AC circuits Analysis

CH9 & CH10

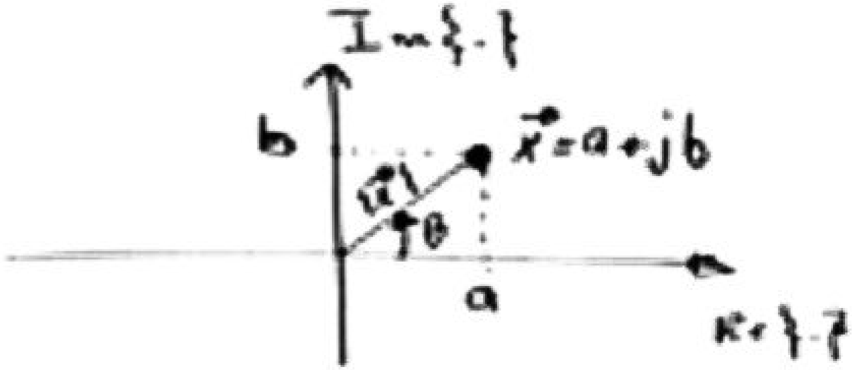
Complex Numbers

- $\vec{x} = a + jb$

rectangular form

$a = Re\{\vec{x}\}$

$b = Im\{\vec{x}\}$



- $\vec{x} = |\vec{x}|\angle\theta$

polar form

- Convert from rectangular form to polar form:

$\vec{x} = a + jb$, $|\vec{x}| = \sqrt{a^2 + b^2}$.

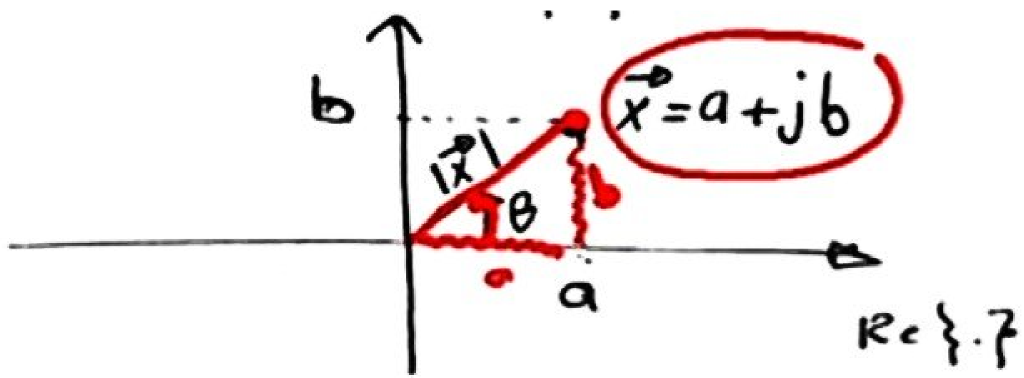
$\theta = \tan^{-1}(b/a)$

$\vec{x} = a + jb$

rectangular form

$a = Re\{\vec{x}\}$

$b = Im\{\vec{x}\}$



- $\vec{x} = |\vec{x}| \angle \theta$

polar form

$|\vec{x}| = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}(b/a)$

- Convert from rectangular form to polar form:

$\vec{x} = a + jb$, $|\vec{x}| = \sqrt{a^2 + b^2}$.

$\theta = \tan^{-1}(b/a)$.

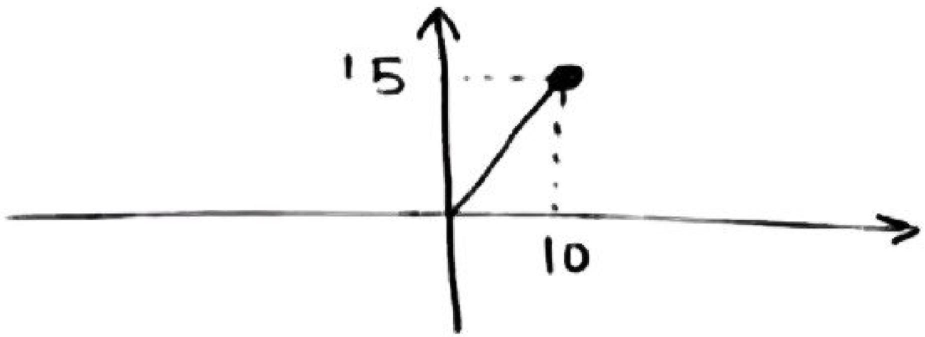
- Convert from polar form to rectangular form:

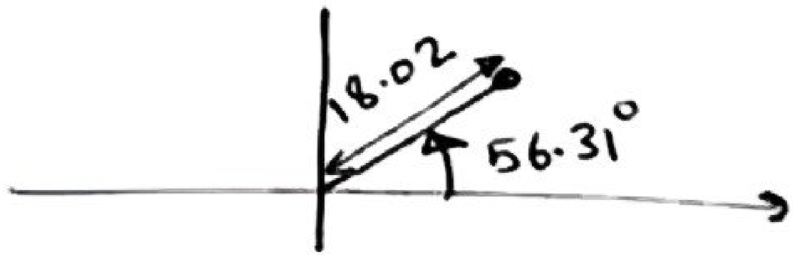
$$\theta = \tan^{-1}(b/a).$$

- Convert from <u>polar form</u> to <u>rectangular form</u>:

$$\vec{x} = |\vec{x}|\angle\theta \quad \Rightarrow \quad \begin{aligned} a &= |\vec{x}|\cos\theta \\ b &= |\vec{x}|\sin\theta \end{aligned}$$

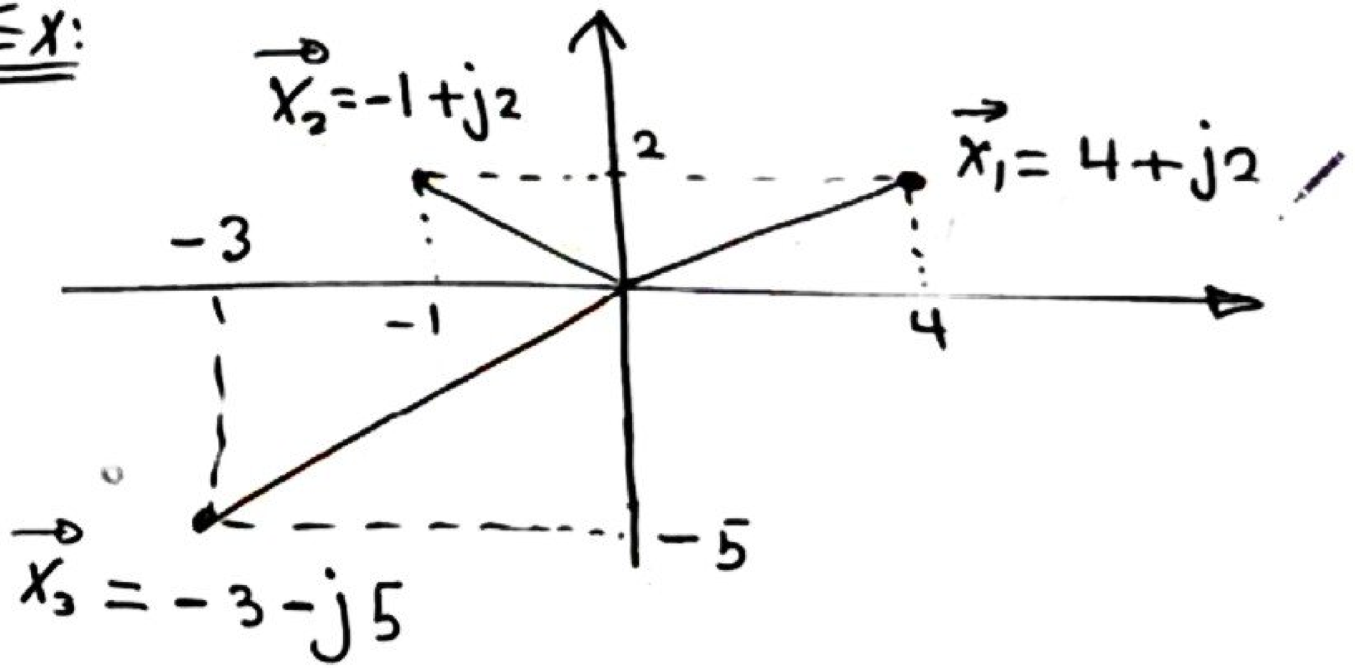
- <u>Ex</u>: $\vec{x} = 10 + j15 \triangleq \sqrt{10^2+15^2}\,\angle\tan^{-1}(15/10) = 18.02\angle 56.31^\circ$





each use

• **Ex:**

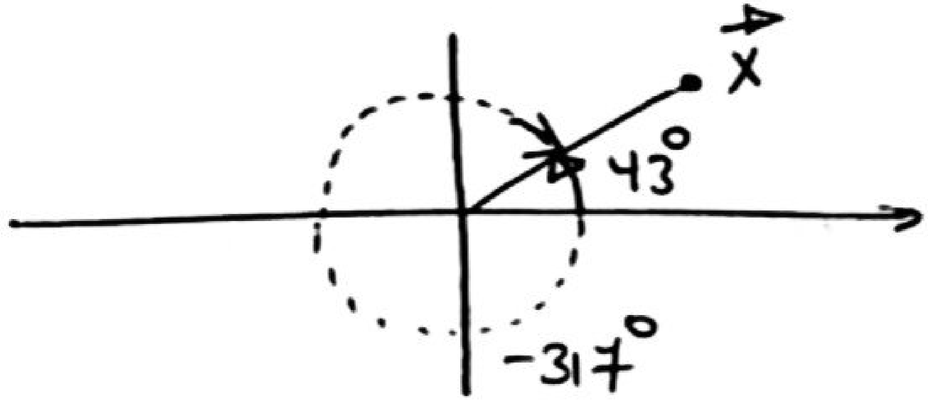


Find the polar form for each of these complex numbers.

• **Ex:** $\vec{y} = 110\angle 93^\circ \Rightarrow \vec{y} = \underbrace{110\cos(93^\circ)}_{=a} + j\,\underbrace{110\sin(93^\circ)}_{=b}$

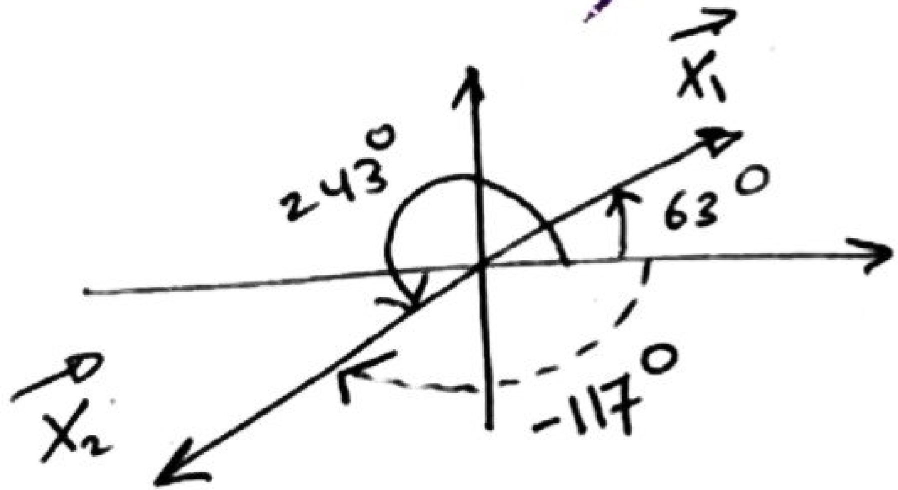
$= -5.76 + j109.849$.

- EX: $\vec{X} = 12\angle 43^\circ \triangleq 12\angle 43^\circ \pm 360^\circ = 12\angle 403^\circ = 12\angle -317^\circ$



- EX $\vec{X}_1 = 5\angle 63^\circ \Longrightarrow \vec{X}_2 = -\vec{X}_1 = 5\angle 63^\circ \pm 180^\circ$

$$= 5\angle 243^\circ$$
$$= 5\angle -117^\circ$$



- sum & subtraction of two complex numbers
(preferred in the rectagular form)

EX: $\vec{X_1} = 3 + j4$, $\vec{X_2} = -9 + j2$

$$\vec{X_1} + \vec{X_2} = \left(3 + (-9)\right) + j\left(4 + 2\right) = -6 + j6 .$$

$$\vec{X_1} - \vec{X_2} = \left(3 - (-9)\right) + j\left(4 - 2\right) = 12 + j2 .$$

- multiplication & division:
(preferred in polar form)

EX : $\vec{X_1} = 120 \angle -89^\circ$, $\vec{X_2} = 60 \angle 112^\circ$

$$\vec{X_1}\,\vec{X_2} = (120)(60) \angle (-89^\circ) + (112^\circ)$$

$-j - 12 + j2$ ✓

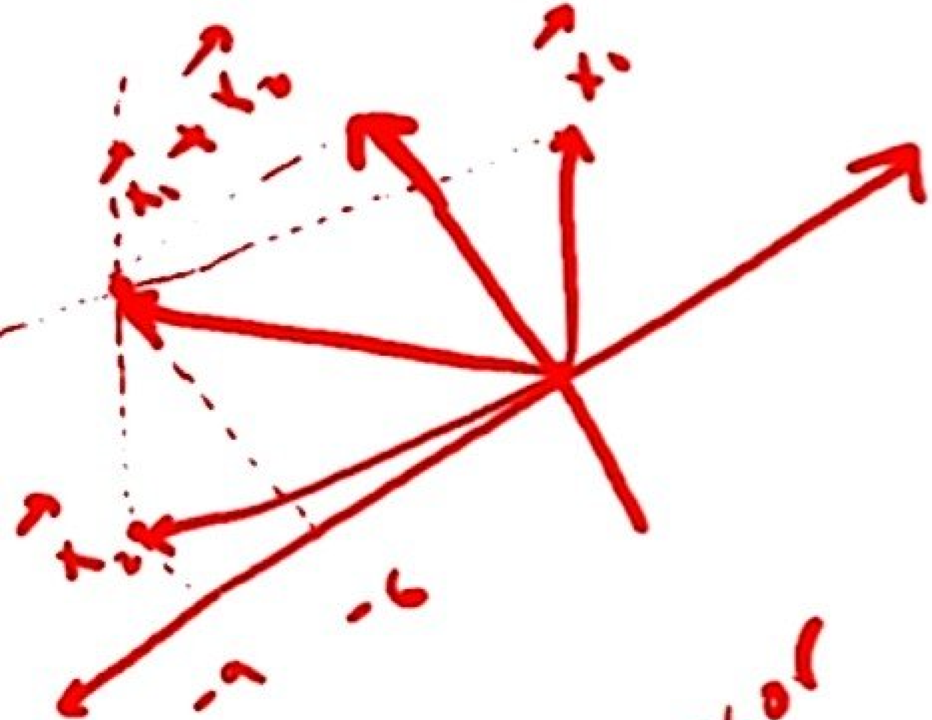
- multiplication & division:
  (preferred in polar form)

