L 17

ch 12

* Rectilinear: straight line motion.
* Distance: S_t: scalar quantity

- Velocity ① $V_{instat} = \frac{ds}{dt}$ $V_{avg} = \frac{\Delta s}{\Delta t}$ $V_{speed} = \frac{S_t}{\Delta t}$ ds a scalar quantity

$a_{avg} = \frac{\Delta V}{\Delta t}$ $a_{instantinuos} = \frac{dv}{dt}$ ② $\longrightarrow$ $\underline{a\,ds = v\,dv}$ important equation. without time.

12.5 prob:

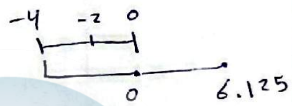
$V = 3t^2 - 6t$ m/s , Located on origin , distanc t traveld when $= 3.5s$

V_{avg} , $V_{sp,avg}$

* $V = \frac{ds}{dt} \longrightarrow ds = v\,dt \longrightarrow \int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$

$s = t^3 - 3t^2$ m

t	0	1	2	3	3.5
s	0	-2	-4	0	6.125

$\rightarrow$ 

* distance t $= \Sigma s = 14.125$ m $\longrightarrow$ we need to check first.

*لازم نتأكد من ال t الي فعلياً غير فيها الجسم اتجاه حركته وعندها بكون V=0

- So when $V = 0$ $t = 2 , 0$ so yes it turned when $t = 2$.

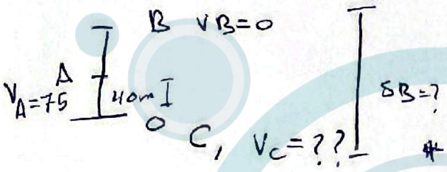
$V_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125}{3.5} = 1.75$ m/s $+\rightarrow$ / $V_{sp/avg} = \frac{14.125}{3.5} = 4.04$ m/s

* accelerations equations: $a_{constant}$

* $V_f = V_i + a_c t$
* $S = S_o + V_o t + \frac{1}{2} a_c t^2$
* $V_f^2 = V_i^2 + 2a_c(S_f - S_i)$ $\rightarrow$ no time in these eqation.

Lit /2.

example 12.4



* no time

Part (A)

* $V_f^2 = V_o^2 + 2a_c(S_f - S_i) \Rightarrow V_B^2 = V_A^2 - 2g(S_B - S_A)$
* $a_c = g = 9.81 \, m/s^2$
  ↳ it will be (−)

$\Rightarrow$

$0 = 75^2 - 2(9.81)(S_B - 40)$

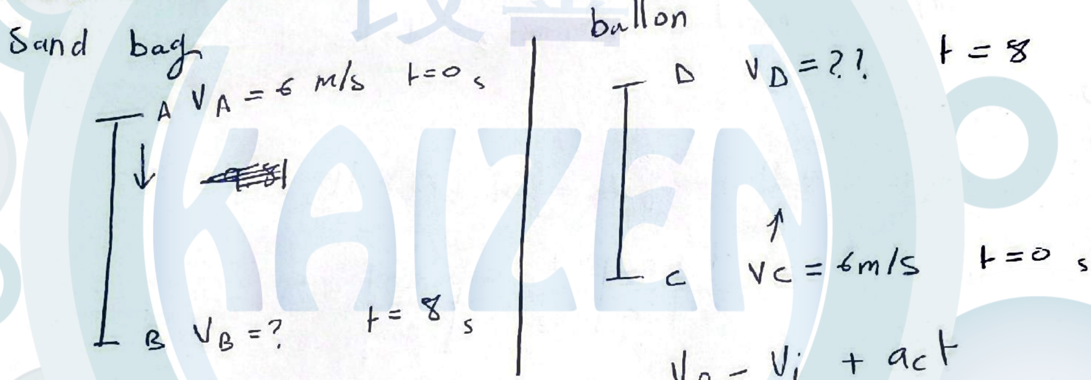
$S_B = 327 \, m.$

Part (B) $\Rightarrow V_c^2 = V_B^2 - 2g(S_c - S_B)$

$= 0 - 2(9.81)(0 - 327)$

$V_c = 80.1 \, m/s \downarrow$

Prob (12-22) * constant spee = acceleration is zero



$V_f = V_i + a_c t$

$V_D = 6 + 8(9.81)$

$= 84.5 \, m/s$

$V_f = V_i + a_c t$

$V_B = +6 - 8(9.81)$

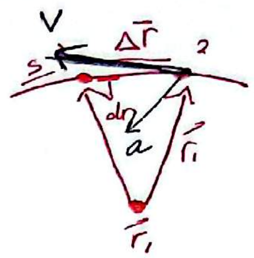
~~84.5 m/s~~

$= 72.5 \, m/s \downarrow$

L 19

12.4

general curvilinear Motion:



