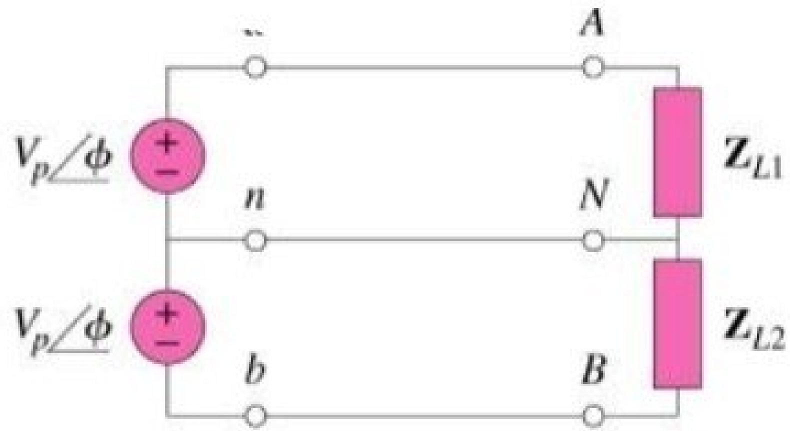
Single-Phase, Two-Phase, Three phase Circuits

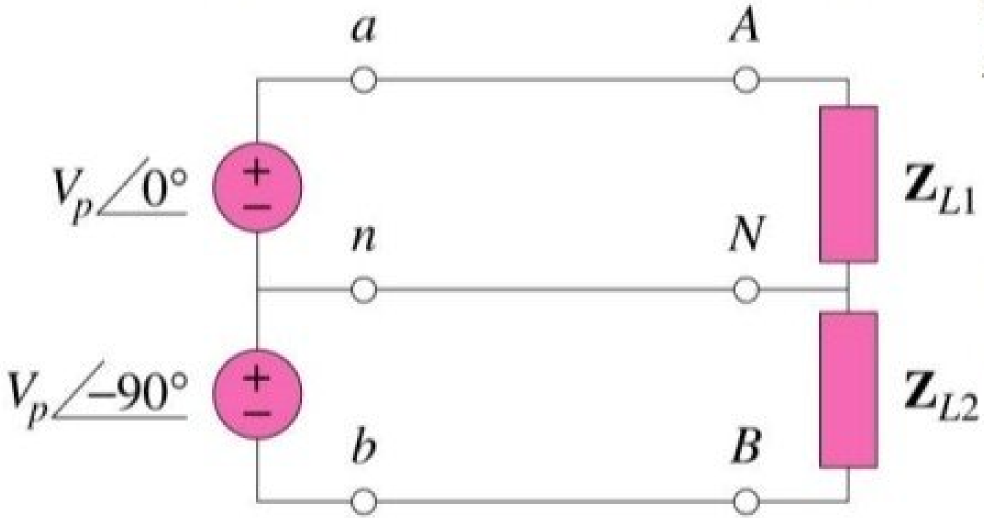
(a)



(b)

a) Single phase systems two-wire type

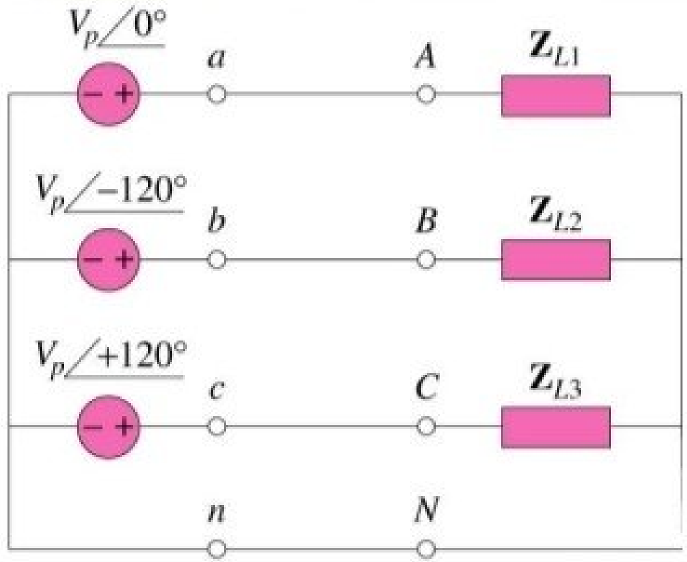
b) Single phase systems three-wire type. Allows connection to both 120 V and 240 V.



Two-phase three-wire system.

The AC sources operate at different phases.

