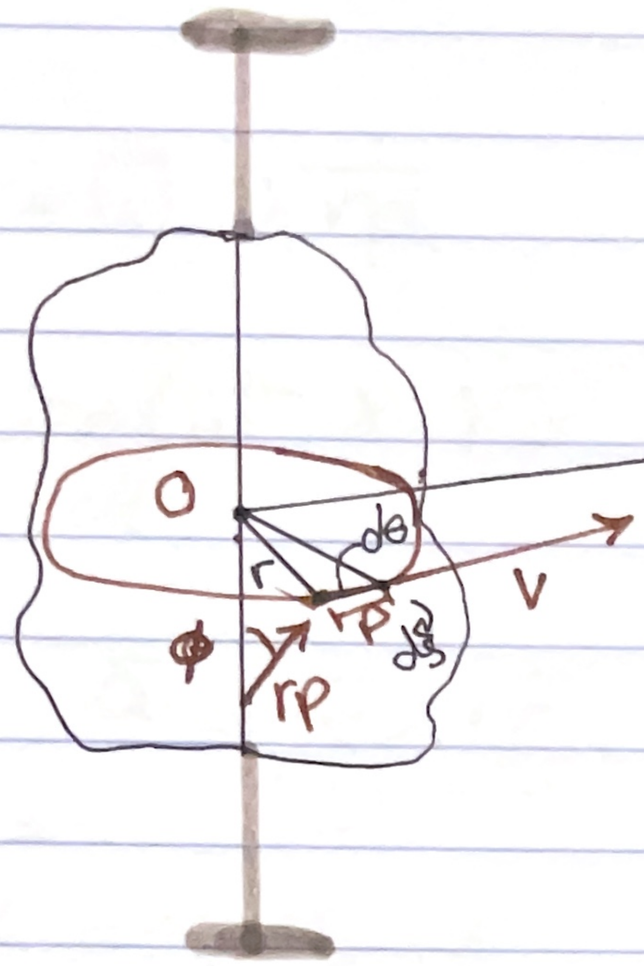
$$w = \frac{d\theta}{dt} \text{ (rad/s)}$$

③ Angular acceleration:

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{dw}{dt} \text{ (rad/s}^2\text{)}$$

Constant angular acc. equations.

$$w = w_0 + \alpha_c t$$
$$\theta = \theta_0 + w_0 t + \frac{1}{2}\alpha_c t^2$$
$$w^2 = w_0^2 + 2\alpha_c(\theta - \theta_0)$$

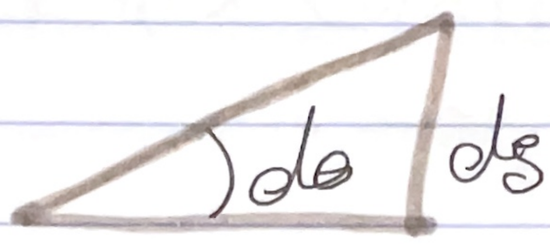


$\tan d\theta = \frac{ds}{r}$ $= d\theta \rightarrow ds = r\,d\theta$

if $d\theta$ is tooooo small then:- $\tan d\theta = d\theta$

$s = r\theta$

velocity (P)



$\frac{ds}{dt} = r\frac{d\theta}{dt}$ → divid by dt

$v = r\omega$ → scalar formulation

↳ Tangent to the curve

$\bar{V} = \bar{\omega} \times \bar{r_p}$

$\bar{r_p}$: and position ~~rotation~~ vector From axis of rotation to point P.

$\bar{V} = \bar{w} \times \bar{r}$

Acceleration (p)

$at = \frac{d}{dt} wr = r\frac{dw}{dt} = r\alpha$

$\bar{at} = r\alpha$ → scalar

↳ time rate change in the $|\bar{V}|$

↳ speed

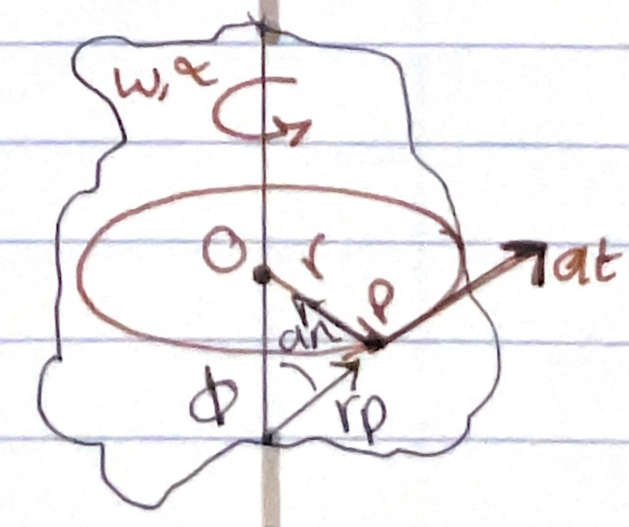
$an = V^2/\rho$

$V = rw$

$\rho = r$

$an = r^2w^2/r$

$an = rw^2$ time rate in ≢ Velocity direction



$\bar{V} = \bar{\omega} \times \bar{rp}$, $\bar{a} = \frac{d\bar{V}}{dt}$ $\quad \bar{v} = \bar{\omega} \times \bar{rp}$

$\bar{a} = \frac{d\bar{\omega}}{dt} \times \bar{rp} = \frac{d\bar{\omega}}{dt} \times rp + \bar{\omega} \times \frac{d\bar{rp}}{dt}$

$\frac{d\bar{\omega}}{dt} \rightarrow \bar{\alpha}$

$\bar{a} = \bar{\alpha} \times \bar{rp} + \bar{\omega} \times (\bar{\omega} \times \bar{rp})$

mag. = $\alpha rp \sin\phi$ ← $at = \alpha rp \sin\phi$

mag. = $an = \omega^2 rp \sin\phi$ $= \omega^2 r$

$\bar{a} = \underbrace{\bar{\alpha} \times \bar{r}}_{at} \underbrace{- \omega^2 \bar{r}}_{an}$

سالب لأنه بعكس اتجاه ال Position Vector.

$|\bar{a}| = \sqrt{at^2 + an^2}$

take a look at pic page 326.

at point A : A' ← is called mesh point

$V_C = V_B$

$(\omega r)_C = (\omega r)_B$

$(a_t)_C = (a_t)_B$

Note:- Normal acceleration is different.

$(\alpha r)_C = (\alpha r)_B$

مشوفي
الرسمات
بالكتاب

Ex 16-20 (Pg 333)

A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)\ rad/s^2$ where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20\ rad/s$ determine the angular velocity of gear B when $t = 2s$.

Solu-

given

$(\alpha)_a = (4t)^3$

$\omega_A = 20$

ω_B at $t = 2$

$V_A = V_B$

$\omega_B r_B = \omega_A r_A$

$\alpha = \frac{d\omega}{dt}$

$d\omega = \alpha\, dt$

$\int_0^\omega d\omega = \int_0^t 4t^3\, dt$

$\omega_F - \omega_i = t^4$

$\omega_F = t^4 + \omega_0$

$= 16 + 20 = 36\ rad/s$

$\omega_B = \frac{(36)(0.05)}{0.15} = 12$

Ex 16-31

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)\ rad$, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3s$.

Solu- $\omega = \frac{d\theta}{dt}$, $V_A = V_B$

$(\omega_C)_A = (\omega_B)_B$

$\omega = 15t^2 + 20t$

at $t = 3$

$\omega = 195$

$\frac{\omega_A r_A}{r_B} = \omega_B$

$\frac{(195)(0.2)}{0.125} = \omega_B$

$\omega_B = 312$

$(\alpha\, dt)_A = (\alpha t)_B$

Ex.F16-6. For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5 rad/s^2$, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pully D which is rigidly attached to gear B.