$\bar{V}_{avg} = \frac{\Delta \bar{r}}{\Delta t}$, $\bar{V}_{inst} = \frac{d\bar{r}}{dt}$

$a_{inst} = \frac{d\bar{V}}{dt}$

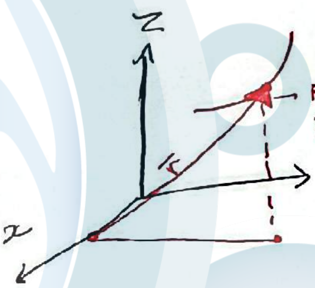
12.5 : Rectangular componants.

12.7 : Normal and Tangential:

12.8 : cylindrical componants. X not included.

* velocity is always tangent to the curve, a is always to inside the curve.

12.5 : Rectangular componants.



position vector
$\bar{r} = (x\hat{i} + y\hat{j} + z\hat{k})$ m

$\bar{V} = \frac{dr}{dt}$

$= (\dot{x}\hat{i} + \dot{y}j + \dot{z}\hat{k})$ m/s

$\dot{x} = \frac{dx}{dt} = \bar{V}_x$

$\dot{y} = \frac{dy}{dt} = \bar{V}_y$

$\dot{z} = \frac{dz}{dt} = \bar{V}_z$

$\bar{a} = \frac{dV}{dt} = \frac{d^2r}{dt^2}$

$a = (\dot{V}_x i + \dot{V}_y j + \dot{V}_z K)$ m/s²

$a = (a_x i + a_y j + a_z K)$ m/s²

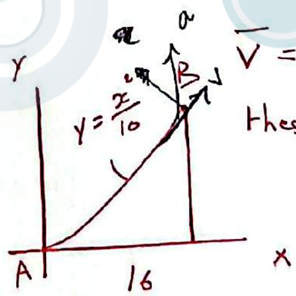
ex - 12.9

$x = 8t$

$y = \frac{x^2}{10}$ we need V, a when t = 2s,

$\bar{V} = (V_x i + V_y j + V_z k)$ m/s

these are Rectangula compon-



* first position vector:

$\bar{r} = (xi + yj)$ ft

$\bar{r} = (8t\, i + \frac{(8t)^2}{10} j)$ ft

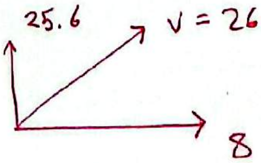
$\bar{V} = \frac{dr}{dt} = (8\hat{i} + 12.8t\,\hat{j})$ ft/s

$\bar{a} = \frac{dv}{dt} = (12.8\, i) = 12.8$ ft/s² $\hat{j}$ $\theta = 90$ دائماً

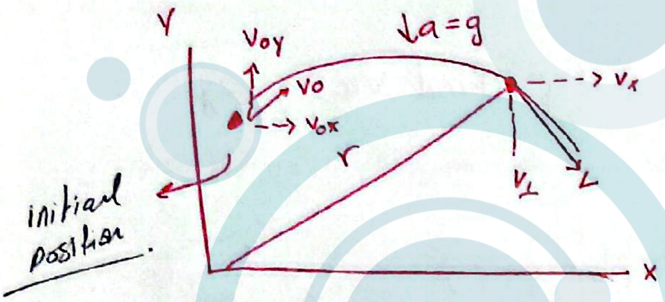
$|\bar{V}| = \sqrt{8^2 + (12.8(2))^2} = 26.8$ ft/s

$v = 26.8$, $\tan^{-1}(\frac{25.6}{8}) = 92.6$

* to find direction



12.6) Motion of projectile.



