Notebook

Control

Sec

Semester

2024

control system

1. Laplace Transforms
   - → partial fractions
   - → Matlab

2. physical systems
   - → electrical
   - → Amplifiers
   - → Mechanical

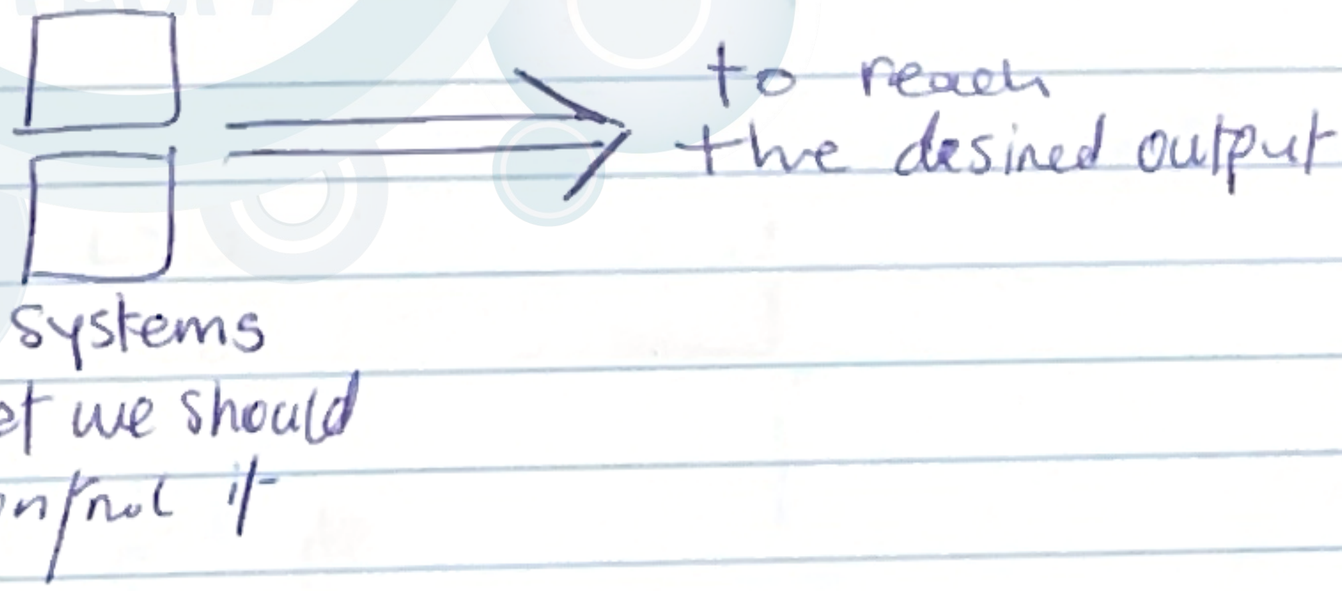
3. Mason's Rule

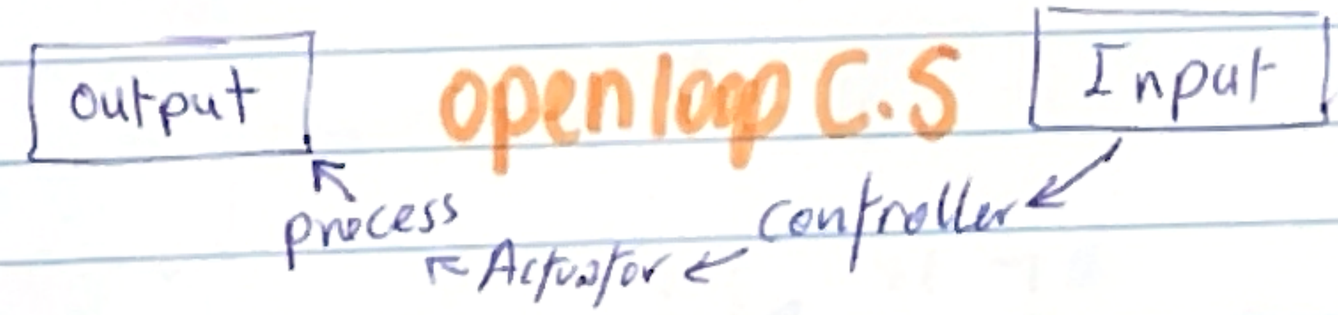
4. First & sec. order systems

5. Root locus

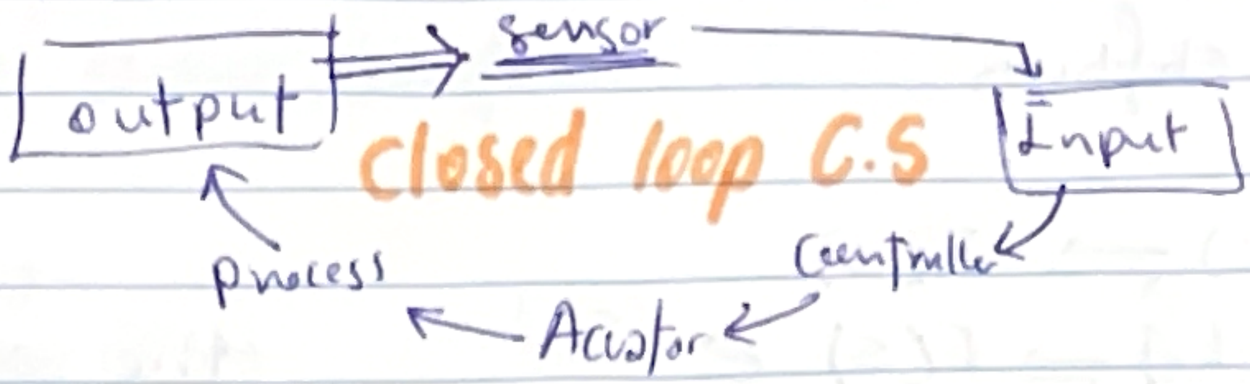
6. Matlab

7. Arduino





desired Input ← مشکلة اکی بتواجهنا بال openC.S إنو ال output مابیقدر نشیك ازا هو فعلن ←
closed loop control sys ال AC قبل



Open loop	Closed loop
- Accuracy is low	- Accuracy is high
- NOT reliable because they are not accurate	- Reliable
- Easy to design	- complex design
- Less expensive	- expensive

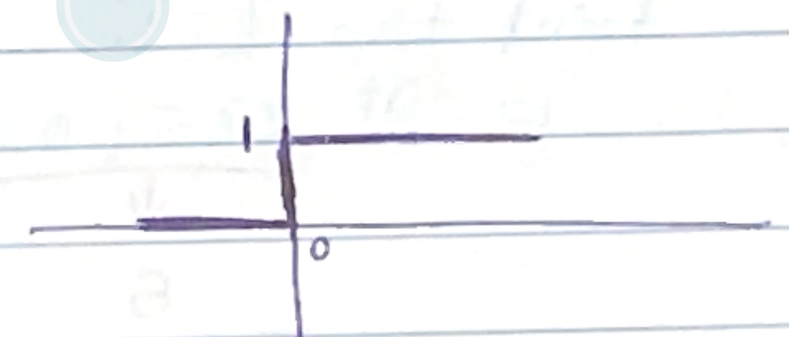
Review of laplace transform :-

When we do a mathmematecal modeling of control systems we get differential equations & the laplace transfor convert the complex ↑ to simple algebric equation

example : Find the laplace transform of unit step func.

$$f(t) = u(t)$$

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



$$F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st}\, dt$$

$$\int_{-\infty}^{0} u(t) = 0$$

$$= \int_{0}^{\infty} 1 \cdot e^{-st}\, dt$$

$$= -\frac{1}{s}\left[e^{-st}\right]_{0}^{\infty} = \frac{1}{s}$$

$$[e^{-\infty} - e^{-0}]$$

$$[-1 - 0]$$

properties of Laplace transform

1. Linearity

$$f_1(t) \rightleftharpoons F_1(s)$$
$$f_2(t) \rightleftharpoons F_2(s)$$

$$a f_1(t) + b f_2(t) \rightleftharpoons a F_1(s) + b F_2(s)$$

↓ constant

example

$$f(t) = u(t) + 2e^{-3t} + 3\sin(2t)$$
$$\frac{1}{s} + \frac{2}{s+3} + \frac{3.2}{s^2+4}$$
$$F(s) = \frac{1}{s} + \frac{2}{s+3} + \frac{6}{s^2+4}$$

2. time scaling

$$f(t) \rightleftharpoons F(s)$$
$$f(at) \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right)$$

example: If the L.T. of $f(t) \rightleftharpoons \frac{1}{s+1}$ then Find the L.T. of $f(2t)$!

$$\mathcal{F}(2t) = \frac{1}{|2|} F\left(\frac{s}{2}\right)$$
$$= \frac{1}{2}\left(\frac{1}{\frac{s}{2}+1}\right) = \frac{1}{s+2}$$

3. time shifting

$$f(t) \rightleftharpoons F(s)$$
$$f(t-t_0) \rightleftharpoons F(s) \cdot e^{-st_0}$$
$$f(t+t_0) \rightleftharpoons F(s) \cdot e^{+st_0}$$

the same sign

we can say that the shifting in time domain is the exponetial multiplication in the frequency domain with the same sign

example 8-

Find L.T. of $u(t-2)$

$$= \frac{1}{s} \cdot e^{-2s}$$

4. frequency shifting

$$f(t) \rightleftharpoons F(s)$$
$$e^{s_0 t} f(t) \rightleftharpoons F(s-s_0)$$
$$e^{-s_0 t} f(t) \rightleftharpoons F(s+s_0)$$

opposite sign

example 8

Find the L.T.

$e^{-at} \cos(\omega t)$

$F(s+s_0)$
$F(s+a)$

$$\frac{s}{s^2+\omega^2}$$

$$= \frac{s+a}{(s+a)^2+\omega^2}$$

5. time differentiation

$$f(t) \rightleftharpoons F(s)$$

$$\frac{d}{dt}\{f(t)\} \rightleftharpoons s.F(s) - F(0^-)$$

Intial value.