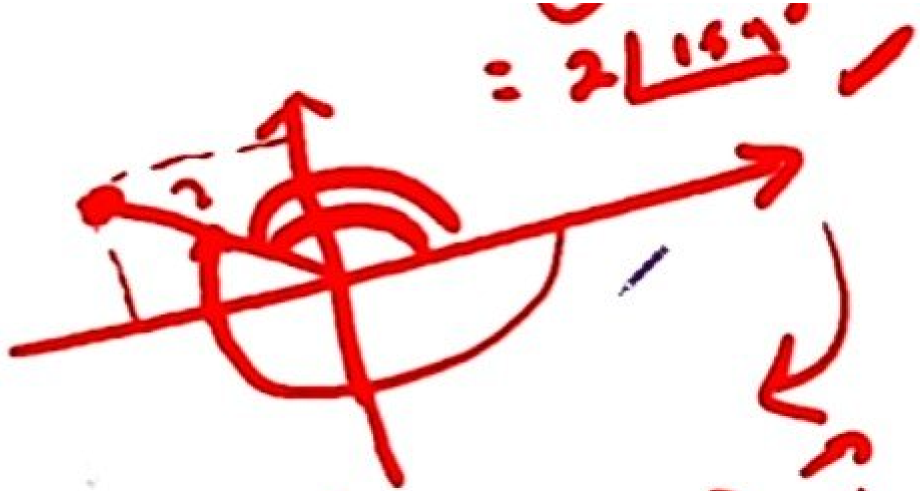
EX : $\vec{X_1} = 120\angle -89^\circ$, $\vec{X_2} = 60\angle 112^\circ$

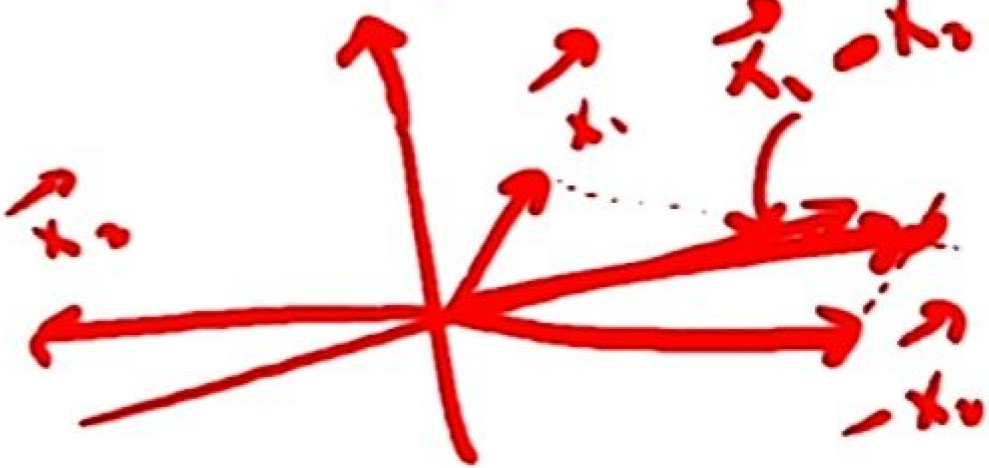
$$\vec{X_1}\ \vec{X_2} = (120)(60)\angle(-89^\circ)+(112^\circ)$$

$$= 7200\angle 23^\circ$$

$$\frac{\vec{X_1}}{\vec{X_2}} = \frac{120\angle -89^\circ}{60\angle 112^\circ} = \frac{120}{60}\angle -89^\circ-(112^\circ) = 2\angle -201^\circ .$$







= phasor Diagram

$\vec{x}_1 - \vec{x}_0$

$= \vec{x}_1 + (-\vec{x}_0)$

$= \vec{x}''$

3

$\sin(\ )$ ← OR → $\cos(\ )$

⊜ Sinusoids Characteristics:

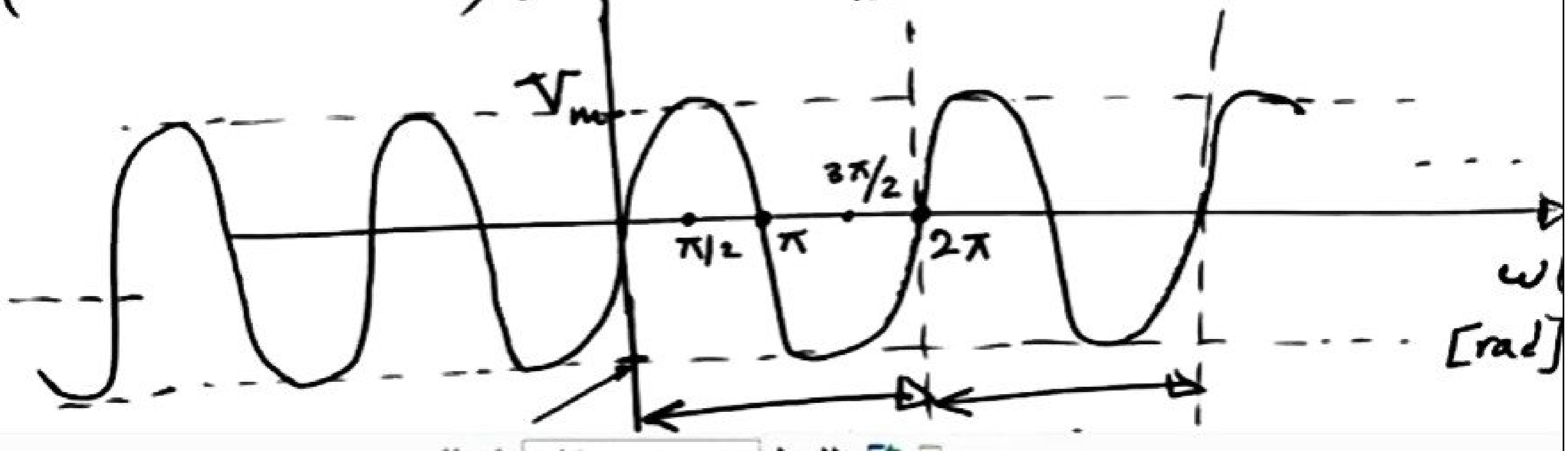
- $v(t) = V_m \sin(\omega t)$

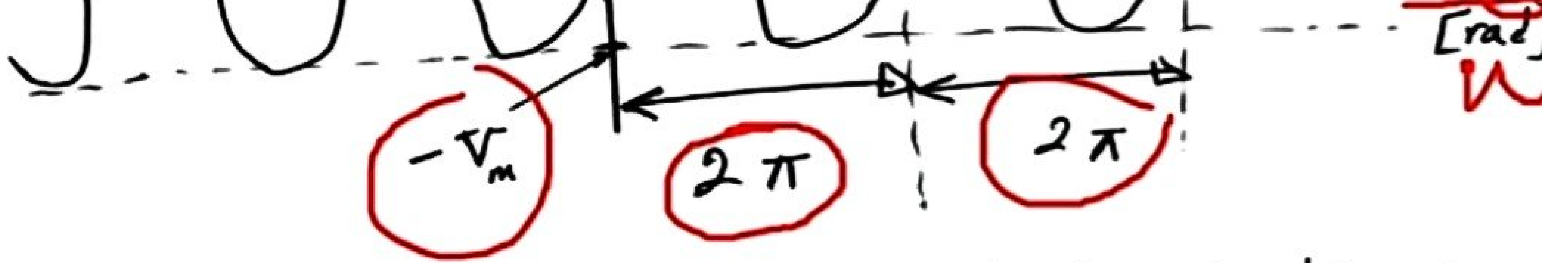
V_m ≗ peak value (amplitude)

ωt ≗ argument [rad]

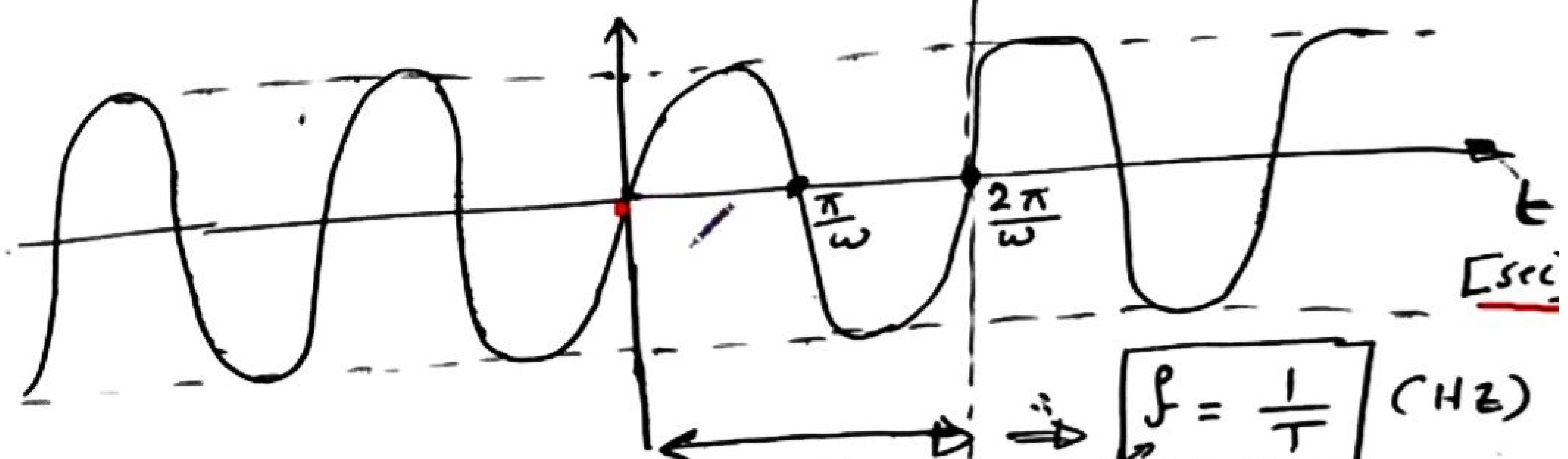
ω ≗ radian frequency [rad/sec]

(plot $v(t)$ vs ωt): $v(t) = V_m \sin(\omega t)$





(plot $v(t)$ Vs t): divide the (ωt)-axis by ω



"period" → T [sec] ⇒ $\boxed{f = \frac{1}{T}}$ (Hz) "frequency"

$$\Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

Ex: $v(t) = 20 \sin(1000t)$ [V], find f & T.

sol: $\omega = 1000$ [rad] $\Rightarrow f = \dfrac{\omega}{2\pi} = \dfrac{1000}{2\pi} = 159.15$ Hz

$\Rightarrow T = \dfrac{1}{f} = 6.283$ msec.

Ex: $i(t) = 10 \cos(100\pi t)$ [A].

$\omega = 100\pi \Rightarrow f = \dfrac{100\pi}{2\pi} = 50$ HZ

$\Rightarrow T = \dfrac{1}{f} = 20$ msec.

- More general case:

$$v(t) = V_m \sin(\omega t + \theta)$$

$V_m \triangleq$ peak value or amplitude.

$\omega t + \theta \triangleq$ argument [rad] or [degree]

$\omega \triangleq$ radian frequency [rad/sec]

$\theta \triangleq$ phase [rad] or [degree]

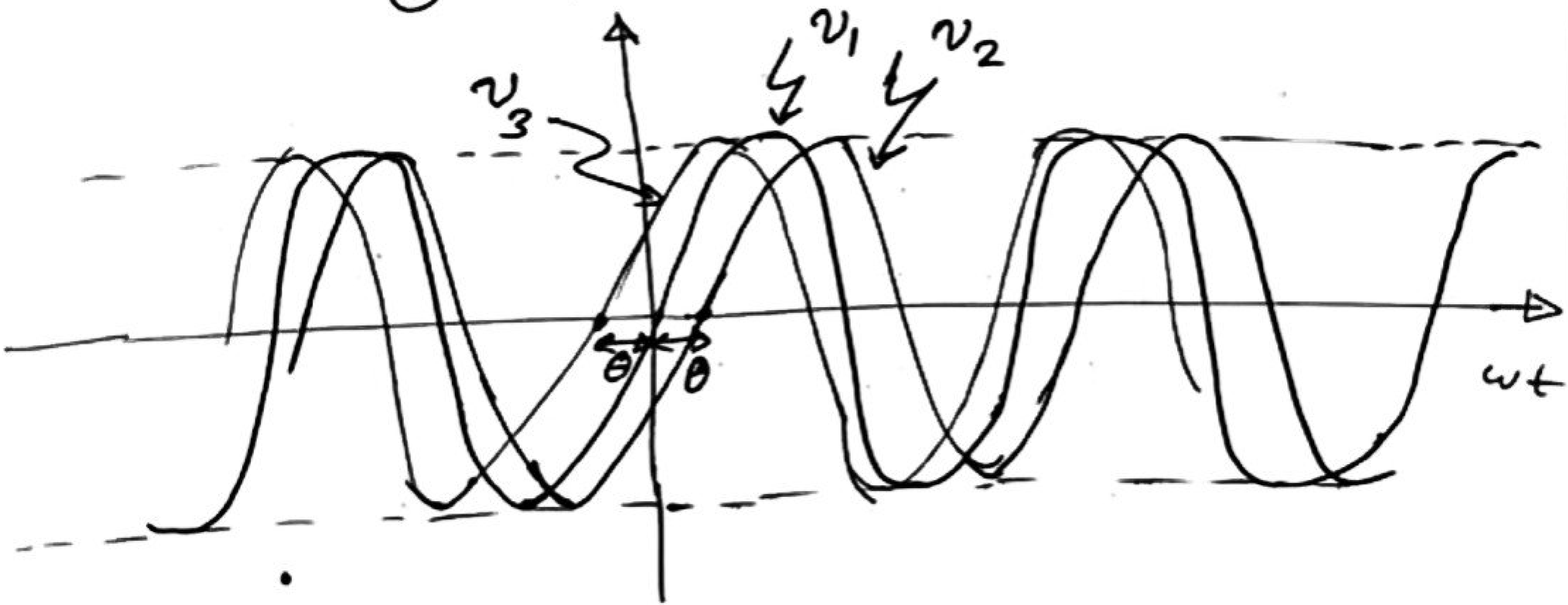
represents the signal shift to the right or to the left.

plot $v(t) = V_m \sin(\omega t + \theta)$ for

① $\theta = 0 \Longrightarrow v_1(t)$

② $\theta < 0 \Longrightarrow v_2(t)$ shift to right

③ $\theta > 0 \Longrightarrow v_3(t)$ " " left



Lagging & Leading

$v_1(t)$ leads $v_2(t)$ by θ.

OR $v_2(t)$ lags $v_1(t)$ by θ.

OR $v_1(t)$ lags $v_2(t)$ by $-\theta \doteq 360° - \theta$.

OR $v_2(t)$ leads $v_1(t)$ by $-\theta \doteq 360° - \theta$.

what about $v_3(t)$ and $v_2(t)$?

