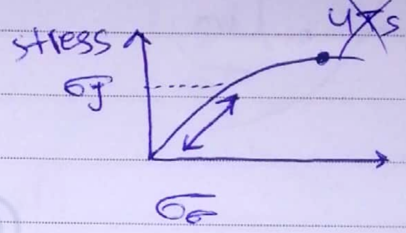
Weekly Notebook

DESIGN

first semester
2021-2022

No.

factor of safety.



$\mu_\sigma = 20$ MPa

σ applied.

/ AISI 1040

material → table → properties.

$n = \dfrac{\text{Property}}{\text{applied stress}} = \dfrac{\mu_S}{\mu_\sigma}$ → yield strength of materials

↳ strength of the applied stress

$n \geq 1$ → safe.

$= \dfrac{800}{20} = 40$ → deterministic FOS (factor of safety)

100 % safe.

Reliability P_s

↓ probalistic FOS

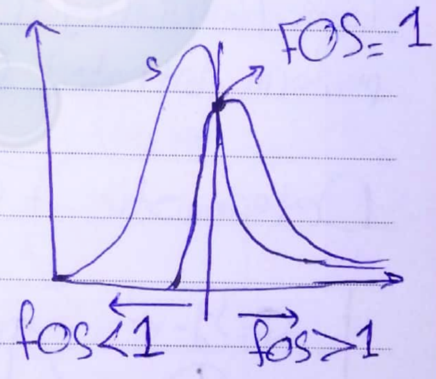
Properties of ductile materials

μ_s

σ_s

S → material (yield strength)

Normal distribution.

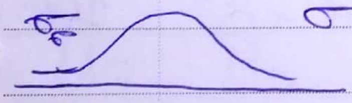


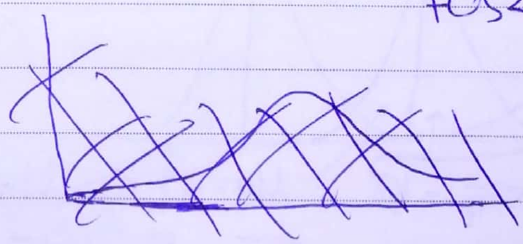
μ_σ

σ_σ

σ → stress

N.D





Q = margin = $S - \sigma$

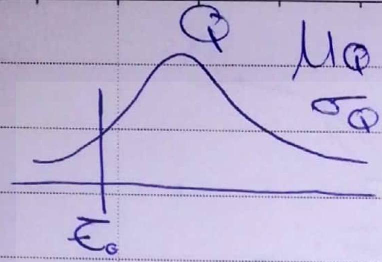
smile for life

* Brittle material

~~u: maximum or ultimate~~ strength

= f: fracture strength

* Ductile $u > f$



$$\varepsilon = \frac{Q - \mu_Q}{\sigma_Q}$$

$$\mu_Q = \mu_s - \mu_\sigma$$

$$\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\mu^2}$$

$$\varepsilon_0 = \frac{-\mu_Q}{\sigma_Q} \longrightarrow P_{z_0} \text{ area} = \text{failure} = P_f$$

$$1 - P_{z_0} = \text{Reliability} = P_R$$

Ex: Consider a structural member ($\mu_s = 40$, σ_s) subjected to a static load that develops a stress σ ($\mu_\sigma = 30$, σ_σ) find the reliability of member.

Note: Reliability is probability that machine element will perform intended function satisfactory 100% reliability

Deterministic FOS = 40/30 = 1.33

$\mu_Q = 40 - 30 = 10$

$\sigma_Q = \sqrt{6^2 + 8^2} = 10$

$\mu_Q = 10$, $\sigma_Q = 10$ $\quad$ $\mu_\sigma = 30$ $\quad$ $\mu_s = 40$,

* $\sigma_\sigma = 8$ $\quad$ $\sigma_s = 6$ *

1-11 2-2
1-12 20-3

No.

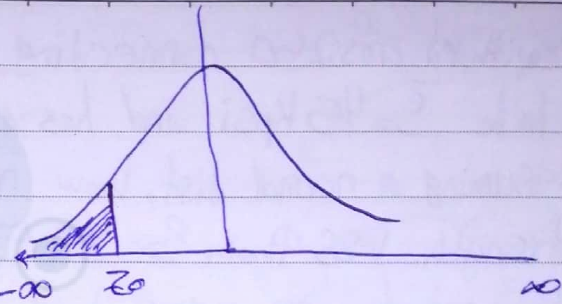
Margin $Q = S - \sigma$

at $Q = 0$

$Z_0 = \frac{0-10}{10} = -1$

$\mu_Q = 10$

$\sigma_Q = 10$



FOS is equivalent to 1.33 gives a reliability of 84.13% which is lower than 100% Reliability needed so it is insufficient for the present design, therefore there's a need to increase this factor.

Ex: A round # 1018 steel rod having yield strength (540, 40) Mpa is subjected to tensile Load (220, 18) kN. Determine the diameter of rod that results in a reliability of .999 ($Z = -3.09$). Table A10. 0.001

Sol:-

تحويل
Pressure = $\frac{\text{Force}}{\text{Area}}$
Newton $\rightarrow$ MPa

$\mu_S = 540$ Mpa $\quad \sigma_S = 40$ MPa

$\mu_\sigma = \frac{220000}{\frac{\pi}{4} d^2}$ MPa $\quad \sigma_\sigma = \frac{18000}{\frac{\pi}{4} d^2}$ MPa

$\mu_Q = \mu_S - \mu_\sigma = 540 - \frac{880000}{\pi d^2}$

$\sigma_Q = \sqrt{\sigma_S - \sigma_\sigma} = \sqrt{40^2 + \left(\frac{72000}{\pi d^2}\right)^2}$

$Z_0 = -\frac{\mu_Q}{\sigma_Q} \Rightarrow 3.09\sqrt{40^2 + \left(\frac{72000}{\pi d^2}\right)^2} = 540 - \frac{880000}{\pi d^2}$

$\boxed{d = 26\text{ mm}}$

Design factor?

No.

In a shipment of 250 connecting rods, the mean tensile strength is found to be $\bar{S} = 45$ Kpsi and has a standard deviation of $\hat{\sigma}_S = 5$ kpsi

ⓐ Assuming a normal dist, how many rods can be expected to have a strength less than $S = 39.5$ Kpsi?

ⓑ How many are expected to have a strength between 39.5 and 59.5 Kpsi?

ⓐ Sol:-

$$Z_{39.5} = \frac{X-\mu}{\sigma} = \frac{39.5-45}{5} = -1.1$$

$$\Phi(-1.1) = \boxed{.1357}$$

$$n = N\,\Phi(z) = 250(.1357) = 33.9 \approx 34 \text{ rods.}$$

ⓑ Sol:-

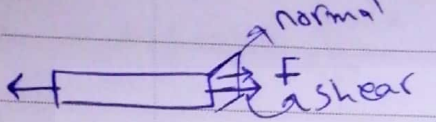
$$Z_{59.5} = \frac{X-\mu}{\sigma} = \frac{59.5-45}{5} = 2.9$$

$$\Phi(2.9) = .99813$$

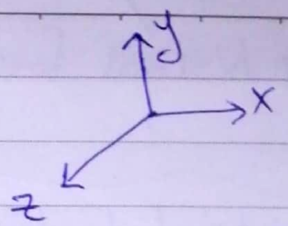
$$n = N\left(\Phi(2.9) - \Phi(-1.1)\right) = 250(.99813 - .1357)$$
$$= 215.5 \text{ rods} \approx 216 \text{ rods.}$$

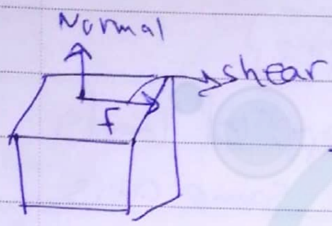
chapter 3

No. Analysis of stress and load.

