WEEKLY

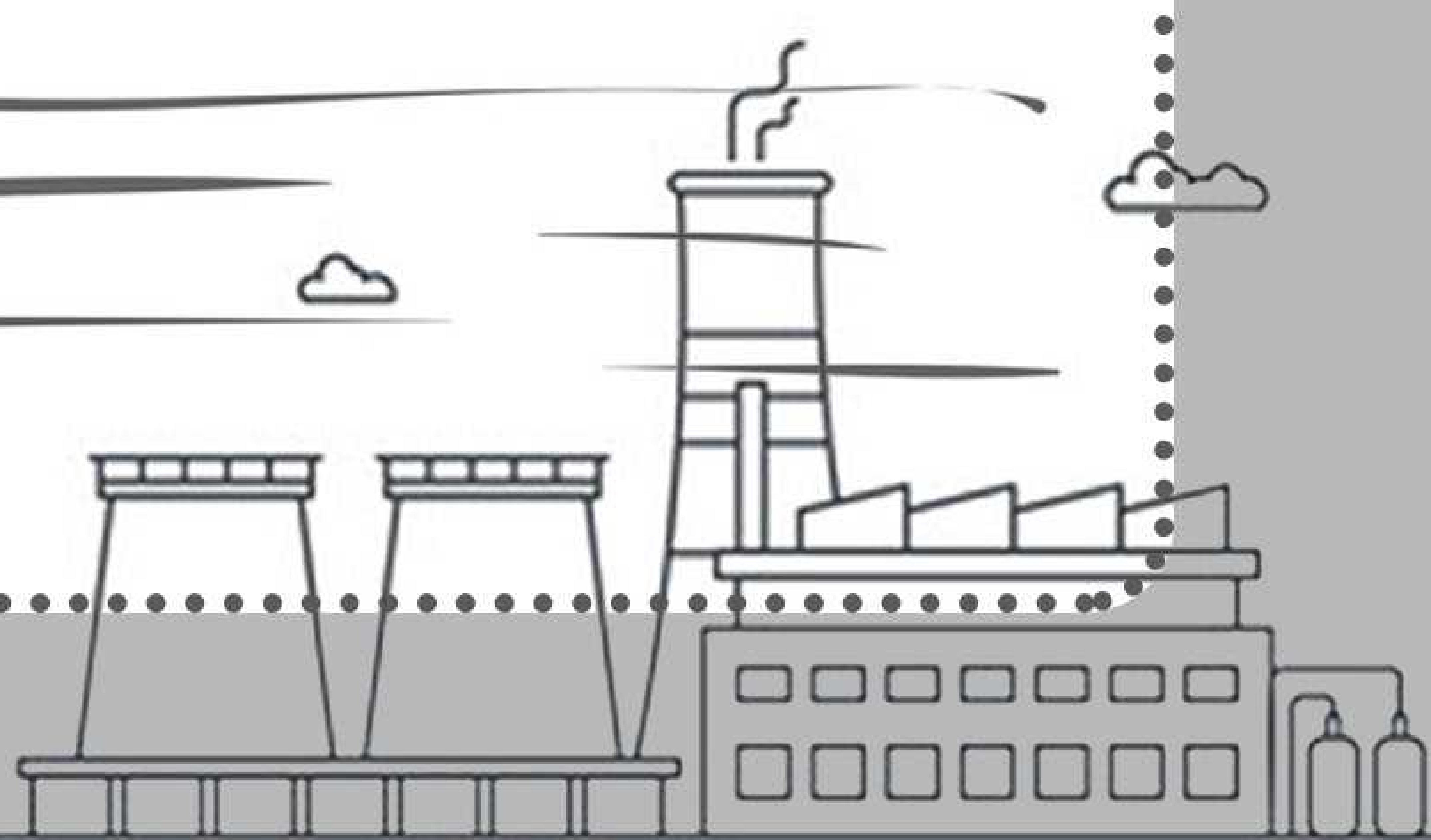
NOTEBOOK

Logistics

second semester

2021

KAIZEN TEAM



* Logistics :

PC&Lm ⇒ suppliers in supply chain.
Distribution center ⇒ used more than central Distributer.
D.C

ال supply chain :

* عزيز ال supplier و نحدد procurement ال quantities و ال quality ب
، ال manufacturer ← عند capacity
و inventory ← MPS ، MRP
و ال Logistics تهتم بـ transportation في (داخلي / خارجي).

Logistics توصيل ← (product service) ازاي نوصل لـ الزبون

- Amazon is a big & complex supply chain.

> - supply chain differ from supply chain management
> + Logistic + Log management

*Demand include D.C & customer.

- Forward logistics towards customer.
- Reverse (backward) " " opposite direction. (because of Awkward pieces)
- Sync : synchronisation (تزامن)
- ERP : تجميع ال supply chain ⇒ Enterprise Resource planning.
→ Technology in logistics definition !

- How many times they deliver service within the range that they order ? (customer satisfaction) → Effectiveness

Cost determined by :
→ value stream mapping
→ Lean management
→ time & motion
→ inventory → MPS

⇒ Efficiency.

⇒ Logistics management is a subsystem "subactivities" of supply chain management.

