Statics

No.

Mechanics

Idealization

particle
- no shape
- no size

Motion: translation

Rigid body
- shape
- size

Motion: → translation
↳ rotation
moment ↙ ↘ torq

Equilibrium

→ statics $\Sigma F = 0$
no motion

→ Dynamics $\Sigma F = 0$
motion constant

Ch. 4

we care only abt rigid bodies now.
no motion ~~ could move

4.1] moment of a force
scalar formulation

No.

Moment: Tendency for a body to rotate at a point which is not on the line of action at the force.

لخط امتداد القوة

* نحدد الاتجاه باليد اليمين (القوة باتجاه الأصابع ومركز الدوران باتجاه باطن اليد (The palm of th hand

2D → counterclockwise

3D → +Z أو -Z على الإبهام للأعلى بكون

*Be careful in what dimention he asks the direction for.

$$M = F * d$$

↳ distance between "o" rotation center and the line of action for the force.

اعملي كل اشي بتقدري عليه عشان تجيبي العلامة
والبعد بين مركز الدوران وخط امتداد القوة.

No.

Ex:

$M = (F)(d)$

Could be

$= 20^{N.m}$, clockwise , $= -20\ N.m$

or

$=$ 20 N.m, counter/clockwise, $= +20\ N.m$

don't forget abt the signs and the unit

these ans are wrong***

$=$ +20N.m counter-clockwise , $=$ −20 N.m, clockwise

x x

* sometimes it is hard to recognise the ⊥ distance, so we make sin and cos components ∴

No.

4.2] talks abt cross product.

4.3] vector formulation (Moment of Force)

2D * $M_o = (F)(d)$

3D * $M_o = \vec{r} \times \vec{F}$

position ↵

vector → it is represent as 6 from point "O" to any point at the line of action of the force. choose the easiest point u could have.

$\vec{r} \rightarrow \langle a, b, c \rangle = (a\hat{i} + b\hat{j} + c\hat{k})m$ *

$|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$ the magnitude

assume:

$\vec{r} = (1\hat{i} - 2\hat{j} + 4\hat{k})$

$\vec{F} = (2\hat{i} + 1\hat{j} - 3\hat{k})$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = i(6 - 4) - j(-3 - 8) + k(5)$$

$= (2\hat{i} - 11\hat{j} + 5\hat{k})\ N \cdot m$

FIVE APPLE

No.

* sometimes we hawe to creat our own suitible Unit vector.

To do that we need to:

$\vec{U}$:

1. creat cuposition vector $\vec{F}$ & then divide it by its magnetud.

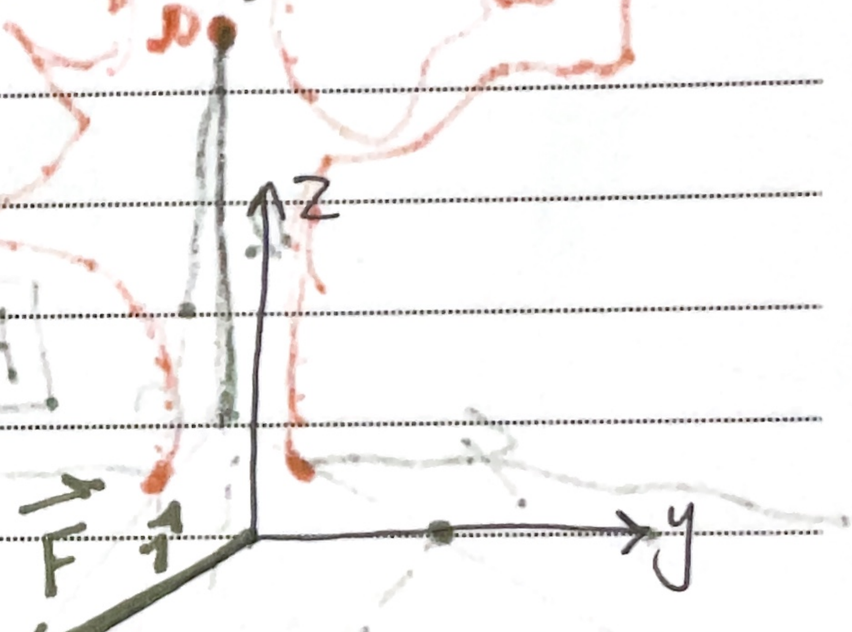
$\vec{F} = 50N$

$\vec{M} = (\vec{F})(\vec{U})$

$\vec{M} = (50)(\vec{U})$

$\vec{M} = (50)\left(\frac{\hat{i}}{1}\right) N \cdot m$

$\vec{M} = 50\hat{i}\, N \cdot m$

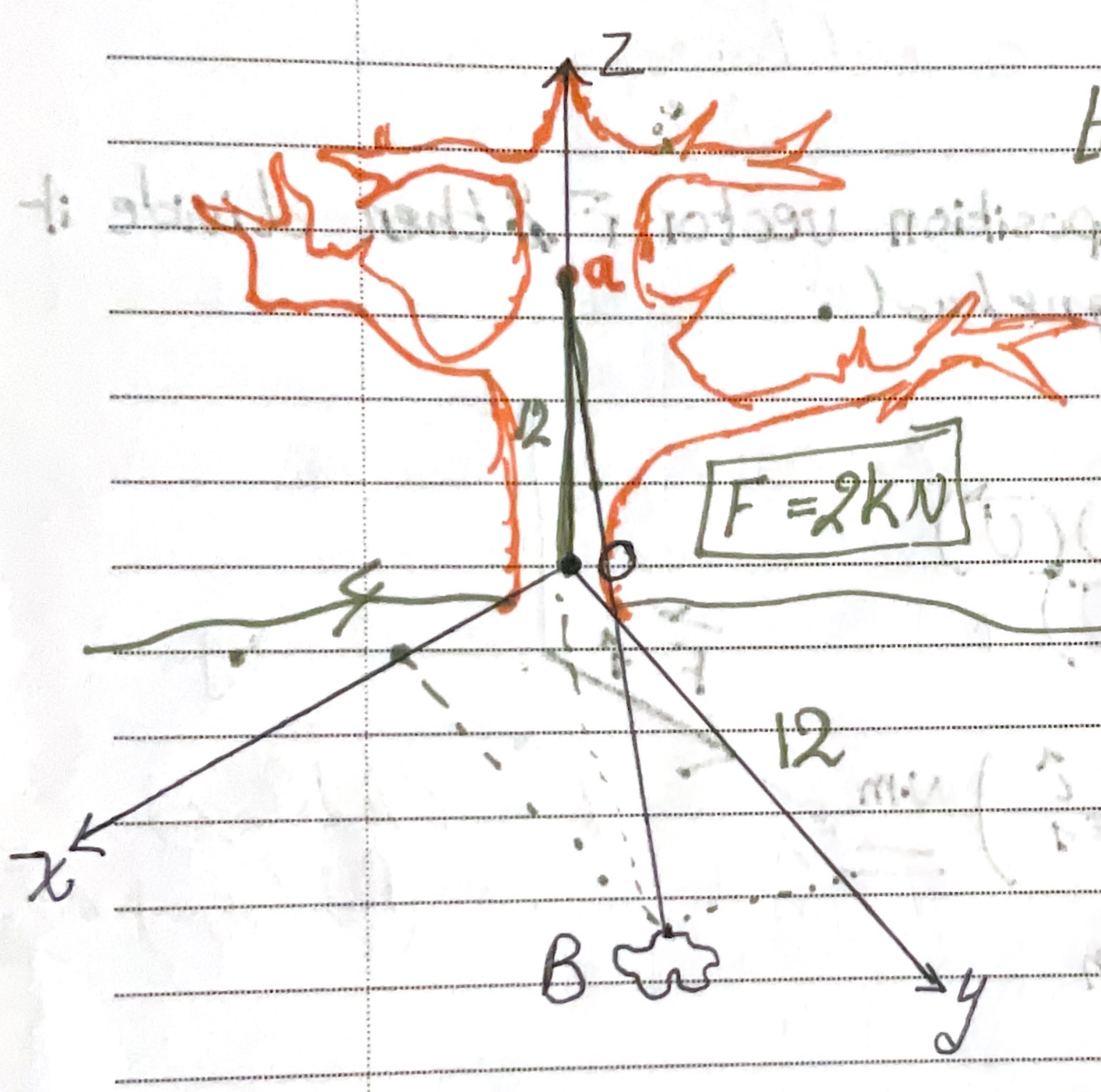


* when he says use cartisian vector u need to use $(\hat{i}, \hat{j}, \hat{k})$

No.

Ex:

Determine the moment produced by the force F in the fig. about point O. Express the result as a cartesian vector.



to determine the moment I need position vector and Force vector

$\vec{M}_O = \vec{r}_{OA} \times \vec{F}$

$\vec{r}_{OA} = (12\hat{k})\ m$

$\vec{F} = F(\vec{U}_{AB})$

~~$\vec{F} = (\quad)$~~

$\vec{U}_{AB} = \frac{\vec{r}_{AB}}{|r_{AB}|}$

$\vec{F} = \langle 0.46\hat{i} + 1.37\hat{j} - 1.37\hat{k} \rangle \frac{kN}{} \vec{U}_{AB} = \frac{\langle -4, 12, 12 \rangle}{\sqrt{16 + 144 + 144}}$

and then

$M_O = \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 0.46 & 1.36 & -1.36 \end{vmatrix} = -16.5\hat{i} + 5.51\hat{j}\ kN.m$

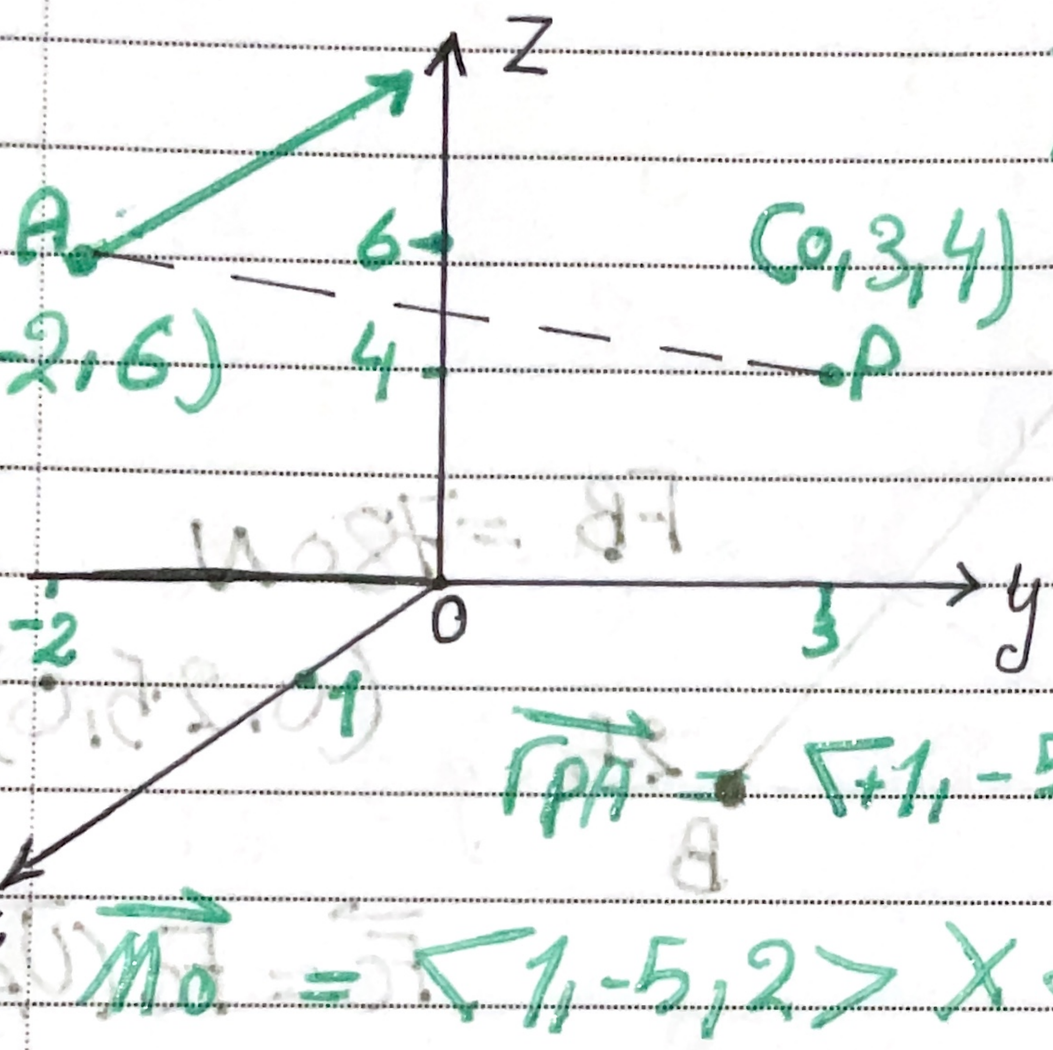
And the direction of $\vec{M}_O$ will be $\perp$ on $\vec{F}$ and $\vec{r}$

FIVE APPLE

No.

Ex:

Determine the moment of the force F about point P. Express the result as a cartesian vector.



$\vec{F} = (-6i + 4j + 8k)\,kN$

$\vec{M}_O = \vec{r}_{PA} \times \vec{F}$

$\vec{r}_{PA} = \langle +1, -5, +2 \rangle m$

$\vec{M}_O = \langle 1, -5, 2 \rangle \times \langle -6, 4, 8 \rangle$

$$\vec{M}_O = \begin{vmatrix} i & j & k \\ 1 & -5 & 2 \\ -6 & 4 & 8 \end{vmatrix} = (-40 - 8)i - (8 + 12)j + (4 - 30)k$$

$$= (-48i - 20j + (-26)k)\,kN$$

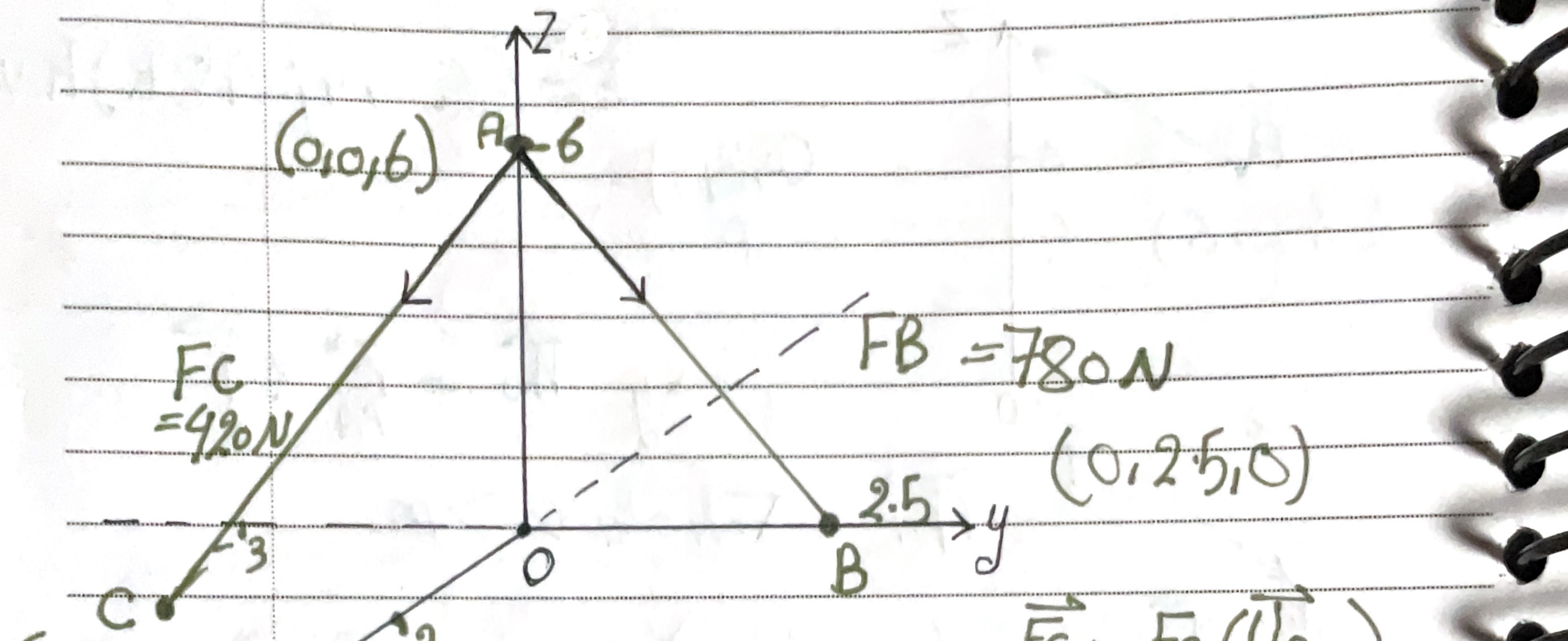
Calcutions are wrong it should be

$(-60i - 25j - 32k)\,kN \cdot m$

FIVE APPLE

No.

Example: determine the $\vec{M}_{net}$ moment abt point O.



$\vec{M}_{O(F_C)} = \vec{r}_{OA} \times \vec{F}_C$

$$\vec{M}_{O(F_C)} = \begin{vmatrix} i & j & K \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix}$$

$\vec{M}_{O(F_C)} = (+1080i - 720j + 0K)$

$\vec{M}_{O(F_C)} = (1080i - 720j)$ N.m

$\vec{U}_{AC} = \dfrac{\vec{r}_{AC}}{|r_{AC}|}$

$\vec{U}_{AC} = \dfrac{2i - 3j - 6K}{\sqrt{4+9+36}}$

$\vec{U}_{AC} = \left\langle \frac{2}{7}, \frac{-3}{7}, \frac{-6}{7} \right\rangle$

$\vec{F}_C = 420\left(\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}\right)$

$\vec{F}_C = (120i - 180j - 360K)$ N

$\vec{r}_{OA} = (0i + 0j + 6K)$ m

FIVE APPLE

No.

$\vec{M}_{O(F_B)} = \vec{r}_{OA} \times \vec{F}_B$ → $\langle 0,0,6 \rangle$

$\vec{M}_O = \begin{vmatrix} i & j & K \\ 0 & 0 & 6 \\ 0 & 300 & -720 \end{vmatrix}$

$\vec{M}_{O_{F_B}} = (-1800i - 0j + 0\hat{K})$

$\vec{M}_{O_{F_B}} = (-1800\hat{i})\ N.m$

$\Sigma \vec{M}_O = \vec{M}_{O_{F_C}} + \vec{M}_{O_{F_B}}$

$\Sigma \vec{M}_O = 1080\hat{i} + (-720j) - 1800i$

$\Sigma \vec{M}_O = (-720i - 720j)\ N.m$

$\vec{F}_B = F_B(\vec{U}_{AB})$

$\vec{U}_{AB} = \dfrac{\vec{r}_{AB}}{|r_{AB}|}$

$\vec{r}_{AB} = \langle 0, 2.5, -6 \rangle m$

$\vec{U}_{AB} = \dfrac{\langle 0, 2.5, -6 \rangle}{\sqrt{6.25 + 36}}$

$\vec{U}_{AB} = \langle 0, \frac{2.5}{\sqrt{42.25}}, \frac{-6}{\sqrt{42.25}} \rangle$

~~$\vec{F}_B = 780 \langle 0, \frac{5}{13}, \frac{-12}{13} \rangle$~~

$\vec{U}_{AB} = \langle 0, \frac{5}{2} * \frac{2}{13}, \frac{-6}{1} * \frac{2}{13} \rangle$

$\vec{U}_{AB} = \langle 0, \frac{5}{13}, \frac{-12}{13} \rangle$

$\vec{F}_B = 780 \langle 0, \frac{5}{13}, \frac{-12}{13} \rangle$

$\vec{F}_B = \langle 0, 300, -720 \rangle N$

No.

Dont Forget:

when it's hard to recognise the $\vec{F}$ (the geometry isn't clear) try to make components of $\vec{F}$ (F_x, F_y) $(\cos, \sin)$

No.

Moment of a couple

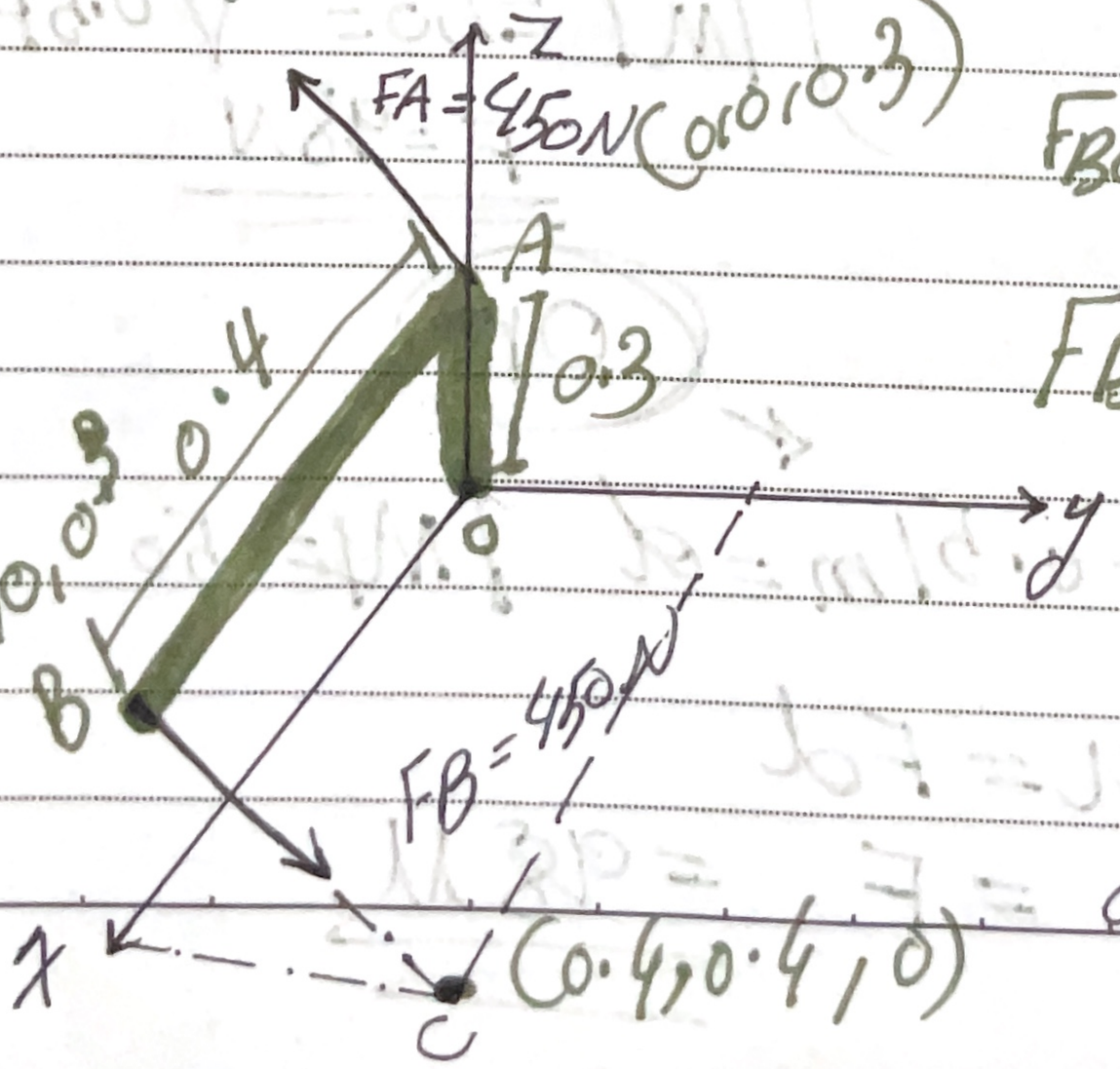
: Two forces, same magnitude diffirent direction.
line of action for forces shouldn't be aligned.

بغض النظر عن مركز الدوران
عزم الإزدواج يظل ثابت.

Ex on Fig 4-30 is imp! (2D que)

The force that I want to choose to calculate the moment of a couple and the position vector to be towards the choosen force.

F4-24. Determine the couple moment acting on the pipe.