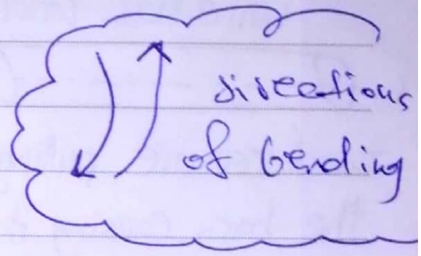


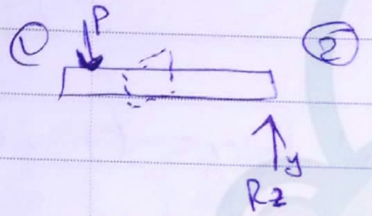
$\sigma \longrightarrow \sigma_{xx} = \frac{F}{A}$



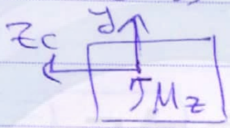


$\longrightarrow \tau_{yx} = \frac{F}{A}$





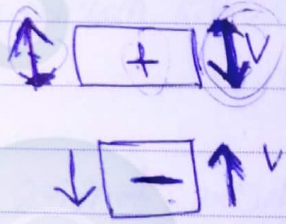
cross section



Load

bending around z ← y و أ

V → shear force.



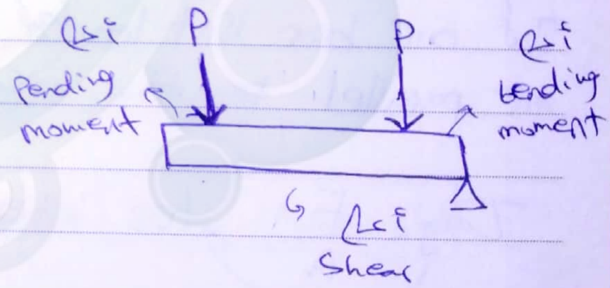
$\vec{M} = \vec{r} \times \vec{F}$

$M_z = r_z \times R_z^y \quad = \hat{i} \times \hat{j} = +\hat{k}$

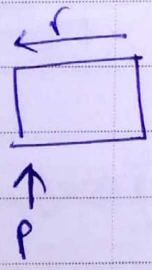
$\sum M_1^z = 0$

$R_z^y \, r_z - P r_3 - P r_4 = 0$

$\sum F_y = 0 \qquad R_1^y + R_z^y - P - P = 0$



$V = \frac{dM}{dx}$



$\vec{M} = r \times P$

$= -\hat{i} \times \hat{j} = -k$

Reaction → external bending Moment.

compressive tensile

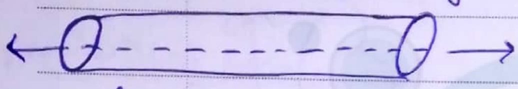
No.

Normal force always → σ_{xx}
→ σ_{yy}
→ σ_{zz}

σ

Normal forces

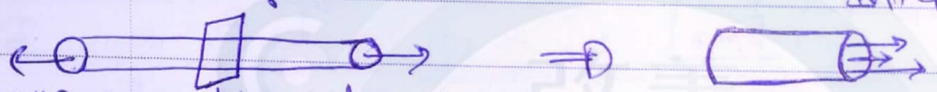
uniaxial loading



*because all of the applied loads are acting along the same axis

Two forces are pulling the bar, causing it to stretch, so internal forces develop

Internal forces N dist
⇓
leads to stress

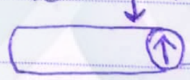


exposing the internal forces by making an imaginary cut.

Shear forces τ

If our bar isn't loaded along its axis means, Internal forces are parallel to the cross section.

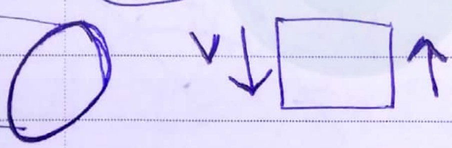
external



internal

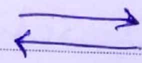
$\tau_{avg} = \frac{F}{A}$ bcz internal forces aren't normally distributed

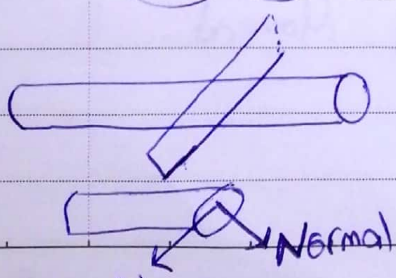
cross section



shear stress on one face of the element. equilibrium??

for rotational equilibrium we must add shear stresses



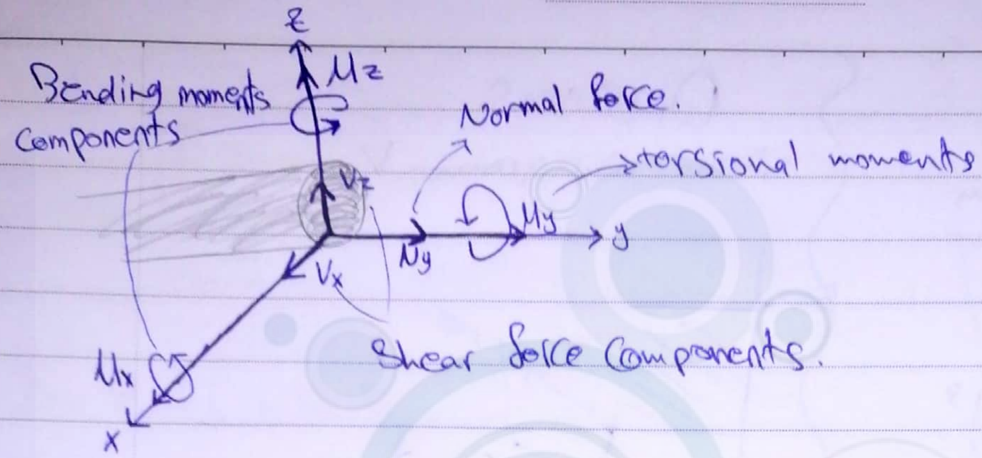


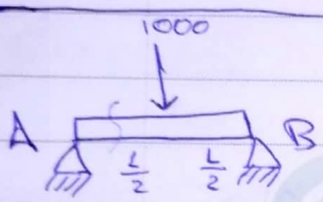
Inclined plane

Normal σ

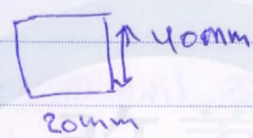
Shear τ

No.