Example:

$$\underbrace{\frac{d^2y(t)}{dt^2}} + 12\frac{dy(t)}{dt} + 32\,y(t) = 32u(t)$$

$$\downarrow$$

$$\left[s^2Y(s) - sY(0^-) - Y'(0^-)\right] + \left[12sY(s) - 12Y(0^-)\right]$$

$$+ 32Y(s) = \frac{32}{s}$$

$$s^2Y(s) + 12sY(s) + 32Y(s) = 32\frac{1}{s}$$

$$Y(s)\left(s^2 + 12s + 32\right) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2+12s+32)}$$

Impulse Response of an L.T.I system

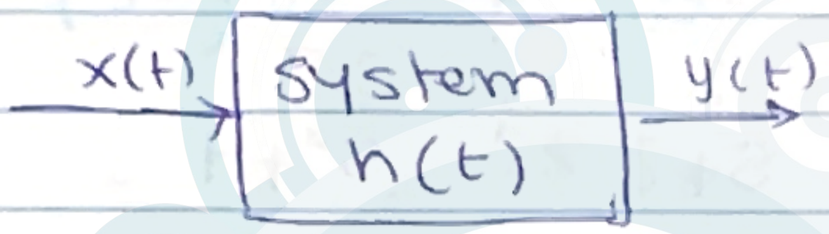


Convolution of Laplace transform

$x(t) \rightleftharpoons x(s)$

$y(t) \rightleftharpoons y(s)$

$x(t) \cdot y(t) \rightleftharpoons x(s) \cdot y(s)$



$y(t) = x(t) \cdot h(t)$

Find The I.L.T of $F(s) = \frac{1}{(s+3)^2}$

$t \rightleftharpoons \frac{1}{s^2}$

e^{-3t} ← because we are shifting +3

Find The I.L.T of $F(s) = \frac{2}{(s+1)(s+2)}$

$$\frac{2}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$\frac{2}{s+2} = A + \frac{B(s+1)}{s+2} \qquad \text{If } s = -1$$

$$2 = A + \frac{1}{1} \longrightarrow A = 2$$

$$\frac{2}{s+1} = B + \frac{A(s+2)}{s+1} \qquad s = -2$$

$$-2 = B + -A \longrightarrow B = -2$$

$$F(s) = \frac{2}{s+1} + \frac{(-2)}{s+2}$$

$$2e^{-t} + -2e^{-2t}$$

$$F(s) = \frac{10}{s.(s+2)(s+3)^2}$$

$$\frac{10}{s.(s+2)(s+3)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{(s+3)^2}$$

$s=0$ $$\frac{10}{(s+2)(s+3)^2} = A + \frac{B(s)}{s+2} + \frac{C(s)}{s+3} + \frac{D(s)}{(s+3)^2}$$

$$A = \frac{5}{9}$$

$s=-2$ $$\frac{10}{s(s+3)^2} = B + \frac{A(s+2)}{s} + \frac{C(s+2)}{(s+3)} + \frac{D(s+2)}{(s+3)^2}$$

$$B = -5$$

multiply by 1

$$\frac{5s}{24} = \frac{5/9}{1} + \frac{-5}{3} + \frac{C}{4} + \frac{D}{16}$$

$$\frac{C}{4} + \frac{D}{16} = \frac{95}{72}$$

multiply by -1

$$\frac{-5}{2} = \frac{5/9}{-1} + \frac{-5}{1} + \frac{C}{2} + \frac{D}{4}$$

$$\frac{55}{18} = \frac{C}{2} + \frac{D}{4}$$

$$C = \frac{40}{9}, \quad D = \frac{10}{3}$$

$$\frac{\frac{5}{9}}{s} + \frac{-5}{s+2} + \frac{\frac{40}{9}}{s+3} + \frac{\frac{10}{3}}{(s+3)^2}$$

$$\frac{5}{9} + -5e^{-2t} + \frac{40}{9}e^{-3t} + \frac{10}{3}te^{-3t}$$

Final value theorem

$$f(\infty) = \lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$$

→ Given $F(s)$

I will multiply it by (s)

then $s = 0$

INITIAL VALUE THEOREM

$$f(0) = \lim_{s \to \infty} sF(s) = \lim_{t \to 0} f(t)$$

→ Given $F(s)$

I will multiply by (s)

then $s = \infty$

PARTIAL FRACTION

Case 1 unrepeated real factor

$$\frac{}{(s+3)(s+4)}$$

Case 2 repeated real factor

$$\frac{}{(s+3)^3}$$

Case 3 unrepeated complex factor

Case 4 repeated complex factor

$$Y(t) = \frac{1}{S(S^2+1)}$$

$$\frac{1}{S(S^2+1)} = \frac{A}{S} + \frac{BS+C}{S^2+1}$$

$$A(S^2+1) + (BS+C)(S) = 1$$

$$AS^2 + A + BS^2 + CS = 1$$

$$(A+B)S^2 + CS + A = 1$$

$$A = 1$$

$$CS = 0 \rightarrow C = 0$$

$$A + B = 0$$

$$1 + B = 0 \rightarrow B = -1$$

$$\frac{1}{S} + \frac{-S}{S^2+1}$$

$$1 - \cos(t)$$ ✓

$$\ddot{x} + 2x = u(t)$$

$$\left[S^2 x(S) - S x(0) - \dot{x}(0)\right] + 2\left[x(S)\right] = \frac{1}{S}$$

$$x(S)\left[S^2 + 2\right] = \frac{1}{S}$$

$$X(S) = \frac{1}{S(S^2+2)} = \frac{A}{S} + \frac{BS+C}{S^2+2}$$

$$A(S^2+2) + (BS+C)(S) = 1$$

$$AS^2 + 2A + BS^2 + CS = 1$$

$$(A+B)S^2 + CS + 2A = 1$$

$$2A = 1 \rightarrow A = \frac{1}{2}$$

$$CS = 0 \rightarrow C = 0$$

$$A + B = 0 \rightarrow B = -\frac{1}{2}$$

$$\frac{\frac{1}{2}}{S} + \frac{-\frac{1}{2}S}{S^2+2}$$

$$\frac{1}{2} - \frac{1}{2}\cos t$$

$$Y(s) = \frac{1}{s^2+2s+3}$$, Find the Final value.

$$\lim_{s \to 0} \frac{s}{s^2+2s+3} \quad \frac{0}{0+2(0)+3} = 0$$

Revise Chapter 1

- Laplace tables → $f(t) \overset{\mathcal{L}}{\rightleftharpoons} F(s)$
- Matlab codes → $f(t)$ Laplace
  - $F(s)$ i Laplace
- Intial value → $F(s)$ → multiply By s then $s = \infty$
- Intial value → $F(s)$ → multiply By s then $s = 0$
- Partial fraction
  - unrepeated real root
  - repeated real ROOT
  - unrepeated complex root
- Applying Laplace
  - $\mathcal{L}(x) = X(s)$
  - $\mathcal{L}(\dot{x}) = sX(s) - x(0)$
  - $\mathcal{L}(\ddot{x}) = s^2X(s) - sx(0) - \dot{x}(0)$

Chapter 2: physical systems

ELECTRICAL SYSTEM

(3 elements)

- Resistors R → $Z_R = R$
- capacitors C → $Z_C = \frac{1}{CS}$
- Inductors L → $Z_L = LS$

Series connection

$= R + \frac{1}{CS} + LS$

parallel connection

$= \frac{1}{Z_{eq}}$

$Tf = \frac{V_o(s)}{V_i(s)}$, (transfer function)

$V_R = IR = I(s) \cdot R$

$V_C = \frac{1}{C}\int i(t)\,dt = I(s) \cdot \frac{1}{SC}$

$V_L = \frac{di(t)}{t} \cdot L = I(s) \cdot S \cdot L$

$V = I(s) \cdot Z$

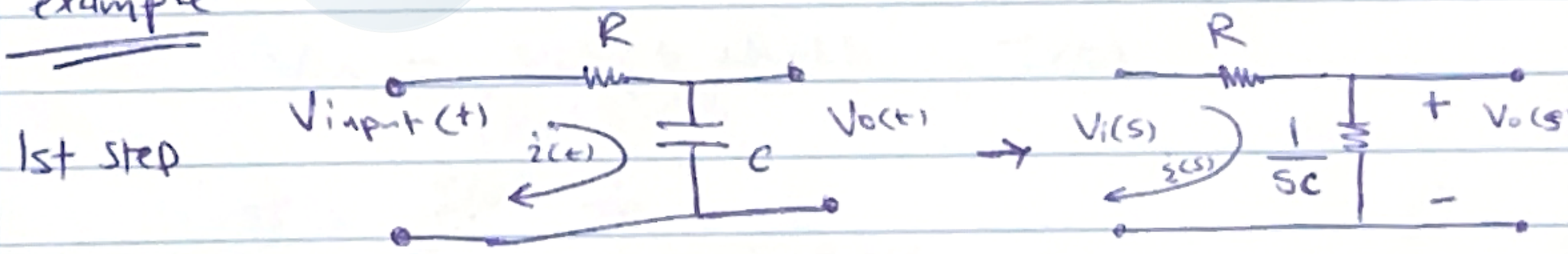
$V_{out\text{-}put}$ $V_{in\text{-}put}$

$V_o = Z_e \cdot I(s)$ $V_i = Z_{eq} \cdot I(s)$

معيه

if they give us function (t) → I should convert it to (Z) function.

example



1st step

2nd step $V_o = I(s) \cdot \frac{R}{SC}$

in Series
$I_R = I_C$

3rd step $V_{in} = I(s) \cdot Z_{eq} = \frac{(R + \frac{1}{SC}) \cdot I(s)}{\frac{R \cdot I}{SC}}$

4th $Tf = \frac{V_o(s)}{V_i(s)} = \frac{I(s) \cdot \frac{1}{SC}}{(R + \frac{1}{SC}) \cdot I(s)}$ $\frac{\frac{1}{SC}}{\frac{RSC+1}{SC}} = \frac{1}{1+RSC}$

example

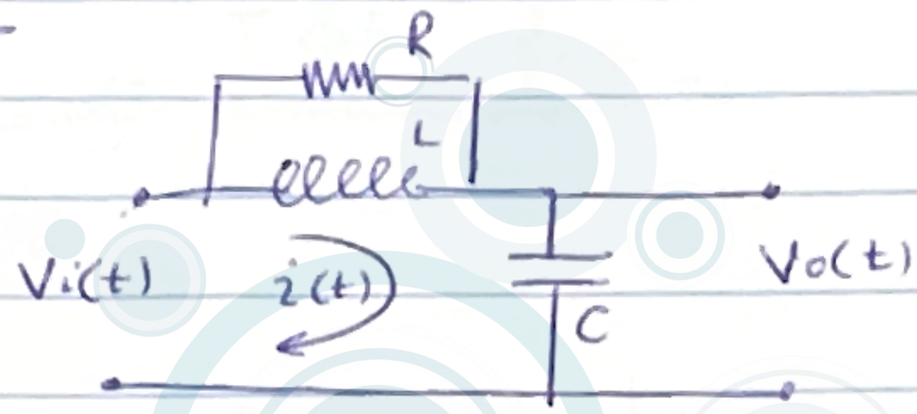
$S = -3$

$S = \pm \frac{1}{RC} = \pm 3$

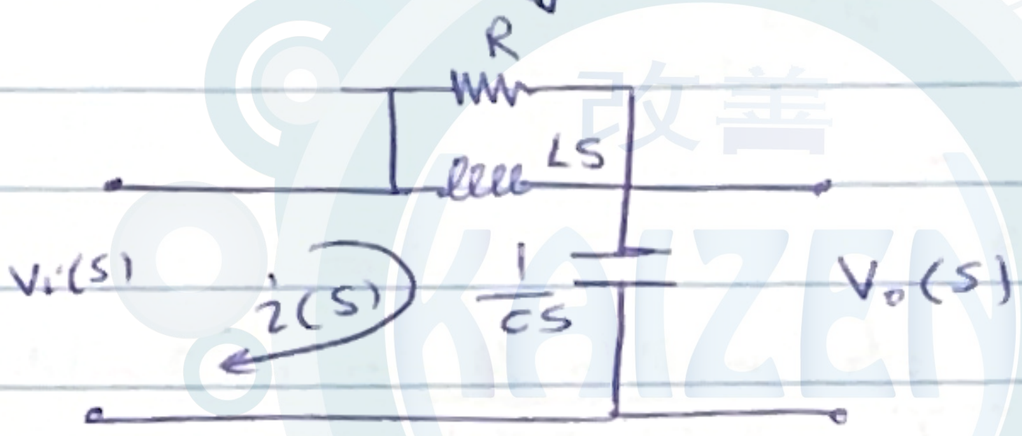
$RC = \frac{1}{3}$ If $R = 1M\Omega$

$C = 333\,pF$

example



convert it to f(s)