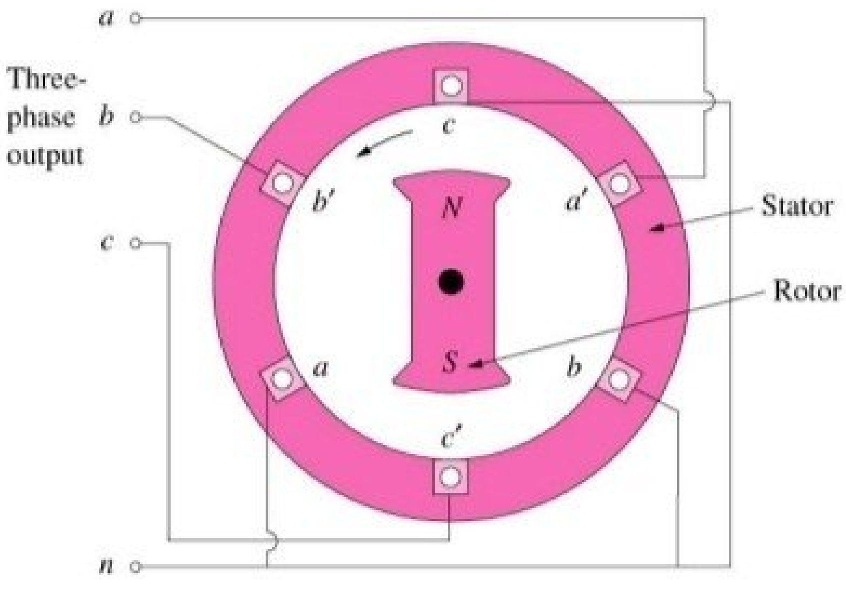
Balanced Three-phase Voltages



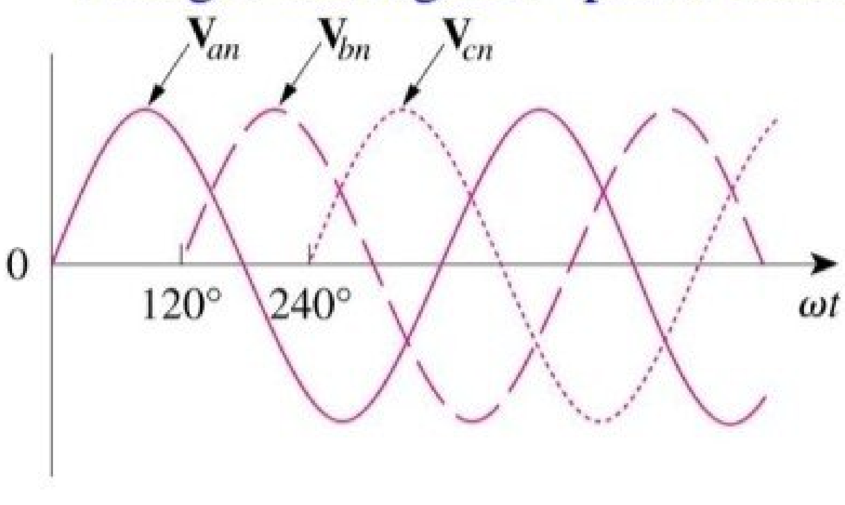
Three-phase four-wire system

Neutral Wire

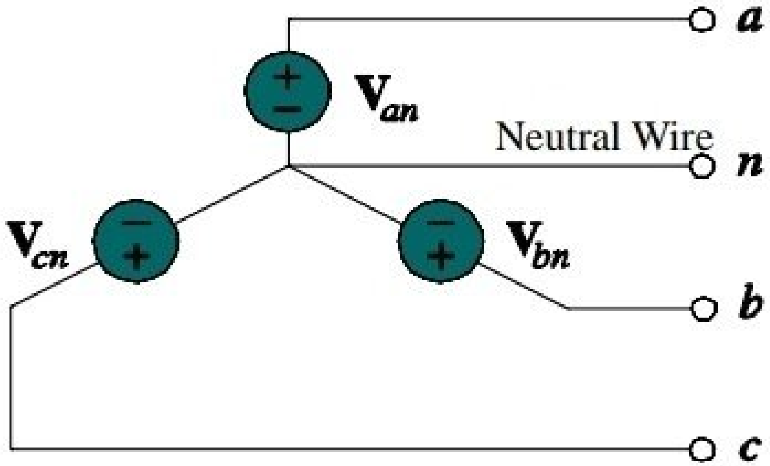
A Three-phase Generator



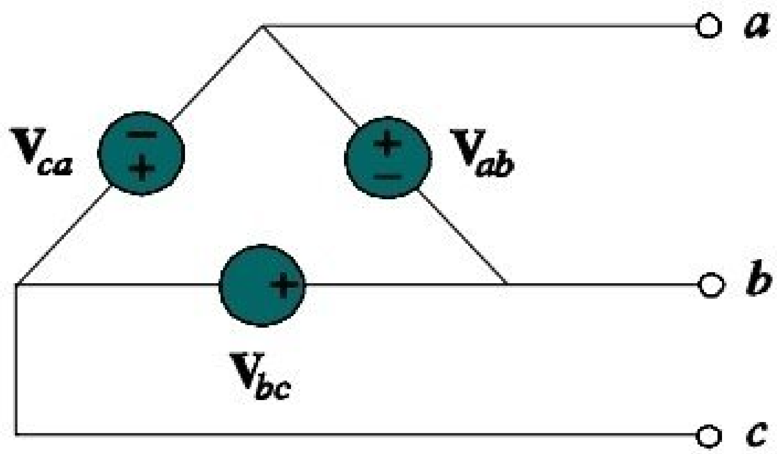
Voltages having 120° phase difference



Balanced Three phase Voltages

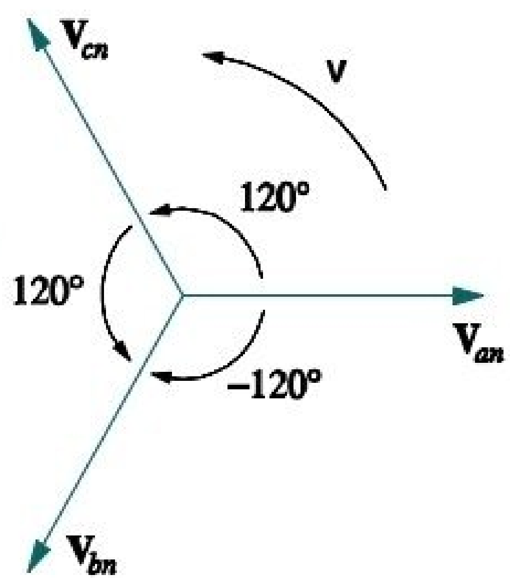


a) Wye Connected Source

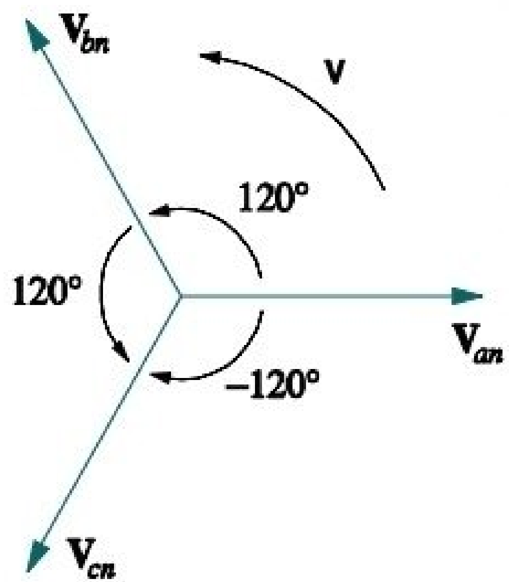


b) Delta Connected Source

$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle -120°$$
$$V_{cn} = V_p \angle -240°$$