$\theta_B = \theta_D$

$\theta_B = (\omega t)_B + \frac{1}{2}\alpha t^2$

V and distance at $t = 3$

$\theta_B = 0 + \frac{1}{2}(1.5)(3)^2$

$= 6.75$

$(\alpha_B) = (\alpha_A)$

$\alpha_A = \frac{\alpha_{rA}}{r_B}$

$V_C = \omega_B r_D = (4.5)(0.125)$

$= 0.56$

$\alpha_B = \frac{(4.5)(0.075)}{0.125} = 1.5 rad/s^2$

$s_C = \theta_C \cdot r_C =$

$(6.7)(0.125) =$

$s_C = 0.844m$

By kinematic equation

$s = \theta_C r_C$

$\omega_B = (\omega_B)_0 + \alpha t$

$\theta_D = \theta_B = \theta_C$

$\omega_B = 0 + (1.5)(3)$

$\omega_B = 4.5$

* constant velocity = zero acceleration
* " angular " = zero angular accele

* The importance of dynamics is to find a usable transformation could be rotational or normal
* don't forget- cartesian means there is smth multiplied by unit vector of the vector itself.

16.35, IF the shaft and plate rotates with a constant angular velocity of $w = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.

Solu:-

$w = 14$ rad/s $\rightarrow \alpha =$ zero.

$V_P = \bar{w} \times \bar{r}$

$\bar{w} = w \bar{U}_{OA}$

$$\bar{w} = (14)\left(\frac{(-0.3i + 0.2j + 0.6k)}{\sqrt{0.3^2 + 0.2^2 + 0.6^2}}\right)$$

$\bar{w} = (-6i + 4j + 12k)$ rad/s

$\bar{r}_{OC} = (-0.3i + 0.4j)$ m

$$\bar{V}_C = \bar{w} \times r = \begin{vmatrix} i & j & k \\ + & - & + \\ -6 & 4 & 12 \\ -0.3 & 0.4 & 0 \end{vmatrix} = (-4.8i - 3.6j - 1.2k)$$ m/s

$a = \underbrace{\alpha \times r} + \bar{w} \times (\bar{w} \times \bar{r})$

Zero $+ (-6i + 4j + 12k) \times (-4.8i - 3.6j - 1.2k)$

$= 38.4i + -64.8j + 40.8k$ m/s^2

pay attention to the direction of α, a, V

Ch 13

Kinetics of a particle

(Force and acceleration)