$V_o = I(s) \cdot \frac{1}{CS}$

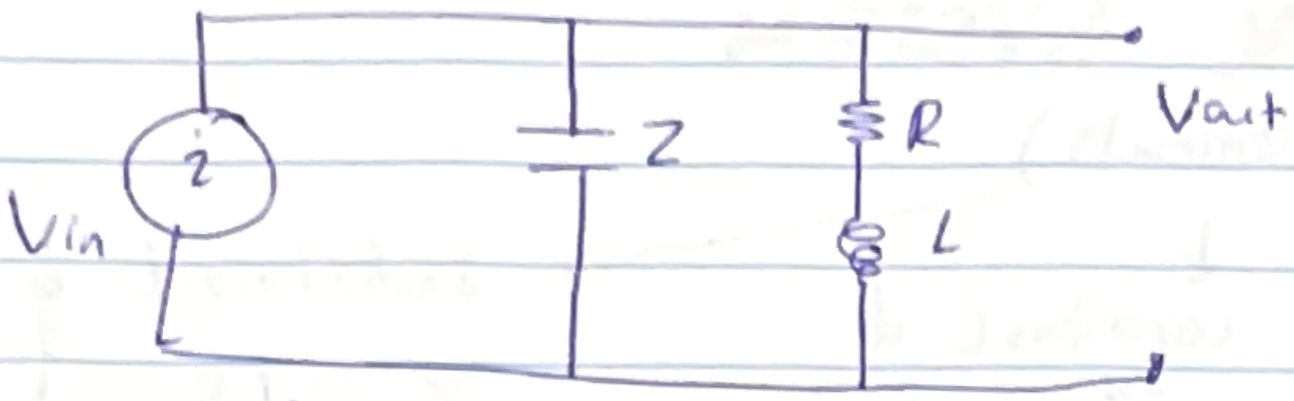
$V_{in} = Z_{eq} \cdot I(s)$

$Z_{eq} = \frac{RLS}{R+LS} + \frac{1}{CS}$

$V_{in} = \frac{RLSCS + R + LS}{(R+LS)CS} \cdot I(s)$

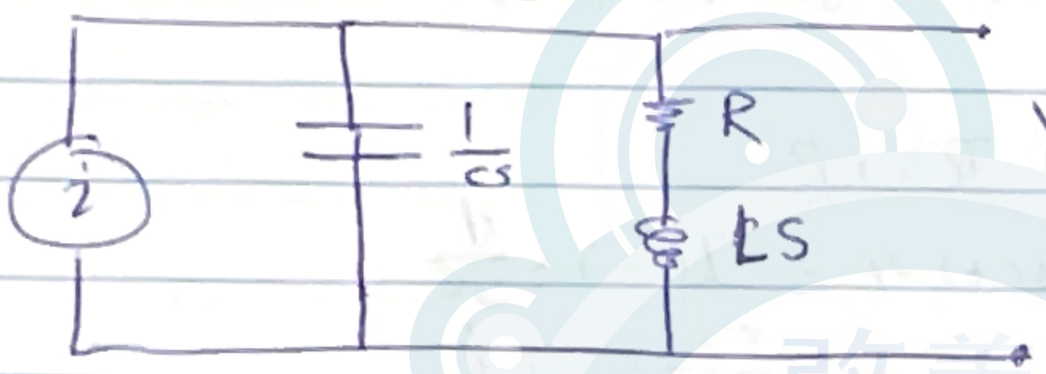
$Tf = \frac{\frac{I(s)}{CS}}{I(s) \cdot \frac{RLCS^2 + R + LS}{(R+LS)CS}}$

Example



When I saw this type of q.'s I Know that TF = 1

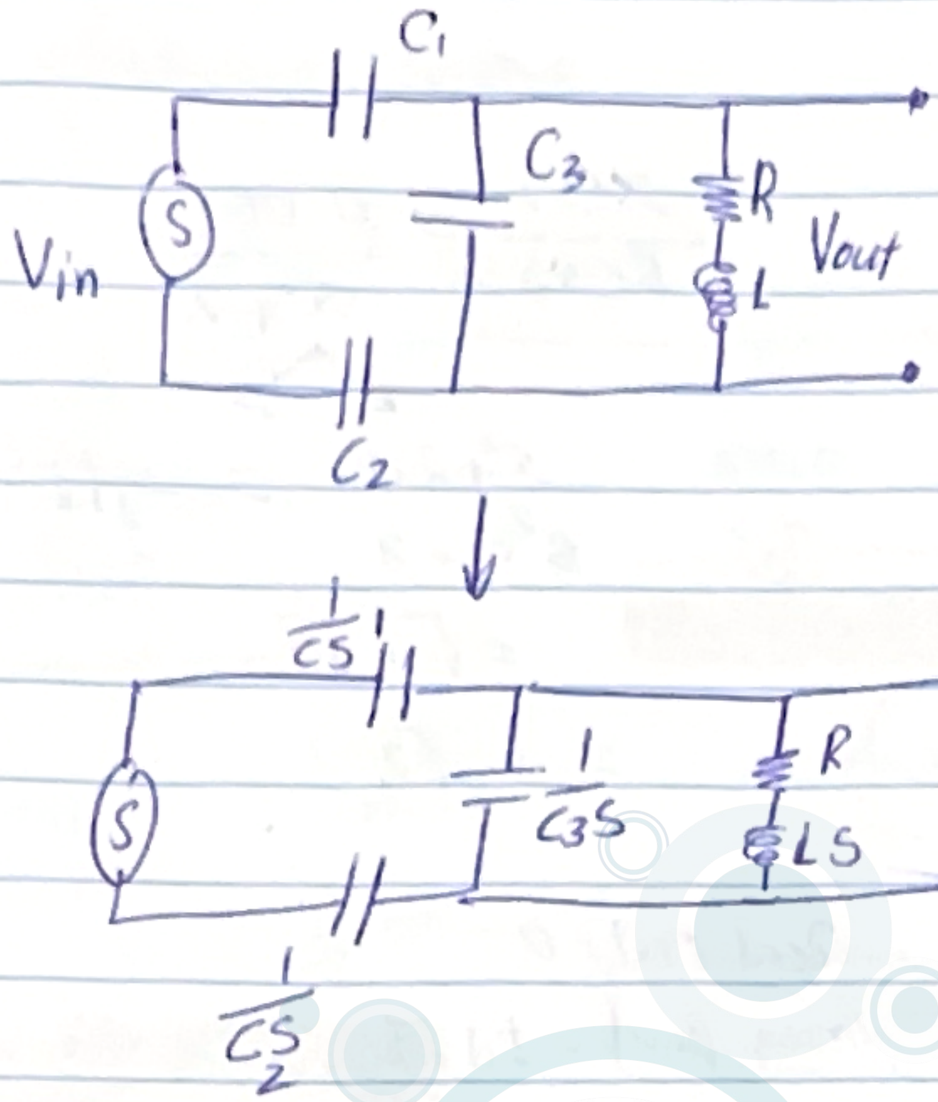
convert it to f(s)



$$V_{out} = I(s) \cdot Z_{eq}$$

$$\frac{(R+LS) * \frac{1}{CS}}{R+LS+\frac{1}{CS}} \cdot I(S) = V_{input}$$

then $\frac{V_{o(s)}}{V_{i(s)}} = 1$



$$V_{out}(S) = \frac{(R+LS)\times\frac{1}{C_3S}}{(R+LS)+\frac{1}{C_3S}} = \frac{\frac{R}{C_3S}+\frac{LS}{C_3S}}{(R+LS)+\frac{1}{SC_3}} = \frac{R+L}{C_3S(R+LS)+1}\times I(S)$$

$$V_{in}(S) = \frac{R+LS+\frac{1}{C_3S}}{(R+LS)+\frac{1}{C_3S}} + \frac{1}{CS_1} + \frac{1}{CS_2}$$

$$Tf = \frac{V_{out}}{V_{in}} = \frac{\frac{(R+LS)\frac{1}{C_3S}}{(R+LS)+\frac{1}{C_3S}}}{\frac{R+LS+\frac{1}{C_3S}}{(R+LS)+\frac{1}{C_3S}} + \frac{1}{CS_1} + \frac{1}{CS_2}}$$

$$= \frac{R+LS+\frac{1}{C_3S}}{(CS_1+CS_2)(R+LS)+\frac{CS_1}{C_3S}+CS_2}$$

$$= \frac{CS_1R+CS_2R+CS_1L+C_2S^2+\frac{CS_1+CS_2}{C_3S}}{C_1SR+C_2SR+\frac{C_1S}{C_3S}+\frac{C_2S}{C_3S}} + \frac{R+LS+\frac{1}{C_3S}}{CS_1R+C_1S^2L}$$

+

Mechanical systems

3 elements

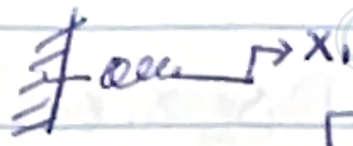
- spring
- mass
- damper

spring

free spring



fixed spring



$F_s = k\Delta x$ (free)

$F_s = kx$ (fixed)

mass



$f_M = M\ddot{x}$

damper



Fixed (x_1)

$F_B = B(\dot{x}_1 - \dot{x}_2)$

free (x_1, x_2)

$F_B = B\dot{x}$

no. of x's ← diff eqn. ال رتبة

in the system

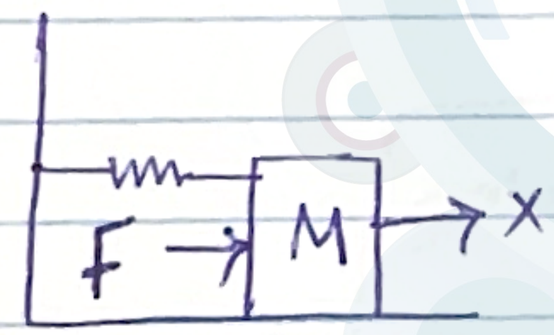


بتكون مكونة من معادلات قوة

- damper + Mass ← الثابت
- Spring + Mass ← الثابت
- damper + f ← الثابت
- spring + F ← الثابت