$\vec{r}_{AB} = 0.4\hat{i}$

$F_{BC} = F_{BC} U_{BC}$

$F_B = 360\hat{j} - 270\hat{k}$ N

$M = \vec{r} \times F$

cross product

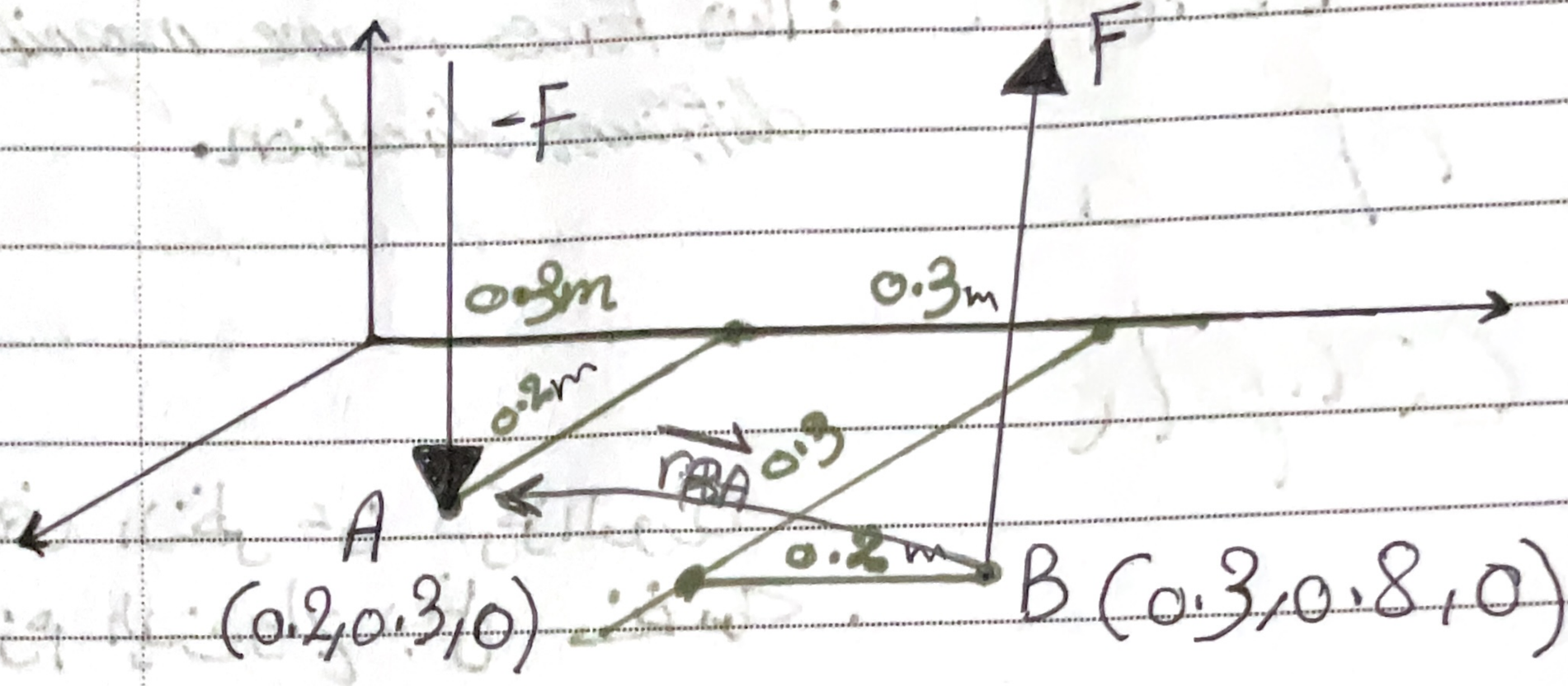
ans $= (108\hat{j} + 144\hat{k})$

FIVE APPLE

N.m

No.

prob 4 - 92/93



$\vec{r_{BA}} = (-0.1\hat{i} - 0.5\hat{j})\ m$

$\vec{F_A} = F_A(U_A) = -F\hat{k}\ N$

$\vec{U} = \hat{k}$

$\vec{M} = \vec{r_{BA}} \times F$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -F \end{vmatrix} \quad (+\ -\ +)$$

$\Rightarrow -0.5F\hat{i} - 0.1F\hat{j}$

$|\vec{M}| = 50 = \sqrt{0.5F^2 + 0.1F^2}$

$F = 98N$

or

$|r_{BA}| = 0.51m = d$ $\quad |M| = 50$

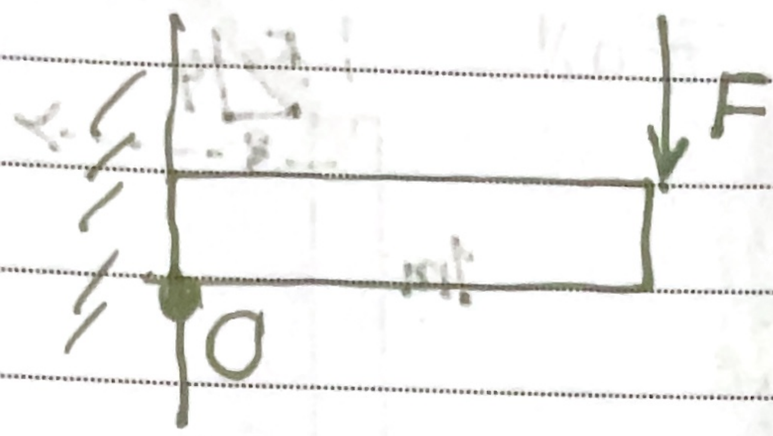
$M = Fd$

$\frac{50}{0.51} = F = 98N$

FIVE APPLE

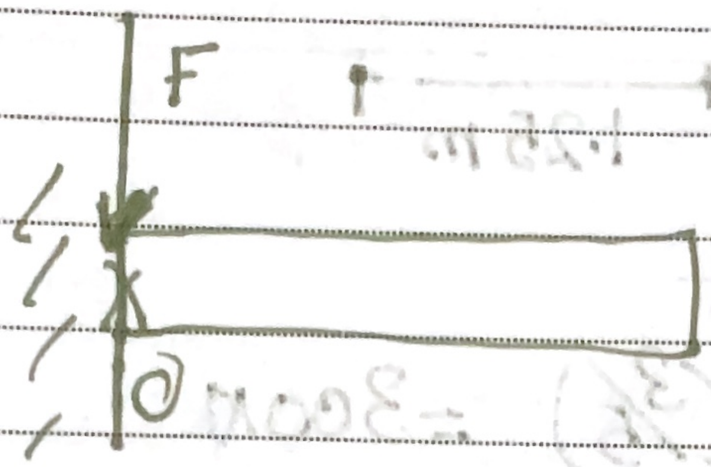
No.

note again: the position vector should be toward the meant force (the force that I want to calculate the Moment at).



sec 4.7

F: Rotation and translation

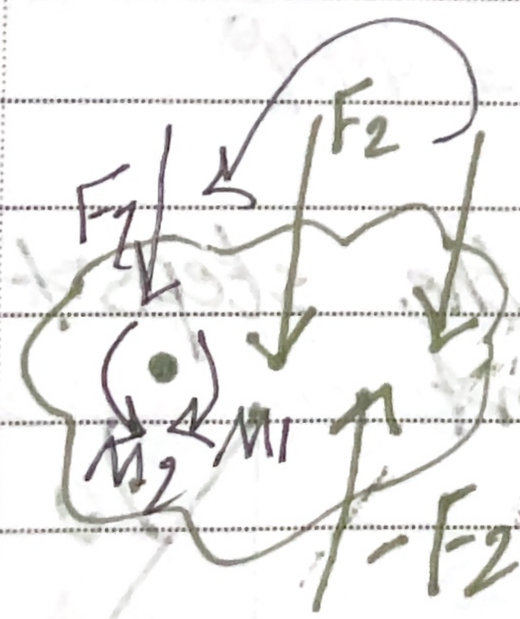


F: it wont rotate it will have a trasplation move.



couple → F: F + couple

we have a rigid body



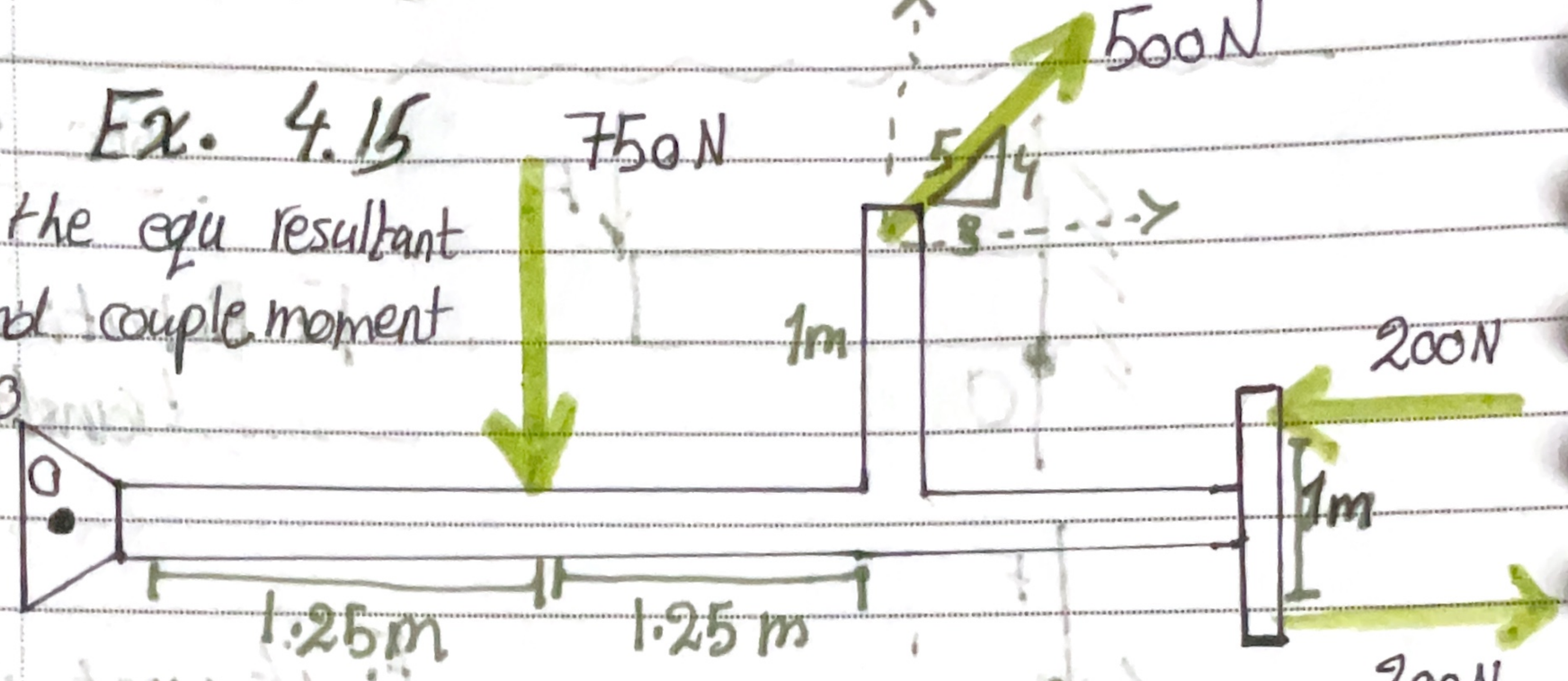
I will make a Moment of couple of F_2 around the O and then I will make M_1 of F_1

No.

* in Moment don't forget that u need the ⊥ distance *

Ex. 4.15

Calculate the equ resultant force and couple moment at point O



Sol: $F_R = \Sigma F$

$\xrightarrow{+} \Sigma F_x = 500(3/5) = 300N$ ←

أنا بفرض إلى اتجاه

$+\uparrow \Sigma F_y = -750 + 500(4/5)$

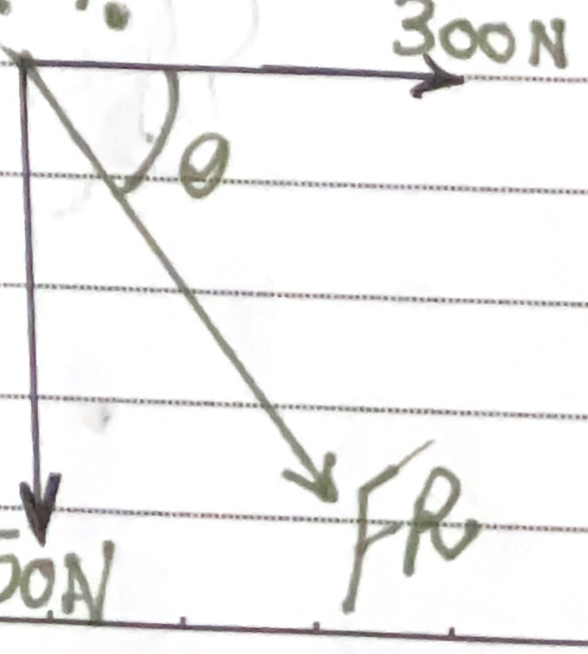
$= -350N$ or $= 350N\downarrow$

$F_R = \sqrt{300^2 + (-350)^2} = 461N$

$at\ \theta = \tan^{-1} 350/300 = 49.4°$

* the unit is more imp than the steps *
* the dr focus on them sm *

* this is longer pay attention to the details plz



No.

now we need to calculate the whole moment at point O.

بر هنو هرجهت الارتجاه

$$\curvearrowleft (+(MR)_{ato} = \Sigma M + M_C^*$$

$$= -750(1.25) + 500(4/5)(2.5) + (-500)(3/5)(1) + 200(1)$$

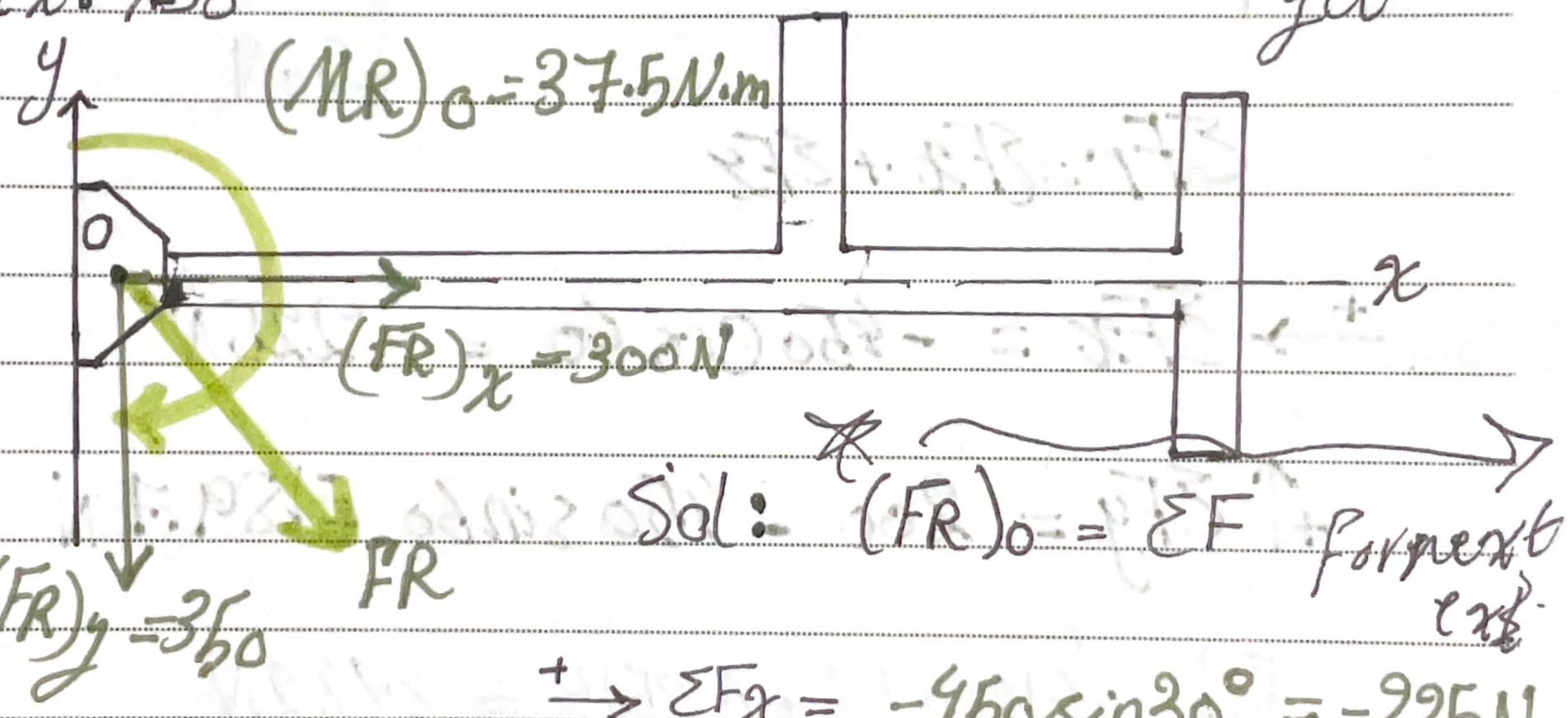
↳ the ⊥ distance dont forget

$$= -37.5\ N.m$$

$$= 37.5\ N.m \curvearrowright$$

Ex: 4-38

it is not answered yet



final sketch of the prev que

Sol: $(FR)_O = \Sigma F$ for next ex

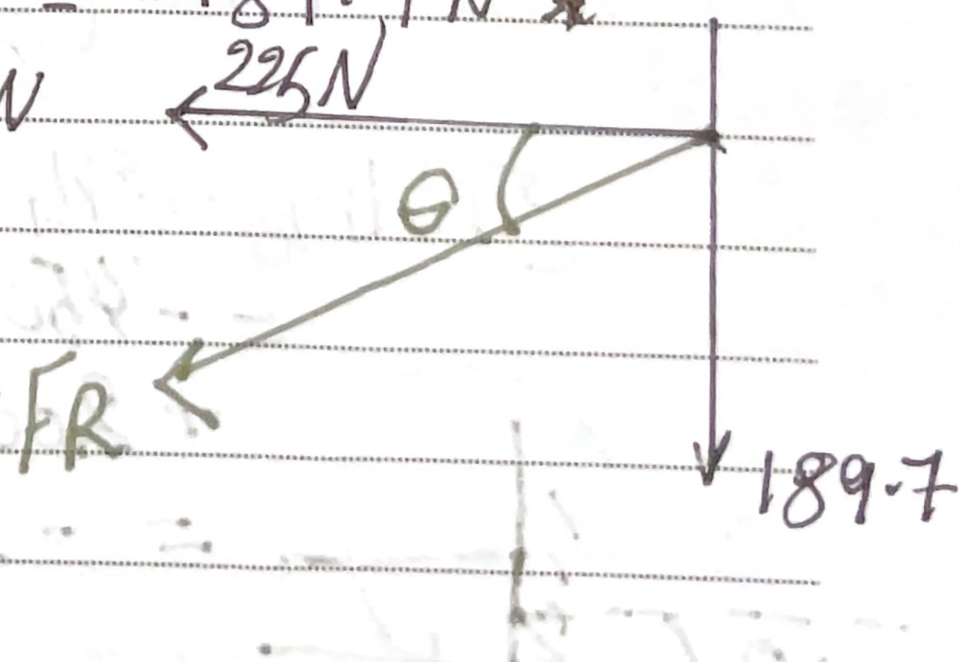
$$\stackrel{+}{\rightarrow} \Sigma F_x = -450\sin 30° = -225N$$

$$+\uparrow \Sigma F_y = -450\cos 30 = -389.7\ N + 200N$$

$$= -189.7N$$

$$FR = \sqrt{225^2 + 189.7^2} = 294.2N$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = 40.14°$$

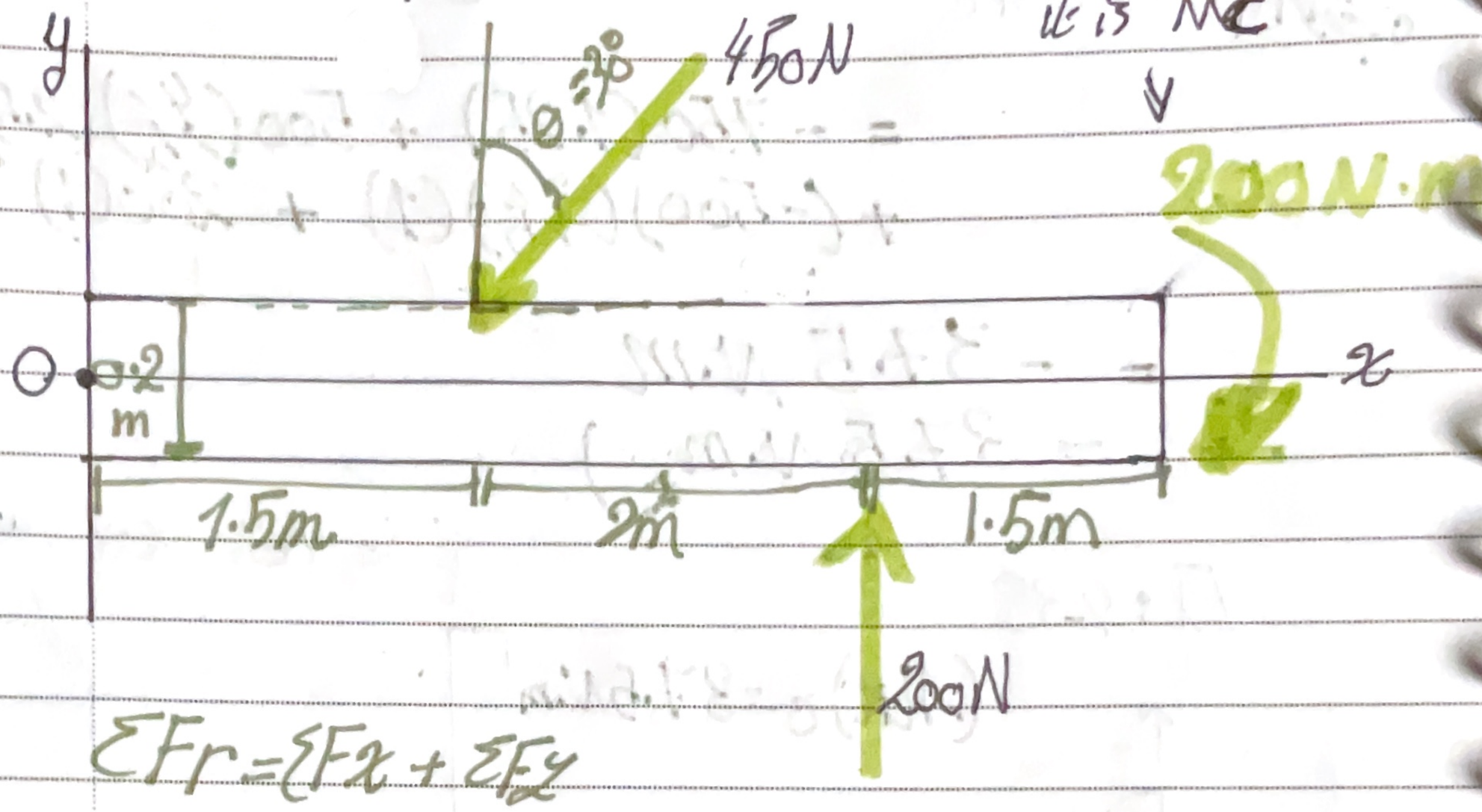


FIVE APPLE

No.

Ex: 4-101

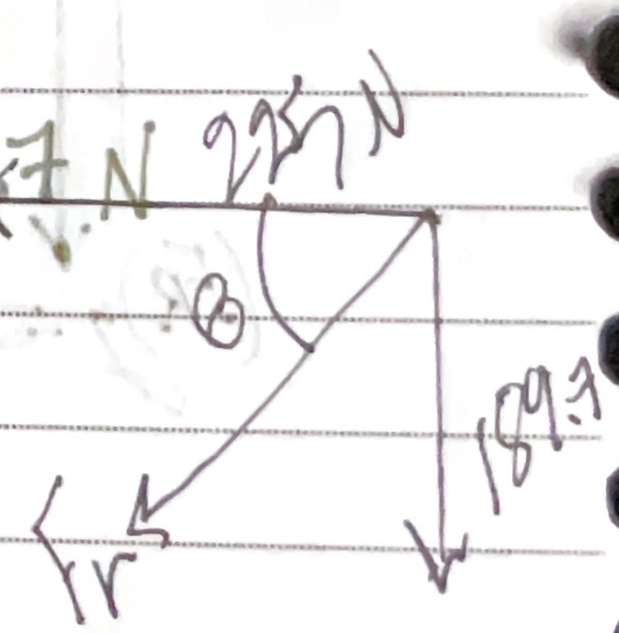
Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.



$\Sigma F_R = \Sigma F_x + \Sigma F_y$

$\xrightarrow{+} \Sigma F_x = -450\cos 60 = -225\,N$

$+\uparrow \Sigma F_y = 200 - 450\sin 60 = -189.7\,N$



$F_R = \sqrt{(-189.7)^2 + (-225)^2} = 294.2\,N$

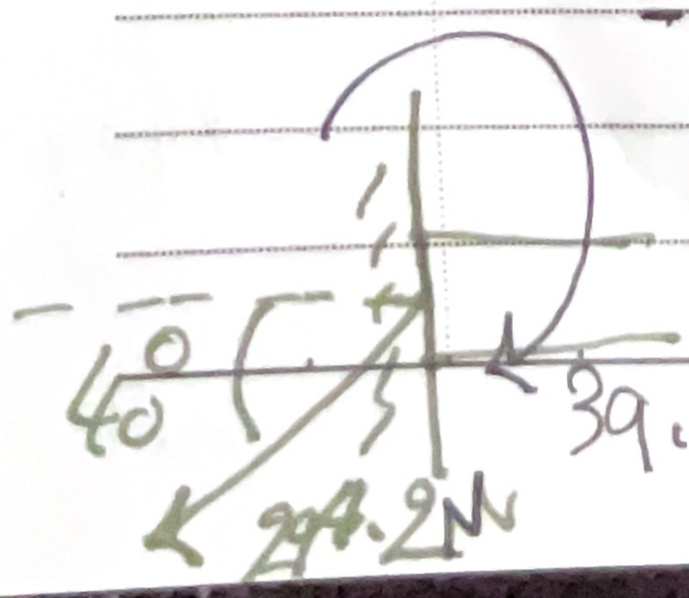
$\theta = \tan^{-1}\left(\frac{F_z}{F_x}\right) = 40.1°$

$\Sigma (M_r)_O = \Sigma M_r + M_C$

$= -450\cos 30(1.5) + 450\sin(30)(0.2)$

$+ 200(3.5) - 200$

$= -39.56\,N\cdot m$



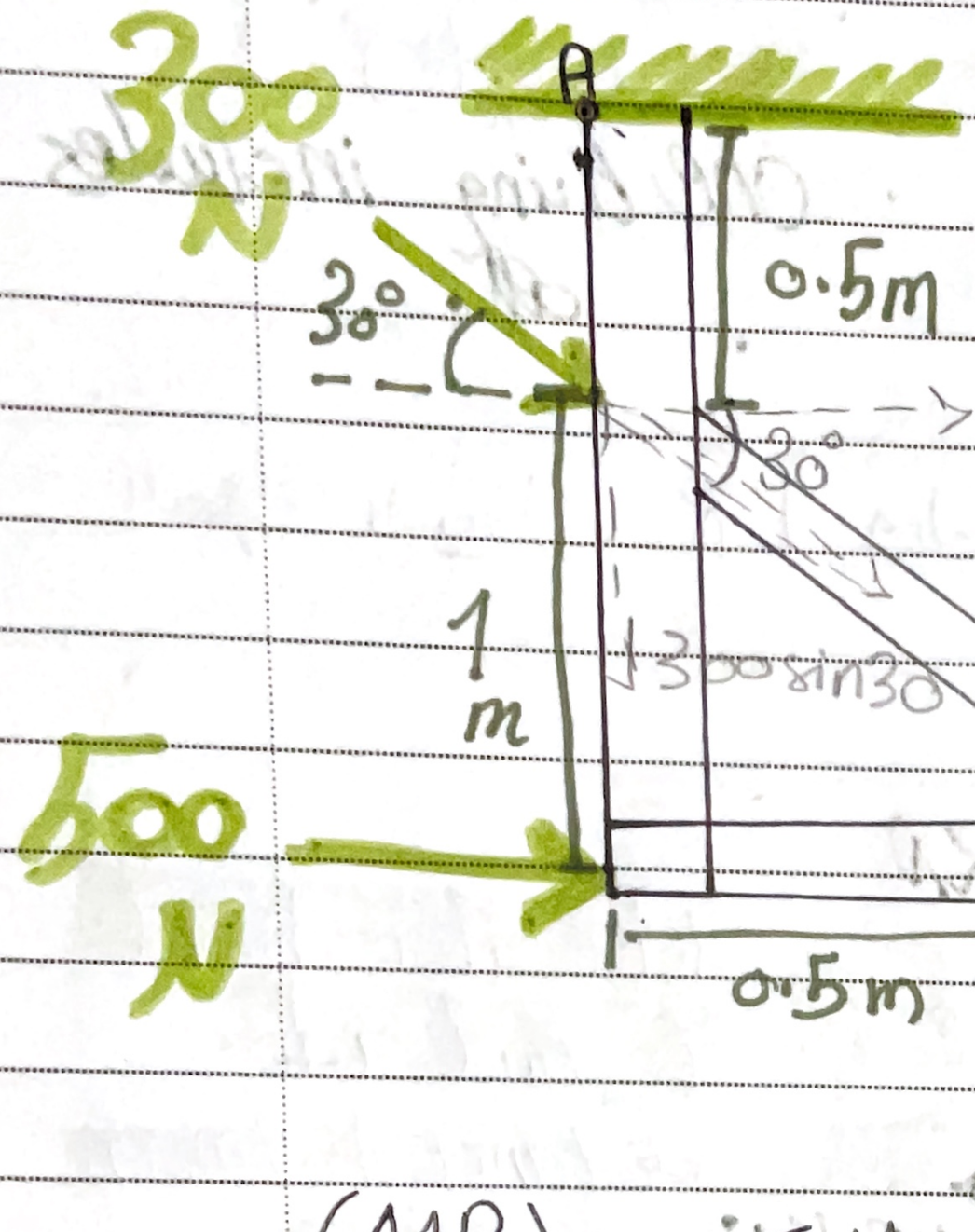
39.56 N.m Final sketch include M and ΣF_R.

FIVE APPLE

No.

Ex 4-105:-

Replace the force system acting on the frame by an equivalent resultant force and couple acting at point A.