Range $= x_f - x_o$

*** Horizontal Motion :** $a_c = 0$

$V_f = V_o + a_c t$, $V_x = (V_o)_x$

$x = x_o + V_o t + \frac{1}{2} a_c t$, $x_f = x_o + V_{o_x}(t)$ → $x_f - x_o = (V_o)_x t$

$x_f - x_o =$ Range

Range $= (V_o)_x t$

$V_f^2 = V_o^2 + 2(a_c)(x - x_o)$, $V_x = (V_o)_x$

*** vertical Motion :** $a_c = -9.81$

$V = V_o + a_c t$

$Y = Y_o + V_o t + \frac{1}{2} a_c t^2$

$V^2 = V_o^2 + 2a_c(Y - Y_o)$

exc - 12.11

$V_{oy} = 0$, $V_{ox} = 12$, $t = 0$; | find t and Range

$Y_o = 0$, $y_f = -1m$, $x_o = 0$, $x_f = R$

*** will use this equation**

$Y = Y_o + (V_o)_y t + \frac{1}{2} a_c t^2$, $x = x_o + (V_o)_x t$

find t

find Range

Range $= (V_o)_x t$

* 12-85 Prob

$t = 2.5\,s$

$y_0 = 0 \quad y_f = -1.2\,m \quad x_0 = 0 \quad x_f = 50$ | find v_A, θ_A

* $x_f = x_0 + (v_{0x})t \Rightarrow 50 = 0 + (v_0)_x(2.5) \qquad v_{0x} = 20\,m/s$

* $y_f = y_0 + v_{0y}t + \frac{1}{2}at^2 \Rightarrow -1.2 = v_{0y}(2.5) + (\frac{1}{2})(-9.81)(2.5)^2$

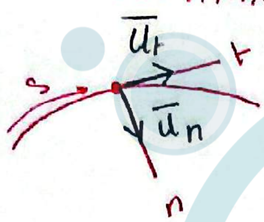
$v_{0y} = \cancel{24.80}\ 12.7$

$v = \cancel{31.85\,m/s}\ 23.7$

$\tan^{-1}\left(\frac{\cancel{24.80}\ 23.7}{20}\right) = 32.41^{\circ}$

L 20 Mech

12.7 - Curvilinear Motion : Normal and Tangtial componant.



t-n : non-fixed cordinate system $\qquad V = \frac{ds}{dt}$

$\bar{V} = \frac{ds}{dt}(\bar{u}_t)$ → * velocity is always tangent to the curve.

$\bar{a} = \frac{d(\bar{V})}{dt} = \frac{d(V\bar{u}_t)}{dt}$ → * use product rule:

$\bar{a} = \frac{dv}{dt}\bar{u}_t + V\frac{d\bar{u}_t}{dt} \Rightarrow \bar{a} = \underbrace{a_t\bar{u}_t}_{\text{change in speed}} + \underbrace{V\dot{u}_t}_{\text{change in direction}}$

$\bar{a}_{tot} = a_t\bar{u}_t + \frac{V^2}{\rho}\bar{u}_n \qquad \rho$ = radius of curve.

$\frac{V^2}{\rho} = a_n$ if $a_n = 0$ only straight line .

12-120 Prob

$a_t = 0.5e^t$, $S = 18$, $\rho = 30$ m

$a = \sqrt{a_t^2 + a_n^2}$

$\int dv = \int 0.5e^t\, dt \Rightarrow v = 0.5(e^t - 1) \Rightarrow v = 19.85$ ✓

$t = 3.7$ s ✓

$ds = v\, dt \Rightarrow \int ds = \int 0.5(e^t - 1)dt \rightarrow \int_0^{18} s =$ ~~[illegible]~~

~~t = 3.5~~

$a_t = 0.5e^t \quad t = 3.7 \rightarrow \boxed{a_t = 20.35\, m/s^2}$ ✓

$a_n = \frac{v^2}{\rho} = 13.14\, m/s^2$ ✓

$a = \sqrt{a_t^2 + a_n^2} = 24.2\, m/s^2$

L 20/2

$$\rho = \frac{\left[\left(1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$,

الأصل ما نستخدمها غير إذا اضطرينا

12-124 Prob)