لكل Mass معادلة قوة

no frictions = no damper





اذا كانت الفورس للمين
كل الفورسز اللي بتكون للسيستم

FBD



$$F = F_s + f_M$$

$$F = Kx + M\ddot{x}$$

$$F(s) = Ms^2X(s) + K\,x(s)$$

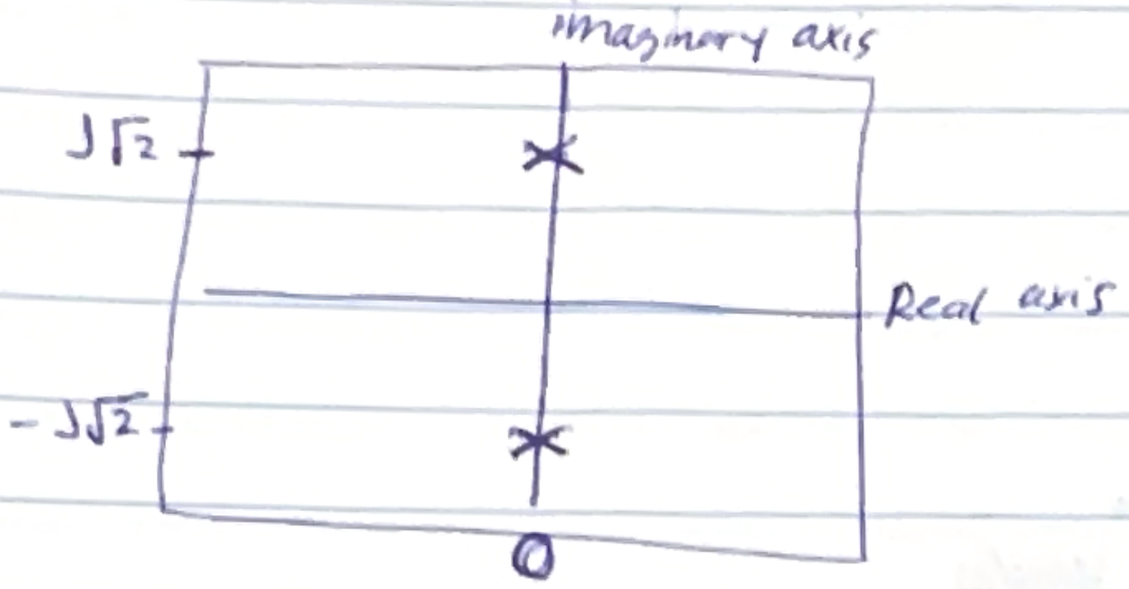
$$Tf = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K} = \frac{1}{1000s^2 + 2000}$$

بنعوض بالأرقام

$$= \frac{\frac{1}{1000}}{s^2 + 2}$$

We may asked to find the transfer function
& M = 1000 kg, K = 2000 Nm⁻¹

pole



$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2+2}$$

$s^2+2=0$ $= -j\sqrt{2}$

$s^2 = -2$

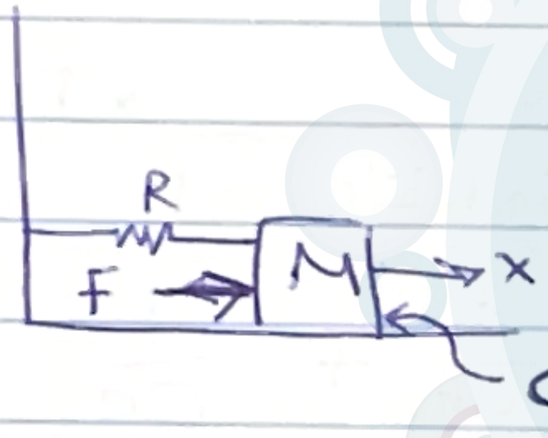
$= \sqrt{-1}\sqrt{2}$

$= +j\sqrt{2}$

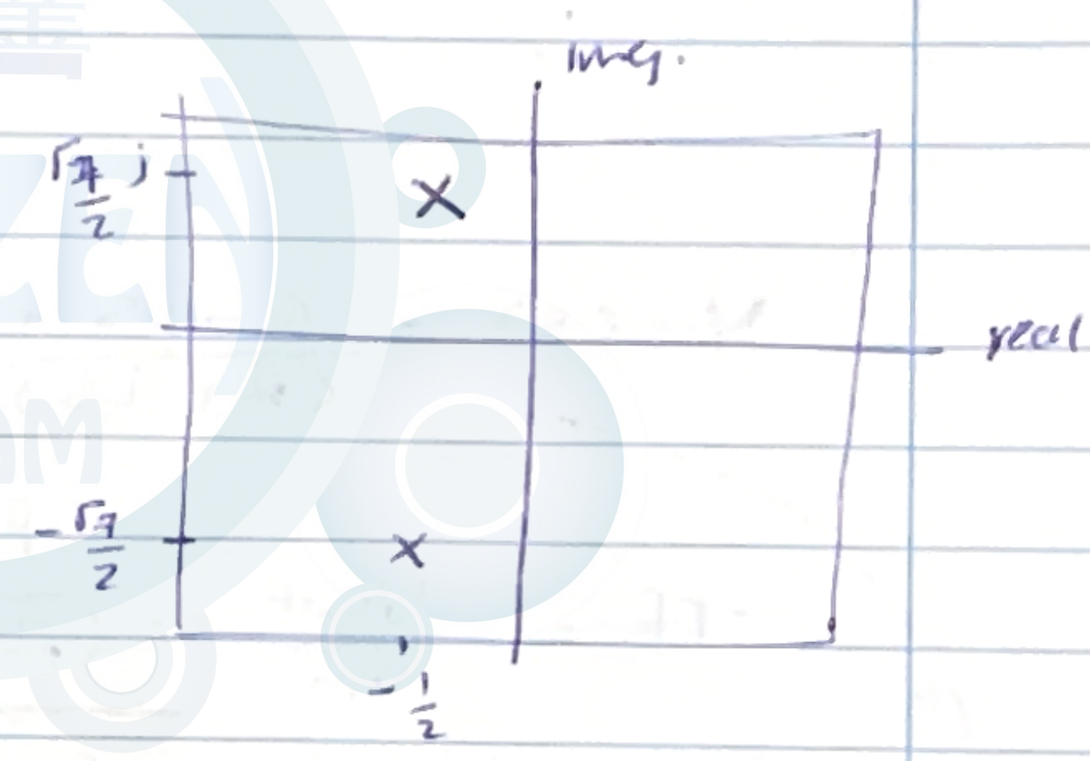
Real part = 0

imag. part = $\pm j\sqrt{2}$

example2







$$F = f_S + f_M + f_D$$

$$F(s) = Kx + M\ddot{x} + D\dot{x}$$

$$F(s) = KX(s) + MS^2X(s) + DSX(s)$$

$$X(s)\{K + Ms^2 + Ds\}$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Ds+K}$$

$M = 1000$ kg

$K = 2000$ N m^{-1}

$C = 1000$

$$\frac{1/1000}{\frac{1000}{1000}s^2 + \frac{1000}{1000}s + \frac{2000}{1000}} = \frac{.001}{s^2+s+2}$$

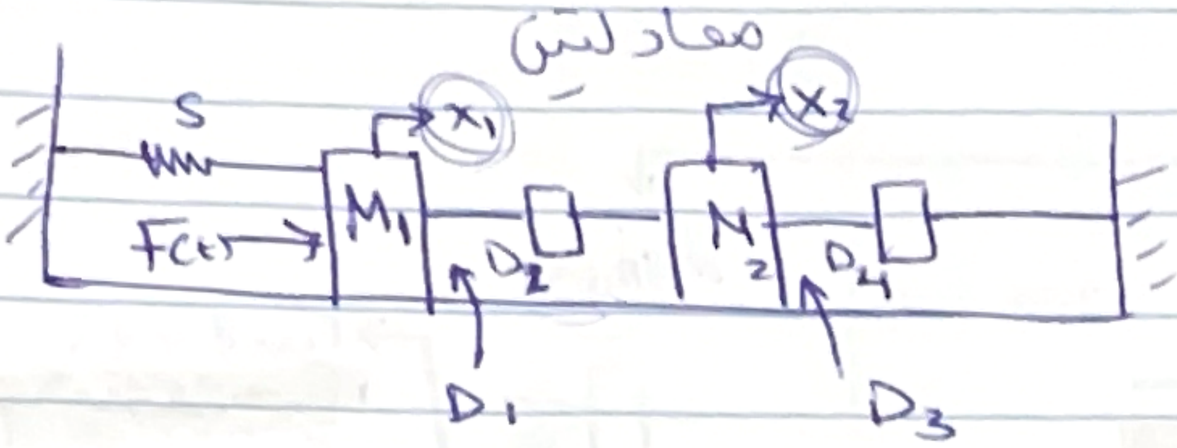
$s^2+s+2=0$

$$\frac{-b \pm \sqrt{-4ac+b^2}}{2a}$$

ازاحة عن السيار

$$-\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

example 6



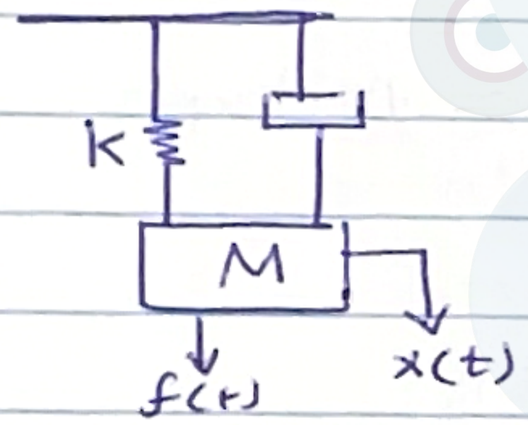
$$f(t) = kx_1 + M\ddot{x}_1 + D_1\dot{x}_1 + D_2(\dot{x}_1 - \dot{x}_2)$$

$$F(s) = kX_1(s) + Ms^2X_1(s) + D_1sX_1(s) + D_2s(X_1(s) - X_2(s))$$

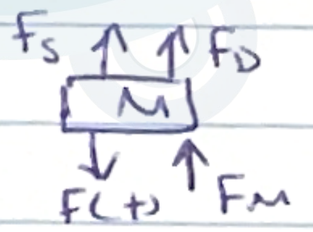
$$0 = M_2\ddot{x}_2 + D_2(x_2 - x_1) + D_3\dot{x}_2 + D_4\dot{x}_2$$