Solu:

$(FR)_A = \Sigma F$

$+\rightarrow \Sigma Fx = +300\cos 30 + 500$

$\Sigma Fx = 759.8\,N$

$+\uparrow \Sigma Fy = +300\sin 30 - 400$

$\Sigma Fy = -550\,N$

$FR = \sqrt{(\Sigma Fx) + (\Sigma Fy)}$

$FR = \sqrt{(759.8)^2 + (-550)^2}$

$FR = 937.9\,N$

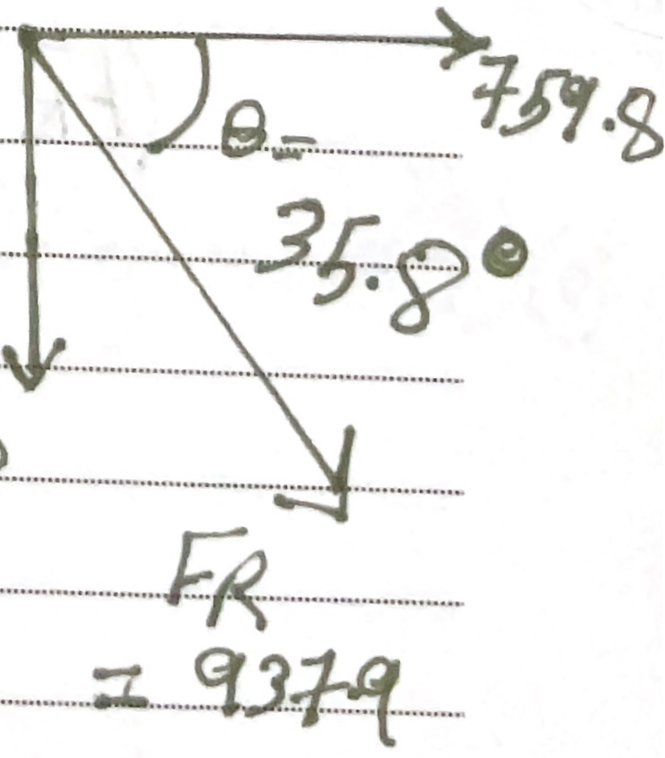
$\theta = \tan^{-1}\left(\frac{|\Sigma Fy|}{|\Sigma Fx|}\right)$

$\theta = 35.8^\circ$

$(MR)_o = \Sigma M$

$(MR)_A = +300\cos 30\,(0.5) + 500\,(1.5) - (300\sin 30)(0) - 400\,(0.5$

$(\Sigma MR)_A = 680\,N.m$



* we dont have L distance on component y (bc he didn't tell my anything abt if it has a thickness in the que so I include it or smth.)

No.

Sec 4.8

Further simplification (one for all)

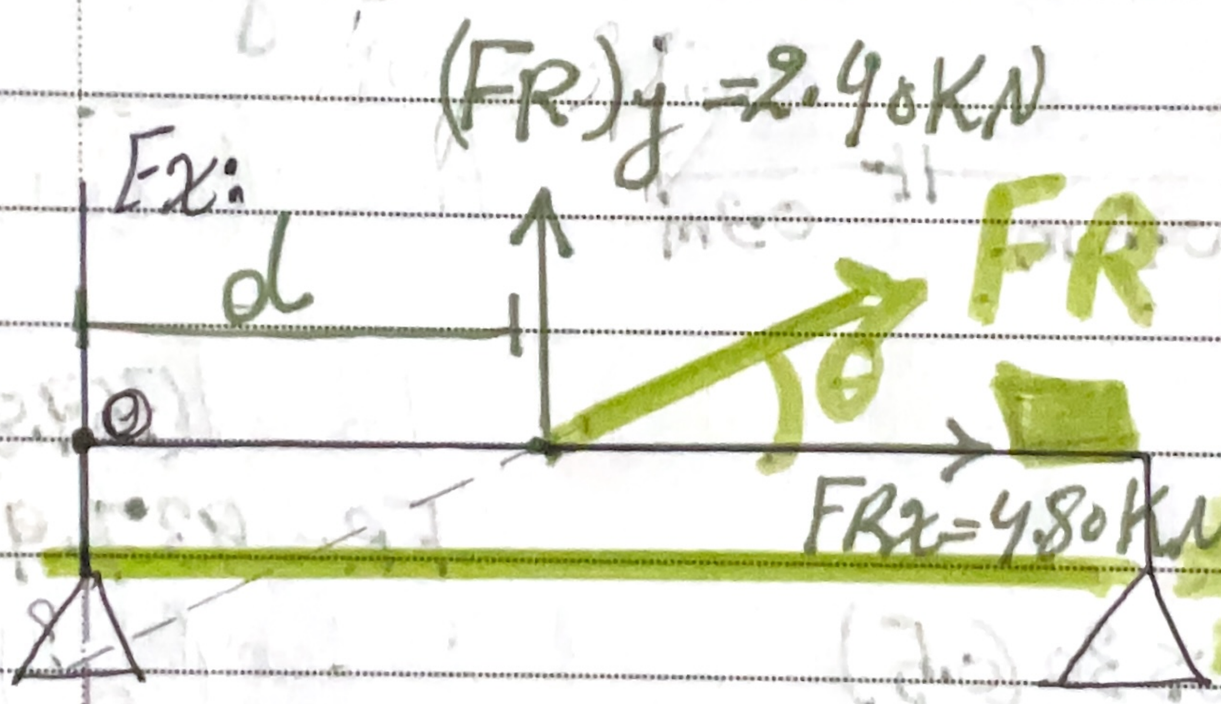
$\Sigma M_R = FR\,d$

$d = \frac{\Sigma M_R}{FR}$

Mcouple والجهة اليمين FR والجهة اليسار

و ΣM_R العادي.

$(FR)_y = 2.90\,KN$



solu:

$(\Sigma M_R)_O = \Sigma M_O + \Sigma M_C$

$(FR)(d) = (F_{Ry}^{*})(d)$

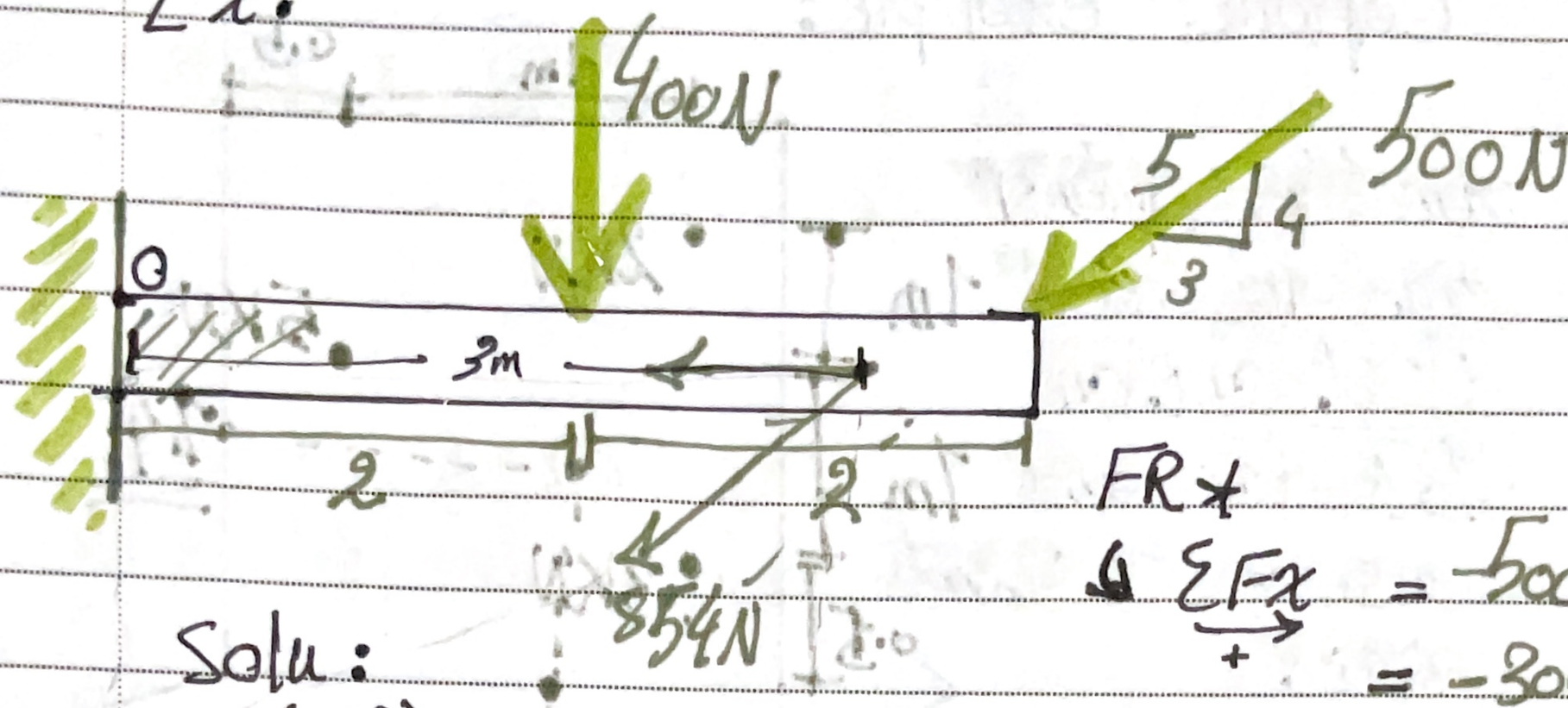
bc FRx Ldistane = zero

* it's like we only have one FR includ the couple, (means we cancel out the couple)
* one thing includes oth.

* bc it is havel to find a l distance between FR and O, we break FR into 2 componant * (and they won't be equal bc of θ).

No.

Ex:



Solu:

$(MR)_O = \Sigma M$

$(F_R)_d = \Sigma M$

$F_R = \sqrt{(F_x)^2 + (F_y)^2}$

$F_R = \sqrt{300^2 + 800^2}$

$F_R = 854.4N$

$\theta = \tan^{-1}\left(\frac{800}{300}\right) = 69.4^\circ$

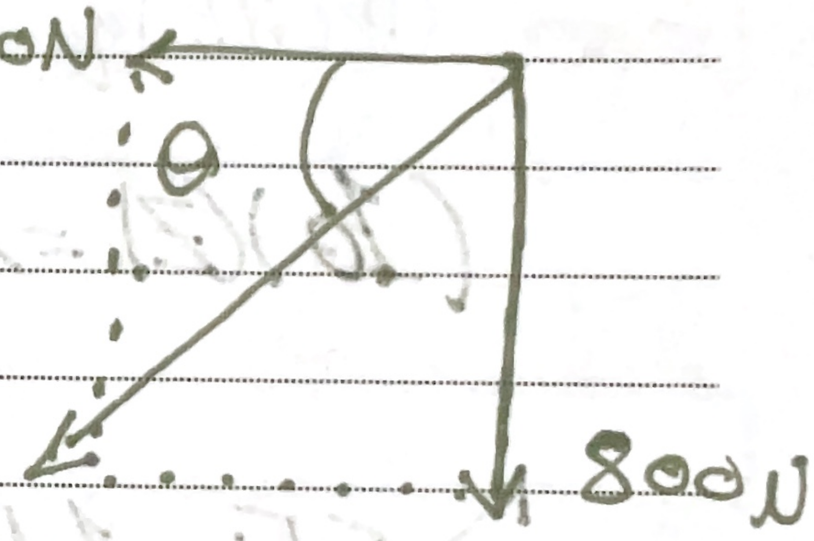
$FR \rightarrow$

$\downarrow \Sigma F_x = -500(3/5)$

$= -300N$

$+\uparrow \Sigma F_y = -400 - 500(4/5)$

$= -800N$



we choose $F_{Ry} = 800$ bc it has 1 distance with origin as F_{Ry} & distance with O = zero

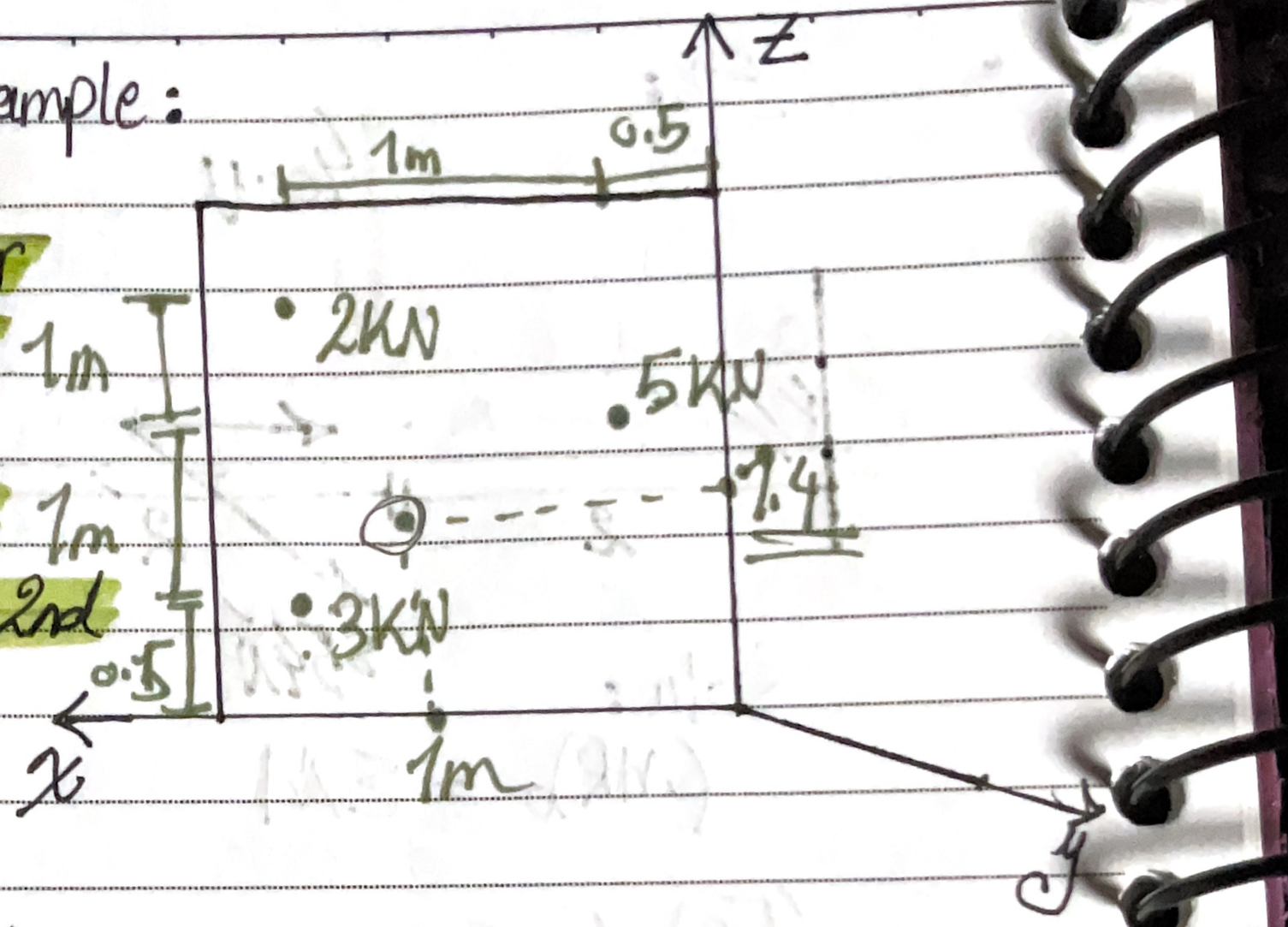
$+\circlearrowleft (800)(d) = -400(2) - 500(4/5)(4) - 500(3/5)(0)$

$d = 3m$

No.

Coplaner example:

*in the coplaner que we need 2d one on 1st axis and one on the 2nd axis.



solu:

① $(MR)_x = M_x$

$(-10)(Z) = 2(2.5) + 5(1.5)$

$Z = 1.4m$

+↑ $\Sigma F_y = -10kN$

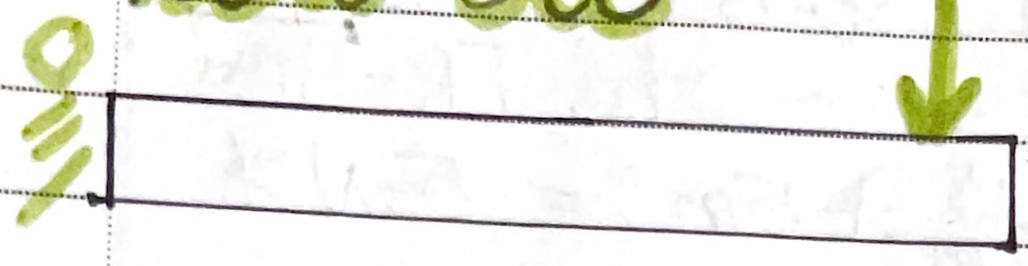
+↻ $(MR)_z = M_z$

$(-10)(x) = -5(0.5) - 2(1.5) - 3(1.5)$

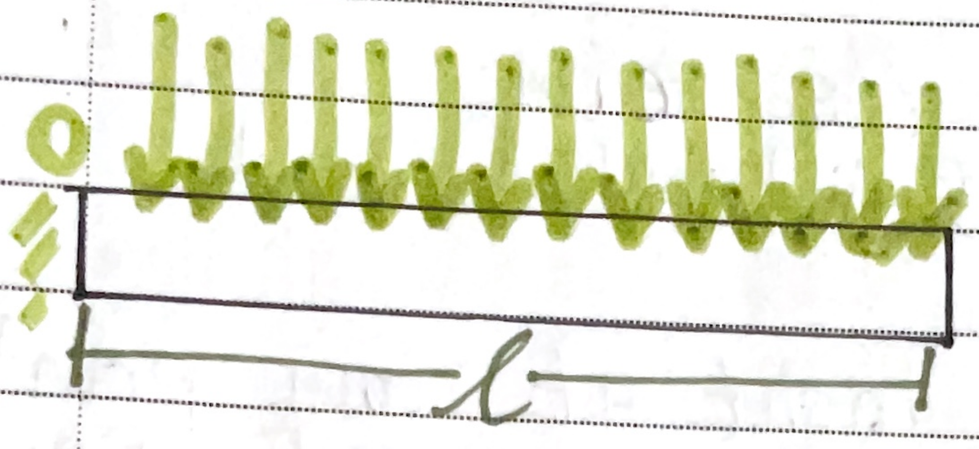
$x = 1m.$

No.

new Sec

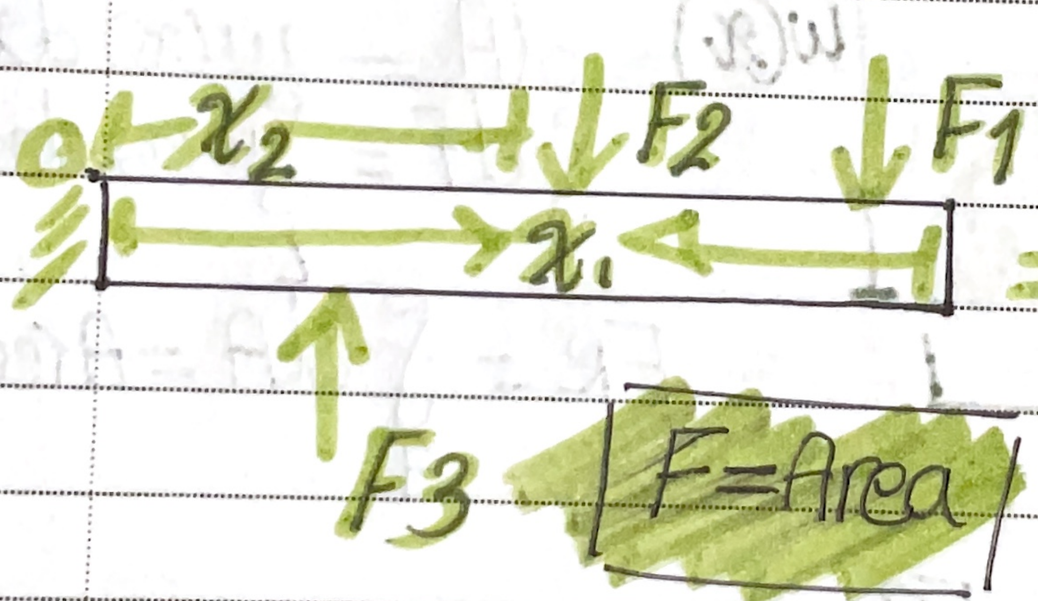


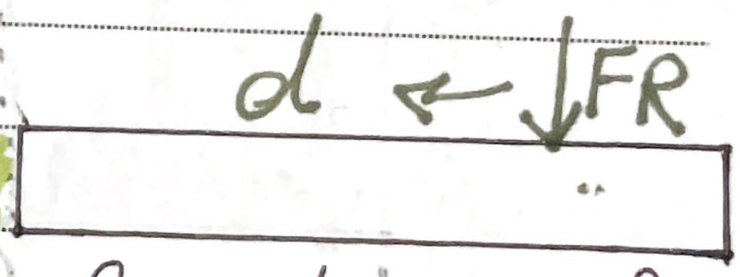
F: concentrated force
"point load"
we used to study this



F: "Distributed load": N/m
القوة موزعة على كامل الجسم
(كامل طوله)

$w(x)$ → the length (a function of the length)
$0 < x \leq l$





* if we have infinite number of loads, we can not simply add them up, that why we make integration

$w(x)$: N/m

Distributed load: wind, Rain, snow ... etc.
we usually use one axis: (x axis).

No.

Smth ex:

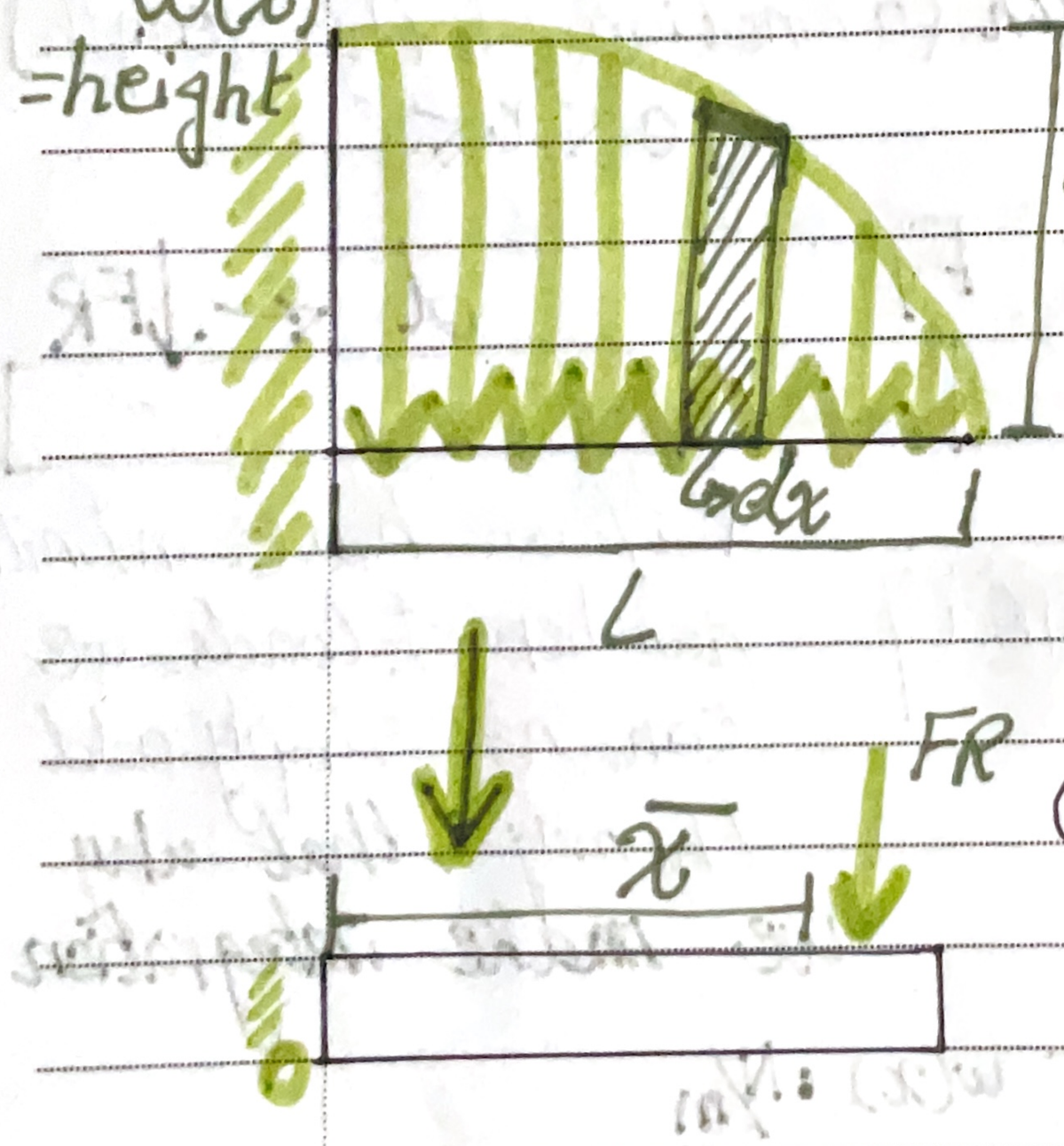
$w(x) = 10\ N/m$, $x = 5$ — الطول بالمتر

$\downarrow FR = w \cdot x$ ← بـ bc $FR = Area$

$\downarrow FR = 10\ N/m * 5m = 50N$ *

* we look after the area *
in the case of distributed load.

$w(x)$ is the height at any point
so we call it a fun of x.



$dA = w(x)\,dx = dF$

$\int dF = \int w(x)\,dx$

$FR = \int_A dA = Area$

$\curvearrowleft (M_R)_O = \Sigma M_O$

$(-FR)(\bar{x}) = -\int (w(x)\,dx)(x)$

note: $\bar{x}$ قيمة ثابتة

distance قيمة متغيرة x و هي

between the point I want

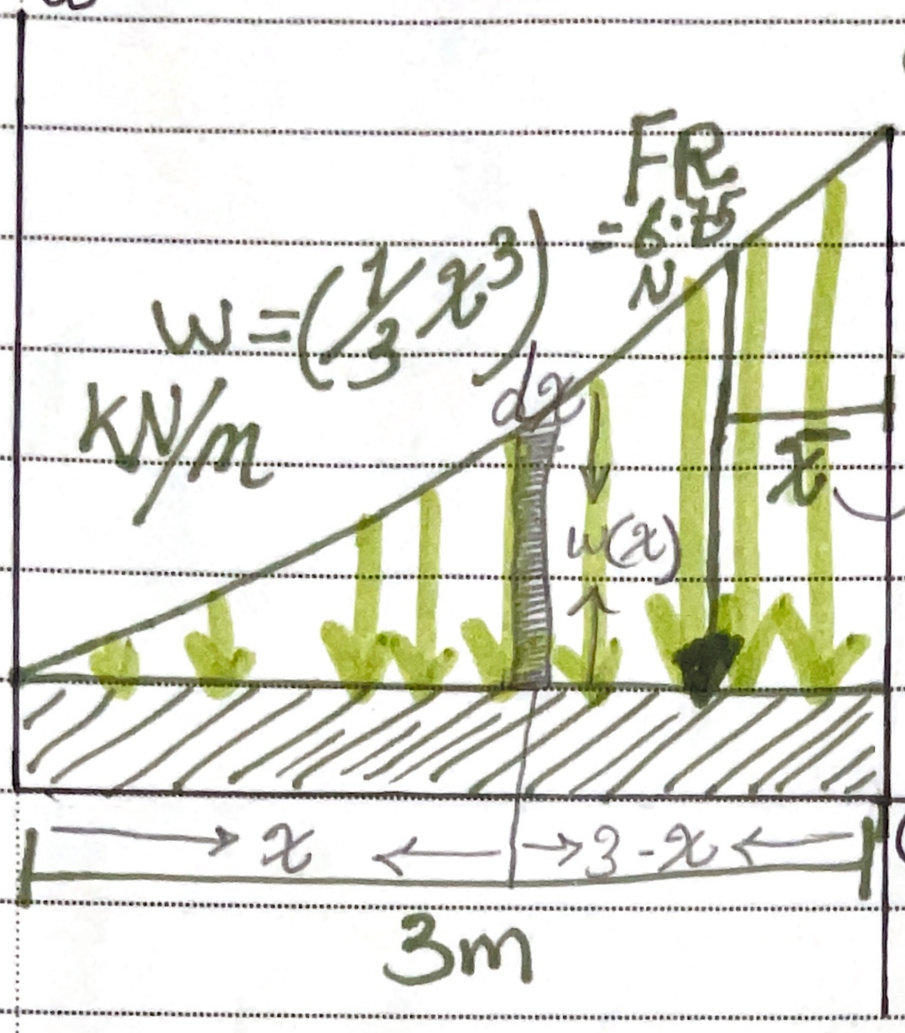
to calculate the area

to point O.

