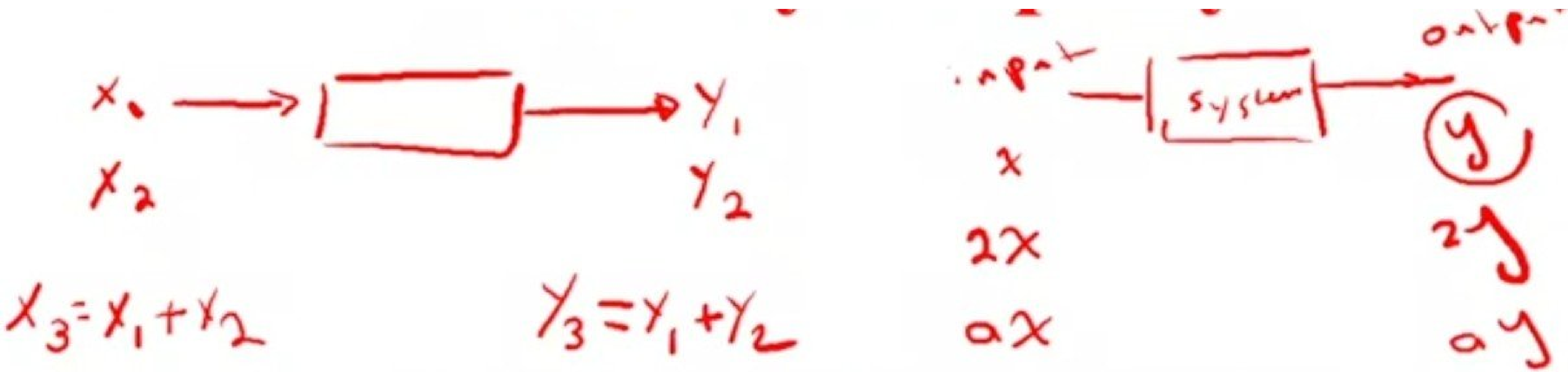


- ❑ Linearity is the property of an element describing a linear relationship between cause (input) and effect (response or output).

- ❑ The linearity property is a combination of both:
  - ➢ the homogeneity (scaling) property and
  - ➢ the additivity property

- The homogeneity property requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) is multiplied by the same constant.
- Check the homogeneity for the resistor:

Ohm's law relates the input i to the output v,

$$v = iR$$

If the current is increased by a constant k, then the voltage increases correspondingly by Type equation here.k; that is,

$$kiR = kv$$

Linearity Property

- The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.
- Check the additivity for the resistor:

$$v_1 = i_1 R$$

and

$$v_2 = i_2 R$$

then applying $(i_1 + i_2)$ gives

$$v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

- As a conclusion, we say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

- Also, the capacitors and the inductors are linear elements because their current-voltage relationship is linear.

Linearity Property

- The independent source are linear elements.

- The dependent source is linear if its output current or voltage is proportional only to *the first power* of a specified current or voltage variable in the circuit