- types of supply chains :-

1 - pull → make to order ⇒ customer pull supply chain
(MTO) ↓ — by make order ⇒ مثل السيارات
ما بجهز الغرض قبل ما — ← مثل كل ما شي بالطلب — بالآخر بتصنع حسب
لأنه ما عندي stock
lead time short . ومعينتر نة و — rose - rise. رفعة مثل
مثل أي تغير بصير وبحجم بصير خفيفة كثير. — porch.
poughati

2 - make to assembly ⇒ restaurants, Dell laptop. ⇒ خليط بين
(MTA) → (push then pull) MTA, MTO

3 - make to stock (MTS) ⇒ ppl ⇒ inventory + safety stock + etc.

- push supply chain ⟶ بيشتغل مسبقاً، عندي stocks.

* pull → demand can't be forecasted because it doesn't have pattern (increasing, decreasing ⇒ trend + so on)

* push (make - to - stock) : forecasting.

* 3 type of flows in supply chain :
⇒ ① product ~~flo~~ may be a reverse flow if it rework → mainly forward.
② Information (feedback) mainly backword.
, requirements
③ money - flow . mainly backword flow. →
ممكن تكون ← forward. مثل كاش، شيك آجل .

⇒ 3rd party Logistics Example : Aramex , Agility.
Like warehouse
↳ Like carrefour
↳ Inventory نوع مستودع
شركة خاصة بتحمل تخزين و نقل للبضائع الي تشتغلها — Assorted : متنوع
(Logistics) شغلة
↳ Allows Company to do core work. → main goal of 3PL.

- Vendor → EDT → تراقب الرتبة سي طريقة كاملا
supply يشحن صنعة من المسير باركة كما تعمل الكمية تقل

- Least amount of inventory → 1. make to order
then
2. make to assembly
then
3. make to stock.

(بالترتيب)

- In - transit inventory: الي بالطريق الكل goods آمرة
- Reasons to hold inventory:
* service level : percentage of orders to be fulfilled right way (without waiting). without Backorder.
trade - off → ...
* reduce transportation cost : → cost ... efficiency → تقلل
* cope with randomness : to ensure that I have quantity that for highest demand.
demand from customer. lead time of supplier variables. - lead time : if supplier supply me with orders by diff. duration each time

زيادة طلب مثل فترة ... قلة ... الشحن

* speculate on price patterns.

* Logistics is responsible on 3 activities :
① Order processing :
② Inventory management : by check quantities to send it.
③ Freight transportation.

transmission : ارسال
stock piles : ...
speculate : ...

Inventory management + it's Policies :
↳ to make decision , When , How much , How .
(Q, R) , (T, S) , (s, S).

* Mode choice in Freight Transportation → know minimum cost & way to transport goods or services.

Like plane, ship, ---

False. → marketing not in activities.

→ Just in Logistics activities: Transportation, cost ⇐ Inventory, ~~...~~ order processing.

Cost → Distance, time, cost.

* difference bw strategic, tactical and operation levels is time. (بمعنى بناخد وقت طويل)

strategic → [illegible] → نسأل Facilities planning

- Resource Allocation: توزيع الـ Resources الـ Facilities

- Capacity Allocation: توزيع [illegible] الـ Facilities

Tactical level. → خدمة الـ [illegible]

· strategy [illegible]

- Supply chain Activities:

ppc, Facilities, Logistics, Business

* Costs in logistics system: 4 essential types

1. Transportation costs (نقل [illegible])
2. Handling costs (تفريغ وتحميل [illegible])
3. Holding Costs (opportunity cost) (cost of capital)
4. Facility rent cost. [illegible] Facility [illegible]

Inventory [illegible]

- Normalizing costs: consider Σ cost per time period / per item

* Comparing costs of carriers or modes (transportation), [illegible]

→ ultimate objective of logistics: minimize total cost when selecting mode of transportation.

type

⇒ often requires ~~trading off~~ inventory + transportation costs.
↳ Like balancing ~~the~~ these costs to know how much to stock in inventory.

* Inventory costs : 2 types. ← [illegible Arabic annotation]

① Holding cost (cost of capital). (opportunity cost)

② Renting cost for space. (warehousing space) or own.

→ f (value held , interest rate). Like → 25% of item for one year.
↳ function. ← معناها 25% على التكلفة زيادة بسبب التخزين.

* Inventory held in 3 places : [illegible Arabic] → 25% of value / year held
↳ (15% - 25%)

① at Plant (مصنع [illegible Arabic]) ال Item الي اتصنعت فيه.

② at warehouse (Distination).

③ in - transit → وهي بالطريق شحنة تعتبر مخزنة.

Plant → time → warehouse (WH).

Problem : A company needs to select between two transportation modes rail and truck, to move goods from plant to WH with obj of minimizing total transportation cost and inventory cost.

علاقة عكسية

لانه اذا حركة أكثر من ارفع تكلفة بحتة [illegible] اقل [illegible] عدد [illegible]

Data	Plant	WH.
safety stock	0	100 units
value of items	\$500/unit	\$512/unit. → handling of materials
space cost	\$20/unit.year	\$25/unit.year.

(اداخل ال Inventory — margin annotation beside safety stock)

per shipment

Rail : - Car capacity = 400 units. ~~[illegible]~~
- transit time = 20 days
- cost per shipment = \$ 1200

* Truck : - Truck capacity = 100 units → per shipment
- transit time = 3 days
- cost per shipment = \$ 700

* production rate at plant 10 units / day.
  Demand at WH 10 units / day.
  Inventory holding cost rate 25% of value / year held.

* **Goal** is to determine the optimal shipment size q (# of units) for each mode and the total annual cost associated with these shipment sizes.

Assumption: shippers will never ship more than a single carload (rail) or a single truckload (truck).

