Control Quizzes

Jana Howell

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s^2+1)}$$

Solution : $\frac{1}{s(s^2+1)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+1}$

$1 = K_1(s^2+1) + (K_2 s + K_3)(s)$

$s=0 \rightarrow K_1 = 1$, $s=1 : 1 = 2 + (K_2 + K_3)$.

$s=2 : 1 = 5 + (2K_2 + K_3)$ (2)

$K_2 = -3 \quad K_3 = 2$.

$1 - \cos t$.

Name: ID:

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s+1)^2}$$

$\frac{1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{(s+1)^2}$

$1 = K(s+1)^2 + K_2(s)(s+1) + K_3(s)$.

$s=0 : 1 = 4K_1 \Rightarrow K_1 = 1/4$.

$s=-1 : 1 = -1K_3 \Rightarrow K_3 = -1$

$s=1 : 1 = 1 + 2K_2 + -1 \quad K_2 = \frac{1}{2}$

$\Rightarrow \frac{1}{4} + \frac{1}{2}e^{-t} + te^{-t}$

Name: ID:

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s^2+4s+13)}$$

$\frac{K_1}{s} + \frac{K_2 s + K_3}{(s+2)^2+3^2} = \frac{1}{s(s^2+4s+13)}$

$\frac{K_2(s+2)}{\dots} + \frac{K_3 - 2K_2}{\dots}$

$K_1((s+2)^2+3^2) + (K_2 s + K_3)(s) = 1$.

$s=0 : 1 = K_1(13) \rightarrow K_1 = 1/13$

$s=1 : 1 = \frac{18}{13} + (K_2 + K_3)$.

$s=-1 : 1 = \frac{10}{13} + K_2 - K_3$

$K_2 = -1/13$

$K_3 = -4/13$

$\frac{1}{13} + \frac{-1}{13} e^{-2t}\cos 3t + \frac{-5}{39} e^{-2t}\sin(3t)$.

Name: ID:

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{(s+3)(s^2+2s+10)}$$

$\frac{K_1}{s+3} + \frac{K_2 s + K_3}{(s+1)^2+3^2}$

$\frac{K_2(s+1)}{\dots} + \frac{K_3 - K_2}{\dots}$

$1 = K_1(s^2+2s+10) + (K_2 s + K_3)(s+3)$.

$s=-3 : 1 = 13K_1 \rightarrow K_1 = 1/13$

$s=0 : 1 = \frac{10}{13} + 3K_3 \rightarrow K_3 = 1/13$

$s=1 : 1 = 1 + 4K_2 + \frac{4}{13} \rightarrow K_2 = -4/13$

$\frac{1}{13}e^{-3t} + \frac{-4}{13}e^{-t}\cos 3t + \frac{5}{39}e^{-t}\sin 3t$.

Find the solution of the following differential equation

$$\dot{X} + 2X + X = 1, \quad \textit{Initial conditions zero}$$

$sX(s) - X(0) + 2X(s) + X(s) = 1/s$

$X(s)(s+2+1) = 1/s$

$X(s) = \frac{1}{(s+3)(s)} = \frac{K_1}{s} + \frac{K_2}{s+3}$.

$1 = K_1(s+3) + K_2(s)$ $\quad s=0 : K_1 = \frac{1}{3}$, $s=-3 : K_2 = -\frac{1}{3}$.

$\frac{1}{3} - \frac{1}{3}e^{-3t}$ #.

Find C(t)

$$C(s) = \frac{5}{(s+1)(s+3)}$$

$5 = K_1(s+3) + K_2(s+1)$ $\quad s=-1 \quad K_1 = 5/2$, $s=-3 \quad K_2 = -5/2$

$\frac{5}{2}e^{-t} + \frac{-5}{2}e^{-3t}$

Write the MATLAB code to find the inverse laplace for

$$C(s) = \frac{5}{(s+1)(s+3)(s^2+2s+3)}$$

Type equation here.

```
syms t s
ilaplace(5/((s+1)*(s+3)*((s^2)+2*s+3)))
```

Find the final value for

$$C(s) = \frac{s+3}{s^2+2s+5}$$

$F(s) = \frac{s+3}{s^2+2s+5}$. Final $t \rightarrow \infty \Rightarrow s \rightarrow 0$

$\lim_{s\to 0} (s)\left(\frac{s+3}{s^2+2s+5}\right) = 0 * \frac{3}{5} = 0$.

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Find the final value for

$$C(s) = \frac{s+1}{s(s+1)}$$

$\frac{K_1}{s} + \frac{K_2}{s+1} \Rightarrow s+1 = K_1(s+1) + K_2 s$.

$s=0 : 1 = K_1$

$s=1 : 2 = 2 + K_2 = K_2 = 0$

$c(t) = 1$.

$\lim_{s\to 0} s.\frac{(s+1)}{(s+1)(s)} = \frac{1}{1} = 1$.

Quiz 1 Industrial control Prof M. Barghash

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Find the initial value for

$$c(s) = \frac{s+2}{s(s+5)}$$

$s \cdot F(s) = \cancel{s} \cdot \frac{(s+2)}{\cancel{s}(s+5)}$, initial value $t=0 \rightarrow s=\infty$

$\lim_{s\to\infty} \frac{s+2}{s+5} = \frac{\infty}{\infty} = 1.$

Name: ID:

Find the final value for the following function using final value theorem

$$C(s) = \frac{s+3}{s(s+1)^2}$$

$\lim_{s\to 0} \cancel{s} \cdot \frac{s+3}{\cancel{s}(s+1)^2} = \frac{3}{1} = 3$

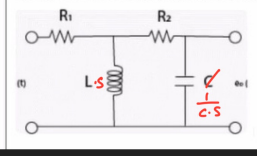
Find c(t) the time response for

$$C(s) = \frac{s+2}{s(s+1)^2}$$

$\frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{(s+1)^2} \quad \cdots$

Quize (2):

Q5) Find the transfer function for (5 points)



$$V_o = I * \frac{1}{C\cdot s} \quad \dots (1)$$

$$Z_{eq} = \left(\frac{\left(\frac{1}{C\cdot s} + R_2\right) * L\cdot s}{\left(\frac{1}{C\cdot s} + R_2\right) + L\cdot s} \right) + R_1$$

$$\frac{\frac{1+R_2C\cdot s}{C\cdot s} * L\cdot s}{\left(\frac{1+R_2C\cdot s}{C\cdot s} + L\cdot s\right)} + R_1 = \left(\frac{(1+R_2Cs) * L\cdot s}{1+R_2C\cdot s + LCs^2} \right) + R_1$$

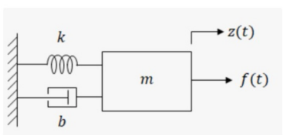
Z_{eq}

$V_{in} = I * Z_{eq}$

$\Rightarrow$ the transfer function.

$$\frac{I}{C\cdot s * I \left(\frac{(1+R_2Cs) * L\cdot s}{(1+R_2Cs+LCs^2)} + R_1 \right)}$$

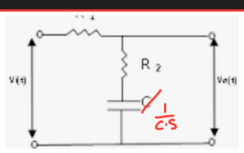
Q2) find the transfer function for the following a) Z(s)/F(s) (4 points)



$$F(t) = M\ddot{x} + b\dot{x} + kx$$

$$Ms^2X(S) + bsX(S) + KX(S)$$

$$\frac{X(S)}{F(x)} = \frac{1}{Ms^2 + bs + k}$$



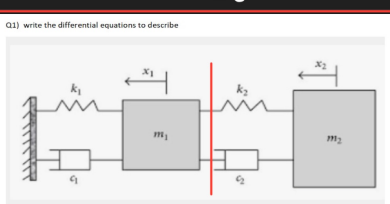
Vo/Vi = $\frac{R2+1/cs}{R2+\frac{1}{cs}+R1}$

$$V_o = I * Z_{R_2C} = I\left(R + \frac{1}{C\cdot s}\right)$$

$$Z_{eq} = R_2 + \frac{1}{C\cdot s} + R_1 = \left(\frac{1 + R_1Cs + R_2C\cdot s}{C\cdot s}\right)$$

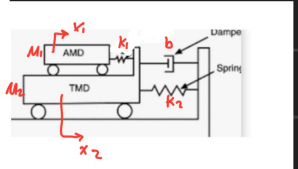
$$\left(\frac{V_o}{V_{in}} = \frac{\left(R_2 + \frac{1}{C\cdot s}\right)}{R_2 + R_1 + \frac{1}{C\cdot s}}\right)$$

Q1) write the differential equations to describe



$$0 = M_1\ddot{x} + c_1\dot{x}_1 + Kx_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2)$$

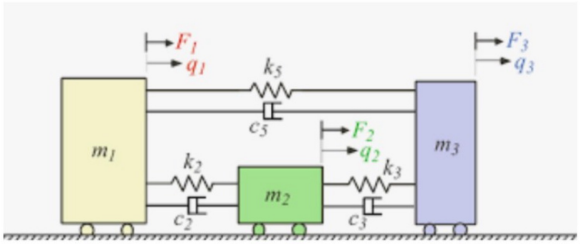
$$0 = M_2\ddot{x} + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)$$



$$0 = M_1\ddot{x}_1 + k_1(x_1 - x_2).$$

$$0 = M_2\ddot{x}_2 + b\dot{x}_2 + k_2x_2 + k_1(x_2 - x_1).$$

Write the differential equaions describing the following system

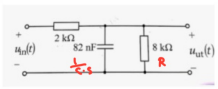


$$F_1 = M\ddot{x}_1 + K_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + K_5(x_1 - x_3) + c_5(\dot{x}_1 - \dot{x}_3).$$

$$F_2 = M_2\ddot{x}_2 + K_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_3) + c_3(\dot{x}_2 - \dot{x}_3)$$

$$F_3 = M_3\ddot{x}_3 + k_3(x_3 - x_2) + c_3(\dot{x}_3 - \dot{x}_2) + k_5(x_3 - x_1) + c_5(\dot{x}_3 - \dot{x}_1)$$

Q2) find the transfer function for the following a) Uut/Uin (4 points)

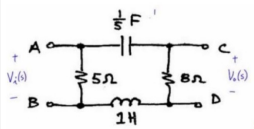


$$V_o = I * \left(\frac{R_1 * \frac{1}{C\cdot s}}{R_1 + \frac{1}{C\cdot s}}\right) = \left(\frac{8000}{6.56 * 10^{-4}s + 1}\right)$$

$$V_{in} = I\left(Z_{RC} + Z_{R_2}\right) = \left(\frac{1000 + 1.3s}{6.56 * 10^{-4}s + 1}\right)$$

$$\frac{V_o}{V_{in}} = \frac{8006}{1.3s + 1000}.$$

Q5) Find the transfer function for (5 points)



(hint 1/5 F series with 8 ohm and 1 H) then all is parallel with 5 ohm)

$$V_o = I * 8.$$

$$Z_{eq(1)} = 8 + \frac{5}{s} + 1.5 = \left(\frac{s^2 + 8s + 5}{s}\right) \qquad \left(\frac{40 + 25 + 5s}{8 + \frac{5}{s} + s + 5}\right)$$

$$Z_{eq} = \frac{\left(\frac{s^2 + 8s + 5}{s}\right) * 5}{\frac{s^2 + 8s + 5 + 5s}{s}} = \frac{(s^2 + 8s + 5)5}{s^2 + 8s + 5 + 5s}.$$

$$\frac{8}{5}\left(\frac{8 + \frac{5}{s} + s + 5}{(8 + \frac{5}{s} + s)}\right) *$$

$$\frac{8(s^2 + 8s + 5 + 5s)}{5(s^2 + 8s + 5)} \Rightarrow$$

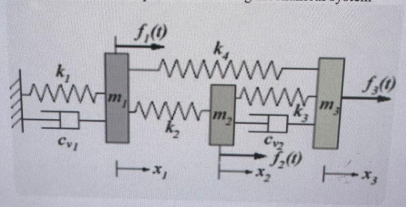
$$\frac{8}{5} + \frac{8}{(8 + \frac{5}{s} + s)} * \frac{8}{8}$$

$$\frac{8}{5}\left(\frac{s^2 + 8s + 5}{s^2 + 8s + 5} + \frac{5s}{s^2 + 8s + 5}\right)$$

$$\frac{8}{5} + \frac{8}{8} * \frac{8}{s + 8 + \frac{5}{s}} = \left(\frac{8}{5} + \frac{8}{s + \frac{5}{s} + 8}\right) *$$

Quiz Industrial control Prof M. Barghash modelling 27/5/2025
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Write the differential equation following mechanical system



$$M_1\ddot{x}_1 + C_{v1}(\dot{x}_1) + Kx_1 + K_2(x_2 - x_1) + K_4(x_3 - x_1) = F_1(s).$$

$$M_2\ddot{x}_2 + C_{v2}(\dot{x}_2 - \dot{x}_3) + K_2(x_2 - x_1) + K_3(x_2 - x_3) = F_2(s).$$

$$M_3\ddot{x}_3 + C_{v2}(\dot{x}_3 - \dot{x}_2) + K_3(x_3 - x_2) + K_4(x_3 - x_1).$$