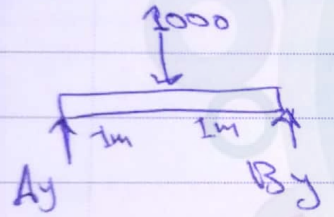
$L = 2m$ cross sectional



ال stress الناتج عن ال bending moment في ال neutral axis = صفر

sol:-

① find reactions At A+B



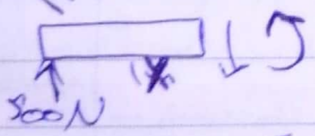
FBD

$\Sigma M_A = 0 \Rightarrow 2By - 1000 = 0 \Rightarrow \cancel{2}By = \frac{1000}{2}$

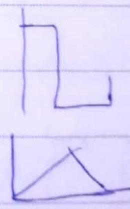
$\boxed{By = 500\ N}$

$\Sigma F_y = 0 \Rightarrow 500 + Ay - 1000 = 0 \Rightarrow \boxed{Ay = 500\ N}$

$0 < x < 1$



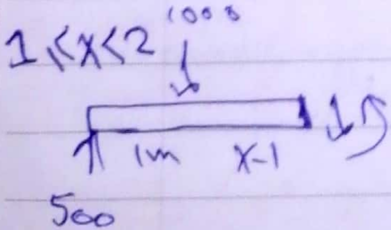
$\Sigma F_y = 0 \Rightarrow V = 500\ N$ *



$\Sigma M = 0 \Rightarrow -500x + M = 0$

$M = 500x\ N \cdot m$ *

$1 < x < 2$



$\Sigma F_y = 0 \Rightarrow 500 - 1000 + V = 0$

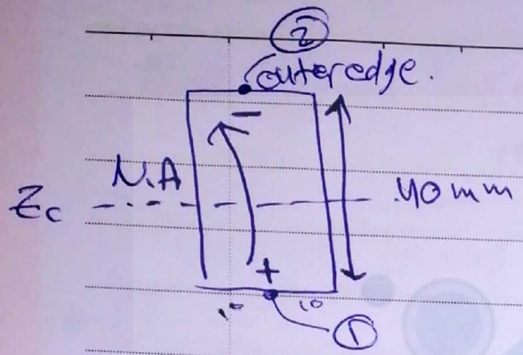
$V = -500\ N$ *

$\Sigma M = 0 \Rightarrow -500x + 1000x - 1000 + M = 0$

$M = -500x + 1000\ N \cdot m$

Neutral axis $\equiv$ centroidal axis

No.



$$I_{zz} = \frac{1}{12}\left(\frac{20}{1000}\right)\left(\frac{40}{1000}\right)^3$$

$$\sigma_{xx} = \frac{M_z y}{I_{zz}} \quad \text{at } ①$$

tensile stress

$$\sigma_{xx} = \frac{500\left(\frac{20}{1000}\right)}{\frac{1}{12}\left(\frac{20}{1000}\right)\left(\frac{40}{1000}\right)^3} \quad \text{tension}$$

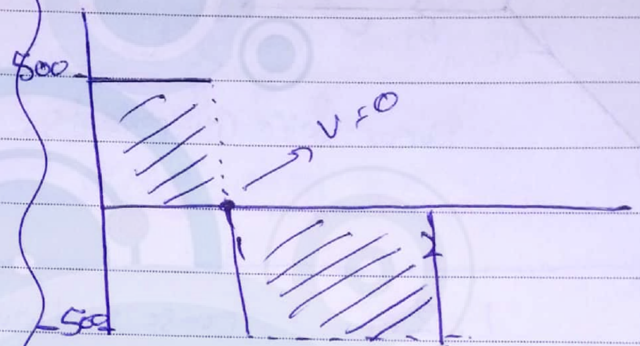
at ② Compressive stress

$$\sigma_{xx} = -\frac{500\left(\frac{20}{1000}\right)}{\frac{1}{12}\left(\frac{20}{1000}\right)\left(\frac{40}{1000}\right)^3}$$

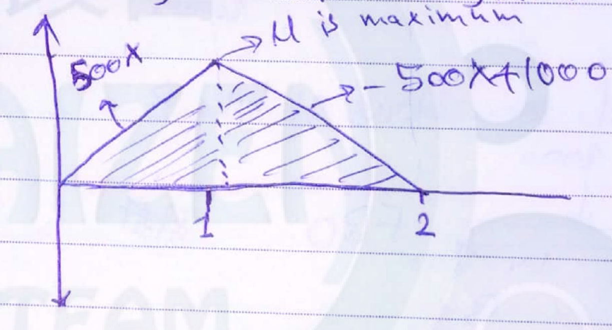
Comp

Diagrams

shear diagram V



Bending moment diagram.



Centroid

$$\bar{x} = \frac{\sum \tilde{x}.A}{\sum A} = 0$$

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A}$$

① for $c_1 + c_2$ dis 1.0 ?

c_1 ~~[illegible]~~

① total Area = A_1 (Top-center) + A_2

C_2 = total L - G or

$C_2 \rightarrow$ same but down-center (top-center)

② for $\tilde{y} \rightarrow$ centroid - required axis in $I_z = I_z' + A\tilde{y}^2$

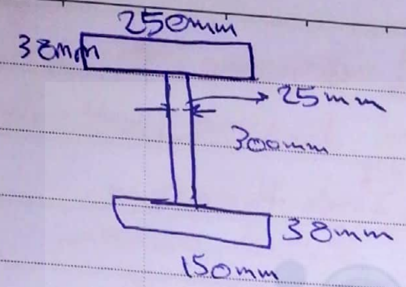
$\rightarrow$ tension top-edge ~~[illegible]~~ required axis

compression down - " "

$$I = \bar{I} + Ad^2$$ parallel axis theorem.

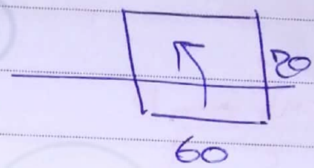
① Shear force / ② bending moment / ③ shear stress / ④ normal stress

No.



$\boxed{I \uparrow \quad \text{stress} \uparrow}$

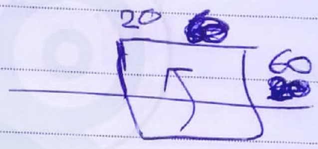
$$I = \sum I_i + A\tilde{y}^2$$



$I = \frac{1}{12} bh^3$

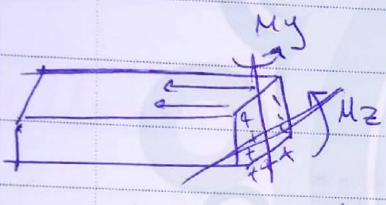
$\frac{1}{12}(60)(20)^3$

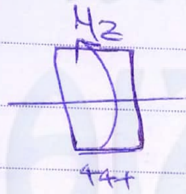
$$\sigma_{xx} = \frac{My}{I_{zz}}$$

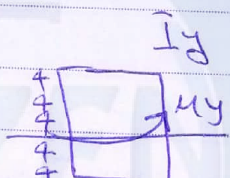


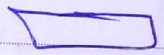
$I = \frac{1}{12}(20)(60)^3$

maximum moment of inertia









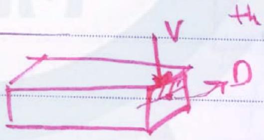
لو كذا

$$\sigma_{xx} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

الـ maximum stress دايما على الأطراف.