حبة
خبة

13.1 Newton's second law of Motion

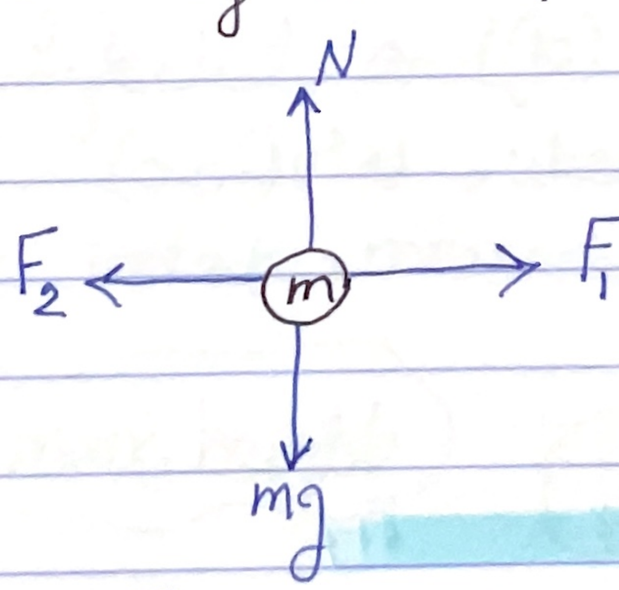
(unbalanced force) → $\Sigma F > 0$

18.2 Equation of motion

$\Sigma F = ma$ ⟶ For a system of particle we need FBD

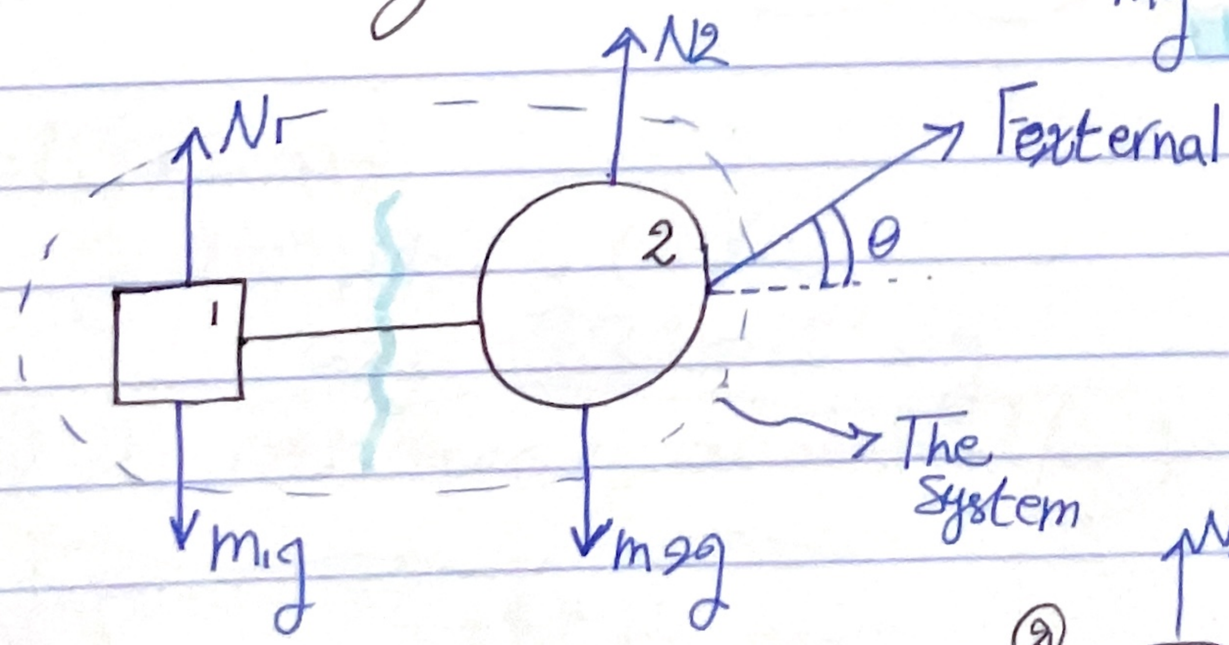
Newton's second law

one single particle we need to determine our system

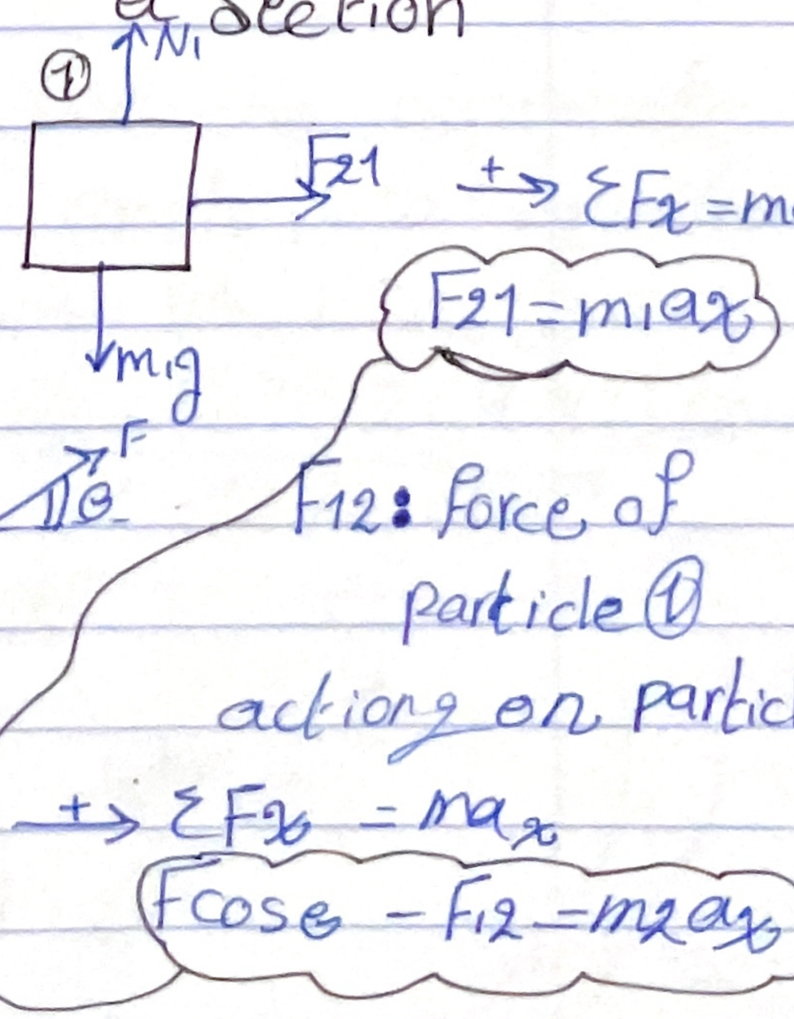


$\xrightarrow{+} \Sigma F_x = m_1 a_x$

$+\uparrow \Sigma F_y = m_1 a_y$



We have to create a Section



$\xrightarrow{+} \Sigma F_x = ma_x$

$F_{21} = m_1 a_x$

F_{12}: Force of particle ① acting on particle ②

$\xrightarrow{+} \Sigma F_x = ma_x$

$F\cos\theta - F_{12} = m_2 a_x$

$\xrightarrow{+} \Sigma F = ma$

$F\cos\theta = (m_1 + m_2)a$

* What about the internal F between the two particles?

Internal Forces

13.4 Equation of Motion:

Rectangular components. → Motion is usually rectilinear
→ Particle moves in a straight line.

$$\Sigma F = m\bar{a}$$

$$\Sigma F_x i + \Sigma F_y j + \Sigma F_z k = m(a_x i + a_y j + a_z k)$$

$$\Sigma F_x = ma_x \;,\; \Sigma F_y = ma_y \;,\; \Sigma F_z = ma_z$$

Ex 13.2

A 10-kg projectile is fired vertically upward from the ground with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (a) atmospheric resistance is neglected; and (b) atmospheric resistance is measured as $F_D = (0.01)v^2 N$, where v is the speed of the projectile at any instant measured in m/s.

Solu:

① Draw the FBD ① max height



$+\uparrow \Sigma F_z = ma_z$

$(-9.81)(10) = (10)(a_z)$

$a_z = -9.81 \text{ m/s}^2 = g$

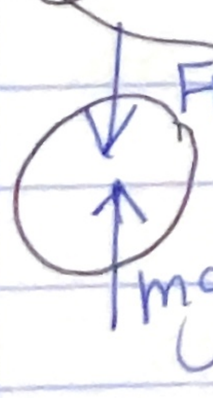
use the appropriate constant "a" equation

$v^2 = v_0^2 + 2a_c(z - z_0)$

$0 = 50^2 - 2(9.81)(z - 0)$

$z = 127 \text{ m}$ #

② Air resistance is accounted for



$+\uparrow \Sigma F_z = ma_z$

$(-0.01)(v^2) - (10)(9.81) = (10)(a_z)$

$a_z = -(0.001v^2 + 9.81) \text{ m/s}^2$

use $a\,ds = v\,dv$

$-(0.001v^2 + 9.81)\,dz = v\,dv \rightarrow \int_0^z dz = \int_{50}^{0} \frac{-v\,dv}{0.001v^2 + 9.81}$

let $u = 0.001v^2 + 9.81$

$du = 0.002v\,dv$

$v\,dv = \frac{1}{0.002}\,du \rightarrow = -\int_{50}^{0} \frac{du}{0.002u} = \frac{-1}{0.002}\ln|0.001v^2 + 9.81| + C$

$z = 113.5 \text{ m}$ #

* تتوقف الفرق بين ما كان في مقاومة الهواء وبين ما في *

13.4 Rectangular components "Equation of motion".

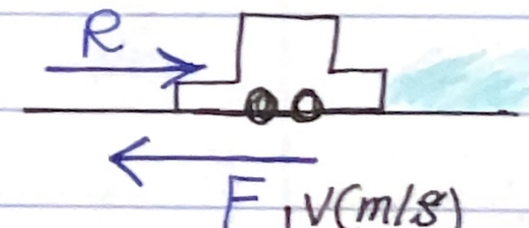
Straight line motion.

1) $V = ds/dt$
, $a = dv/dt$
, $a\,ds = V\,dv$

2) if acceleration was constant. we use the constant acc equs.

Ex 13-30,

The 1.5 Mg sports car has a tractive force of F = 4.5 kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.