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} \quad \therefore \frac{\int w(x)\,dx\,(x)}{FR}$$

No.

ex:-



Solu:

$$+\downarrow FR = \int_0^3 w(x)\,dx$$

$$= \int_0^3 \left(\frac{1}{3}x^3\right) dx$$

$$= \frac{1}{12} x^4 \Big|_0^3$$

$$FR = 6.75\,N$$

$$(+\circlearrowleft)(M_R)_O = \Sigma M_O$$

$$dA = dF$$

$$(6.75)(\bar{x}) = \int_0^3 w(x)\,dx\,(3-x)$$

$$\bar{x} = \int_0^3 \frac{\left(\frac{1}{3}\right)(x^3)(3-x)\,dx}{6.75}$$

$$\bar{x} = 0.6m \text{ From } O$$

$$M_O = (FR)\bar{x}$$

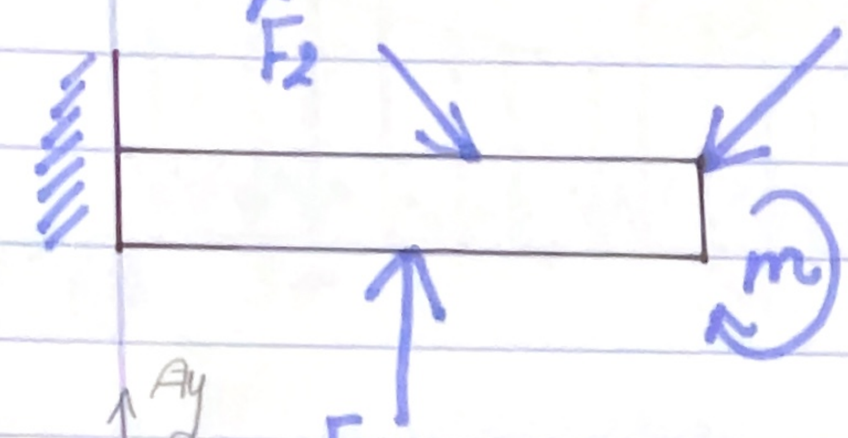
$$= (6.75)(0.6) = 4.05\,KN.m$$

$(MR)_O \rightarrow$ From it we find the direction.

FIVE APPLE

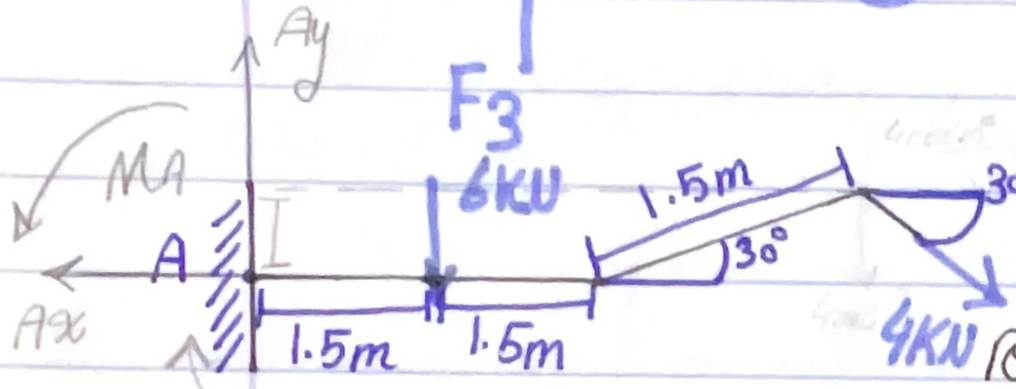
Ch.5 Equilibrium of a Rigid body

Equilibrium means no acceleration, $a=0$. Why? bc of supports.



$\Sigma F=0, \Sigma M=0$ Supports: ① Reactionary force ② Moment

*take a look at the types of supports that we r goin to study. (only: 1,3,4,8,10 are included).



Ex: 5-10 Determine the components of the support-reactions at the fixed support A on the cantileveral beam.

1] The First ever step is to draw the Free body diagram. (FBD)

2] Apply the equilibrium equations.

Solu:-

$\pm\rightarrow \Sigma F_x=0$ ①

$4\cos 30 - A_x = 0$

$A_x = 3.46\ KN$

$+\uparrow \Sigma F_y=0$ ②

$-4\sin 30 - 6 + A_y = 0$

$A_y = 8\ kN$

$\circlearrowleft + \Sigma M_A = 0$ ③

$M_A - 6(1.5) - 4\sin 30^\circ(3+1.5\cos 30^\circ) - 4\cos 30^\circ(1.5\sin 30^\circ) = Zero$

$M_A = 20.2\ KN.m$ → bc the L distance is only this I (we don't include the 1.5+1.5 with it).

All prob solu must include a FBD.

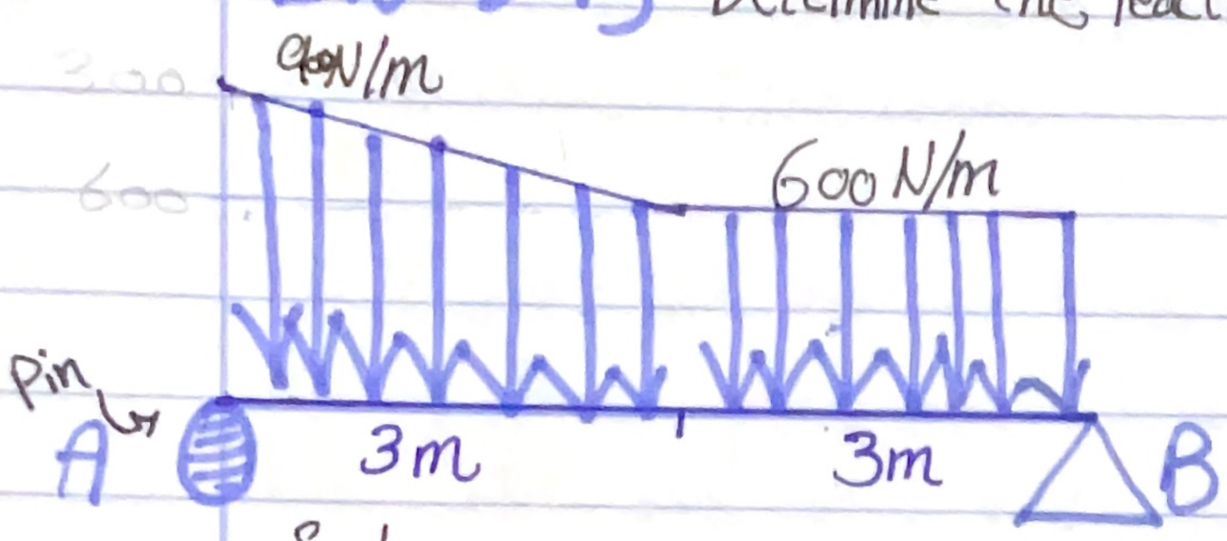
طالما طلع جوابك بالفرض $\pm\rightarrow +\uparrow \circlearrowleft +$ إذاً الفرض 100% صح.

Remember:

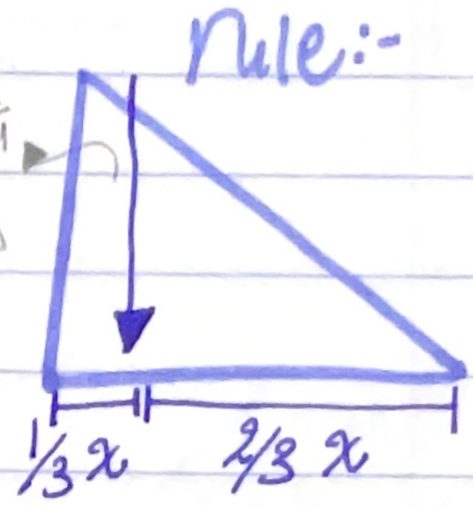
Roller prevent movement on y, so we replace it with reactionary Fy.

Pin prevent movement on x, so we replace it with reactionary Fx.

Ex: 5-13 Determine the reactions at the supports.

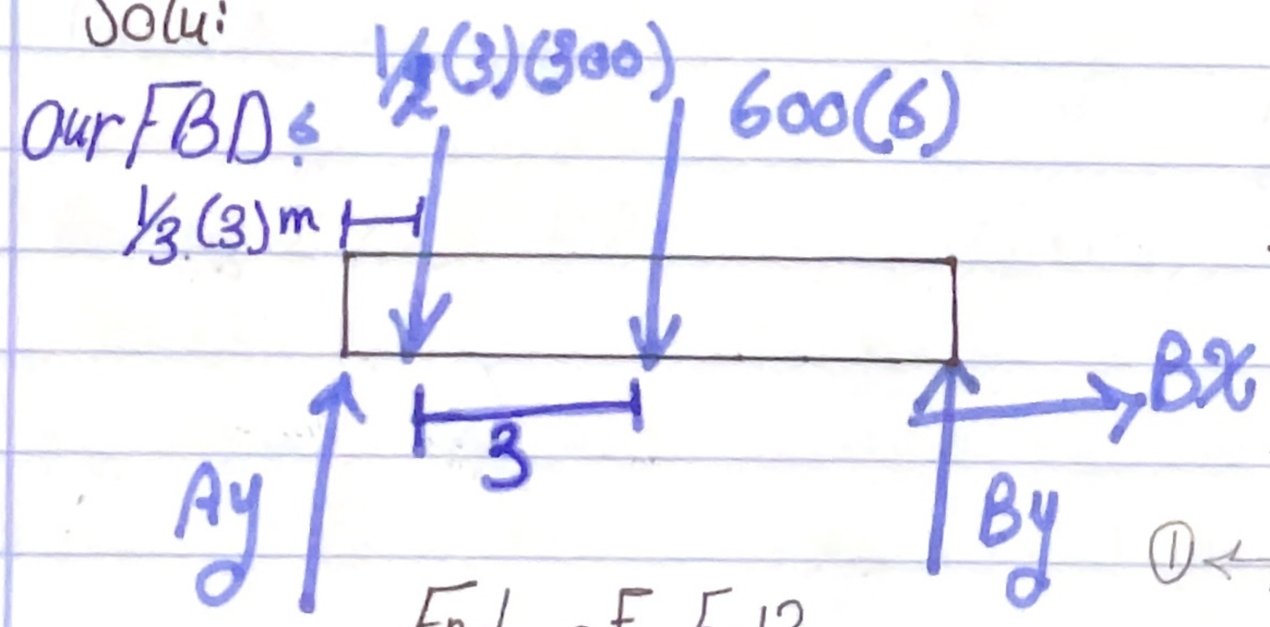


Rule:-



Solu:

Our FBD:



$\xrightarrow{+} \Sigma Fx = 0$

$Bx = 0$ ★

$+\uparrow \Sigma Fy = 0$

① $\leftarrow$ [$Ay + By - 600(6) - \frac{1}{2}(3)(300) = 0$

↳ Two variables one equation (we need other equation) So:

$\circlearrowleft^{+} \Sigma MB = 0$

$-Ay(6) + \frac{1}{2}(3)(300)(5) + 600(6)(3) + (By)(0) = 0$

$Ay = 2175\ N$ → plug it in eq①

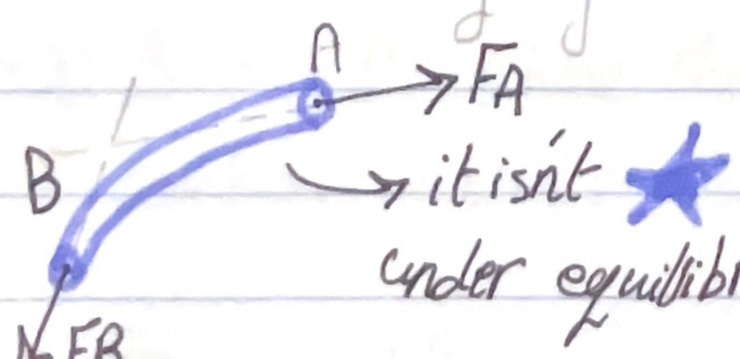
$By = 1875\ N$.

End of 5-13.

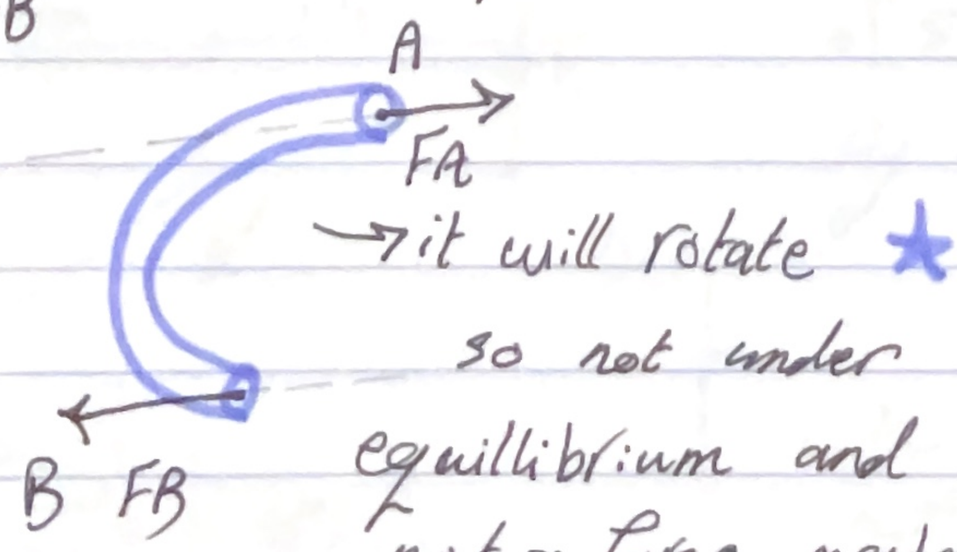
5.4 Two and three force members.

★ we will only study Two force members

we r going to simplify it



→ it isn't ★ under equilibrium



→ it will rotate ★ so not under equillibrium and not a force members.

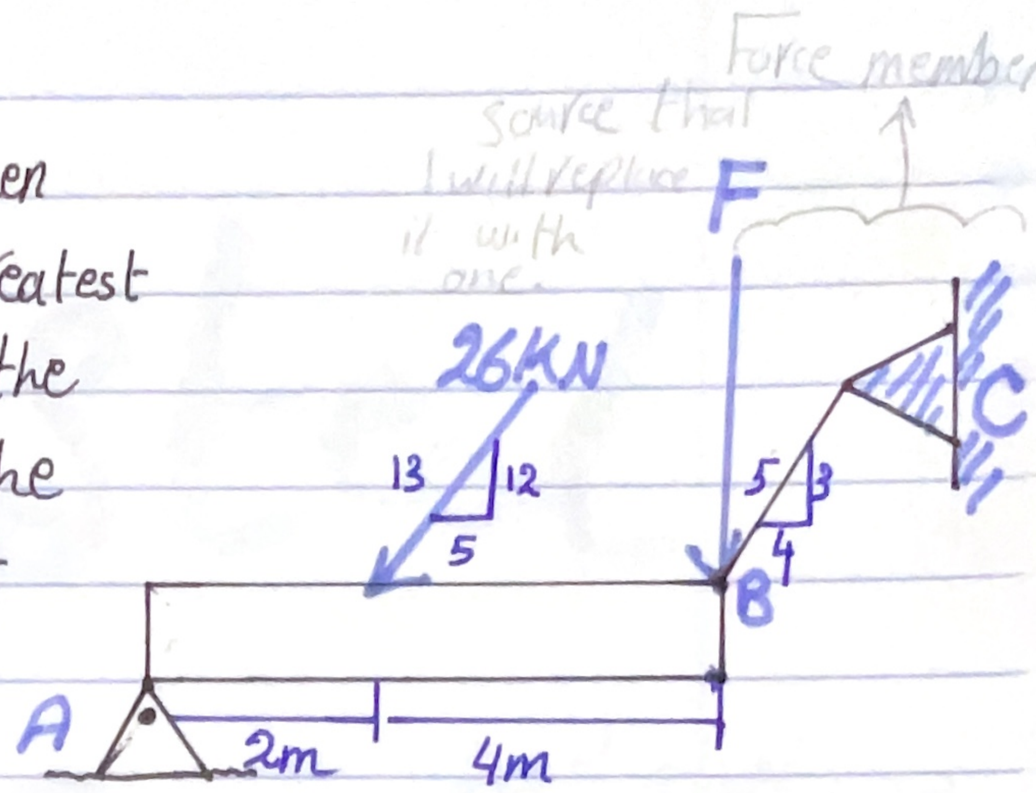
The only case of force m



→ same line of action.

we will replace em with 1 force member.

Ex 5-48: If rope BC will fail when tension becomes 50kN, determine the greatest vertical load F that can be applied to the beam at B. what is the magnitude of the reaction at A for this loading? Neglect the thickness of the beam.



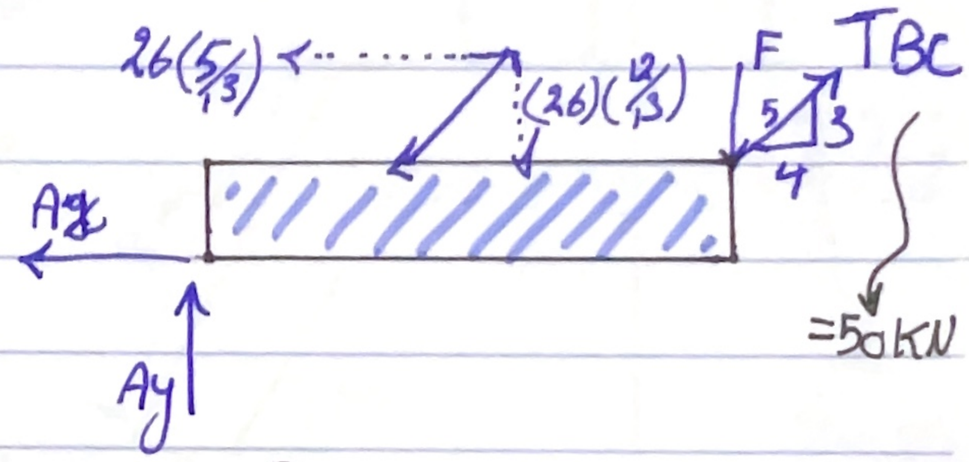
Idon't care abt tensive force coming from the wall I just care abt the one that goes towards the wall (this could change according to the ex).

How can I find F? بج اشتغل بذكاء واختار نقطة يسقط عندها اجر القوة بأي طريقة

o = distance ⊥ كانت (بالعادة بختار نقطة بتكون عشان استغيد ويطلع عندي متغير ① واقدر احل المعادلة بمجهول).

Solu:

First we need our FBD



$\circlearrowleft^{+}\Sigma M_A = 0$

$0 = +26(5/3)(0) - 26(12/13)(2) - F(6) + 50(3/5)(6).$

$\boxed{F = 22\,kN}$

$+\uparrow \Sigma F_y = \text{Zero}$

$-26(12/13) - 22 + 50(3/5) + A_y = 0$

$\boxed{A_y = 16\,kN}$

$\xrightarrow{+} \Sigma F_x = 0$

$-26(5/13) - A_x + 50(4/5) = 0 \rightarrow \boxed{A_x = 30\,kN}$

Solu: $\circlearrowleft^{+}\Sigma M_A = 0$

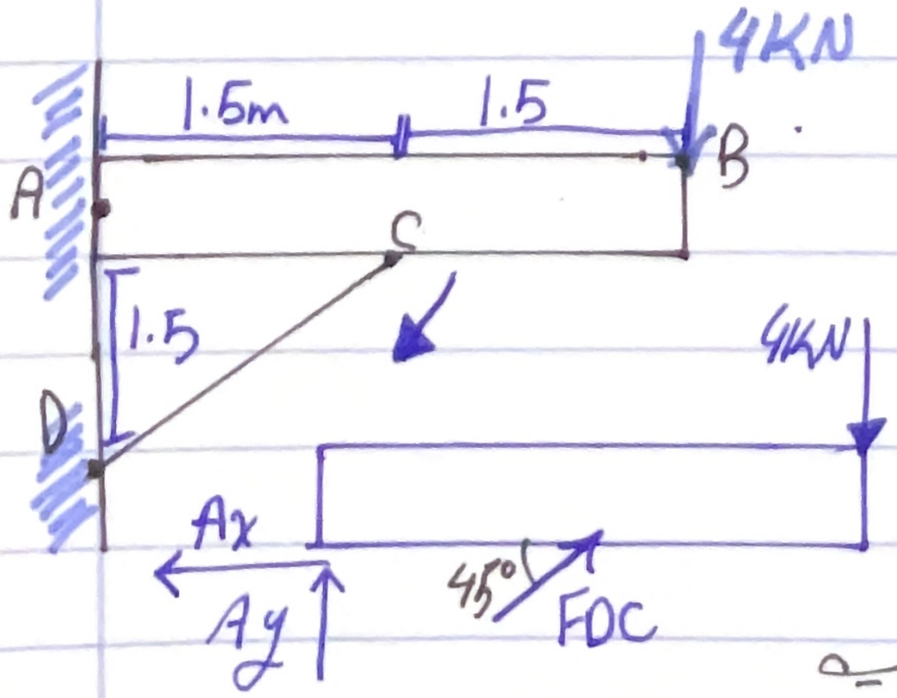
$0 = -4(3) + F_{CD}(\sin 45^\circ)(1.5$

$\boxed{F_{DC} = 11.31\,kN}$

$\xrightarrow{+} \Sigma F_x = 0$

$-A_x + 11.31(\cos 45^\circ) = 0, \boxed{A_x = 8\,kN}$

Ex 5-2 Determine the horizontal and vertical components of reaction at the pin A and the reaction on the beam at C.



$+\uparrow \Sigma F_y = \text{Zero}$

$A_y + 11.31(\sin 45^\circ) - 4 = 0$

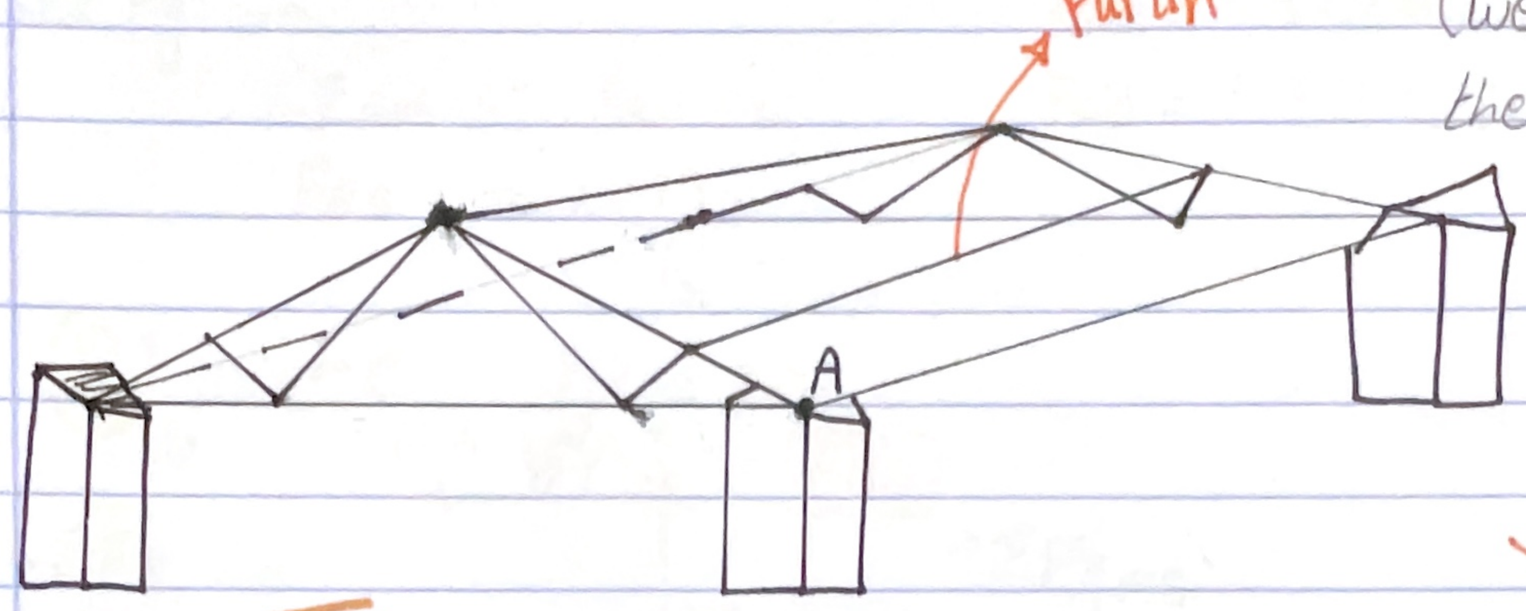
$\boxed{A_y = -4\,kN = 4\,kN\downarrow}$

طلع سالب لذلك فرضي خطأ و بين بسوية هيك (بعكس الإتجاه اللي فرضته).

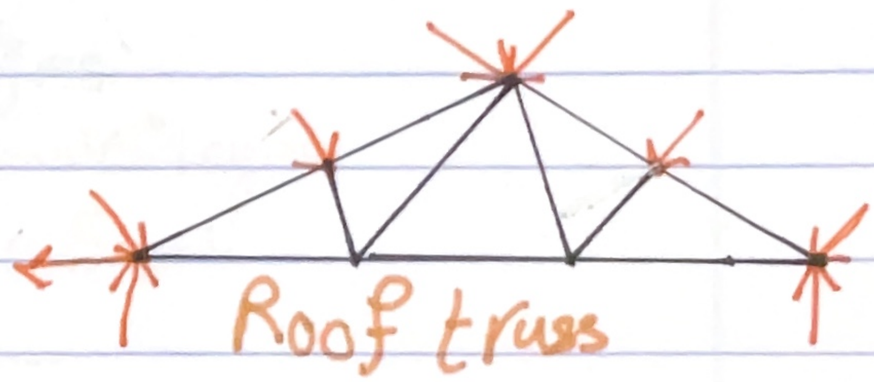
Ch.6

Structural Analysis

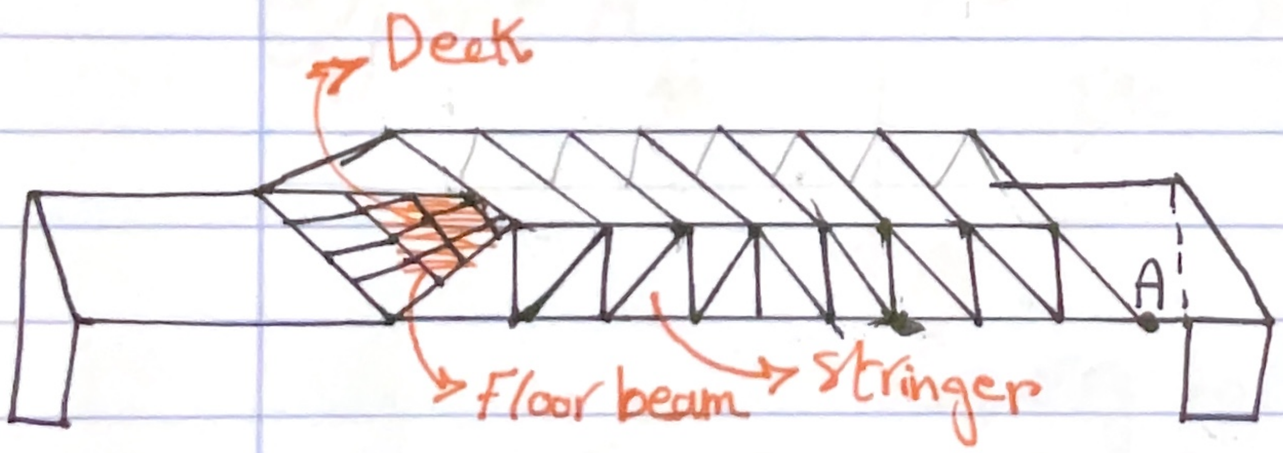
we r going to focus on trusses ~~and the bridges~~ bc they don't rotate.
(we don't calculate the moment rn.)