Sol:

Steps:

[1] Determine annual inventory space cost at the plant.
[2] " " " holding " " " "
[3] " " " space " " " WH.
[4] " " " holding " " " "
[5] Determine in-transit inventory cost for Rail / " truck.
[6] " transportation costs Rail / truck.
[7] total annual cost expensions R / T
[8] Determine optimal q for each mode.
[9] Choose mode

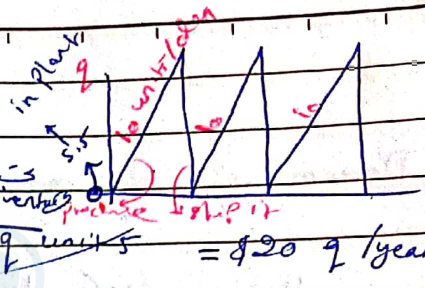
Solution :

1) space cost at plant →

annual space cost = $\frac{\$20}{\text{unit.year}}$ * q units = $20 q /year.

→ year

$/year



average = q/2 ← assurance

q: max. inventory level → space cost

2) Holding cost at plant.

- rate of charge per item of inventory held at plant for a year = $ 500 * 25% = 125 $/unit.year

- Avg inventory level = $\frac{q}{2}$

(125 $ /unit.year) (q/2 units) = 62.5 q $ /year.

3) Space cost at WH.

→ depend on max. inventory level

max inventory level = q + 100 units

Space Cost = ($ 25 /unit.year)(q+100) units

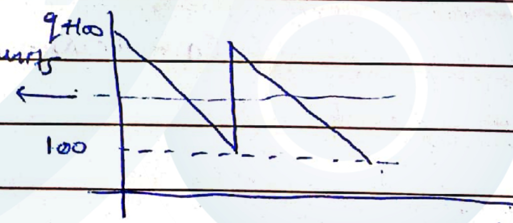
100 = S.S → safety stock.

= (25 q + 2500) $/yr.

4) Holding cost at WH.

Avg inventory. Avg inventory level = 100 + $\frac{q}{2}$ units

100



Holding cost.

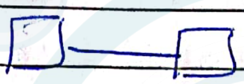
each item held on year charged at rate of $ 512 * 25%

= $128 / unit.year

Holding cost = $\frac{\$128}{\text{unit.yr}}$ * (100 + $\frac{q}{2}$) units

5) Determine in transit inventory costs.

use $\frac{506\ \$}{\text{unit}} = \frac{500 + 512}{2}$



each item charged at $\frac{\$506}{\text{unit}} * 25\% / \text{yr} = \$126.5/\text{unit.yr}$

for Rail: each item spends 20 days amount to be shipped for whole yr = production rate * period

$= 10 \frac{\text{unit}}{\text{day}} * \frac{365\ \text{day}}{\text{yr}} = 3650 \frac{\text{unit}}{\text{yr}}$

In transit cost = amount shipped (unit/yr) * in transit time * in transit holding cost.

$= 3650 \frac{\cancel{\text{unit}}}{\text{yr}} * 20\ \cancel{\text{days}} * \frac{1\ \cancel{\text{yr}}}{365\ \cancel{\text{day}}} * 126.5\ \$/\cancel{\text{unit}}.\cancel{\text{yr}}$

$= 25300\ \$/\text{yr}$

for Truck:

In-transit cost truck $= \left(3650 \frac{\text{units}}{\text{year}}\right) * \left(3\ \text{days} * \frac{1\ \text{yr}}{365\ \text{day}}\right) * \left(126.5\ \$/\text{unit.yr}\right)$

$= 3{,}795\ \$/\text{year}$

⑥ Transportation cost.

Rail: every shipment cost = \$1200.

# of shipments $= \frac{3650}{9}$ shipment/yr.

↳ كم شحنة بدي

[illegible]

Transportation cost Rail $= \left(\frac{\$\,1200}{\text{shipment}}\right) * \left(\frac{3650}{q}\ \frac{\text{shipment}}{yr}\right)$

$= \frac{4,380,000}{q}$ $/yr.

$\Rightarrow$ ~~From~~ Transportation cost for Truck $= \left(\frac{\$\,700}{\text{shipment}}\right) * \left(\frac{3650}{q}\ \frac{\text{shipment}}{yr}\right)$

[illegible]

$= \frac{2,555,000}{q}$ $/yr.

7) * Total annual cost $\Rightarrow$ $TCR(q) = 20q + 62.5q + 25q + 2500$
$+ 64q + 12800 + 25300$
$+ \frac{4380000}{q}$

$$= \boxed{171.5q + 40600 + \frac{4380000}{q}}$$

from inventory costs — from (in-transit) cost

for optimum $q \Rightarrow$ Derive $\rightarrow$ (8)

* for Truck :

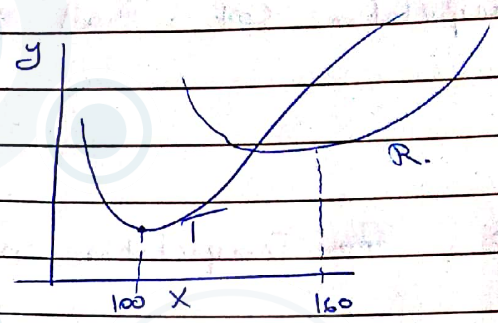
$TCT(q) = 20q + 62.5q + 25q + 2500 + 64q + 12800 + 3795 + \frac{2555000}{q}$

$$= \boxed{171.5q + 19095 + \frac{2555000}{q}}$$

Scanned with CamScanner

In Excel:

q	TCT	TCR
10	$=171.5q+\_\_$	⋮
20	⋮	⋮
⋮	Expand.	