$$0 = M_2s^2X_2(s) + D_2s(X_2(s) - X_1(s)) + D_3sX_2(s) + D_4s(X_2(s))$$

Example 7



FBD

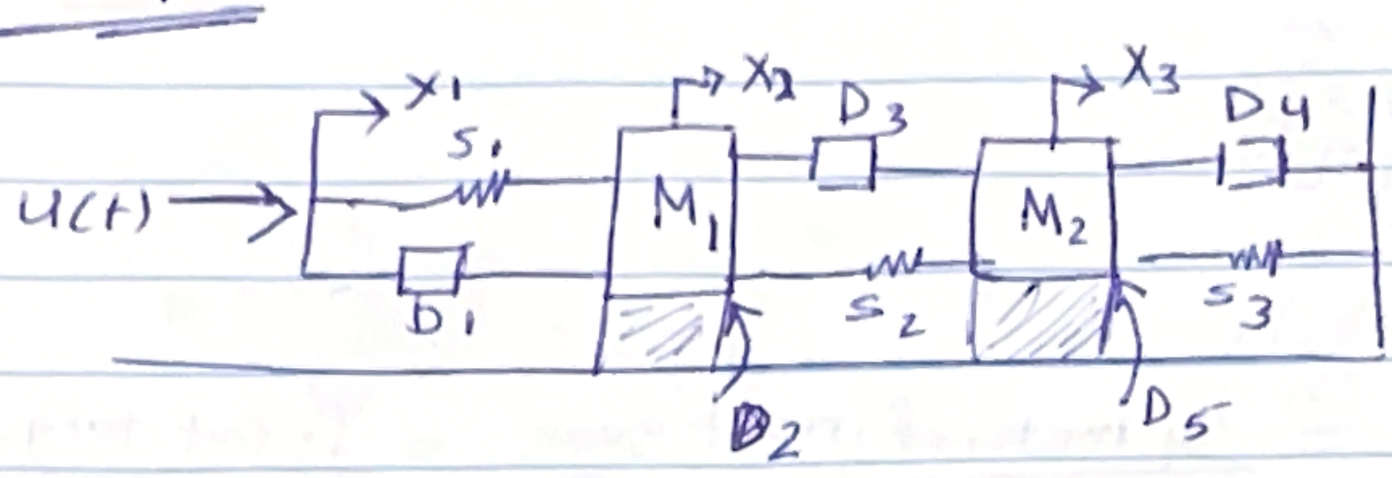


$$F(t) = kx_1 + M\ddot{x}_1 + D\dot{x}_1$$

$$F(s) = kX(s) + Ms^2X(s) + DsX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

example 8



3 equations

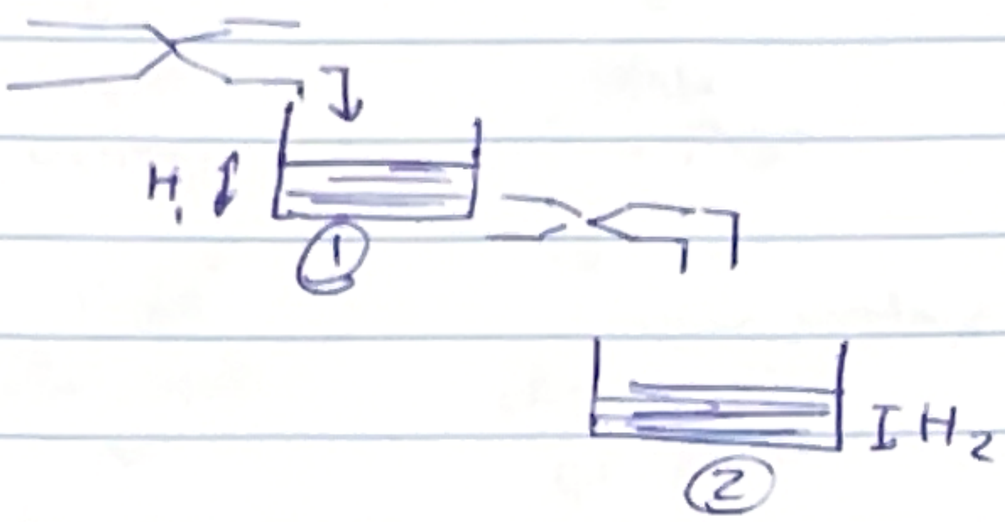
$F(t) = K(x_1 - x_2) + D_1(\dot{x}_1 - \dot{x}_2) +$ ~~[illegible]~~

$$0 = M_1 \ddot{x}_2 + D_2 \dot{x}_2 + K_2(x_2 - x_3) + D_3(\dot{x}_2 - \dot{x}_3) + K_1(x_2 - x_1) + D_1(\dot{x}_2 - \dot{x}_1)$$

$$0 = D_3(\dot{x}_3 - \dot{x}_2) + K_2(x_3 - x_2) + D_5 \dot{x}_3 + M_2 \ddot{x}_3 + K_3(x_3) + D_4(\dot{x}_3)$$

KAIZEN TEAM

Tanks



① عند كل تانك المعادلة

$$\frac{C\,dh}{dt} = q_{in} - q_{out}$$

② عند كل Valve المعادلة

$$\frac{C_1\,dh}{dt} = q_1 - q_2$$

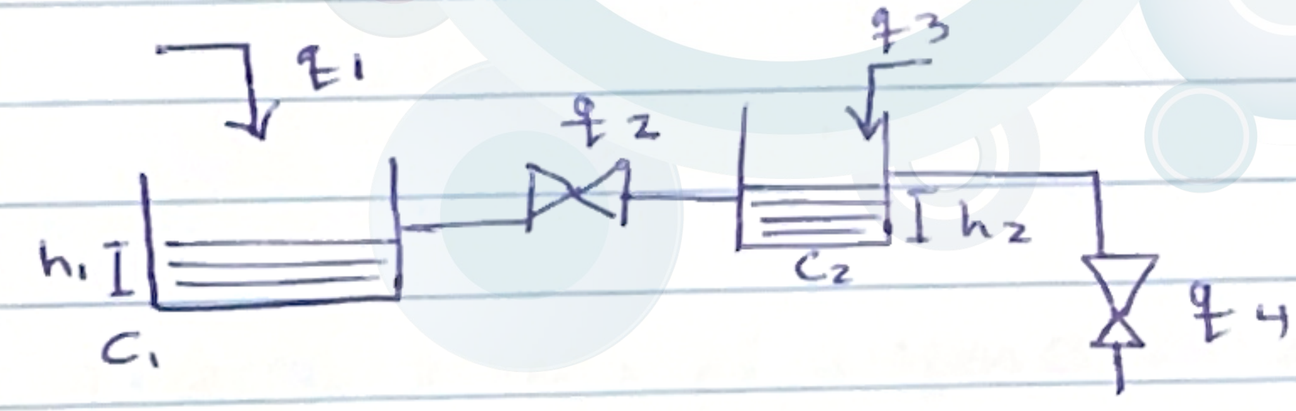
$$\frac{C_2\,dh}{dt} = q_2 - 0$$

$$R = \frac{\Delta h}{q}$$

$$R_1 = \frac{h_1 - h_0}{q_1}$$

$$R_2 = \frac{h_2 - h_1}{q_2}$$

example 1



$$\frac{C_1\,dh}{dt} = q_1 - q_2$$

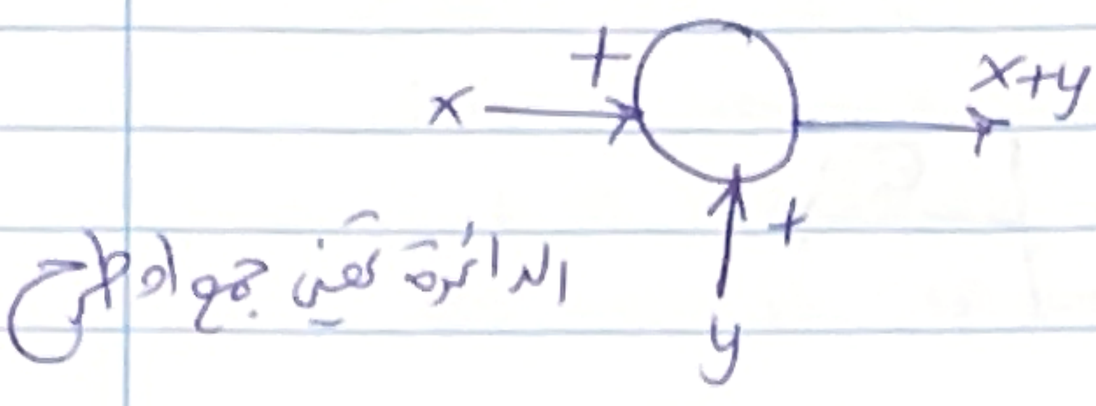
$$\frac{C_2\,dh}{dt} = (q_2 + q_3) - q_4$$

in — out

$$R_1 = \frac{h_1 - h_0}{q_1}, \quad R_2 = \frac{h_2 - h_1}{q_2}, \quad R_3 = \frac{h_3 - h_0}{q_3}$$

$$R_4 = \frac{h_2 - 0}{q_4}$$

(Block diagram reduction, signal flow chart)



الدائرة تعني جمع او طرح



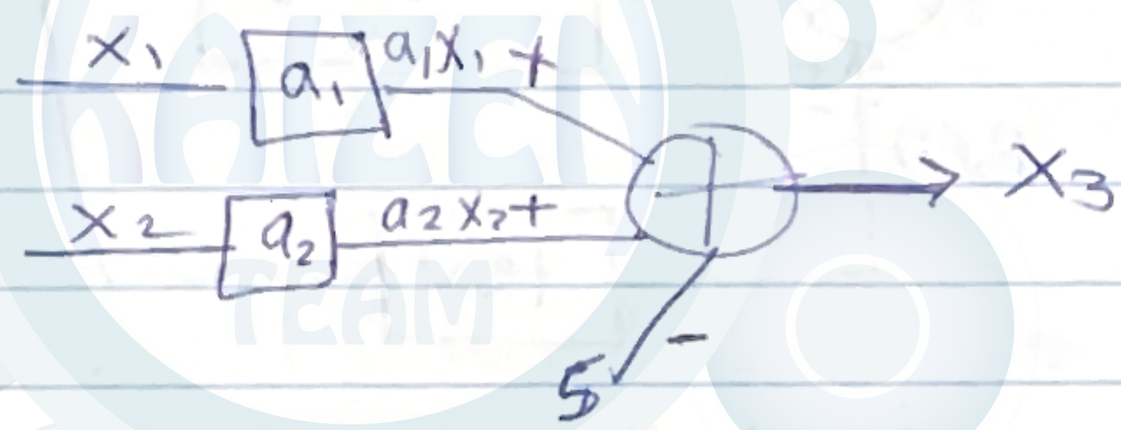
to distribute the signal to several branches.

الفاريابل ياتي الكرمن خاكتو

example1

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$

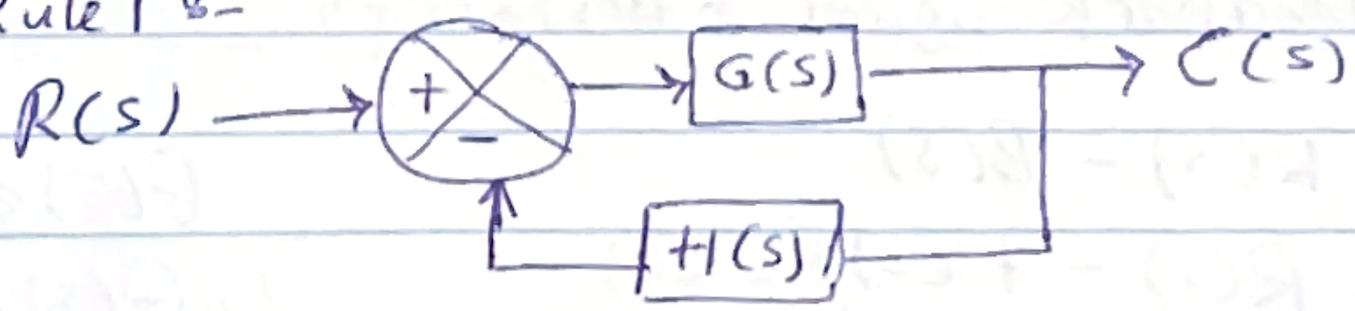
transfer function



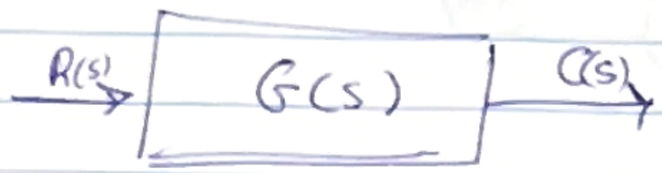
Reduction of Multiple Subsystem

- When the system is complex (having multiple subsys) we need to reduce the complex block digigram representation into a single block.