Two compare two sinusoids:

1. Both must have same ω.
2. Both must be "cosine" or "sine"
3. Both must have +ve amplitude.

The conversion between them is:

$$\boxed{\begin{aligned} -\sin(\omega t) &= \sin(\omega t \pm 180^\circ) \\ -\cos(\omega t) &= \cos(\omega t \pm 180^\circ) \\ \pm\cos(\omega t) &= \sin(\omega t \pm 90^\circ) \\ \pm\sin(\omega t) &= \cos(\omega t \mp 90^\circ) \end{aligned}}$$

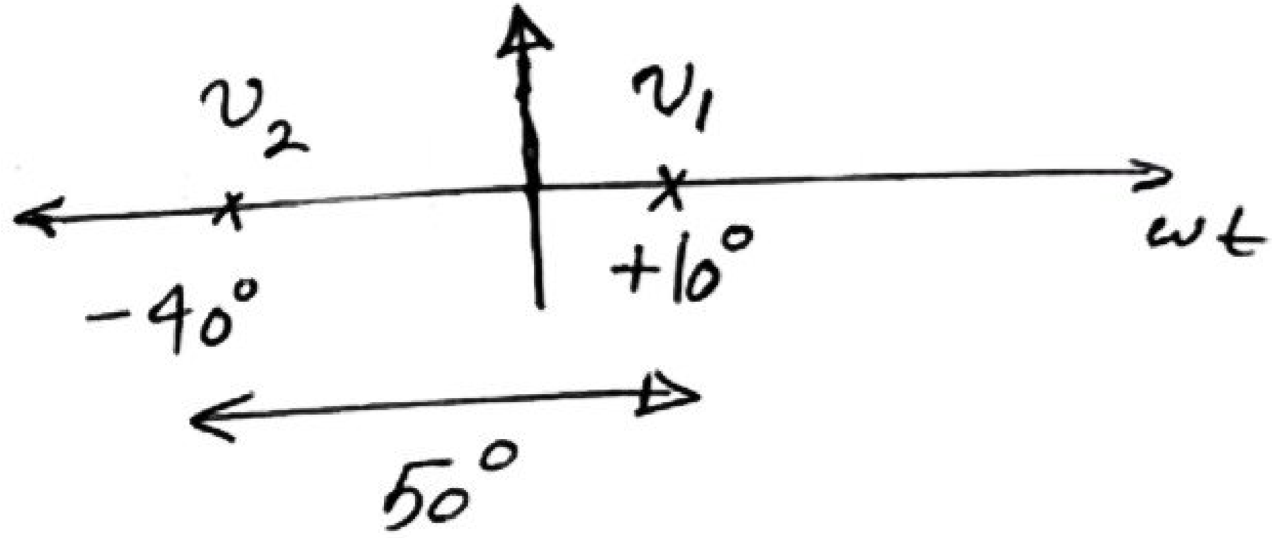
Ex: $v_1(t) = 10\cos(1000\pi t - 10^\circ)$

Ex: $v_1(t) = 10\cos(1000\pi t - 10^\circ)$

$v_2(t) = 2\cos(1000\pi t + 40^\circ)$

which leads?, which lags?

sol.



v_1 lags v_2 by 50°

v_2 leads v_1 by 50°

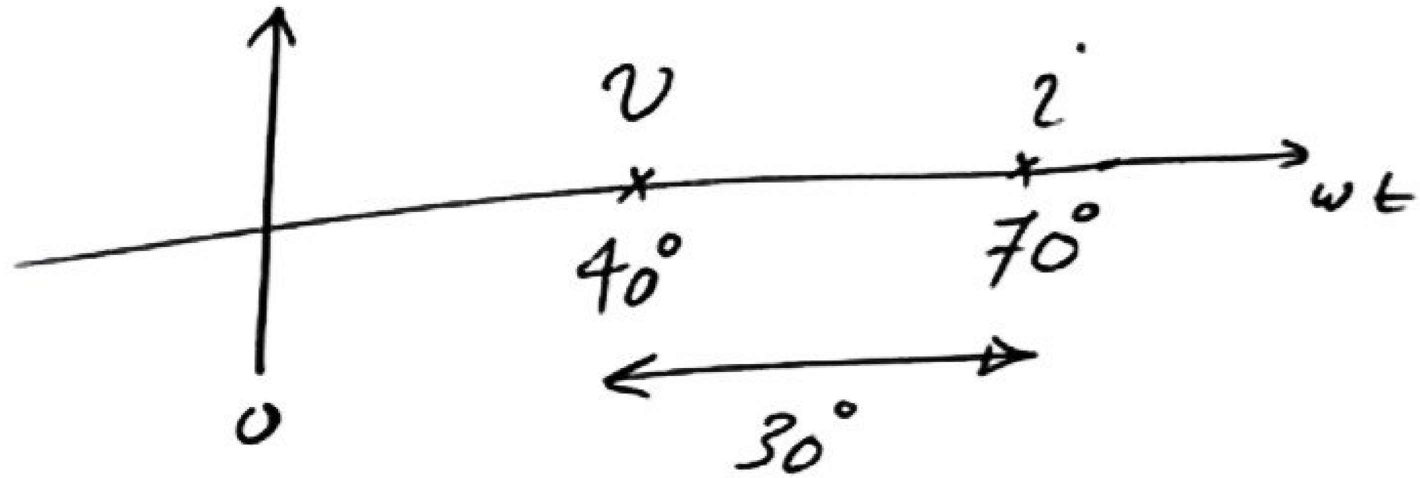
or v_1 leads v_2 by $-50^\circ = 310^\circ$

Ex: $i(t) = -2\cos(100\pi t + 20^\circ)$ [A]

$v(t) = 12\sin(100\pi t - 40^\circ)$ [V]

sol: $i(t) = 2\sin(100\pi t + 20^\circ - 90^\circ)$

$= 2\sin(100\pi t - 70^\circ)$



v leads i by 30°

or i lags v by 30°.

Ex: $v(t) = 2\cos(200\pi t + 30^\circ)$

Find $v(0.01)$?

sol: $v(0.01) = 2\cos(2\pi + 30^\circ)$

$\triangleq 2\cos\left(2\pi + 30^\circ \frac{\pi}{180^\circ}\right) = \sqrt{3}$.

Your calculator on [rad]

oR

$\triangleq 2\cos(360^\circ + 30^\circ) = \sqrt{3}$.

Your calculator on [degree]

AC circuits:

Electric circuits that are driven by AC (sinusoidal) sources.

Why AC?

- Electric power delivered to homes and industry is AC.
- In the late 1800's, AC won out his battle with DC due to its efficiency

DC

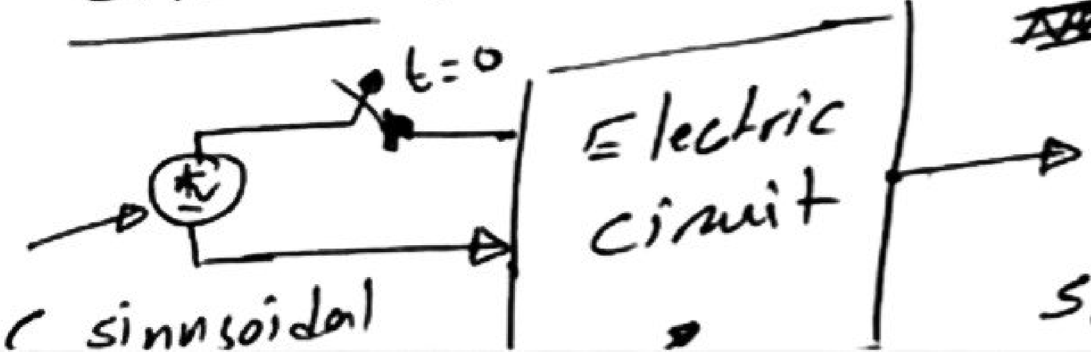
AC

Why AC?

- Electric power delivered to homes and industry is AC.
- In the late 1800's, AC won out his battle with DC due to its efficiency for long distance transmission.

Steady-state Analysis of AC (sinusoidal)

circuits:



Transient response (t-s)

$+$

steady-state response (ss-s)

AC sinusoidal

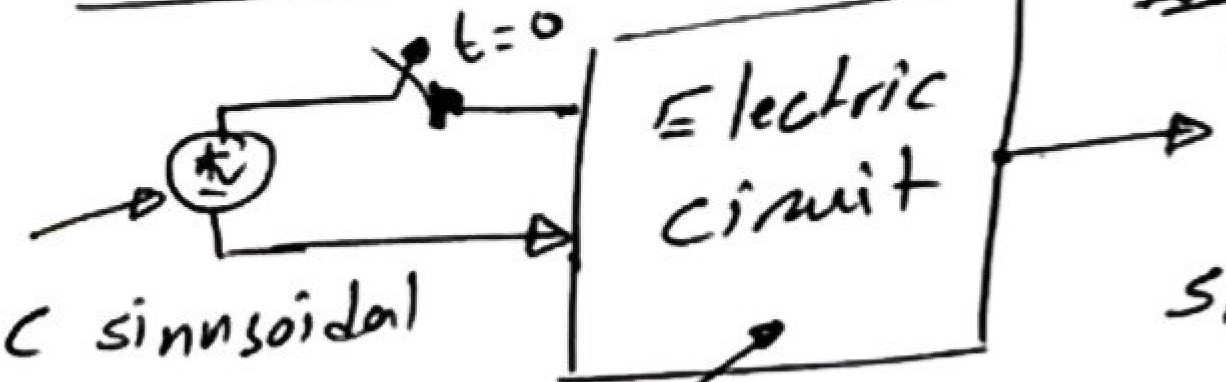
the

AC

- In the late 1800's, AC [illegible] battle with DC due to its efficiency for long distance transmission.

Steady-state Analysis of AC (sinusoidal)

Circuits:



AC sinusoidal source

with intial conditions

Transient response (t-s)
+
steady-state response (ss-s)

- we need only the ss-s because the t-s is temporary and will die out.

⇒ So, we assume that AC circuits are under the steady-state and then analyze them.

To simplify analysing the steady-state AC circuits, we convert them from <u>time-domain</u> to <u>complex-domain</u> (phasor domain). So, instead of dealing with integro-diff-equations we deal with algabraic-equations but with complex-numbers.

... we deal with ... equations we deal with algebraic equations but with complex-numbers.

$(2\angle 30^\circ)\ \vec{I}_1 + (3+j)\ ...$

$\vec{I}_1 \cdots\cdots i_1(t)$

③ phasors:

<u>Def:</u> a phasor is a complex number that represents the amplitude and phase of a sinusoid. (the sinusoid has to be written as cosine with +ve amplitude)

Let $v(t) = V_m \cos(\omega t + \theta_v)$

$= Re\{ V_m \angle \omega t + \theta_v \}$

$= Re\{ (V_m \angle \theta_v)(1 \angle \omega t) \}$

this is the phasor $\vec{V} = V_m \angle \theta_v$

the ckt operate at same ω, so need to carry this.

$$v(t) = V_m \cos(\omega t + \theta_v) \Leftrightarrow \vec{V} = V_m \angle \theta_v$$

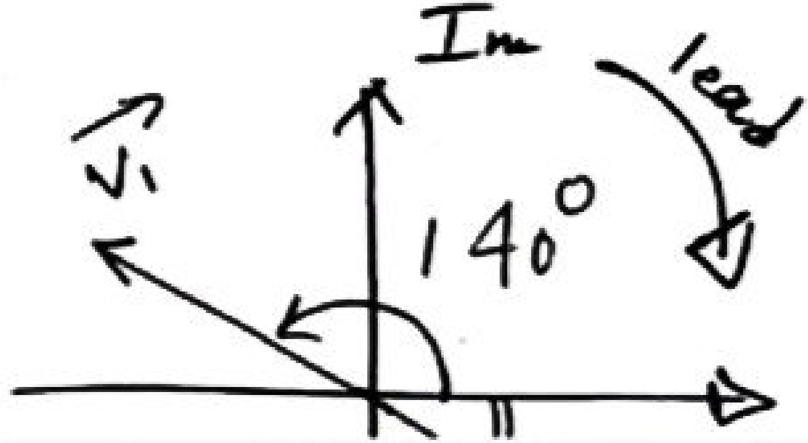
the same for currents:

$$i(t) = I_m \cos(\omega t + \theta_i) \Leftrightarrow \vec{I} = I_m \angle \theta_i$$

Ex: • $i_1(t) = 6\cos(50t - 40^\circ)$ A

$\Rightarrow \vec{I}_1 = 6 \angle -40^\circ$ A

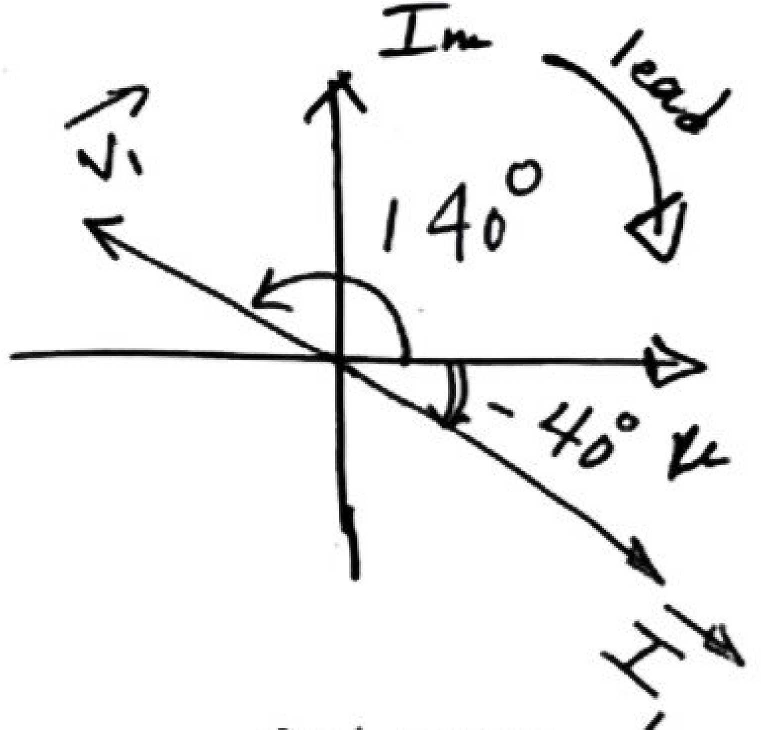
$\hat{=} \; 6 \angle 320^\circ$ A



Ex: • $i_1(t) = 6\cos(50t - 40^\circ)$ A

$\Rightarrow \vec{I_1} = 6\angle -40^\circ$ A

$\doteq 6\angle 320^\circ$ A



(phasor diagram)

• $v_1(t) = -4\sin(30t + 50^\circ)$

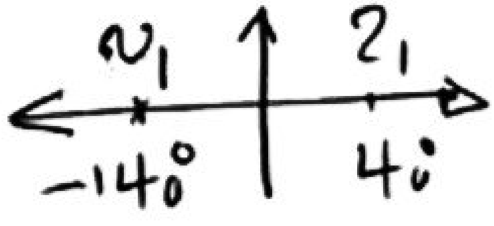
$\Rightarrow \cancel{-4\angle 50^\circ}$

$v_1(t) = -4\sin(30t + 50^\circ)$

$= 4\cos(30t + 50^\circ + 90^\circ)$

$= 4\cos(30t + 140^\circ)$

$\Rightarrow \vec{V_1} = 4\angle 140^\circ$



Ex: $\vec{I} = 30\angle{-20^\circ}$ mA   Phasor

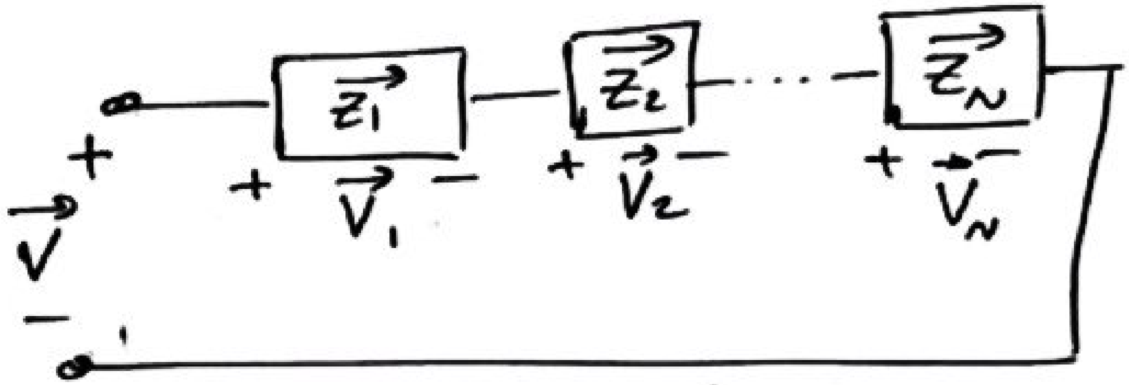
$\Rightarrow i(t) = 30\cos(\omega t - 20^\circ)$ mA.