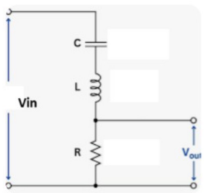


$$V_o = I \cdot R$$

$$Zeq = R + \frac{1}{C \cdot S} + L \cdot S.$$

$$\frac{V_o}{V_o} = \frac{R}{R + \frac{1}{C \cdot S} + L \cdot S}.$$

Q2) Find the transfer function for the following circuit (5 points)

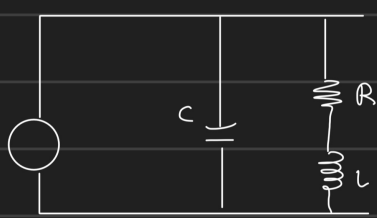


$$V_o = IR$$

$$Zeq = R + \frac{1}{C \cdot S} + L \cdot S$$

$$Vin = I\left(R + \frac{1}{C \cdot S} + L \cdot S\right).$$

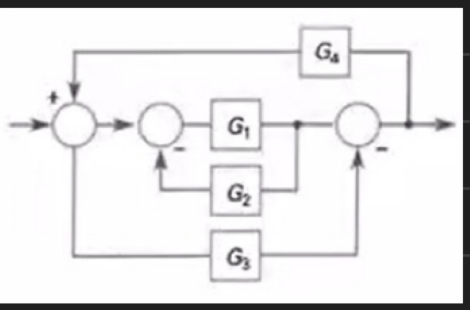
$$\frac{V_o}{Vin} = \frac{R}{\left(R + \frac{1}{C \cdot S} + L \cdot S\right)}.$$



$$T.F = \frac{R + L \cdot S}{\frac{(R + LS)}{C \cdot S}}$$
$$R + LS + \frac{1}{C \cdot S}$$

$$Zeq = \frac{\frac{(R+LS)}{CS}}{\left(R + LS + \frac{1}{CS}\right)\frac{CS}{CS}} \left(\frac{R + LS}{RCS + CLS^2 + 1}\right)$$

Quize (3) Block diagram.



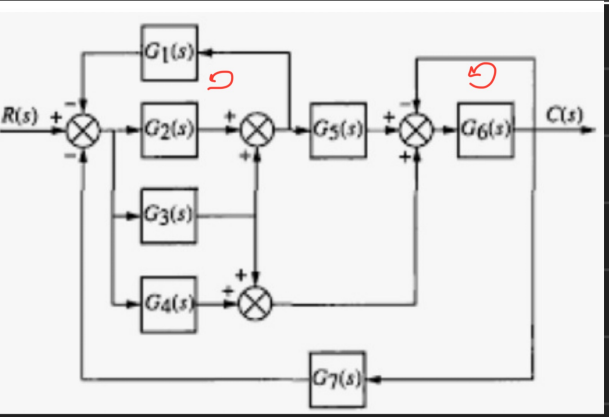
Pathes ① P_1 G ② P_2 $-G_3$

Δ ① $\Delta 1 = 1$ ② $\Delta 2 = 1-(-G_1G_2)$

loops $[-G_1G_2 \;,\; G_1G_4 \;,\; -G_3G_4]$

nontouching loops $\Rightarrow G_1G_2G_3G_4$

$$T.F = \frac{G_1 - G_3(1+G_1G_2)}{1+G_1G_2+G_3G_4-G_1G_4+G_1G_2G_3G_4}$$

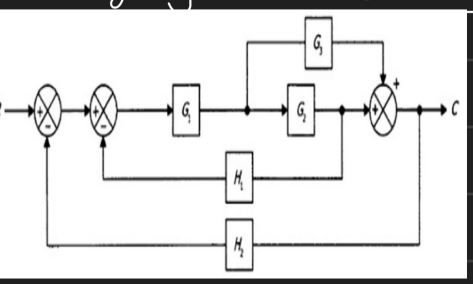


Pathes $[G_1G_5G_6 \;,\; G_3G_5G_6 \;,\; G_4G_6 \;,\; G_3G_6]$ $\Delta = 1$

loops $[-G_1G_2 \;,\; -G_6 \;,\; -G_2G_5G_6G_7 \;,\; -G_3G_5G_6G_7$

$-G_4G_6G_7 \;,\; -G_3G_6G_7]$

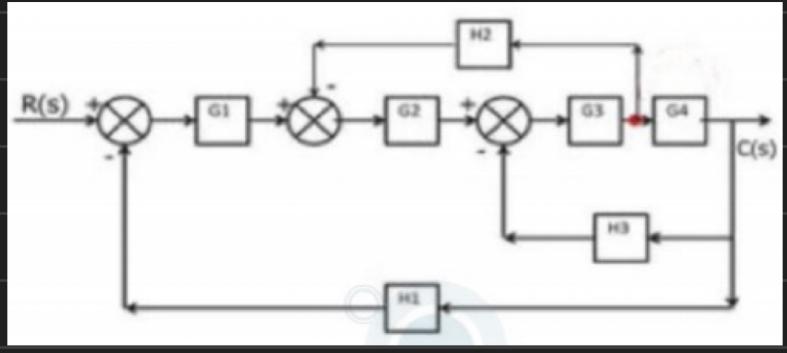
nontouching $G_1G_2G_6$



no nontouching loops.

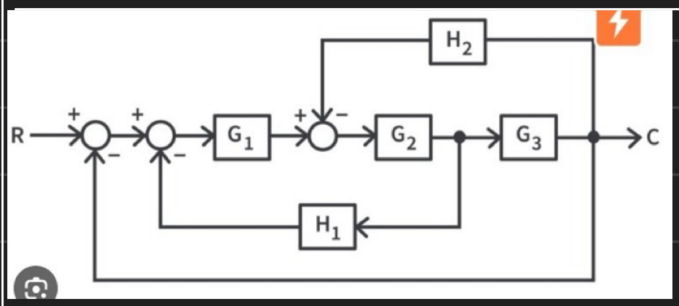
Pathes $[G_1G_2 \;,\; G_1G_3]$ $\Delta = 1$

loops $[-G_1G_2H_1 \;,\; -G_1G_2H_2 \;,\; -G_1G_3H_2]$



Pathes $[G_1G_2G_3G_4]$ $\Delta = 1$

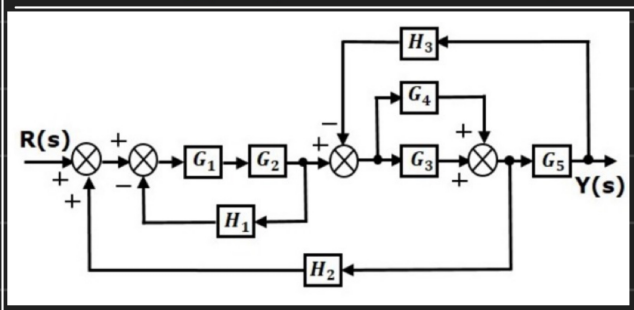
loops $[-G_2G_3H_2 \;,\; -G_3G_4H_3 \;,\; -G_1G_2G_3G_4H_1]$



Pathes $[G_1G_2G_3]$ $\Delta = 1$

loops $[-G_1G_2H_1 \;,\; -G_1G_2G_3 \;,\; -G_2G_3H_2]$

$$\Delta P = \frac{G_1G_2G_3}{1+G_1G_2H_1+G_1G_2G_3+G_2G_3H_2}$$



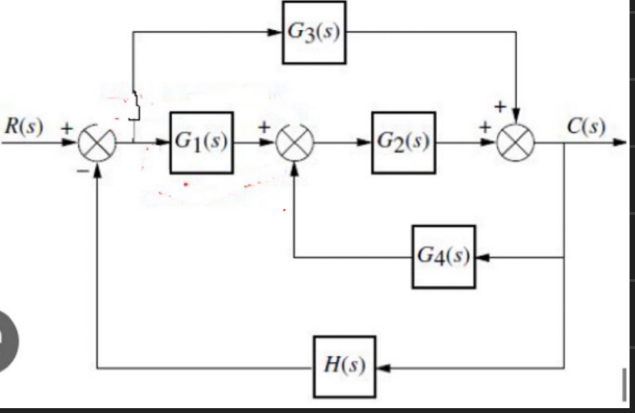
Pathes $[G_1G_2G_3G_5 \;,\; G_1G_2G_4G_5]$.

loops $[-G_1G_2H_1 \;,\; G_1G_2G_3H_2 \;,\; G_1G_2G_4H_2 \;,\; -G_3G_5H_3 \;,$

$-G_4G_5H_3]$.

nontouching loops $[G_1G_2H_1G_3G_5H_3 \;,\; G_1G_2H_1G_4G_5H_3]$

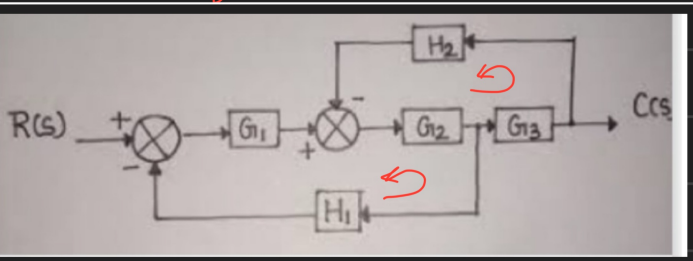
$$1-\Sigma loops + non.$$



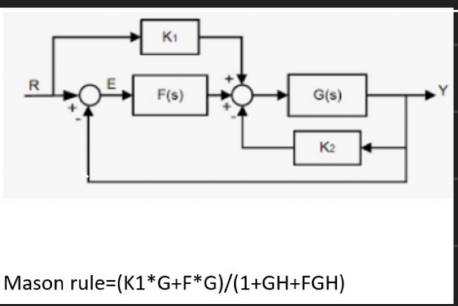
Pathes $[G_1G_2 \;,\; G_3]$ $\Delta_1=1$, $\Delta_2=1$.

Loops $[G_2G_4 \;,\; -G_1G_2H \;,\; -G_3H]$

no nontouching loops .



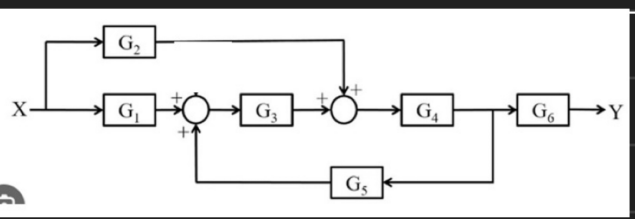
pathes $[G_1G_2G_3]$, loops $[-G_2G_3H_2 \;,\; -G_1G_2G_3H_1]$



Mason rule=(K1*G+F*G)/(1+GH+FGH)

Pathes $[KG \;,\; FG]$.

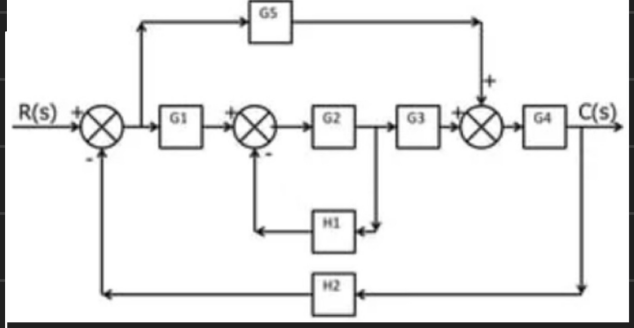
loops $[FG \;,\; GK$



Pathes $[G_1G_3G_4G_6 \;,\; G_2G_4G_6]$

loops $[G_3G_4G_5]$

$$T.F = \frac{G_1G_3G_4G_6 + G_2G_4G_6}{1 - G_3G_4G_5}$$

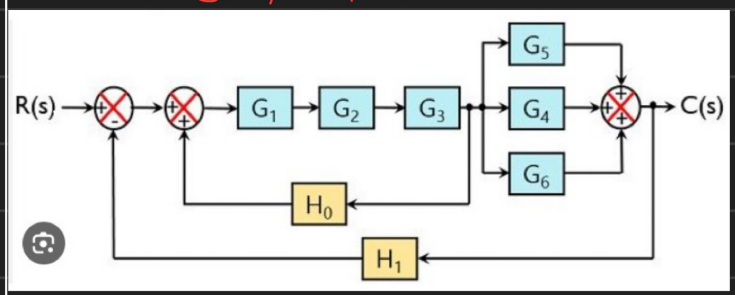


Pathes $[G_1G_2G_3G_4 \;,\; G_5G_4]$ $\Delta_1=1$, $\Delta_2=1+G_2H_1$

Loops $[-G_2H_1 \;,\; -G_1G_2G_3G_4H_2 \;,\; -G_5G_4H_2]$

nontouching $[G_2H_1G_5G_4H_2]$.

$$T.F = \frac{G_1G_2G_3G_4 - G_5G_4(1+G_2H_1)}{1 - \Sigma[\text{loops}] + G_2H_1G_5G_4H_2} .$$

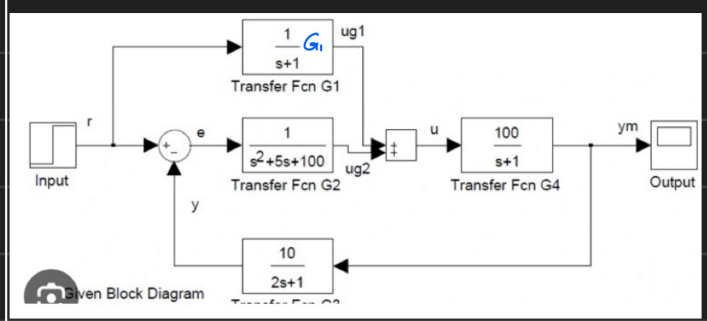


Pathes $[G_1G_2G_3G_5 \;,\; G_1G_2G_3G_4 \;,\; G_1G_2G_3G_6]$ $\Delta=1$

Loops $[G_1G_2G_3H_0 \;,\; -G_1G_2G_3G_5H_1 \;,\; -G_1G_2G_4H_1$

$-G_1G_2G_6H_1]$

no nontouching loops .