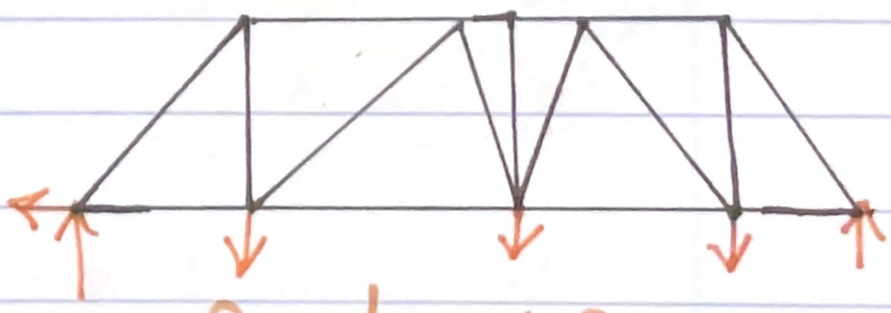
Truss



Roof truss



bridge



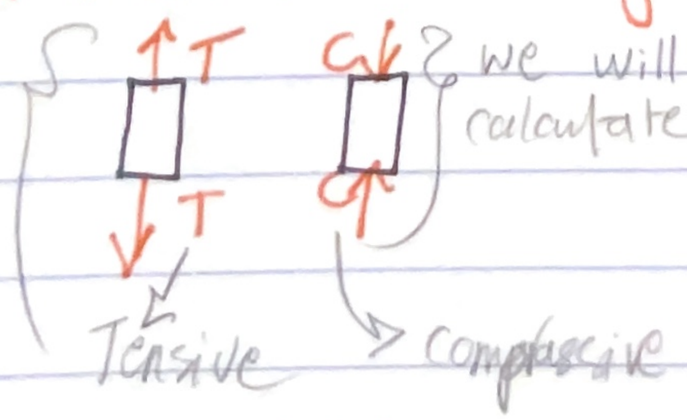
Bridg truss

* رح نحلل لقوة ↕

*All loading are on joints

* the members are joind all together in a smooth way (no friction)

we will calculate only one

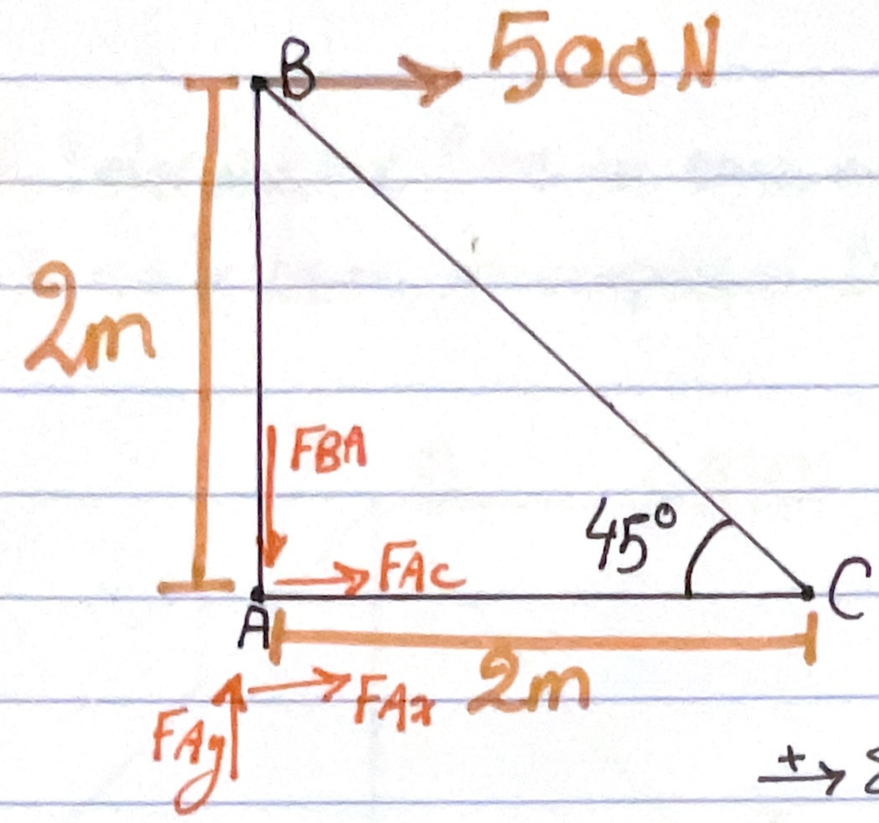


we will calculate only one

members hold a force on it.

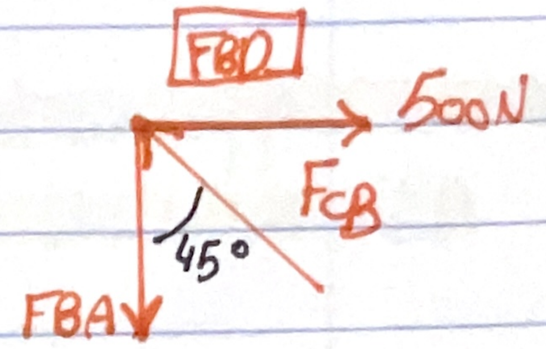
* لازم عدد المجاهيل ما يتعدى ال 2
لأنو عندي بس معادلتين ($\Sigma F_y=0$, $\Sigma F_x=0$)

* لا تنسى العزوم حافي دوران وحافي ΣM.



we r smart so we start ant From joint B. (bc I have 2var. 1known).

① joint B



$\pm\rightarrow \Sigma F_x = 0$

$500 - F_{CB}\sin 45° = 0$

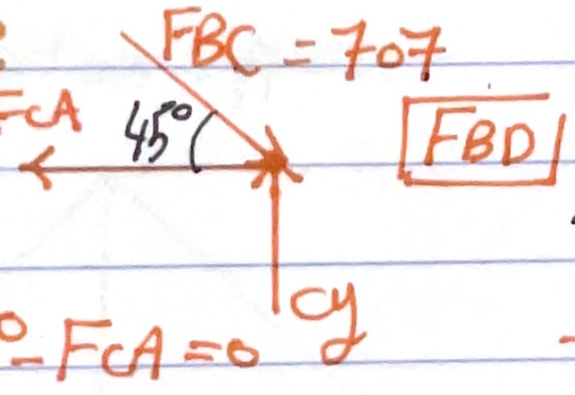
$F_{CB} = 707\,N\ (C)$ ↳ compressive force

$+\uparrow \Sigma F_y = 0$

$707\cos 45° - F_{BA} = 0$

$F_{BA} = 500\,N\ (T)$ ↳ Tensive force

② joint C



$\pm\rightarrow \Sigma F_x = 0$

$707\cos 45° - F_{CA} = 0$

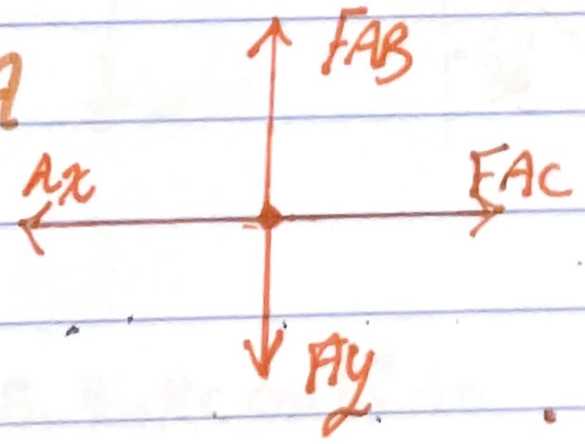
$F_{CA} = 500\,N\ (T)$

$\uparrow \Sigma F_y = 0$

$-707\sin 45° + c_y = 0$

$c_y = 500\,N$

③ joint A



FBD

$\pm\rightarrow \Sigma F_x = 0$

$-A_x + F_{AC} = 0$ $\quad F_{AC} = -500\,N$

$A_x = 500\,N$ ← عكس العزم

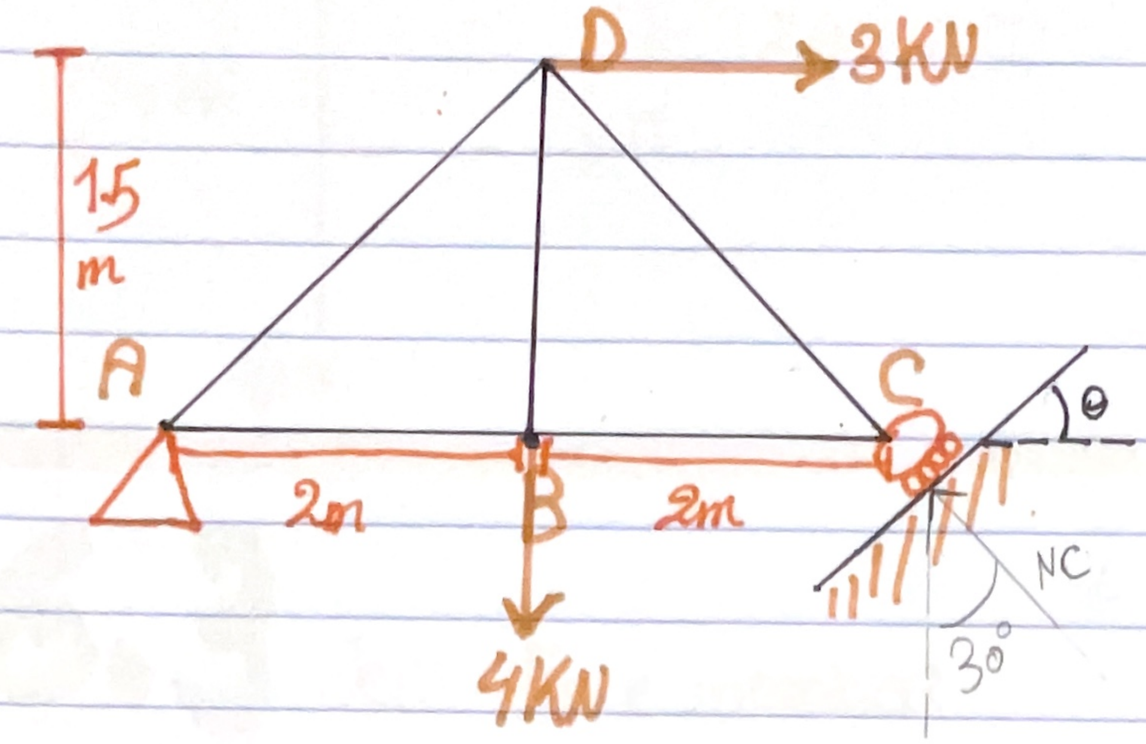
$A_x = 500\,N \rightarrow$

$+\uparrow \Sigma F_y = 0$

$F_{AB} = 500\,N$ $\quad F_{AB} - A_y = 0 \rightarrow A_y = -500\,N$

$A_y = 500\,N\ \uparrow$

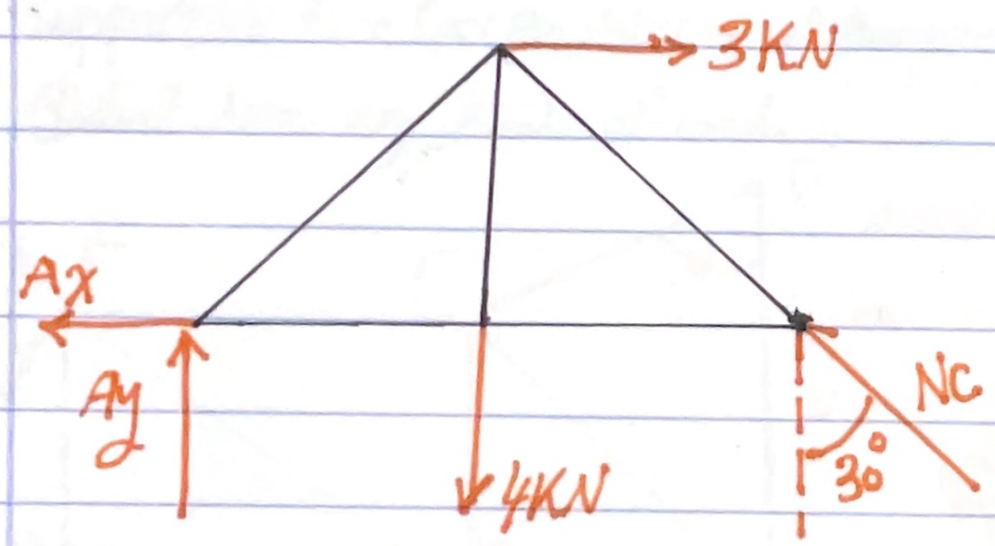
prob 6-6. Determine the force in each member of the truss, and state if the members are in tesion or compression. Set $\theta = 30°$



★ To find the support reactions: Draw the whole FBD.

Don't be solid be practically smartness, the point is in the normal case we have alot of unknowen Variabels that we can't find from ch6 rules. So I went back to find support reactions and place $\Sigma M = 0$ at a smart point so we can continoue easly.

★ just reactions supports!



$\overset{+}{\circlearrowleft} \Sigma M_N = 0$

$-Ay(4) + 4(2) - 3(1.5) = Zero$

$Ay = 0.875 kN.$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$-Ax + 3 - NC\sin 30° = 0] \rightarrow ①$

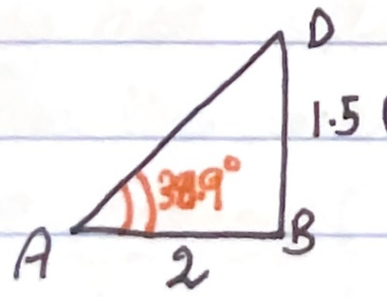
$NC = 3.6 KN$

$Ax = 1.2 KN.$

$+\uparrow \Sigma F_y = 0$

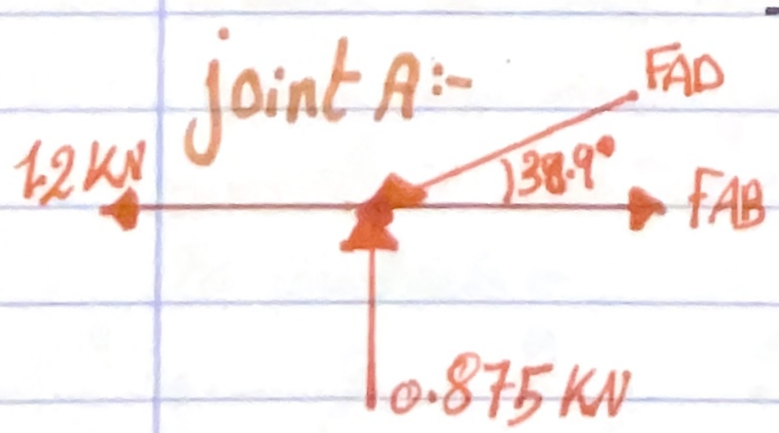
$0.875 - 4 + NC\cos 30° = 0$

$NC = 3.6 KN.$] → back to ①



$\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.9°$

joint A:-



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$FAB - FAD\cos 36.9° - 1.2 = Zero \rightarrow ①$

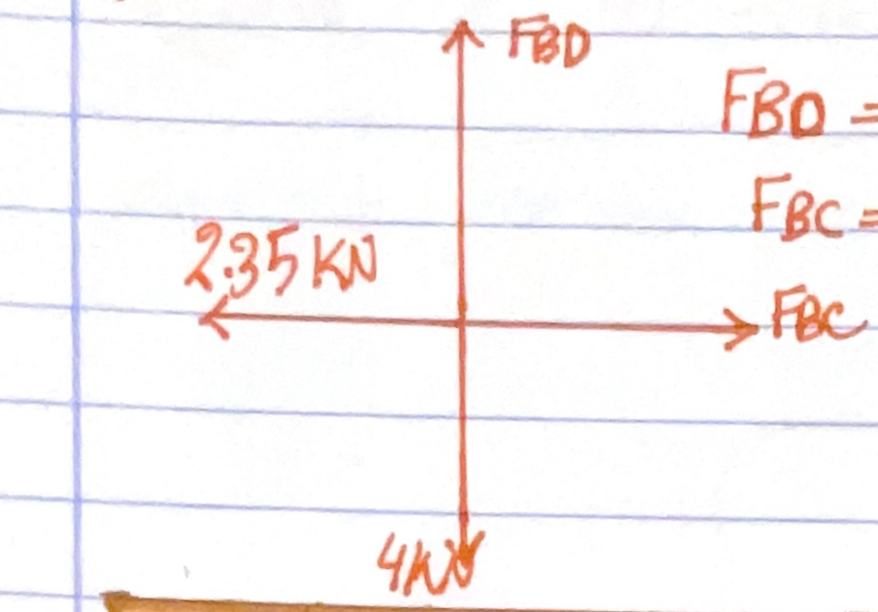
$FAB = 2.35 KN (T)$ ← plug in eqn ①

$+\uparrow \Sigma F_y = 0$

$FAD\sin 36.9° + 0.875 = 0$

$FAD = -1.45 kN = 1.45 KN (C)$

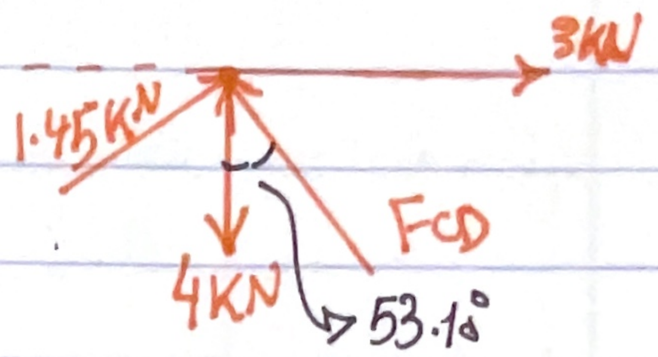
joint B:



$F_{BD} = 4\,KN(T)$

$F_{BC} = 2.35\,KN(T)$

joint D:



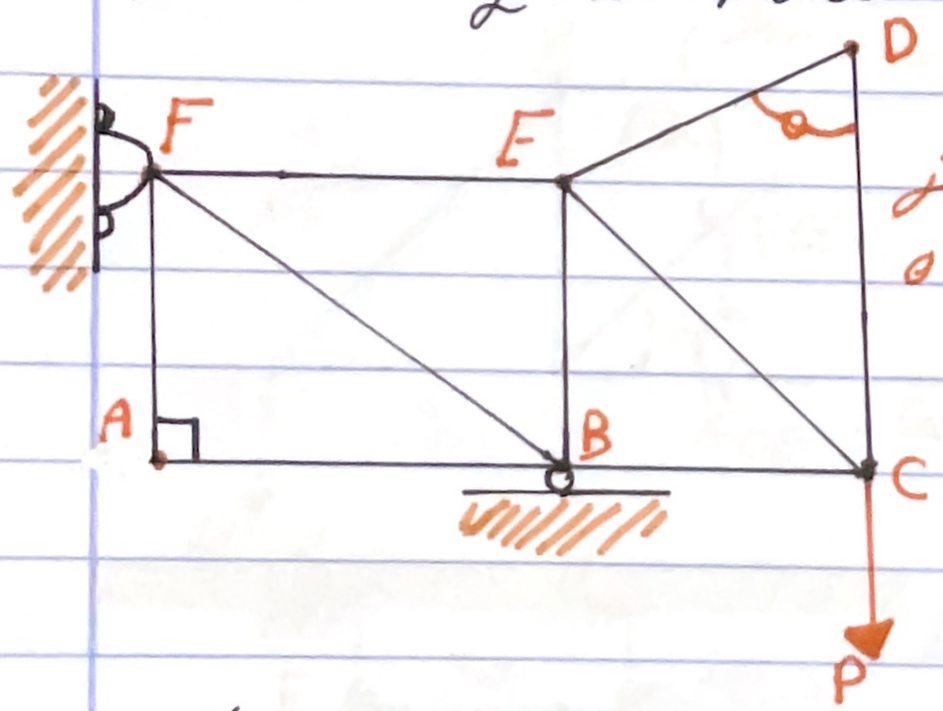
$\xrightarrow{+} \Sigma F_x = 0$

$3 + 1.45\cos 36.9° - F_{CD}\sin 53.1° = 0$

$F_{CD} = 2.25\,kN\,(C)$

6.3 Zero - Force members

we consider a member as a Zero-force member when it ① not a supportive force (not pin, roller, ... not hanging on anything). ② not and don't have an ex force ③ don't have any kind of load.



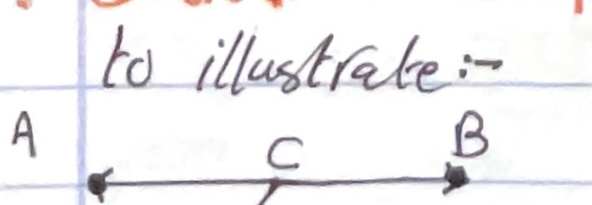
member mean two joints, we start from the joint that suceed the pre notes to the other point.

DE, DC, AF, AB } Zero - Force members

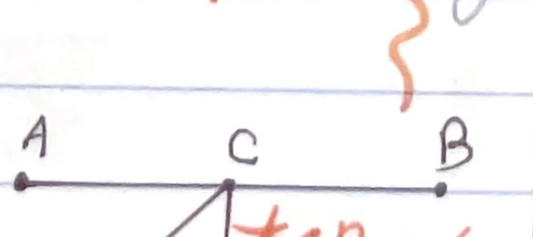
another note :-

When there is 2 members going out from the same joint and they are collinear (نقطتين على خط واحد): there should be only one member going from that joint to consider it Zero Force member:

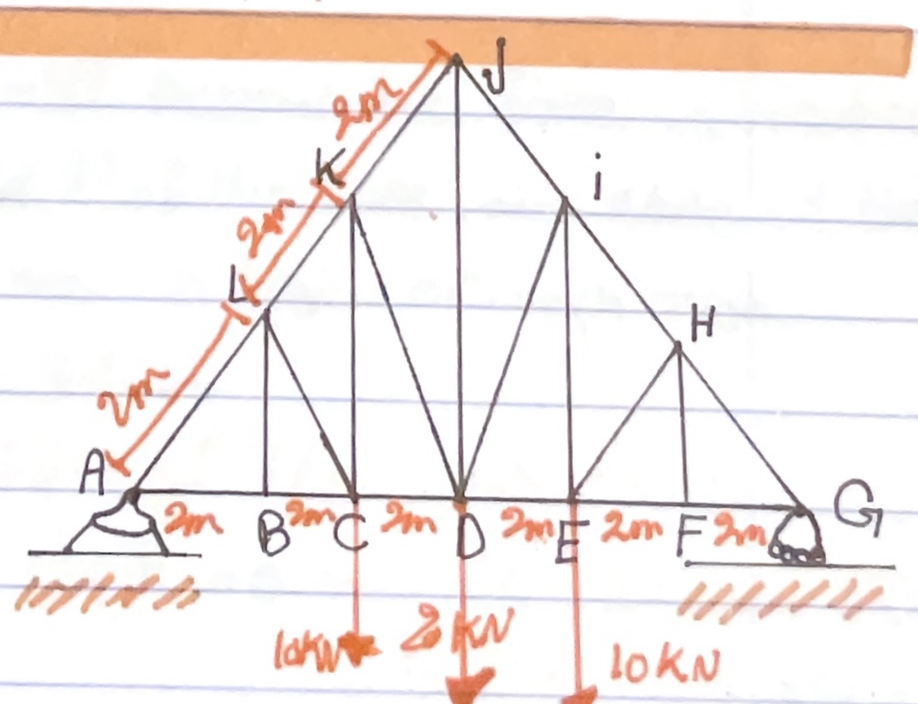
to illustrate :-



CD is * Zero Force mem.



*CD isn't a Zero Force mem.



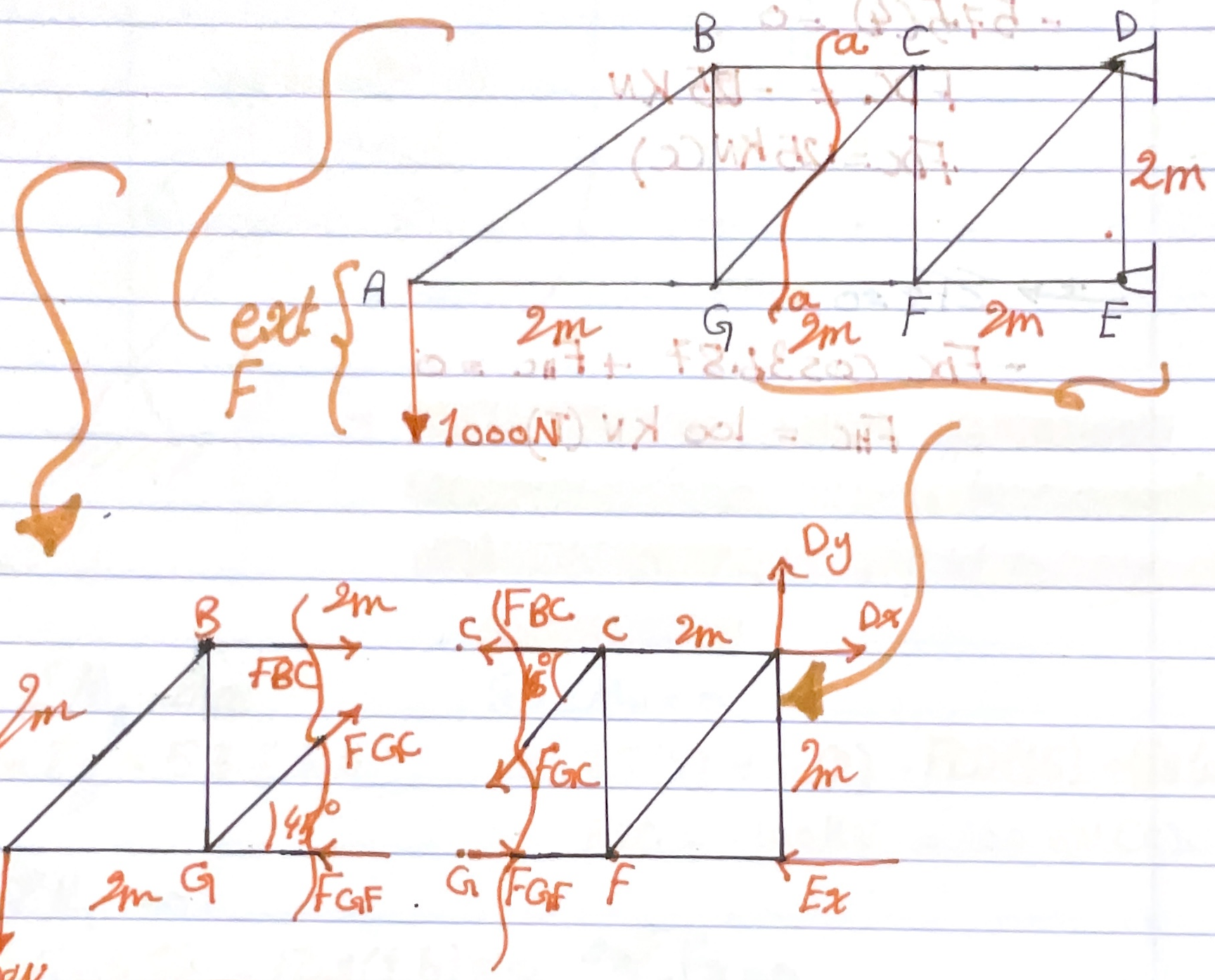
عندي هيك تماثل بالشكل لازم يكون في تماثل بالشكل وبالقوة المطبقة عليها.