Shear Stress

$V \Rightarrow \tau = \frac{VQ}{Ib}$ when the load isn't Normally distributed.

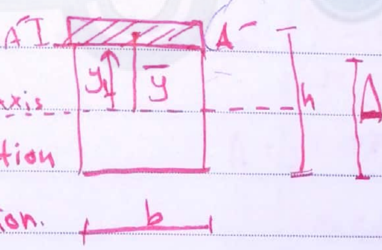


the force isn't N.D

$\tau = ?$ so we use

$\tau = \frac{VQ}{Ib}$

$Q = \bar{y}A'$ $\quad Q = \frac{b}{2}(c^2 - y_1^2)$



$\boxed{h = c + c = 2c}$

Shear force acting at cross section

V: Internal shear (at) the cross section.

Q: First moment of area (can be found by 2 methods)

$I = \frac{1}{12} b (2c)^3$

I: Moment of Inertia for the whole shape

b: width of the cross section.

أعلى shear stress في الـ neutral axis

$\tau = \frac{V}{2I}(c^2 - y_1^2)$

$\tau = \frac{3V}{2A}\left(1 - \frac{y_1^2}{c^2}\right)$ max when $y_1 = 0$

$\tau = \frac{V}{A}$ avg shear stress over cross section

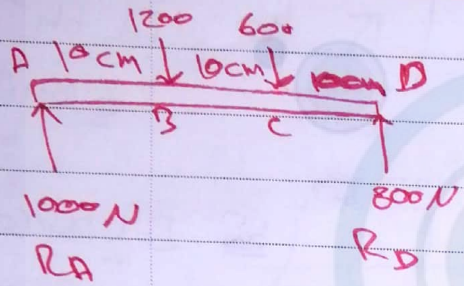
neutral axis الـ $y_1 = 0$

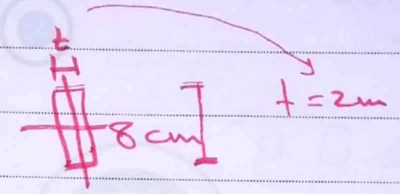
smi)e for life

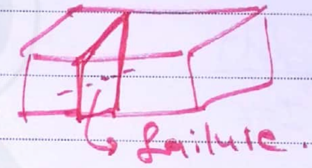
$\tau_{max} = \frac{3V}{2A}$ | max Shear stress ~~occurs~~ occurs at the N.A and zero at the top & bottom surface of the beam.

No.

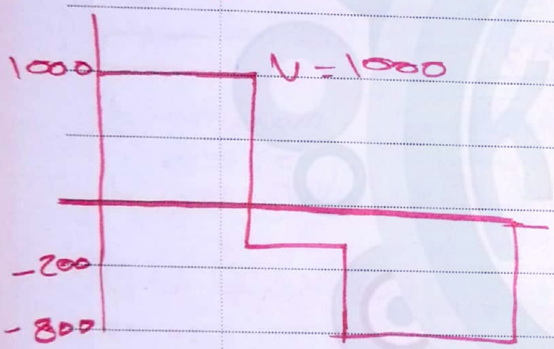
$\tau = \frac{VQ}{Ib}$ → we use this when there's composite bodies + Parallel ~~...~~ axis theorem



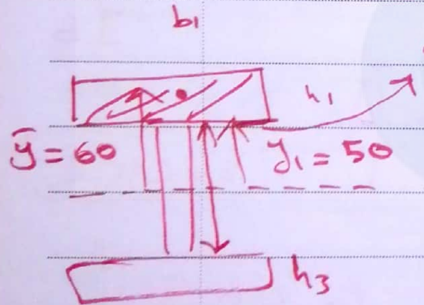




$V = \frac{dM}{dx}$



$\tau_{max} = \frac{3V}{2A} = \frac{3\times1500}{2\times2\times8\times10^{-4}}$



lower surface of the upper flange

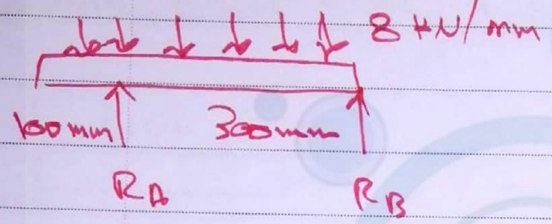
N.A

يعني السؤال بسيط يعطيني V
أنا بعمل Shear diagram
بطلع الـ maximum
Shear.

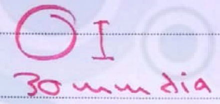
$\tau = \frac{VQ}{Ib} = \frac{500 * 120 * 10^{-6}}{16.2 * 10^{-6} * .1}$

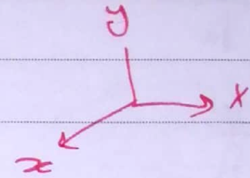
No.

for the beam in figure. find the locations and magnitudes of the maximum tensile stress due to V.



cross section.





$$n = \frac{\sigma_y}{\sigma_{max}} = \frac{800 \times 10^6}{26.8 \times 10^9} < 1$$

so failure.

$$n = \frac{400 \times 10^6}{2.51 \times 10^9} < 1 \text{ failure.}$$

Concentrated load.

$8 \times 400 = 3200 \text{ kN}$

$\sum M_B = 0$

$R_A \times 300 - 3200 \times 200 = 0$

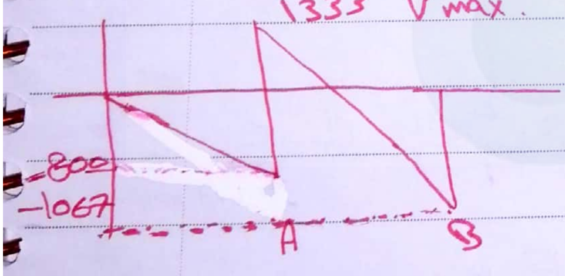
$R_A = \frac{+200 \times 3200}{300} = 2133$

$\sum F_y = 0$

$2133 + R_B - 3200 = 0$

$R_B = 1067 \text{ N}$

Now shear diagram



$$\tau_{max} = \frac{4 V_{max}}{3A} = \frac{4(1333)}{3\left(\frac{\pi}{4}\left(\frac{30}{1000}\right)^2\right)}$$

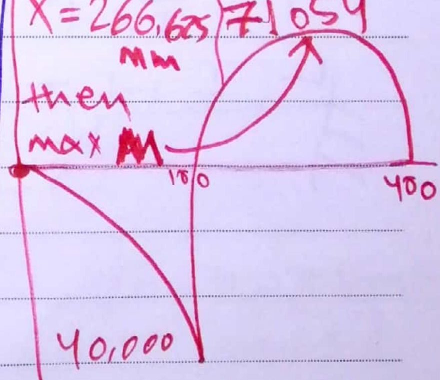
to find max bending moment.