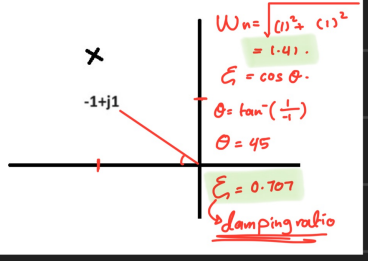


$$T.F = \frac{\frac{100}{(s+1)^2} + \frac{100}{(s^2+5s+100)(s+1)}}{\frac{1001}{(s^2+5s+100)(s+1)(2s+1)}} = \frac{\frac{100(s^2+5s+100)+100(s+1)}{(s^2+5s+100)(s+1)^2}}{\frac{1001}{(s^2+5s+100)(s+1)(2s+1)}}$$

$$\frac{100\left((s^2+5s+100)+(s+1)\right)}{1001} * \frac{(2s+1)}{(s+1)}$$

Quiz (4): first & Second Order.

For the following pole find the settling time



$W_n = \sqrt{(1)^2 + (1)^2} = (1.4)$.

$\xi = \cos\theta$.

$\theta = \tan^{-1}(\frac{1}{1})$

$\theta = 45$

$\xi = 0.707$ damping ratio

① $T_s = \frac{4}{0.707 * 1.41} = 4.012$ s.

② $T_p = \frac{\pi}{1.41\sqrt{1-0.707^2}} = 3.11$ s.

③ $MP = e^{\frac{-\pi * 0.707}{\sqrt{1-0.707^2}}} * 100 = 4.5\%$

④ $T_r = \frac{\pi - \theta}{1.41\sqrt{1-0.707^2}} = 2.34$ s. ($45 * \frac{\pi}{180} = 0.78$)

For the following system find the rise time, settling time and percent overshoot

$$\frac{c}{R} = \frac{s+3}{s^2+s+4.25}$$

$W_n = \sqrt{4.25} = 2.06$, $\xi = \frac{1}{2*2.06} = 0.24$

① $T_r \Rightarrow \theta = \cos^{-1}(0.24) = 76.113 \Rightarrow T_r = \frac{\pi - 76.113 * \frac{\pi}{180}}{2.06\sqrt{1-0.24^2}} = 0.906$ s

② $T_p \Rightarrow \frac{\pi}{2.06\sqrt{1-0.24^2}} = 1.57$

③ $T_s = \frac{4}{2.06 * 0.24} = 8.09$ s , ④ $MP = e^{\frac{-\pi * 0.24}{\sqrt{1-0.24^2}}} * 100 = 45.99$.

rise time for the following system

$$G(s) = \frac{7}{s^2+14s+40}$$

$(\frac{14}{2})^2 > 40 \longrightarrow$ first order

$$\frac{7}{(s+10)(s+4)} = K_1 e^{-10t} + K_2 e^{-4t}$$

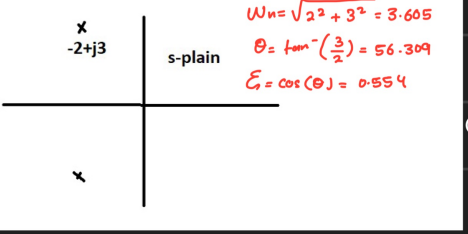
$t_1 = 0.4$, $t_1 = 1$

$\neq \frac{K_2}{s+4} \Rightarrow \frac{K_2}{s+\frac{1}{\tau}}$.

$4 = \frac{1}{\tau} \Rightarrow \tau = \frac{1}{4}$. , $T_r = 2.2 * \frac{1}{4} = 0.55$

$T_s = 4\tau = 1$.

For the following system with roots shown in the following s-plane find the rise time, settling time and percent overshoot



$W_n = \sqrt{2^2+3^2} = 3.605$

$\theta = \tan^{-1}(\frac{3}{2}) = 56.309$

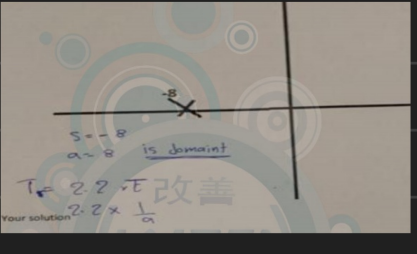
$\xi = \cos(\theta) = 0.554$

① $T_s = \frac{4}{3.605 * 0.554} = 2.05$ s.

② $MP = e^{\frac{-\pi * 0.554}{\sqrt{1-0.554^2}}} * 100 = 12.36\%$

③ $T_r = \frac{\pi - (56.309 * \frac{\pi}{180})}{3.605\sqrt{1-0.554^2}} = 0.719$ s. ,

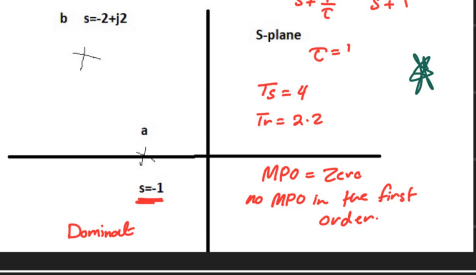
④ $T_p = \frac{\pi}{3.605\sqrt{1-0.554^2}} = 1.046$ s



$$\frac{K}{s+8} = \frac{K}{s+\frac{1}{\tau}}$$

$\tau = \frac{1}{8}$

Q5) a) For a and b poles in the following fins the associated damping ratio and percent opevershoot (5 points)



$\frac{K}{s+\frac{1}{\tau}} = \frac{K}{s+1}$

$\tau = 1$

$T_s = 4$

$T_r = 2.2$

MPO = Zero no MPO in the first order.

$W = \sqrt{2^2+2^2} = \sqrt{8}$

$\xi =$

For the following system find the rise time, settling time and percent overshoot

$W_n = \sqrt{13} = 3.605$

$\xi = \frac{4}{2*3.605} = 0.554$

$\theta = \cos^{-1}(0.554)$

$$\frac{c}{R} = \frac{s+3}{s^2+4s+13}$$ complex

① $T_s = \frac{4}{3.605 * 0.554} = 2.05$ s.

② $MP = e^{\frac{-\pi * 0.554}{\sqrt{1-0.554^2}}} * 100 = 12.36\%$

③ $T_r = \frac{\pi - (56.309 * \frac{\pi}{180})}{3.605\sqrt{1-0.554^2}} = 0.719$ s.

④ $T_p = \frac{\pi}{3.605\sqrt{1-0.554^2}} = 1.046$ s

For the following system find the rise time, settling time and percent overshoot

$$\frac{s+3}{(s+3)(s+6)} = K_1 e^{-3t} + K_2 e^{-6t}$$

Dominat

$$\frac{c}{R} = \frac{s+3}{s^2+9s+18}$$ $\sqrt{18} =$

$\frac{K_1}{s+\frac{1}{\tau}} = \tau = \frac{1}{3} = 0.33$, $T_s = 1.33$ s , $T_r = 0.733$.

Q1) for the following pole what is the percent overshoot

$W_n = \sqrt{6} = 2.45$

$\xi = \frac{2}{2*2.45} = 0.408$ complex

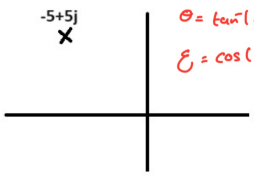
$G(s) = \frac{k}{s^2+2s+6}$

$\theta = \cos^{-1}(0.408) = 65.9$

$T_s = \frac{4}{2.45 * 0.408} = 4s$, $MPO = e^{\frac{-\pi * 0.408}{\sqrt{1-0.408^2}}} * 100 = 24.56\%$

$T_p = \frac{\pi}{2.45\sqrt{1-0.408^2}} = 1.433$ s , $T_r = \frac{\pi - (65.9 * \frac{\pi}{180})}{2.45\sqrt{1-0.408^2}} = 0.9088$ s

Q1) for the following pole what is the peak time Tp



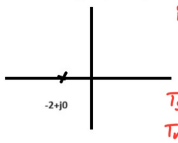
$W_n = \sqrt{5^2+5^2} = 7.07$

$\theta = \tan^{-1}(\frac{5}{5}) = 45$

$\xi = \cos(45) = 0.707$

Q1)find the percent overshoot and settling time related to pole in the figure

(note: remember this pole is first order not second order , initially draw the expected time response before answering the question)



Real root $\frac{K}{s+2} = \frac{K}{s+\frac{1}{\tau}}$

$\tau = \frac{1}{2} = 0.5$

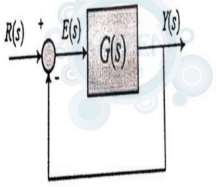
$T_s = 4 * 0.5 = 2$

$T_r = 2.2 * 0.5 = 1.1$

Quiz (5) Steady state error :

Q1) find the K value to achieve a steady state error Of < 0.1 for a unit step input

$$G(s) = \frac{K(s+1)}{(s+5)(s+7)}$$



$e_{ss} = \frac{1}{1+K_p} < 0.1$, $K_p = \lim_{s\to 0} G(s)$

① $N=0$

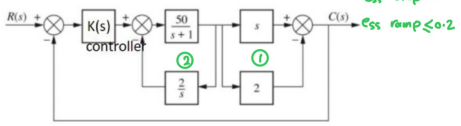
$K_p = \frac{K}{35} \Rightarrow e_{ss} = \frac{1}{\frac{35}{35}+\frac{K}{35}} = \frac{35}{35+K} < 0.1$

$350 < 35+K \Rightarrow K > 315$

② $N \geq 1$ $e=0$ *

$K_p = \lim_{s\to 0} \frac{K}{s} * \left(\frac{s+1}{(s+5)(s+7)}\right) = \infty$

Q2) for the following system design the steady state error for a step input to be zero and for a ramp input to be less than 2 (design the controller)? (note K(s) is any function of s to be suitably designed) (10 points)



e_{ss} step $=0$

e_{ss} ramp ≤ 0.2

① Parallel $(S-2)$

② feed back $\left(\frac{\frac{50}{s+1}}{1+\frac{100}{s(s+1)}}\right)$

$\frac{50s}{s^2+s+100}$

① & ② series $\Rightarrow \left[\frac{50s(s-2)}{s^2+s+100} = G(s)\right]$. *

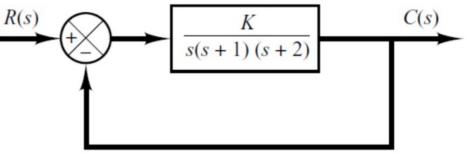
e_{ss} step $=0 \Rightarrow \frac{1}{1+K_p}$, $K_p = \lim_{s\to 0} \frac{K}{s^2} * \frac{50s(s-2)}{s^2+s+100} = \infty$

for ramp :

$e_{ss} = \frac{1}{K_v} < 2$, $K_v > \frac{1}{2} = \lim_{s\to 0} s.\left(\frac{K}{s^2}\right) * \frac{50s(s-2)}{s^2+s+100} = \frac{-100K}{100} = \frac{1}{2}$

$K = \frac{-1}{2}$, $e_{ss} = 0 \Rightarrow K_v = \infty$ when $\frac{K}{s^3}$

For following system what is the steady state error for R= 3u(t) +5t



$e_{ss} = \frac{3}{1+K_p} + \frac{5}{K_v}$

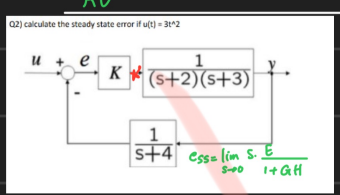
$K_p = \lim_{s\to 0} G(s) = \frac{K}{0} = \infty$

$K_v = \lim_{s\to 0} s.G(s) = \frac{K}{2} \Rightarrow e_{ss} = \frac{3}{\infty} + \frac{10}{K} = \frac{10}{K}$.