Ex: $\vec{V} = -3 + j4$ V , Find $v(t)$.

sol: $\vec{V} = \sqrt{9+16}\ \angle \tan^{-1}\left(\frac{4}{-3}\right) = 5\angle 126.87^\circ$

$\rightarrow v(t) = 5\cos(\omega t + 126.87^\circ)$

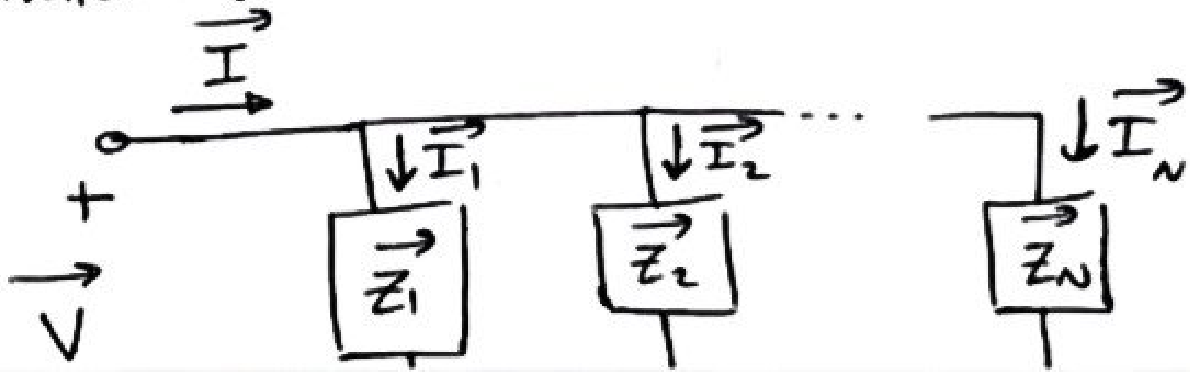
• Impedances in series & voltage division:



$$\vec{Z}_{eq} = \vec{Z}_1 + \vec{Z}_2 + \cdots + \vec{Z}_N .$$

$$\vec{V}_n = \frac{\vec{Z}_n}{\vec{Z}_1 + \vec{Z}_2 + \cdots + \vec{Z}_N} \quad \text{(voltage division)}$$

• Impedances in parallel & current division:



$$\vec{Z}_{eq} = \frac{1}{\cdots}$$

sol: $i(t) = i_1(t) + i_2(t)$

$$i_1(t) = 4\cos(wt + 30^\circ) \Rightarrow \vec{I_1} = 4\angle 30^\circ$$

$$i_2(t) = 5\sin(wt - 20^\circ)$$
$$= 5\cos(wt - 110^\circ) \Rightarrow \vec{I_2} = 5\angle -110^\circ$$

$$\therefore \Rightarrow i(t) \Rightarrow \vec{I} = \vec{I_1} + \vec{I_2}$$
$$= 4\angle 30^\circ + 5\angle -110^\circ$$
$$\triangleq 3.218\angle -56.91^\circ$$

$$\Rightarrow i(t) = 3.218\cos(wt - 56.91^\circ)$$

Lect #3

- In complex-domain, ckT elements converted to impedances.

what is "Impedance"?

- Impedance : can be defd as complex resistance

$$\vec{Z} = R + jX = |\vec{Z}| \angle \theta_Z \quad [\Omega]$$

resistance (dissipate energy) originally

Reactance (stores energy) originally OR

(...) energy

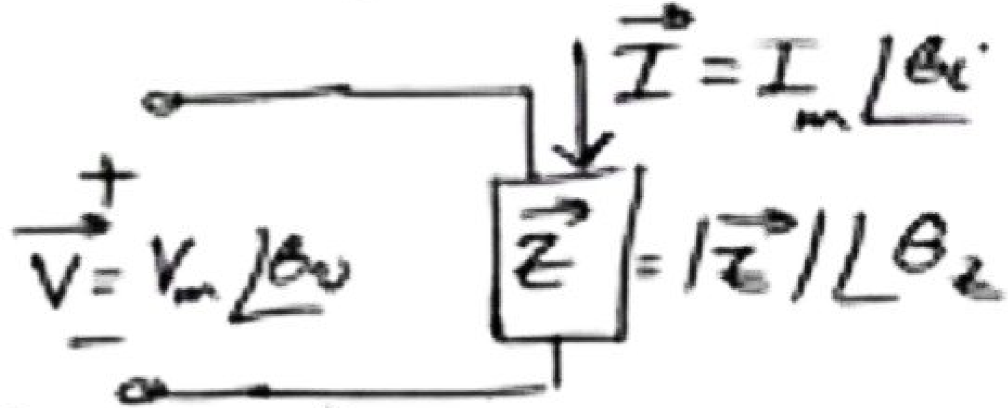
(stores energy)

originally

originally OR

<u>Ohm's Law is directly applied</u>

$$\boxed{\vec{V} = \vec{Z}\,\vec{I}}$$



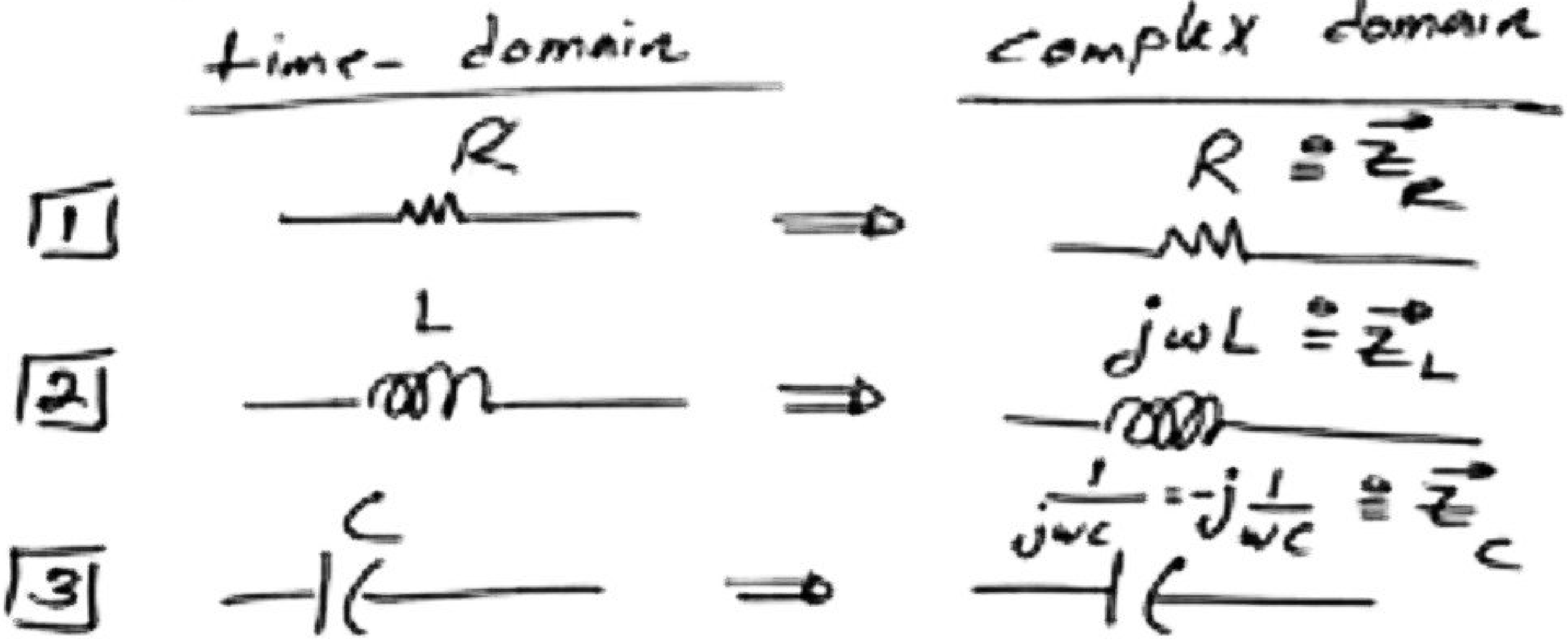
$$\Rightarrow V_m \angle \theta_v = \left(|\vec{Z}| \angle \theta_z\right)\left(I_m \angle \theta_i\right)$$

$$V_m = |\vec{Z}|\, I_m$$

$$\theta_v = \theta_i + \theta_z$$

→ find the impedances for R, L, C

② Convert CKT elements



Note: $\vec{Z}$ refers to the impedance
(will be defined later).

proof: Let $v(t) = V_m \cos(\omega t + \theta_v) \Rightarrow \vec{V} = V_m \angle\theta_v$.

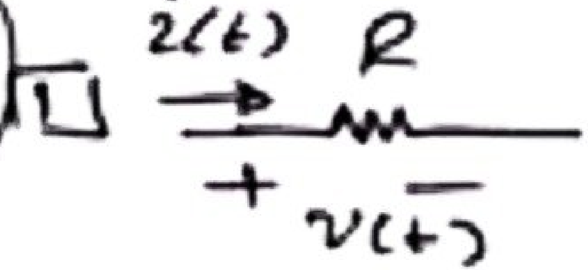
$i(t) = I_m \cos(\omega t + \theta_i) \Rightarrow \vec{I} = I_m \angle\theta_i$

Note: $\vec{Z}$ refers to the impedance (will be defined later).

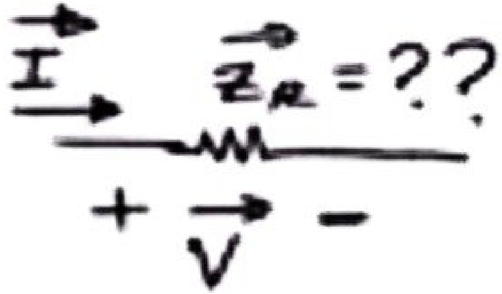
proof: Let $v(t) = V_m \cos(\omega t + \theta_v) \Rightarrow \vec{V} = V_m \angle \theta_v$.

$$i(t) = I_m \cos(\omega t + \theta_i) \Rightarrow \vec{I} = I_m \angle \theta_i$$

Resistive load



$\Longrightarrow$



- $v(t) = R\, i(t)$

$$V_m \cos(\omega t + \theta_v) = R I_m \cos(\omega t + \theta_i)$$

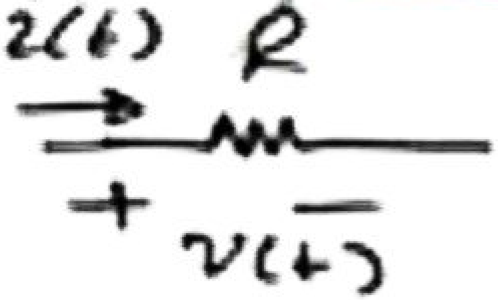
$$\Downarrow$$

$$V_m \angle \theta_v = R \underbrace{I_m \angle \theta_i}_{= \vec{I}}$$

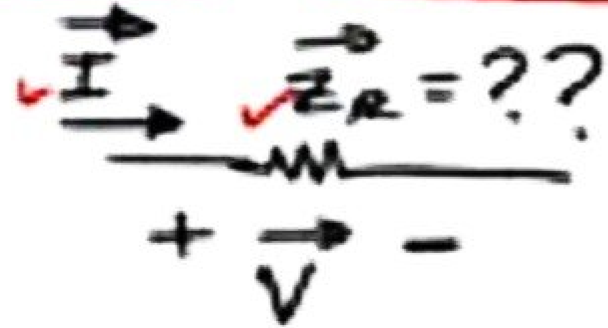
$$\Rightarrow \vec{V} = R \vec{I}$$

Note that $\theta_v = \theta_i$

Resistive load [1]



$\Longrightarrow$



- $v(t) = R\,i(t)$

$$V_m \cos(\omega t + \theta_v) = R I_m \cos(\omega t + \theta_i)$$

$$\Downarrow$$

$$V_m \angle \theta_v = R \underbrace{I_m \angle \theta_i}_{= \vec{I}}$$

$$\Longrightarrow \quad \vec{V} = R\, \vec{I}$$

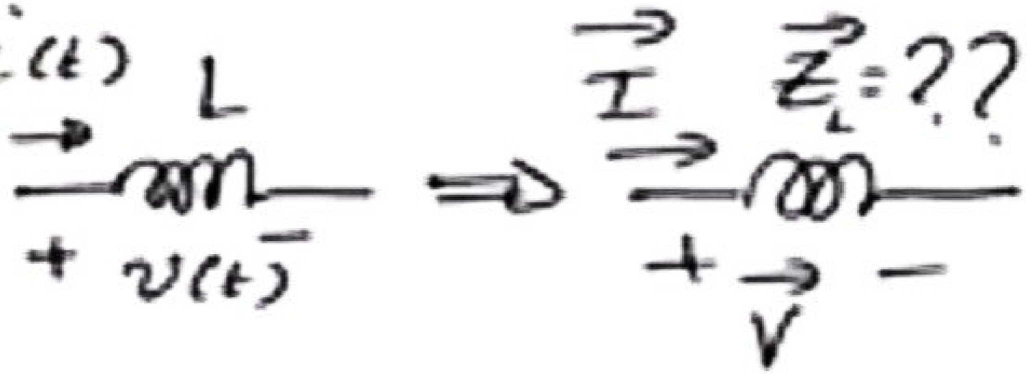
$$\boxed{\vec{Z}_R = R} \quad (2)$$

$$\theta_Z = 0$$

$\Longrightarrow$ Note that $\theta_v = \theta_i$ for resistive load
$\vec{I}$ & $\vec{V}$ are in phase

pure inductive load

$\boxed{2}$



- $v(t) = L \dfrac{di(t)}{dt}$

$\Rightarrow \quad V_m \cos(\omega t + \theta_v) = -L I_m \omega \sin(\omega t + \theta_i)$

$\Rightarrow \quad V_m \cos(\omega t + \theta_v) = \omega L I_m \cos(\omega t + \theta_i + 90^\circ)$

$$\Downarrow$$

$$V_m \angle \theta_v = \omega L I_m \angle \theta_i + 90^\circ$$

$\Rightarrow \quad V_m \angle \theta_v = (\omega L \angle 90^\circ)(I_m \angle \theta_i)$

$$\Downarrow$$

$$V_m \angle \theta_v = \omega L I_m \angle \theta_i + 90^\circ$$

$$\Rightarrow V_m \angle \theta_v = (\omega L \angle 90^\circ)(I_m \angle \theta_i)$$

$$\Leftrightarrow \vec{V} = \vec{Z}_L \vec{I}$$

$$\Rightarrow \boxed{\vec{Z}_L = \omega L \angle 90^\circ = j\omega L}$$

$\theta_z = 90^\circ$

Also note $\theta_v = \theta_i + 90^\circ$

$\Rightarrow \theta_i = \theta_v - 90^\circ$ (shifted to the left by 90°)