$V = 0 \quad 100 \leq X \leq 400$

$0 = 2133 - 8X$

$X = 266.625 \text{ mm}$

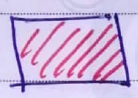
then max M



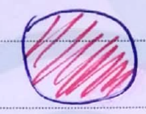
τ_{max} | cross section

~~cross section~~ | ~~Maximum shear stress (τ_{max})~~

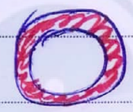
$\frac{3}{2}\frac{V}{A}$



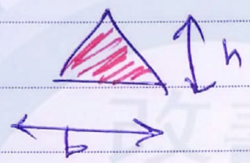
$\frac{4}{3}\frac{V}{A}$



$2\frac{V}{A} \rightarrow \pi(d_2-d_1)^2$

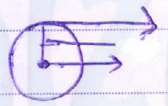


$\frac{3}{4}\sqrt{4+\frac{b^2}{h^2}}\frac{V}{A}$

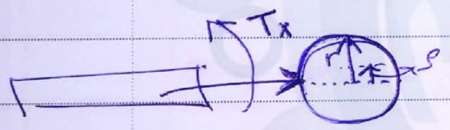


Torsion

$T \rightarrow \rho\ \tau$



τ_{max} (max shear stress)



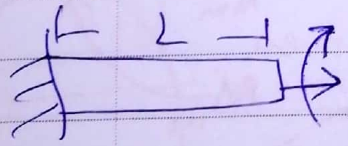
Torsion due to shear stress

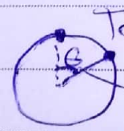
$\tau = \frac{T\rho}{J}$ $\qquad J = \frac{\pi}{32}d^4$

J → Polar moment of Inertial

$\tau_{max} = \frac{Tr}{J}$

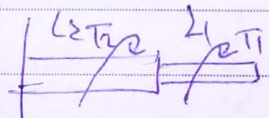
$T = \vec{r} \times \vec{F}$





Torsion → twisting angle.

$\theta = TL$ → remains constant



$G = \frac{E}{2(1+\nu)}$

$\sigma = E\epsilon$

$\theta = \frac{TL}{GJ}$

T: torque
L: bar length
G: shear modulus
J: polar moment of Inertia

$\tau = G\gamma$

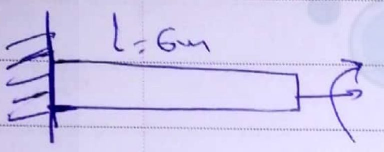
$\gamma = \frac{\rho\theta}{L}$ shear strain

$\rho = R$ → max shear strain.

rectangle / hollow / circle.

No.

Ex: what's the maximum Diameter of a solid shaft which will not twist more than 3° in a length of 6m when subjected to a torque of 12 KN-m. what's the maximum shear stress

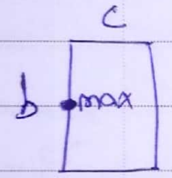


$$\tau = \frac{T\rho}{J}$$
shear stress

$$\frac{T}{J} = \frac{\theta G}{L} = \frac{\tau}{r}$$

$$\frac{\frac{3\pi}{180} * 82*10^9}{6} = \frac{12*10^3}{\frac{\pi d^4}{32}} \Rightarrow d = 114.32 \text{ mm}$$

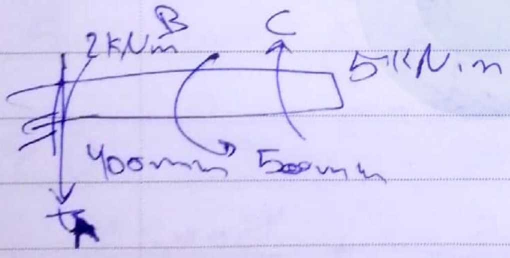
max shear stress in rectangle.

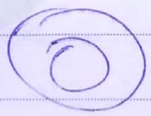


$$\frac{\theta}{L} = \frac{T}{\beta b c^3 G}$$

$$\tau_{max} = \frac{T}{\alpha b c^2} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right)$$

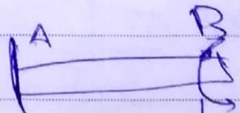
calculate the angle of twist at C with respect to A
$G = 80$ GPa



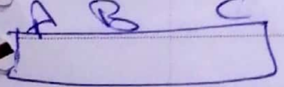


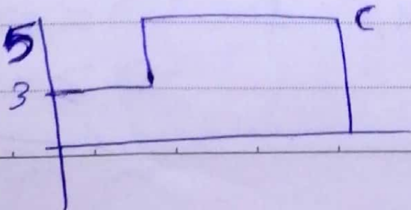
$D = 75$ mm
$d = 60$ mm

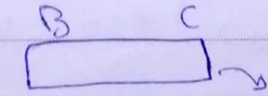
$\Sigma T = 0 \quad -5 + 2 + T_A = 0$
$T_A = 3$ kN.m



$$\theta = \frac{T_{AB} L_{AB}}{GJ} = \frac{3*10^3 * 400*10^{-3}}{80*10^9 * J}$$







$$\theta = \frac{5*10^3 * 500*10^{-3}}{80*10^9 * J}$$

$$\frac{T_{BC} L_{BC}}{80*10^9 * J}$$

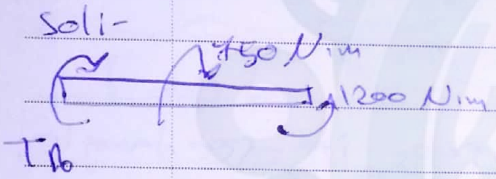
smile for life

No.

$$\theta_{A/C} = \frac{3*10^3 * 400*10^{-3}}{80*10^9 * J} + \frac{5*10^3 * 500*10^{-3}}{80*10^9 * J}$$

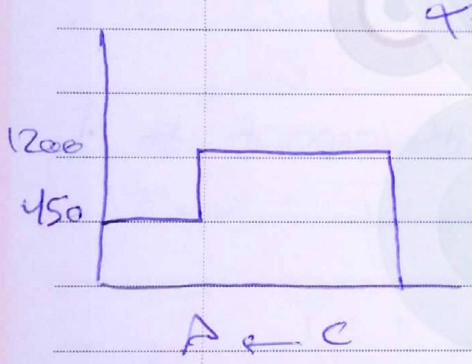
$\theta_{C/A} = -\theta_{A/C}$ C.W

Ex: A solid steel shaft.