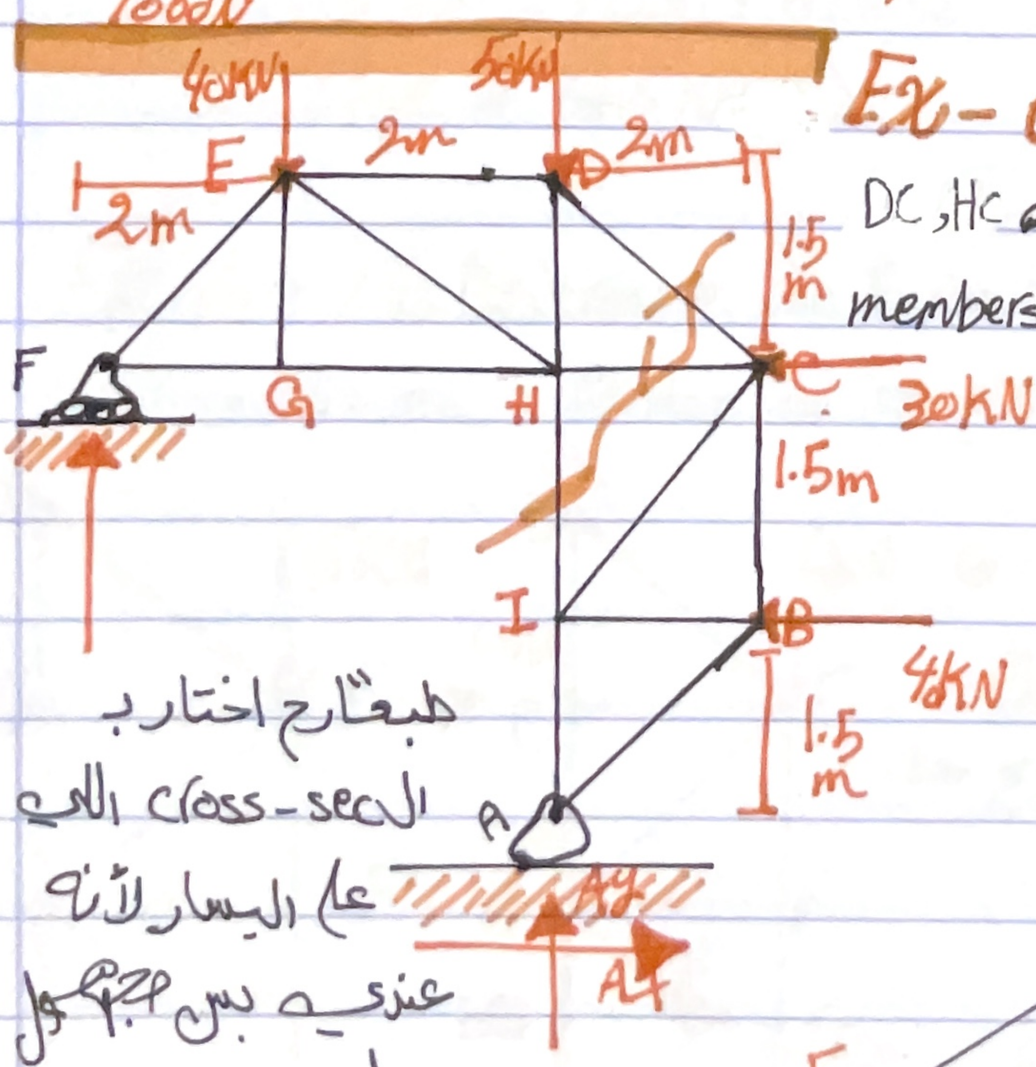
LB, HF, CL, EH Zero - Force mems.

6.4 The method of sections

if u didn't get Zero force
[illegible]

Main rule: the cross-section should pass 3 mems or less only.





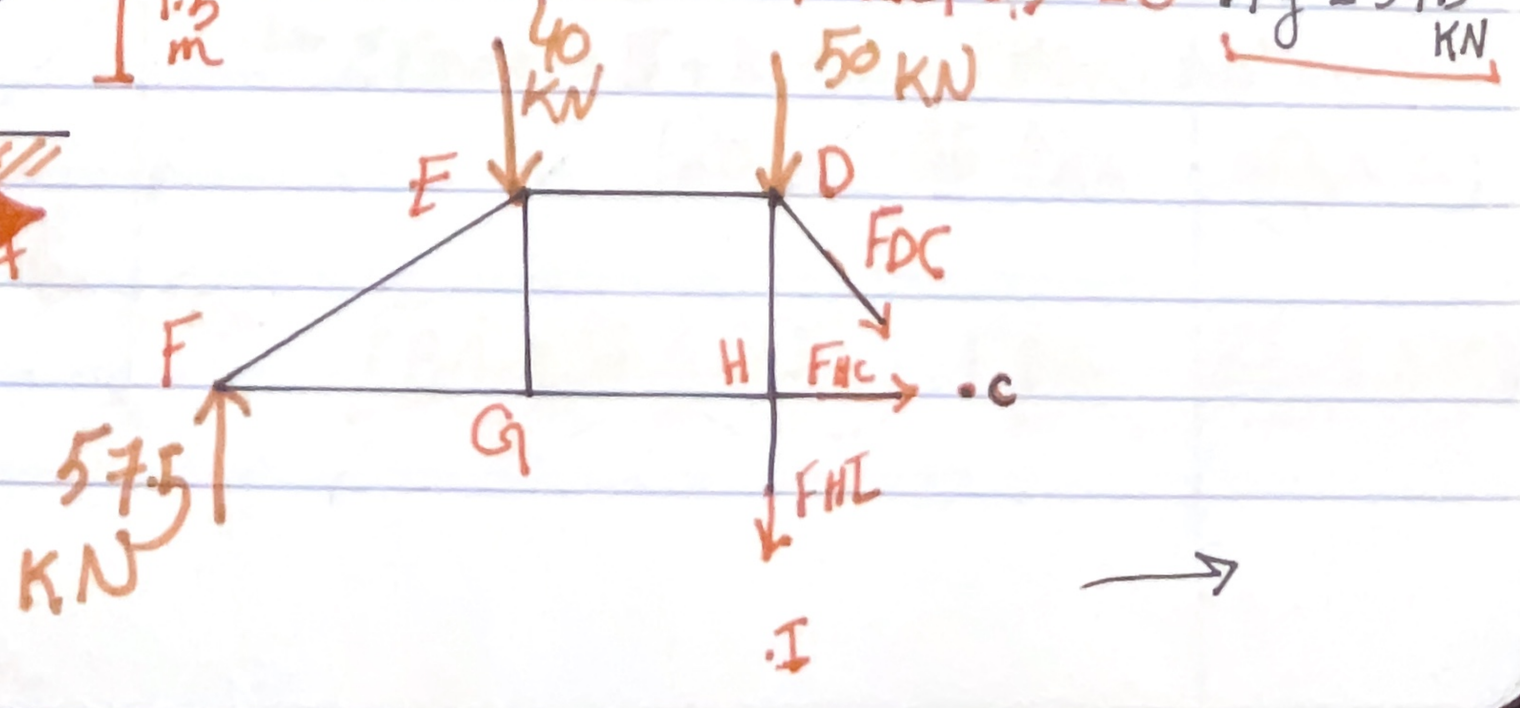
طبعاً رح اختار ب
الـ cross-sec اللي
على اليسار لأنو
عندي بس 3 مجاهيل
و احد.

Ex- 6-41 Determin the force in members DC, HC and HI of the truss, and state if the members are in tesion or compression.

Solu:-

FBD of the whole truss.

$\circlearrowleft + \sum M = 0 \rightarrow -Fy(4) + 40(2) + 30(3) + 40(1.5) = 0 \rightarrow Fy = 575$ KN



$\oplus \circlearrowleft \Sigma M_C = 0$

$F_{HI}(2) + 50(3) + 40(4) - 57.5(6) = 0$

$F_{HI} = 42.5\ KN\ (T)$ ← هنوض النقطة البداية وعلى اتجاه حركتي اذا بتفاجئوها او بتتأكد منها.

$\oplus \circlearrowleft \Sigma M_H = 0$

$-F_{DC}\cos 36.87(1.5) + 40(2)$ كنا فاضيتها خطأ ولكن عدلناها على الرسمة (الرسمة صح)

$-575(4) = 0$

$F_{DC} = -125\ KN$ →

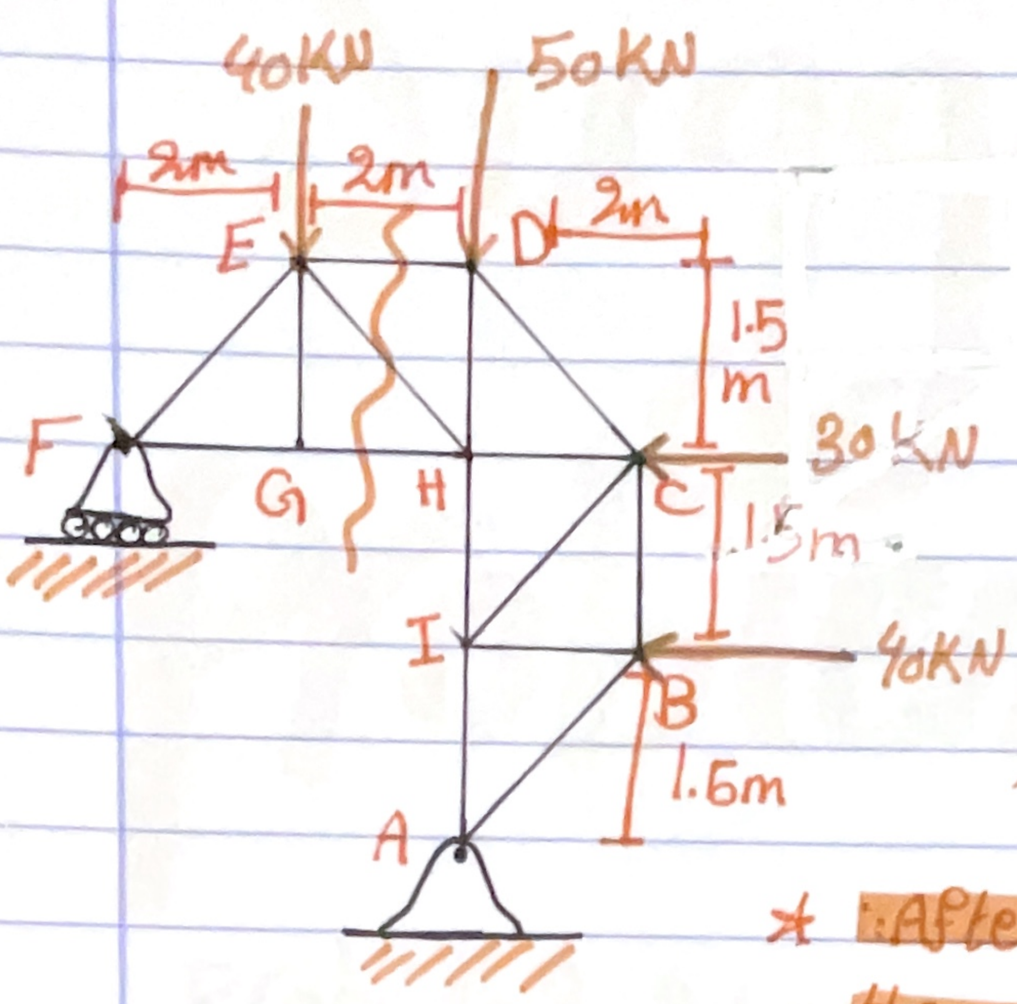
$F_{DC} = 125\ KN\ (C)$

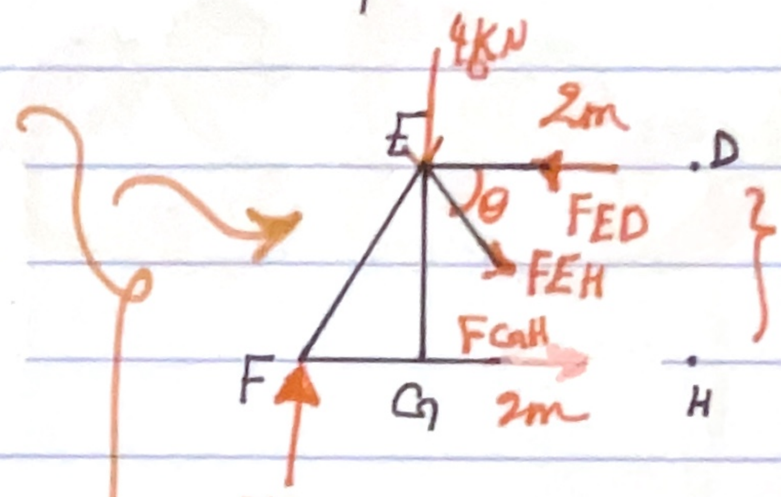
$\rightarrow \Sigma F_x = 0$

$-F_{DC}\cos 36.87 + F_{HC} = 0$

$F_{HC} = 100\ KN\ (T)$

Ex 6-41. Determine the force in members ED, EH and GH of the truss and state if the members are in tension or compression.





Solu:-

$\circlearrowleft + \Sigma M_A = Zero$

$\rightarrow F_y = 57.5\ KN$

$\circlearrowleft + \Sigma M_E = 0$

$-57.5(2) + F_{GH}(1.5) = 0$

$F_{GH} = 76.6\ KN (T).$

$\circlearrowleft + \Sigma M_H = 0$

$-57.5(4) + 40(2) - F_{ED}(1.5) + F_{EH}(0) + F_{GH}(0) = 0$

$F_{ED} = -100KN = 100\ KN\ (C).$

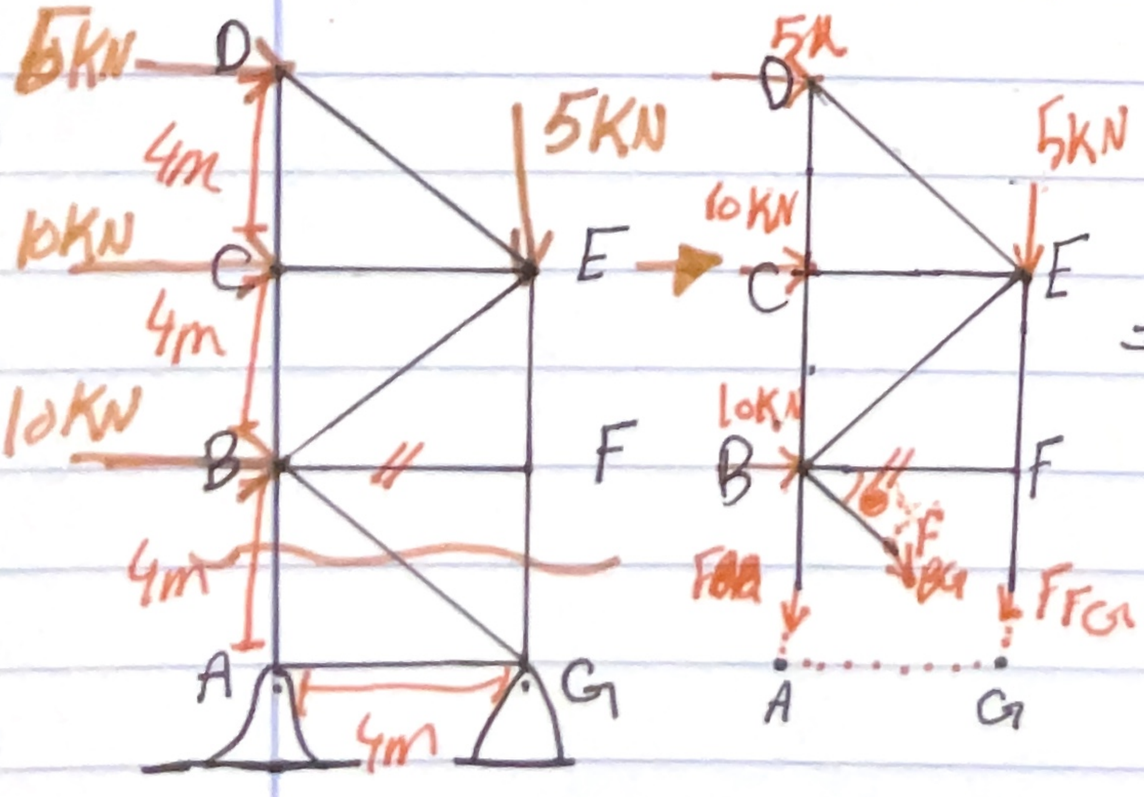
$\pm \rightarrow \Sigma F_x = 0$

$76.6 - 100 + F_{EH}\cos\theta = 0$

$F_{EH} = 29KN\ (T).$

* : After section I always assume the member forces to be tension and after if it was ⊖ I just flip it and say it is Compression.

Ex 6-47. Determine the force in members BF, BG and AB, and state if the members are in tension or compression. Soln:- $F_{FB} = Zero$



$\circlearrowleft + \Sigma M_G = 0$

$-5(12) - 10(8) - 10(4) + F_{BA}(4) = 0 \rightarrow F_{BA} = 45KN\ (T)$

$\pm \rightarrow \Sigma F_x = 0 \rightarrow 5 + 10 + 10 + F_{BG}\cos 45^\circ = 0$

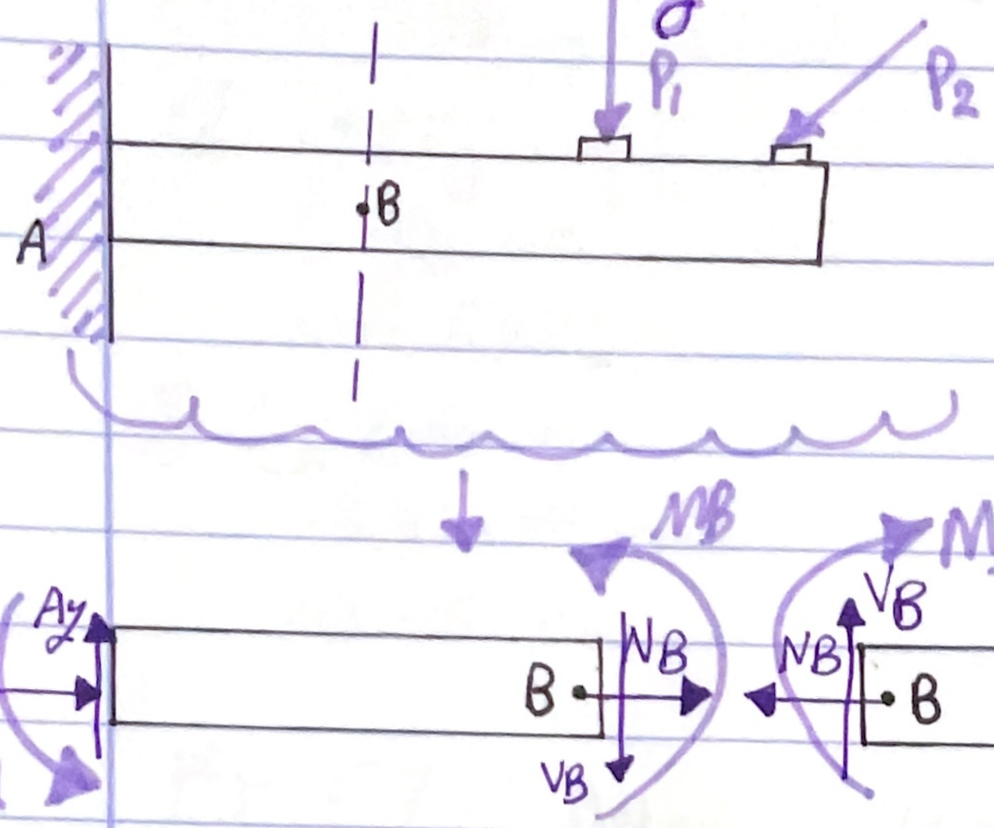
$F_{BG} = -35.4KN = 35KN\ (C)$

$F_{BA} = 45\ KN (T),\ F_{BG} = 35.4\ KN (C)$

member BF is a zero force mem.

Ch.7 Internal Forces

7.1 Internal loading Developed in Structural members.



لح نعمل قطع عند نقطة معينة وهمية ونتيجة هذا القطع لح ينشأ عندي قوى زي N, V, M ← دقوة المحورية وقوة القص وزخم كل واحد الا حبايبه.

take a look at page 343.

NB دائما خارجة من النقطة بشكل ⊥

VB بفرضها بشكل clockwhise يخليها تدور

MB بفرضها بجيث تعطيلي حركة حرف "U"

بختار الجزء الأسهل بالحل (اللي مافيه reactions كتير)

(VB shear force قوة القص)

Ex 7.1 p.g:346

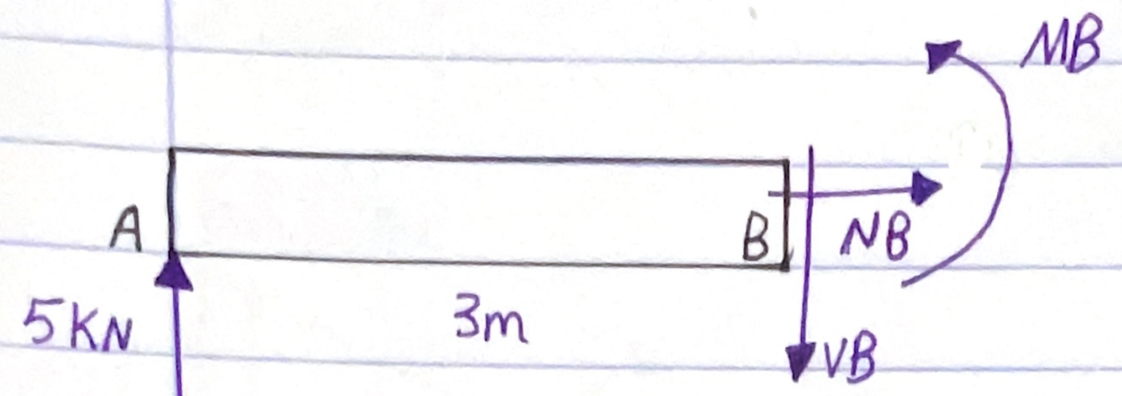
Determine the normal force, shear force, and bending moment acting just to the left, point B, and just to the right, point C, of the 6-KN force on the beam in Fig. 7-4a.

Solu:-

1. Draw the FBD of the whole member.
2. Find your required reactions.
3. Make your section.

$\circlearrowleft \Sigma M_D = 0 \rightarrow -A_y(9) + 6(6) + 9 = 0 \;,\; A_y = 5KN$

★ Section A-B



Solu:

① $\xrightarrow{+} \Sigma F_x = 0$

$N_B = 0$

② $+\uparrow \Sigma F_y = 0$

$5 - V_B = 0$

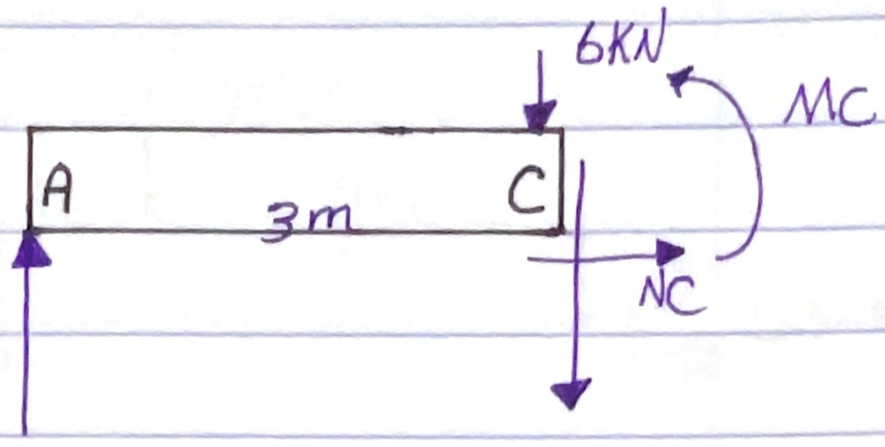
$V_B = 5\,KN \downarrow$

③ $\circlearrowleft_{+} \Sigma M_B = 0$

$-5(3) + M_B = 0$

$M_B = 15\,KN.m$

★ Section A-C



solu:-

1. $\overrightarrow{+} \Sigma F_x = 0$, $N_C = Zero$

2. $+\uparrow \Sigma F_y = 0$

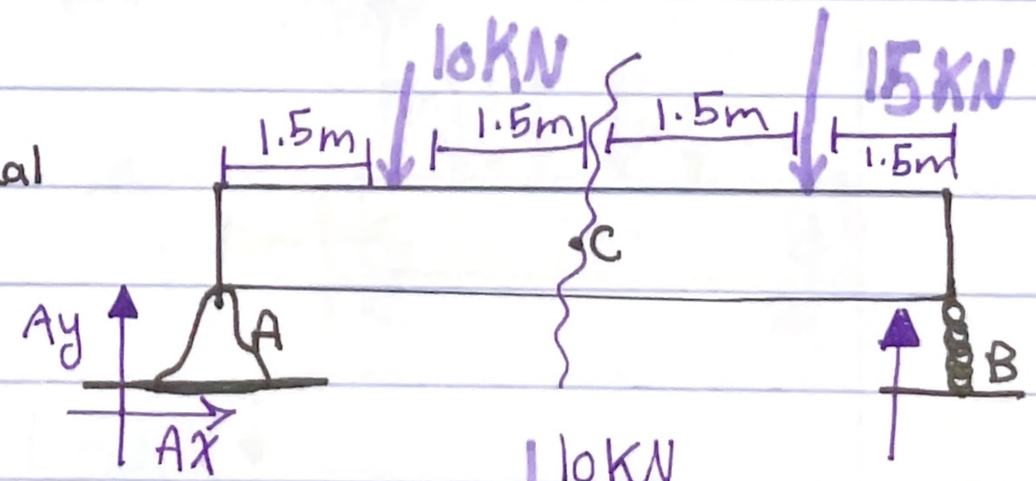
$5 - 6 - V_C = 0$

$V_C = -1\,KN = 1\,KN \uparrow$

3. $M_c = 0$

$-5(3) + M_c = 0 \rightarrow M_C = 15\,KN.m$

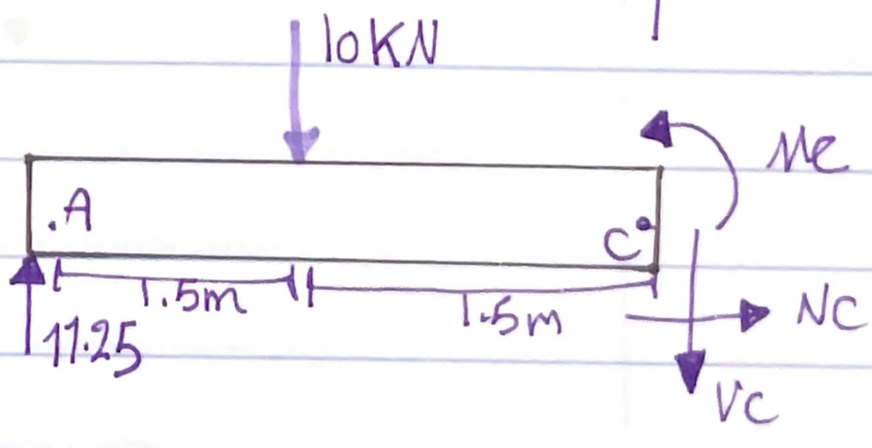
Ex F7-1 Determine the normal force, shear force and moment at point C.