* design k to achive 0.1 e_{ss} in ramp :

$e_{ss} = \frac{1}{K_v} = 0.1 \Rightarrow K_v = 10 = \frac{K}{2} \Rightarrow K = 20$ *

Q2) calculate the steady state error if u(t) = 3t^2

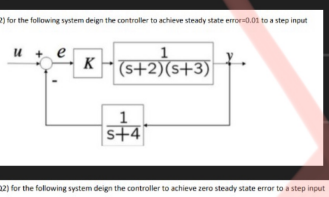


$E = \frac{6}{s^3}$, $1+GH = 1+\frac{K}{(s+2)(s+3)(s+4)}$

$\lim_{s\to 0} s.\frac{6}{s^3}.\frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}} = \infty$.

$e_{ss} = \lim_{s\to 0} s.\frac{E}{1+GH}$

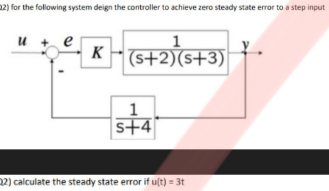
Q2) for the following system deign the controller to achieve steady state error=0.01 to a step input



$e_{ss} = \lim_{s\to 0} s.\frac{1}{s}.\left[\frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}}\right]$ * $\frac{1}{1+\frac{K}{24}}$

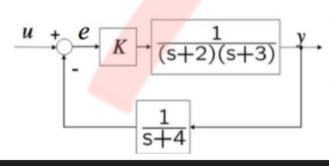
$\frac{24}{24+K} = 0.01 = 2376$

Q2) for the following system deign the controller to achieve zero steady state error to a step input



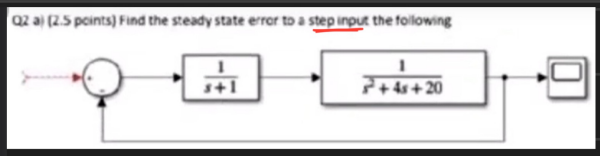
$e_{ss} = \frac{1}{1+K_p} = 0$, $K_p = \infty \Rightarrow K = \frac{1}{s}$

Q2) calculate the steady state error if u(t) = 3t



$e_{ss} = \lim \frac{3}{s^2} * \frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}} * s = \infty$

Q2 a) (2.5 points) Find the steady state error to a step input the following



$e_{ss} = \frac{1}{1+K_p}$, $K_p = \lim_{s\to 0} G(s)$, $G(s) = \frac{1}{s+1} * \frac{1}{s^2+4s+20} = \frac{1}{(s+1)(s^2+4s+20)}$

$K_p = \lim_{s\to 0} G(s) = \frac{1}{1\times 20} = \frac{1}{20} \Rightarrow e_{ss} = \frac{1}{1+\frac{1}{20}} = \frac{20}{21}$ error of step unit

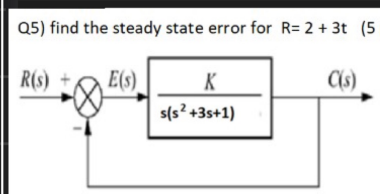
OR

(design a controler) to achive $e_{ss} = 1$ when the input $\left(\frac{1}{s^2}\right)$ ramp.

$e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s\to 0} G(s)\cdot s$, $G(s) = \frac{1}{(s+1)(s^2+4s+20)}$

$K_v = \lim_{s\to 0} G(s)\cdot s\left(\frac{K}{s}\right)$ ramp $= \left[K_v = \frac{K}{20}\right] \Rightarrow e_{ss} = \frac{1}{\frac{K}{20}} = 1$, $K = 20$

Q5) find the steady state error for R= 2 + 3t (5 points)

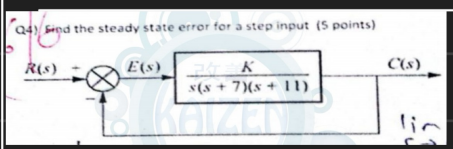


$\frac{2}{1+K_p} + \frac{3}{K_v}$

$K_p = \lim_{s\to 0} G(s) = \frac{K}{0} = \infty$

$K_v = \lim_{s\to 0} s.G(s) = \frac{K}{1} \Rightarrow e_{ss} = 3/K$.

Q4) Find the steady state error for a step input (5 points)

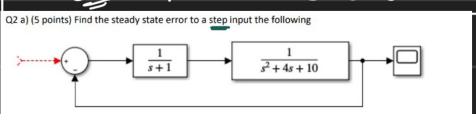


$e_{ss} = \frac{1}{1+K_p}$ $e_{ss} = 0$.

$K_p = \lim_{s\to 0} G(s) = \infty$

OR N=1 : Zero error

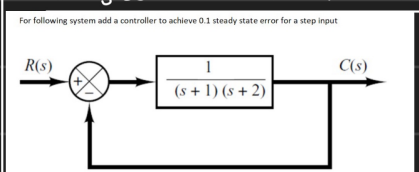
Q2 a) (5 points) Find the steady state error to a step input the following



$G(s) = \frac{1}{(s+1)(s^2+4s+10)}$

$K_p = \lim_{s\to 0} G(s) = \frac{1}{10}$, $e_{ss} = \frac{10}{11}$

For following system add a controller to achieve 0.1 steady state error for a step input

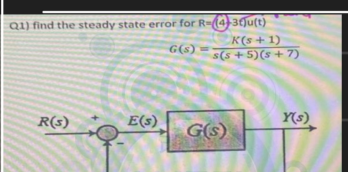


$e_{ss} = \frac{1}{1+K_P} = 0.1$ $K_p = \lim_{s\to 0} K.G(s) = \frac{K}{2}$

$10 = 1+\frac{K}{2} = K = 18$

Q1) find the steady state error for R= 4+3tu(t)

$G(s) = \frac{K(s+1)}{s(s+5)(s+7)}$

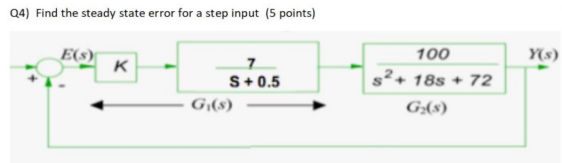


$\frac{4}{1+K_P} - \frac{3}{K_v}$, $K_p = \infty$

$K_v = \frac{K}{35}$

$e_{ss} = \frac{35*3}{K} = \frac{105}{K}$

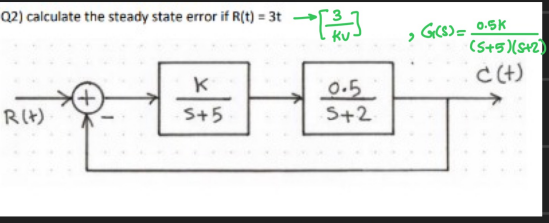
Q4) Find the steady state error for a step input (5 points)



$G(s) = \frac{700K}{(s+\frac{1}{2})(s^2+18s+72)}$

$K_p = \lim_{s\to 0} G_1(s) = \frac{350}{9}K$ $e = \frac{9}{9+350K}$

Q2) calculate the steady state error if R(t) = 3t

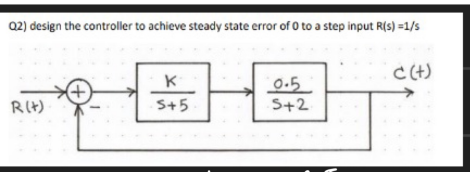


$\rightarrow \left[\frac{3}{K_V}\right]$, $G(s) = \frac{0.5K}{(s+5)(s+2)}$

$K_V = \lim_{s\to 0} s\,G(s) = 0$

$e_{ss} = \infty$

Q2) design the controller to achieve steady state error of 0 to a step input R(s) =1/s

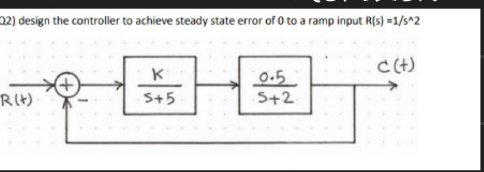


$e_{ss} = \frac{1}{1+K_p} \Rightarrow K_p = \infty$

if $K = \frac{1}{s}$

Then $K_p = \lim_{s\to 0} \frac{0.5}{s(s+5)(s+2)}$

Q2) design the controller to achieve steady state error of 0 to a ramp input R(s) =1/s^2

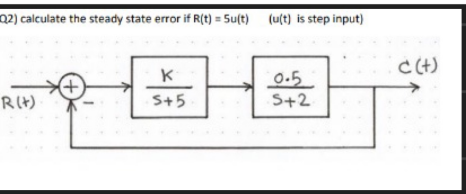


$e_{ss} = \frac{1}{K_V}$

$K_V = \lim_{s\to 0} s \cdot G(s)$

$K = \frac{1}{s^2}$

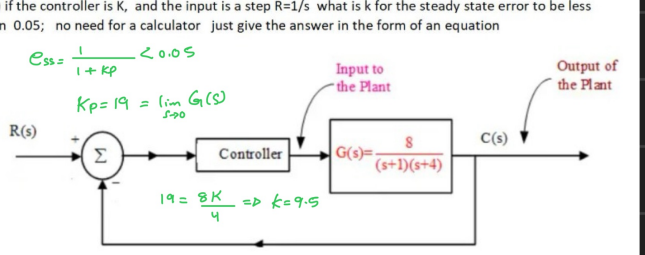
Q2) calculate the steady state error if R(t) = 5u(t) (u(t) is step input)



$e_{ss} = \frac{5}{1+K_p}$, $K_p = \frac{0.5K}{10}$

$K_p = \frac{K}{20} \Rightarrow e_{ss} = \frac{100}{20+K}$

Q1) if the controller is K, and the input is a step R=1/s what is k for the steady state error to be less than 0.05; no need for a calculator just give the answer in the form of an equation

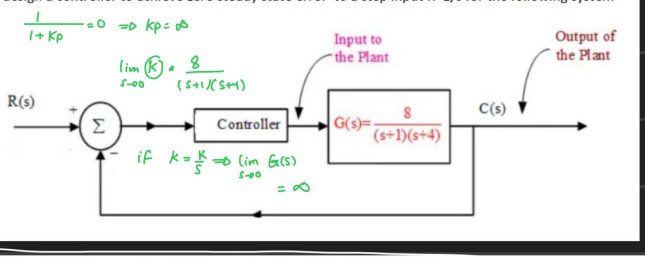


$e_{ss} = \frac{1}{1+K_p} < 0.05$

$K_p = 19 = \lim_{s\to 0} G(s)$

$19 = \frac{8K}{4} \Rightarrow K = 9.5$

Q1) design a controller to achieve zero steady state error to a step input R=1/s for the following system



$\frac{1}{1+K_p} = 0 \Rightarrow K_p = \infty$

$\lim_{s\to 0} (K) \cdot \frac{8}{(s+1)(s+4)}$

if $K = \frac{K}{s} \Rightarrow \lim_{s\to 0} G(s) = \infty$

Q4) Find the steady state error for a ramp input 1/s^2 (5 points)

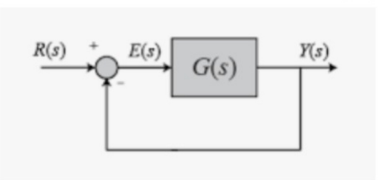


$e_{ss} = \frac{1}{K_V}$, $K_V = \lim_{s\to 0} s \frac{700K}{s(s^2+18s+72)} = \frac{700K}{72} \Rightarrow e = \frac{72}{700K}$

An industrial conveyor belt system uses a **proportional controller** with gain K to regulate speed. The dynamics of the system (plant) are modeled by the open-loop transfer function:

$$G(s) = \frac{1}{s(2s+3)}$$

$e_{ss} = \frac{1}{K_V} = 0.05$



$K_V = \lim_{s\to 0} s \cdot G(s) \cdot K$

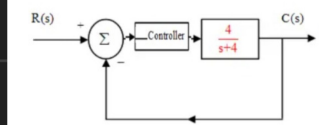
$K_V = \frac{K}{3}$ $\frac{3}{K} = 0.05$

$K = 60$

Add a suitable controller to have the steady state error for a ramp input R(t)=t , to be equal to 0.05

Q1) design a controller to achieve zero steady state error to a ramp input R=1/s^2 for the following system

$e_{ss} = \frac{1}{K_V} = 0$, $K_V = \infty$, $K_V = \lim_{s\to 0} s \cdot (K) \cdot \frac{4}{s+4}$, if controller $= \frac{K}{s^2}$

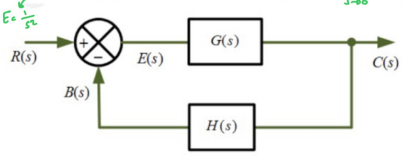


$K_V = \lim_{s\to 0} s \cdot \frac{K}{s^2} \cdot \frac{4}{s+4} = \infty$

Q3) for the following system find the added controller to achieve zero steady state error to a ramp input R(t)=t , H(s)=1, R(s) = 1/(s^2+2s+3) (5 points)