Solu:-

$\overleftarrow{+}\ \Sigma F_x = ma_x$

$4500 - R = 1500(a_x)$ ..... ①

$a = \frac{dv}{dt} = \frac{d(-0.05t^2 + 3t)}{dt}$

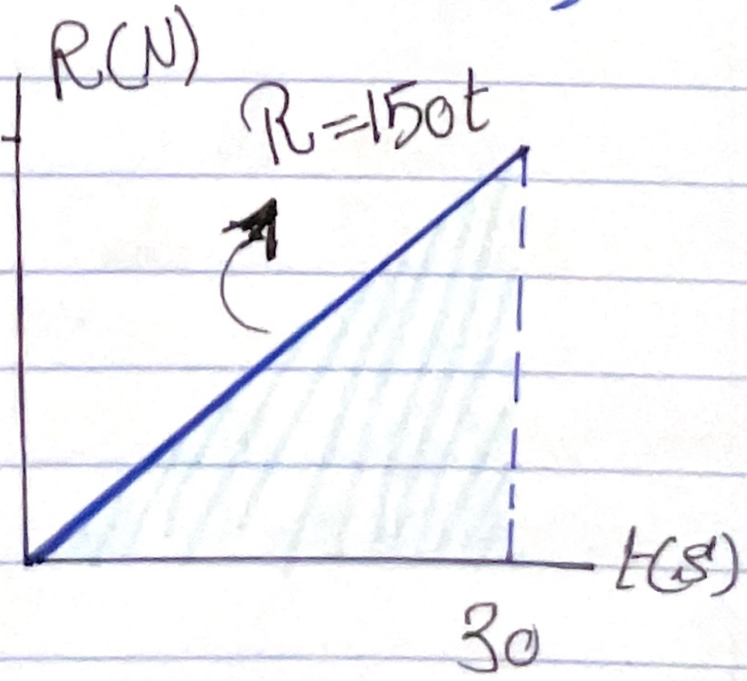
$a = (-0.1t + 3)\ m/s^2$

plug into ①

$4500 - R = 1500(-0.1t + 3) \rightarrow R = (150t)\ N$

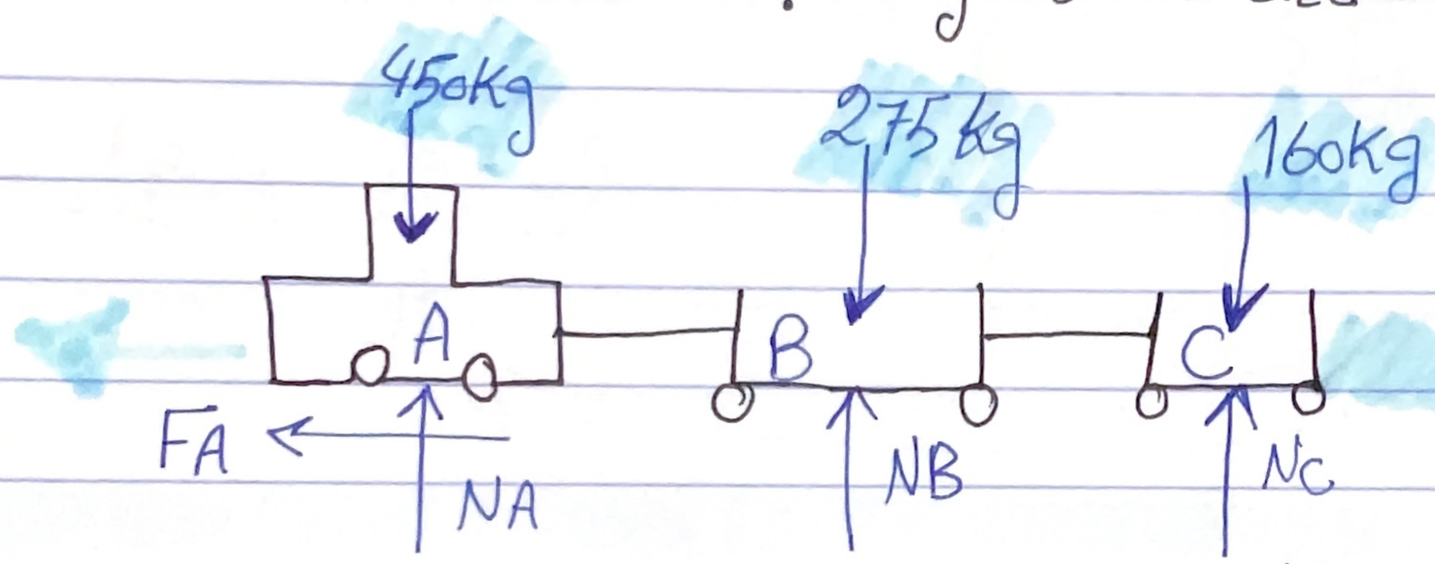
at $t = 0 \rightarrow R = 0$

at $t = 30 \rightarrow R = 4500\ N$



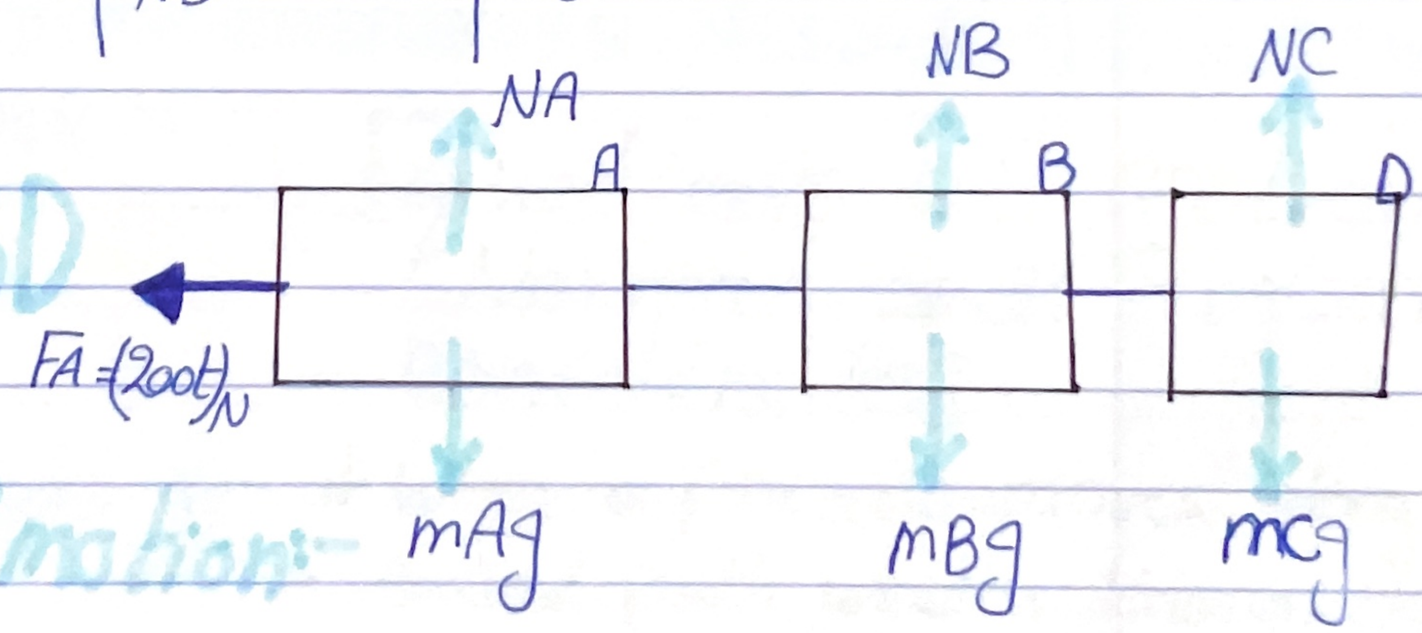
EX 13.3,

The baggage truck A shown in the photo has a mass of 450kg and tows a 275kg cart B and a 160kg cart C. For a short time the driving frictional force developed at the wheels of the truck is $F_A = (200t)$ N, where t is in seconds. If the truck starts from rest, determine its speed in 2 seconds. Also, what is the horizontal force acting on the coupling between the truck and cart B at this instant? Neglect the size of the truck and carts.



Soln:

1. Draw the FBD



2. Equation of motion:-

(1)