8) optimal q for each mode.

Rail $\ast$ $TCR'(q) = 171.5 + 0 - \dfrac{4380000}{q^2} = 0.$

$$q_R = \sqrt{\frac{4380000}{171.5}} \simeq 160 \text{ units}$$

دي الـ Rail's الـ capacity هي

$$TCR(q_R = 160) = 171.5 \ast 160 + 40600 + \frac{4380000}{160}$$

$$= \$95,415 \text{ /yr.}$$

Truck $\ast$ $TCT'(q) = 171.5 - \dfrac{2555000}{q^2}$

$$q_T = \sqrt{\frac{2555000}{171.5}} = 122 \text{ units.}$$

capacity هي

So I need 2 Trucks ← Capacity < q ← Truck

∴ Shipment cost $= 2 \ast 700 \frac{\$}{\text{Shipment}} \ast \dfrac{3650}{q}$

$$= \frac{510000}{q}$$

$$TCT(q_T = 122) = 171.5 \ast 122 + 19095 + \frac{510000}{122}$$

$$= \$81,903.$$

أنا هنا دخلت بالاعتبار Inventory cost وتجاهلنا

. تجاهلنا In-transit cost

If maximum of Truck is optimum.

$TCT(q_T = 100) = \$61,795$ ⇒ so this is minimum possible cost. (q on capacity of Truck)

max. cap. ...

$TCT(q_T = 99) = \boxed{\$61881.58}$

So Truck is better than Rail (according of cost).

addition:

Shipment	Inventory cost	Transp. cost	Total cost
160 Rail	68,040	27375	95,415
400	↑ 109,200.	↓ 10,950	120,150
140	↓ 64,610	↑ 31,286	95,896.

∴ Transportation Rates and Costs by mode.

*Fixed costs : infrastructure (port , Equipment , right of way ( ), airports , Planes , ships , machine , vehicle , ...)

*variable cost (operating) : fuel , maintenance , driver , —

mode	Fixed	variable	Fixed cost components (why it's low or Hight)	varible cost component
(ships) water	MH	L	vehicle ( )	Low speed, high capacity, fuel efficiency
Rail	( ) H	L	rail roads, control systems	fuel efficiency, Small crew
Truck	L	H	Trucks, cross dock, terminals	Drivers, fuel,
(planes) Air	H	MH	vehicle, terminals, Facilities	maintainance, Fuel, ...

Five Apple

$max = q + 100$

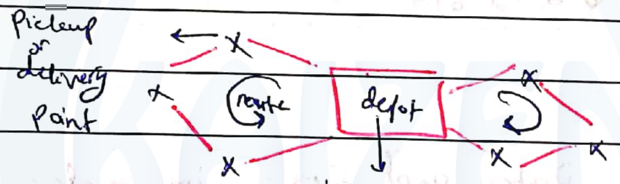
$min = 100 = s.s$ [illegible]

$avg = \dfrac{q + 100 + 100}{2} = \dfrac{q}{2} + \dfrac{200}{2}$

$= \dfrac{q}{2} + \underline{100}$ [illegible]

* Short - haul (نقل) transportation : (freight) [illegible].
  when pickup / delivery of goods in relatively small area. using fleet not vehicles.
  unless stated otherwise, vehicles are stationed at single depot.

- Routes include several pickup and/or delivery point
- Routes performed in a single work shift.



i.e : warehouse, port, terminal, plant itself.

* Decisions in short transportation :

[illegible] - strategic : where to locate the depot / warehouse "terminal". → Location problem / facility Location
[illegible] - tactical : fleet sizing ([illegible]) ⇒ how many (trucks) (Cars) to have.
- operational : vehicle routing and scheduling problem "VRSP"
  ↳ route : where each vehicle is responsible.

* we will focus on operational prob.
* according to data known beforehand type.
  ↳ static vehicle routing prob : all customers requests (locations, amount of goods) is known in advance.
  ↳ Dynamic vehicle routing prob : (VRP) not all information is known in advance.

Five Apple

(in DVRP) - Routes planned for set of requests known in advance, dynamically updated to incorporate new requests.

*

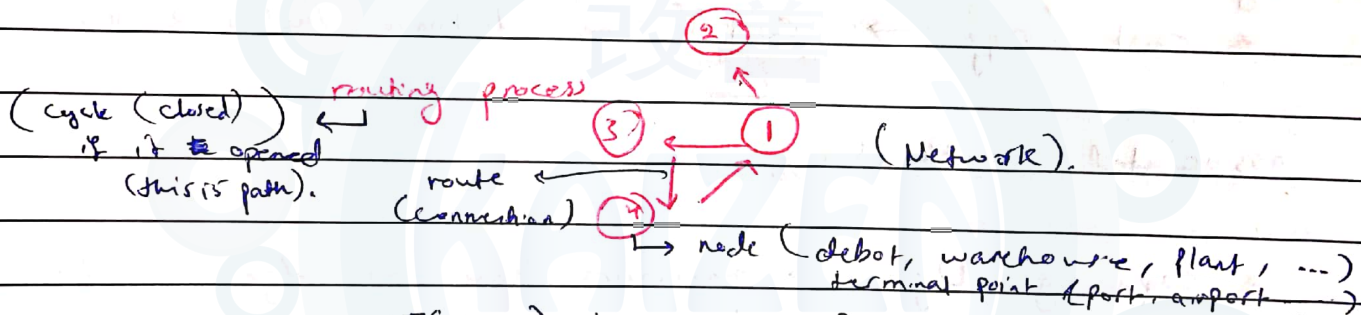
restaurant ⟶ Delivery man. ⟶

get orders — معروفة data

لو مفقود بدونين مرة ربع (planned خطط) لذلك لما نرجع لازم في orders تانية كذلك. static problem.