Solu:-

★ Ax will be Zero at anypoint we take M. ★ [Ax = Zero]

Section A-C →



$\circlearrowleft_{+} \Sigma M_B = 0$

$-A_y(6) + 10(4.5) + 15(1.5) = 0$

$\boxed{A_y = 11.25\,kN}$

$\xrightarrow{+} \Sigma F_x = 0$, $\boxed{N_C = 0}$

$+\uparrow \Sigma F_y = 0$, $11.25 - 10 - N_C = 0$

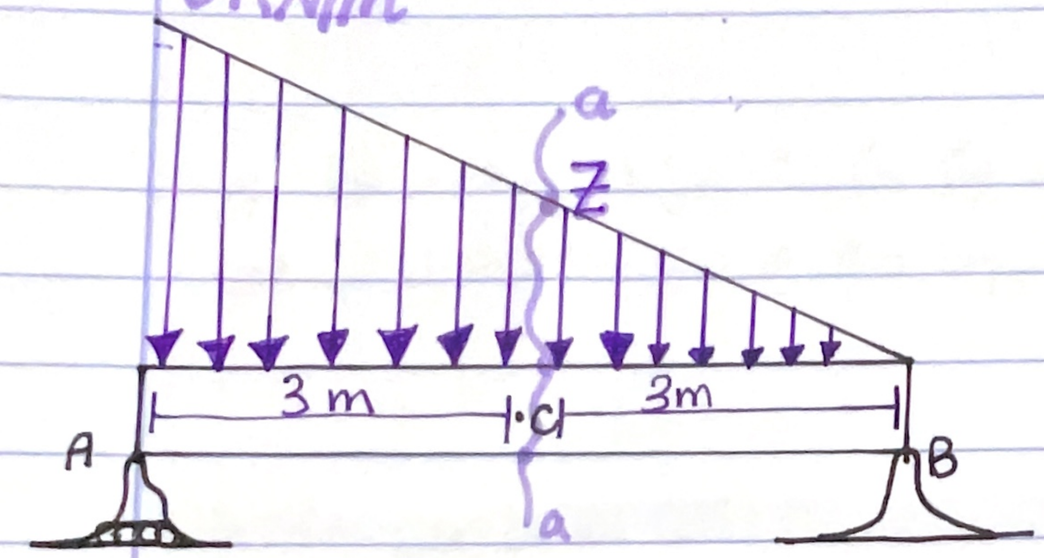
$\boxed{V_C = 1.25\,kN \downarrow}$

$\circlearrowleft_{+} \Sigma M_c = 0$

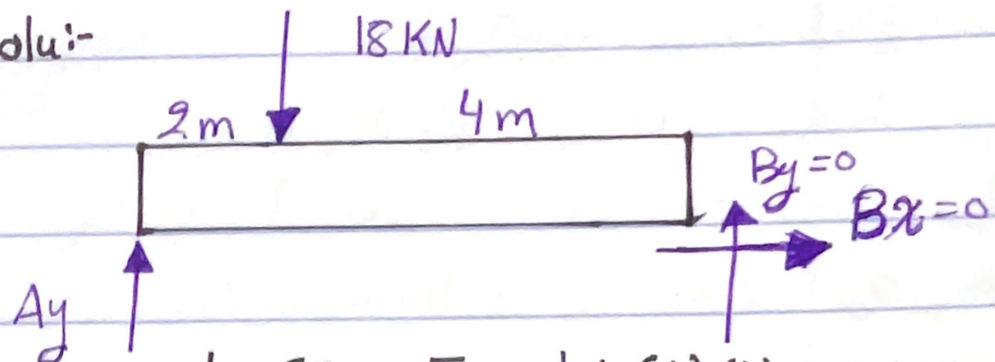
$-11.25(3) + 10(1.5) + M_C = 0$

$\boxed{M_C = 18.75\,KN.m}$

Ex-7-15. Determine the internal normal force, shear force, and moment at point C.



Solu:-



$+\downarrow \Sigma F = F_R = \frac{1}{2}(6)(6) = 18\,KN$

at $\frac{1}{3}(6) = 2m$

$\circlearrowleft + \Sigma M_A = 0$

$B_y(6) - 18(2) = 0$

$B_y = 6\,KN$

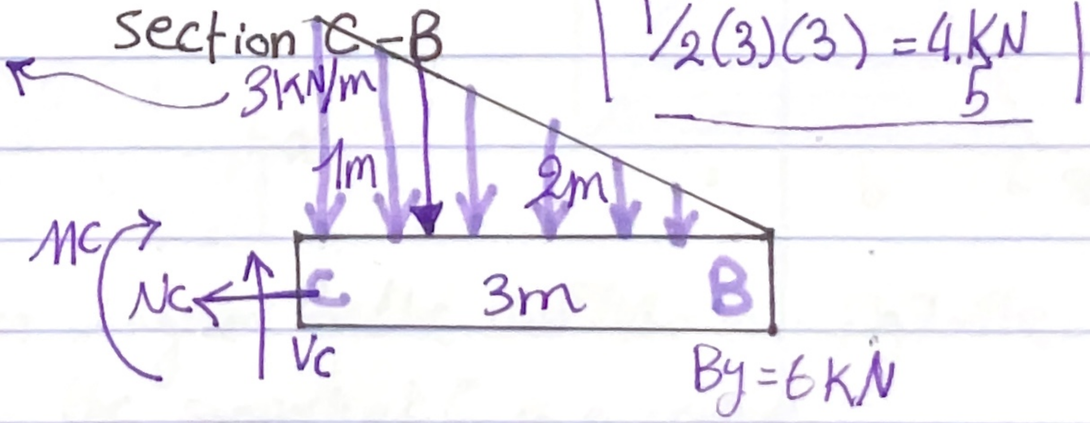
Z بتجيبها من تشابه المثلثات

بين المثلث الكبير والصغير

$\frac{6}{Z} = \frac{6}{3}$ $Z = 3\,KN/m$

Section C-B

$\frac{1}{2}(3)(3) = 4.5\,KN$



$\pm \rightarrow \Sigma F_x = 0$

$N_C = Zero$

$+\uparrow \Sigma F_y = 0$

$V_C + 6 - 4.5 = 0$

$V_C = -1.5\,KN = 1.5\,KN\downarrow$

$\circlearrowleft + \Sigma M_C = 0$

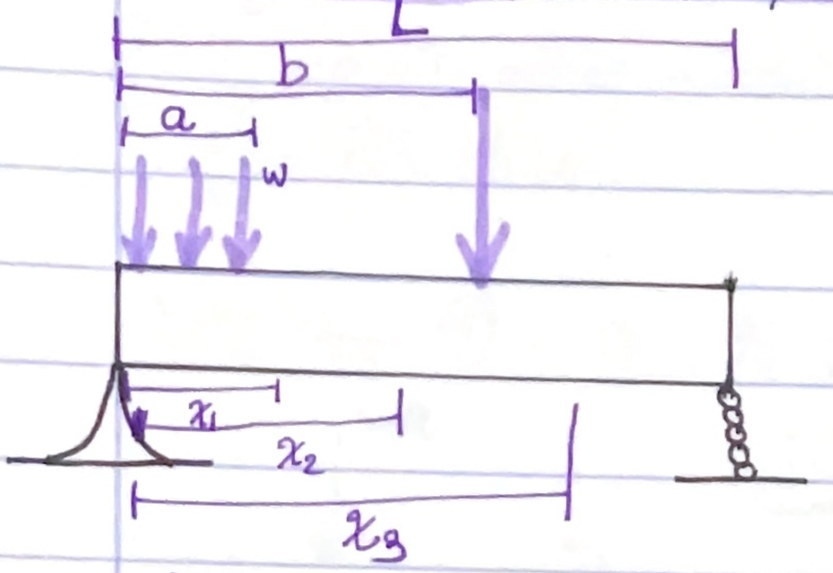
$6(3) - 4.5(1) - M_C = 0$

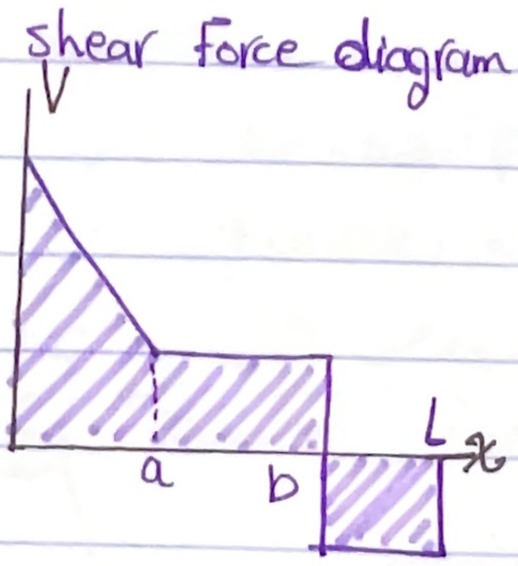
$M_C = 13.5\,KN.m.$

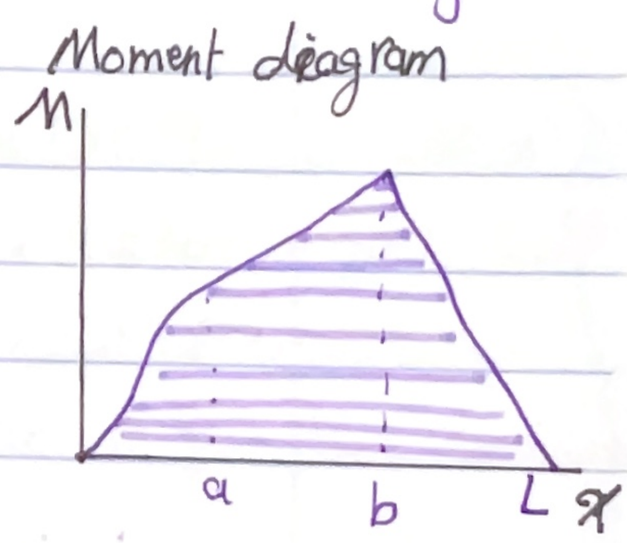
7.2 Shear and Moment Equations and Diagrams

كنت زمان اعمل see من نقطة وحرة في ال Beam ولكن ما بقدر احسب انه ال V و M و N رح تكون نفسها إذا قطعت see آخر من البيم واحكي هون نفسها هون لذلك في هذا ال section بركز على إنها يعمل لها Fun of x بحيث x هي بتعبر عن الطول وبين اجيب هذا الإقران هاعيتها بعوض قيم الطول المختلفة واحكي هون V بتساوي كذا و هون M تساوي هيك وهكذا.

* In this section if M and V = ⊖ it is ok! don't a dare to change it!

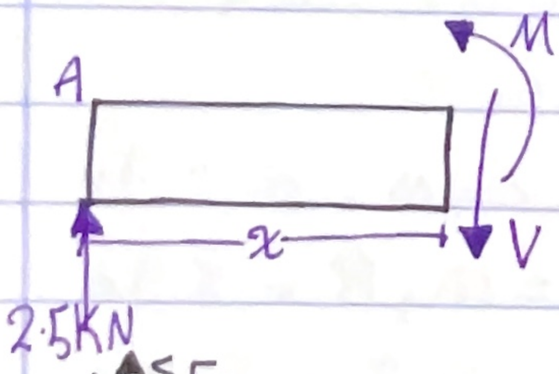






Ex 7.6. Draw the shear and moment diagram for the shaft shown in fig 7-11a. The support at A is a thrust bearing and the support at C is a journal.

Section [1]:- $0 < x \leq 2m$



$+\uparrow \Sigma F_y = 0$

$2.5 - V = 0, V = 2.5KN$

at $x = 0, M = 0$

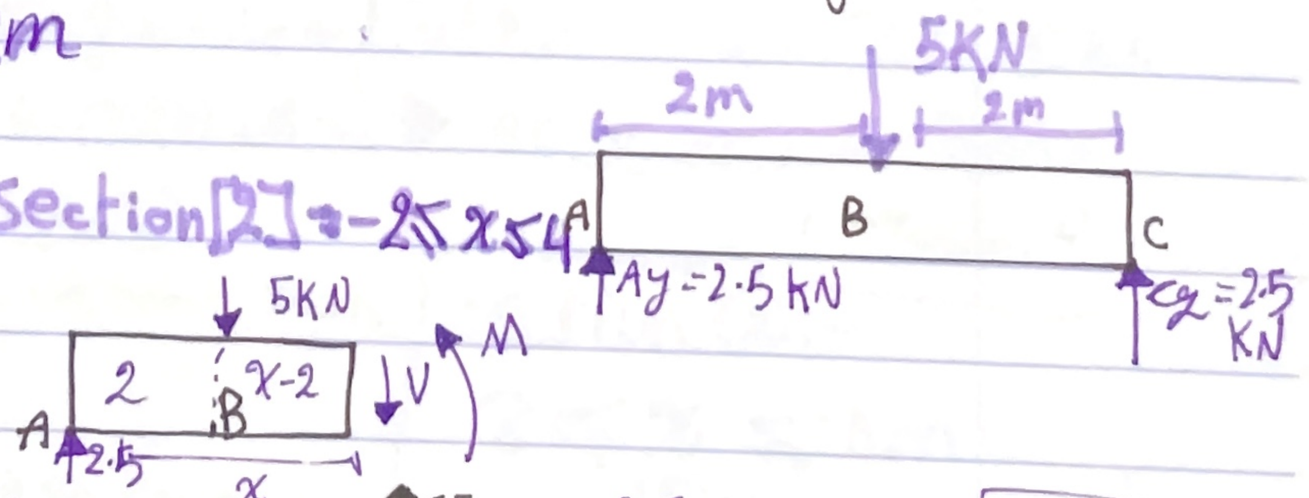
$\circlearrowleft^{+} \Sigma M_A = 0$

$M - V(x) = 0$

$M = 2.5(x)$ KN.m

at $x = 2, M = 5$ KN.m

Section [2]:- $2 \leq x \leq 4$



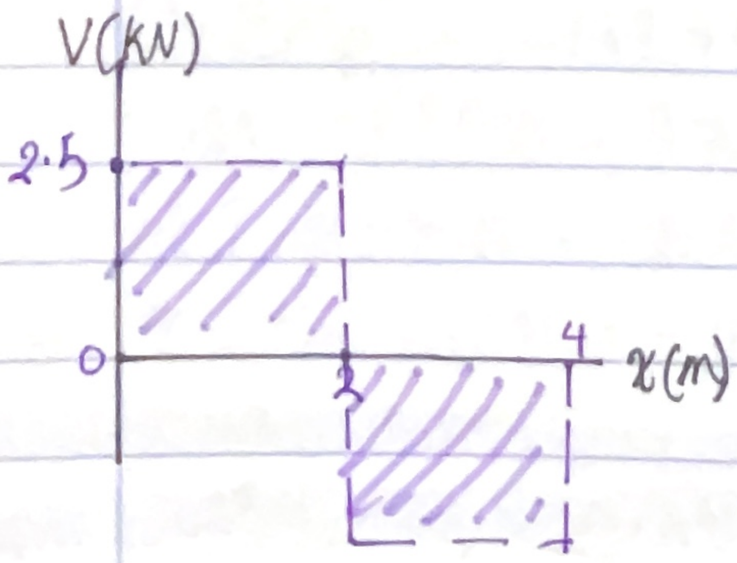
$+\uparrow \Sigma F_y = 0, 2.5 - 5 - V = 0, \boxed{V = -2.5\ KN}$

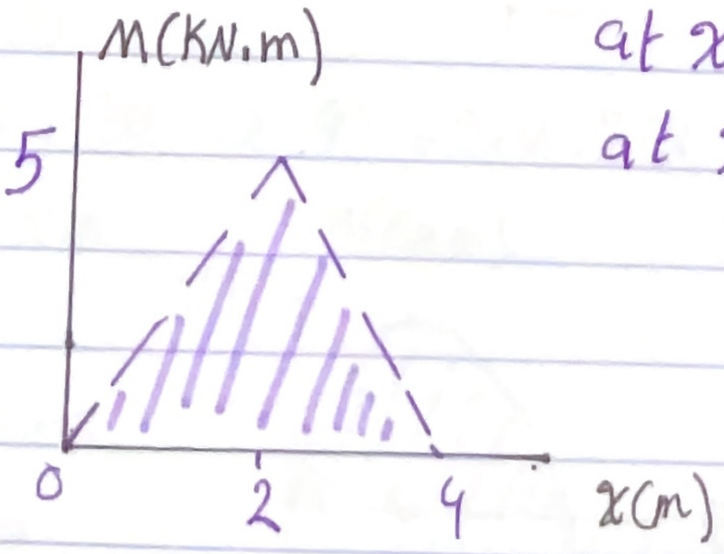
$\circlearrowleft^{+} \Sigma M_A = 0, (-2.5)(-)(x) - 5(2) + M = 0$

$M = (10 - 2.5x)\ KN$

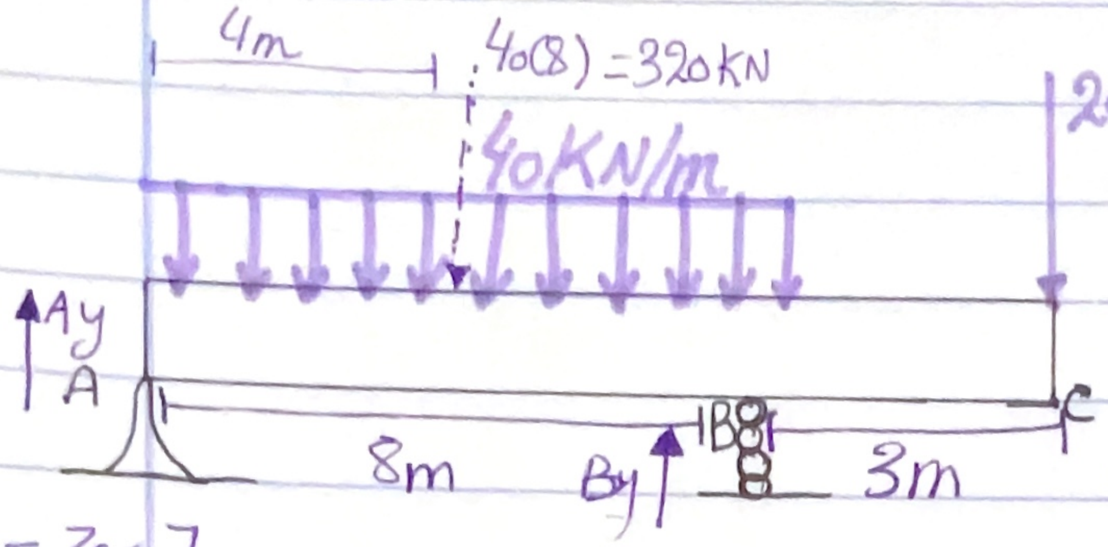
at $x = 2 \longrightarrow M = 5$ KN.m

at $x = 4 \longrightarrow M =$ Zero.





Ex- 7-55, Draw the shear and moment diagrams for the beam.



This que called over-Hanging beam. (which describes the supported beam from 2 points, one in the biggening and one between the 2 ends and the 2end isn't supported by any reaction forces).

[Ax = zero]

$\circlearrowleft^{+} \Sigma M_A = 0, \; -150 + By(8) - 32o(4) - 2o(11) = 0$

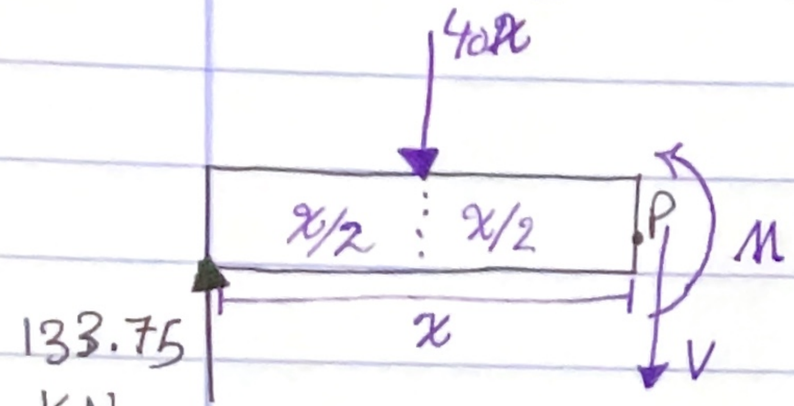
$By = 2o6.25\,kN$

$+\uparrow \Sigma Fy = 0, \; Ay - 32o + 2o6.25 - 2o = 0$

$Ay = 133.75\,kN$

* بالقوة الموزعة انا اول اثبت بحلها لقوة وحدة عشان اطلع ال reactions وبعدين بس اطلعهم برجع القوة الموزعة كما كانت عادي وبعدين بعمل Moment و shear force عشان اطلع ال Section diagrams.

Section ①:- $0 < x < 8m$



بعد ما اعمل Point load بعتبرها لازم تكون بالمركز عشان العشاكل منتظم عشان حيلك في $x/2$.

$\circlearrowleft^{+} \Sigma M_p = 0, \; -133.75(x) + 4o(x)(x) + M = 0$

$M = 133.75x - 2ox^2$ ← we need 3 points.

at $x = 0, \; M = 0$

at $x = 8, \; M = 223.6\,kN.m$

at $x = 3.34 \rightarrow M_{max} = -210\,kN.m$

$+\uparrow \Sigma Fy = 0, \; -133.75 - 4ox = V$

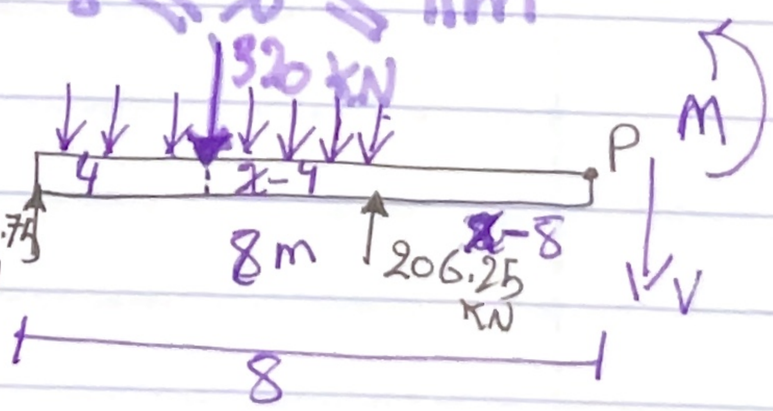
at $x = 0 \rightarrow V = 133.75\,kN$

at $x = 8 \rightarrow V = -186.75\,kN$

at $V = 0, \; x = 3.34m$ ← $max_{mom.}$

Section ②:-

$8 \leq x \leq 11m$



$+\uparrow \Sigma Fy = 0, \; 133.75 - 32o + 2o6.25 - V = 0$

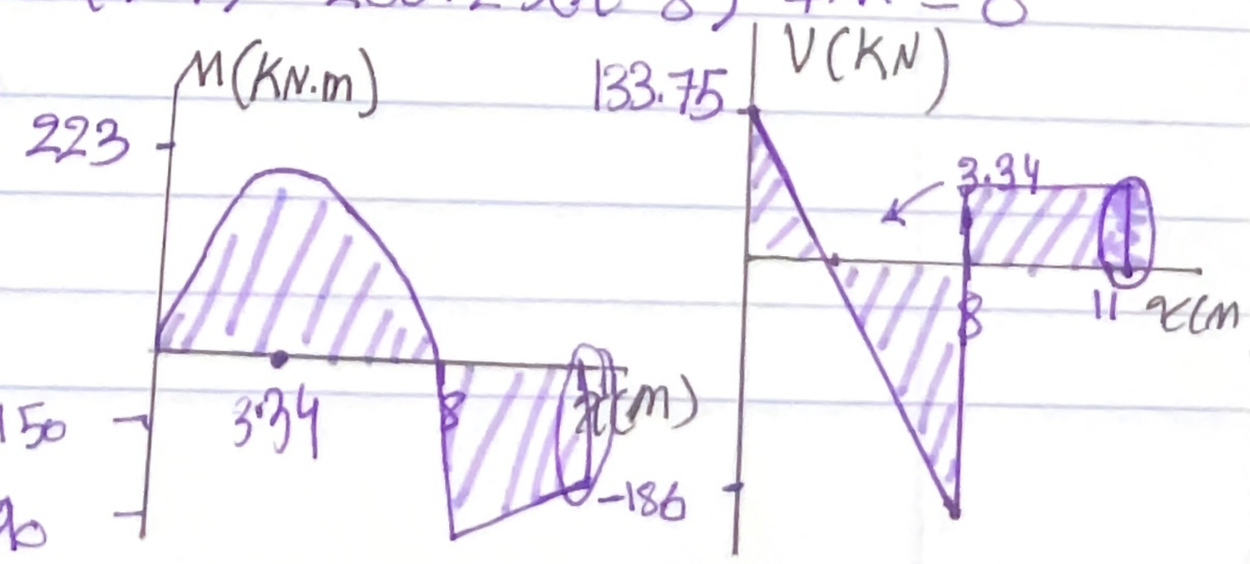
$V = 2o\,kN$ — ①

$\circlearrowleft^{+} \Sigma M_p = 0, \; -133.75(x) + 32o(x-4) - 2o6.25(x-8) + M = 0$

$M = (2ox - 37o)\,kN.m$

at $x = 8 \rightarrow M = -2o\,kN.m$

at $x = 11m \rightarrow M = -15o\,kN.m$



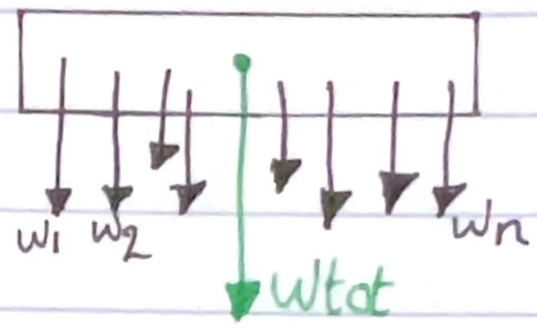
بعد ال sec بكون عندي M وقوة ما بكون حاسبتها بالقطع بعد ديري بالك قبول باثروا بال diagram ثم الي بسكرولي الرسمات زي نهاية ال diagrams

ch.9

Center of gravity and centroid

Center of gravity: The point through which the whole weight of a body acts.