$\Rightarrow$ $\vec{I}$ lags $\vec{V}$ by 90°

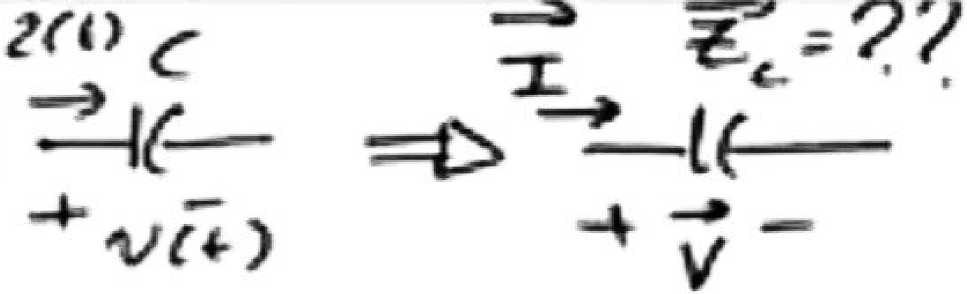
is lagging element

always for pure

. Im

pure capacitive load

$\boxed{3}$



$$i(t) = C\,\frac{dv}{dt}$$

$$\Rightarrow I_m \cos(\omega t + \theta_i) = -C\,\omega V_m \sin(\omega t + \theta_v)$$

$$\Rightarrow I_m \cos(\omega t + \theta_i) = \omega C\, V_m \cos(\omega t + \theta_v + 90^\circ)$$

$$\Downarrow$$

$$I_m \angle \theta_i = \omega C V_m \angle \theta_v + 90^\circ$$

$$I_m \angle \theta_i = (\omega C \angle 90^\circ)\, V_m \angle \theta_v$$

$$\Rightarrow V_m \angle \theta_v = \left(\frac{1}{\omega C \angle 90^\circ}\right)\left(I_m \angle \theta_i\right)$$

$$\Rightarrow V_m \angle \theta_v = \left(\frac{1}{\omega C} \angle -90^\circ\right)\left(I_m \angle \theta_i\right)$$

$$\Rightarrow V_m \angle\theta_v = \left(\frac{1\angle 0}{\omega C\angle 90^\circ}\right)\left(I_m\angle\theta_i\right)$$

$$\Rightarrow V_m \angle\theta_z = \left(\frac{1}{\omega C}\angle -90^\circ\right)\left(I_m\angle\theta_i\right)$$

$$\Rightarrow \vec{V} = \vec{Z}_c\,\vec{I} \Rightarrow \boxed{\vec{Z}_c = \frac{1}{\omega C}\angle -90^\circ = -j\frac{1}{\omega C}}$$

$$\theta_z = -90^\circ$$

Also note: $\theta_i = \theta_v + 90^\circ$

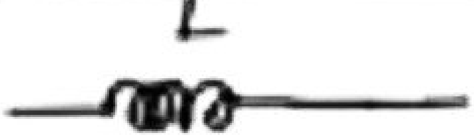
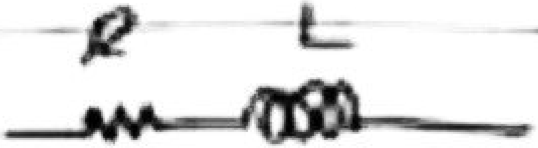
(i shifted to the right)

$\Rightarrow$ $\vec{I}$ leads $\vec{V}$ by 90°

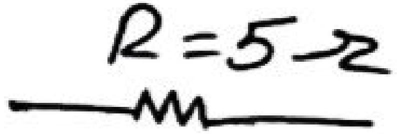
it is leading element

always for pure capacitive load

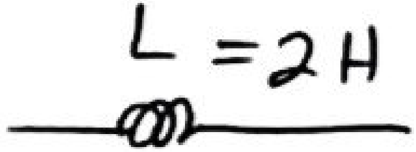
(4)

Time	$\vec{Z}$	Load Type	θ_Z	$\vec{I}$ & $\vec{V}$
	R	pure resistive	0	In phase
	$j\omega L = jX_L$	pure inductive	$90°$	$\vec{I}$ lags $\vec{V}$ by $90°$
	$-j\frac{1}{\omega C} = -jX_C$	pure capacitive	$-90°$	$\vec{I}$ leads $\vec{V}$ by $90°$
	$R + jX_L$	inductive (not purely)	+ve	$\vec{I}$ lags $\vec{V}$ by θ_Z
	$R - jX_C$	capacitive (not purely)	-ve	$\vec{I}$ leads $\vec{V}$ by θ_Z

Ex: If $\omega = 10$ rad/sec, specify the equivalent load for each of the following impedances

① $\vec{Z} = 5\,\Omega \;\therefore\Rightarrow$ 

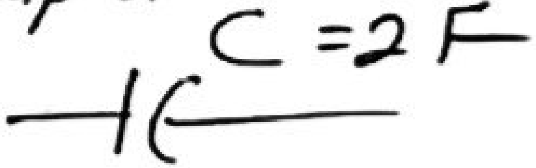
(pure resistive)

② $\vec{Z} = j20\,\Omega \;\therefore\Rightarrow$ 

pure inductive $\quad j20 = j\omega L \Rightarrow L = \frac{20}{\omega} = 2\,H$

③ $\vec{Z} = \frac{1}{j20}\,\Omega$

$\vec{Z} = -j\frac{1}{20} \;\therefore\Rightarrow$ pure capacitive



$-j\frac{1}{20} = -j\frac{1}{\omega C}$

$$-j\frac{1}{20} = -j\frac{1}{wC}$$

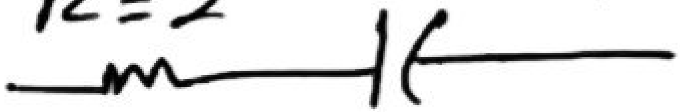
$\Rightarrow 20 = wC$

$\Rightarrow C = 2\ F$

(4) $\vec{Z} = 4 + j3 \Rightarrow$ inductive load

$wL = 3$

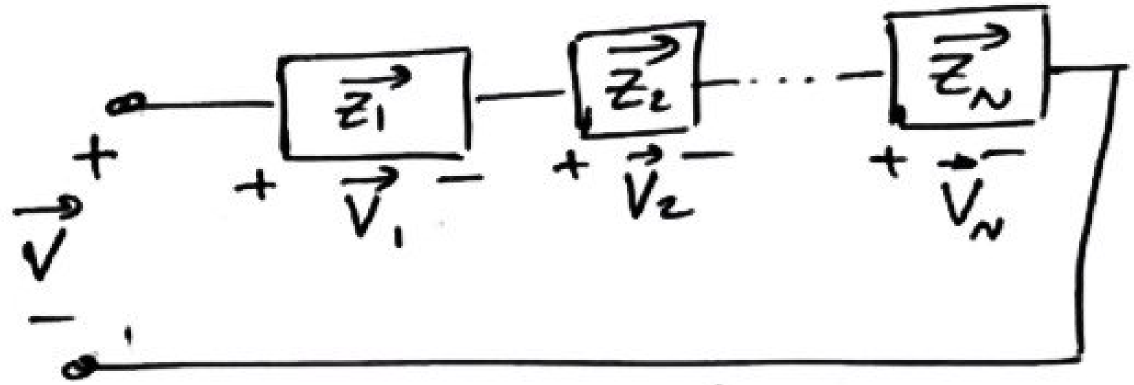
(5) $\vec{Z} = 2 - j2\ \Omega \Rightarrow$ capacitive load



$$-j2 = -j\frac{1}{wC}$$

$$2 = \frac{1}{wC} \Rightarrow C = \frac{1}{20}$$ (8)

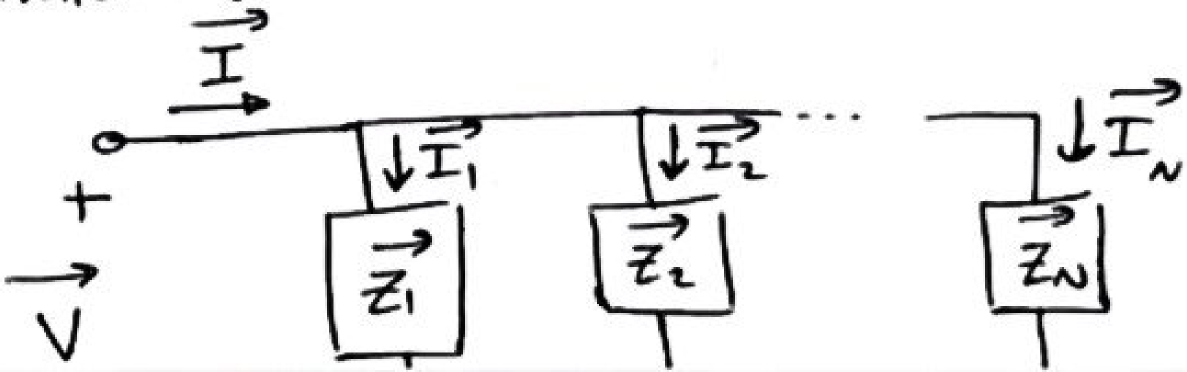
- Impedances in series & voltage division:



$$\vec{Z}_{eq} = \vec{Z}_1 + \vec{Z}_2 + \cdots + \vec{Z}_N .$$

$$\vec{V}_n = \frac{\vec{Z}_n}{\vec{Z}_1 + \vec{Z}_2 + \cdots + \vec{Z}_N} \quad \text{(voltage division)}$$

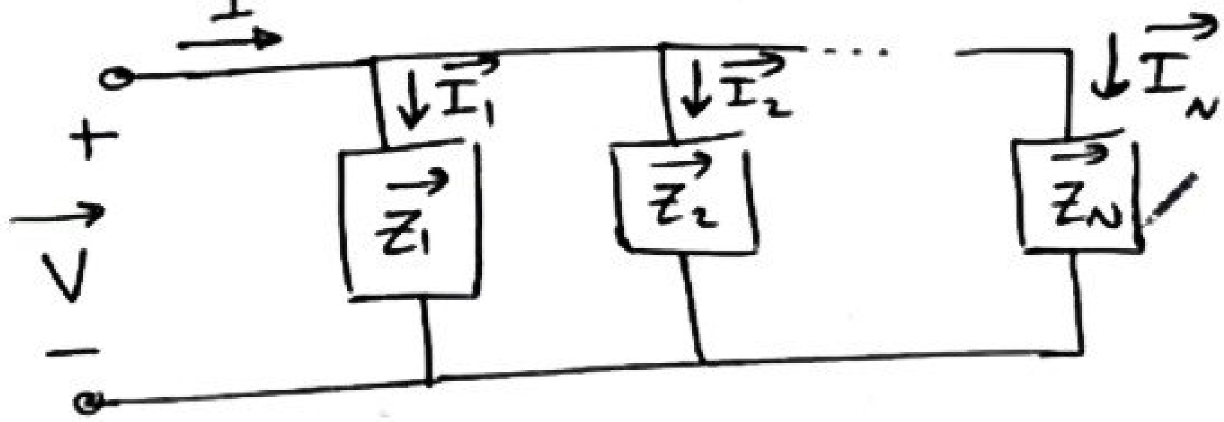
- Impedances in parallel & current division:



$$\vec{Z}_{eq} = \frac{1}{\quad}$$

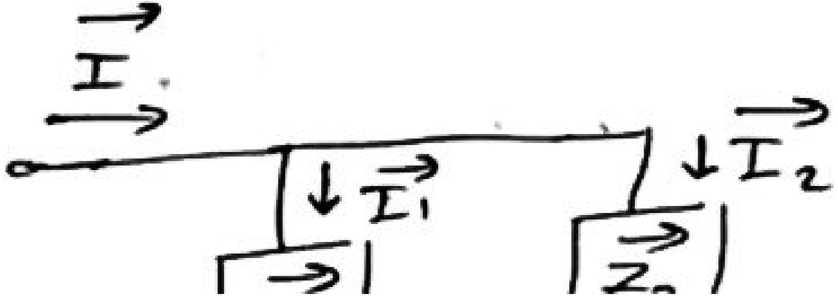
$$\vec{V}_n = \frac{\vec{Z}_n}{\vec{Z}_1 + \vec{Z}_2 + \cdots + \vec{Z}_N} \quad \text{(voltage division)}$$

- Impedances in parallel & current division:

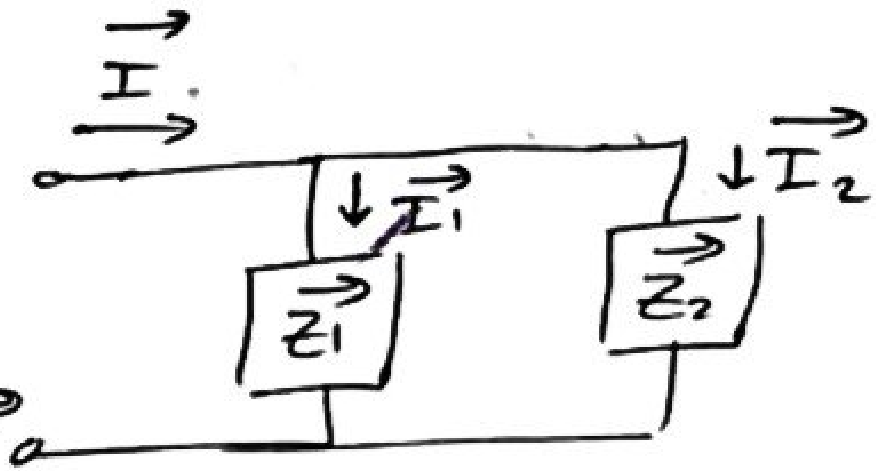


$$\vec{Z}_{eq} = \frac{1}{\frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} + \cdots + \frac{1}{\vec{Z}_N}}$$

$$\vec{I}_n = \frac{1/\vec{Z}_n}{1/\vec{Z}_{eq}} \vec{I} \quad \text{(current division)}$$



eq



$$\vec{Z}_{eq} = \frac{\vec{Z_1}\,\vec{Z_2}}{\vec{Z_1}+\vec{Z_2}}$$

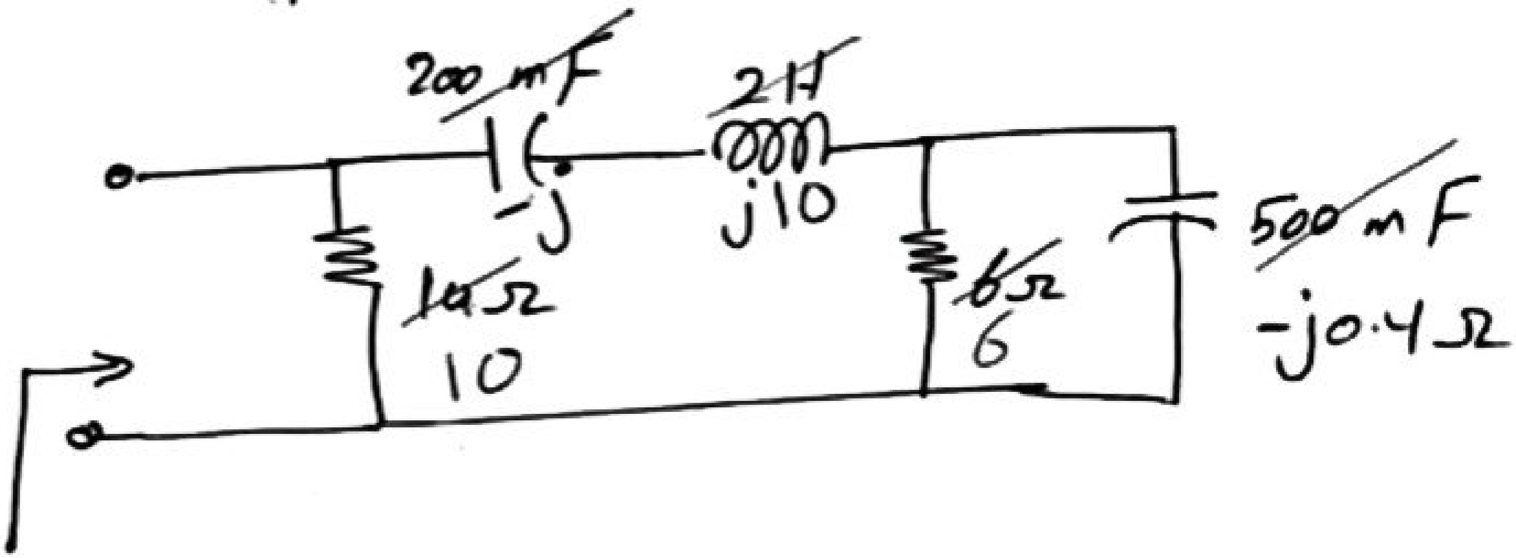
$$\vec{I}_1 = \frac{\vec{Z_2}}{\vec{Z_1}+\vec{Z_2}}\vec{I} \quad , \quad \vec{I}_2 = \frac{\vec{Z_1}}{\vec{Z_1}+\vec{Z_2}}\vec{I}$$

Dr. Yazid Khattabi

1

Ex: Find the equivalent impedance.