$Y = 0.4x^2$

$x_A = 2 \quad Y_A = 16$

$V = 8$

$\frac{dv}{dt} = a_t = 4 m/s^2$

$a = \sqrt{a_t^2 + a_n^2}$

$a_n = \frac{V^2}{\rho}$

$$\rho = \frac{\left[\left(1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$Y = 0.4x^2$

$y' = 0.8x = 1.6$

$y'' = 0.8 = 0.8$

$$\rho = \frac{\left[\left(1+(1.6)^2\right)\right]^{3/2}}{0.8} \quad \rho = 8.4$$

$a_n = 7.6 \ m/s^2$

$a = 8.6 \ m/s^2$

12-127 Prob

$V_t = 20 m/s \qquad \rho = 29.62$

$a_{tot} = 14$

$a_n = 14 \sin 75 = 13.5 \ m/s$, $a_t = 14 \cos 75 = 3.6 \ m/s$

$a_n = \frac{V^2}{\rho} \Rightarrow 13.5 = \frac{20^2}{\rho} \Rightarrow \rho = 29.62$

L 21 Mech

Ch16

* Planar Kinematics of a rigid body:

16.1 : ✓

16.2 : Translation

constant coordinate system (y, x), Translating coordinate systems (y', x')

* $r_{B/A} \Rightarrow * \; r_A + r_{B/A} = r_B \rightarrow v = \frac{dr}{dt}$

* $\underbrace{r_{B/A}}_{\text{constant}} = r_B - r_A \rightarrow v_A = v_B \;/\; a_A = a_B$

16.3 : Rotation about a fixed Axis: use Right hand method.

angular velocity $\Rightarrow \omega = \frac{d\theta}{dt}$ rad/s , $\alpha = \frac{d\omega}{dt}$ → acceleration / نفس الاتجاه ; → deceleration / عكس الاتجاه

* constant Angular acceleration

* موجودين بالكتاب

1) $\omega = \omega_0 + a_c t$

2) $\theta = \theta_0 + \omega \ldots$

3) . . . . . .

* Motion of P

$\tan\theta = \frac{ds}{r}$

لو الزاوية صغيرة لان $\theta = \tan$

so $d\theta = \frac{ds}{r} \Rightarrow s = r\theta$

$\frac{ds}{dt} = \frac{r\,d\theta}{dt}$

$\Rightarrow v = r\omega$ → angular velocity

tangent to curve

$\frac{dv}{dt} = a_t = \frac{d\omega r}{dt} = r\frac{d\omega}{dt}$

$a_t = r\alpha$

* 3d :

$\bar{v} = \bar{\omega} \times \bar{r}_P$

* You learn by mistakes. Dr Yazan

L21/2

$$a_n = V^2/R \Rightarrow a_n = \frac{\omega^2 r^2}{r} \Rightarrow a_n = \omega^2 r$$

time rate change in direction.

$$\bar{V} = \bar{\omega} \times \vec{r}_p$$

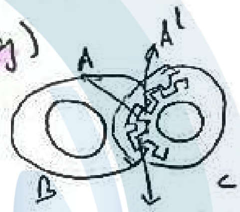
$$\alpha = \frac{d\omega}{dt} \times r_p + \frac{dr_p}{dt} \times \omega \Longrightarrow a_{resultant} = \alpha \times r_p + \omega \times (\omega \times r_p)$$

(تبسيط

$$a = a_{resultant} = \overset{a_t}{\alpha \times r} - \overset{a_n}{\omega^2 r}$$

* r_p: it is any position vector from center to P
if we have angle ($r_p \sin\theta$)

- two rotating bodies: (Meshing)



$$(a_{t_B})_A = (a_{t_C})_{A'}$$

* for tangential acceleration $r\alpha_B = r\alpha_C$

$$(V_B)_A = (V_C)_{A'}$$

$$(\omega r)_B = (\omega r)_C$$

* normal accelerations are not equal

* هيك شرح التشابتر خلص

L22 Mech

F16-6

$\left.\begin{array}{l}\omega_0 A = 0 \\ \omega_0 B = 0\end{array}\right]$ ~~start~~ starting From rest