Rule 1 :-



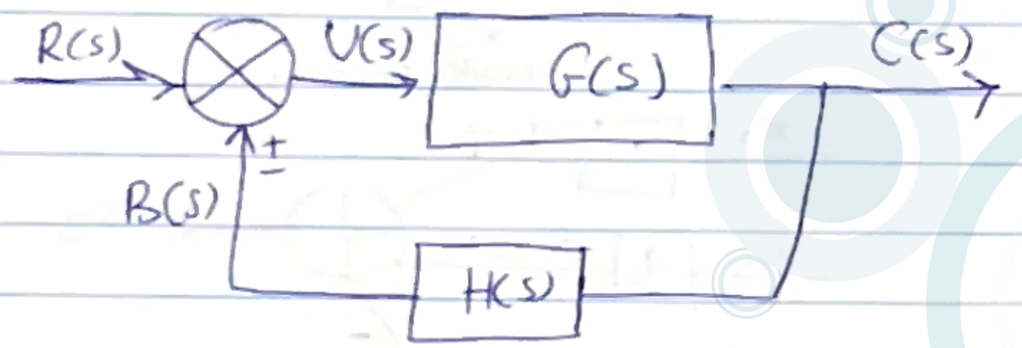
Transfer function of an OPEN SYS.



$$C(s) = G(s) * R(s)$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{\text{output}}{\text{input}}$$

Transfer Function of a closed SYS.



positive feedback , negative feedback

if negative feedback :-

U(s) = Actuating Signal

error signal in case of unity feedback sys

B(s) = feedback signal = H(s) * C(s)

$$U(s) = R(s) - B(s)$$

$$U(s) = R(s) - H(s) * C(s)$$

$$C(s) = U(s) * G(s)$$

$$C(s) = [R(s) - H(s) * C(s)] * G(s)$$

$$\frac{G(s)}{1 + G(s) * H(s)} = \frac{C(s)}{R(s)}$$

what we mean by zeros & poles

zero : the zero of a transfer function are the frequencies (value of s) for which the numerator of the transfer function becomes zero

poles : poles of a transfer function are the frequencies (value of s) for which the denominator of the TF becomes zero

example

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = \frac{dx(t)}{dt} + 2x(t)$$

where x(t) input , y(t) is the output

$$s^2y(s) + 7sy(s) + 12y(s) = sx(s) + 2x(s)$$

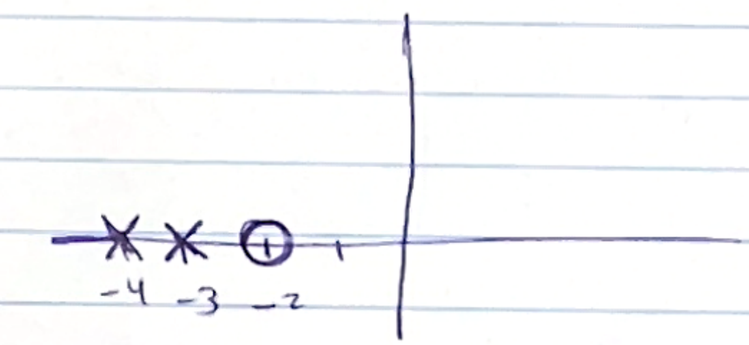
$$y(s)[s^2 + 7s + 12] = x(s)[s + 2]$$

$$\frac{y(s)}{x(s)} = \frac{s+2}{s^2+7s+12} = \frac{s+2}{(s+3)(s+4)}$$

zeros of this Tf → $s+2=0 \Rightarrow -2$

poles of this Tf → $-3, -4$

pole-zero diagram



BLOCK Diagram REDUCTION

Rule 1 : Representation of a closed loop system

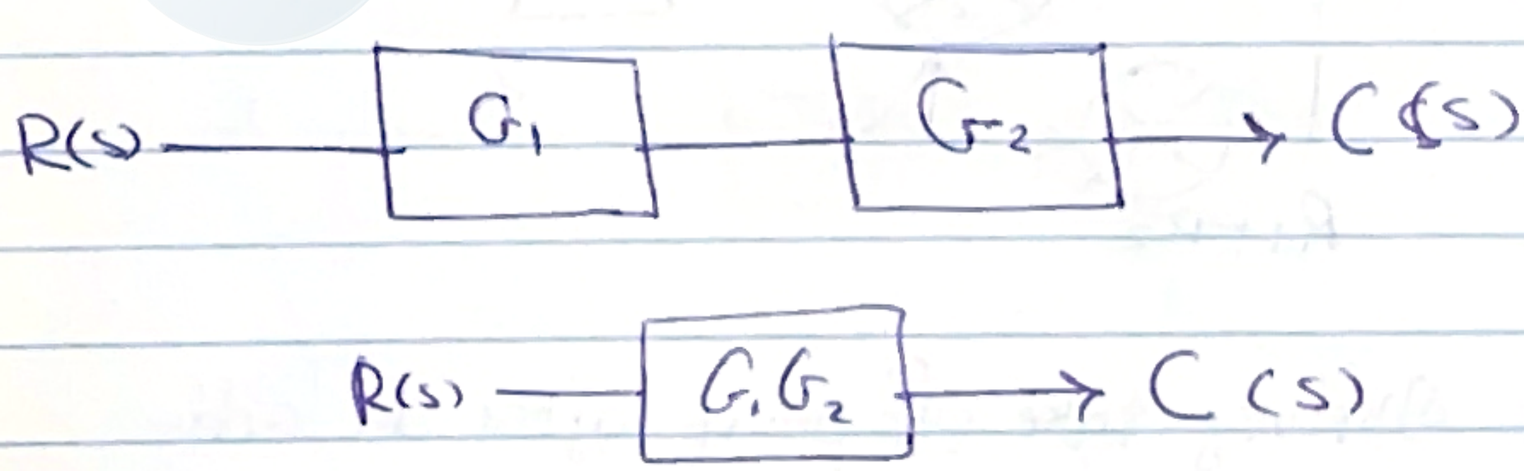


$$\frac{G(s)}{1 + G(s) \cdot H(s)} \rightsquigarrow \frac{G}{1+GH} = \frac{C(s)}{R(s)}$$

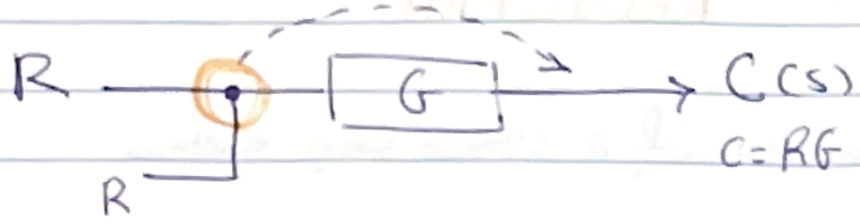


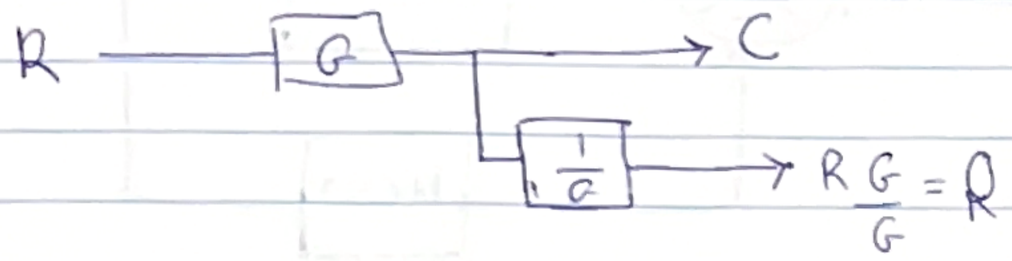
$$\frac{G}{1-GH} = \frac{C(s)}{R(s)}$$

Rule 2 : When the blocks are connected in series we should multiply them



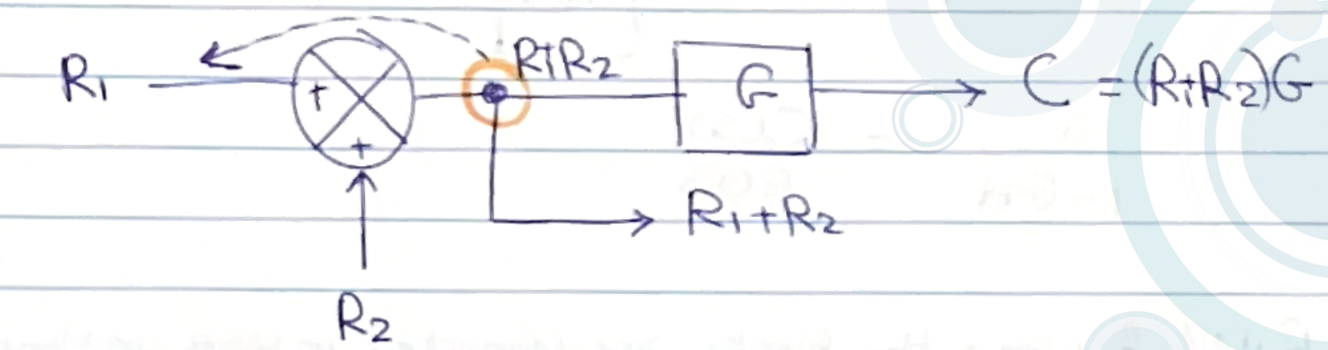
Rule 5 : Shifting of take off point after a block