$\xrightarrow{+} \Sigma F_x = ma_x$

$(200t) = (450 + 275 + 160)(a_x)$

$a = (0.226t)\ m/s^2$

$a = dv/dt$

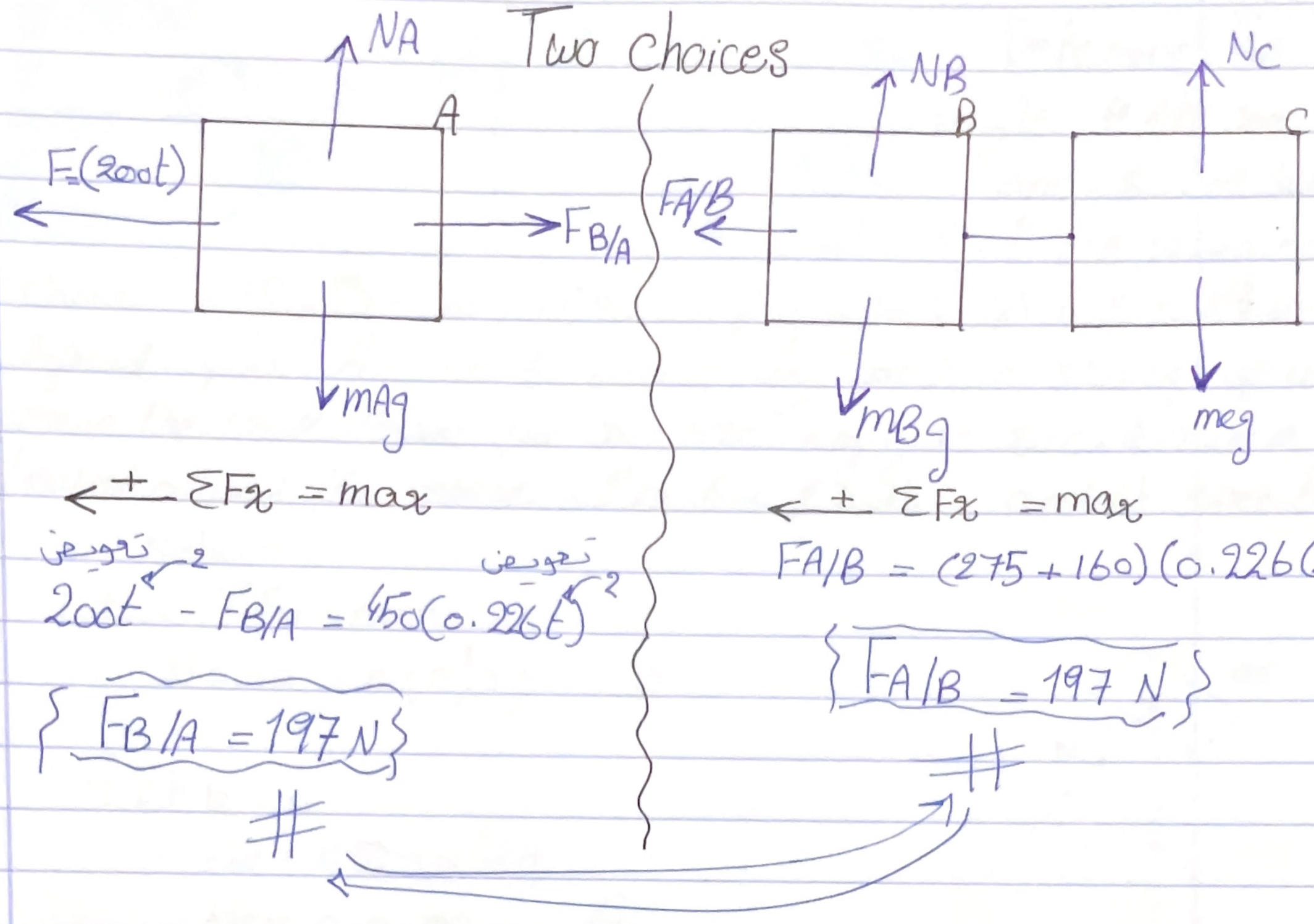
$dv = a\,dt$

$\int_0^v dv = \int_0^t (0.22t)\,dt$

$v = (0.113t^2)\ m/s$

at $t = 2s \rightarrow v = 0.452\ m/s$

The Force given is the (frictional driving force):
A Force that is need for the car to make it move.
it is not the frictional force that is opp to the origin force nonono A.

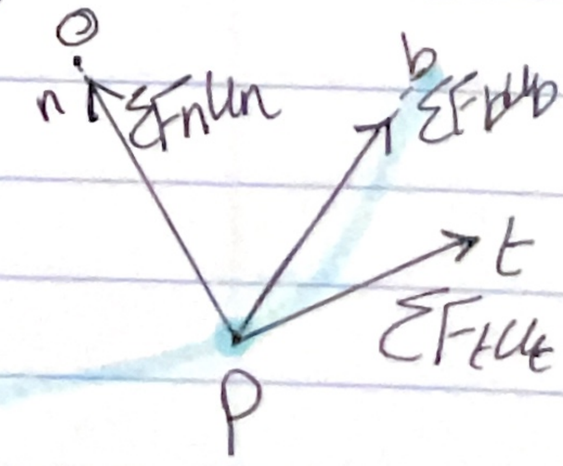


13.5 Equations of motion: Normal and Tangential Coordinates

* When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal and binormal directions, note that there is no motion of the particle in the binormal direction, since the particle is constrained to move along the path. We have:-

$\Sigma F = ma \rightarrow \Sigma F_t u_t + \Sigma F_n u_n + \Sigma F_b u_b = ma_t + ma_n$

Then →:- $\Sigma F_t = ma_t$, $\Sigma F_n = ma_n$, $\Sigma F_b = 0$



don't Forget

$a_t = dv/dt$

$a_n = v^2/\rho$

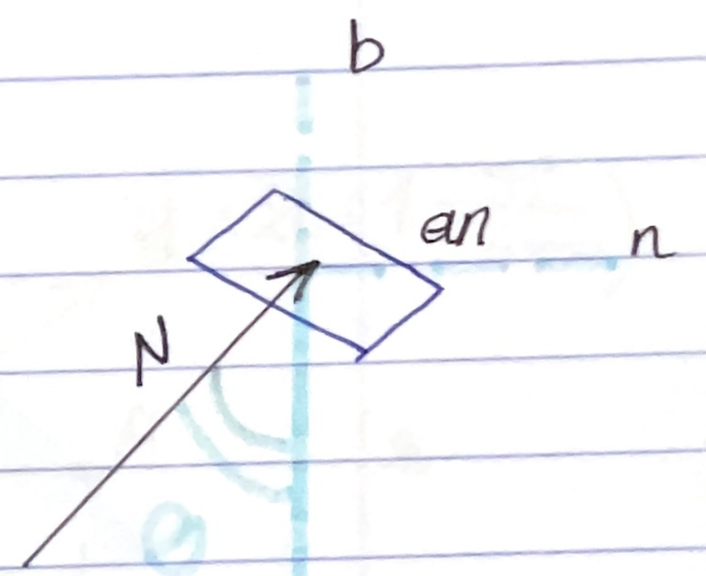
Ex 13.6,

Determine the banking angle θ the race track one, so the wheels of the racing cars shown in (Fig 13 - 12a) → (See the fig in the book) will not have to depend upon Friction to prevent any car from sliding up or down the track. Assume the car have negligible size, a mass m, and travel around the curve of radius ρ with a constant speed V.

Solu:-

$$\xrightarrow{+} \quad \Sigma F_n = ma_n$$

$$N\sin\theta = m(m^2/\rho) \text{ --- } (1)$$



$$+\uparrow \Sigma F_b = 0$$

$$-W + N\cos\theta = 0$$

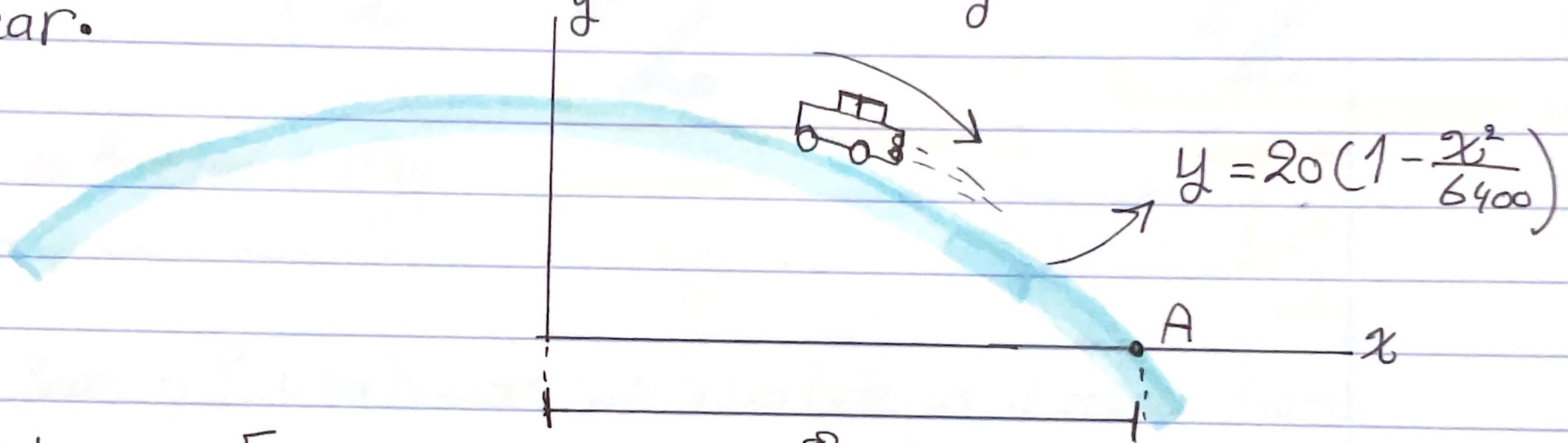
$$N\cos\theta = mg \text{ --- } (2)$$

divide (2) by (1)

$$\frac{N\sin\theta = mV^2/\rho}{N\cos\theta = mg} \Big\} \quad \tan\theta = V^2/\rho g \rightarrow \theta = \tan^{-1} V^2/\rho g$$