$\alpha_A = 4.5$

$V_P = \omega_D r_D \Rightarrow \omega_D = \omega_B$

$V_A = V_B$

$\omega_A r_A = \omega_B r_B$

$(13.5)(0.075) = \omega_B (0.225)$

* $\omega_B = 4.5$ rad/s

$\omega_B = \omega_D$

$\omega_A = \omega_{0A} + \alpha_c t$

$\omega_A = 0 + 4.5(3)$

$\omega_A = 13.5$ rad/s

سؤال مهم ربط كل *
انواع الثابتة وقوانينها

$V_P = 4.5 \times 0.125 = 0.562$ m/s

مربوطين بمعنى الأكسس

For $\theta \Rightarrow S = r\theta \Rightarrow S = r\theta_D$, $\theta_D = \theta_B$

$\alpha_A r_A = \alpha_B r_B$

$\alpha_B = 1.5$

$\theta_B = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + 0 + \frac{1}{2}\alpha_B (3)^2 \Rightarrow$

$\theta_B = 6.75$ rad/s

$S = (0.125)(6.75) = 0.844$ m

22/2

Prob 16-20

$\alpha_A = 4t^3$ * not constant acceleration

$(\omega_A)_0 = 20$

v_B at $t=2$ $\Rightarrow$ $\underline{v_B = \omega_B r_B}$ $\Rightarrow$ $(v_B) = (v_A)$

$\omega_B r_B = \underline{\omega_A r_A}$

we can find it first.

$\int_{20}^{\omega} \omega_A = \int \alpha_A = t^4$

$-20 + \omega_A = 16 \Rightarrow \omega_A = 36$

$\omega_B (0.15) = 36 (0.05)$

$\omega_B = 12$ rad/s.

Prob 16-35

$\omega = 14$ rad/s   we need to find V and a for point C

unit vectors W [illegible]

$\bar{v} = \bar{\omega} \times \bar{r}_c$

$\bar{U}_{OA} \Rightarrow \bar{\omega}_{OA}$

$\bar{r}_c$ from origin $= (-0.3i + 0.4j)$

$$\bar{U}_{OA} = \frac{r_{OA}}{|r_{OA}|} = \frac{-0.3i + 0.2j + 0.6k}{0.7} = 0$$

$$\bar{\omega}_{OA} = \left(\frac{-0.3i + 0.2j + 0.6k}{0.7}\right)(14) = (-6i + 4j + 12k)$$

$$\begin{vmatrix} i & j & k \\ -6 & 4 & 12 \\ -0.3 & 0.4 & 0 \end{vmatrix} = -4.8i - 3.6 - 1.2k$$

$$a_c = \underbrace{\overset{\text{zero}}{\alpha_c} \times r_c}_{\text{zero}} + \omega_c \times (\omega_c \times \bar{r}_c)$$

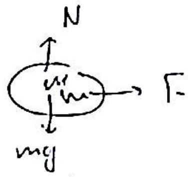
$$= (38.4i - 64.8j + 40.8\hat{k})\ \text{rad/s}^2$$

L23 Mech

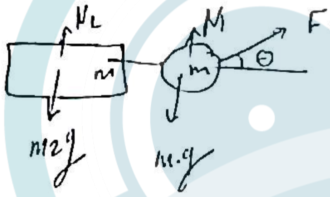
Ch 13: Kinetic

(the Cause of the moment → Force | $\Sigma F = ma$)

13.2 1) system composed of one particle:

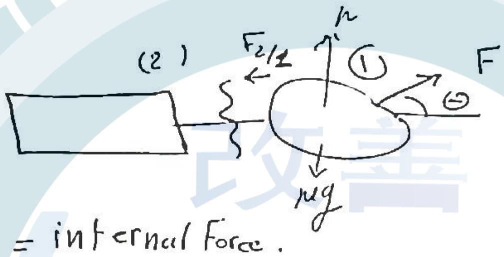


13.3) system composed more than one particle:



$\xrightarrow{+} \Sigma F_x = m_{total}\, a_x$

$F\cos\theta = (m_1 + m_2)\, a_x$



$\xrightarrow{+} \Sigma F_x = m a_x$

$F\cos\theta - F_{2/1} = m_1 (a_x)$

$F_{2/1}$ = internal Force.

13.4) equations of Motion: Rectangular coordinates.