$e_{ss} = \lim_{s\to 0} s \cdot \frac{E}{1+G \cdot H} = 0$

$E = \frac{1}{s^2}$



$e_{ss} = \frac{1}{K_V} = 0$

$K_V = \infty = \lim_{s\to 0} s(K) \frac{1}{s^2+2s+3}$

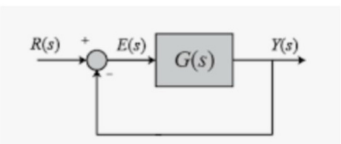
if $K = \frac{1}{s^2} \Rightarrow$ ✓

Find the steady state error for the following system for an input of

R(t) =5+3t $e_{ss} = \frac{5}{1+K_p} + \frac{3}{K_V} = \frac{5}{\infty} + \frac{3}{5} = 0.6$

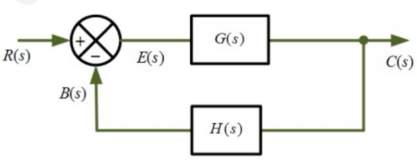
A unity feedback system has the open-loop transfer function:

$$G(s) = \frac{10}{s(s+2)}$$



Q3) for the following system find the added controller to achieve zero steady state error to a parabolic input R(t)=t^2/2 , H(s)=1, R(s) = 1/(s^2+2s+3) (5 points)

$e_{ss} = \frac{1}{K_a}$



$K_a = \lim_{s\to 0} s^2 \cdot (K) \cdot \frac{1}{s^2+2s+3}$

$K = \frac{K}{s^2} \Rightarrow \infty$

$e = 0$

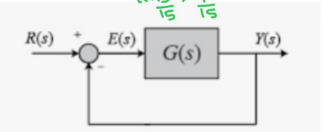
Add a controller for the following system to achieve steady state error 0f less than 0.5 for

R(t) =t $e_{ss} = \frac{1}{K_V} < 2 \Rightarrow K_V > 2$ $K_V = \lim_{s\to 0} s \cdot G(s) \cdot K$

Consider a feedback control system with the following open-loop transfer function:

$K \cdot \frac{5(3)}{2} > 2$ $G(s) = \frac{5(s+3)}{s(s+2)}$

$K \cdot \frac{15}{15} > \frac{4}{15}$ $K > \frac{4}{15}$ ✓



An industrial temperature control system uses a proportional controller with gain K. The process dynamics are modeled as:

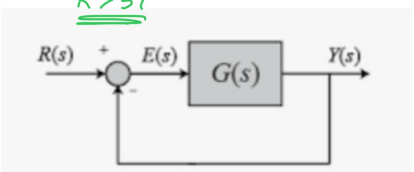
$K_p = K \cdot \frac{1}{3} > 19$ $G(s) = \frac{1}{(s+1)(s+3)}$

$K > 57$

$e_{ss} = \frac{1}{1+K_p} < 0.05$

$1 + K_p > 20$

$K_p > 19$



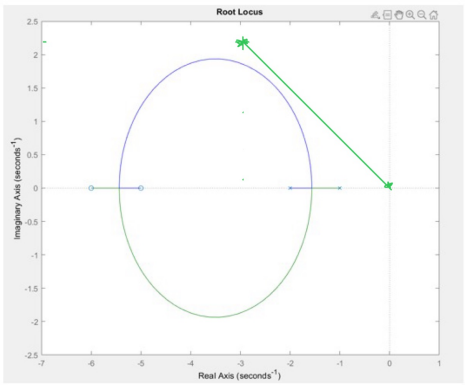
Calculate the value of K that limits the steady-state error to less than **0.05**.

Quiz (6) Root locus.

Q2) a) For the following system $G = \frac{(s+5)(s+6)}{(s+1)(s+2)}$, H=1 design the feedback controller K to achieve a damping ratio of > 0.8. (you must use the root locus figure below) (7.5 points)

$\xi > 0.8$ $\theta = \cos^{-1}(0.8)$ $\theta = 36.87$

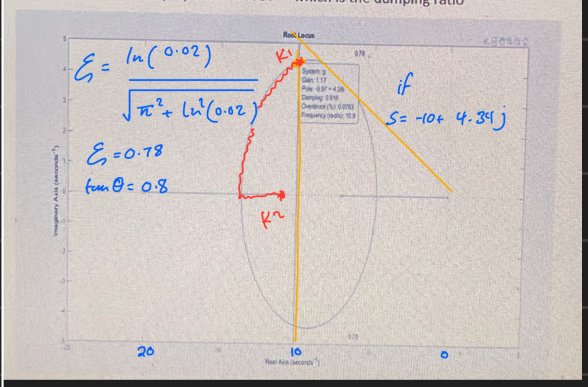
$\tan(\theta) = 0.75$ $x = 3$ $y = 2.25$.



all the points are satisfying (under the line)

Q2) a)(6 points) design K for the system $GH(s)=\frac{(s+11)(s+20)}{s(s+7)}$ to achieve a settling time < 0.4 and percent overshoot < 2% (use the root locus shown below

-ln(0.02)/(pi^2+ln(0.02)^2)^0.5 =0.7797 which is the damping ratio



settling < 0.4

$\frac{4}{\omega\xi} = 0.4$

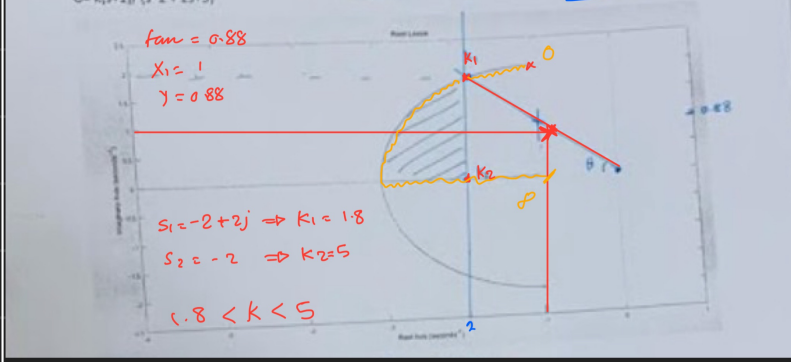
$\omega\xi = 10$ *

$K_2 = \infty$

Determine the value of k to achieve settling time less < 2 and damping ratio less the 0.75 for

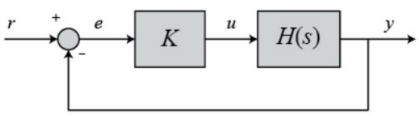
$G = k(s+1)/(s^2+2s+5)$

$T_s < 2$ $\xi = 0.75$



$PO\% \leq 25\% \Rightarrow \xi = 0.404$

Q6)What is the) a) For the following system $H(s) = \frac{(s+7)(s+8)}{s(s+5)}$ design the feedback controller k to achieve settling time < 0.666. (you must use the root locus figure below) (5 points)



Q21) find the K value to achieve settling time <3.2 and damping ratio > 0.9

$G(s) = \frac{K(s+7)}{s(s+5)(s+9)}$

$\omega\xi = 1.25$

$\tan = 0.484$

$x = 2$

$y = 0.968$

$1.09 < K < 6.32$

$6.32 < K < 10.76$

$K = 10.76$

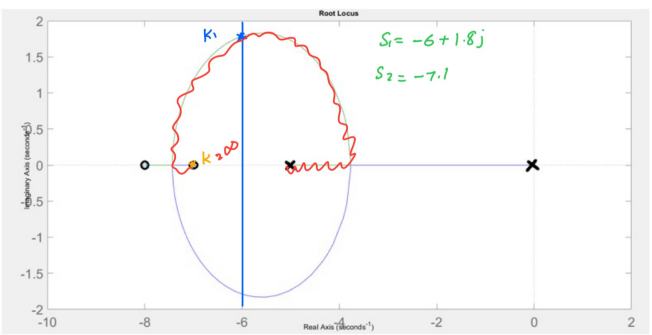
$-2.83 \pm 1.8j$

$26.84°$

$K = 6.37$

$S = -1.25$

$K = \frac{1}{|GH|}$



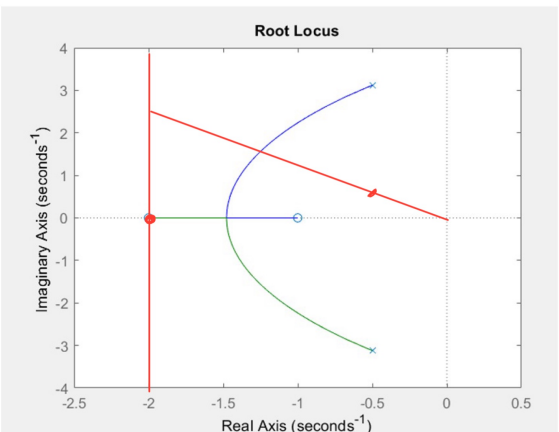
$K = \infty$ ⇐ القطب

$K = 0$ ⇐ القطب

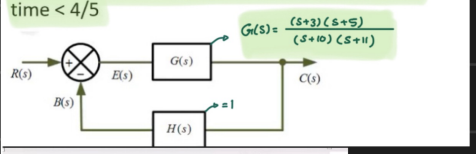
For the following system determine K that achieves a damping ratio of > 0.7 and settling time < 2 s

GH(s)= (s+1)(s+2)/((s^2 +s +10))

$\tan = 1.02$



Q5) For the following system design a controller K to achieve a damping ratio of > 0.95 and settling time < 4/5



in Matlab :

tan (a cos (0.95))

=0.3287

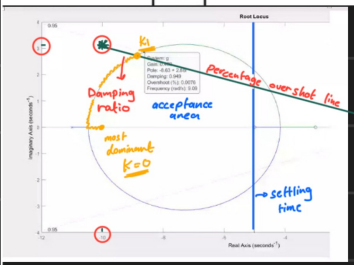
Theta = a cos (0.95) * (180 / pi)

= 18.1949

ج) (10) بمقدار (X) اذا مستبيت عين على

امستبي عين (Y) بمقدار

10 * 0.3287 = 3.287

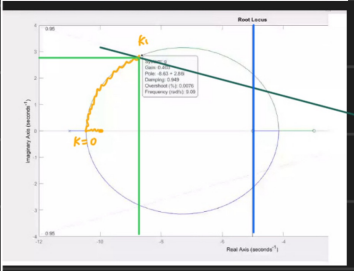


settling time : $\frac{4}{5} = -4 \Rightarrow Ts = -5$

(0,0)

the solution $0 < K < K_1$

$S_1 = -8.8 + j2.75$ in matlab



$S = -8.8 + 2.7i$

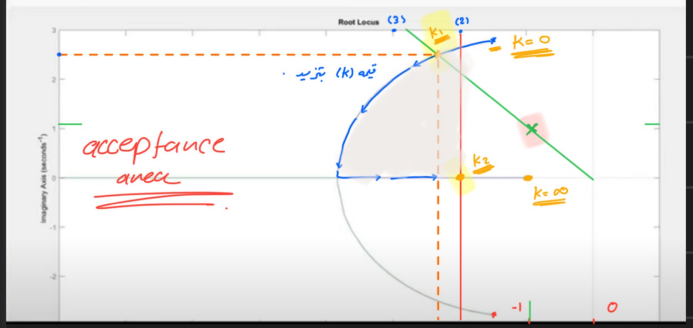
g = (s+3)*(s+5) / ((s+10)*(s+11))

k1 = abs (1/g) = 0.3451

fainal answer $0 < K < 0.345$

Fo: he following system determine K that achieves a damping ratio of > 0.7 and settling time < 2s R

GH(s)= (s+1)/ (s^2 +3s +10)



$\xi > 0.7 \Rightarrow \theta = \cos^{-1}(0.7) = 45.57$ Scale = tan(θ) = 1.02

if $x = 1 \Rightarrow y = 1.02$

settling time < 2 $\Rightarrow \frac{4}{\omega\xi} = 2 \Rightarrow \omega\xi = 2$ (*)

$S_1 = -2.3 + j2.45$

$S_2 = -2$

$\Rightarrow K = \frac{1}{|GH(s)|} = \left|\frac{s^2+3s+10}{(s+1)}\right|$

$K_1 = 1.652$, $K_2 = 8 \Rightarrow K_1 < K < K_2$

$1.652 < K < K_2$.

Quiz (7) Matlab

write MATLAB code using ODE45 to solve the following differential equations

$\ddot{x} + x = \log(t), \quad \dot{x}(0) = 0, \; x(0) = 0$

Plot(y) and plot(x) on same figure for a time interval 0 to 20 seconds.

