by Eng. Zena Qasem

Linear Congruential Method

$$X_{i+1} = (aX_i + c) \bmod m,$$

Exponential Distribution

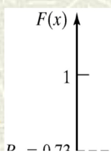
$$X_i = F^{-1}(R_i) = -\frac{1}{\lambda}\ln(1-R_i)$$

$= -\mu \ln(1-R_i)$

Uniform Distribution [UN(a,b)] (X = a + (b-a)R)

An Empirical Discrete Distribution

- p(0) = P(X=0) = 0.50
- p(1) = P(X=1) = 0.30
- p(2) = P(X=2) = 0.20 → عشرين % من الحالات



tive

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 1 \\ 0.8, & 1 \le x < 2 \\ 1.0, & 2 \le x \end{cases}$$

A Discrete Uniform Distribution

$$X = roundup(kR)$$

Direct Transformations

$$Z_1 = \sqrt{-2\ln R_1}\cos(2\pi R_2)$$

$$Z_2 = \sqrt{-2\ln R_1}\sin(2\pi R_2)$$

$X_i = \mu + \sigma Z_i$

1)

Linear Congruential Method

مطلوب ال Simple Method

[Techniques]

$$X_{i+1} = (aX_i + c) \bmod m,$$

- Use $X_0 = 27$, $a = 17$, $c = 43$, and $m = 100$.
- The X_i and R_i values are:

$X_1 = (\overset{a*X+c}{17*27+43}) \bmod 100 = 502 \bmod 100 = 2, \quad R_1 = 0.02;$

$X_2 = (17*2+43) \bmod 100 = 77, \quad R_2 = 0.77;$

$X_3 = (17*77+43) \bmod 100 = 52, \quad R_3 = 0.52;$

...

$\frac{2}{m} = \frac{2}{100}$

$X_0 = 27 \quad a = 17 \quad c = 43 \quad mod = 100$

$X_1 = A \times X_0 + C \bmod 100$

$X_1 = 17 \times 27 + 43 \bmod 100$

$502 \bmod 100$

$5(100) + 5$

$x_2 = 17 \times 2 + 43 \mod 100$

$0(100) + 77$

$x_3 = 17 \times 77 + 43 \mod 100$

1352

$13(100) + 52$

mod → reminder of division

so $13 \div 5 = \underline{2}.6$

$2 \times 5 = 10$

$\boxed{\text{3 left}}$ (باقي قسمة)

2)

Random-Variate Generation

عليه ٦ أو ٧ علامات بالفاينل

1 Exponential Distribution

$$X_i = F^{-1}(R_i) = -\frac{1}{\lambda}\ln(1-R_i)$$

R_i is number I Generated

ex: 0.02

$R_2 = 0.77$

if $\lambda = 0.5$

$$x_i = -\frac{1}{\lambda}\ln(1-R_i)$$

$$= \frac{-1}{0.5}\ln(1-0.77)$$

$$= -\frac{1}{0.5}\ln 0.23$$

$$= \frac{-1}{0.5} \times -1.469$$

$$= 2.939$$

$= -\mu\ln(1-R_i)$

2 Uniform Distribution [UN(a,b)] $(X = a + (b\text{-}a)R)$

$R_2 = 0.77$

$X = a + (b - a) R$

if $a = 10$

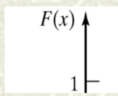
$b = 20$

$X = 10 (10 \times 0.77)$

$= 17.7$

3 An Empirical Discrete Distribution

- $p(0) = P(X=0) = 0.50$
- $p(1) = P(X=1) = 0.30$
- $p(2) = P(X=2) = 0.20$



$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 1 \\ 0.8, & 1 \le x < 2 \\ 1.0, & 2 \le x \end{cases}$$

commulative

$R_1 : 0.02$

$R_2 : 0.77$

$R_3 : 0.52$

$x_1 = 0$

$x_2 = 1$

$x_3 = 1$

4 A Discrete Uniform Distribution

$$p(x) = \frac{1}{k}, \quad x = 1, 2, \ldots, k$$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{i}{k}, & i \le x < i+1, \ for \ \ i \in [1, k-1] \\ 1, & k \le x \end{cases}$$

$$if \quad \frac{i-1}{k} < R \le \frac{i}{k} \quad \Rightarrow \quad X = i$$

$$or \quad X = roundup(kR)$$

if $k=10$

$10 \times 0.02 = 0.2 \simeq 1$

$10 \times 0.52 = 5.2 \simeq 6$

$10 \times .77 = 7.7 \simeq 8$

$X_1 = 1$

$X_2 = 8$

$X_3 = 6$

k: number of possible outcome

Direct Transformations

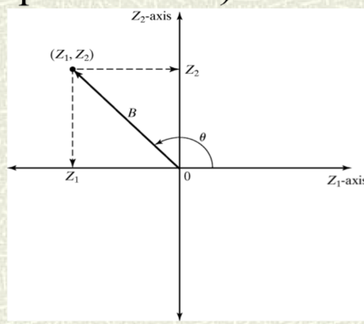
$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dx$$

- Consider two normal variables Z_1 and Z_2

$$B^2 = Z_1^2 + Z_2^2 \sim \boxed{Chi-square} \text{ with two degrees of freedom}$$

(Exponential with parameter 2)

$$\theta = \tan^{-1}\left(\frac{Z_1}{Z_2}\right) \sim Uniform[0, 2\pi]$$



الطول → $B = \sqrt{-2\ln R}$

they are two Random Normally distributed

$$Z_1 = \sqrt{-2\ln R_1}\cos(2\pi R_2)$$

$$Z_2 = \sqrt{-2\ln R_1}\sin(2\pi R_2)$$

$Z_1 = \sqrt{-2\ln R_1} \cos(2\pi R_2$

if $R_1 = 0.02$ $\quad R_2 = 0.77$

$\sqrt{-2\ln R_1} = 2.797$

→ rad (WTF

$Z_1 = 2.797 \cos(2\pi R_2)$ $\quad 2\pi = 360°$

$= 2.797 \times 0.125 = 0.3506$

$x = z$

if $\mu = 0$ $\sigma = 1$ we stop here

if not : $X_i = \mu + \sigma Z_i$

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If the first 3 random numbers are R1: 0.1007, 0.9003, 0.21 R2: 0.5731, 0.3412, 0.7771 generate the first 3 standard normal numbers, then generate the first n(7,1)=sigma*Z+mean=1*Z+7.

1- standard Normal :

we have 3 R so 6 pairs

the equation only requires 3

$Z_1 = \sqrt{-2 \ln R_1} \cos(360 R_2)$

$Z_2 = \sqrt{-2 \ln R_1} \sin(360 R_2)$

$Z_3 = \sqrt{-2 \ln R_1} \cos(360 R_3)$

Final

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Evaluate the first 3 random numbers R, R1, R2, R3 using linear congruential method if X0 =10, M=103, a=13 c =17

$X_0 = 10$ $M = 103$ $a = 13$ $c = 17$

$X_1 = a X_0 + C \mod M$

$= 13(10) + 17 \mod 103$

$147 \mod 103$

$= 44$

$R_1 = \frac{44}{103} = 0.427$

$X_2 = 13(44) + 17 \mod 103$