1) FBD (example) 13.2

$+\uparrow \Sigma F_y = m a_y$

$-mg = m a_y \Rightarrow (10)(-9.81) = 10\, a_y$

$a_y = -9.81\ m/s^2$

$V_0 = 50\ m/s$ $\quad V_f = 0\ m/s$ (use constant acceleration equation)

$V_f^2 = V_0^2 + 2a(\Delta S)$

$0 = 50^2 + 2(-9.81)\,\Delta S \rightarrow (S - 0)$

$S = 127\ m$ (height)

$S = 113.5\ m$ أقل من الارتفاع الي قبل بسبب وجود F_D نقل الارتفاع

b) $+\uparrow \Sigma F_y = m a_y$

$-mg - F_D = m a_y$

$-10(9.81) - 0.01V^2 = 10\, a_y \Rightarrow a_y = -(0.001V^2 + 9.81)\ m/s^2$

When we don't have (t) $\rightarrow a\,ds = V\,dv \Rightarrow \int_0^s ds = \int_{50}^{0} \frac{V}{a}\,dv \Rightarrow \int$

Prob 13-26:

$1.5\,Mg \rightarrow 1500\,kg$

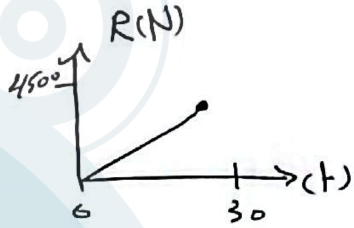
$F = 4.5\,kN$

$\xrightarrow{+} F_x = m a_x$

$F - R = m a_x$

$4{,}500 - R = 1500 \times a_x$

$R = (150t)\,N \Rightarrow = \underline{4500\,N}$



$V = (-0.05t^2 + 3t)\ m/s$

$a = \frac{dv}{dt} = -0.1t + 3\ m/s \quad , t = 30$

$a = -0.1(30) + 3 \Rightarrow a = 0$

Mech L26

before this Lecture go back and solve Prob 13-5 and 13-4 they were mentioned in the previous Lecture L25 : ملاحظة

ex 13.6

$\sum F_a = ma$ $\quad N\sin\theta = ma_n \longrightarrow N\sin\theta = m\,v^2/\rho$ (1)

$+\uparrow \sum F_b \Rightarrow N\cos\theta - mg = 0 \qquad N\cos\theta = mg$ (2)

divided one by 2

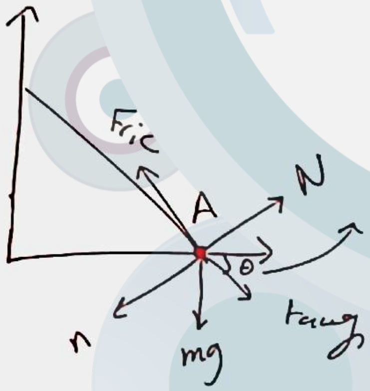
$1/2 = \tan\theta = v^2/\rho g \qquad \theta = \tan^{-1}(v^2/\rho g)$ كل ما زادت السرعة بتزيد θ

Prob 13-69

$a_t = 3\,m/s^2 \quad m = 0.8 \quad m = 800\,kg \quad v_f = 9\,m/s$

$Y = 20\left(1 - \frac{x^2}{6400}\right)$

We can know the angle from the slope.



$\theta = 26.57$

$\frac{dy}{dx} = -0.00\,63\,x \quad x = 80$

$= -0.5$

$\tan^{-1}(-0.5) = 26.570$

$+\swarrow \sum F_n = ma_n \longrightarrow -N + mg\cos\theta = m\,v^2/\rho$

$-N + 800(9.81)\cos 26.57 = \frac{800\,a}{223.6}$

$N = 6.729\ kN$

$\searrow^{+} \sum F_t = ma_t$

$-Fric + mg\sin\theta = ma_t$

$-Fric + 800(9.81)(\sin 26.57) = 800(3)$

$Fric = 1.11\ kN$

* We need to find ρ by this equation:

$$\rho\ \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{d^2y/dx^2} = 223.6$$

$Y = 20\left(1 - \frac{x^2}{400}\right) \longleftarrow$

Mech L27

Chapter 14) Kinetics of a particle : work and Energy.

$$du = F\,ds\cos\theta = \bar{F}\cdot d\bar{r}$$ $\theta = 90$, the work is zero.