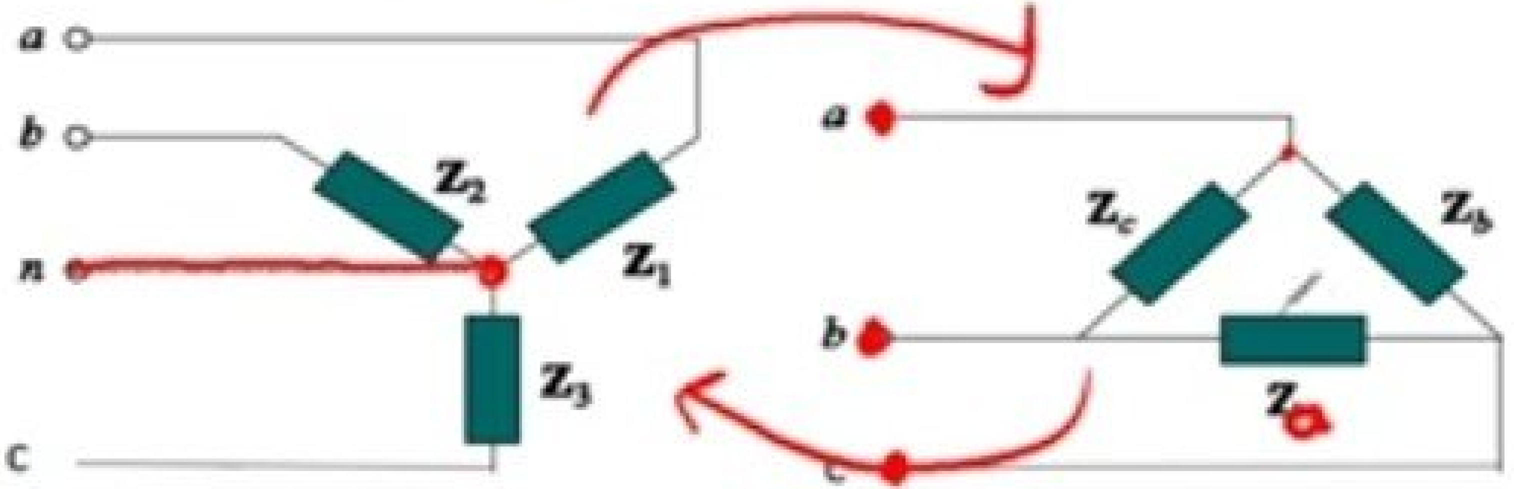


a) abc or positive sequence



$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle +120°$$
$$V_{cn} = V_p \angle +240°$$

b) acb or negative sequence



a) Wye-connected load b) Delta-connected load

Balanced Impedance Conversion:

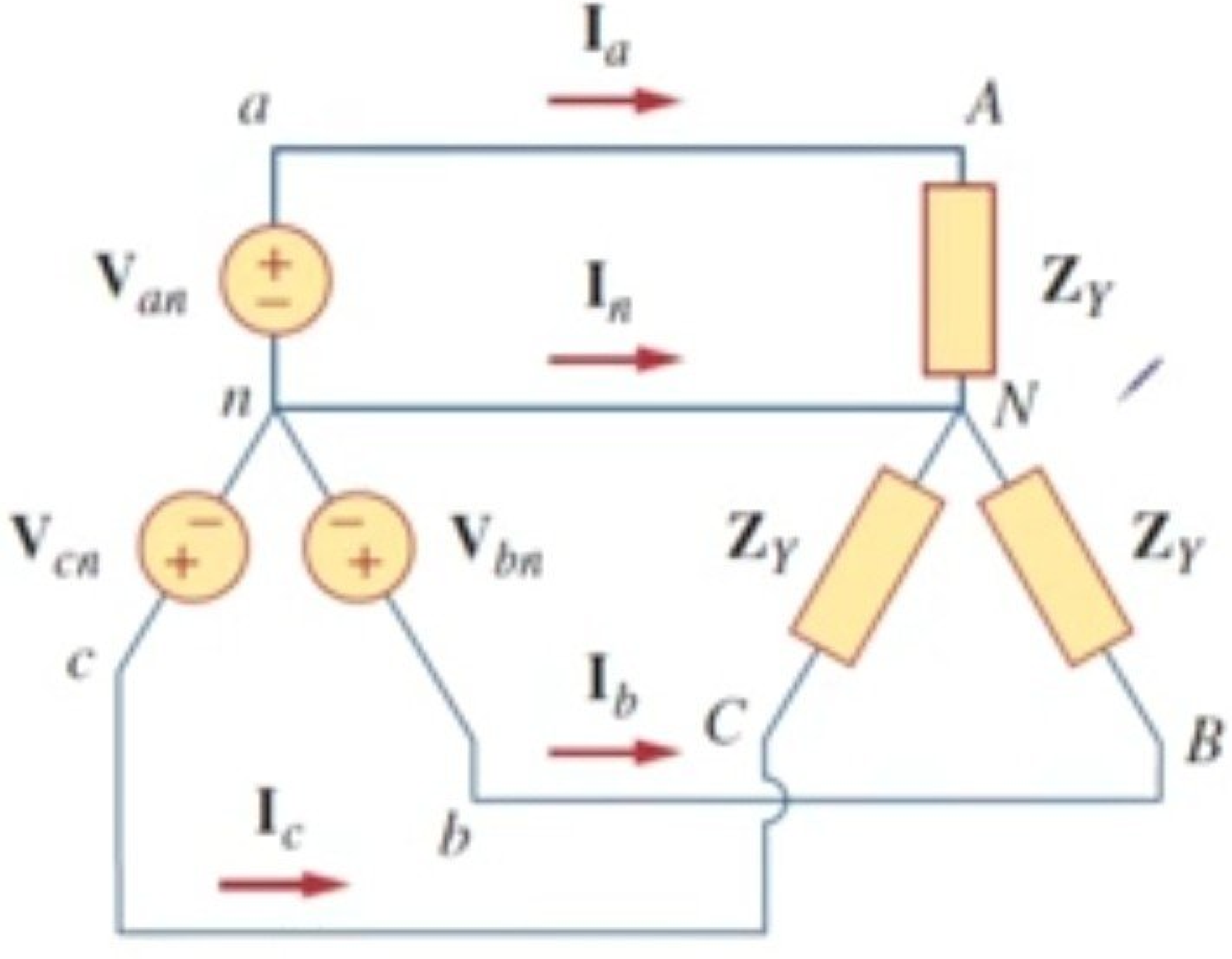
Conversion of Delta circuit to Wye or Wye to Delta.