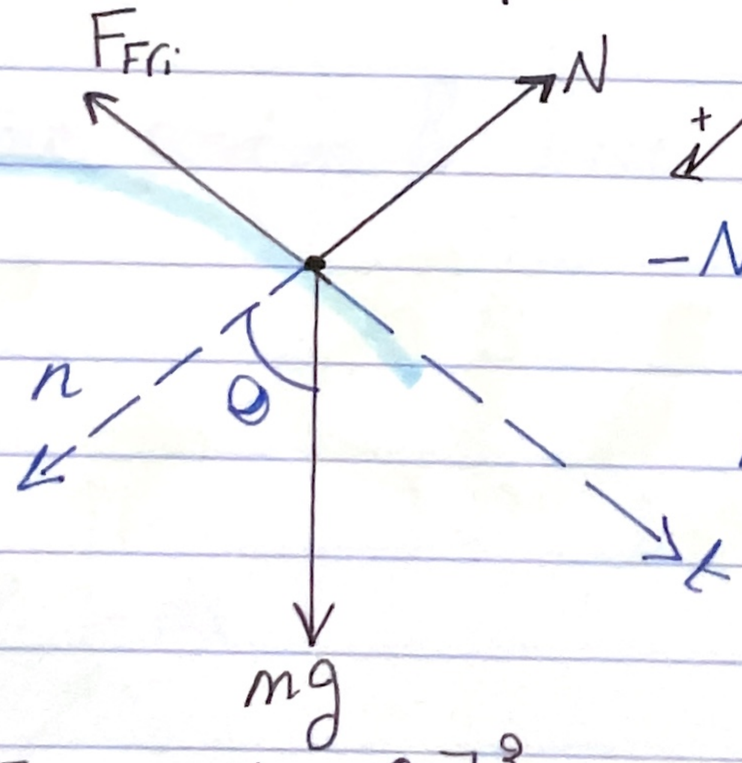
Ex 13-69

The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9m/s and increasing its speed at $3 m/s^2$. Determine both the resultant normal force and the resultant Frictional force that all the wheels of the car exert on the road at this instant Neglect the size of the car.



* he gave me Kinematics info and want kinetics *

Solu:-

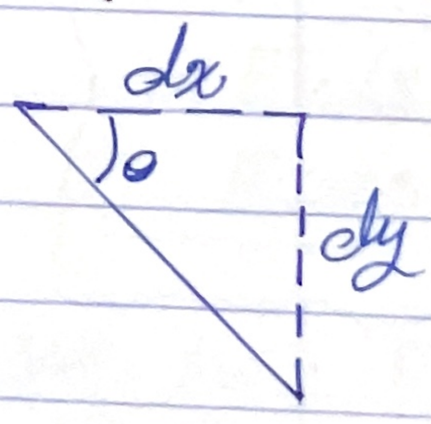


$+\swarrow \Sigma F_n = ma_n$

$-N + mg\cos\theta = m(V^2/\rho)$

$N = 6729 N$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = 223.61 m$



$\frac{dy}{dx} = -6.24(10^{-3})x$ at $x = 80m$

$\frac{dy}{dx} = 0.5 = \tan\theta$

$\theta = 26.56°$

$\frac{d^2y}{dx^2} = -6.24(10)^{-3}$

$+\downarrow \Sigma F_t = ma_t$

$-F_{Fri} + mg\sin\theta = ma_t$??

$\hookrightarrow F_{Fr} = 1109 N$

Ch 14, Kinetics of a particle: Work and Energy

$$\sum U_{1-2} = \underbrace{\tfrac{1}{2}mv_2^2}_{T_2} - \underbrace{\tfrac{1}{2}mv_1^2}_{T_1}$$

T → Kinetic energy

→ Sum of work done on a particle as it moves from position 1 to position 2

we have position & force information.

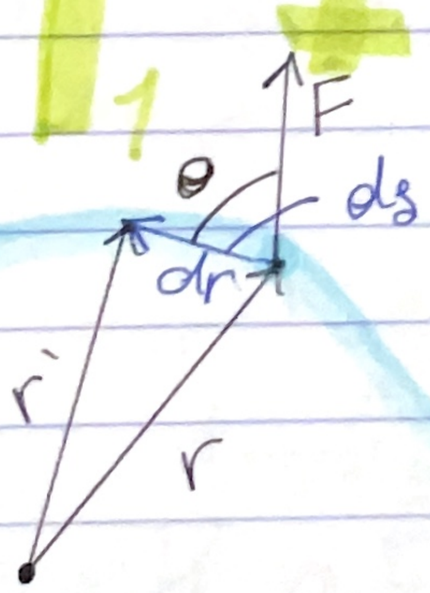
$$T_1 + \sum U_{1-2} = T_2$$

Where du ↓

$= F\,ds\cos\theta$

$= F\cos\theta\,ds$

We have F, U, s



dr ↓ زي كانها straight-line بتعامل معها

إذا كانت θ متغيرة ↙

$$U_{1-2} = \int_{s_1}^{s_2} F\cdot dr = \int_{s_1}^{s_2} F\cos\theta\, ds$$

* إذا كانت الزاوية (θ) ثابتة .

$$U_{1-2} = Fc\cos\theta\,(S_2 - S_1)$$

work of weight

لازم تأثير القوة بس عالمحور ←

$$U_{1-2} = \int F \cdot dr = \int_{r_1}^{r_2} (-Wj)\cdot(dx\,i + dy\,j + dz\,k)$$

$$U_{1-2} = \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1)$$

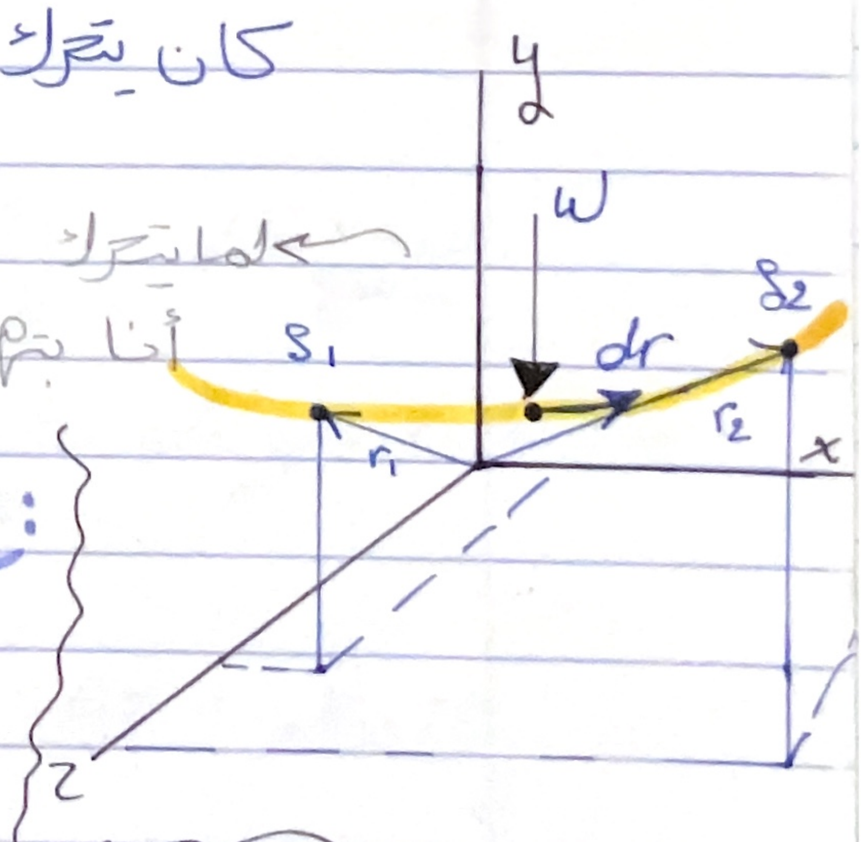
$$U_{1-2} = -W\Delta y$$

Be carefull

↳ اذا كانت الحركة للأسفل ف شغل الوزن موجب واذا كان يتحرك للأعلى ف شغل الوزن ح يكون سالب *

