Control.

Jana Househ.

Control

Laplace part (1):

$f(t) \rightarrow$ find Laplace : $\mathcal{L}\, f(t) = F(s)$

" " inverse : $\mathcal{L}^{-1} F(s) = f(t)$

*] 3 cases for partial fraction:

- unrepeted factor ⇒ كل اشي بالمقام مكرر. $(s)(s+2)(s+3)$.
- repeted factor ⇒ قوس اله قوة بالمقام. $(s+8)^2 / (s)^2$
- Complex ⇒ عبارة تربيعية لن تحلل بالمقام (s^2+5s+3).

Ex: بسط / $(s^2)(s+2)(s+3)(s^2+2s+5)$

repeted — unrepeted — complex

① unrepeted : $\frac{8(s+3)(s+8)}{(s)(s+2)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+4}$

$\mathcal{L}^{-1}(F(s)) \rightarrow f(t)$.

$\left.\frac{8(s+3)(s+8)}{(s+2)(s+4)}\right|_{s=0} = 24$, ----- $k_2 = -12$, $k_3 = -4$

$f(t) = 24 - 12e^{-2t} - 4e^{-4t}$

Laplace part (2): Repeted Roots.

Ex: $\frac{8(s+1)}{(s+2)^2} = \frac{k_1}{s+2} + \frac{k_2}{(s+2)^2}$, ⇒ بهاي الحالة بستخدم توحيد المقامات

$8(s+1) = k(s+2) + k$ → $s=0 \Rightarrow 8 = 2k_1 + k_2$; $s=-2 \Rightarrow -8 = k_2$ ⇒ $k_1 = 8$, $k_2 = -8$

ans: $8e^{-2t} - 8te^{-2t} = f(t)$.

② Complex: متحل بإكمال مربع

Ex: $\frac{k_1 s + k_2}{s^2+2s+5} = \frac{k_1 s + k_2 + k_1 - k_1}{s^2+2s+1+4} = \frac{k_1(s+1) + (k_2 - k_1)}{(s+1)^2 + (2)^2}$

$\frac{k_1(s+1)}{(s+1)^2+2^2} + \frac{k_2 - k_1}{(s+1)^2+2^2} = k_1 \cos 2t \cdot e^{-t} + \left(\frac{k_2 - k_1}{2}\right) \sin 2t \cdot e^{-t}$

Laplace part (3):

Important case: find ($\mathcal{L}$) then ($\mathcal{L}^{-1}$) : $X(s) \rightarrow x(t)$

$\ddot{x} + 3\dot{x} + 2x = 5\sin(t)$. , Apply using Laplace to find $X(t)$.

$\mathcal{L}(\ddot{x} + 3\dot{x} + 2x) = \mathcal{L}(5\sin(t))$ → from table $= 5\left(\frac{1}{s^2+1}\right)$

$\ddot{x} = s^2 X(s) - s x(0) - \dot{x}(0)$

$\dot{x} = s X(s) - x(0)$

$x = X(s)$.

, $x(0) = 1$, $\dot{x}(0) = 0$

$\left(s^2 X(s) - s\cancel{x(0)} - \cancel{\dot{x}(0)}\right) + 3\left(sX(s) - \cancel{x(0)}\right) + 2(X(s))$

$X(s)(s^2+3s+2) - s - 3 = \frac{5}{s^2+1}$.

$X(s) = \frac{5}{(s^2+1)(s^2+3s+2)} + \frac{(s+3)}{(s^2+3s+2)}$. → Partial fraction the ($\mathcal{L}^{-1}$).

Part (A) — Part (B).

Part (A): $\frac{5}{(s^2+1)(s+2)(s+1)} = \frac{c_1 s + c_2}{s^2+1} + \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$

$5 = (c_1 s + c_2)(s+2)(s+1) + k_1(s^2+1)(s+2) + k_2(s^2+1)(s+1)$.

① $s=-2$: $5 = k_2(5)(-1) \Rightarrow \boxed{k_2 = -1}$.

② $s=0$: $5 = c_2(2) + k_1(2) + \cancel{k_2} \Rightarrow 3 = c_2 + k_1$.

③ $s=1$: $\cancel{5} = (c_1+c_2)(6) + k_1(6) + \cancel{(-1)}\cancel{(4)} \Rightarrow 3 = 2(c_1+c_2) + 2k_1$

④ $s=-1$: $5 = k_1(2) \rightarrow \boxed{k_1 = 5/2}$

$\boxed{c_2 = 1/2} \Rightarrow \boxed{c_1 = -3/2}$.

Part (B) $= \frac{(s+3)}{(s^2+3s+2)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$.

$(s+3) = k_1(s+2) + k_2(s+1)$.

① $s=-2$: $1 = -k_2$ $\boxed{k_2 = -1}$

② $s=-1$: $2 = k_1$ $\boxed{k_1 = 2}$

the fainal answer:

$\boxed{\frac{(-\frac{3}{2})s + (\frac{1}{2})}{s^2+1}} + \frac{(\frac{5}{2})}{(s+1)} + \frac{-1}{(s+2)} + \frac{2}{(s+1)} + \frac{-1}{(s+2)}$ (9/2, −2)

$\left(\frac{9}{2}\right)e^{-t} - (2)e^{-2t} + \mathcal{L}^{-1}\left[\frac{-(\frac{3}{2})s}{s^2+1} + \frac{1/2}{s^2+1}\right]$

$= \left(\frac{9}{2}\right)e^{-t} - (2)e^{-2t} + \left(\frac{-3}{2}\right)\cos t + \left(\frac{1}{2}\right)\sin t. = x(t)$.

Laplace part (4):

① final value theorem (FVT), ② Initial value theorem (IVT).

↳ $F(S)$ → find ⇄ FVT. / IVT.

$F(S) \cdot (S) \longrightarrow \lim$ $S \to 0$ (fainal) / $S \to \infty$ (initial).

Ex: $F(S) = \frac{S+3}{(S)(S+2)}$, find the (FVT) then (IVT).

$\lim_{S\to 0} \left((S) \cdot \frac{S+3}{(S)(S+2)} \right) = 3/2$. , $\lim_{S\to \infty} \left((S) \cdot \frac{S+3}{(S)(S+2)} \right) = \frac{\infty}{\infty} = 1$

The system start from (1) and goes to ($3/2$).

---

* $F(S) \to f(t)$: $\mathcal{L}^{-}$
$f(t) \to F(S)$: $\mathcal{L}$

Matlab code: write the matlab code

Ex: $f(t) = te^{-4t} \Rightarrow (\mathcal{L})$:

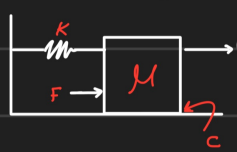
```
syms t s
Laplac(t*exp(-4*t)) = -4/(S+1)^2
```

→ case (1).

Ex: $F(S) = \frac{S(S+6)}{(S+3)(S^2+6S+18)} \Rightarrow \mathcal{L}^{-}$:

```
syms t s
ilaplace ......
```

Mchanical system:



- Mass M → always with one x
- Spring K
- Damper D, C. → can be fixed or free.

from mass

* عدد الـ (X) الموجودين بالرسمة = عدد المعادلات اللي رح نكتبها.

$M \longrightarrow M \cdot \ddot{x}$
$k \longrightarrow k \cdot x$
$c \longrightarrow c \cdot \dot{x}$

$\ddot{x} = s^2 X(S)$
$\dot{x} = s\, X(S)$
$x = X(S)$

(M, K, C) تتأثر باتجاه عكسي (F) دائما

*] Damper → fixed : effected by only one (x)
free : 〃 〃 more than one (x).

—]— , fixed.

[1] نحدد نتجه العناصر اللي عندي
[2] نحدد كل عنصر (free or fixed)

find the transfer function $= \frac{\text{output}}{\text{input}} = \frac{X(S)}{F(S)}$.

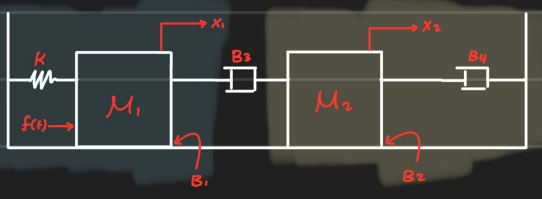
the solution: (one (x) means one equation)

$F = M\ddot{x} + C\dot{x} + Kx \Rightarrow Ms^2X(S) + Cs\,X(S) + K\,X(S)$

$F(S) = X(S)(Ms^2 + Cs + K) \Rightarrow \frac{X(S)}{F(S)} = \frac{1}{Ms^2 + Cs + K}$.

Anothe case:

will have two equations.



equation (1) for (X_1) : $F = M_1\ddot{x}_1 + k_1x_1 + B_1\dot{x}_1 + B_3(\dot{x}_1 - \dot{x}_2)$.

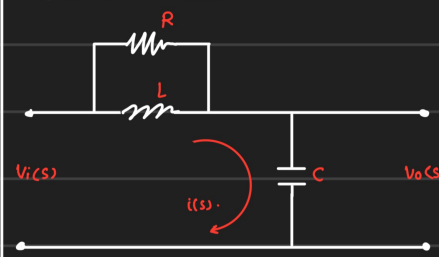
〃 (2) 〃 (X_2) : $0 = M_2\ddot{x}_2 + B_3(\dot{x}_2 - \dot{x}_1) + B_4\dot{x}_2 + B_2\dot{x}_2$

$F(S) = M_1S^2X_1(S) + KX_1(S) + B_1S\,X_1(S) + B_3S(X_1(S) - X(S)_2)$.

$0 = M_2S^2X_2(S) + B_3S(X_2(S) - X_1(S)) + B_4S\,X_2(S) + B_2S\,X_2(S)$.

Electrical system:

* find the transfer function: $\frac{\text{output}}{\text{input}}$.



* impedance :
$Z_R = R$
$Z_C = \frac{1}{Cs}$
$Z_L = L \cdot s$.

*] Zeq. in :

series: $Z_{eq} = Z_R + Z_C + Z_L$

parallel: $\frac{1}{Z_{eq}} = \frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L}$.

in case of two element: $\left(Z_{eq} = \frac{Z_1 * Z_2}{Z_1 + Z_2}\right)$

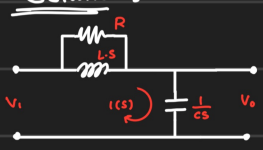
Step (1): find the impedance for each element.