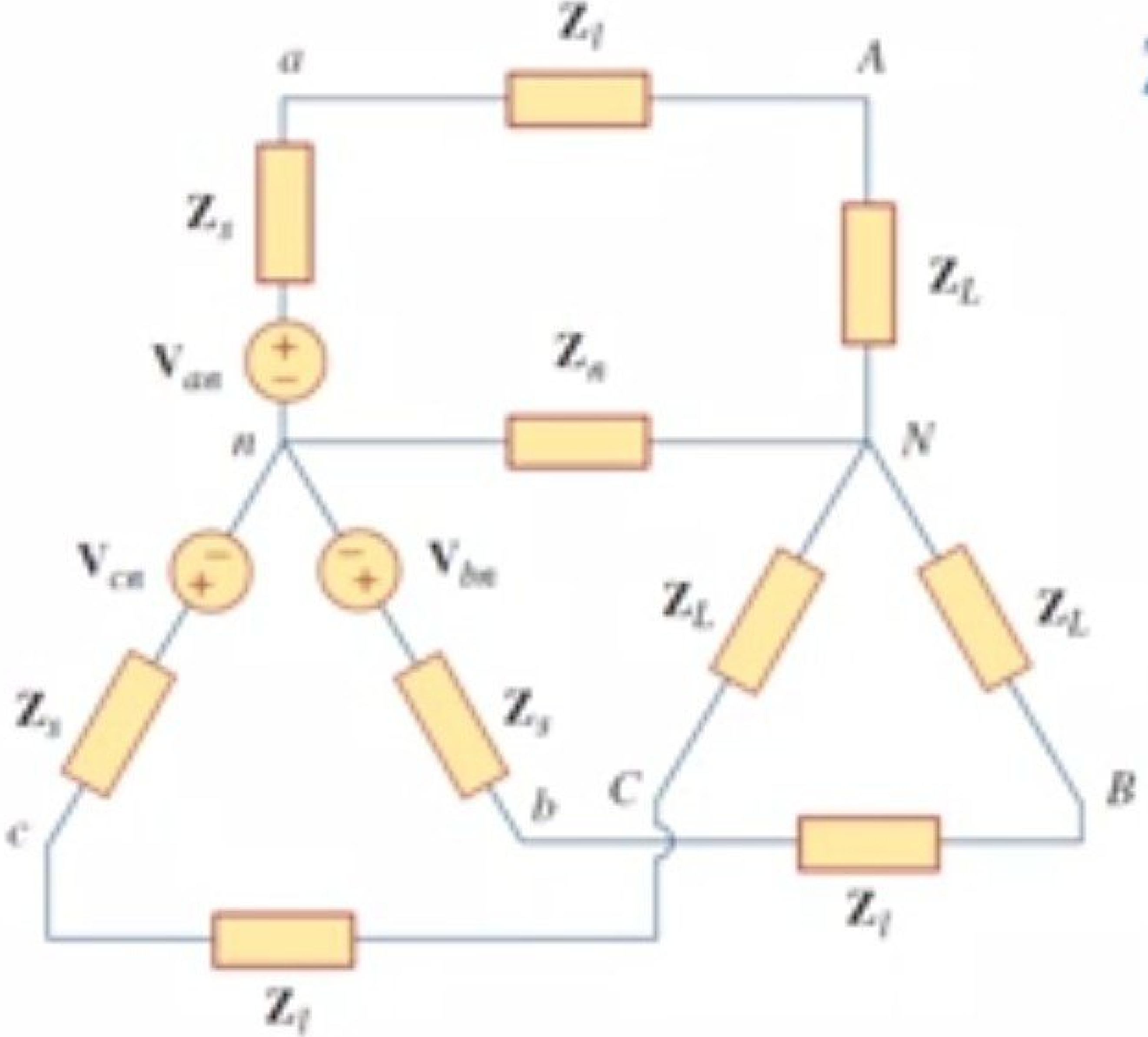
$$Z_Y = Z_1 = Z_2 = Z_3$$

$$Z_\Delta = Z_a = Z_b = Z_c$$

$$Z_\Delta = 3Z_Y \qquad Z_Y = \frac{1}{3} Z_\Delta$$

BALANCED Y-Y CONNECTION

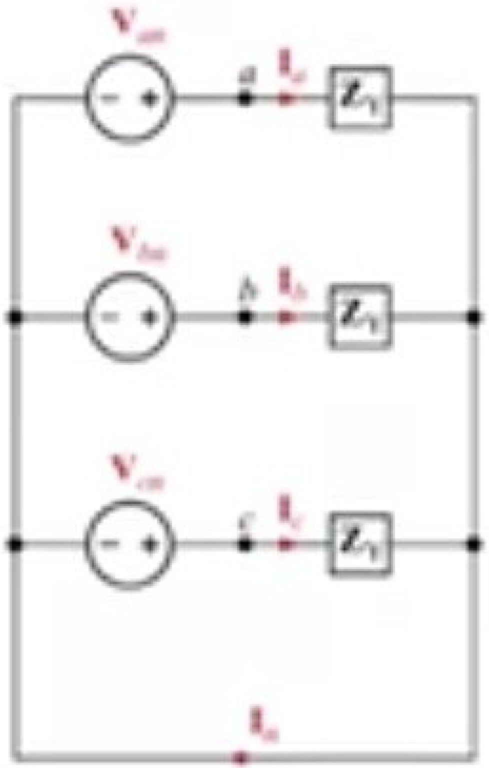




$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

Example: For an abc sequence, balanced Y-Y three phase circuit $V_{ab} = 208\angle -30^{\circ}$

Determine the phase voltages.



Balanced Y - Y

The phasor diagram could be rotated by any angle

$$V_{ab} = 208\angle -30^{\circ}$$

$$\therefore V_{an} = \frac{|V_{ab}|}{\sqrt{3}}\angle(-30^{\circ} - 30^{\circ})$$

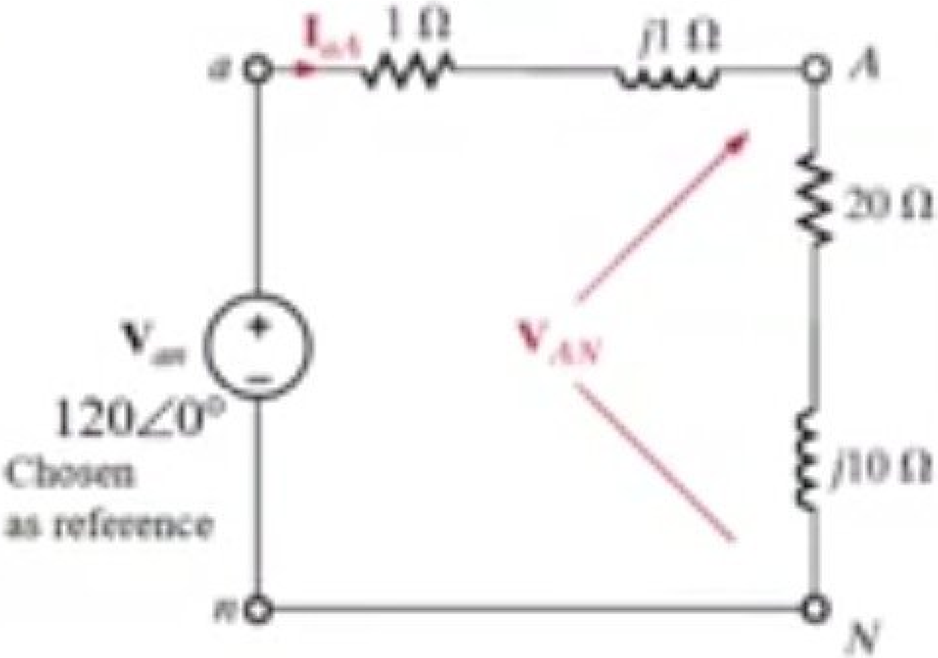
$$V_{an} = 120\angle -60^{\circ}$$

$$V_{bn} = 120\angle -180^{\circ}$$

$$V_{an} = 120\angle 60^{\circ}$$

source $|V_{phase}| = 120(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 20 + j10\Omega$

Determine line currents and load voltages.



$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