بالكتاب فيه سالب ← ليه؟ لأنه ignore it focus on the note*

← لما يتحرك الجسم من نقطة لنقطة أفقيًا هاا الشغل بتغير الشغل لأنه أنا بتهمني ال distance ا



work of a spring force:

$$U_{1-2} = -\left(\tfrac{1}{2}KS_2^2 - \tfrac{1}{2}KS_1^2\right)$$

K → string ثابت

* اذا السؤال حكى string يعني في stroching force

السالب فيه القانون ← بسبب الزاوية انها 180°

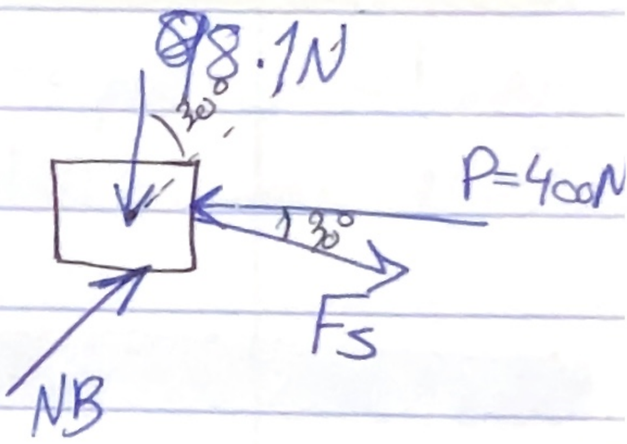
← $\cos 180° = -1$

لما اشتقينا هذا القانون من القانون الأصلية

Ex 14.1,

The 10-Kg block shown in Fig 14-6a. rests on the smooth incline. If the spring is originally stretched 0.5m. determine the total work done by all the forces acting on the block when a horizontal force $P = 400\,N$ pushes the block up the plane $s = 2m$

Solu: open the book at page 183 to study the fig (the dimentions are on the fig tho)



First all the time → FBD

the force is constant in the Horizontal side

So

$$U_P = 400(2m\cos 30°) = 692.8\,J$$

$\rightarrow = F.r$

Spring force $S_1 = 0.5m$, $S_2 = 0.5m + 2m$ (2m = Point لأنه شريت)

$$U_s = -\left[\tfrac{1}{2}(30)(2.5)^2 - \tfrac{1}{2}(30)(0.5)^2\right] = -90\,J$$

$\rightarrow = -\left(\tfrac{1}{2}Ks_2^2 - \tfrac{1}{2}Ks_1^2\right)$

Weight

$$U_w = (98.1)(2\sin 30°) = -98.1\,J$$

$\rightarrow = W\Delta y$ (لأنه لفوق بتطلع)

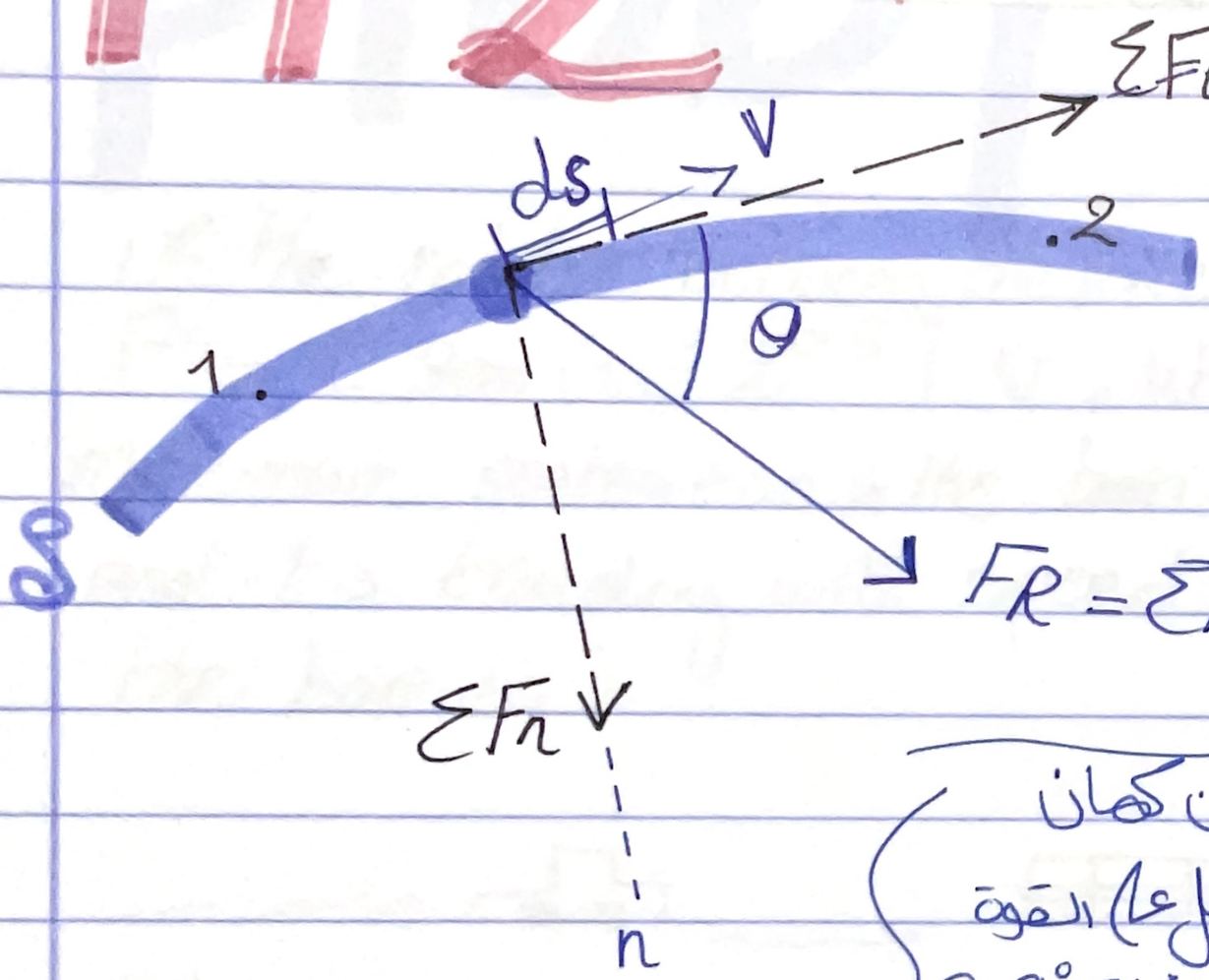
Normal force $N_B = 0$, no work → it's always ⊥ to the displacement