ODE45 : Numerical answer :

Convention ⇒ there is a second order

x_1 , x_2 $\qquad \ddot{x} + x = \log(t)$

$\dot{x}_1 = x_2$ —① $\qquad \dot{x}_2 + x_1 = \log(t)$

$\ddot{x} = \dot{x}_2$ $\qquad \dot{x}_2 = \log(t) - x_1$ ... ②

```
function  dx = sos(t,x)
[m,n] = size x ;
  dx = zeros (m,n) ;
dx(1) = x(2) ;
dx(2) = log10(t) - x(1) ;
tspan = [0 , 20]
x0 = [0;0];
[tx] = ode45 (at sos, tspan, x0)
 plot (t,x)
 plot (t, x(:,1))
```

Normal first order equation :

$\dot{x} + ax = b \Rightarrow \dot{x} + 2x = b \qquad \dot{x} = b - ax$

```
function  dx = sos(t,x)
  a = 2 ;
  b = 4 ;
dx = b - a*x
tspan = [0 20];
x0 = [0]
[tx] = ode45 (at sos, tspan, x0)
 plot (t,x)
```

Changing parameters :

$\dot{x} + ax = b \qquad \dot{x} = b - ax$

```
function  dx = sos(t,x,param)
a = param (1)
b = param (2)
dx = b - a*x
tspan = [0 20];
x0 = [0] ;
param [ , ] ;
[tx] = ode45 (at sos, tspan, x0, [], param)
plot (t,x)
```

Symbolic :

write MATLAB symbolic code to solve the following differential equations

$\ddot{x}y + y = \log(t), \quad \ddot{y}x + \dot{y} = t, \qquad \dot{x}(0) = 0, \; \dot{y}(0) = 0, x(0) = 0, \; y(0) = 0$

x(t) , y(t)

```
Syms  x(t)  y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
       , diff(y,t,2)*x + diff(y,t) == t ] ;
dx = diff (x,t) ;
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0, x(0)==0, y(0)==0];
sol = dsolve (eqn, cond)
```

في حال لغى المعادلة الثانية

```
Syms  x(t)  y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
       , diff(y,t,2)*x + diff(y,t) == t ] ;
dx = diff (x,t) ;
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0, x(0)==0, y(0)==0];
sol = dsolve (eqn, cond)
```

جس منلغي هاي .

Solve the following differential equation using ODE45 for t from 0 to 5

$$\dddot{W} + \ddot{W} + \dot{W} + 2W = \sin(t) \quad W(0) = 0 \quad \dot{W}(0) = 1 \quad \ddot{W}(0) = 2;$$

$w_2 = \dot{w}_1$ --- ①

$w_3 = \dot{w}_2 = \ddot{w}_1$ --- ②

$\dot{w}_3 = \ddot{w}_2 = \dddot{w}_1$

$\dot{w}_3 + w_3 + w_2 + 2w_1 = \sin t$

$\dot{w}_3 = \sin t - w_3 - w_2 - 2w_1$

```
function dw = sos(t,w)
[m,n] = size(w);
dw = zeros(m,n);
dw(1) = w(2);
dw(2) = w(3);
dw(3) = sin t - w(3) - w(2) - 2*w(1)
tspan = [0 5];
w0 = [0;1;2];
[t w] = ode45(@sos, tspan, w0)
```

Solve the following differential equation using symbolic

$$\ddot{X} + 3X + 2t = 0 \quad X(0) = 0 \quad \dot{X}(0) = 1;$$

```
syms x(t)
eqn = [diff(x,t,2) + 3*x + 2*t == 0];
dx = diff(x,t);
cond = [x(0)==0, dx(0)==1];
sol = dsolve(eqn, cond)
```

Solve the following differential equation using symbolic

$$\dddot{W} + \ddot{W} + \dot{W} + 2W = \sin(t) \quad W(0) = 0 \quad \dot{W}(0) = 1 \quad \ddot{W}(0) = 2;$$

```
syms w(t)
eqn = [diff(w,t,3) + diff(w,t,2) + diff(w,t)
       + 2*w == sin(t)];
dw = diff(w,t);
dw2 = diff(w,t,2);
cond = [w(0)==0, dw(1)==1, dw2(0)==2];
sol = dsolve(eqn, cond).
```

Solve the following differential equation using ODE45

$$\ddot{X} + 3X + 2t = 0 \quad X(0) = 0 \quad \dot{X}(0) = 1;$$

$x_2 = \dot{x}_1$ --- ① , $\dot{x}_2 = \ddot{x}_1$

$\dot{x}_2 = -2t - 3x$

```
function dx = sos(t,x)
[m,n] = size(x);
dx = zeros(m,n);
dx(1) = x(2);
dx(2) = -3*x - 2*t
tspan = [0 5];
x0 = [0;1]
[t w] = ode45(@sos, tspan, x0);
plot(t,x)
```

$\ddot{x} + 1 = t \qquad x(0)=0 \;,\; \dot{x}(0)=0$

① ODE 45 :

$x_2 = \dot{x}_1 \quad \cdots ① \qquad \dot{x}_2 = \ddot{x}_1 \Rightarrow \quad \dot{x}_2 = t - 1 \quad \cdots ②$

```
function dx = sos(t,x)
[m,n] = size(x);
dx = zeros(m,n);
dx(1) = x(2);
dx(2) = t-1;
```

```
tspan = [0 10];
x0 = [0;0];
[t x] = ode45(@sos,tspan,x0)
plot(t,x)
```

② Symbolic

```
syms x(t)
eqn = [diff(x,t,2)+1 == t];
dx = diff(x,t);
Cond = [x(0)==0, dx(0)==0];
sol = dsolve(eqn, cond).
```

$$\ddot{x} + x = \log(t) \qquad \dot{x}(0)=0 \;,\; x(0)=0$$

Symbolic :

```
syms x(t)
eqn = [diff(x,t,2) + x == log10(t)];
dx = diff(x,t);
cond = [dx(0)==0, x(0)==0];
sol = dsolve(eqn, cond)
```

③ Tool box :

Example : $y'' + 6y' + 5y = 4u' + 3u.$ ① Transfer function.

$s^2 Y(s) + 6s\,Y(s) + 5Y(s) = 4s\,U(s) + 3U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}.$

```
>> num = [4 3];
>> den = [1 6 5];
>> G = tf(num,den)
```

G = tf([4 3],[1 6 5])

$$G = \frac{1}{(s+3)(s^2+2)}$$

to find r.locus & step response

```
num = [1];
den = conv([1 3],[1 0 2]);
g = tf(num,den)
g = tf(1, con([1 3],[1 0 2])).
rlocus(g)
step(g)
```

$$G = \frac{1}{s^2+2s+7}$$

to find r.locus & step response

```
num = [1];
den = [1 2 7];
g = tf(num,den);
rlocus(g);
step(g);
```

11/8

Matlab

Control system Toolbox :

Transfer function assuming zero initial condition

Example : $y'' + 6y' + 5y = 4u' + 3u$. ① Transfer function.

$s^2 Y(s) + 6s\,Y(s) + 5Y(s) = 4s\,U(s) + 3U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}$.

```
>> num = [4 3];
>> den = [1 6 5];
>> G = tf(num, den)
```

G = tf([4 3], [1 6 5])

② Zero-pole-gain (ZPK)

$\frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5} = \frac{4(s+0.75)}{(s+1)(s+5)}$, G = zpk([-0.75], [-1, -5], 4)

* Normal : s = tf('s')

G = (4*s + 3)/((s^2) + 6*s + 5)

③ State-Space Model (ss)

$\frac{Y}{R} = \frac{4s+3}{s^2+6s+5}$, $\frac{X}{R} = \left(\frac{1}{s^2+6s+5}\right)$, $\frac{Y}{X} = (4s+3)$.

$R = x'' + 6x' + 5x$, $Y = 4x' + 3x$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]R.$$

G = ss([0 1; -5 -6], [0; 1], [3, 4], 0)

Symbolic :

⇒ Solve the diff. equ. $\frac{dy}{dx} = e^{-\frac{1}{x}} \frac{1}{x^2}$:

```
syms y(x)
eqn = diff(y) == exp(-1/x)/x^2
cond = [y(0) == 1];
sol(x) = dsolve(eqn, cond)
```

⇒ system of differential equ.

$\frac{dy}{dt} = z$, $\frac{dz}{dt} = y$.

```
syms y(t) z(t)
eqns = [diff(y,t) == z   diff(z,t) == -y]
sol = dsolve(eqn)
sol.z
sol.y
```

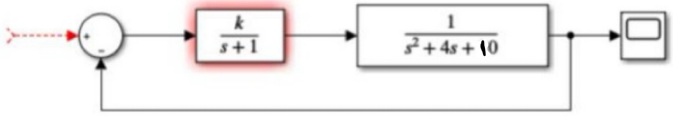
$(1+t^2)\ddot{w} + 2t\dot{w} + 3w = 2$ 2 order

$t_0 = 0$, $t_f = 5$, $w(t_0) = 0$, $\dot{w}(t_0) = 1$

```
function dx = diff(t, x)
[m, n] = size(x)
dx = zeros(m, n)
dx(1) = x(2)
dx(2) = (2 - 2*t*x(2) - 3*x(1))/(1+t^2)
tspan = [0 5]
x0 = [0; 1]
[t, x] = ode23(@diff, tspan, x0)
plot(t, x(:,1))
plot(t, x(:,1), t, x(:,2))
```

Quiz (8): Routh Hurwitz

Q3(5 points) Use the Routh Hurwitz array to find the



Solution

s^3	1	14	0		
s^2	5	k+10	0		
s^1	(60-k)/5	0	0		
s^0	k+10	0	0		

$60 > k > 0$

K=.....

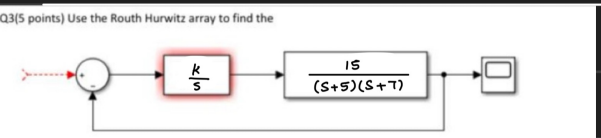
$$\frac{\frac{k}{(s+1)(s^2+4s+10)}}{1+\frac{k}{(s+1)(s^2+4s+10)}} \Rightarrow \frac{k}{(s+1)(s^2+4s+10)+k} = \frac{k}{s^3+5s^2+14s+(k+10)}$$

$\frac{70-(k+10)}{5} = \frac{60-k}{5}$,

$\frac{60-k}{5} > 0 \Rightarrow 60 > k$ / $k+10 > 0$

$k > -10$ can't take the negative value.

Q3(5 points) Use the Routh Hurwitz array to find the



Solution

s^3	1	35	0		
s^2	12	15k	0		
s^1	A	0			
s^0	15k	0			

$28 > K > 0$

K=.....

$$\frac{15k}{s(s+5)(s+7)+15k} = \frac{15k}{s^3+12s^2+35s+15k}$$

$A = \frac{420-15k}{12}$, $\frac{420-15k}{12} > 0$

$420 > 15k \Rightarrow 28 > k$, $15k > 0$, $k > 0$

$s^3 + s^2 + 2s + k = 0$, what's (k) for stability?

Solution

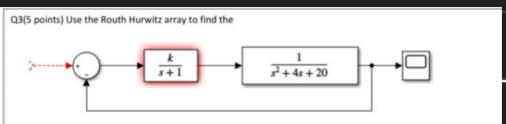
s^3	1	2	0		
s^2	1	k	0		
s^1	2-k	0	0		
s^0	k	0	0		

$k > 0$

$2 > k$

$2 > k > 0$

Q3(5 points) Use the Routh Hurwitz array to find the



Solution

s^3	1	24	0		
s^2	5	K+20	0		
s^1	A	0	0		
s^0	K+20	0	0		

K=.....

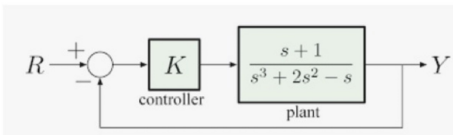
$$\frac{k}{s^3+5s^2+24s+(20+k)}$$

$A = \frac{120-k-20}{5} = \frac{100-k}{5}$

$K+20 > 0 \Rightarrow k > -20 \Rightarrow k > 0$ ∴ no (-ve) value.