Because circuit is balanced data on any one phase are sufficient

$$I_{bB} = 5.06\angle -120 - 27.65°(A)rms$$

$$I_{cC} = 5.06\angle 120 - 27.65°(A)rms$$

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$V_{BN} = 113.15\angle -121.08°(V)rms$$

$$V_{CN} = 113.15\angle 118.92°(V)rms$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{cn} = 120\angle 120°$$

Abc sequence

Example: For an abc sequence, balanced Y - Y three phase circuit

$V_{an} = 120\angle 90^{\circ}(V)rms$. Find the line voltages

V_{ab} leads V_{an} by 30°

$$V_{ab} = \sqrt{3}\times 120\angle 120^{\circ}(V)rms$$

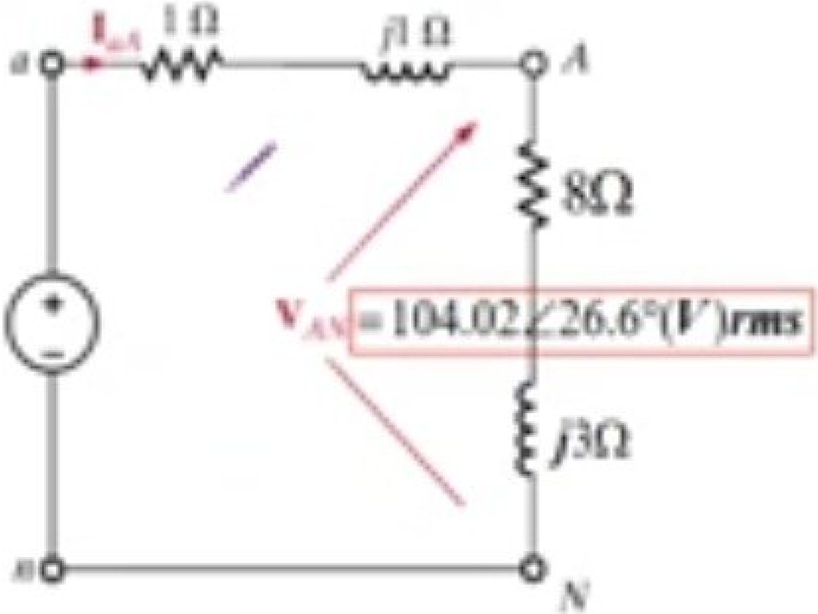
$$V_{bc} = \sqrt{3}\times 120\angle 0^{\circ}(V)rms$$

$$V_{ca} = \sqrt{3}\times 120\angle 240^{\circ}(V)rms$$

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

load, $|V_{phase}| = 104.02\angle 26.6°(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 8 + j3\Omega$