$\omega = 5$ rad/sec



$\vec{Z}_{eq} = ??$

convert the elements to impedances:

$R \longrightarrow R$

$L \longrightarrow j\omega L$

$C \longrightarrow -j\frac{1}{\omega C}$

$(6)(-j0.4) \triangleq 0.3991\angle -86.18^\circ$

- $6 \parallel -j0.4 = \dfrac{(6)(-j0.4)}{6 - j0.4} \triangleq \boxed{0.3991\angle -86.18^\circ}$.

- $0.3991\angle -86.18^\circ + j10 - j \triangleq 8.602\angle 89.823^\circ$

- $(8.602\angle 89.823^\circ) \parallel 10 \triangleq 6.511\angle 49.19^\circ$

$$\triangleq 4.255 + j4.93 \ \Omega$$

equivalent to

$R_{eq} = 4.255$ $\quad$ $L_{eq} = \dfrac{4.93}{5}$

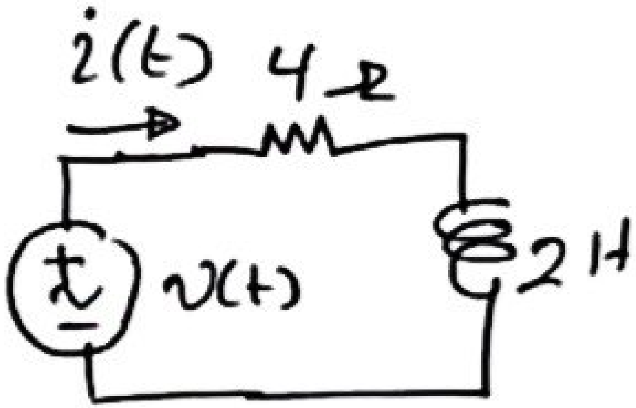
"lagging impedance"

Dr. Yazid Khattabi

Ex: Find $i(t)$

(a) using time-domain.

(b) using complex-domain.



$$v(t) = 10\cos(50t + 20^\circ)$$

sol: (a) using time-domain.

KVL: $-v(t) + 4i(t) + 2\frac{di}{dt} = 0$

$$\Rightarrow \quad 2\frac{di}{dt} + 4i(t) = 10\cos(50t + 20^\circ)$$

we need to solve 1st order diff-equation !!

$\Rightarrow$ proposed solution:

$$i(t) = I_m \cos(50t + \phi)$$

$\Rightarrow$ proposed solution:

$$i(t) = I_m \cos(50t + \phi)$$

$$\Rightarrow \underbrace{-100 I_m}_{=B} \sin(50t + \phi) + \underbrace{4 I_m}_{=A} \cos(50t + \phi)$$

$$= 10 \cos(50t + 20^\circ)$$

<u>using</u>: $A \cos(\alpha) + B \sin(\alpha) = \sqrt{A^2 + B^2} \cos(\alpha - \tan^{-1}(B/A))$

$$\therefore \Rightarrow \sqrt{16 I_m^2 + 10^4 I_m^2} \cos\left(50t + \phi - \tan^{-1}\left(\frac{-100 \cancel{I_m}}{4 \cancel{I_m}}\right)\right)$$

$$= 10 \cos(50t + 20^\circ)$$

$$\therefore \Rightarrow \sqrt{10^4 + 16}\, I_m \cos(50t + \phi + 87.7^\circ)$$

$$= 10 \cos(50t + 20^\circ)$$

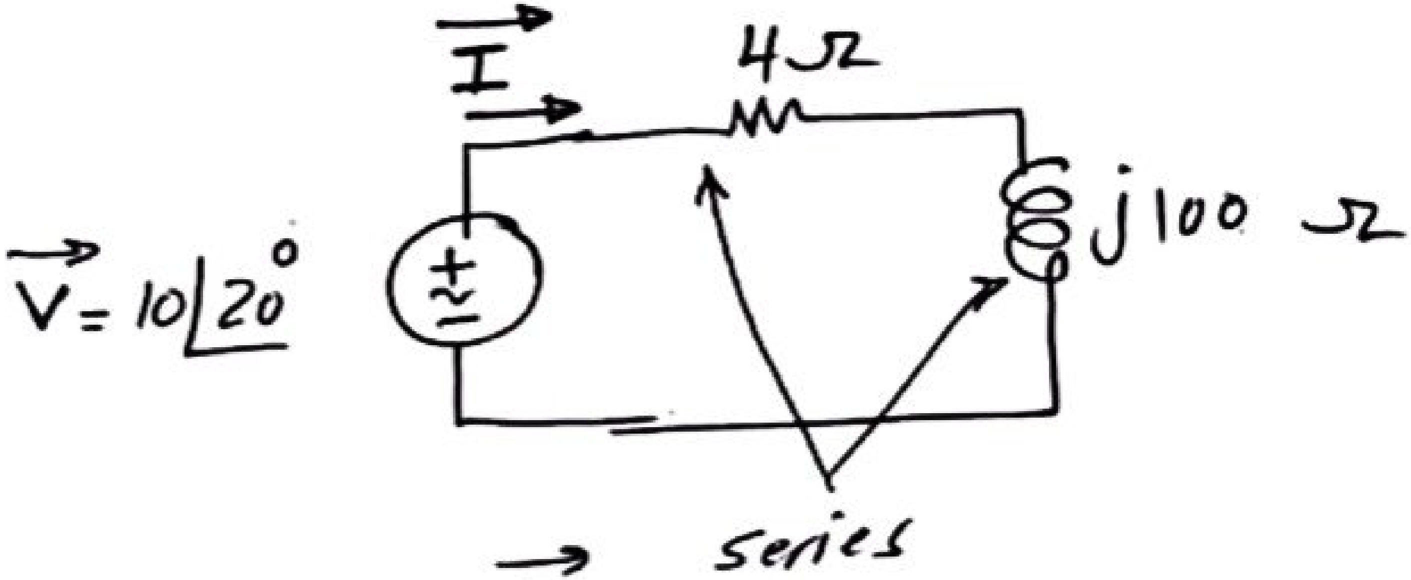
$$\therefore \Rightarrow I_m = \frac{10}{\sqrt{10^4 + 16}} = 0.1 \quad , \quad \phi = 20^\circ - 87.7^\circ = 67.7^\circ$$

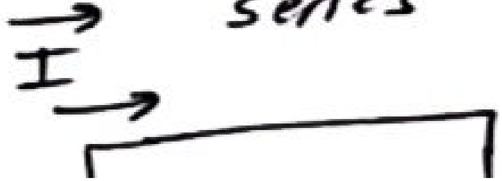
(b) using complex domain:

$$R = 4\,\Omega \longrightarrow \vec{Z}_R = 4\,\Omega$$

$$L = 2\,H \longrightarrow \vec{Z}_L = j\omega L = j100\,\Omega$$

$$v(t) = 10\cos(50t + 20^\circ) \rightarrow \vec{V} = 10\angle 20^\circ \;\; V$$

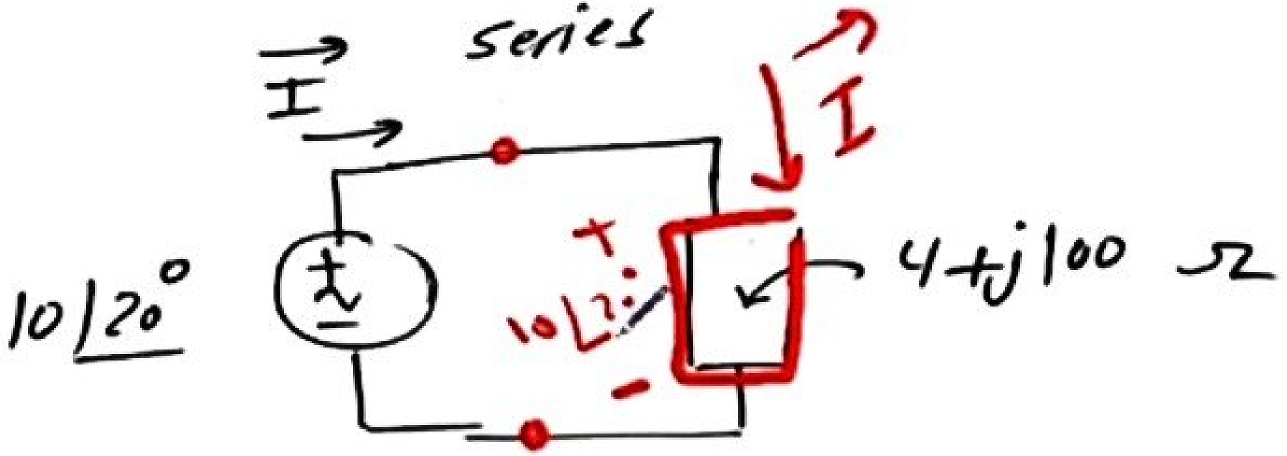




Ω's la

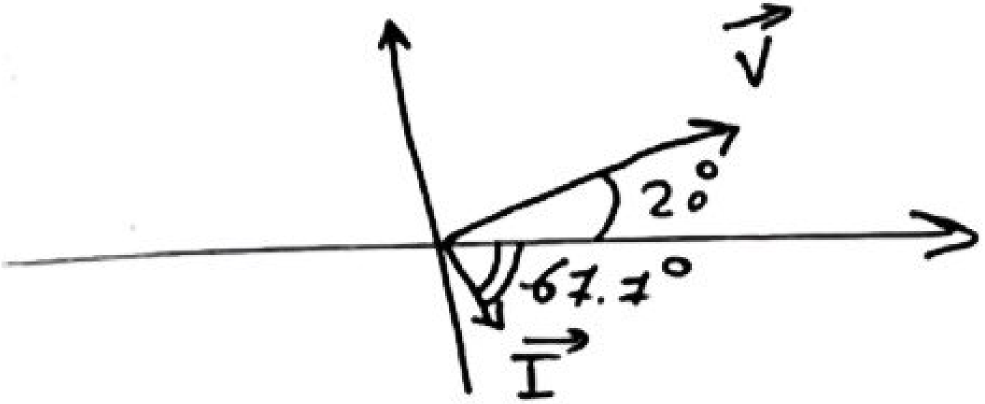
$$\vec{I} = \frac{10\angle 20^\circ \ \text{V}}{4+j100\ \Omega}$$

$$= \boxed{0.1\angle -67.7^\circ}\ \text{A}$$

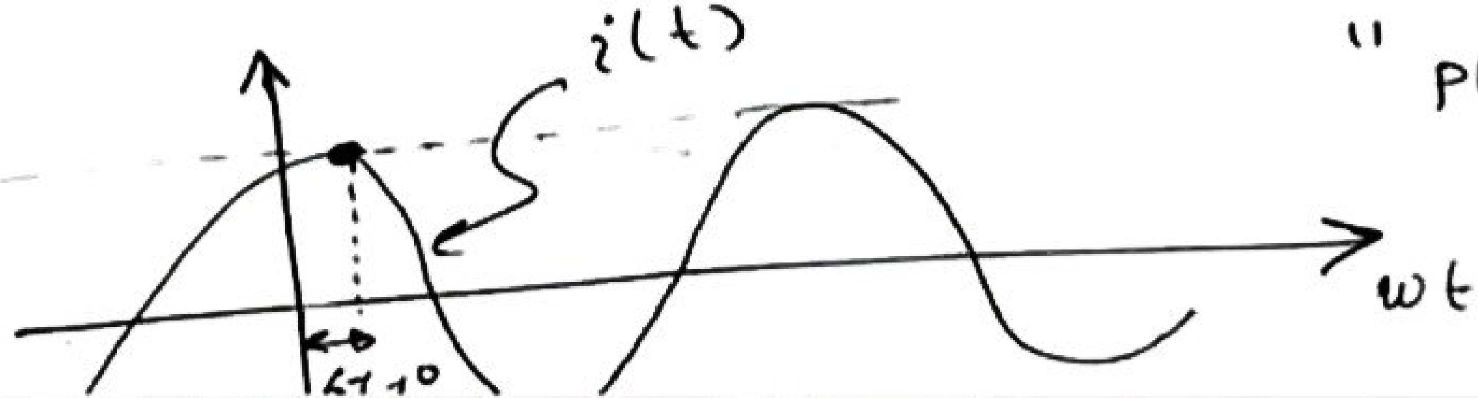


$$\Rightarrow i(t) = 0.1\cos(50t - 67.7^\circ)\ \text{A}$$ ✓

i lags v by 67.7°



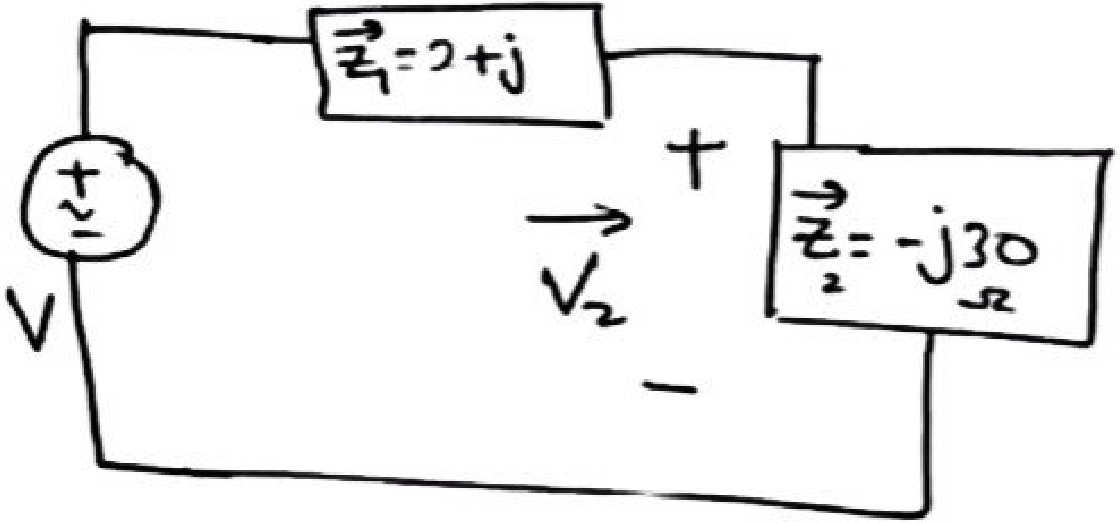
"Phasor Diagram"



Note: In complex domain, we can apply all circuit analysis techniques we have learnt in CH#1 $\longrightarrow$ CH#5 but we deal will complex quantities.