$\sum T = 0 \Rightarrow +T_A - 750 + 1200 = 0$

$T_A = -450$ N.m



$$\tau_{max} = \frac{Tr}{J} = \frac{1200 * \frac{D}{2}}{\frac{\pi}{32}(D^4)} = 60*10^6$$

$33.79 > D$

$D = \cancel{\ldots}$ mm

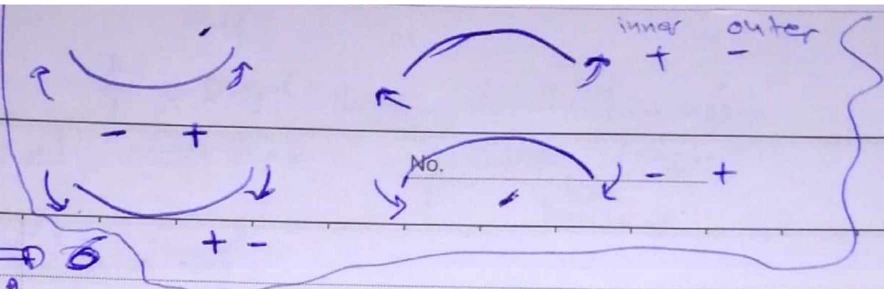
$$\theta_{C/A} = \frac{TL}{GJ} \Rightarrow \frac{4\pi}{180} = \frac{1200*2.5}{83*10^9 * \frac{\pi}{32} D^4} + \frac{450*2.5}{83*10^9 * \frac{\pi}{32} D^4}$$

$J = \frac{\pi}{32}(75^2 - 60^4) * 10^{-12}$ Mm^4 $\Rightarrow D = 51.9$ mm

$D \geq 51.9$ mm

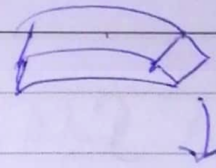
we take this

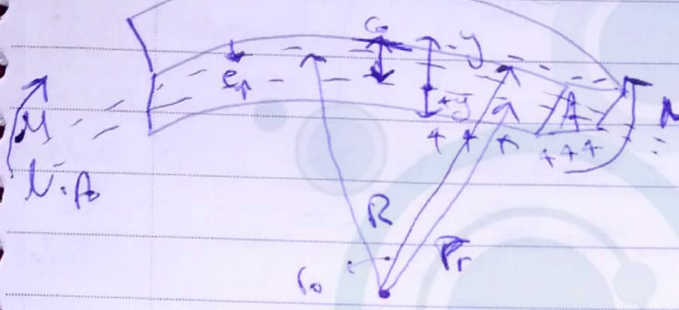
D



$M \Rightarrow \sigma$

$e = R - r_n$





composite Area.

(1) ΣA

(2) centroid $\rightarrow \frac{\Sigma \tilde{y} A}{\Sigma A}$ reference is O (center)

(3) Neutral axis (N.A) $r_n\square / r_n\triangle \Rightarrow \frac{\Sigma A}{\Sigma r_n\square + \Sigma r_n\triangle}$

inner

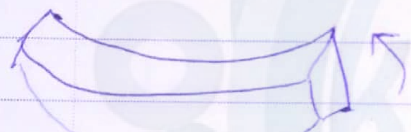
$\sigma_i = +\frac{M c_i}{A e r_i}$

outer

$\sigma = -\frac{M c_o}{A e r_o}$

$C_i = r_n - r_i$

$C_o = r_o - r_n$



inner

$\sigma = -\frac{M c}{A e r_o}$

outer

$\sigma = +\frac{M c}{A e r_o}$

The curved bar has a cross-sectional area ... If it's subjected to bending moments of 4 kN.m, determine the max normal stress.

$\Sigma A = \square + \triangle = 50 * 50 + \frac{1}{2} * 50 * 30 = 3.25 * 10^{-3}\ m^2$

$R = \frac{225(50*50) + 260*(\frac{1}{2}*50*30)}{50*50 + \frac{1}{2}*50*30} \Rightarrow R = 233.08\ mm.$

$r_n = \frac{\Sigma A_i}{\Sigma \int \frac{dA}{r}} = \cancel{\frac{50*50 + \frac{1}{2}*50*30}{182}} \quad \frac{3.25*10^{-3}}{.011 + .0028867} = 0.23142\ m$

بحرين

↑ stress للثني لعوامل بين straight beam and curved

الأول نجوزها s.f

$$\int \frac{dA}{r} = b \ln \frac{r_o}{r_i} = 50 \ln\left(\frac{250}{200}\right)$$

50

$\sigma = \frac{My}{I}$ composite

straight beam في حالة

neutral يقع على centroid R

$$\int \frac{dA}{r} = \frac{b r_o}{(r_o - r_i)} \ln \frac{r_o}{r_1} - b = \frac{50 * 280}{280-250} \ln \frac{280}{250} - 50$$

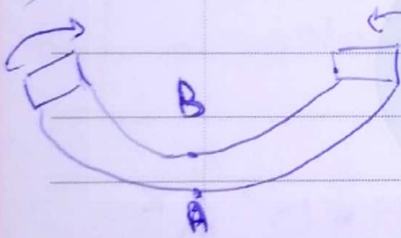
$$e = .23142 - .223308 = .00166\,m$$

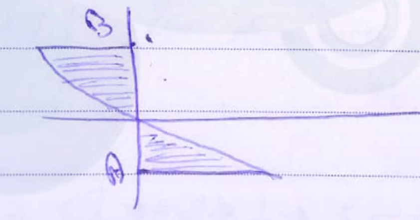
$$C_i = 0.23142 - 0.2 =$$

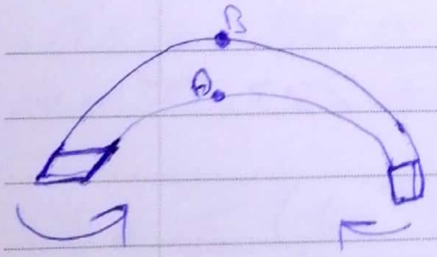
$$C_o = 0.280 - 0.23142 =$$

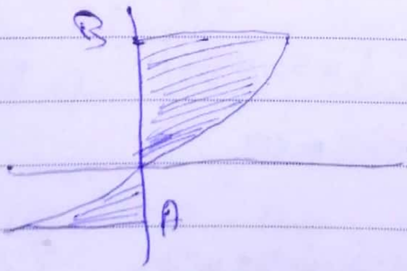
$$\sigma_i = \frac{-4*10^3 * (.23142 - 0.2)}{(3.25*10^{-3})(0.00166)(0.2)}$$

$$\sigma_o = \frac{4*10^3 * (0.28 - 0.23142)}{(3.25*10^{-3})(.00166)(0.28)}$$





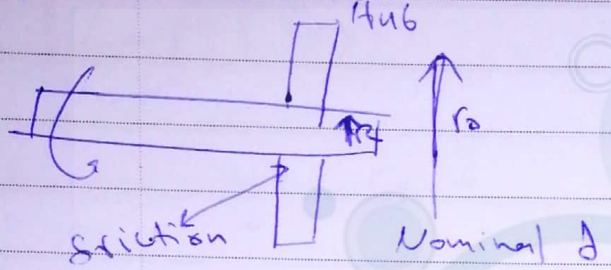


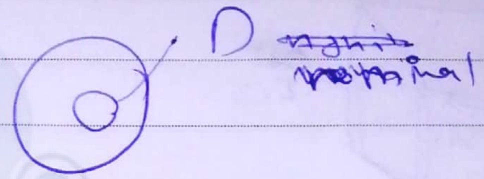


Contact planes.

No.

Fit Interferance.