Determine source phase voltages



Currents are not required. Use inverse voltage divider

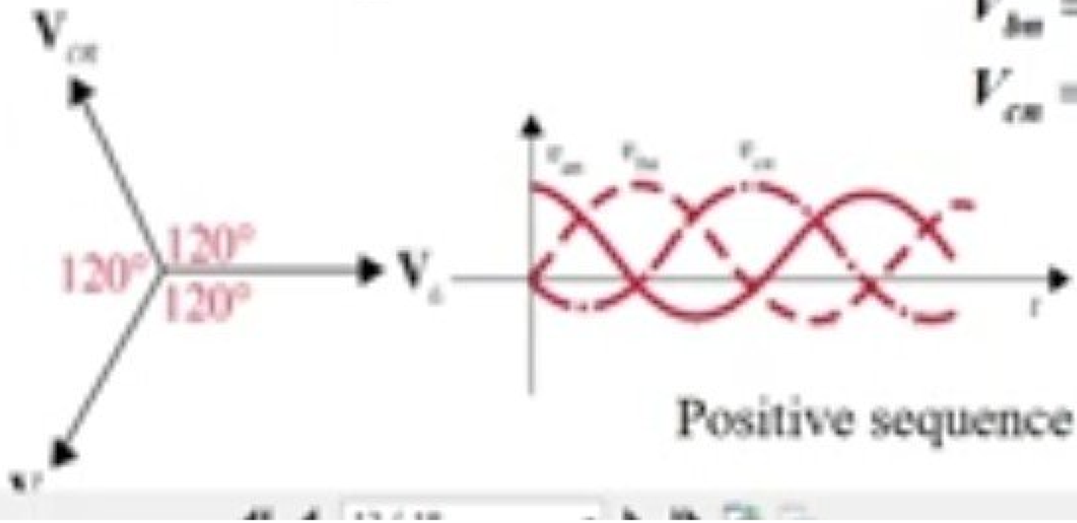
$$V_{an} = \frac{(8 + j3) + (1 + j1)}{8 + j3} V_{AN}$$

$$\frac{9 + j4}{8 + j3} \times \frac{8 - j3}{8 - j3} = \frac{84 + j5}{73} = 1.15\angle 3.41°$$

$$V_{an} = 120\angle 30°$$

$$V_{bn} = 120\angle -90°$$

$$V_{cn} = 120\angle 150°$$

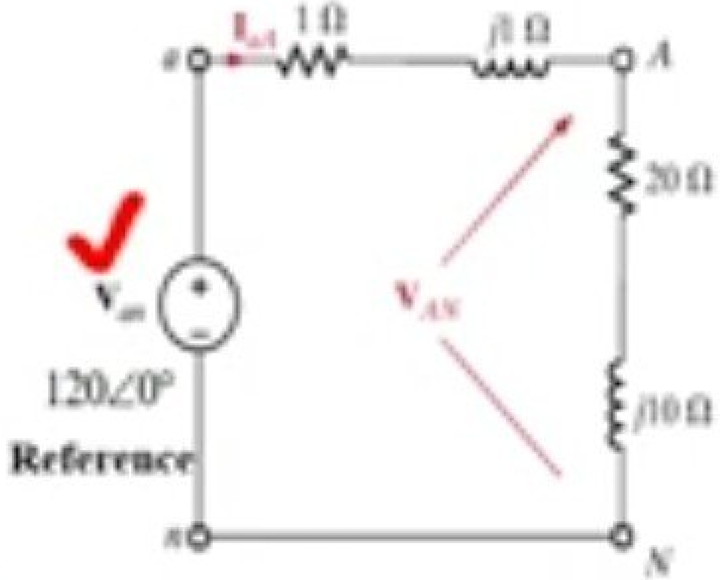


Positive sequence a-b-c

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

$$\text{source } |V_{phase}| = 120(V)_{rms},\ Z_{line} = 1 + j1\Omega,\ Z_{phase} = 20 + j10\Omega$$

Determine real and reactive power per phase at the load and total real, reactive and complex power at the source



$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$S_{phase} = V_{AN} I_{aA}^* = 113.15\angle -1.08° \times 5.06\angle 27.65°$$

$$S_{phase} = 572.54\angle 26.57° = 512 + j256.09(VA)rms$$

$$S_{\text{source phase}} = V_{an} \times I_{aA}^* = 120\angle 0° \times 5.06\angle 27.65°$$

$$S_{\text{source phase}} = 607.2\angle 27.65°$$

$$= 537.86 + j281.78VA$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{cn} = 120\angle 120°$$