Ex:

$\vec{V}_s = 20\angle 30^\circ$ V

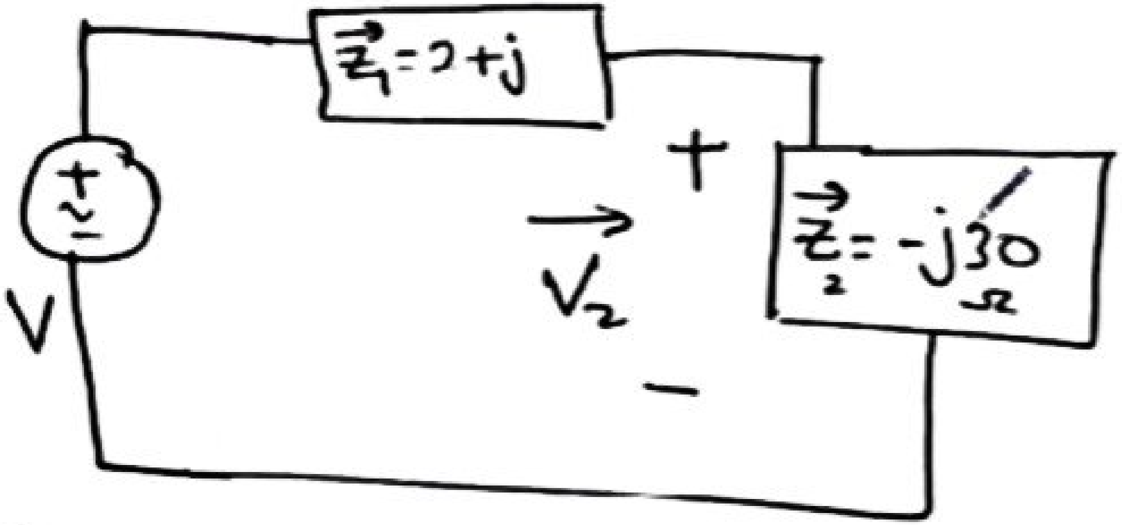


Find $\vec{V}_2$??

sol: By voltage division:

Ex:

$\vec{V}_s = 20\angle 30^\circ \ V$



Find $\vec{V}_2$??

sol: By voltage division:

$$\vec{V}_2 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{V}_s = \frac{-j30}{(2+j)+(-j30)} 20\angle 30^\circ$$

$$\triangleq 19.224 + j9.34$$

$$\triangleq 21.374\angle 25.91^\circ$$

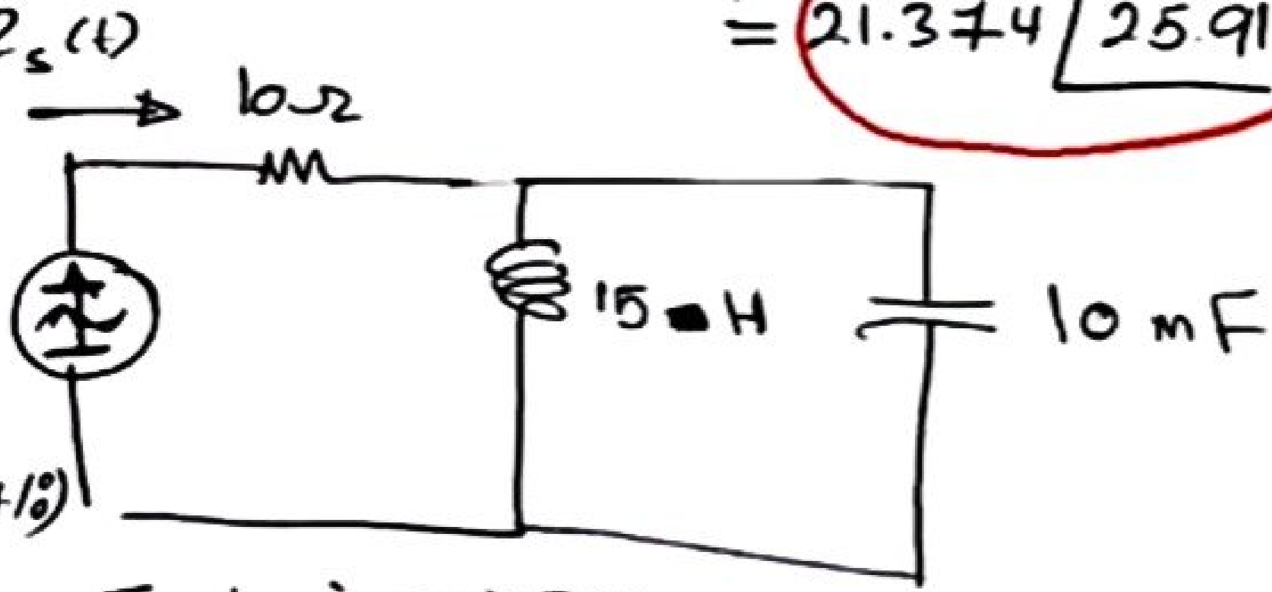
$i_s(t)$

10Ω

Ex:

$\triangleq 21.374\angle 25.91^\circ$

Ex:

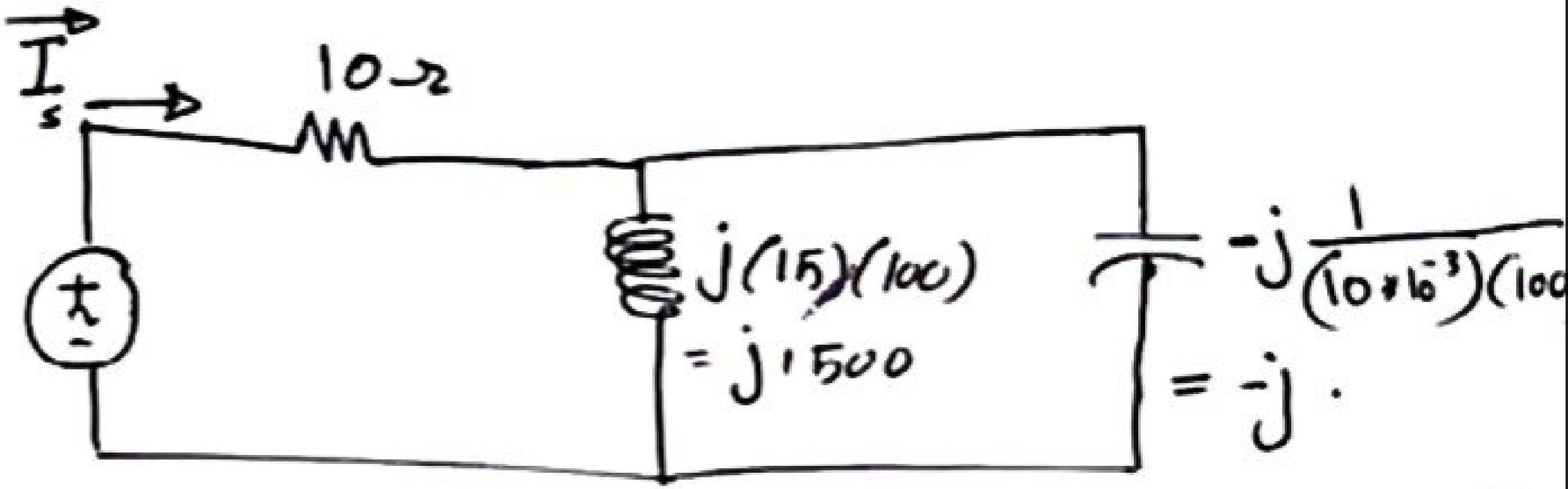


$v_s(t) =$
$20\cos(100t+10^\circ)$

"ω"

Find $i_s(t)$??

Sol:



$\vec{V}_s = 20\angle 10^\circ$

(10)

$$\vec{I}_s = \frac{\vec{V}_s}{\vec{Z}_{eq}}, \quad \vec{Z}_{eq} = \left(-j \,//\, j1500\right) + (10)$$

$$= \frac{(-j)(j1500)}{-j + j1500} + 10$$

$$\simeq 10 - j1 \simeq 10.05\angle -5.71^\circ$$

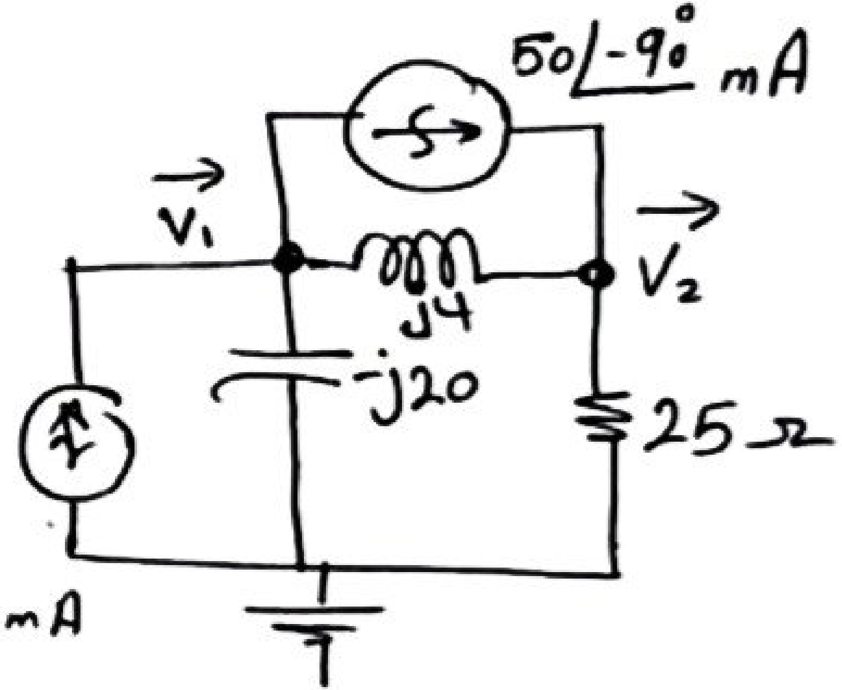
$$\vec{I}_s = \frac{20\angle 0^\circ}{10.05\angle -5.71^\circ} = 2\angle 5.71^\circ$$

$$\cancel{\Rightarrow i_s(t) = 10.05\angle -5.}$$

$$\Rightarrow \boxed{i_s(t) = \cancel{10.05\cos(100 - 5.71^\circ)}}$$

Ex: ~~[illegible]~~ Nodal analysis

Find $\vec{V}_1$ & $\vec{V}_2$ using Nodal analysis.



$= 20\angle 0^\circ$ mA

sol: Node ①:

$$20\,mA = \frac{\vec{V}_1 - 0}{-j20} + \frac{\vec{V}_1 - \vec{V}_2}{j4} + 50\angle -9^\circ\,mA$$

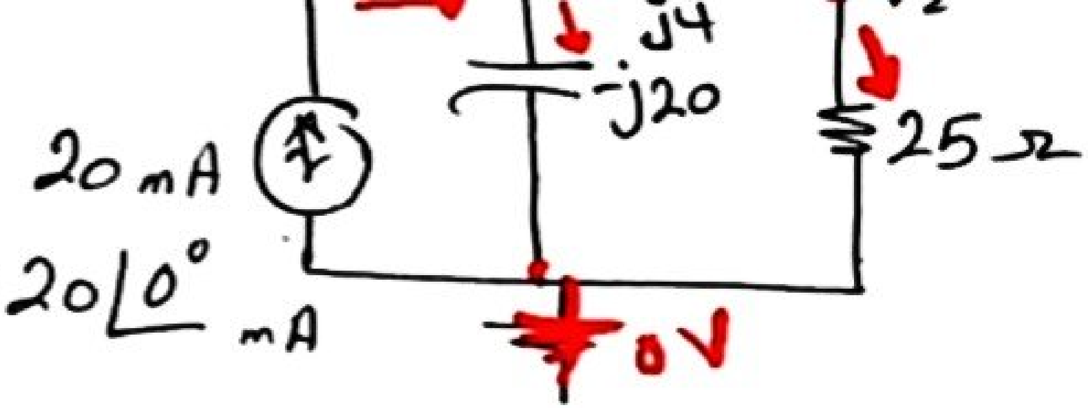
$$\Rightarrow \left(\frac{1}{-j20} + \frac{1}{j4}\right)\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 20\,mA - 50\angle -9^\circ\,mA$$

$$\boxed{-j\frac{1}{5}\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 53.85\angle 68.19^\circ} \quad (1)$$

Node #2: $50\angle -9^\circ\,mA = \frac{\vec{V}_2 - \vec{V}_1}{j4} + \frac{\vec{V}_2 - 0}{}$

Find $\vec{V}_1$ & $\vec{V}_2$

using Nodal analysis.



$= 20\angle 0^\circ$ mA

sol: Node ①:

$$20\,mA = \frac{\vec{V}_1 - 0}{-j20} + \frac{\vec{V}_1 - \vec{V}_2}{j4} + 50\angle -9^\circ\ mA$$

$$\Rightarrow \left(\frac{1}{-j20} + \frac{1}{j4}\right)\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 20\,mA - 50\angle -9^\circ\ mA$$

$$\boxed{-j\frac{1}{5}\vec{V}_1 - \frac{1}{j4}\vec{V}_2 = 53.85\angle 68.19^\circ} \quad (1)$$

Node #2:

$$50\angle -9^\circ\ mA = \frac{\vec{V}_2 - \vec{V}_1}{j4} + \frac{\vec{V}_2 - 0}{25}$$

$$\Rightarrow \boxed{j0.25\vec{V}_1 + 0.253\angle -80.91\ \vec{V}_2 = -50\angle -9^\circ\ mA} \quad (2)$$

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Example 10.3

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0$$

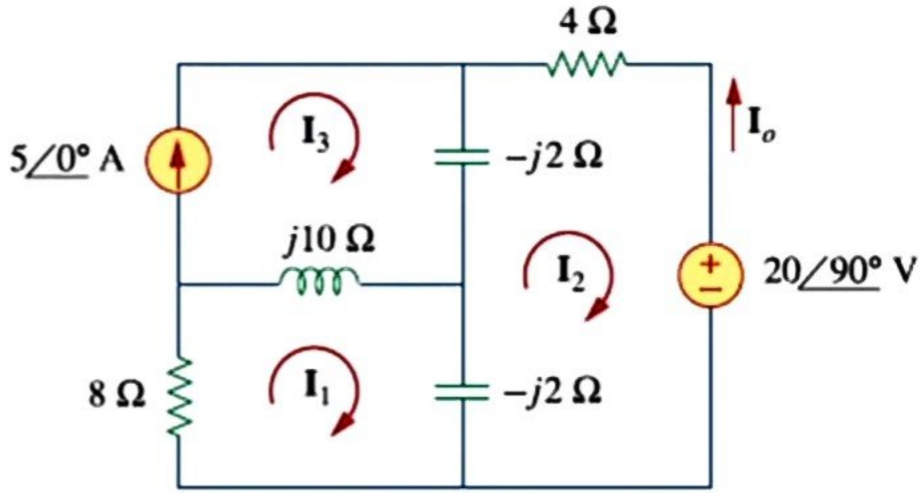
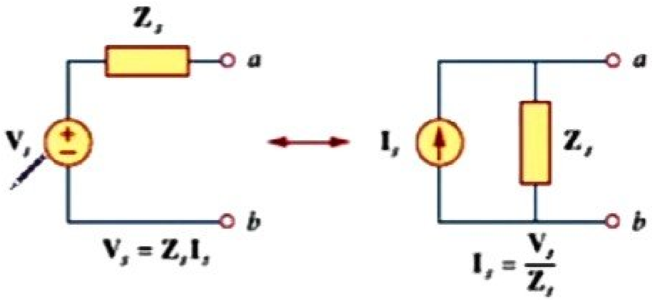