$100 > k \Rightarrow 100 > k > 0$

Q1) Determine K for stability



$$\frac{K(s+1)}{s^3+2s^2+(k-1)s+k}$$

s^3	1	k-1	0		
s^2	2	k	0		
s^1	(A)	0	0		
s^0	k	0	0		

$A = \frac{2k-2-k}{2} = \frac{k-2}{2}$

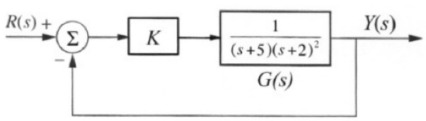
$\frac{k-2}{2} > 0$

$k > 2$

$k > 0$

$k > 2$

Q1) find k for stability for following system, what is the auxiliary equation and the point of cross with the imaginary axis.



$$\frac{k}{s^3+9s^2+24s+(k+20)}$$

s^3	1	24	0		
s^2	9	K+20	0		
s^1	A	0	0		
s^0	K+20	0	0		

$A = \frac{216-k-20}{9} = \frac{196-k}{9}$

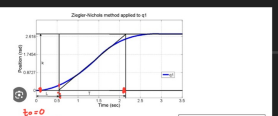
$K+20 > 0$ $k > 0$, no -ve

$196-k > 0 \Rightarrow 196 > k > 0$

* Anxiliary: $9s^2 + (196+20) = 9s^2 + 216 = 0$

$s^2 + 24 = 0 \Rightarrow s = \pm\sqrt{24}\,j$

Quiz (9) PID :



$t_0 = 0$
$t_1 = 0.55$
$t_2 = 2.15$

The response when we changes the input from 0.1 to 0.3 KW the output changed according to the figure

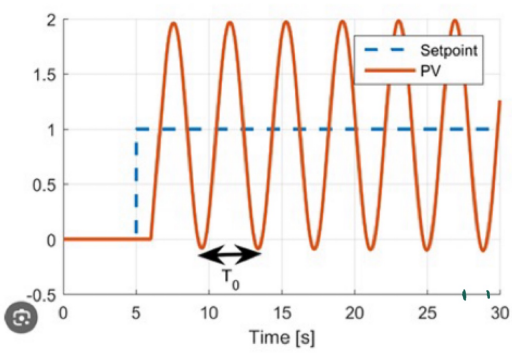
	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o \tau_o}$	/	/
PI	$\frac{0.9\nu_o}{K_o \tau_o}$	$3\tau_o$	/
PID	$\frac{1.2\nu_o}{K_o \tau_o}$	$2\tau_o$	$0.5\tau_o$

$V_o = 1.6$ s

$T_o = 0.55$ s

$K_o = \frac{2.75}{0.3 - 0.1} = 13.75$ s

Q1)Design the PID controller using Zeigler Nichols Oscillation method



Solution $K = 6$

$P_{cr} = 4$, $K_{cr} = 6$

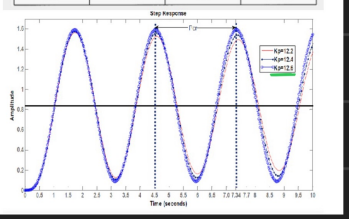
$K_I = 3.6$

$T_i = 2$

$T_d = 0.5$

Q) Tune the PID controller using the zeigler nichols oscillation technique

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

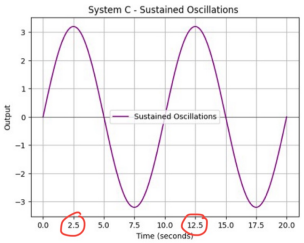


$P_{cr} = 2.84$ s

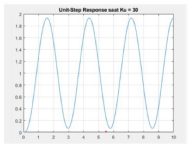
$K_{cr} = 12.6$ s

The system has modified proportional controller in step and obtained sustained oscillation at K=3.5

Evaluate the PID



$P_{cr} = 10$, $K = 3.5$



The response above is obtained for the system for the PID auto tuning Ziegler Nichols oscillation method. What is the PID controller

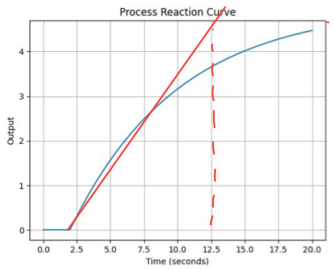
Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$P_{cr} = 5.81 - 2.95 = 3.86$

$K_{cr} = 30$

$K_p = 18$, $T_i = 1.43$ ...

Evaluate the PID parameters using Ziegler Nichols response method if time t=0, I modified the input u from 0.25 to 1.25 and I got the following output response curve, the output started from 0 and final value is 5



$t_0 = 0$
$t_1 = 2$
$t_2 = 12.5$
$U_0 = 10.5$
$T_0 = 2$
$K_0 = \frac{5}{1} = 5$

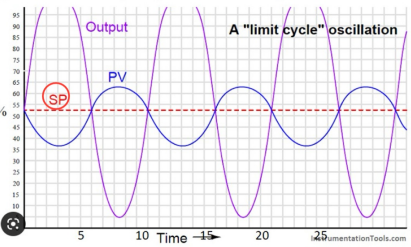
	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o \tau_o}$		
PI	$\frac{0.9\nu_o}{K_o \tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o \tau_o}$	$2\tau_o$	$0.5\tau_o$

$K_p = \frac{1.2 * 10.5}{5 * 2} = 1.26$

$T_n = 4$



Q) Tune the PID controller using the zeigler nichols oscillation technique



	K_p	T_r	T_d
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o \tau_o}$		
PI	$\frac{0.9\nu_o}{K_o \tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o \tau_o}$	$2\tau_o$	$0.5\tau_o$

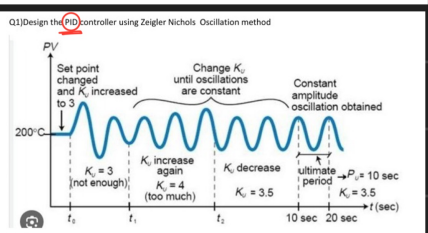
$P_{cr} = 11$

$K_{rc} = 3$

$K_p = 1.8$

$T_n = 5.5$.

Q1)Design the PID controller using Zeigler Nichols Oscillation method



Solution

Table-2

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$P_{cr} = 20 - 10 = 10$ S.

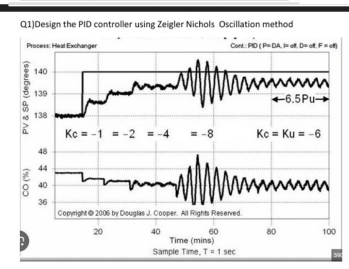
$T_d = 1.25$

$T_i = 5$ S

$K_{cr} = 3.5$

$K_p = 0.6 * 3.5 = 2.1$

Q1)Design the PID controller using Zeigler Nichols Oscillation method



Solution

Table-2

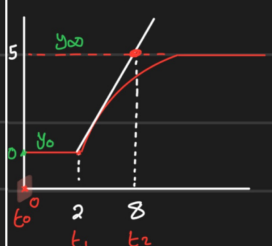
Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$\frac{19}{7} = 2.85$ min $* 6.5 = 18.525$.

$\frac{200}{7} = 28.5 * 6.5 = 184.6$



Example : find the fainal PID ? $u_\infty = 1.25$, $u_0 = 0.25$



① $V_o = t_2 - t_1 = 6$

② $T_o = t_1 - t_0 = 2$

③ $K_o = \frac{y_\infty - y_0}{u_\infty - u_0} = \frac{5}{1} = 5$

(,) given

وبعدين على الجدول .

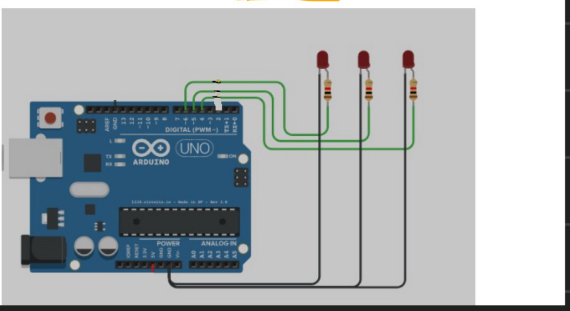
PID : $\frac{1.2\,V_o}{k_o T_o} = \frac{1.2 * 6}{5 * 2}$

$2\tau_o = 4$

$0.5\,T_o = 1$

Ard.

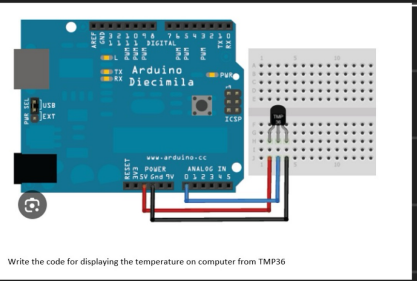
Q) Write the code for the light to start from left to right; separated by 0.1 seconds



```
Const int LED=6, LED2=5, LED3=4;

voidsetup ()
{ pin Mode (LED1, OUTPUT);
  pin Mode (LED2, OUTPUT);
  Pin Mode (LED3, OUTPUT); }

void Loop ()
{ digital Write (LED1, HIGH);
    delay (100);
  digital Write (LED2, HIGH);
    delay (100);
  digital Write (LED3, HIGH); }
```



Write the code for displaying the temperature on computer from TMP36

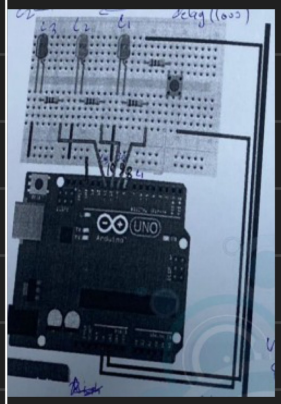
```
Const int Tpin = 0;

Void setup()
{ Serial. begin (9600); }

void Loop()
{ double voltage, Temperature;
  voltage = get voltage (Tpin);
  Temperature = (Voltage - 0.5) * 100;
  Serial. print ("T is");
  Serial. println(Temperature);
    delay (1000); }
```

```
double get voltage (int Tpin)
{ return (analog Read
    (Tpin) * 5/1024); }
```

For the LED, Code for the lighting sequance right then middle then left then middle, separated by (1s).



```
Const int LED1 = 8, LED2=9, LED3=10;

void setup()
{ pin Mode (LED1, OUTPUT);
  pin Mode (LED2, OUTPUT);
  pin Mode (LED3=OUTPUT); }

void loop()
{ digital Write (LED1, HIGH);
  digital Write (LED2, LOW);
  digital Write (LED3, LOW);
    delay (1000);
  digital Write (LED1, LOW);
  digital Write (LED2, HIGH);
  digital Write (LED3, LOW);
    delay (1000);

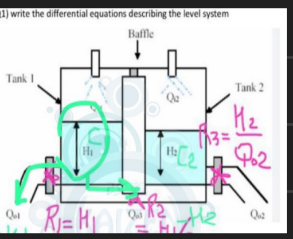
  digital Write (LED1, LOW);
  digital Write (LED2, LOW);
  digital Write (LED3, HIGH);
    delay (1000);

  digital Write (LED1, LOW);
  digital Write (LED2, HIGH);
  digital Write (LED3, LOW);
    delay (1000); }
```

Quiz & Tanks / Modelling &

Gear : $\frac{\text{Teeth A}}{\text{Teeth B}} = \frac{\text{Speed (B)}}{\text{Speed (A)}}$ ...

1) write the differential equations describing the level system

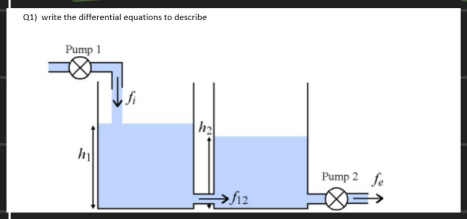


$$\text{tank (1)} = Q_{i1} - Q_{01} - Q_{03} = C\frac{dh}{dt}$$

$$\text{tank (2)} = Q_{i2} + Q_{03} - Q_{02} = C\frac{dh}{dt}$$

$$R_1 = \frac{H_1}{Q_{01}}, \quad R_2 = \frac{H_1 - H_2}{Q_{03}}, \quad R_3 = \frac{H_2}{Q_{02}}$$

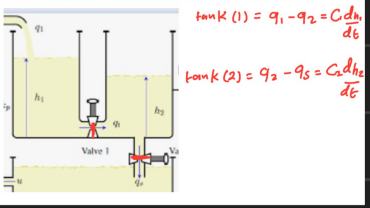
Q1) write the differential equations to describe



$$\text{tank (1)} = C_1\frac{dh_1}{dt} = f_i - f_{12}$$

$$\text{tank (2)} = C_2\frac{dh_2}{dt} = f_{12} - f_e$$

Q) Write the equations describing the following system add needed symbols



tank (1) = $q_1 - q_2 = C_1\frac{dh_1}{dt}$

tank (2) = $q_2 - q_3 = C_2\frac{dh_2}{dt}$

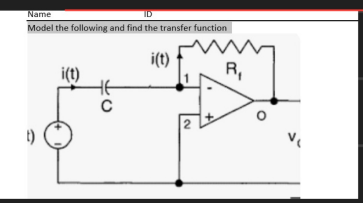
$$R_1 = \frac{h_1 - h_2}{Q_2}$$

$$R_2 = \frac{h_2}{Q_3}$$

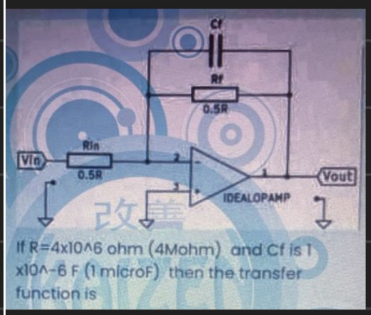
$$R_1 = \frac{-h_1}{f_i}, \quad R_2 = \frac{h_2}{f_e}.$$

Name ID

Model the following and find the transfer function



$$T.f = \frac{-Z_2}{Z_1} = -R_fC.S.$$



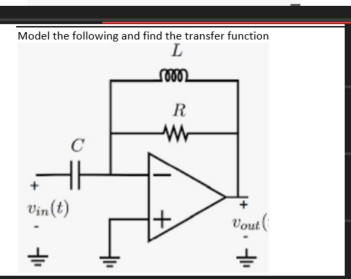
$$Z_2 = \frac{\frac{R_f}{C.S}}{R_f + \frac{1}{C.S}} = \frac{R_f}{R_fCS+1}$$

$$Z_1 = R_{in}$$

$$T.f = -\left(\frac{\frac{R_f}{R_f.C.S+1}}{R_{in}}\right).$$

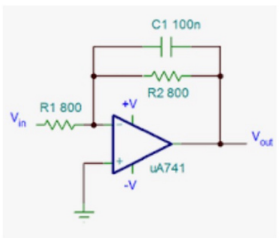
$$T.f = \frac{2*10^6}{(1+2*10^6*1*10^{-6}*S)\,2*10^6} = \frac{1}{1+2S}.$$