Step (2): the relationship between (Vo & Vi) with the element.

Step (3): transfer equation.

* Vo = is the voltage of the element that parallel to it.
* Vi : is the voltage of all elements.

Solution:



Vo = Voltage of (C)

$Vo = I * \frac{1}{C \cdot s}$ ... ①

Vi = voltage for all element : $Vi = I * Zeq$. --- ②

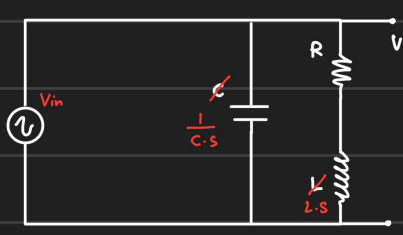
$Zeq = \frac{RLs}{R+Ls} * \frac{Cs}{Cs} + \frac{1}{C \cdot s} * \frac{R+Ls}{R+Ls} = \frac{RLs^2C + (R+Ls)}{LCs^2 + RCs}$

$T.f = \frac{Vo}{Vi} = \frac{I * (\frac{1}{C \cdot s})}{I\left(\frac{RLs}{R+Ls} + \frac{1}{C \cdot s}\right)} = \frac{1}{C \cdot s} * \frac{1}{\left(\frac{RLs}{R+Ls} + \frac{1}{C \cdot s}\right)}$

$\frac{1}{\frac{RLCs^2}{R+Ls} + 1} = T.f$, $\Rightarrow$

Example of Electrical systems:

* find the transfer function:



$V_o = I\,Z_{eq}$:

$Z_{RL} = (R + L\cdot s)$.

$$I_{Zeq} = \left(\frac{\frac{1}{C\cdot S} * (R+L\cdot S)}{\frac{1}{C\cdot S} + (R+L\cdot S)} \right) * I$$

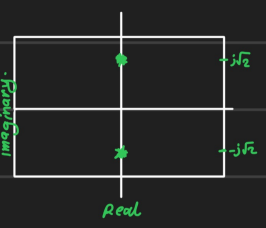
$\& = V_{in}$.

then the transfer function

$$\frac{V_o}{V_{in}} = \frac{I \cdot Z_{eq}}{I \cdot Z_{eq}} = 1$$ ✱

because they have the same element.

Lec (6) transfer functions.

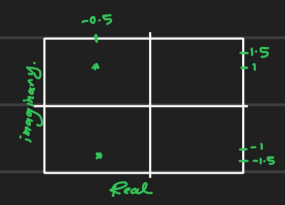


Example (2) $\frac{X(S)}{F(S)} = \frac{0.001}{s^2+2}$.

complex root ، sys. without complex is slow

$s^2 = 2 \rightarrow s = \sqrt{2}\,j$ Sinosudal behavior.

we care about dominant جذر سائد



Example (3). $\frac{X(S)}{F(S)} = \frac{0.001}{s^2+s+2}$.

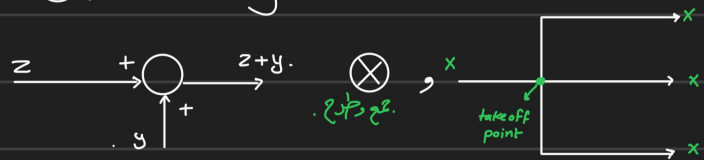
$(0.5) = \frac{1}{2}$ (S coefficient) ، $e^{-at} = e^{-0.5t}$

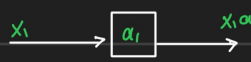
in the imaginary it's frequancy.

Sin (1.4t) or cos (1.4t).

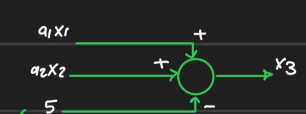
if I have a Real Root → will be in x-axis with no vaibration.

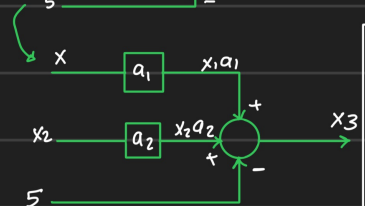
✱ Block diagram:



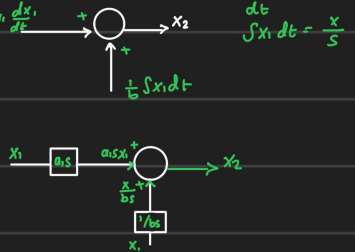


Example : $X_3 = a_1 X_1 + a_2 X_2 - 5$. →



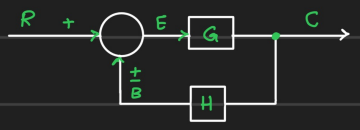


$X_2 = a_1 \left(\frac{dx_1}{dt}\right) + \frac{1}{b}\int x_1\,dt$



$\frac{dx_1}{dt} = sX$.

$\int x_1\,dt = \frac{x}{s}$



H: feedback tranfer function.

① $\frac{C}{R}$: closed − loop $= \left[\frac{G}{1 \pm GH}\right]$ فعكس الاشارات بالمقام.

② $\frac{E}{R}$: error ratio $= \left[\frac{1}{1 \pm GH}\right]$

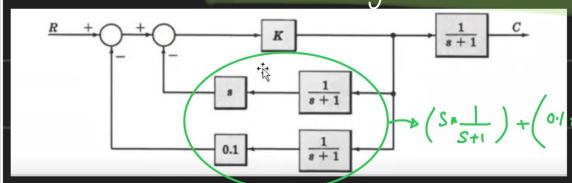
two block in series :

مربعين على التوالي ← ضرب

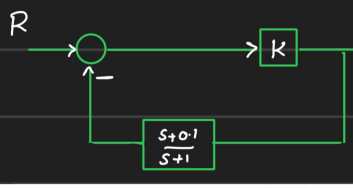
" على التوازي ← جمع وطرح.

Lec (7) T.

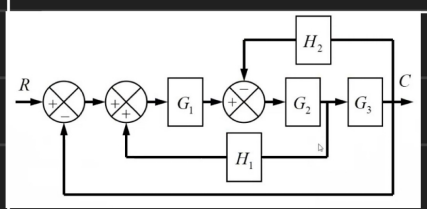
Block diagram reduction :

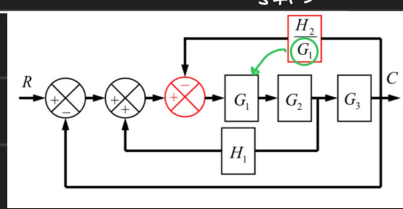


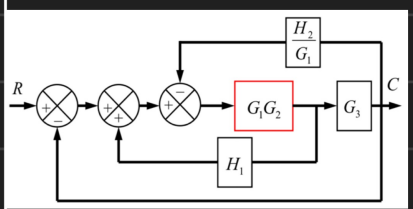
$\left(s * \frac{1}{s+1}\right) + \left(0.1 * \frac{1}{s+1}\right) = \frac{s+0.1}{s+1}$

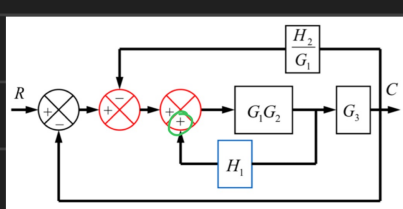


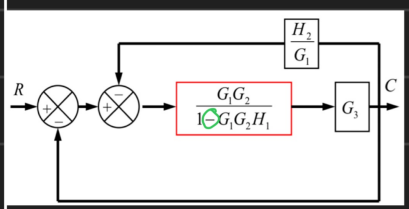
$\frac{C}{R} = \frac{K}{1 + K\left(\frac{s+0.1}{s+1}\right)}$

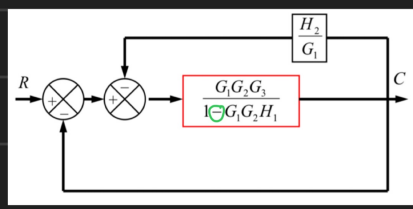


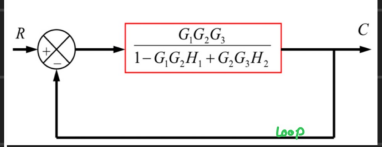


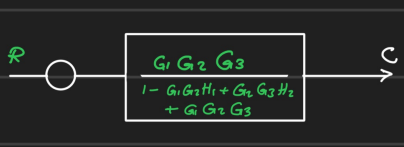




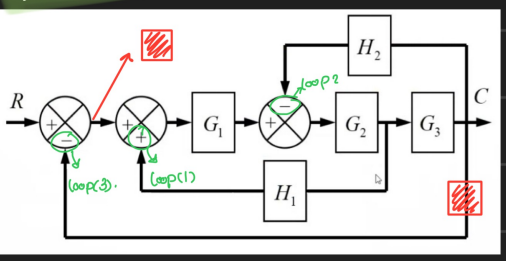








Lec(7)



Solving it using Masons Rule :

$$\frac{C}{R} = \frac{\sum P_i \Delta i}{\Delta} = \frac{G_1G_2G_3 * \Delta i}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

P_i (path from R to C)

Δi

$\Delta = 1 - (\text{loops مجموع}) + (\text{2 loop مضروب (اذا ما في بينهم مشترك)})$

G_2 مشترك بينهم كلهم $+0$

بالعادة منخط بالمكان الفاضي عشان اشوف block مشو مشترك بعديم بجوزه (1).