اذا لا حالات استقبل نغير طريقة تتبع ، dynamic.

* transportation prob are defined on graphs, which are symbolic representation of networks and their connectivity.

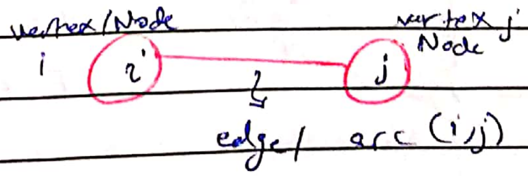


Graph : $G(V, E)$ is a set of vertices / nodes (V/N) connected by a set of edges / arcs (E/A).

Notation $G = (N, A)$.

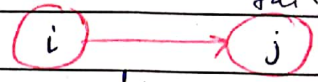
- vertex : terminal point / intersection point of graph.
  - ⇒ Represents a city, Customer location, depot, road intersection.
  - First + Final location.

- Edges : Link between two vertices. Edges / arcs often referred to as (i, j) connected vertex i to vertex j.



- Arrow representation direction.
  - * Head node i
  - * Tail node (j)

Head tail



directional connection. (i, j) $i \to j$

- Assumed bidirectional if no arrow. (اذا رسم الطريق .)

↳ 

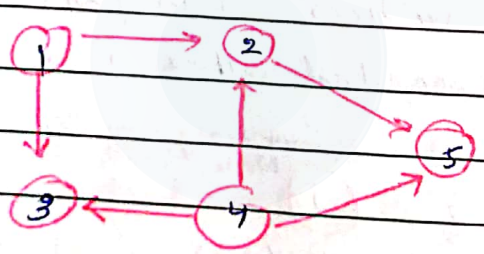
(symmetric if the both routes the same cost)

- Edges / arcs have number of characteristics :
  - * cost "C_{ij}" → cost (in terms of Time, distance, fuel, Risk,) associated with (i, j)
  - * capacity "U_{ij}" → maximum flow on edge (i, j).
    ↳ trafic , amount of .

** order of a graph : number of vertices (nodes) , $|V|$
$V = \{1, 2, 3, 4, 5, \ldots\} = \{$Amman, Aqaba ---$\}$ ↳ as Listing
$|V| = 5$

** size of graph : number of edges , $|E|$

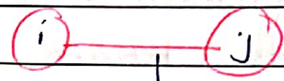
Ex :



⇒ $G = (V, E)$
$V = \{1, 2, 3, 4, 5\}$
$E = \{(1, 2), (1, 3), (2, 5), (4, 2), (4, 3), (4, 5)\}$
order of $G = 5$ $|V|$
directional / non-direc. > routes (يعني links) عدد ⇐ ~~size~~ size of $G = 6$ $|E|$

* Structural properties of graph :

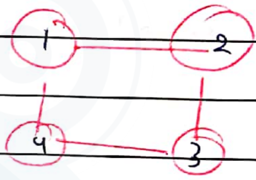
- Undirected vs directed :
  → Networks without specified direction.
  ↓
  a connection between i & j implies a connection btw i & j .



Also referred to as a $\boxed{\text{symmetrical}}$ network. (symmetrical edge, ...)

the same cost from 1 → 2 (or 2 → 1)

if differs → bidirectional & assymetric.



Ex :

symmetric

- move in both direction.
- cost is the same in both.

bidirectional.

asymmetric

- move in both directions.
- Cost isn't equal. (3, 5)

(bidirectional).

* In transportation problem ; sometimes symmetry used to represent whether cost is same when traveling from i to j as when traveling from j to i.

- asymmetry example → traffic may be higher in one direction resulting in higher travel time.

- بالرغم انه bidirectional بس ال cost مش ثابت بالعكس فيه اختلاف بين الاتجاهين ،

→ . $\boxed{\text{asymmetric}}$ هي

* symmetric & asymmetric both are bidirectional, cost that determine

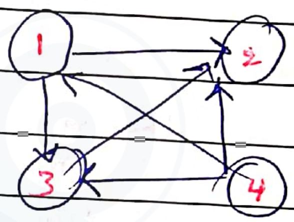
which one of them.

∴ Final Conclusion:

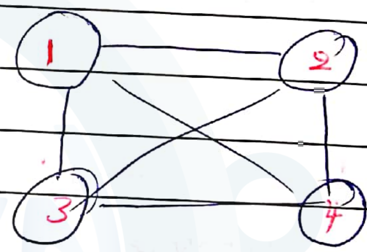
sym. or asym. = bidirectional] in indirected

but directed just in one direction.

- Completeness: a graph is complete if an edge exists between every pair of vertices. (regardless of direction).



Complete indirected ⟸



* Paths, Cycles, Trees:

For the following definition, assume there are (n) vertices in ($\underline{\underline{V}}$), and ($\underline{\underline{m}}$) edges in (E.)