Total work → $U_T = 692.8 - 90 - 98.1 = 505\,J$

لما يكون ال path تبع الجسم straight بتكون ثابتة (لفوق) اما لو لمسار curved فهي ليست ثابتة

اذا كانت ال path بتطلع لفوق فشغل الوزن بيكون ⊖ / ولما يكون بتنزل لتحت واخد شغل الوزن بيكون ⊕

14.2 principle of work and Energy



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\Sigma F_t = m v \frac{dv}{ds} = \Sigma F_t = ma_t$$

$$a\,ds = v\,dv$$

جرمنو هون كمان
حافي شغل ع القوة
العمودية لانا $\theta = 90°$
$\cos 90° = 0$

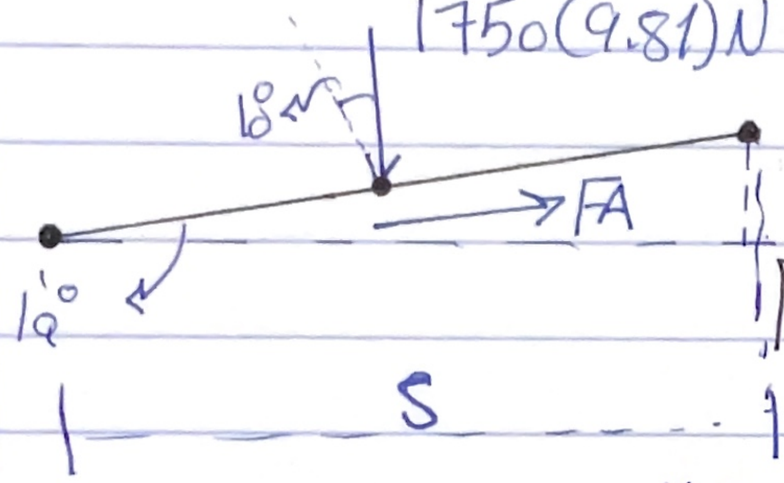
Ex 14.2

The 1750 kg automobile shown in Fig 14-10a travels down the 10° inclined road at a speed of 6 m/s. If the driver jams on the brakes causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.

Solu:-

FBD first

- consider (•) is the automobile



$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2} m v_1^2 + \Sigma U_{1-2} = \frac{1}{2} m v_2^2$ (Zero)

$FA = \mu_k N$

$+\nearrow \Sigma F_n = \text{Zero}$

$NA - mg\cos 10 = 0$

$NA - 1750(9.81)\cos 10 = 0$

$\boxed{NA = 16007\,N}$

$\left(\frac{1}{2}\right)(1750)(6)^2 + (1750)(9.8)(s)(\sin 10)$

$(8453.5)(5) = 0$

$\boxed{s = -5.76\,m\ \#}$

Friction → F (opp the direction of the particle)

Normal ← F (perp to the surface)

Static Friction const

يعبر التزان الجسم وعدم تحركه بسبب Friction

Kinitics Friction ثابت

يعبر عن تحرك الجسم بعكس اتجاهه بسبب Friction

هل عندي علاقة تربط بين Friction و Normal ؟

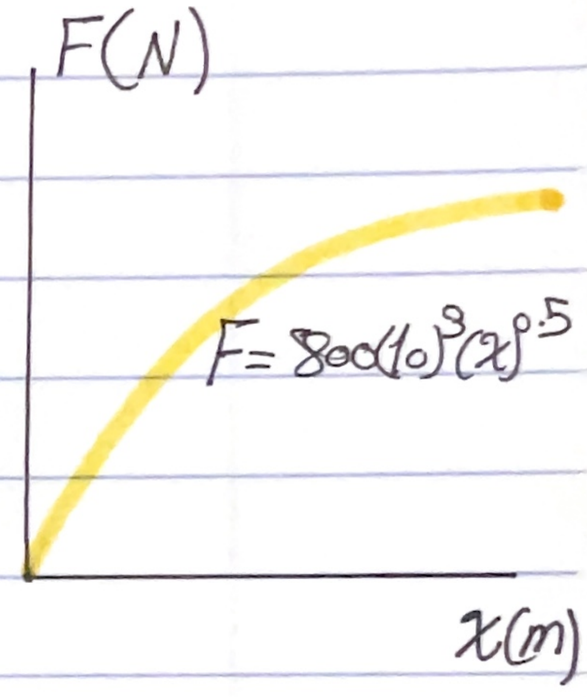
$F = \mu_k N$

work on Weight is positive bc its goun down (in the same direction)

Prob 14-1,

For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = [8\,00(10)^3 x^{0.5}]\ N$, where x is in m, determine the car's maximum penteration in the barrier. The car has a weight of 2Mg and it is traveling with speed of 20m/s just before it hits the barrier.





Solu:-

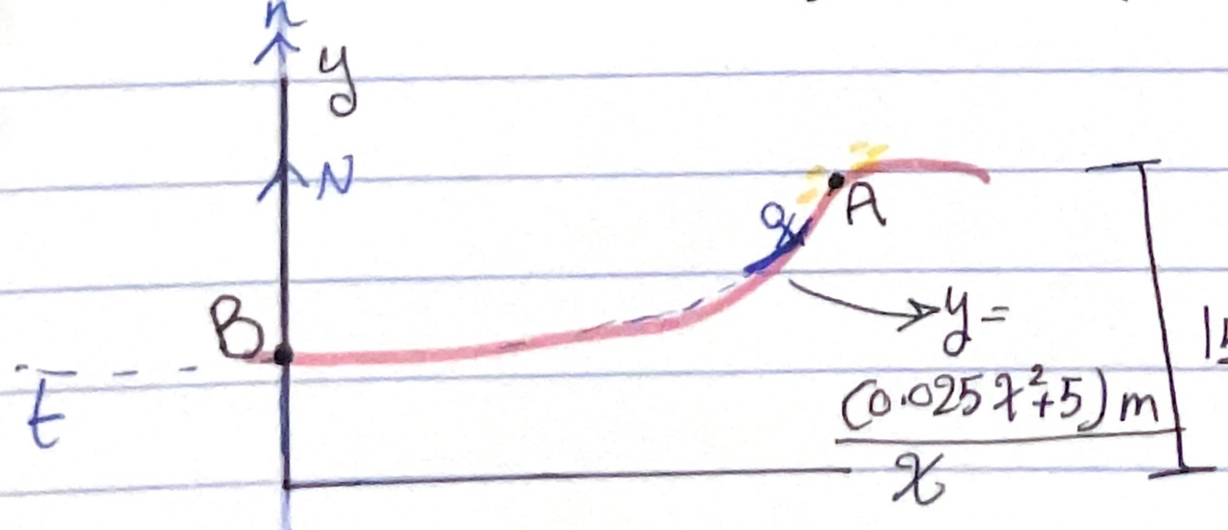
$$T_1 + \Sigma U_{1-2} = T_2$$

$$(1/2)(2000)(20)^2 - \int_0^x 800(x)^{0.5}(10)^3\,dx = \text{Zero}$$

$$x = 0.825m$$

Prob 14-38

If the 60kg skier passes point A with a speed of 5m/s. determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$(1/2)(60)(5)^2 + (60)(9.81)(10) = (1/2)(60)(V_2)^2$$

$$V_2 = 14.87\ m/s$$

$$+\uparrow \Sigma F_n = ma_n,\quad N - W = m(V_B^2/\rho)$$

$$N = 1250\ N$$

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\frac{dy}{dx} = 0.05x \rightarrow at\ x = 0 \rightarrow 0$$

$$\frac{d^2y}{dx^2} = 0.05$$

$$\rho = \frac{1}{0.05}$$

$$\rho = 20m$$

(لانه الحركة على curve يتوجب علينا معادلات تتناسب مع n, t)