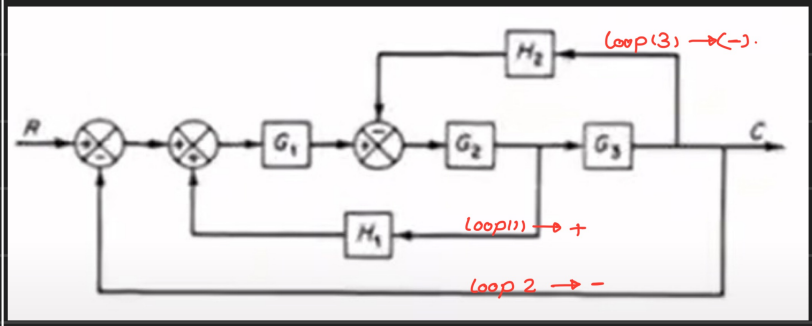
$\Delta i \leftarrow$ نفس Δ لكنه لو في اشي مشترك مع المسار خط جوز

$\Delta i = (1 - 0 - 0 - 0) = 1.$



instead of





$P = G_1G_2G_3$

$\Delta k = (1 - 0 - 0 - 0)$

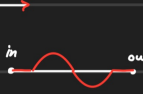
$\Delta = 1 - (G_1G_2H_1 - G_1G_2G_3 - G_2G_3H_2).$

* paths ال متحدد input → output.
* Δk متحدد (1 − Σ loop nontouching).
* loops كل متحدد مع المسار خارج.
* Δ متحدد.

Masons Role : to find the transfer function = $\frac{\text{output}}{\text{input}}$



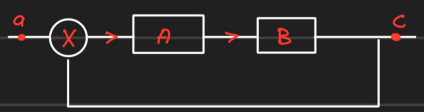
$T.f = \frac{\sum P.\Delta k}{\Delta}$, P: path



, to the same point.



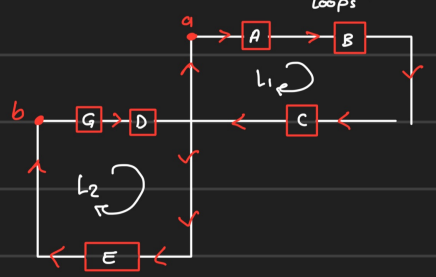
Δk : (1 − (Σ loops (untouching to the choosen path).)



$P = A \cdot B$

$\Delta k = (1 - 0) = 1.$

$\Delta = 1 - (\Sigma \text{loops}) + 2 \text{ nontouching loops} - 3 \text{ nontouching loops}.$



$L_1 = A \cdot B \cdot C$

$L_2 = G \cdot D \cdot E$

nontoughing loops.

Lec (8)

Steady State Response :

- any value over the target value is called overshot.

for Example [25-250] if I get 270 then it's overshot &

the time from (10% − 90%) : 250 − 25 = 225 (10% − 90%) : Rise time.

the time when it's stable with (2%) from 250 ⇒

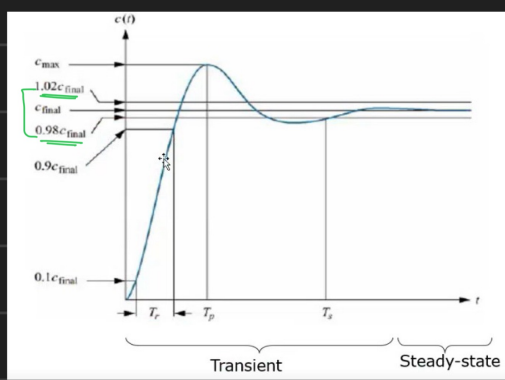
then the sys. reach the stability. (settling)

if it reached 300

then the overshot

$= (50/225) * 100$

* from 98% to 102% stability.



① First Order System using partial fraction.

*3 The fast response effect goes fast

so we will have the dominant root

we care about complex root. (best behavior).

First Order is about Real Roots.

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}, \quad R(s) = \frac{1}{s} \text{ (step response)}.$$

$$C(s) = \frac{1}{\tau s + 1} R(s) = \frac{1}{s(\tau s + 1)} = \frac{1}{s} - \frac{1}{s + \frac{1}{\tau}}$$

① First Order System

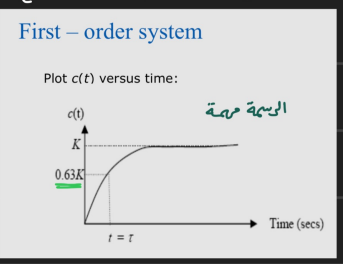
then with inverse Laplace transform, $c(t) = 1 - e^{-\frac{t}{\tau}}$

if $t = \tau \Rightarrow$ step response $c(t) = (1 - 0.37) = 0.63$.

in case of having only one real root (we could have 9 roots & one of them real).

Settling time when $t = 4\tau$.

$[\tau = RC]$ → used here in time.



τ: could be used to measure the capacitor.

very slow sys.

Rise Time: (from step response to go from (10% to 90%)

$T_r = 2.2\tau$.

Settling time: (2% from fainal value). $\tau_s = 4\tau$.

x-axis. على ويقدر (0.63 * final value) نوخذ عند طريقه (τ) منحسب

Lec (9) Tu.

② Second – Order System: way much better.

this system has a slight overshot.

We will two complex root:

$G(s) = \frac{b}{s^2 + as + b} \rightarrow \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$, ξ: determain if it's complex.

complex $\Rightarrow (\frac{1}{2}a)^2 < b$.

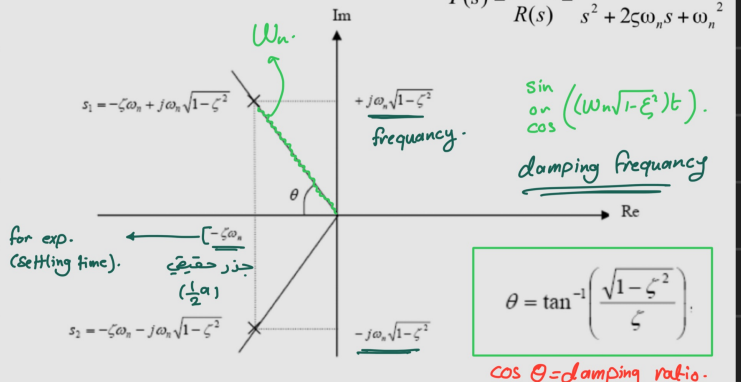
$\omega_n = \sqrt{b}$, $2\xi\omega_n = a$, $\xi = \frac{a}{2\sqrt{b}}$

- $\xi = 1 \Rightarrow$ repeated
- $\xi > 1 \Rightarrow$ real.
- $\xi < 1 \Rightarrow$ complex

Poles;		
	$-\omega_n\zeta + \omega_n\sqrt{\zeta^2 - 1}$	Poles are complex if ζ< 1!
	$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$	

Second – Order System

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



Sin or cos $((\omega_n\sqrt{1-\xi^2})t)$.

damping frequency

$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$.

cos θ = damping ratio.

Mapping the poles into s-plane

---

damping ratio → at x-axis = 1, y-axis = zero (cosθ)

$\xi = \frac{a}{2\sqrt{b}}$

- $\xi = 1 \Rightarrow$ repeated Critically damped.
- $\xi > 1 \Rightarrow$ real → Over damped, 2 real roots.
- $\xi < 1 \Rightarrow$ complex → $(0 < \xi < 1)$ → Underdamped. (wanted)

$\xi = 0$ → Undamped.

Given that the closed loop TF

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

to find rise time $\Rightarrow T(0.9) - T(0.1)$.

" " settling time $\Rightarrow -\xi\omega_n t = -4 \Rightarrow t = \frac{4}{\xi\omega_n}$

(A) For transient response, we have 4 specifications:

(a) Tr – rise time $= \frac{\pi - \theta}{\omega_n\sqrt{1-\zeta^2}}$ → $\cos^{-1}(\xi)$. in Radian.

(b) Tp – peak time $= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

(c) %MP – percentage maximum overshoot $= e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} x100\%$

(d) Ts – settling time (2% error) $= \frac{4}{\zeta\omega_n}$

لازم معامل $s^2 = 1$.

Example for Underdamped

$G(s) = \frac{36}{s^2 + 4.2s + 36}$: $\omega_n = 6$, $\xi = 0.35$

$2\xi\omega_n$: damping ratio = 4.2.

Example: Underdamped:

$G(s) = \frac{100}{s^2 + 15s + 100}$, $\omega_n = 10$, $\xi = 0.75$

$T_s = \frac{4}{0.75 * 10} = 0.533s$, $OS\% = e^{-\frac{\pi * 0.75}{\sqrt{1-0.75^2}}} = 2.838\%$

$T_p = \frac{\pi}{10\sqrt{1-0.75^2}} = 0.475s$

Overdamped Response

$G(s) = \frac{b}{s^2 + as + b}$

a = 9

$C(s) = R * G(s)$.

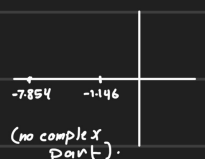
Overdamped system

$R(s) = \frac{1}{s}$ → $\frac{9}{s^2 + 9s + 9}$ → $C(s)$

2 poles. No zeros.

$\xi > 1$

$C(s) = \frac{9}{s(s^2 + 9s + 9)} = \frac{9}{s(s + 7.854)(s + 1.146)} = k_1 + k_2 e^{-7.854t} + k_3 e^{-1.146t}$

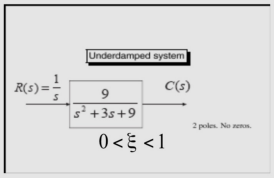


ξ: Damping ratio → makes sys. slower.