Path: a path in G is a sequence of vertices $\{i_1, i_2, \dots, i_p\}$ such that

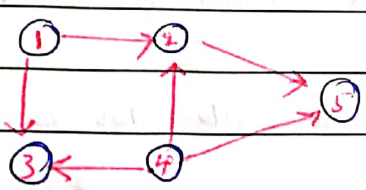
- $i_k \in V \quad \forall k = 1, \dots, p$
- i_k unique $\forall k = 1, \dots, p$ ← no vertex visited more than once.
- (i_k, i_{k+1}) or $(i_{k+1}, i_k) \in E$

Path from 1 to 5:

$\{1, 2, 5\}$ feasible path

$\{1, 2, 4, 5\}$ not feasible path

← Ex:



Directed, not complete

edges مش مرسومة زي ---- و 1,5 و 1,4

$\{1, 3, 5\}$
$\{1, 4, 5\}$ } not paths (مش (1,4) ,(4,3) لانه ما في بين)
$\{1, 2, 4, 3, 1, 2, 5\}$

- Directed path:

Additional condition : $(i_k, i_{k+1}) \in E \;\forall\; k = 1, \dots, p-1$
is a feasible sequence of decisions For users.
according to cost we
decide Which path to go (OR prob. (min cost))

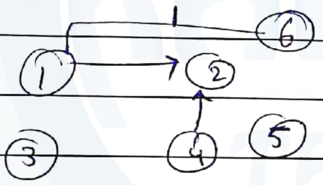
* in previous ex, $\{1, 2, 5\}$ is a directed path ⇒ feasible.

- Related concept : Connectedness

Connected : two vertices i & j are connected in G if
there exists a path from i to j.
↳ regardless direction or feasibility.

- Adjecency : i & j connected by a path of length 1.

Ex:

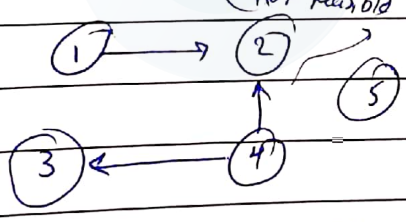


- nodes 1 and 4 are connected
but not adjacent.
- nodes 1 and 2 are adjacent.
so By default connected
- nodes 1 and 6 are adjacent.

- A graph is connected if every pair of vertices
in the graph are connected.

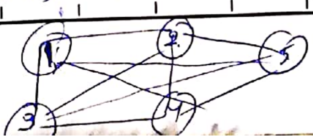
(not feasible) 4 ← 2 in

Ex:



Not connected.
(if 5 connected then path is
connected).

(complete if (1,5) (3,5) → مش path.) Not complete.



- لو 2 nodes اذا ملهمش Path مش connected -

آخر مثال ← (1,4) ملهمش path مثلا ② .

- لو 2 nodes ايا كان بينهم Path يبقى Connected

آخر مثال — (1,4) ملهمش path عشان كدا ④ .

complete = by default connected path.

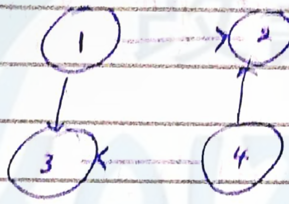
* cycle :

set of edges that form a closed loop in G, you can begin walking at some vertex, follow only edges in the cycle regardless of their direction, then return to original vertex.

path ≠ cycle

← then →

ble or not-feasible



⇒ not feasible cycle.

(لو عايزة اروح من 2 ← 4)

Direction مش مظبوط ←

لو فيه cycle وعكس Direction → not feasible

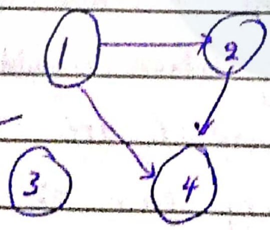
that - cycle is a sequence of vertices $\{i_1, i_2, \ldots, i_p, i_1\}$

1 ⟶ * $i_k \in V \quad \forall k = 1, \ldots, p$ (p: path)

* i_k unique $\forall k = 1, \ldots, p$ (يعني مفيش تكرار)

* (i_k, i_{k+1}) or $(i_{k+1}, i_k) \in E \quad \forall k = 1, \ldots, p-1$

⇒ either (i_p, i_1) or $(i_1, i_p) \in E$ (i.e either feasible or no)



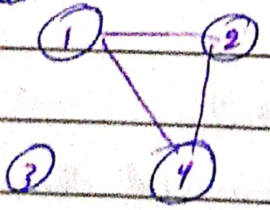
$\{1, 2, 4, 1\}$

a cycle (not feasible), directed

ected

not

jacent



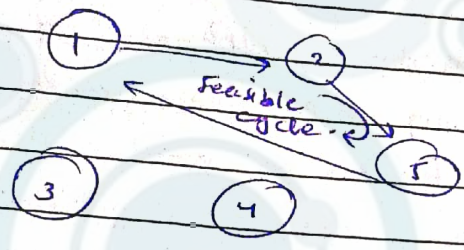
undirected

$\{1, 2, 4, 1\}$

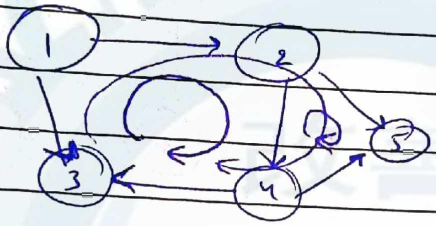
feasible cycle

nodes in undirected cycle by default feasible cycle

Ex :



Ex :



∃ cycle ? none is feasible

1 - 2 - 5 path

2 - 5 - 4 - 3 - 1 - 2 } cycle.