$D = d$

$$\sigma = P = \frac{\delta E}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}\right]$$

$E \rightarrow 200$ GPa

$\delta_{min} = d_{min} - D_{max}$

$\delta_{max} = d_{max} - D_{min}$

Ex:

$d_{max} = 2*0.5 + 2.3*10^{-3}$

$D_{mini} = 2*.5 + 0 = 1$

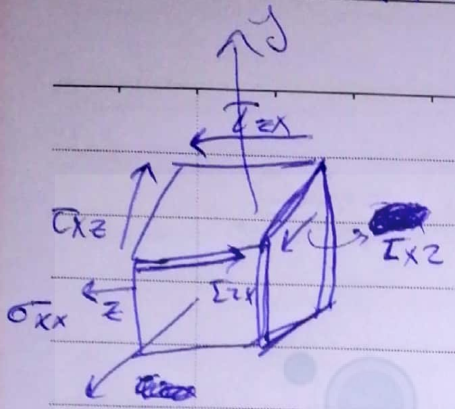
Friction $= \mu N$

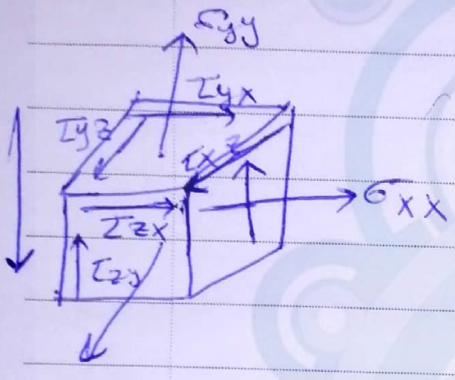
$N = P\, A_{contact}$

$= P\, 2*R*$ thickness.

Principal stresses

No.





$3\times3 = 9$

$\tau_{yx} = \tau_{xy}$

$\tau_{zx} = \tau_{xz}$

$\tau_{yz} = \tau_{zy}$

$9-3=6$ components.

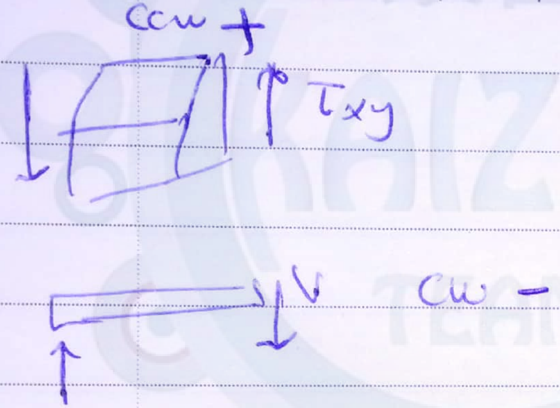
Symmetric

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

plane stress

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\tau_{x}, \tau_{z}, = 0$ bcz it's plane stress



* shear stress = 0 when the plane is principal.

* الـ max shear sts plane بكون الـ normal

$$\sigma_{avg} = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

$$= \frac{-40+70}{2} = 15$$

No.

max and mini normal stresses are known as principal stresses.

* shear stress = 0 when the plane is principal means σ is max.

ال plane الي max shear عليها ال normal بيكون

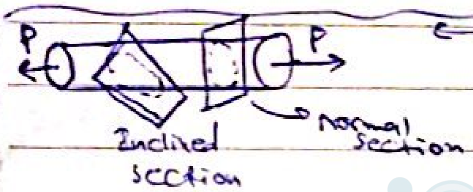
$$\sigma_{avg} = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

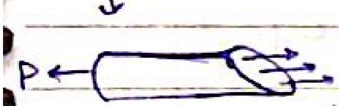
$$= \frac{-40+70}{2} = 15$$

ccw +

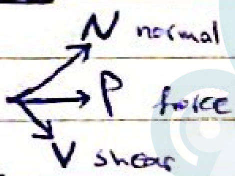
cw −

σ on Inclined sections





$\sigma = \frac{F}{A}$

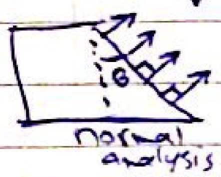


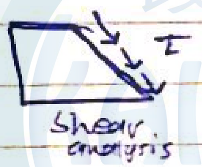
σ and τ at any θ of inclination

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

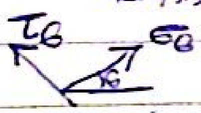
$$\tau_{x_1y_1} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$





σ_{max} and $\tau = 0$ principal

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



τ_{max} and $\sigma = \frac{\sigma_x + \sigma_y}{2}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$ σ_θ will be max at $\theta = 0$

$\tau_\theta = -\frac{\sigma_x}{2}\sin 2\theta$ τ_θ will be max at $\theta = 45°$

لتحديد ال max و mini لل σ نعوض

ال θ الي بتحقق $\sigma_{x_1} \leftarrow$ max

$\cancel{\text{XXX}}$ $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

نطلع ال θ من المعادلة ونعوضها بالمعادلة σ_{x_1} وبتطلع ال max σ_{x_1}

$\frac{d\sigma_{x_1}}{d\theta} = 0$ مكان $\theta \leftarrow$ الي بتحقق σ_{x_1} max

مكان $\theta \leftarrow$ الي بتحقق max shear

$\frac{d\tau_{x_1y_1}}{d\theta} = 0 \Rightarrow \tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$

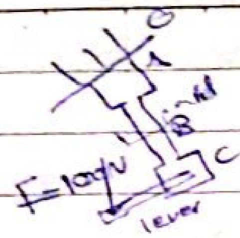
لتحديد ال max, mini shear

smile...

Scanned with CamScanner

$r \times F$

$\hat{i} \times -\hat{j}$ No.



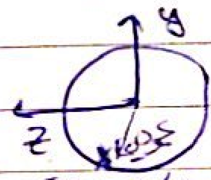
AISI 1035 steel forged & heat treated.

$\sigma_y = 55.8\,MPa$

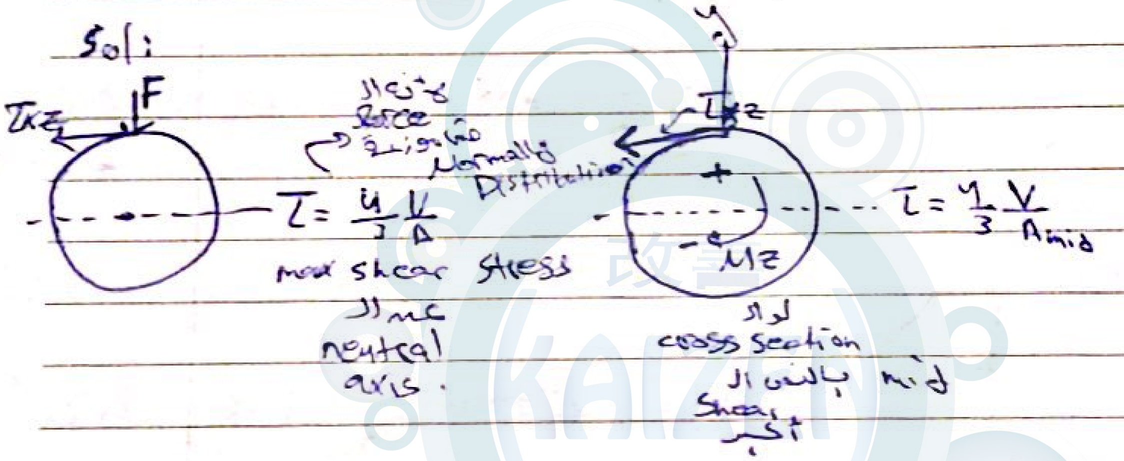
- 1- τ_{max}
- 2- Tresca yielding criteria.