Because circuit is balanced data on any one phase are sufficient

Abc sequence

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

$$P_{\text{total source}} = 3 \times 537.86(W)$$

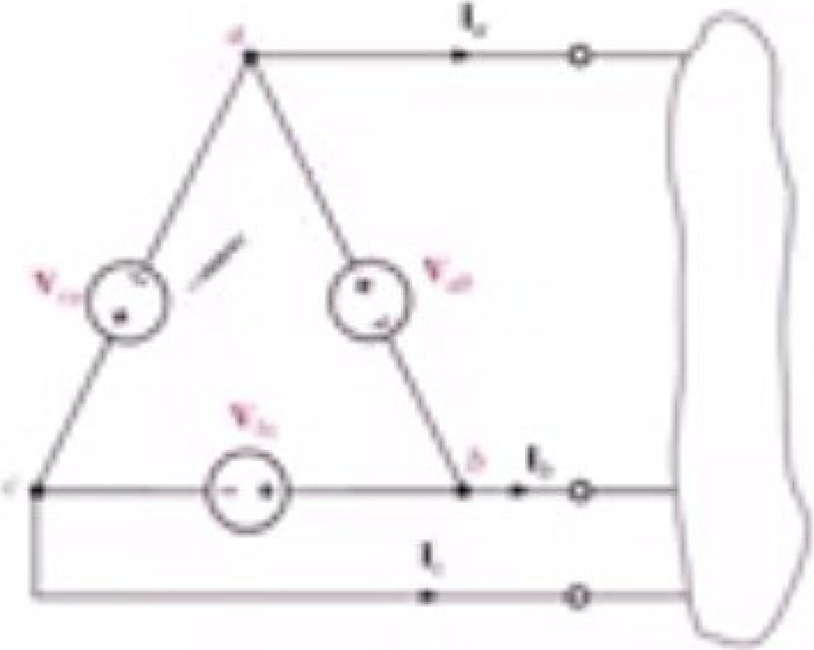
$$Q_{\text{total source}} = 3 \times 281.78(VA)$$

$$S_{\text{total source}} = P_{\text{total source}} + Q_{\text{total source}}$$

$$= 1613.6 + j845.2(VA)$$

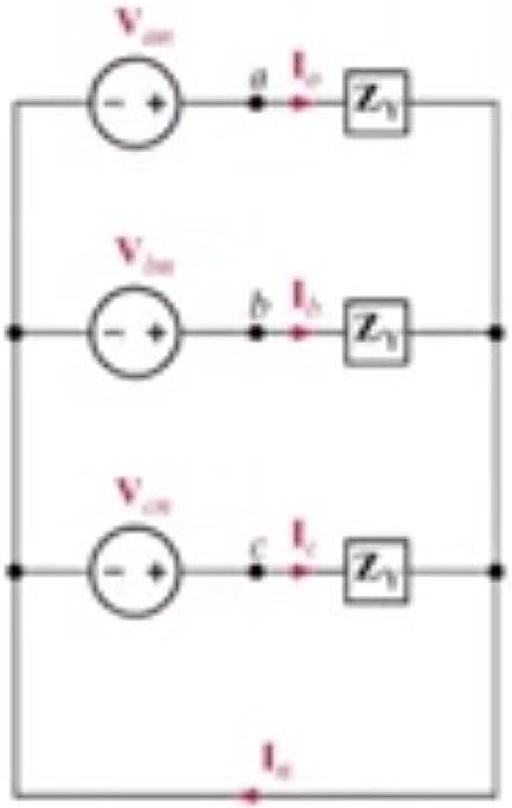
$$|S_{\text{total source}}| = 1821.6(VA)$$

DELTA CONNECTED SOURCES


(a)

Line voltages = phase voltages

$$\left.\begin{aligned} V_{ab} &= V_L\angle 0^\circ \\ V_{bc} &= V_L\angle -120^\circ \\ V_{ca} &= V_L\angle 120^\circ \end{aligned}\right\} \Rightarrow$$



Balanced Y - Y

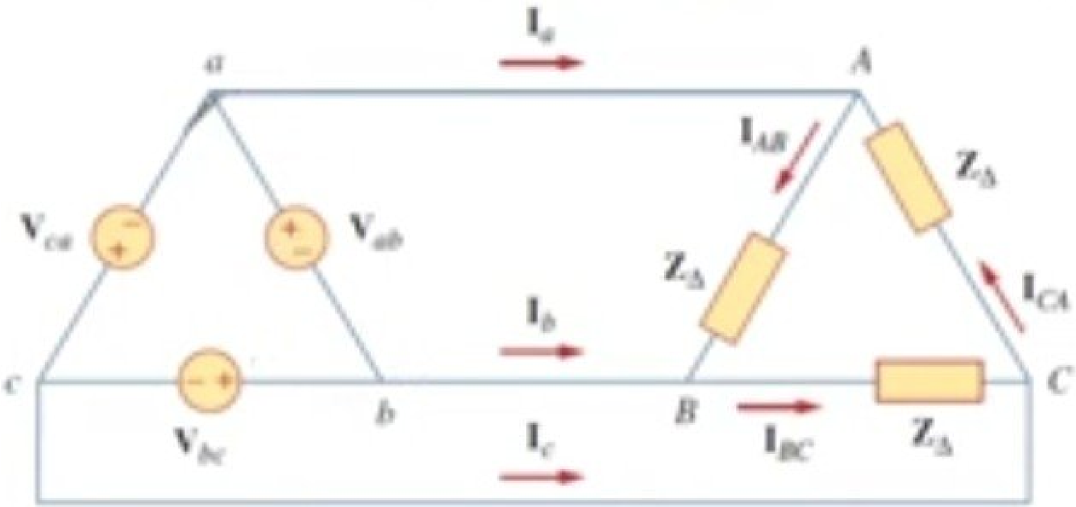
The phasor diagram could be rotated by any angle

$$V_{ab} = 208\angle -30^\circ$$

$$\therefore V_{an} = \frac{|V_{ab}|}{\sqrt{3}}\angle(-30^\circ - 30^\circ)$$

$$V_{an} = 120\angle -60^\circ$$
$$V_{bn} = 120\angle -180^\circ$$
$$V_{an} = 120\angle 60^\circ$$