$\boxed{1 - 2 - 5 - 4 - 2.}$ → cycle → قصده

لو حذفت النقطة ← cycle يبقى (2-5-4-2)

1 - 2 - 5 - 1 not cycle (5-1 not have an edge)

→ 5 - 4 - 3 - 1 → path عادي

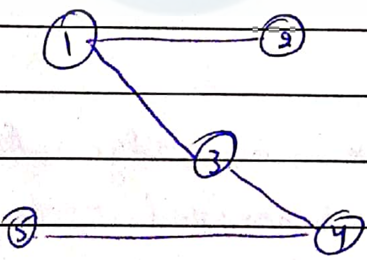
(arc) 5-1 مش

* Tree :

is a Collection of edges Such that vertices are Connected & no cycle exists.

Equivalent def : n vertices in the network. ⇒ A tree is a set of n-1 edges such that all vertices in the network are connected.

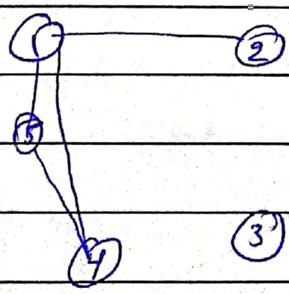
ex :



4 edges , 5 nodes (all connected)

⇒ tree ✓

ex :

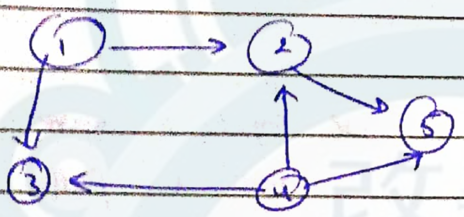


5 nodes & 4 edges but not tree (there is a cycle and (3) not connected)

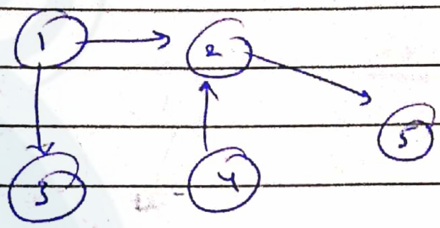
Scanned with CamScanner

- Important property of a tree: defines for each pair of vertices, a unique (not necessarily directed) path between them. (feasible or not)

Ex: Directed network.

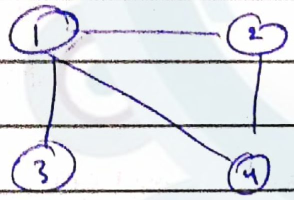


"not tree"
(لانه فيه cycle edges)
nodes آخر سهم

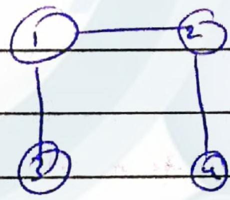


"Directed tree"

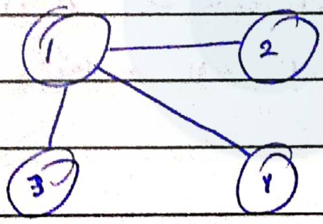
ex: undirected network.



"not tree"



"yes"



"yes"

- what happened if you remove an edge from tree?
some vertex is no Longer Connected → graph is Tree-edge
= unconnected (not connected graph).

- if you add an edge to tree?
creates a cycle → no longer tree.

* minimum cost path models :
many transportation decision models require finding the min cost path(s) between pair(s) of vertices.
⇒ Cost can be time, distance, $, risk.

* Applications:
- personal travel : road network. ⇒ person want to determine what's the minimum time path between origin + destination.
- Global supply chain: Given the network model For a ~~representing~~ shipper representing its full set of transportation services for a particular (type) commodity.
Q: determine the min. cost (time) transport option from origin to destination.

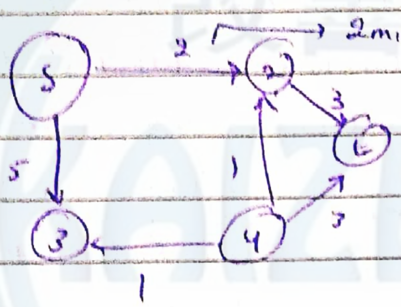
* min cost path prob. variants (times):
- single-pair shortest path problem.
⇒ Find the shortest path between a single source node "S", and destination node "t".
- single - source shortest path problem.
⇒ to find shortest path between single source node "S" and all other vertices, nodes.
- single - destination SP problem:
⇒ to find shortest path from all vertices to a given destination node. "t"
- All-pair shortest path problem "SPP".
⇒ to find shortest path between all pair of vertices.

- Formulation definition & (regardless of types recently defined)
Given a graph $G = (V, E)$ with costs C_{ij} for each $(i,j) \in E$, a source node "s", and a destination node "t", Find the path P from s to t that minimizes "total cost" for the path.

$$\sum_{(k,l) \in P} C_{kl}$$

Ex.

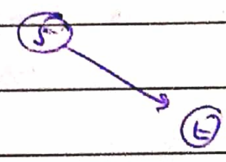
2 → 2mins, 2hrs, 2$, ... (any unit of cost)



* Path from s to t :
{s, 2, t} → cost = 5
{s, 3, 4, t} → cost = 9
{s, 2, 4, t} → cost = 6
etc. (regardless of feasibility)

* Solving minimum cost path problem.
1. Dijkstra's algorithm : Single-pair, Single-source, single-destination.

to find single source shortest path (sp).