Underdamped Response

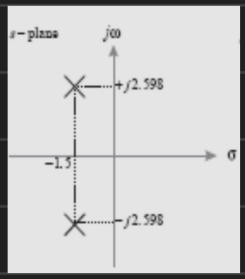
$G(s)=\frac{b}{s^2+as+b}$

a = 3



$c(t)=K_1+e^{-1.5t}(K_2\cos 2.598t+K_3\sin 2.598t)$

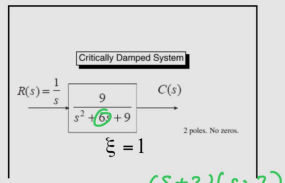
s = 0; s = -1.5 ± j2.598 *(two complex poles)*



Critically Damped System

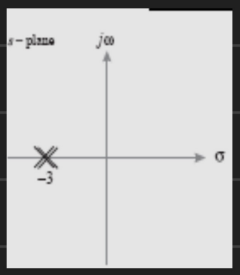
$G(s)=\frac{b}{s^2+as+b}$

a = 6



$c(t)=K_1+K_2e^{-3t}+K_3te^{-3t}$

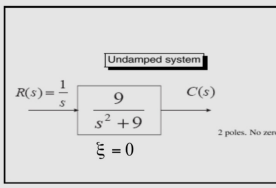
S = 0; s = -3,-3 *(two real and equal poles)*



Undamped Response

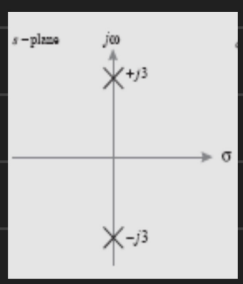
$G(s)=\frac{b}{s^2+as+b}$

a = 0



$c(t)=K_1+K_2\cos 3t$

s = 0; s = ± j3 *(two imaginary poles)*



Conclusion

System	Pole-zero Plot	Response
(a) $\frac{b}{s^2+as+b}$ General		
(b) $\frac{9}{s^2+9s+9}$ Overdamped	s-plane, poles at −7.854, −1.146	$c(t)=1+0.171e^{-7.854t}-1.171e^{-1.146t}$
(c) $\frac{9}{s^2+2s+9}$ Underdamped	s-plane, poles at $-1\pm j\sqrt{8}$	$c(t)=1-e^{-t}(\cos\sqrt{8}t+\frac{\sqrt{8}}{8}\sin\sqrt{8}t)$ $=1-1.06e^{-t}\cos(\sqrt{8}t-19.47^\circ)$
(d) $\frac{9}{s^2+9}$ Undamped	s-plane, poles at $\pm j3$	$c(t)=1-\cos 3t$
(e) $\frac{9}{s^2+6s+9}$ Critically damped	s-plane, pole at −3	$c(t)=1-3te^{-3t}-e^{-3t}$

real part ⇒ Settling time

imagenary ⇒ frequancy.

① First & Second order system:

First Order : $\frac{1}{\tau s+1}$, $\left[\tau=\frac{1}{a}\right]$.

$T_r=2.2\tau$. , with Zero MP %

$T_s=4\tau$.

] first order equation

Example $\frac{1}{3s+2}$, $\tau=3 \Rightarrow a=\frac{1}{3}=0.33$.

$T_r=2.2*3=6.6$, $T_s=4*3=12$ s

Second Order

Example : $\frac{C}{R}=\frac{s+3}{s^2+s+4.25}$ $\omega_n=\sqrt{4.25}=2.06$.

$2\xi\omega_n=1 \Rightarrow \frac{1}{2*2.06}=0.2427$

$\cos\theta=\xi$, $T_s=\frac{4}{\xi\omega_n}$

$\omega_n=\sqrt{real^2+imag^2}$,

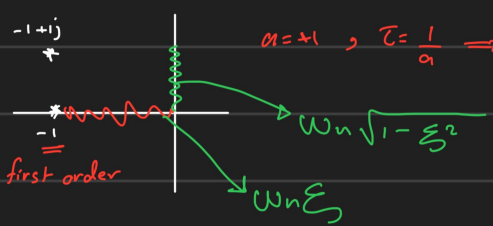
$T_r=\frac{\pi-\theta}{\omega_n\sqrt{1-\xi^2}}$, $T_p=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

] Second order equations

$MP=e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}*100$

$\theta=\tan^{-1}(y/x)$ in Radian $\theta*\frac{\pi}{180}$

in cause first & Second Order in the same quastion.



$a=+1$, $\tau=\frac{1}{a} \Rightarrow T_r=2.2\tau$.

$T_s=4\tau$.

Lec (10)

Steady State Error Analysis :

Design in General around Critical damped.

The fainal value is an Indicator for the accuracy.

accuracy ⇒ amount of error ⇒ diviation from fainal value.

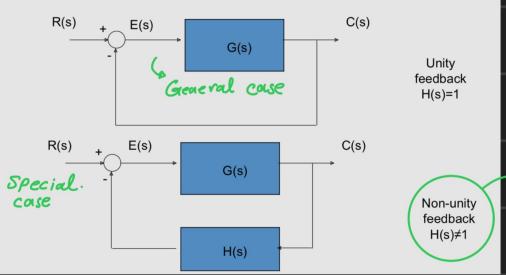
(*s) اشتقاق ← (اساس الحل) المسألة متعلقة بجذر المقام الي قيمته صفر
(*1/s) تكامل ← (1/∞ = 0) عشان اوصل (∞) عشان اوصل (*1/s) بمعني التكامل

Test Waveform for evaluating steady-state error

types of input

Waveform	Name	Physical interpretation	Time function	Laplace transform
r(t)	Step	Constant position	1	1/s
r(t)	Ramp	Constant velocity	t	1/s²
r(t)	Parabola	Constant acceleration	½t²	1/s³

Steady-state error analysis



case (2) if R(S) = 250

f(CS) = 2 . ⇒ then

E(S) = 0 when C(S)=125

Complicated.

Unity feedback ⇒ E(S) = R(S) − C(S) : sys. error.

Non-Unity feed back ⇒ E(S) = R(S) − H(S) · C(S) : Actuating error.

By Final Value Theorem:

steady-state-error $e_{ss} = \lim_{t\to\infty} e(t) \cong \lim_{s\to 0} sE(s)$ ← from fainal value theorem.

$\frac{1}{s} = \frac{1}{\infty}$ = zero

Unity feedback : $E(S) = R(S) - C(S)$, $\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$

$G(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^N\prod_{j=1}^{\theta}(s+p_j)}$ ← type N ← for ∞

$E(S) = \frac{R(S)}{1+G(S)}$

high value for G(s) to get small error.

or ∞

$G(S) = \frac{\text{constant}}{\text{zero}} = 0 \Rightarrow E(S) = \frac{1}{0} = \infty \Rightarrow$ most powerful control tool.

Case (1) ⇒ $R(S) = (1/s)$. $e_{ss} = \lim_{s\to 0} sE(S) = s * \frac{1/s}{1+G(S)} = \lim_{s\to 0} \frac{1}{1+G(S)}$

assuming $k_b = \lim_{s\to 0} G(S)$ ⇒ $\lim_{s\to 0} \frac{1}{1+G(S)} = \boxed{\frac{1}{1+k_b}}$ k_b: static position Error constant.

flow to get (∞)

① if N=0 ⇒ k_p = constant $e_{ss} = \frac{1}{1+k_p}$ = finite = constant

② if N ≥ 1 ⇒ k_p = infinite $e_{ss} = \frac{1}{1+k_p} = \frac{1}{\infty} = 0$

as the type of sys. increase (N ≥ 1) the steady state error goes to zero.

Case (2) : $R(S) = (1/s^2)$

static velocity error constant.

$e_{ss} = \lim_{s\to 0} s * \left(\frac{1/s^2}{1+G(S)}\right) = \lim_{s\to 0} \frac{1}{s + s\cdot G(S)}$, $K_v = \lim_{s\to 0} s\cdot G(S)$

$e_{ss} = \frac{1}{\lim_{s\to 0} sG(S)} = \frac{1}{K_v}$, $K_v = s\,\frac{\pi(s+z_i)}{\pi(s+p_j)}$.

① if N=0 , $K_v = 0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

② if N=1 , K_v = finite ⇒ $e_{ss} = \frac{1}{K_v}$ = finite.

③ if N ≥ 2 , K_v = infinit ⇒ $e_{ss} = \frac{1}{\infty} = 0$.

نكمل الحالات المطلوب
$K = \infty$
عشان بدنا نوصل e_{ss} للصفر.

Case (3) : $R(S) = (1/s^3)$

$e_{ss} = \lim_{s\to 0} s * \frac{1/s^3}{1+G(S)} = \lim_{s\to 0} \frac{1}{s^2 + s^2 G(S)} = \frac{1}{\lim_{s\to 0} s^2 G(S)} = \frac{1}{K_a}$.

K_a: static acceleration error constant $\left[K_a = \lim_{s\to 0} s^2 G(S)\right]$

① N=0 , $K_a = 0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

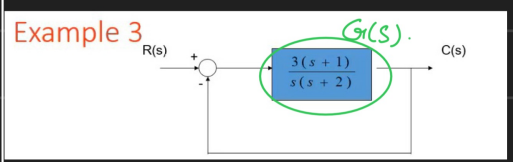
② N=1 , $K_a = 0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

③ N=2 , K_a = constant ⇒ $e_{ss} = \frac{1}{\text{constant}}$ = finite.

④ N ≥ 3 , K_a = infinit ⇒ $e_{ss} = \frac{1}{\infty} = 0$

increasing sys. type (N) will accommodate more different inputs.

Example 3



if $R(t) = (2+3t)u(t)$.

Find steady-state error :

$R(t) = (2+3t)u(t) \Rightarrow \underbrace{\frac{2}{s}}_{\text{case (1)}} + \underbrace{\frac{3}{s^2}}_{\text{case (2)}} = \left[\frac{2}{1+k_p} + \frac{3}{K_v}\right]$.

يتم حل هذا المثال بالطريقة ادا

[1] $k_p = \lim_{s\to 0} G(S) = \infty$, [2] $K_v = \lim_{s\to 0} sG(S) = 3/2$.

$\Rightarrow e_{ss} = \frac{2}{1+\infty} + \frac{3}{3/2} = 2$ #

Steady state error :

Types of errors :

① Step unit $e_{ss} = \frac{1}{1+k_p}$, $k_p = \lim_{s\to 0} G(S)$

② ramp. $e_{ss} = \frac{1}{K_v}$ $K_v = \lim_{s\to 0} G(S)\cdot s$

③ purbolic $e_{ss} = \frac{1}{K_a}$ $K_a = \lim_{s\to 0} G(S)\cdot s^2$

Example : find the $e_{ss} = 3u(t) + 2t$

3u(t) → step unit = $\frac{1*(3)}{1+k_p}$,

2t → ramp = $\frac{1*(2)}{K_v}$,

Confroler

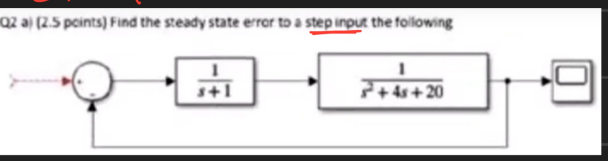
ramp : $K_v = \lim_{s\to 0} G(S)\cdot s * \frac{k}{s}$.

step : $k_p = \lim_{s\to 0} G(S) * k$.

purabolic : $K_a = \lim_{s\to 0} G(S)\cdot s^2 * \frac{k}{s^2}$

Example :

Q2 a) (2.5 points) Find the steady state error to a step input the following



$e_{ss} = \frac{1}{1+k_p}$, $k_p = \lim_{s\to 0} G(s)$, $G(s) = \frac{1}{s+1} * \frac{1}{s^2+4s+20} = \frac{1}{(s+1)(s^2+4s+20)}$

$k_p = \lim_{s\to 0} G(s) = \frac{1}{1*20} = \frac{1}{20} \Rightarrow e_{ss} = \frac{1}{1+\frac{1}{20}} = \frac{20}{21}$ error of step unit

OR

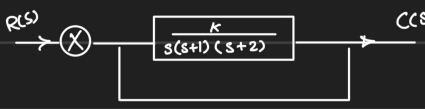
(design a controler) k to achive $e_{ss} = 1$ when the input $(\frac{1}{s^2})$ ramp.