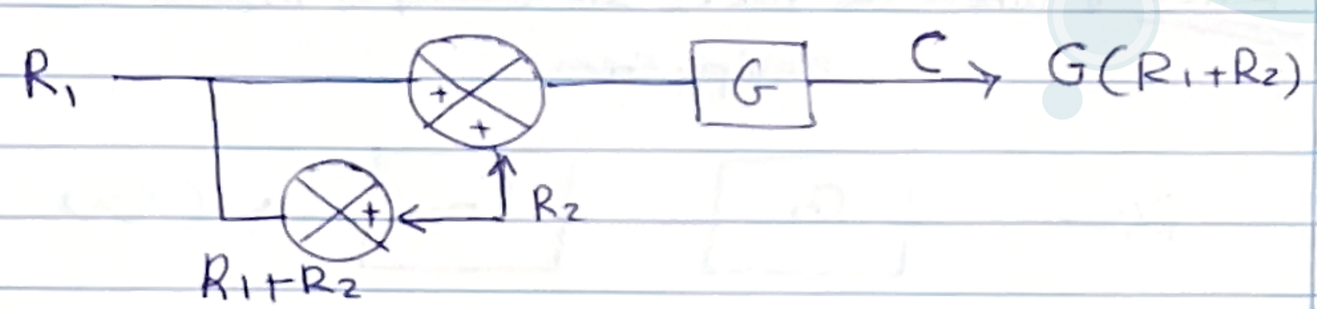




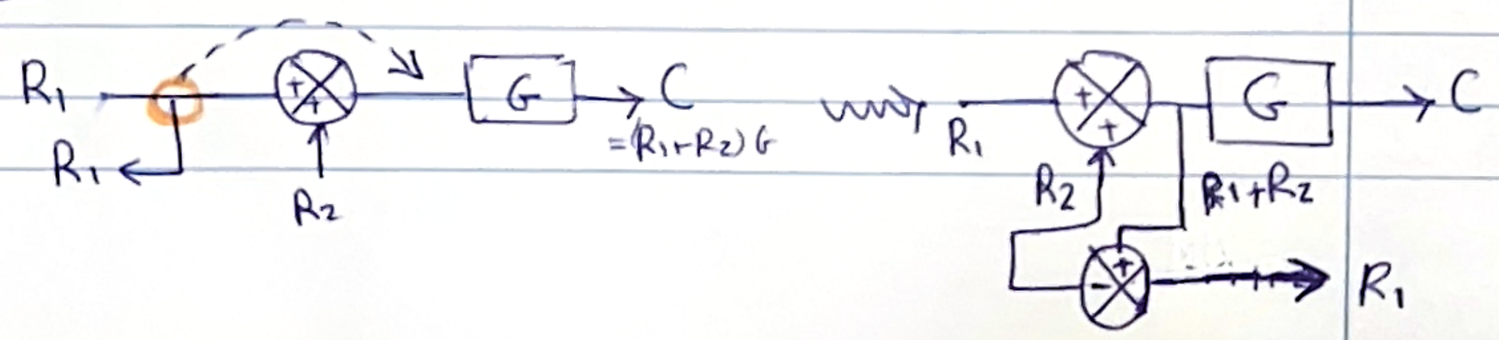
If we want to shift the take off point after the block, then in that case we should divide the take off point with the gain of this block

Rule 6 : Shifting take off point befor an adder

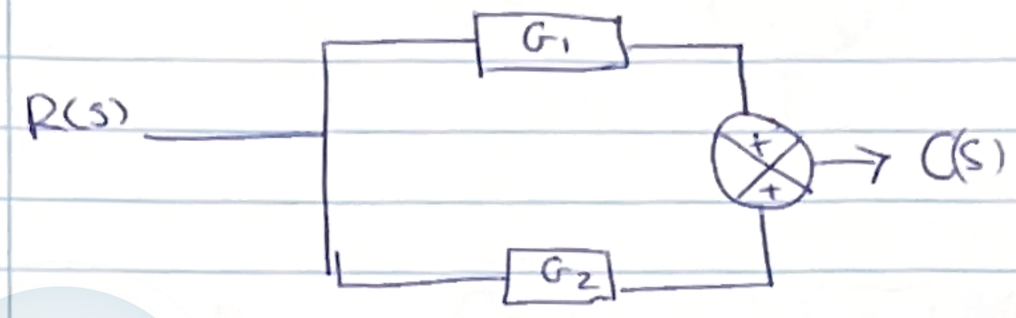


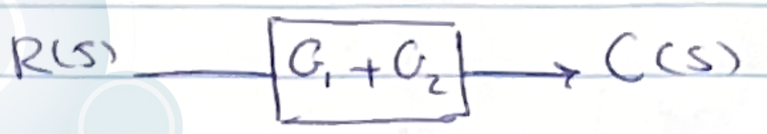


Rule 7 : Shifting take off point after an adder



Rule 3 : When blocks are added into parallel, their individual gains are added





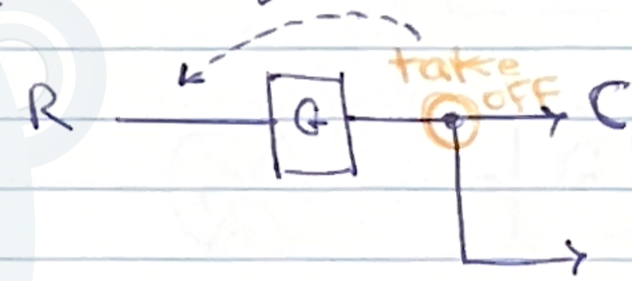
Proof :-

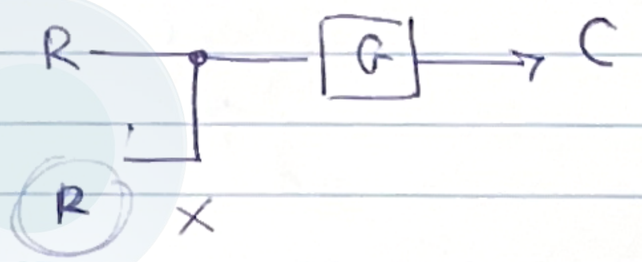
$C = RG_1 + RG_2$

$C = R(G_1 + G_2)$

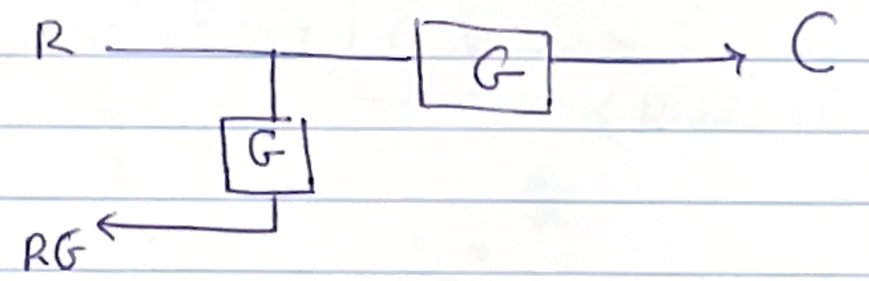
$\frac{C}{R} = G_1 + G_2$

Rule 4 : Shifting of take off points befor a block



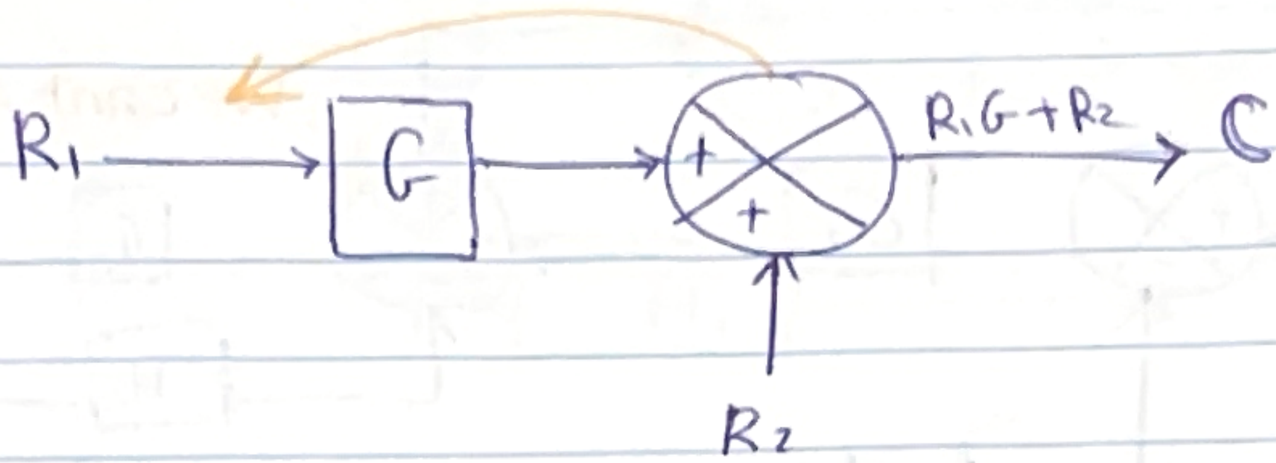


SO WE WILL ADD G



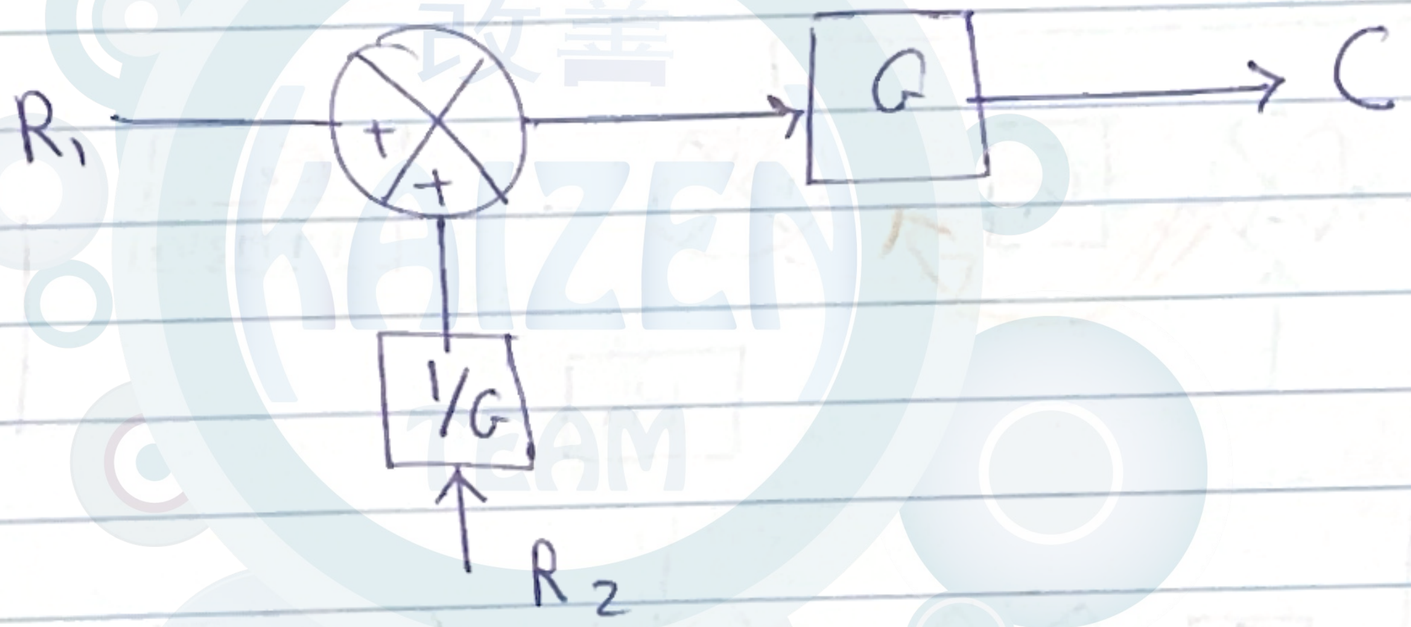
SO If we want to shift the take off point befor the block we need to multiply the take off point with the gain of this block