Balanced DELTA-to-DELTA CONNECTION



Line voltages = phase voltages

$$\left.\begin{aligned} V_{ab} &= V_L\angle 0^\circ \\ V_{bc} &= V_L\angle -120^\circ \\ V_{ca} &= V_L\angle 120^\circ \end{aligned}\right\} \Rightarrow$$

assuming there is no line impedances,

$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \quad \mathbf{V}_{ca} = \mathbf{V}_{CA}$

Hence, the phase currents are

$$\boxed{\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}}$$

V V

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_a\underline{/-120^\circ}$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_a\underline{/+120^\circ}$$

assuming there is no line impedances,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Hence, the phase currents are

$$\boxed{\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta} = \mathbf{I}_{AB}\angle -120^\circ$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta} = \mathbf{I}_{AB}\angle +120^\circ$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_a\angle +120^\circ$$

Balanced DELTA-to-DELTA CONNECTION

Example 12.4

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\underline{/0^\circ}$ V. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\underline{/-36.87^\circ}\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\underline{/0^\circ}}{25\underline{/-36.87}} = 13.2\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 13.2\underline{/-83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 13.2\underline{/156.87^\circ}\ \text{A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = (13.2\underline{/36.87^\circ})(\sqrt{3}\underline{/-30^\circ}) = 22.86\underline{/6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 22.86\underline{/-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 22.86\underline{/126.87^\circ}\ \text{A}$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 13.2\angle 156.87^\circ \text{ A}$$

phase OR Load currents

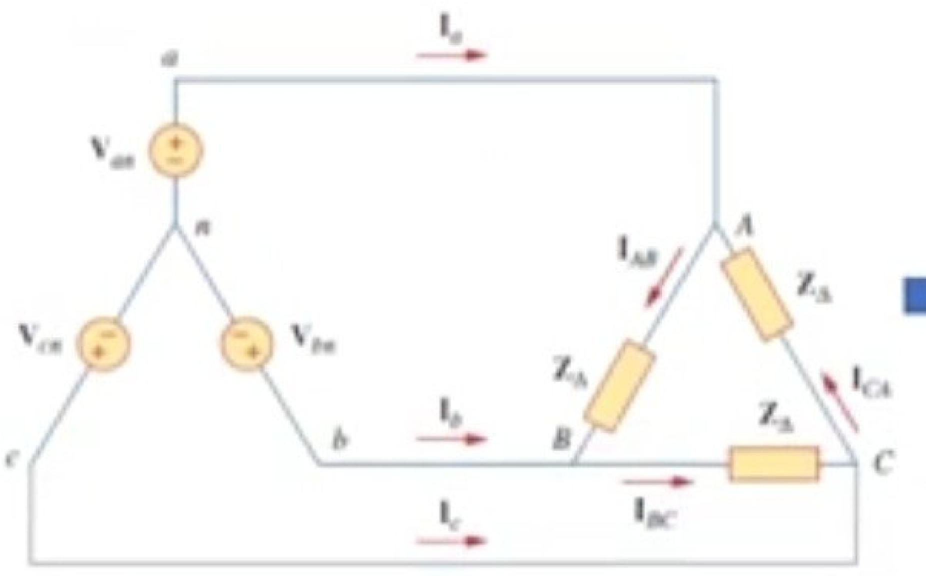
For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = (13.2\angle 36.87^\circ)(\sqrt{3}\angle -30^\circ) = 22.86\angle 6.87^\circ \text{ A}$$

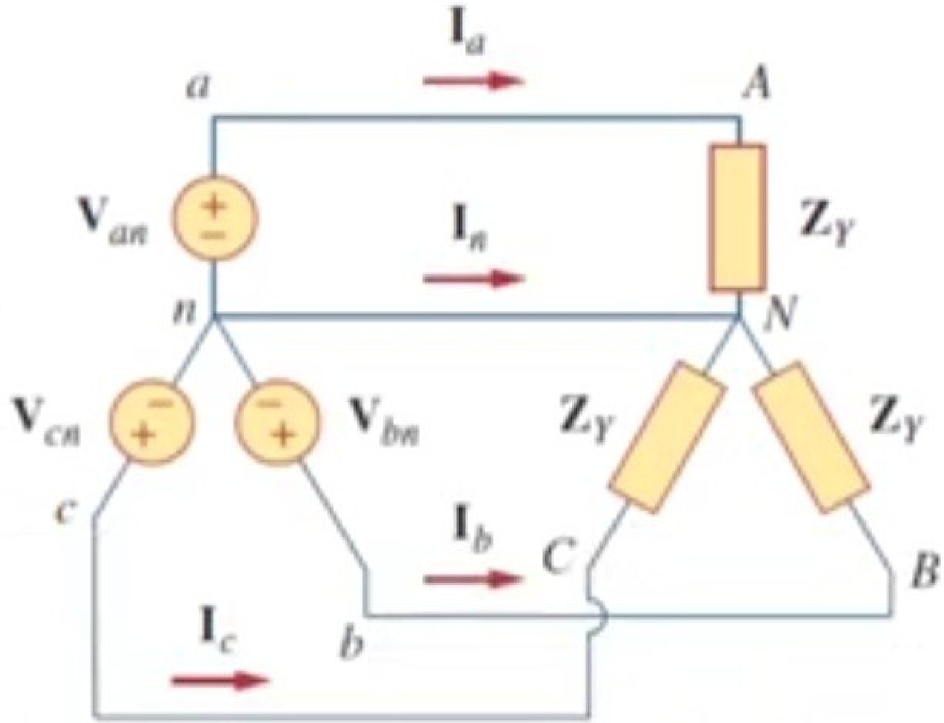
$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 22.86\angle -113.13^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 22.86\angle 126.87^\circ \text{ A}$$

Balanced Wye-Delta Connection



An alternative way of analyzing the Y-Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load. Using the Δ-Y transformation formula in Eq. (12.8),



Balanced Delta-Wye Connection