$e_{ss} = \frac{1}{k_v}$, $k_v = \lim_{s\to 0} G(s)\cdot s$, $G(s) = \frac{1}{(s+1)(s^2+4s+20)}$

$k_v = \lim_{s\to 0} G(s)\cdot s \left(\frac{k}{s}\right)$ ramp $= [k_v = \frac{k}{20}] \Rightarrow e_{ss} = \frac{1}{\frac{k}{20}} = 1$, $k = 20$

(s) منقسم عليها ثمان مرة بالجواب

Example : Find e_{ss} for $R(s) = 3u(t) + 5t$



$e_{ss} = \frac{3}{1+k_p} + \frac{5}{k_v}$

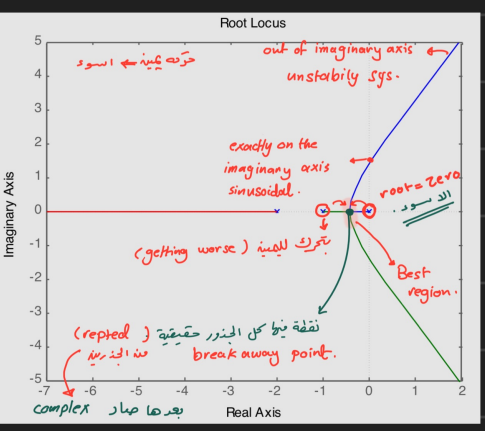
$k_p = \lim_{s\to 0} G(s) = \frac{K}{0} = (\infty)$, $k_v = \lim_{s\to 0} G(s)\cdot s = (\frac{K}{2})$.

$e_{ss} = \frac{3}{1+\infty} + \frac{5}{\frac{K}{2}} = 0 + \frac{10}{K}$, $e_{ss} = \frac{10}{K}$

28/7

Lec (11)M Root Locus

about the dominant أقل واحد سالبية



Used to Expresses performance :

In the best region the damping ratio ≈ 1 or 0.9

$G(s)\cdot H(s) = (-1)$ ... (A)

-1 ⇒ 180 مع مكررات الزاوية

180 ± 360 , $180 \pm (2)(360)$ ....

For example : Point $(-2, +3j)$

$\angle G(s)H(s) = \pm 180(2K+1)$ ... (B)

you will not achive the performance $\Rightarrow e^{-2t}$, $-2t = -4 \Rightarrow t = 2$.

الخطوط والنقاط المرسومة هي فقط النقاط الي بقدر احققها

assuming $G(s) = \frac{K}{s(s+1)(s+2)} = G(s)H(s)$ because $H(s) = 1$.

any complex value can be presented by the magnitude exp θ.

بالنهاية بوصل لصيغة انه زوايا البسط ناقص زوايا المقام .

$\angle G(s)H(s) = \angle k - \angle s - \angle(s+1) - \angle(s+2)$.

لو عوضنا بنقطة وطلعت مع (+ve) بتكون قيمتها = صفر

$\angle G(s)H(s) = \angle k - \angle s - \angle(s+1) - \angle(s+2)$.

Let's take point (-3) : $\angle G(s)H(s) = 0 - \angle -3 - \angle -2 - \angle -1 = -180 - 180 - 180$

$-180 - 360° \Rightarrow$. 180 من مكررات

لسيت من مكررات 180

point $(-1.5) \Rightarrow \angle G(s)H(s) = 0 - 180 - 180 + 0 = -360$ → not part of Root Locus.

part (2)

the magnitude of $G(s)H(s) = -1 \Rightarrow |G(s)H(s)| = 1 \Rightarrow K = \frac{1}{|G(s)H(s)|}$

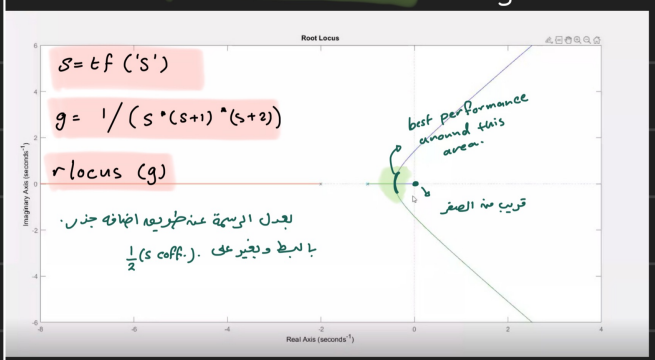
if $(s = -0.25)$: $G(s) = \frac{K}{s(s+1)(s+2)} = \frac{K}{|(-0.25)(0.75)(1.75)|} = 1 = \frac{K}{|-0.328|} = 1$

$K = 0.328$.

we can know the performance from rised time.

29.

Lec (12) Root Locus (2) using Matlab.



The type of questions is to determine the value of (K) such that ----

① if it's damping ratio ⇒ $\xi = \cos\theta \Rightarrow \theta = \cos^{-1}(\xi)$

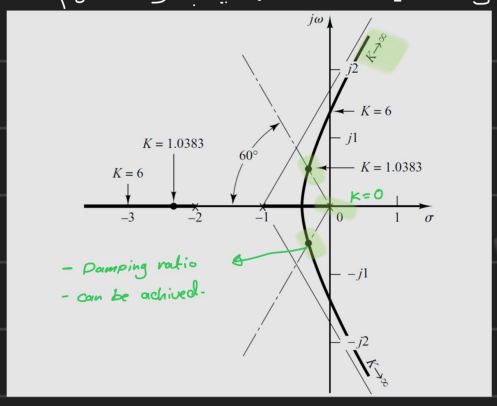
ومنرسم قيمه الزاويه خود رسمة (root.loc)

② Persent of Overshot ⇒ $\xi = \frac{-\ln(\frac{\%MP}{100})}{\sqrt{\pi^2 + \ln^2(\frac{MP\%}{100})}}$.

all points with the same (θ) have the same damping ratio & persent of overshot.

البعد عن المركز فقط يأثر على (frequancy).

تقاطع خطوط (θ) مع الرسمة هي فقط النقاط الي بقدر احققها



two roots :

① $S_1 = -0.3337 + j\,0.5780$

② $S_2 = -0.3337 - j\,0.5780$

Lec (12) Root Locus (2)

① $S_1 = -0.3337 + j0.5780$

② $S_2 = -0.3337 - j0.5780$

then we use it to find the value of (K). #

$$K = \left| S(S+1)(S+2) \right|_{S=-0.3337+j0.5780} = \left| -1.03829 - 1.28 \times 10^{-3} j \right|$$

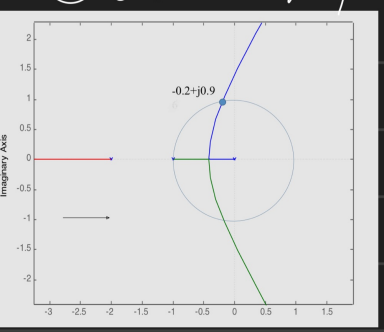
$K = 1.0383$

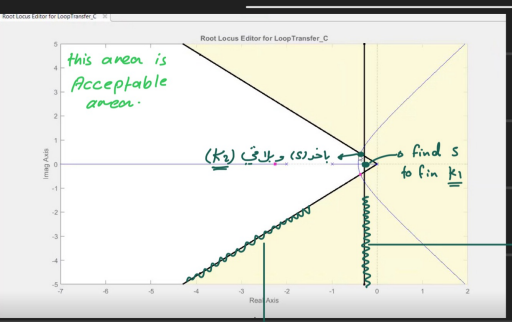
the type of questions is to determine the value of (K) such that ----

③ Natural frequency :

البعد عن المركز

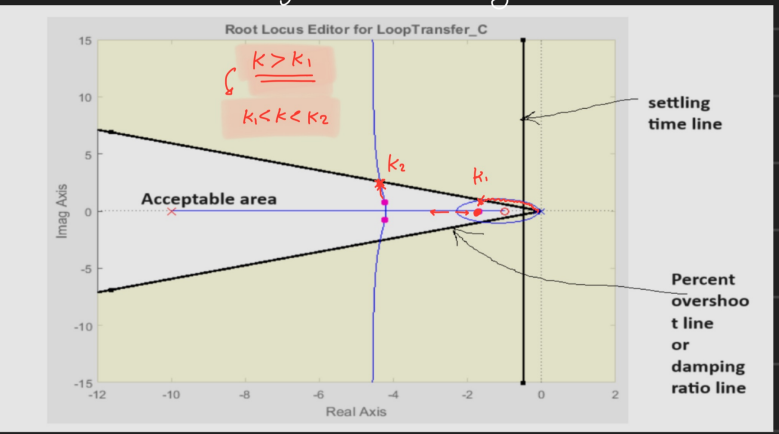
النقاط اللي على نفس الدائرة عندهم نفس البعد

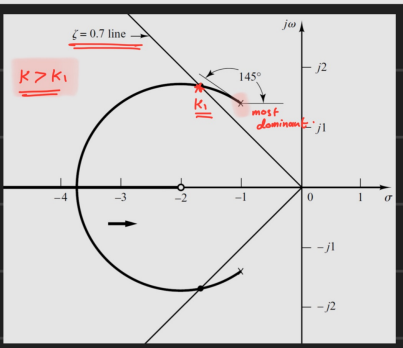




Lec (13). Root Locus (3).

we care about damping ratio or settling time.