Model the following and find the transfer function



$$T.f = \frac{-Z_2}{Z_1} = \frac{\frac{R.L.S}{R+L.S}}{\frac{1}{C.S}} = \frac{RLCS^2}{R+L.S}$$

Q1) Find transfer function Vout/Vin for

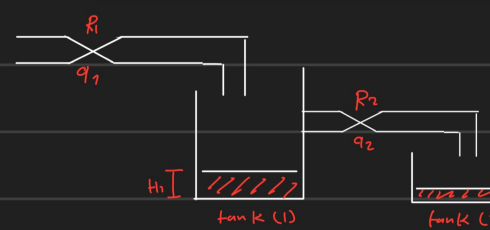


$$Z_2 = \frac{R_2}{R_2C.S+1}$$

$$Z_1 = R_1$$

$$T.f = -\frac{\frac{R_2}{R_2C.S+1}}{R_1}$$

$$T.f = -\frac{800}{(800*100*10^{-9}S+1)*800} = \frac{800}{800+0.064S}.$$

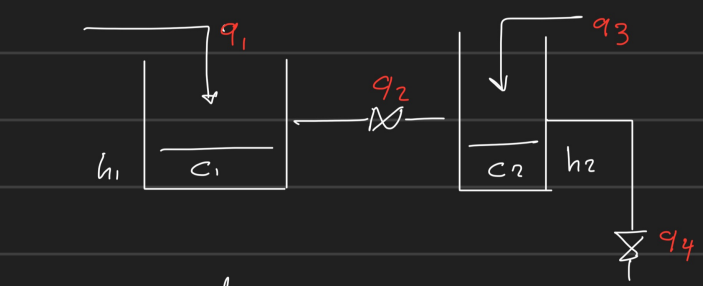


[1] tank (1) $\Rightarrow \frac{C_1 dH_1}{dt} = q_1 - q_2$

tank (2) $\Rightarrow \frac{C_2 dH_2}{dt} = q_2 - 0.$

[2] $R_1 = \frac{-H_1}{q_1}$

$R_2 = \frac{H_1 - H_2}{q_2}$

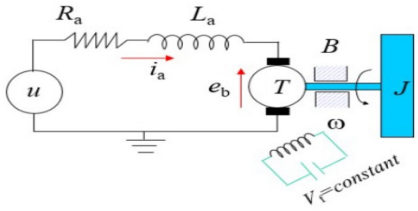


$$\text{tank (1)} = \frac{C_1 dh_1}{dt} = q_1 - q_2$$

$$\text{tank (2)} = C_2\frac{dh_2}{dt} = q_2 + q_3 - q_4$$

$$R_1 = \frac{-h_1}{q_1}, \quad R_2 = \frac{h_1 - h_2}{q_2}, \quad R_3 = \frac{-h_2}{q_3}, \quad R_4 = \frac{h_2}{q_4}$$

Q5) Figure below is a schematic for an armature controlled dc motor. Find the transfer function w/u where w is the rotational velocity and u is the voltage. (10 points)



$J\omega s$

$T_{motor} = J\dot{\omega} + B\omega$

$= K_t I \Rightarrow I = T/K_t$

$e_b = K_b \omega$

$-u + RI + (La \cdot S)I + e_b = 0$

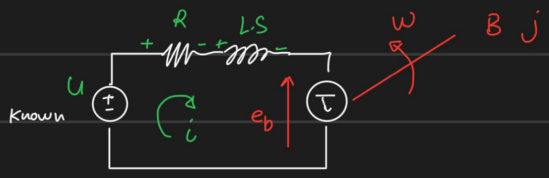
$U(S) = (R + La \cdot S)I + (Kb \cdot \omega)$

$$\frac{(R + La \cdot S)(Jws + B\omega)}{K_t} + (Kb \cdot \omega)$$

$$u = \omega\left(\frac{La J \cdot S^2 + (La \cdot B + RJ)S + BR + K_b K_t}{K_t}\right)$$

$$\frac{\omega}{u} =$$

*] DC motor :



$-u + RI + LSI + e_b = 0$

$T = K_t * I$, $I = T/K_t$, $e_b = K_b \omega$.

$T = J\dot{\omega} + B\omega$

$u = \frac{Jws + B\omega}{K_t}(R + LS) + (K_b \omega)$.

$$u = \omega\left(\frac{JLS^2 + (JR + BL)*S + BR + K_b K_t}{K_t}\right)$$

$$\left(\frac{\omega}{u} = \frac{K_t}{JLS^2 + (JR + BL)S + BR + K_b K_t}\right)$$

Arduino :

on , off
input . output



[0v — 5v
0 — 1024]

* Every Arduino "Sketch" has :

```
// variable declarations

const int LED 13 ;

void setup ()
{
  // configuration of pins . etc .
}
 void Loop ()
{
// what the program does , in the continuous loop
}
```

Example : blink

```
const int LED = 13 ;                    (pin13) مع وصلة

void Setup ()
{ pin Mode (LED, OUTPUT) ;              OUTPUT
}                                       INPUT.
 Void Loop ()
{ digital Write (LED, HIGH) ;           5V
  delay (1000);                         delay (1000)
                                        يتأخر ثانية واحدة
  digital Write (LED, LOW);
  delay (1000) ;
}
```

button

```
    void Loop (0)                       يجب لازم أكتبي قبل حرف منه
{                                       Const int Button = ;
Val = digitalRead (Button);             pin Mod (Button, INPUT)
 if (val == HIGH) {
{
 digital Write (LED, HIGH) ;
}
  else {
        digital Write (LED, LOW);
       }
 }
```



Morse code

LED will light long. ← الاشارة الطويلة

النقطة ← اشارة قصيرة

```
void dot ()
{ blink (200, 200) ; }

void dash ()
{ blink (600, 200) ; }

void letterspace ()
{ delay (400) ; }

void wordspace ()
{ delay (800) ; }
```

```
void blink (int ontime , int offtime ).
{
 digital Write (LED, HIGH) ;
  delay (ontime ) ;
 digital Write (LED, LOW) ;
  delay (offtime ) ;
}
```

Loop!

perhaps letter functions :

```
void morse_s ()
{ dot () ; dot () ; dot () ; letterspace () ; }

void morse_o ()
{ dash () ; dash () ; dash () ; Letter space () ; }
```

Arduino : Temperature Sensors

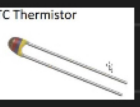
TMP36



⇒ مصمم بطريقة تجريبية عشان يعين Voltage .

يستقبل من الاطراف (Zero & 5 V)

NTC Thermistor

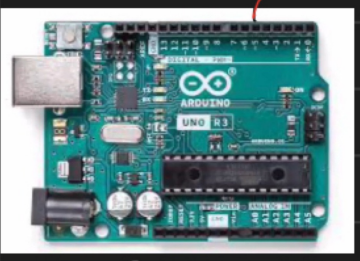


⇒ تتغير المقاومة فيه (تنزل مع الحرارة)

```
// We'll use analog input 0 to read Temperature Data const int t
void setup()
 { Serial.begin(9600); }
void loop()
{ float voltage, degreesC, degreesF;
voltage = getVoltage(temperaturePin);
 // Now we'll convert the voltage to degrees Celsius.
// This formula comes from the temperature sensor datasheet:
 degreesC = (voltage - 0.5) * 100.0;
 // Send data from the Arduino to the serial monitor window
Serial.print("voltage: ");
Serial.print(voltage);
Serial.print(" deg C: ");
Serial.println(degreesC);
delay(1000);
// repeat once per second (change as you wish!) }
float getVoltage(int pin)
 { return (analogRead(pin) * 0.004882814); }
// This equation converts the 0 to 1023 value that analogRead()
// returns, into a 0.0 to 5.0 value that is the true voltage
// being read at that pin.
```

```
// Read Temerature Values from NTC Thermistor
 const int temperaturePin = 0;                    const int temp = 0
 void setup()
{ Serial.begin(9600); }
 void loop()
 { int temperature = getTemp();
Serial.print("Temperature Value: ");
Serial.print(temperature);
 Serial.println("*C");
delay(1000);
 }
 double getTemp()
{
// Inputs ADC Value from Thermistor and outputs Temperature in Celsius int RawADC =
analogRead(temperaturePin);
long Resistance;
 double Temp;
 // Assuming a 10k Thermistor. Calculation is actually: Resistance = (1024/ADC)
Resistance=((10240000/RawADC) - 10000);
 // Utilizes the Steinhart-Hart Thermistor Equation:
// Temperature in Kelvin = 1 / {A + B[ln(R)] + C[ln(R)]^3}
// where A = 0.001129148, B = 0.000234125 and C = 8.76741E-08 Temp = log(Resistance);
 Temp = 1 / (0.001129148 + (0.000234125 * Temp) + (0.0000000876741 * Temp * Temp *
Temp)); Temp = Temp - 273.15;
// Convert Kelvin to Celsius return Temp;
 // Return the Temperature
 }
```

Arduino :



digital [1 0]
on / off ⇒ on = 5V , off = 0 V

input : Button 1 0 Output LED 1 0 :

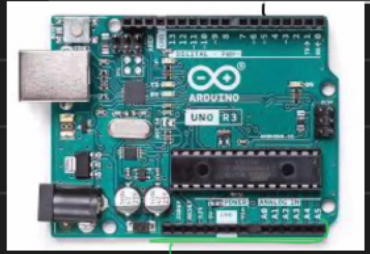
```
const int Button = 10;   (1) منشيها لا تغير
, LED = 9;
void setup( )
{ pinMode (Button , INPUT);
  PinMode (LED , OUTPUT);
  { Void loop ( )
    { int x;
      x= digitalRead (Button);
if (x==1)
digitalWrite (LED, HIGH);
else
digitalWrite (LED, LOW);
}
```

resistance في دايما (LED) بالنسبة لـ

كيف بدخله بالمعادلة ؟؟

في حال كان على (9) و Button على (10).

① Digital pins. بيوخذو قيمة [0 / 1].



↳ Analog pin

* Analog pin [1024]

using analog Read ②

analog Write

delay (1000);

$$R_{total} = 1000 + RT = \frac{V}{I} = \frac{1024}{\frac{V}{1000}}$$

```
float get Temp (int Tpin)
{ int V= analogRead (Tpin);
  float R=(1024.0000 /v)-10000
  float logR= log(R);
  return (1/(0.001129148 + 0.000234125 log(R)
          + 8.76741 E8*logR*logR*logR) }
```

— 273.15 → if it's in (C)



5V Ground Ao

THP 36 . : Ground. الطرف القصير مع الـ



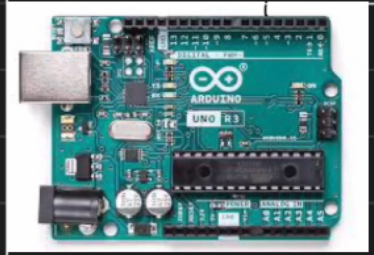
اليسار مع (5V)

الوسط مع Ao

```
Const int Tpin= 0;
Void setup ( )
{ Serial.begin (9600);
  }
Void Loop ( )
{ float voltage , temprature;
voltage = get voltage (Tpin);
temperature =(voltage - 0.5)*100;
Serial.print (" T is ");
Serial.println (temperature);
delay (1000);
}
float get voltage (int Tpin)
{ return (analogRead (Tpin) * 5/1024)
```

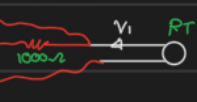
① 0 1024 / 0V 5V

⇒ 512 → 2.5V = *5/1024.





② NTC Thermistor



voltage div

5V → 1024

```
const int Tpin = 0;
Void setup ( )
{ Serial.begin (9600);
  { Void loop (
    { float Temp;
      Temp = get Temp (Tpin);
      Serial.Print ("T is ");
      Serial.print (Temp);
      Serial.println (Temp);
      delay (1000);
    {
```

① Button by LED :

```
const int Button = 12;
const int LED = 13;
void setup ( )
{ pinMode (Button, INPUT);
  pinMode (LED, OUTPUT); }
void loop ( )
{ val = digitalRead (Button);
    if (val == HIGH)
     { digitalWrite (LED, HIGH); }
    else
    { digitalWrite (LED, LOW); }
  }
```

② Blink LED

```
const int LED = 13;
void setup( )
{ pinMode (LED, OUTPUT); }
  void loop ( )
{ digitalWrite (LED, HIGH);
   delay (1000);
   digitalWrite (LED, LOW);
   delay (1000);
}.
```

③ TMP = 36.