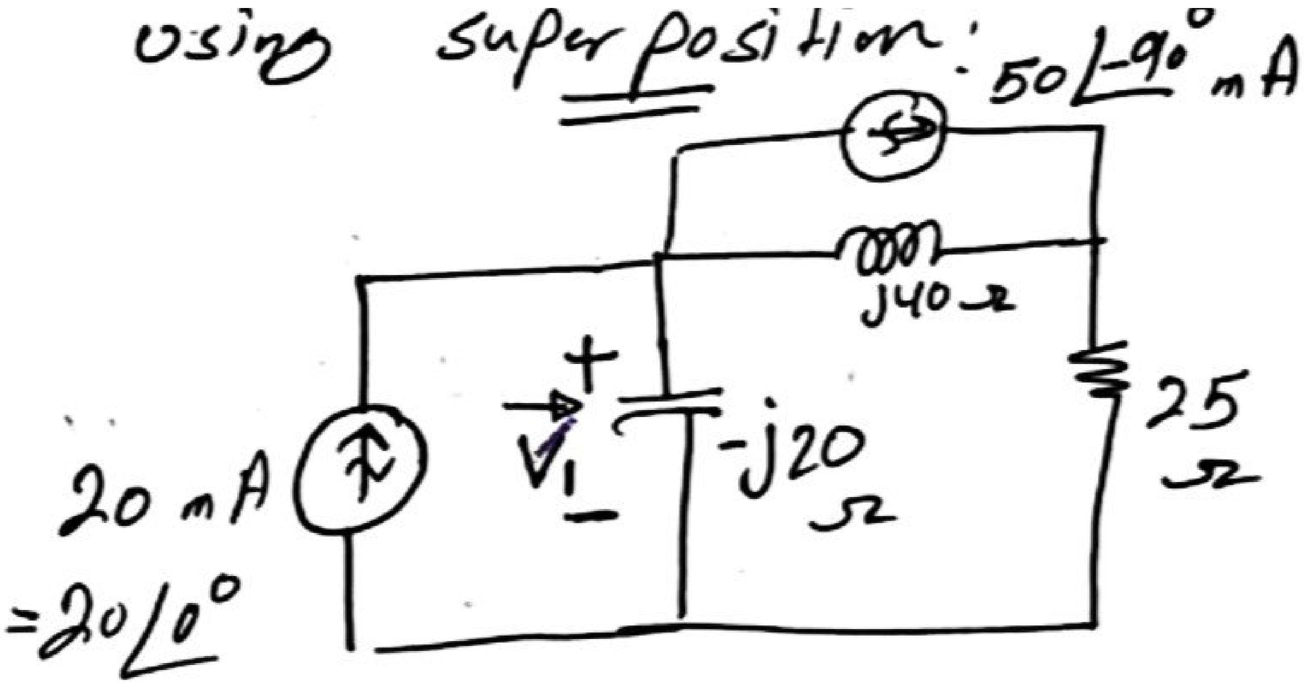
Figure 10.7
For Example 10.3.

Source Transformation

$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

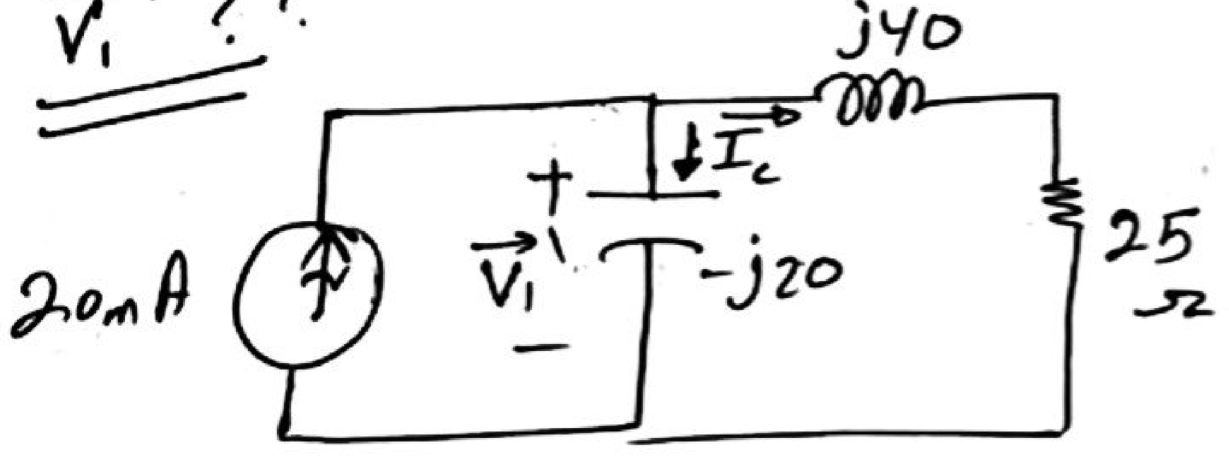


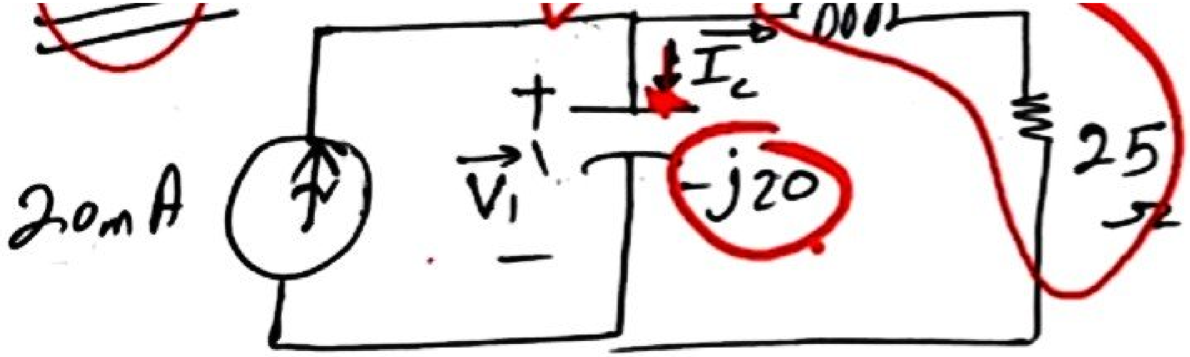
Ex: Find $\vec{V_1}$ using superposition:



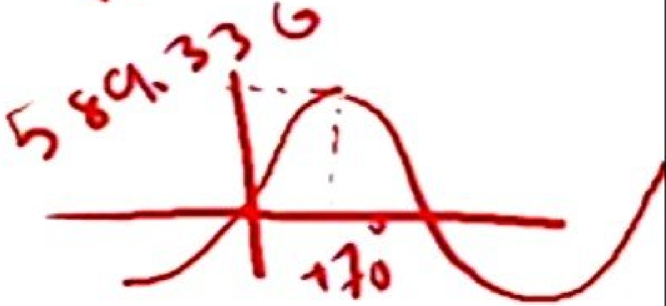
sol: $\vec{V_1} = \vec{V_1}' + \vec{V_1}''$

$\vec{V_1}'$??





Curr
Divi



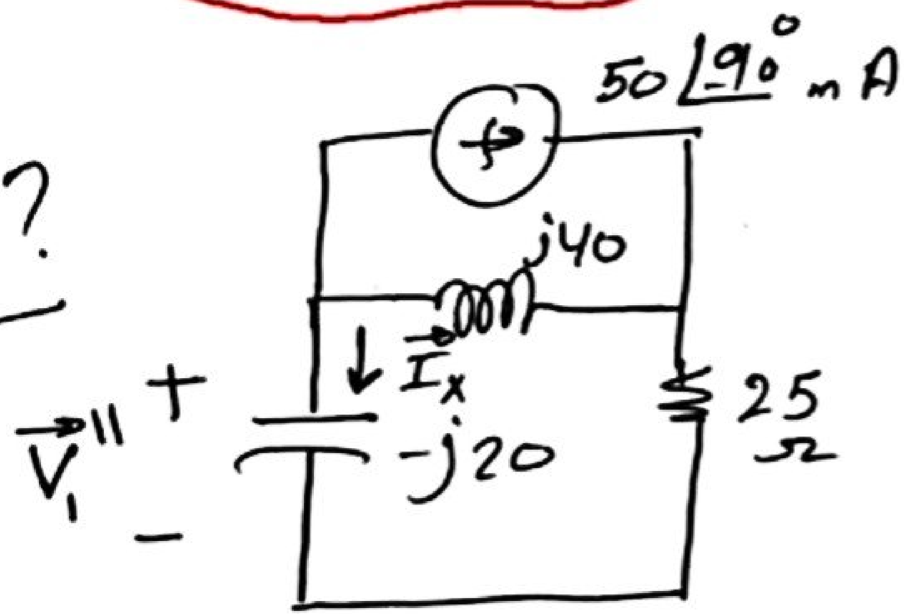
$$\vec{I}_c = \frac{(25+j40)}{(25+j40)-j20}\,20\,mA$$

$$= 29.467\angle 19.334^\circ \; mA$$

$$\Rightarrow \vec{V}_1' = (-j20)\,\vec{I}_c$$

$$= 589.336\angle -70.66^\circ \; mA$$

$\vec{V}_1''$??

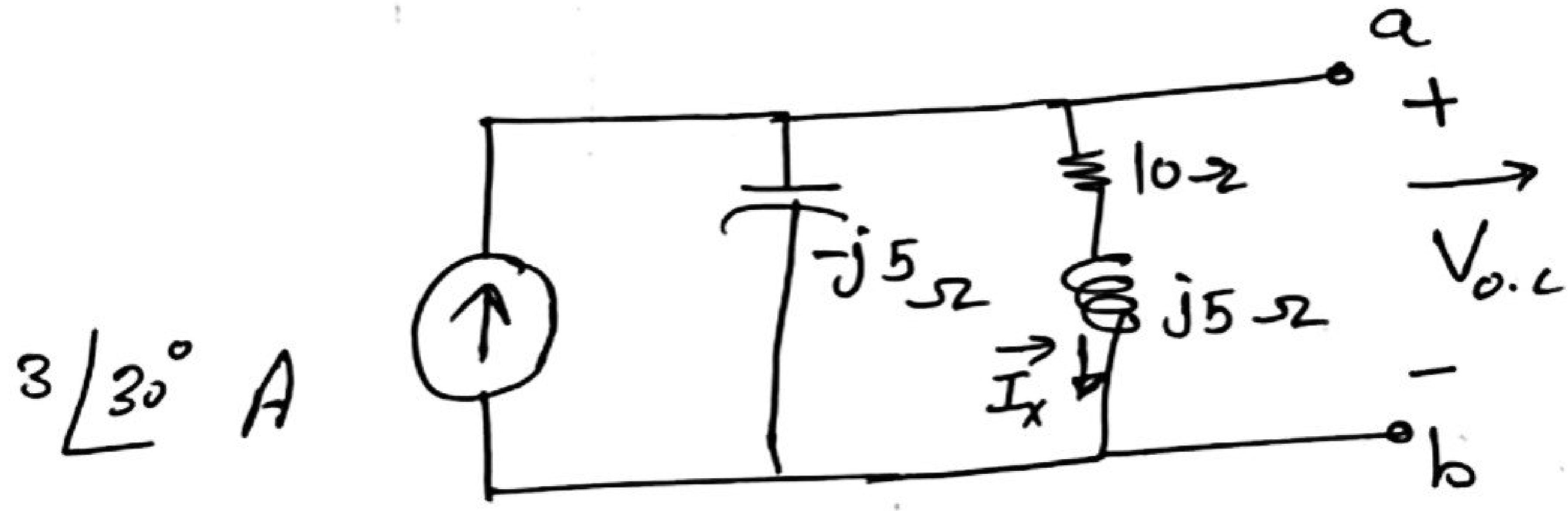


$$\vec{I}_x = \frac{(j40)(50\angle -90^\circ)\,mA}{j40+(25-j20)}$$

$$= \text{- - - - - -}$$

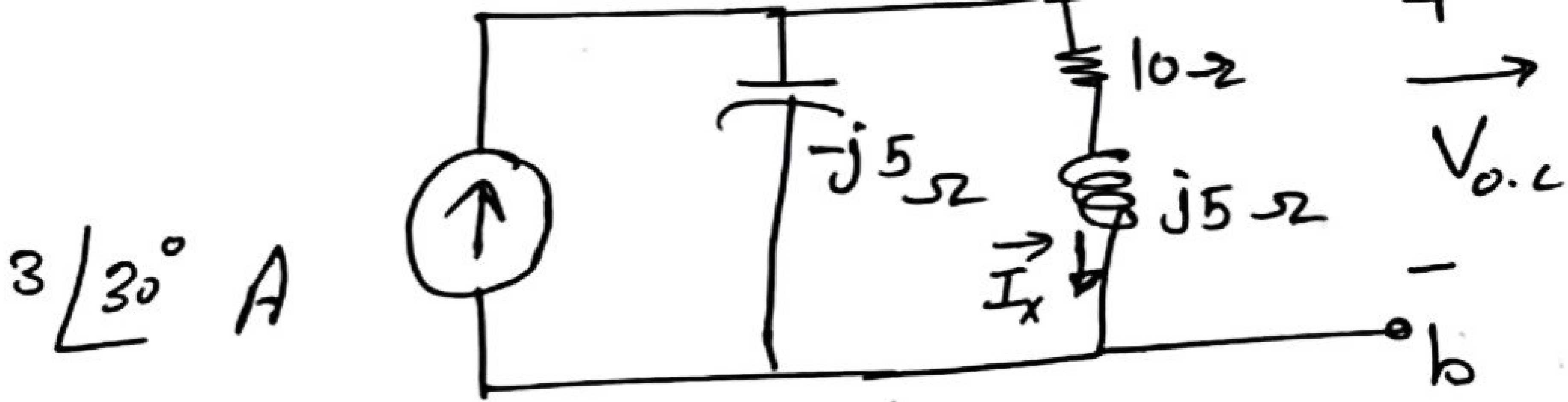
Ex: For the CKT shown find the Thevenin equivalent seen to the left of a and b.

~~Then find $\vec{Z}_L$ for max power~~



$$\vec{Z}_{th} = (-j5) \,//\, (10+j5) = \frac{(-j5)(10+j5)}{-j5+10+j5} = 2.5 - j5 \ \Omega$$

"kill the current source"



$$\vec{Z}_{th} \underset{\uparrow}{=} (-j5) \,//\, (10+j5) = \frac{(-j5)(10+j5)}{-j5+10+j5} = 2.5-j5\ \Omega$$

" kill the current source first "

$$\vec{V}_{th} = \vec{V}_{o.c} = (10+j5)\left[\overbrace{\frac{-j5}{-j5+10+j5}\, 3\angle 30^\circ}^{=\vec{I}_x}\right]$$