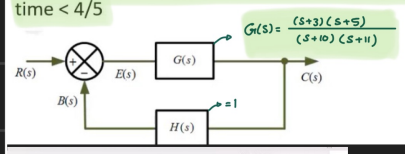


acceptable area من K اذا طلعت

$K < K_1$ بعيد عن

Lec (14) Root Locus (4)

Q5) For the following system design a controller K to achieve a damping ratio of > 0.95 and settling time < 4/5



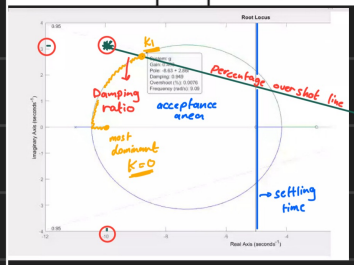
in Matlab :

tan (a cos (0.95))
=0.3287

Theta = a cos (0.95) * (180 / pi)
= 18.1949

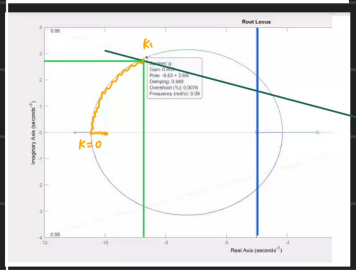
اذا مشيت على على (X) بمقدار (Y) لازم امشي على (Z) بمقدار

$10 * 0.3287 = 3.287$



settling time : $\frac{4}{5} = -4 \Rightarrow Ts = -5$

the solution $0 < K < K_1$



$S_1 = -8.8 + j2.75$ in matlab

$S = -8.8 + 2.7i$

$g = (s+3)*(s+5) / ((s+10)*(s+11))$

$k_1 = abs(1/g) = 0.3451$

fairal answer $0 < K < 0.34$

Root Locus : find K ← damping ratio, settling time.

① if settling time < 3 $\Rightarrow -3t = -4 \Rightarrow (t = 4/3)$ ---- ①

② Damping ratio or %MP $\Rightarrow$ $\xi = \frac{\ln\left(\frac{MP\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{MP\%}{100}\right)}}$ we care about (ξ).

ξ given.

$\xi = \cos\theta \Rightarrow \theta = \cos^{-1}(0.7)$. --- ②

then $\tan(\theta) = \ldots$

$\tan(\cos^{-1}(\xi)) = C \Rightarrow$ --- ③

نحدد قيمة ل (X) ونفوض بـ (C) عشان نحدد قيمة (Y)

منشوف تقاطع الرسمة مع الخطين اللي عندنا.

find the (S)'s value for K_1 & K_2 --- ④

$\xi > X \Rightarrow$ always in the acceptance region.

$Ts < X \Rightarrow$ ⑤

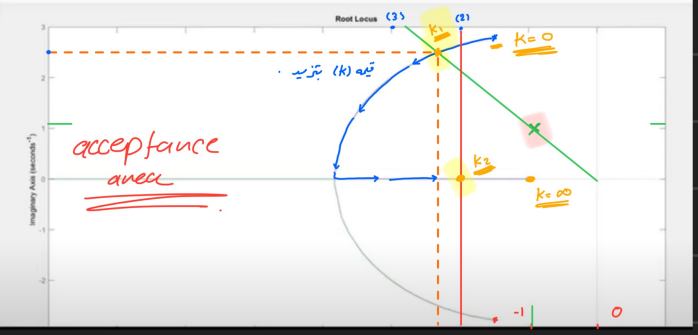
مزوج مع اتجاه بأي اتجاه فيمرر بتكبير كل ما نمشي القيمة بزيد

$$K = \frac{1}{|G(s)H(s)|}$$ --- ⑥

Root locus :

For the following system determine K that achieves a damping ratio of > 0.7 and settling time < 2 s

GH(s)= (s+1)/ (s^2 +3s +10)



$\xi > 0.7 \Rightarrow \theta = \cos^{-1}(0.7) = 45.57$ ↘ Scale $= \tan(\theta) = 1.02$

if $x = 1 \Rightarrow y = 1.02$

Settling time $< 2 \Rightarrow \frac{4}{\omega\xi} = 2 \Rightarrow \omega\xi = 2$ (A)

$S_1 = -2.3 + j\,2.45$

$S_2 = -2$

$\Rightarrow K = \frac{1}{|GH(s)|} = \left|\frac{s^2+3s+10}{(s+1)}\right|$

$K_1 = 1.652$, $K_2 = 8 \Rightarrow K_1 < K < K_2$

$1.652 < K < K_2$

11/8

Matlab

Control system Toolbox :

Transfer function assuming zero initial condition

Example : $y'' + 6y' + 5y = 4u' + 3u$. ① Transfer function.

$s^2 Y(s) + 6s\,Y(s) + 5Y(s) = 4s\,U(s) + 3U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}$.

```
>> num = [4 3];
>> den = [1 6 5];        G = tf([4 3],[1 6 5])
>> G = tf(num,den)
```

② Zero-pole-gain (ZPK)

$\frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5} = \frac{4(s+0.75)}{(s+1)(s+5)}$, `G = zpk([-0.75],[-1,-5],4)`

* Normal : `s = tf('s')`

```
G = (4*s + 3)/((s^2) + 6*s + 5)
```

③ State-Space Model (ss)

$\frac{Y}{R} = \frac{4s+3}{s^2+6s+5}$, $\frac{X}{R} = \left(\frac{1}{s^2+6s+5}\right)$, $\frac{Y}{X} = (4s+3)$.

$R = x'' + 6x' + 5x$, $Y = 4x' + 3x$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]R.$$

```
G = ss([0 1; -5 -6],[0; 1],[3,4],0)
```

Symbolic :

⇒ Solve the diff. equ. $\frac{dy}{dx} = e^{-\frac{1}{x}} \frac{1}{x^2}$:

```
syms y(x)
eqn = diff(y) == exp(-1/x)/x^2
cond = [y(0) == 1];
sol(x) = dsolve(eqn, cond)
```

⇒ system of differential equ.

$\frac{dy}{dt} = z$, $\frac{dz}{dt} = y$.

```
syms y(t) z(t)
eqns = [diff(y,t) == z        diff(z,t) == -y]
sol = dsolve(eqn)
sol.z
sol.y
```

$(1+t^2)\ddot{w} + 2t\dot{w} + 3w = 2$ 2 order

$t_0 = 0$, $t_f = 5$, $w(t_0) = 0$, $\dot{w}(t_0) = 1$

```
function dx = diff(t, x)
[m, n] = size(x)
dx = zeros(m, n)
dx(1) = x(2)
dx(2) = (2 - 2*t*x(2) - 3*x(1))/(1+t^2)
tspan = [0 5]
x0 = [0; 1]
[t, x] = ode23(@diff, tspan, x0)
plot(t, x(:,1))
plot(t, x(:,1), t, x(:,2))
```

write MATLAB code using ODE45 to solve the following differential equations

$\ddot{x} + x = \log(t), \quad \dot{x}(0) = 0, \; x(0) = 0$

Plot(y) and plot(x) on same figure for a time interval 0 to 20 seconds.

ODE45 : Numerical answer :

convention ⇒ there is a second order

x_1 , x_2 $\qquad \ddot{x} + x = \log(t)$

$\dot{x}_1 = x_2$ —① $\qquad \dot{x}_2 + x_1 = \log(t)$

$\ddot{x} = \dot{x}_2$ $\qquad \dot{x}_2 = \log(t) - x_1$ ... ②

```
function   dx = sos(t,x)
[m,n] = size x ;
  dx = zeros (m,n) ;
dx(1) = x(2) ;
dx(2) = log10(t) - x(1) ;
```

```
tspan = [0 , 20]
x0 = [0;0];
[tx] = ode45 (at sos, tspan, x0)
  plot (t,x)
  plot (t, x(:,1))
```

Normal first order equation :

$\dot{x} + ax = b \Rightarrow \dot{x} + 2x = b \qquad \dot{x} = b - ax$

```
function   dx = sos(t,x)
  a = 2 ;
  b = 4 ;
  dx = b - a*x
tspan = [0  20];
  x0 = [0]
[tx] = ode45 (at sos, tspan, x0)
  plot (t,x)
```

Changing parameters :

$\dot{x} + ax = b \qquad \dot{x} = b - ax$

```
function   dx = sos(t,x, param )
  a = param (1)
  b = param (2)
  dx = b - a*x
tspan = [0  20];
  x0 = [0] ;
  param [  ,  ];
[tx] = ode45 (at sos, tspan, x0, [ ], param )
  plot (t,x)
```

Symbolic :

write MATLAB symbolic code to solve the following differential equations

$\ddot{x}y + y = \log(t), \quad \ddot{y}x + \dot{y} = t, \quad \dot{x}(0) = 0, \; \dot{y}(0) = 0, x(0) = 0, \; y(0) = 0$

x(t) , y(t)

```
Syms  x(t)  y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
       , diff(y,t,2)*x + diff(y,t) == t ] ;
dx = diff (x,t) ;
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0 , x(0)==0 , y(0)==0 ];
sol = dsolve (eqn, cond)
```

في حال لغى المعادلة الثانية

```
Syms  x(t)  y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
       , diff(y,t,2)*x + diff(y,t) == t ] ;
dx = diff (x,t) ;          جس ضلغي هاي .
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0 , x(0)==0 , y(0)==0 ];
sol = dsolve (eqn, cond)
```

Lec (4/8)

Routh hurwtz :

عشان الفي الجذور الموجبة لل Polynomials . حتى لو في رموز

Steps:

1. Block-diagram reduction : $\left[\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s_1 + a_0}\right]$

$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s_1 + a_0 = 0$

parameter = (highest power + 1)

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	–	–	–
s	–	–	–
s^0	–	–	–

Example : $\frac{1000}{s^3 + 10s^2 + 31s + 1030}$

s^3	1	31	0
s^2	10	1030	0
s	–	–	–
s^0	–	–	–

$\Rightarrow \frac{\text{row}(2)}{10}$

s^3	1	31	0
s^2	1	103	0
s^1	–	–	–
s^0	–	–	–

$\frac{(1*31)-(1*103)}{1} = -72$, $\frac{(1*0)-(1*0)}{1} = 0$

s^3	1	31
s^2	1	103
s^1	$\frac{31\times1-1\times103}{1} = -72$	$\frac{0\times1-1\times0}{1} = 0$
s^0	$\frac{-72\times103-1\times0}{-72} = 103$	$\frac{-72\times0-1\times0}{-72} = 0$

عدد الجذور الموجبة يساوي عدد تغيرات الاشارة في العامود الاول .