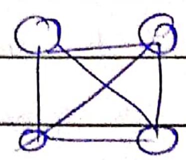


- requires $C_{ij} \geq 0$ (+ve)
(لو كان ال cost (-ve) ما بقدر اطبقها عليها [illegible])
[illegible] revenue ال cost [illegible]

(بنوقف لما نلاقي ال optimum path) ← interest

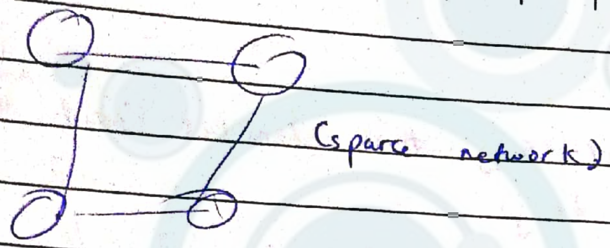
- pros : ① Fast for dense network (m → n) (edges, nodes) (عدد ال nodes)
(Cardinal edges)
② can be truncated. if path of interest has been found.

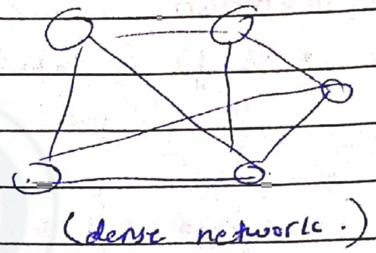
we should write a code to solve this prob on software by this algorithm. → needs programming



(dense network)
edges = 6
nodes = 4 !!

- cons : Alternative methods faster for sparce network.
dense عكس ←





- preparing is needed
  - for efficient implementation.

2. Bellman Ford Algorithm : single-source shortest path, allows (not requires) $c_{ij} \leq 0$, pape's Modification. → performance ↑

*pros :
- Fast for sparce networks
- simple to implement

*Cost if I pay it "loss" (-ve), if I gain it (+ve)

*cons :
- can't be truncated.
- may be slow on dense networks

- route : we should return to source node. (same of cycle)
- path : we should go from source node to destination node..

-

* Dijkstra algorithm :
it's a Labeling algorithm in which, at each ~~iteraction~~ iteraction a different vertex p is given a permanent Label v(j) that indicates the cost of the shortest path from s to p.

$\Rightarrow$ Assumption: $c_{ij} \geq 0 \ \forall (i,j) \in E$ → in terms of Cost.

E: Edges that belongs to network

- Notation :

* $V(j)$: label for vertex j that denotes cost of path from s to j.

* $pred(j)$: predecessor of node j on shortest path from S to j.

* PERM : set of permanently labeled nodes.
  ($\overline{PERM}$ not permanently labeled nodes)
  ↳ : at each iteration one permanent and other not.

- Initialization :

- $V(j) = \begin{cases} 0 & , j = s \\ \infty & , j \neq s \end{cases}$ ⟹ very expensive cost in initialization of program cause I solve minimizing prob.

- $pred(j) = \begin{cases} 0 & , j = s \\ -1 & , j \neq s \end{cases}$
  $\rightarrow$ بداية مسار.
  $\rightarrow$ [illegible] pred v [illegible]

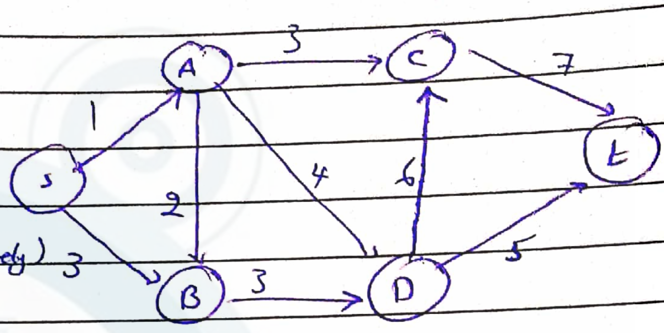
- $PERM = \{\}$, $\overline{PERM} = V$
  ↳ List of permanent nodes. ⟹ empty → [illegible] ولا node في البداية

- Iteration : while $\overline{PERM} \neq \emptyset$ "not empty"

1. Let $p \in \overline{PERM}$ be node for which $v(p) = \min\{v(j), j \in \overline{PERM}\}$
2. $PERM \leftarrow PERM + \{p\}$
3. $\overline{PERM} \leftarrow \overline{PERM} \setminus \{p\}$ "\ : exclude"
4. For all $(p,j) \in E$, $j \in \overline{PERM}$ Leading from p
   if $v(p) + c_{pj} < v(j)$ then
   $v(j) = v(p) + c_{pj}$ "else do not anything"
   $pred(j) = p$

Five Apple

Ex:



v(j) → cost of shortest path from s to j.

pred(j) → node before (immediately) j in path.

v(j) [pred(j)] for each node.

$3 + 0 \overset{?}{\leq} 3 \Leftarrow C_{SB} + v(S) \overset{?}{\leq} v(B)$

iteration	S	A	B	C	D	t	P	PERM	$\overline{PERM}$
initialization	0[0]	∞[-1]	∞[-1]	∞[-1]	∞[-1]	∞[-1]	∞[-1]	{}	S, A, B, C, D, t
1	0[0]	1[S]	3[S]				S	S	A, B, C, D, t
2				4[A]	5[A]		A	S, A	B, C, D, t
3							B	S, A, B	C, D, t

possible nodes to go from S

which node نشتغل عليه؟

أقل cost من B , A

$v(A) + C_{AC} = 1 + 3 = 4 < v(C)$ ✓ ∴ change cost of C to be 4

(A→B) نشوف لو B ... (illegible)

أقل cost بدل PERM

in iteration "3" if I take from B → D, cost = 6 which is greater from A → D so I don't do any thing.