A force 100 N applied at D near the end of the lever.
The bar OAB

Y X Z



Sol:



- ازاكان الناتج عمودي عليه بيكون Torque باتجاه السطح تسببه bending

* لاحظ قديش صاحب ال cross section ال stress بين N (اخترال mid يكون كبيرا).

$$\tau_{xz} = \frac{32 \times \left(\frac{25}{2000}\right)}{\frac{\pi}{32}\left(\frac{25}{1000}\right)^4} =$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= 16.84\ MPa$$

$$M_z = \frac{350}{1000} \times 100 = 35\ N.m$$

$$\sigma_{xx} = \frac{35 * \frac{25}{2000}}{\frac{\pi}{64} * \left(\frac{25}{1000}\right)^4} = 22.8\ MPa$$

smile...

τ_{max} 2x? Tresca   No. τ لحاله Shear stress.

$16.84 = \frac{55.8}{2}$ it will not yield.

factor safety $n = \frac{\sigma_y/2}{\tau_{max}} = \frac{27.9}{16.84}$

Ex: A Bar is AISI 1020 hot-rolled steel $S_y = 331$ MPa

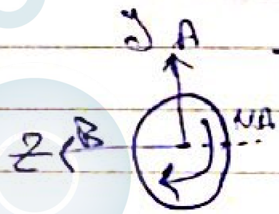
- F = .55 kN
- P = 8.0 kN
- T = 30 Nm

لو طلب مني احسب n
بحسب عند ال max

find:
- Principle stresses and max shear stress.
- factor of safety (n) at A.

for a beam of diameter d the max distance from the neutral axis is $\frac{d}{2}$

P ⇒ $\sigma_{xx} = \frac{P}{A} = \frac{8*10^3}{\frac{\pi}{4}*\left(\frac{20}{1000}\right)^2}$



$\vec{r} \times \vec{F}$

$\hat{i} \times -\hat{j} = -\hat{k}$

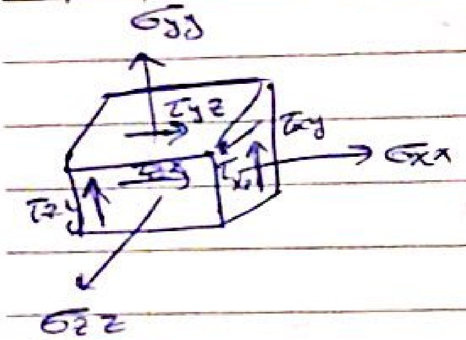
M=0 عند N.A

T ⇒ $\tau_{xz} = \frac{30 * \frac{20}{2000}}{\frac{\pi}{32}\left(\frac{20}{1000}\right)^4} = 19.1$ MPa

F ⇒

$$\sigma_{xx} = \frac{8*10^3}{\frac{\pi}{4}\left(\frac{20}{1000}\right)^2} + \frac{0.55*1000*\frac{100}{1000}*\frac{20}{2000}}{\frac{\pi}{64}\left(\frac{20}{1000}\right)^4} = \cancel{45}.5\text{ MPa}$$

95.5 MPa



Tri-axial state.

x-y

$$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau_{xy}^2}$$ τ_{max} x-y

y-z

$$\sigma_{1,2} = \frac{\sigma_{yy}+\sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{yy}-\sigma_{zz}}{2}\right)^2+\tau_{yz}^2}$$ τ_{max} y-z

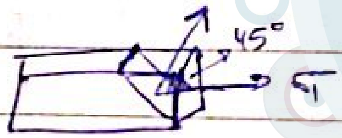
~~Z-x~~

$$\cancel{\sigma_{1,2} = \frac{\sigma_{zz}+\sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}-\sigma_{yy}}{4}\right)^2+\tau_{yz}^2}} \quad \cancel{\tau_{max}\ y-z}$$

z-x

$$\sigma_{1,2} = \frac{\sigma_{zz}+\sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}-\sigma_{xx}}{2}\right)^2+\tau_{zx}^2}$$ τ_{max} z-x

Uni-axial state.



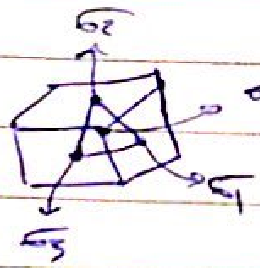
الزاوية بين ال principal plane

و ال shear ← 45°

Failure theories :-

1. Tresca ← $\tau_{max} = \frac{\sigma_y}{2}$ $n = \frac{\sigma_y/2}{\tau_{max}}$
2. Von misses

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$



$\sigma_{normal} = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_y \qquad n = \frac{\frac{\sqrt{2}}{3}\sigma_y}{\tau_{oct}}$$

3-8 Elastic Strain in 3-D

No.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu(\sigma_{yy} + \sigma_{zz})}{E}$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu(\sigma_{xx} + \sigma_{zz})}{E}$$

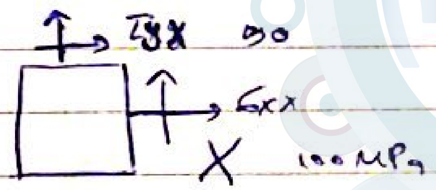
$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu(\sigma_{xx} + \sigma_{yy})}{E}$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

$n = \dfrac{\sigma_y}{2\tau_{max}}$ Tresca

$n = \dfrac{\frac{\sqrt{2}\,\sigma_y}{3}}{\tau_{oct}}$ von misses



Mohr's Circle

only for 2D

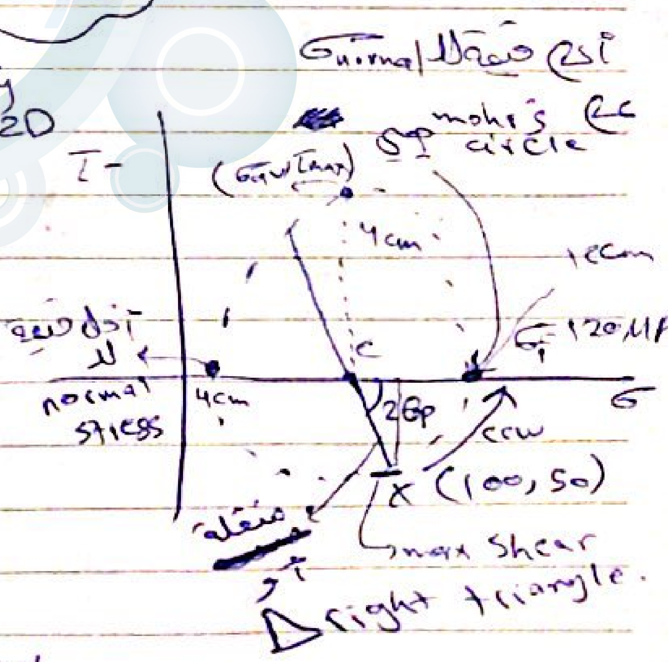
center $(\sigma_{avg}, \tau_{max})$

X (σ_{xx}, τ_{xy})

y (σ_{yy}, τ_{yx})

x ccw (+)

y cw (−)



$$\tan 2\theta_p = \frac{50}{80-100}$$

$$\theta_p = \tan^{-1}\left(\frac{50}{2}\right)/2 \quad \text{ccw}$$