* Special Cases :

Zero in the first column ... (1)

Entire row is zero ... (2).

Example : the characteristic equation $[s^4 + 6s^3 + 11s^2 + 6s + K = 0]$

s^4	1	11	K	0
s^3	6	6	0	0
s^2	10	K		
s^1	$\frac{60-6K}{10}$			
s^0	K			

(1) : $(6*11)-(1*6) = 60/6 = 10$

(2) : $\frac{(10*6)-(6\cdot K)}{10} = \frac{60-6K}{10}$

for stability : $\frac{60-6K}{10} > 0$

$K > 0$

$0 < K < 10$

row 3

$10s^2 + 10 = \text{zero}$

$s^2 = -1$ j

Example : Construct a Routh table & find the # of Roots with positive real parts : $[2s^3 + 4s^2 + 4s + 12 = 0]$

s^3	2	4	0
s^2	4	12	0
s^1	-2	0	
s^0	12		

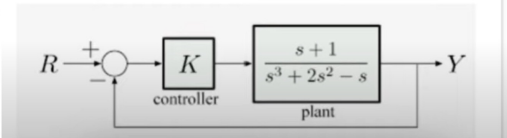
2 changes of sign in the first column

So 2 real Roots positive .

* Stability :

determin (K) for stability :

Q1) Determine K for stability



feed back :

$\frac{G(s)}{1 \pm G(s).H(s)}$

$\frac{\left(\frac{K(s+1)}{s^3+2s-s}\right)}{1+\left(\frac{K(s+1)}{s^3+2s-s}\right)} \Rightarrow \frac{K(s+1)}{(s^3+2s^2-s)+K(s+1)} = \frac{K(s+1)}{s^3+2s^2-s+Ks+K}$

$s^3 + 2s^2 - s + Ks + K = 0 \Rightarrow s^3 + 2s + (K-1)s + K = 0$

تنسيق القسمة

s^3	1	(K-1)	0
s^2	2	K	0
s^1	$\frac{K-2}{2}$	0	
s^0	K		

(1) : $\frac{2(K-1)-K}{2} = \frac{2K-2-K}{2} = \frac{K-2}{2}$

(1) (2) (3)

* دايماً الضرب مع اول عامود

in Stability the answer should be positive.

$\frac{K-2}{2} \geq 0 \Rightarrow K \geq 2$

$K \geq 0$

$\Big] \; K \geq 2$

$K \geq 0$.

PID ⟹ [proportional Integral Derivative].

$$C_{pid} = (K_p e(t)) + \left(K_i \int e(t)\,dt\right) + \left(K_d \frac{de(t)}{dt}\right).$$

(بالنسبة) (بالتفاضل)

↳ amount of the electrical power

ما بيشتغل الا (٣) مكونات سوا بنفس الوقت.

K_p : to get less rise time

$K_d \frac{d}{dt}$: less overshot

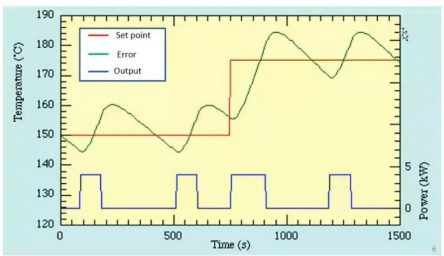
$K_i \int$: more accuracy.

The Characteristics of P, I, and D controllers

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp ↑	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

On-Off Control

- This is the simplest form of control.

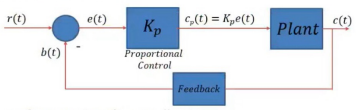


① when the temp. decrease ⇒ will switch to full power.

not accurate.

Proportional Control (P)

- In *proportional mode*, there is a continuous linear relation between value of the controlled variable and position of the final control element.



- Output of proportional controller is

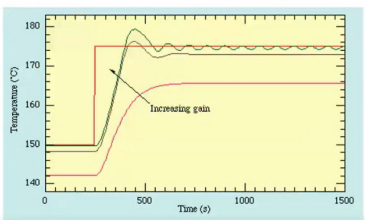
$$c_p(t) = K_p e(t) = \text{power}$$

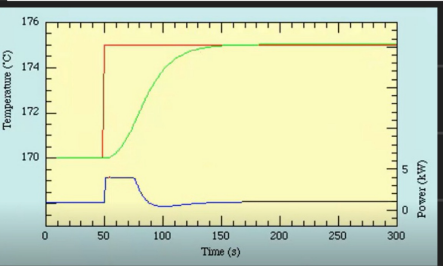
- The transfer function can be written as

$$\frac{C_p(s)}{E(s)} = K_p$$

Proportional Controllers (P)

- As the gain is increased the system responds faster to changes in set-point but becomes progressively underdamped and eventually unstable.

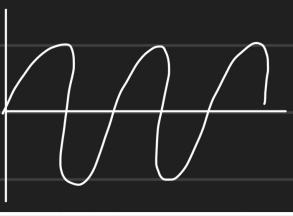




PID part (3) :

PID :

type (1)

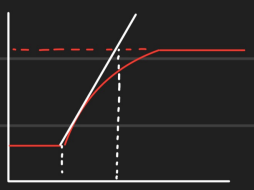


Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

K_{cr} ? given

P_{cr} ?

type (2)



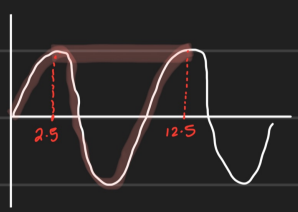
	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

V_o ?

K_o ?

T_o ?

Example & find the PID : ($K_{cr} = 3.5$).



$\Rightarrow P_{cr} = 12.5 - 2.5 = 10$

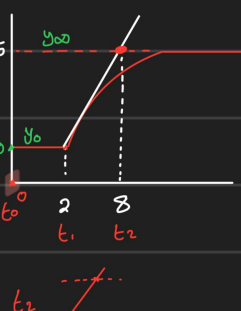
وبتكمل القيم عند الجدول .

$PID = 0.6\, K_{cr} = 0.6 * 3.5 = 2.1$

$0.5 P_{cr} = 0.5 * 10 = 5$

$0.125 P_{cr} = 0.125 * 10 = 1.25$.

Example : find the fainal PID ? $y_\infty = 1.25$, $y_o = 0.25$



① $V_o = t_2 - t_1 = 6$

② $T_o = t_1 - t_o = 2$

③ $K_o = \frac{y_\infty - y_o}{u_\infty - u_o} = \frac{5}{1} = 5$

(,) given

وبعدين على الجدول .

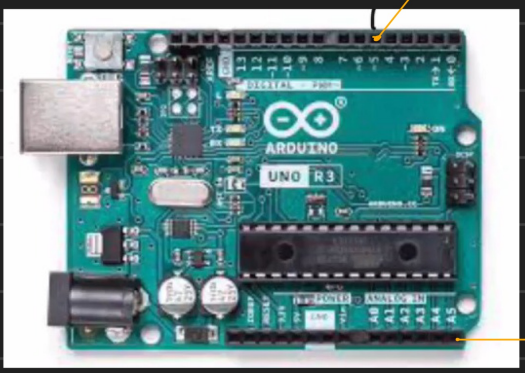
$PID : \frac{1.2\, V_o}{k_o T_o} = \frac{1.2 * 6}{5 * 2}$

$2\tau_o = 4$

$0.5\tau_o = 1$

Arduino :

on , off
input , output



0V — 5V
[0 — 1024]

* Every Arduino "Sketch" has :

```
// variable declarations

const int LED 13 ;

void setup ()
{
  // configuration of pins . etc .
}
  void Loop ()
{
  // what the program does , in the continuous loop
}
```

Example : blink

```
const int LED = 13 ;                    (Pin13) مع وصلة

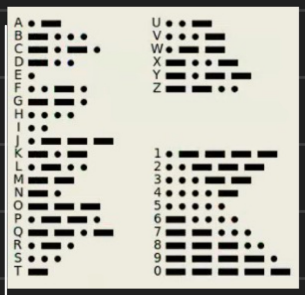
void Setup ()
{ pinMode (LED, OUTPUT) ;        OUTPUT
}                                INPUT.
  Void Loop ()
{
  digitalWrite (LED, HIGH) ;        5V
   delay (1000);                    delay (1000)
                                    تأخر ثانية واحدة
   digitalWrite (LED, LOW);
   delay (1000) ;
}
```

---

button

```
    void Loop (0)
{
val = digitalRead (Button);
  if (val == HIGH) {
{
  digitalWrite (LED, HIGH);
}
  else {
       digitalWrite (LED, LOW);
       }
}
```

يجب لازم نكون نمثل متغير له
const int Button = ;
pinMod (Button, INPUT)



Morse code

LED will ligth long. ← الاشارة الطويلة

النقطة ← اشارة قصيرة

```
void dot ()
{ blink (200, 200) ; }

void dash ()
{ blink (600, 200) ; }

void letterspace ()
{delay (400) ; }

void wordspace ()
{ delay (800) ; }
```

```
void blink (int ontime , int offtime ).
{
  digitalWrite (LED, HIGH);
    delay (ontime );
  digitalWrite (LED, LOW);
    delay (offtime );
}
```

Loop

---

perhaps letter functions :

```
void morse_s ()
{ dot () ; dot () ; dot (); letterspace (); }

void morse_o ()
{ dash () ; dash () ; dash () ; Letterspace () ; }
```