Rule 8 : Shifting of adder befor a block

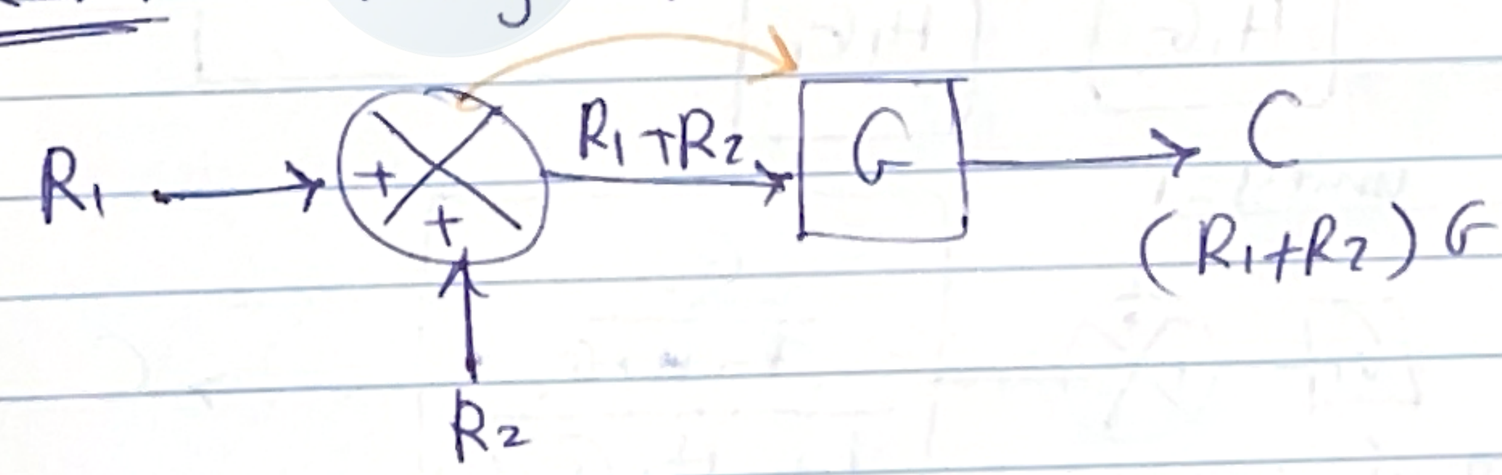


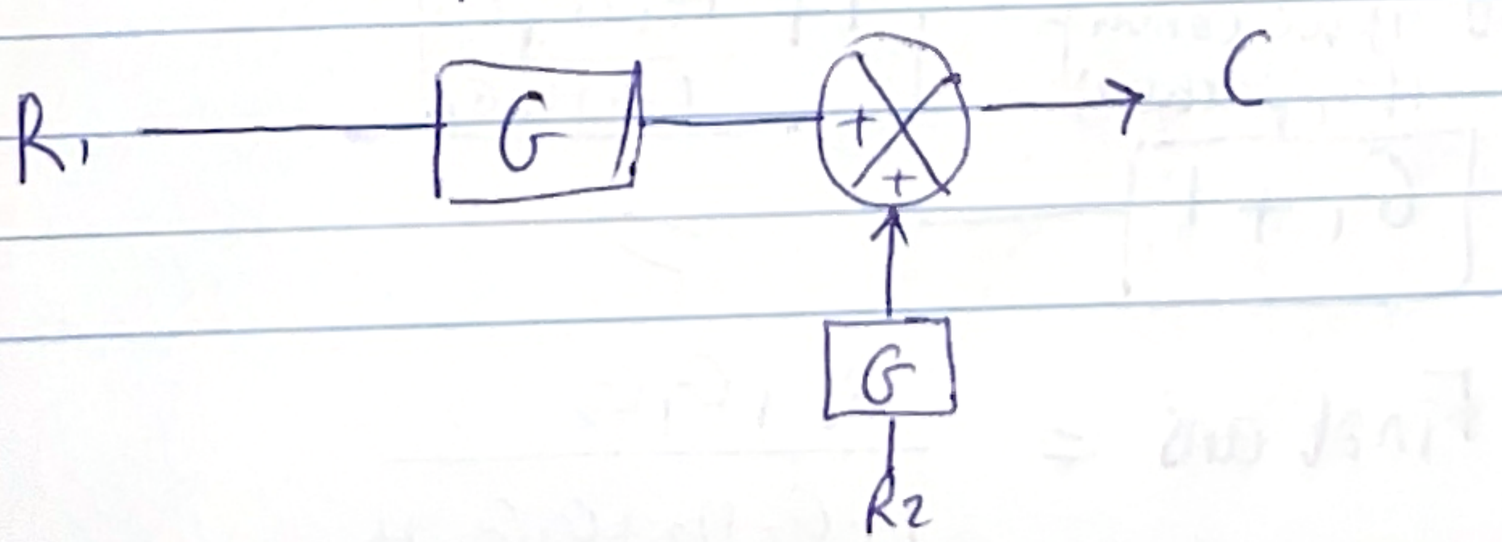


So I should multiply R_2 by $\frac{1}{G}$

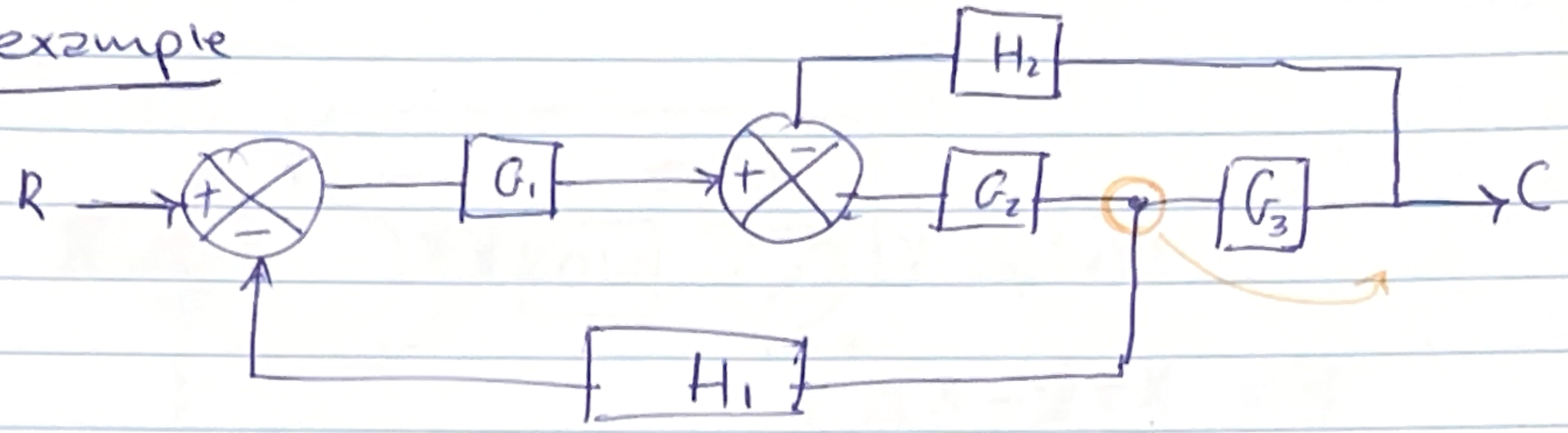


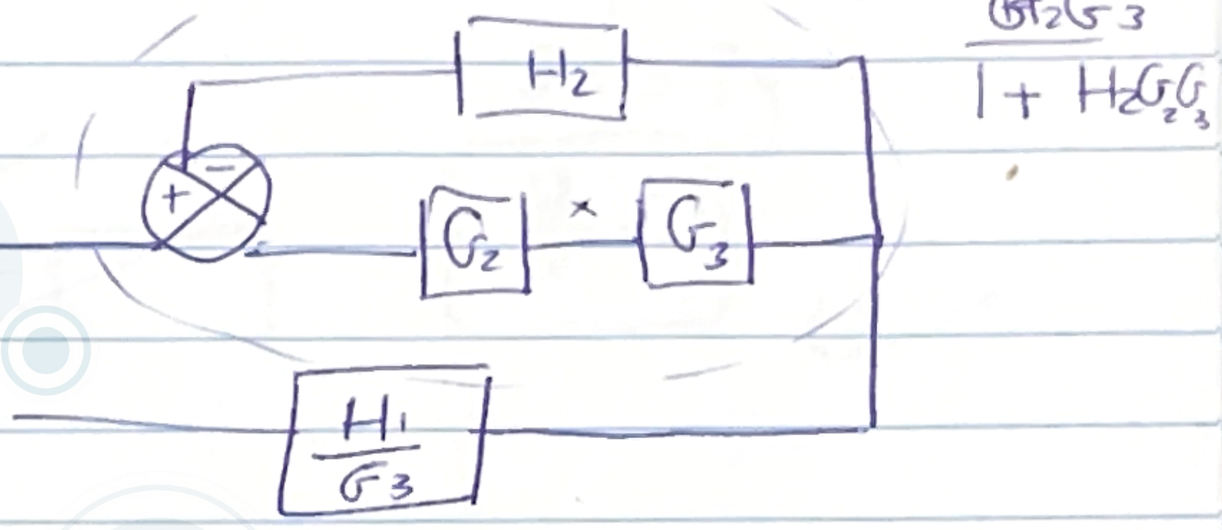
Rule 9 : Shifting of adder after a block



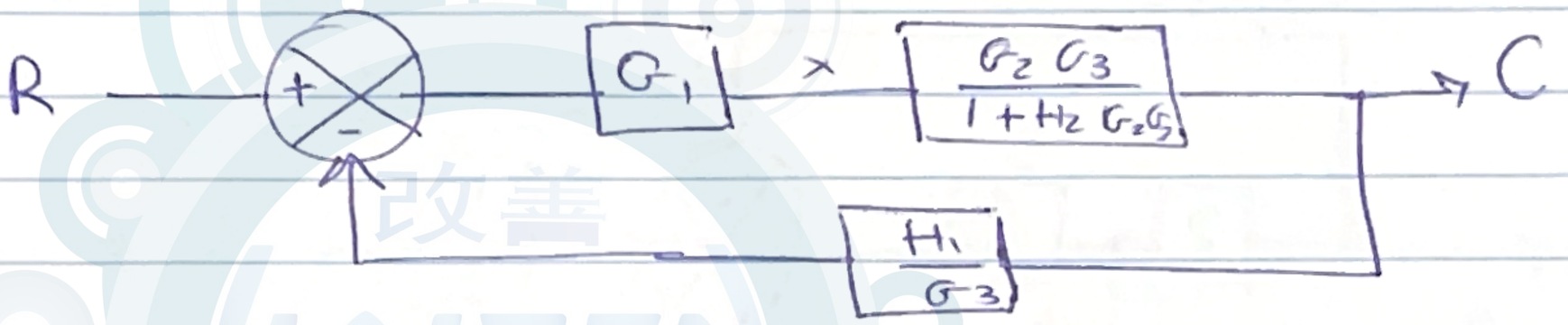


example





$$\frac{G_2G_3}{1+H_2G_2G_3}$$



$$\frac{\dfrac{G_1G_2G_3}{1+H_2G_2G_3}}{1+\dfrac{H_1}{G_3}\times\dfrac{G_1G_2G_3}{1+H_2G_2G_3}} = \frac{G_1G_2G_3}{1+H_1G_1G_2}$$

$$\frac{G_1G_2G_3}{1+H_2G_2G_3+\dfrac{H_1G_1G_2}{1+H_2G_2G_3}(1+H_2G_2G_3)}$$