Centroid:- (Mainly on 2D objects)
The point at which the total area of a plane figure. (rectangle, tringle, circle, ...) is assumed to be connected.



center of gravity, mass, area is calculated using the principle of moments.

★ Moment of the total weight, mass, area, ... etc.

The type of the object that we deal with:-

1) Disscrete bodies:-

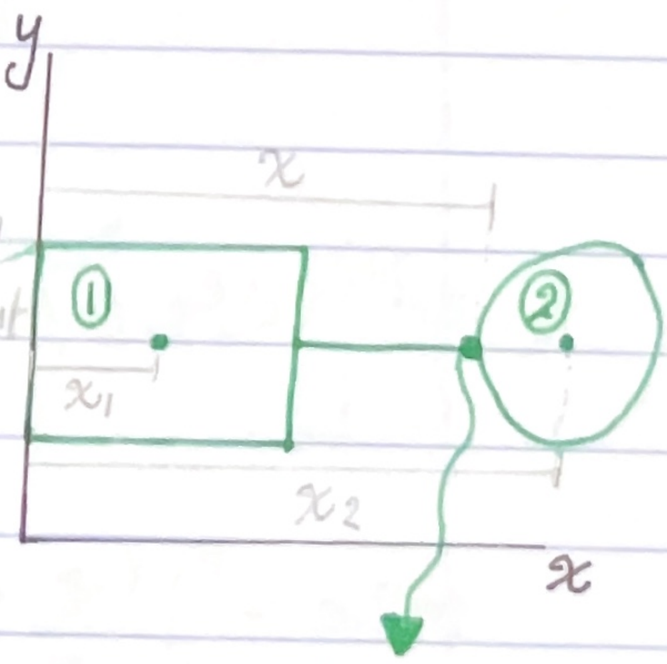
$M_{tot}\,\bar{x} = \sum m_i x_i$

$= m_1 x_1 + m_2 x_2$

$\bar{x} = \dfrac{m_1 x_1 + m_2 x_2}{M_{total}}$

I don't know where $\bar{x}$ is, but I know $x_1 x_2$

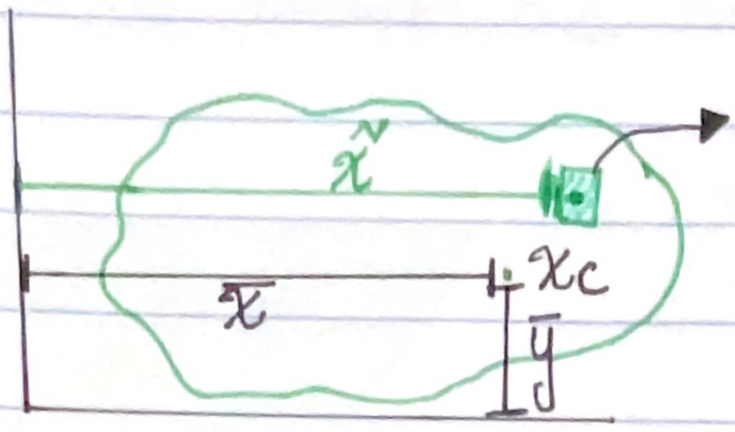
center of mass



could be center of mass so we call it $\bar{x}$

★ What if we deal with smth don't have an organised shape.

2) Continous shapes:-



Differential element (dw)

$\tilde{x}$: (x tilda) the location from the point (axis) that we are taking the moment about to the centroid of the element (dw).
$\tilde{x}$ it is the variable.

$\bar{x}$:- the location at the centroid of the total shape. (location from the opp axis to the center of the whole object.).

Solu:- we have to know the diffrance between $\bar{x}, \tilde{x}$.

$$w\bar{x} = \int \tilde{x}\, dw$$

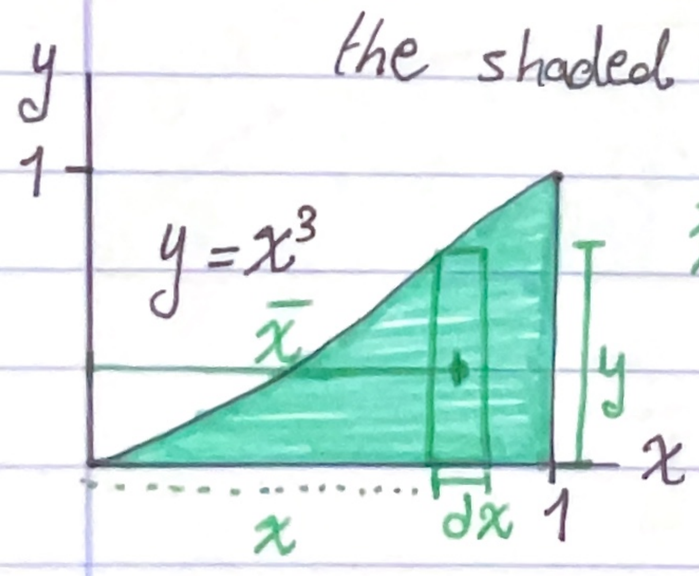
$$\rightarrow \bar{x} = \frac{\int \tilde{x}\, dw}{\int dw}$$

w:- the rate of the whole shape.

, we might need this:-

$$\bar{y} = \frac{\int \tilde{y}\, dw}{\int dw}$$

F9-2, Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area.



Solu:-

$\bar{x} = x$

$dA = y\,dx$

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \int \tilde{y}\, dA / dA$$

$$\bar{x} = \frac{\int y \cdot x\, dx}{\int y\, dx} = \int_0^1 x \cdot x^3\, dx \Big/ \int_0^1 x^3\, dx$$

$$= \int_0^1 y/2\,(y)\, dx \Big/ \int y\, dx$$

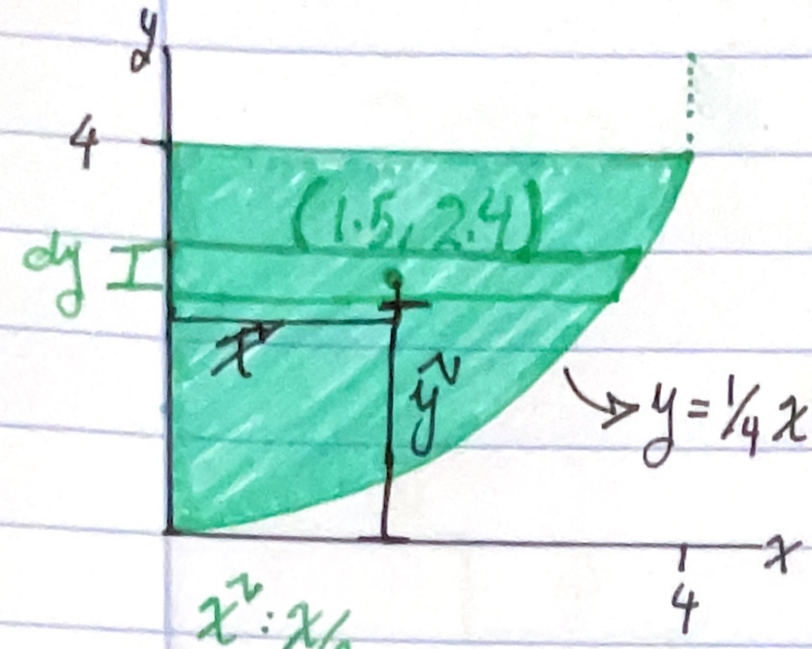
$$\bar{x} = \frac{x^5}{5}\Big|_0^1 \div \frac{x^4}{4}\Big|_0^1 = 0.8m$$

$$= \int_0^1 x^3/2 \cdot (x^3)\, dx \Big/ \int_0^1 x^3\, dx$$

$\bar{y} = 0.286m$

9-9 locate the centroid $\bar{x}$ of the shaded area.

9-10 locate the centroid $\bar{y}$ of the shaded area.



$\tilde{x}: x/2$

$\tilde{y}: y$, $dA = x\,dy$

Solu:-

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int (x/2)(x\,dy)}{\int x\,dy}$$

$$\bar{x} = \frac{\int_0^4 \frac{x^2}{2}\,dy}{\int_0^4 x\,dy} = \frac{\int_0^4 \left(\frac{2\sqrt{2}}{2}\right)(2\sqrt{y})\,dy}{\int_0^4 2\sqrt{y}\,dy}$$

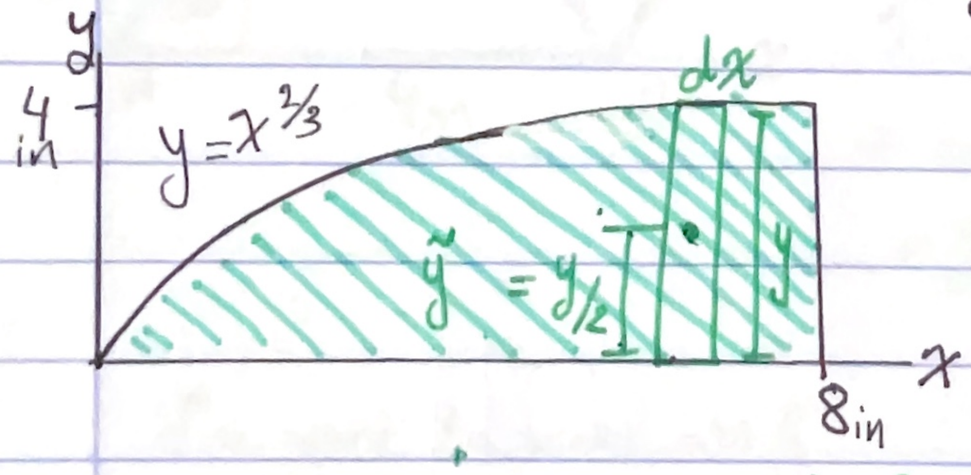
$$\boxed{\bar{x} = 1.5\text{m}}$$

$$\bar{y} = \int \tilde{y}\,dA \Big/ \int dA$$

$$\bar{y} = \int y\,x\,dy \Big/ \int x\,dy$$

$$\bar{y} = \int_0^4 y\,2\sqrt{y}\,dy \Big/ \int_0^4 2\sqrt{y}\,dy = 2.4\text{m}$$

9-17 locate the centroid $\bar{y}$ of the area.



$dA = y\,dx$

Solu:-

$$\bar{y} = \int \tilde{y}\,dA \Big/ \int dA$$

$$\bar{y} = \frac{\int (y/2)(y\,dx)}{\int y\,(dx)}$$

$$\bar{y} = \frac{\int_0^8 \left(\frac{x^{2/3}}{2}\right)(x^{2/3})\,dx}{\int_0^8 x^{2/3}\,dx} = 2.85\text{ in}$$

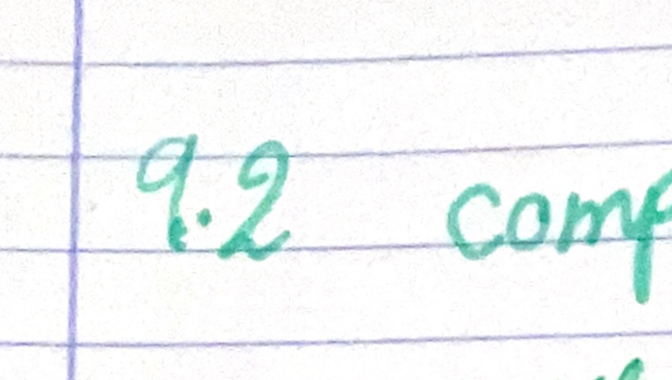
(this is not the

9.2 composite bodies

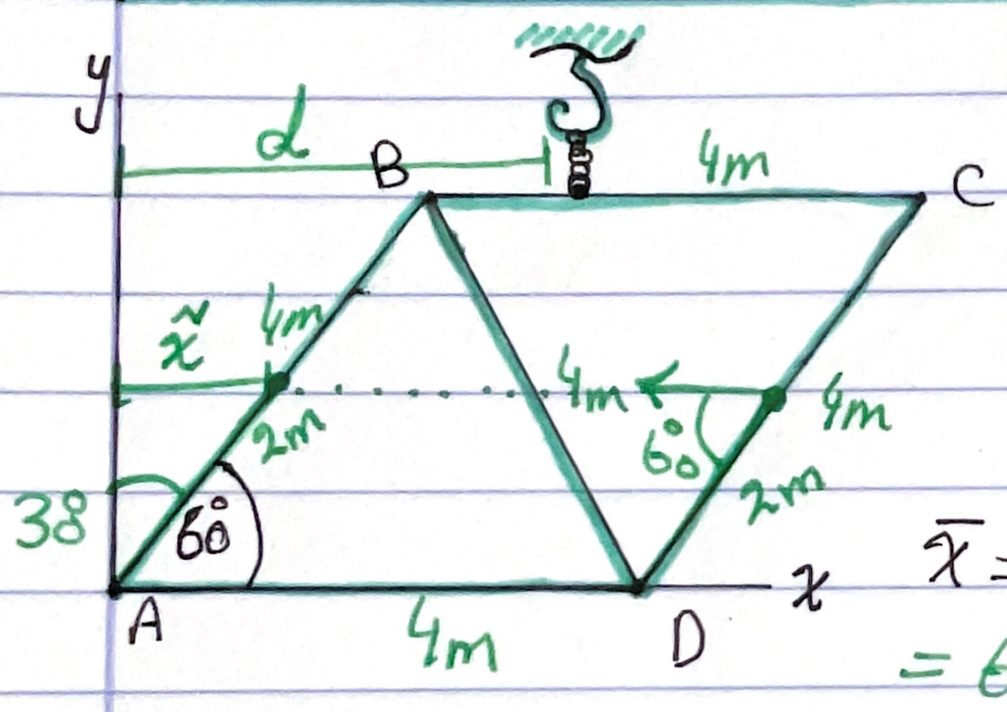
Composite bodies:-

أكثر من جسم بدي احسب centroid أو moment او اي حاجة

$$\bar{x} = \frac{\Sigma \tilde{x}_i A_i}{\Sigma A_i}$$



9-51. The truss is made from Five members each having a length of 4m and a mass of 7kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip(rotate) when it is lifted.



Solu:-

$\bar{x} = \Sigma \tilde{x}_i L / \Sigma L$ = 60/20 = 3m

Element	length	$\tilde{x}$	$\tilde{x}L$
1	4	2sin30	4
2	4	2	8
3	4	2+2cos60°	12
4	4	4	16
5	4	4+2cos60	20
	20		60

if u want to search abt $\tilde{y}$

1. 2cos30
2. 0
3. 2cos30
4. 4cos30
5. 2cos30

Ch.10 Moment of Inertia (I)

Inertia says: "I care abt area".
Area says: "I care to be HIGH".

$$\text{stress} = \frac{F}{A} = \frac{MC}{I}$$

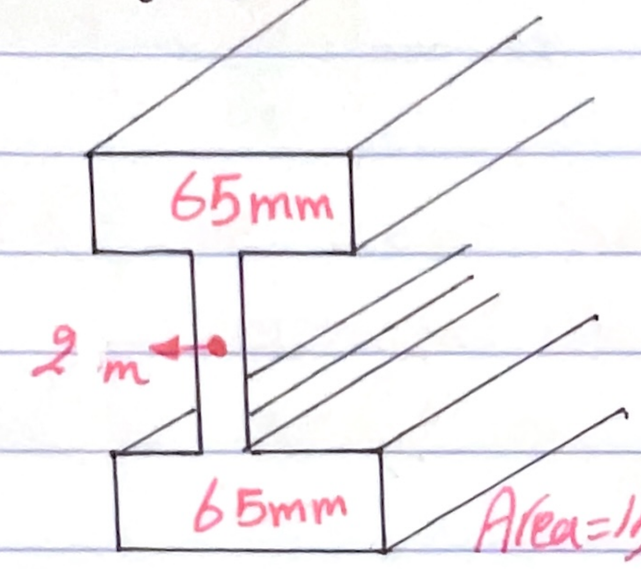
(moment, Inertia, centroid)

I: second moment of area: $A(d)^2$

$I = m^2 (m)^2 ? = m^4 ?!$ ← A مرتين بـ distance ال بضرب !

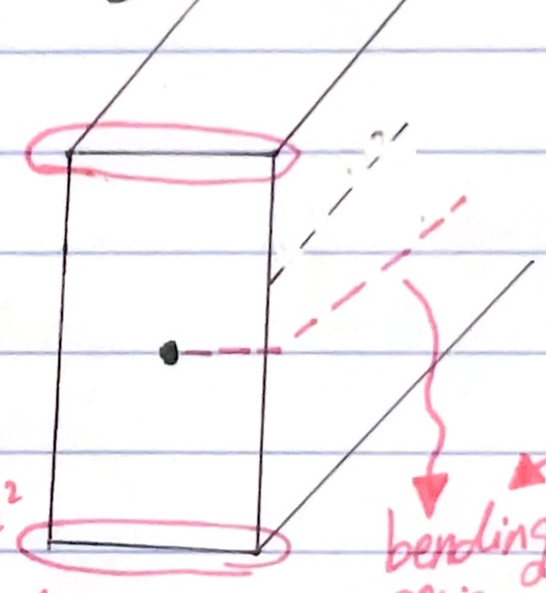
if m: distance, and m^2: Area, and m^3: Volume so what is m^4?

★ m^4 Physicly doesn't have any meaning but it just refers to the Inertia.

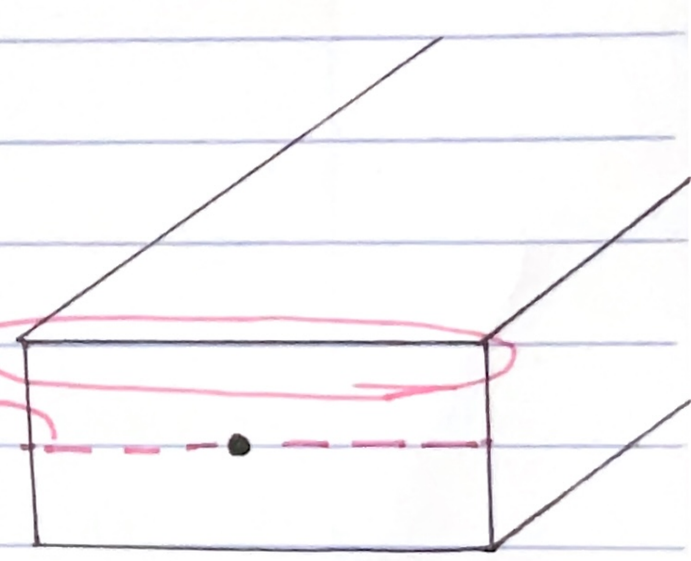


Area = 150 mm^2

I ↑↑ max



Area = 150 mm^2



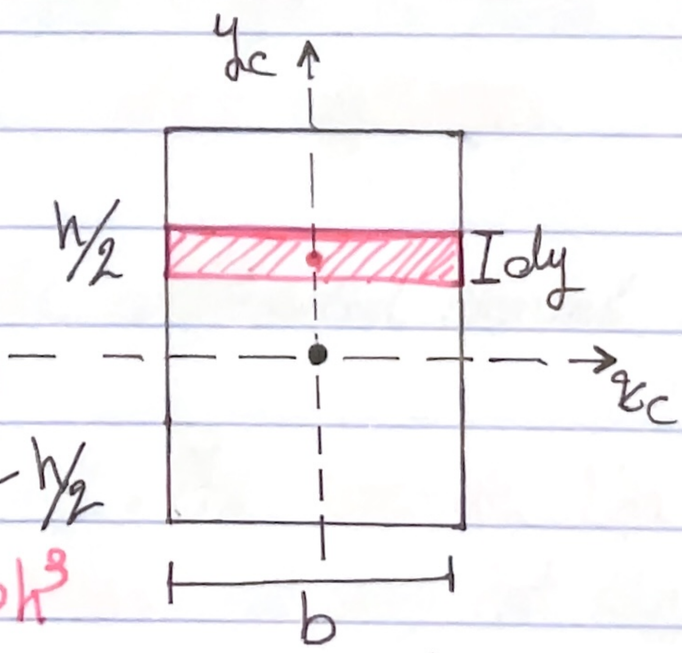
Area = 150 mm^2

I ↓↓ min

Area is the same! the point is where the centroid is located.

$dA = b\,dy \rightarrow \left[I_x = \int_{-h/2}^{h/2} y^2 \, b\,dy \right]$

$I_{xc} = \frac{1}{12} bh^3$ / $I_{y_c} = \frac{1}{12} bh^3$



b: width
h: height

★ parallel axis theorem:

$I_x = I_{xc} + A d^2$

$I_x = \frac{1}{12} bh^3 + (bh)\left(\frac{h}{2}\right)^2$

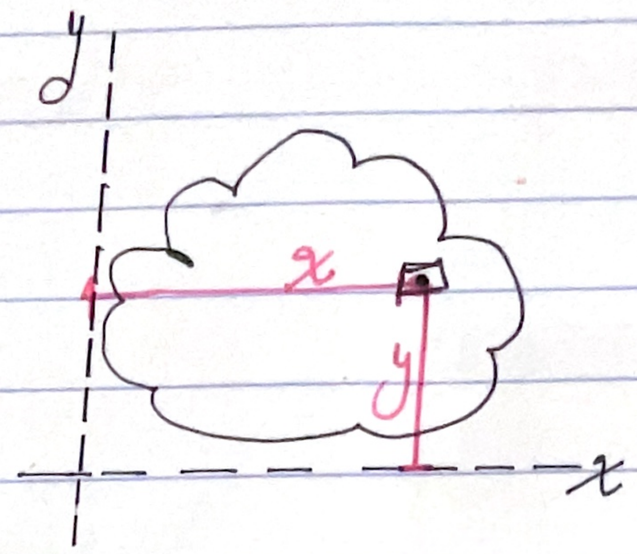
$I_x = \frac{bh^3}{3}$

don't forget u have this ↗

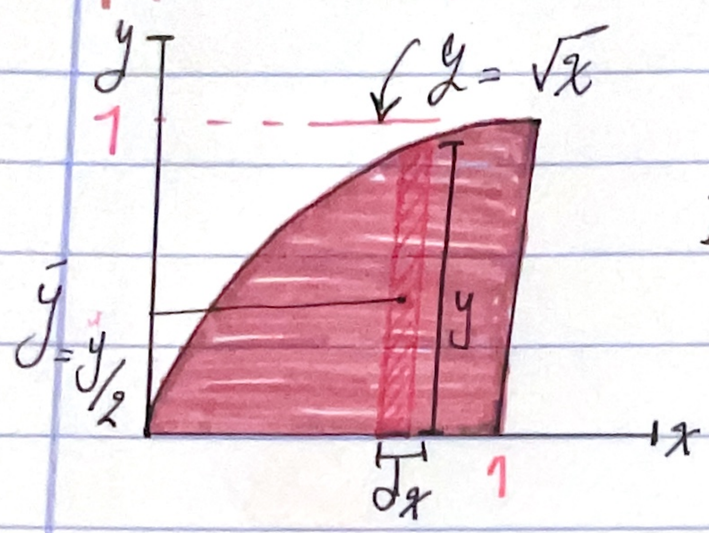
I: second moment at area (Ad^2).

$$I_x = \int_A y'^2 dA$$

$$I_y = \int_A x'^2 dA$$



Prob 10-5:-



What is $I_x . I_y$?

Soln:-

$$I_x = \int y^2 dA$$

$$= \int \left(\frac{y}{2}\right)^2 y \, dx$$

$$I_x = \int_0^1 \left(\frac{x^{\frac{1}{2}}}{4}\right)^2 (x^{\frac{1}{2}}) dx$$

$$\boxed{I_x = 0.133 \, m^4}$$

$$I_y = \int x'^2 dA$$

$$= \int x^2 y \, dx$$

$$I_y = \int_0^1 x^2 (x^{\frac{1}{2}}) dx$$

$$\boxed{I_y = 0.284 \, m^4}$$

* For sure u can take a horizontal chunk tho. But be carefull abt $\bar{y}$ and $\bar{x}$.

imp!

* If I want to find I_x, I have to do dx!



* If I want to find I_y, I have to do dy! dy:

* like differantial element like $I_{_}$ *

* Also I can't just take $\bar{y}/2$, $\bar{x}/2$ because I'm finding the moment at each chunck in the element not only at the differantial ele that I assumed!

U have to know more so read page 532 procedure for Analysis.

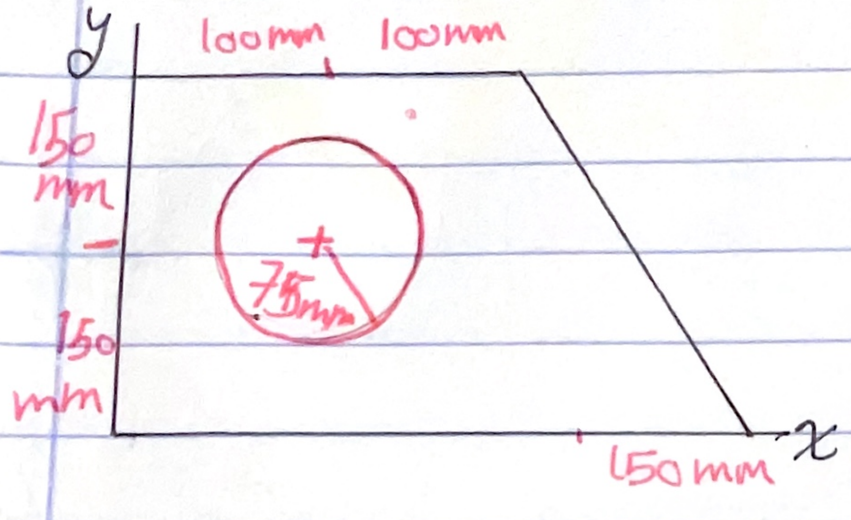
10.4 Moments of Inertia for composite Areas

take a look at ex 10.4 page 541.

parallel axis theorem

$$\begin{cases} I_x = I_{xc} + A d_y^2 \\ I_y = I_{yc} + A d_x^2 \end{cases}$$

10-32. Determine the moment of Inertia I_x of the shaded area about the x axis.



Soln:-

using parallel axis Th and I_{xo}, $I_{x\triangle}$, $I_{x\square}$

$$I_x = I_{xc} + A d_y^2$$

$$I_{xo} = \frac{\pi}{4} R^4, \quad I_{x\square} = \frac{1}{12} bH^3$$

$$I_{x\triangle} = \frac{1}{36} bh^3$$

$I_{x\,total} = I_{x\square} + I_{x\triangle} - I_{xo}$

1) $I_{x\square} =$

$I_{x\square} = I_{xc} + A d_y^2$

$I_{x\square} = \frac{1}{12} bh^3 + A d_y^2$

$I_{x\square} = \frac{1}{12}(200)(300)^3 + 200(300)(150)^2$

$I_{x\square} = 1.3515(10)^9 \text{ mm}^4$

2) $I_{x\triangle} =$

$I_{x\triangle} = \frac{1}{36}(150)(300)^3 + \frac{1}{2}(150)(300)(\frac{1}{3} * 300)^2$

$I_{x\triangle} = 0.3375(10)^9 \text{ mm}^4$

3) $I_{xo} =$

$I_{xo} = (\frac{\pi}{4})(75)^4 + \pi(75)^2(150)^2$

$I_{xo} = 0.79(10)^9 \text{ mm}^4$

$I_{x\,total} =$

$1.3515(10)^9 + 0.3375(10)^9 - 0.79(10)^9 = 0.899 \text{ mm}^4$

* try doing I_y!

remember, $I_y = I_{yc} + A d_x^2$

$I_{y\square} = \frac{1}{12} hb^3$

$I_{y\triangle} = \frac{1}{36} hb^3$

$I_{yo} = (\frac{\pi}{4})(R)^4$.