Arduino : Temperature Sensors

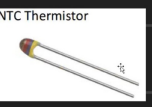
TMP36



⇒ مُصمم بطريقة تحديداً عشان يقيس Voltage

(Zero & 5 V) بستقبل من الاطراف

NTC Thermistor



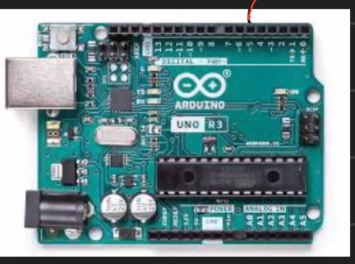
⇒ بتتغير المقاومة فيه (بتنزل مع الحرارة)

```
// We'll use analog input 0 to read Temperature Data const int t
void setup()
 { Serial.begin(9600); }
void loop()
{ float voltage, degreesC, degreesF;
voltage = getVoltage(temperaturePin);
 // Now we'll convert the voltage to degrees Celsius.
// This formula comes from the temperature sensor datasheet:
 degreesC = (voltage - 0.5) * 100.0;
 // Send data from the Arduino to the serial monitor window
Serial.print("voltage: ");
Serial.print(voltage);
Serial.print(" deg C: ");
Serial.println(degreesC);
delay(1000);
// repeat once per second (change as you wish!) }
float getVoltage(int pin)
 { return (analogRead(pin) * 0.004882814); }
// This equation converts the 0 to 1023 value that analogRead()
// returns, into a 0.0 to 5.0 value that is the true voltage
// being read at that pin.
```

```
// Read Temerature Values from NTC Thermistor
 const int temperaturePin = 0;
 void setup()
{ Serial.begin(9600); }
 void loop()
 { int temperature = getTemp();
Serial.print("Temperature Value: ");
Serial.print(temperature);
 Serial.println("*C");
delay(1000);
 }
 double getTemp()
{
// Inputs ADC Value from Thermistor and outputs Temperature in Celsius int RawADC =
analogRead(temperaturePin);
long Resistance;
 double Temp;
 // Assuming a 10k Thermistor. Calculation is actually: Resistance = (1024/ADC)
Resistance=((10240000/RawADC) - 10000);
 // Utilizes the Steinhart-Hart Thermistor Equation:
// Temperature in Kelvin = 1 / {A + B[ln(R)] + C[ln(R)]^3}
// where A = 0.001129148, B = 0.000234125 and C = 8.76741E-08 Temp = log(Resistance);
 Temp = 1 / (0.001129148 + (0.000234125 * Temp) + (0.0000000876741 * Temp * Temp *
Temp)); Temp = Temp - 273.15;
// Convert Kelvin to Celsius return Temp;
 // Return the Temperature
 }
```

const int temp = 0

Arduino :



digital [1 0]
on / off ⇒ on = 5V , off = 0 V

input : Button 1 0

Output LED 1 0 :

```
const int Button = 10;(1)
                 , LED = 9;
void setup( )
{ pinMode(Button, INPUT);
  pinMode(LED, OUTPUT);
  { Void loop( )
    { int X;
      X= digitalRead (Button);
if (x==1)
digitalWrite (LED, HIGH);
else
digitalWrite (LED, LOW);
}
```

(1) منشيرها لا تغير

resistance بالسبة لـ (LED) دايما في

كيف بدخله بالمعادلة ؟؟

في حال كان على (9) و Button على (10)

① Digital pins.
بيوخذو قيمة
[0 / 1].



Analog pin

* Analog pin [1024]
using analog Read
analog Write
delay (1000);

②

$$R_{total} = 1000 + RT = \frac{V}{I} = \frac{1024}{\frac{V}{1000}}$$

```
float get Temp (int Tpin)
{ int V= analogRead (Tpin);
  float R=(1024.0000 /V)-10000
  float logR= log(R);
  return (1/(0.001129148 + 0.000234125 log(R)
          + 8.76741 E8*logR *logR *logR)) }
- 273.15 → if it's in (C)
```



TMP 36 . : Ground. الطرف اليمين مع ال



اليسار مع (5V)

الوسط مع Ao

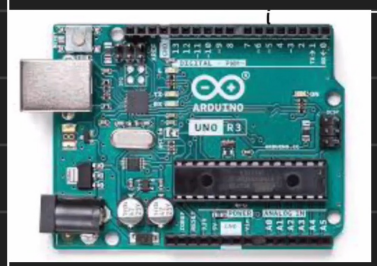
```
Const int Tpin= 0;
Void setup ( )
{ Serial.begin (9600);
  }
Void loop ( )
{ float voltage, temprature;
voltage = getvoltage (Tpin);
temperature =(voltage - 0.5)*100;
Serial.print (" T is ");
Serial.println (temperature);
delay (1000);
}
float getvoltage (int Tpin)
{ return (analogRead (Tpin) * 5/1024) }
```

①

0 1024
0V 5V

⇒ 512 → 2.5V
= * 5/1024.

حاتيه





②

NTC Thermistor

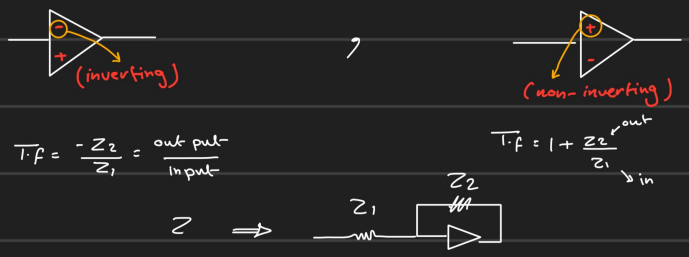


voltage div
5V → 1024

```
const int Tpin = 0;
Void setup ( )
{ Serial.begin (9600);
  { Void loop (
    {float Temp;
    Temp = getTemp (Tpin);
    Serial.Print ("T is ");
    Serial.print (Temp);
    Serial.println (Temp);
    delay (1000);
    {
```

Electronic sys., Active circuit. (AC)

$T.f = \frac{output}{input}$ Cases: 1] Inverting 2] non-inverting.

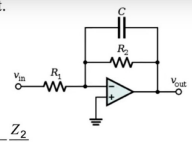


$T.f = \frac{-Z_2}{Z_1} = \frac{out\,put}{input}$

$T.f = 1 + \frac{Z_2}{Z_1}$ ← out / ↘ in

2 ⇒

- Find out the transfer function of the following circuit.



$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$

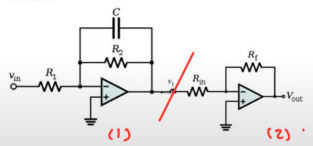
24

inverting

$Z_2 = \frac{R_2 * \frac{1}{C \cdot S}}{R_2 + \frac{1}{C \cdot S}}$ $\quad T.f = \frac{-Z_2}{Z_1}$

$Z_1 = -R$

- Find out the transfer function of the following circuit.

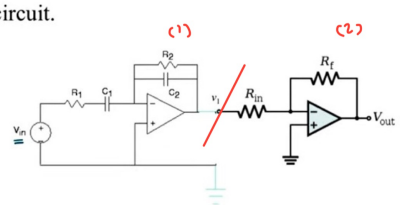


$T.f_{(tot)} = T.f_{(1)} * T.f_{(2)}$.

(1): $\frac{-R_2}{1 + R_2 C \cdot S}$ $\quad$ (2): $\frac{-R_f}{R_{in}}$

$\frac{R_1}{}$ ← يعني مقسوم على

Find out the transfer function of the following circuit.



(2) $\frac{-R_f}{R_{in}}$.

(1) $= \frac{R_2}{1 + R_2 C \cdot S}$

$\left(R_1 + \frac{1}{C_1 \cdot S}\right)$

Gear ratio:

$\frac{\#\text{ of teeth of input gear}}{\text{" " " output "}} = \frac{\text{Input Torque}}{\text{Output Torque}} = \frac{\text{outp. speed}}{\text{inp. speed.}}$

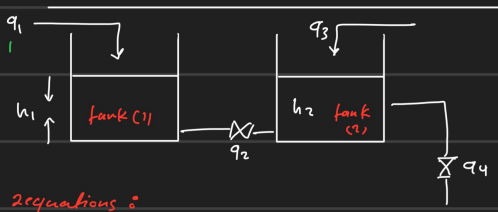
Tanks :

q : الرمز معادلة خاصية فيزيا

$$q = \frac{h_{in} - h_{out}}{q_1} \quad \text{...(1)}$$

$$C \frac{dh}{dt} = q_{in} - q_{out} \quad \text{-- (2)}$$





2 equations :

$$\text{tank (1)}: C_1 \frac{dh}{dt} = q_1 - q_2$$

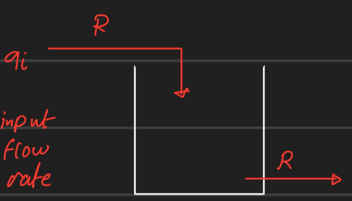
$$\text{tank (2)}: C_2 \frac{dh}{dt} = q_3 + q_2 - q_4$$

Step (1).

for (q) => (R).

$$R_1 = \frac{h_{in} - h_{out}}{q_1} = \frac{0 - h_1}{q_1}$$

$$R_2 = \frac{h_1 - h_2}{q_2}, \quad R_3 = \frac{-h_2}{q_3}, \quad R_4 = \frac{h_2}{q_4}$$



R: Resistance of valve

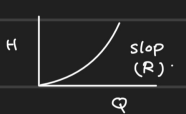
anti proprtional relation ship

$q_o \uparrow \quad R \downarrow$

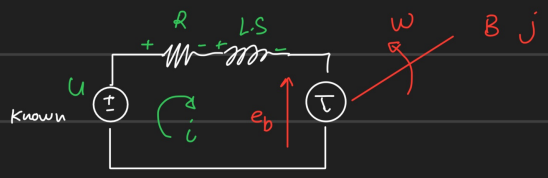
$q_o \propto \frac{1}{R}$

$q_o \uparrow \quad H \uparrow$

$q_o \propto H$



*] DC motor :



$$-u + RI + LSI + e_b = 0$$

$$T = K_t * I, \quad I = T/K_t, \quad e_b = K_b w.$$

$$T = J\dot{w} + Bw$$

$$u = \frac{Jws + Bw}{K_t}(R + LS) + (K_b w).$$

$$u = w\left(\frac{JLS^2 + (JR + BL)S + BR + K_b K_t}{K_t}\right)$$

$$\left(\frac{w}{u} = \frac{K_t}{JLS^2 + (JR + BL)S + BR + K_b K_t}\right)$$