* $U_{1-2} = \int_{s_1}^{s_2} F\cos\theta\, ds$ → θ is constant → rectelinear. → θ is not constant → curvelinear.

- Work of a constant force , θ is constant:

* $U_{1-2} = F_c\cos\theta\,(S_2 - S_1)$

- Work of a weight :

$U_{1-2} = -W\Delta y$ → because we care of vertical distance only.

$W = mg$

- Work of spring Force :

$U_{1-2} = -\left(\frac{1}{2}kS_2^2 - \frac{1}{2}kS_1^2\right)$, it is usually negative.

k = spring steffness $\quad U_{1-2} = \frac{1}{2}k\,(S_1^2 - S_2^2)$ * نفس القانون بس بسطناه

14.2) principle of work and energy :

* $U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$, $\frac{1}{2}mV^2 = T_{\text{kinetic energy}}$.

* $U_{1-2} = T_2 - T_1$

* $T_1 + \sum U_{1-2} = T_2$

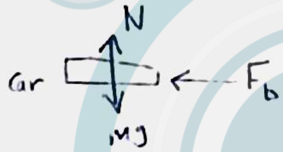
Mech L28

14-2 prob

we will change the unit put the same solution.

$F = [800 (10)^3 x^{1/2}]\,N$, mass $= 2\,Mg$, $V_0 = 20\,m/s$

stop $\rightarrow T_2 = $ zero, $V_f = $ zero



$T_1 + \sum U_{1-2} = T_2$

$T_1 + \sum U_{1-2} = $ zero

$(\frac{1}{2})(2000)(20)^2 + \int_0^x 800 \cdot 10^3 x^{\frac{1}{2}} \cos 18\, dx = $ zero .

$x = 0.825\,m$

14-38 prob

mass $= 60\,kg$

$V_1 = 5\,m/s$

we need to find V_f , Normal force at point B

Sol :

$T_1 + \sum U = T_2$, we have only work of weight

* on B , $x = 0$

So $y = 5\,m$

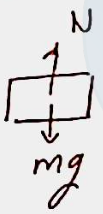
$\Delta y = 15 - 5 = 10$

(+) لأنها نازل

$(\frac{1}{2})(60)(5)^2 + mg\,\Delta y = \frac{1}{2}\, 60 (V_f)^2$

$(\frac{1}{2})(60)(5)^2 + (60)(9.81)(10) = (\frac{1}{2})(60) V_f^2$

$V_f = 14.87\,m/s$ ✓



$+\uparrow \sum F \Rightarrow N - mg = m\,a_n$

$N - mg = m\,V^2/\rho$

$\rho = \frac{(1 + (\frac{dy}{dx})^2)^{3/2}}{d^2y/dx^2}$, at $x = 0$ in point B

$\rho = 20\,m$

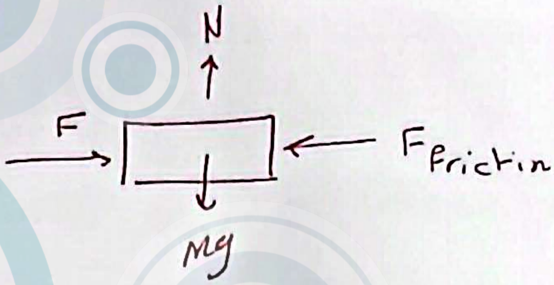
$N - (60)(9.81) = \frac{60 \cdot 14.87^2}{20}$

$N = 1250\,N$ ✓

L28/2

Prob 14-10

mass $= 20\,kg$ **finds,**



$v_f = 15$

$v_i = 6 \quad s = 0$

$\mu_k = 0.3$

solution:

$F_{Fric} = \mu_k N = 0.3 \times 196.2 = 58.86\,N$

$N - mg = 0 \qquad N = 196.2\,N$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\left(\tfrac{1}{2}\right)(20)(6)^2 + \int_0^s 50 s^{1/2}\, ds - 58.86 \cdot s = \tfrac{1}{2}(20)(15)^2$$

$\cos 0 = 1$

$s = 20.52\,m$ ✓

Done ✓