- A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

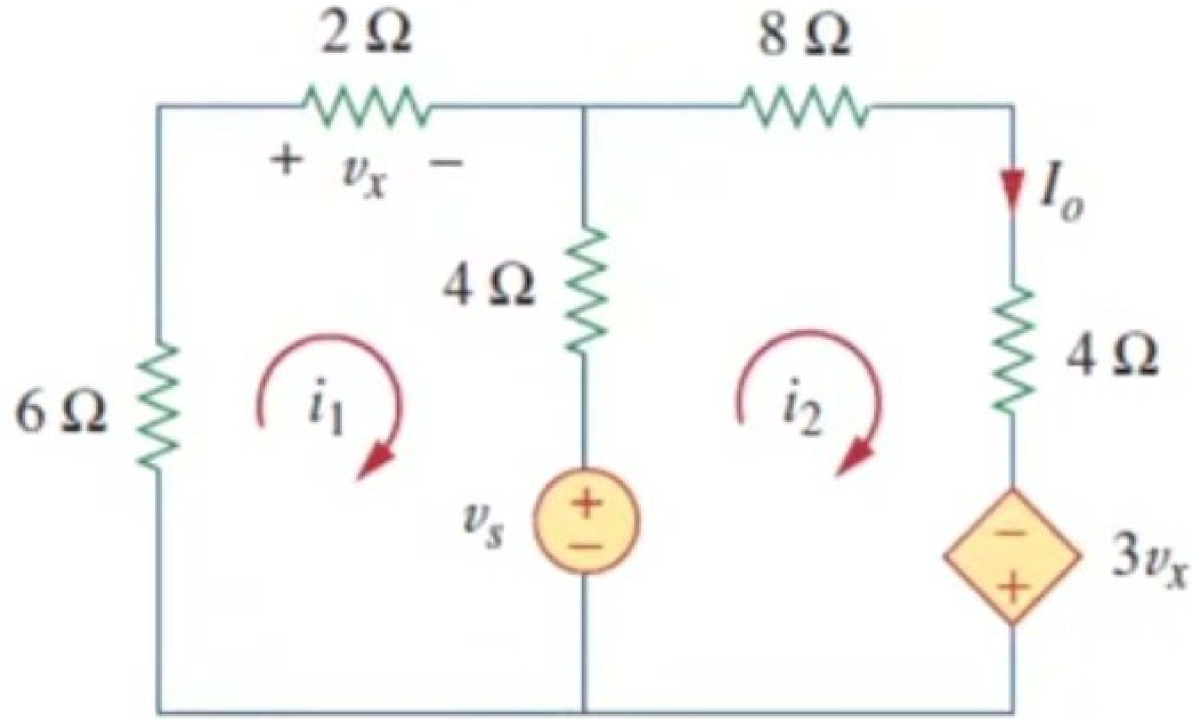
Linearity Property

- A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

- A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Linearity Property

- Example: in the following circuit, it has been found through some circuit analysis that the current $I_o = \frac{12}{76}$ when $v_s = 12$ V. If $v_s = 24$ V, what is I_o ??

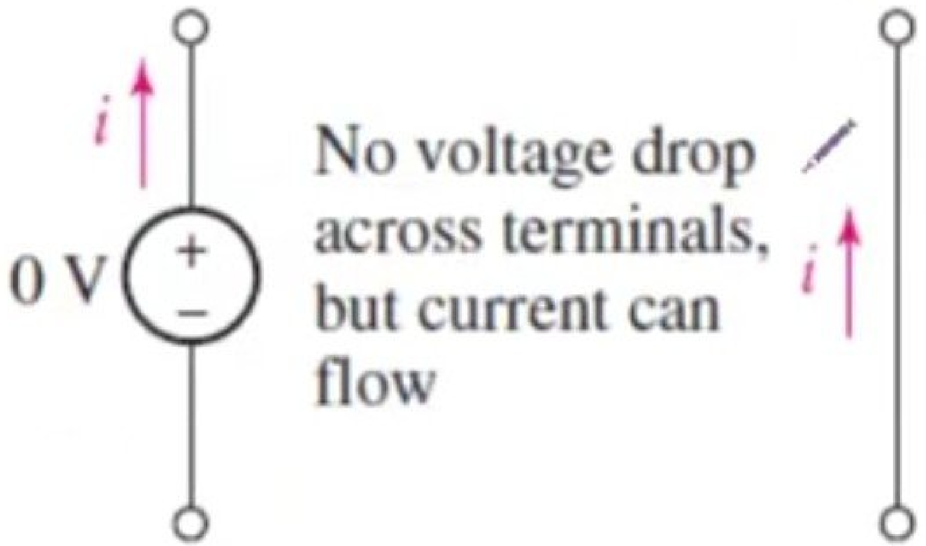


- **Techniques that simplify the analysis of linear circuits:**
  - ✓ Superposition
  - ✓ Source Transformation
  - ✓ Thevenin's Theorem
  - ✓ Norton's Theorem

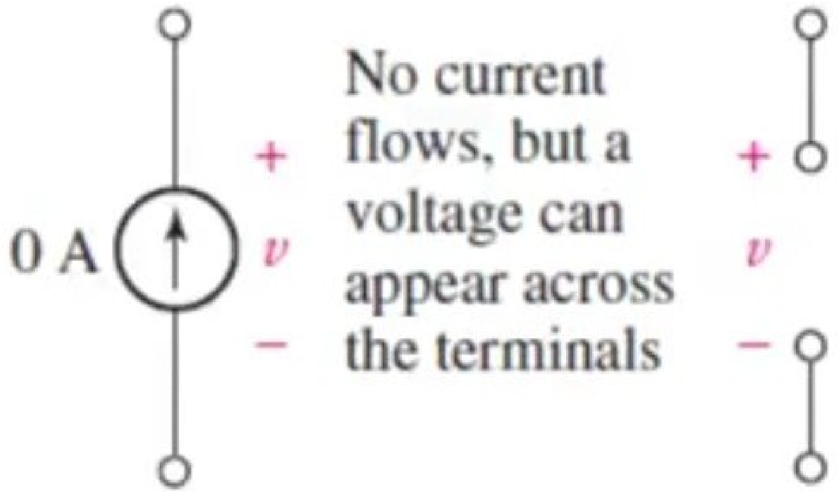
- The superposition is a technique applied to analyze linear circuits with <u>two or more independent sources</u>.

- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element <u>due to each independent source acting alone.</u>

- To kill (turn off) an independent voltage source, replace it by S.C

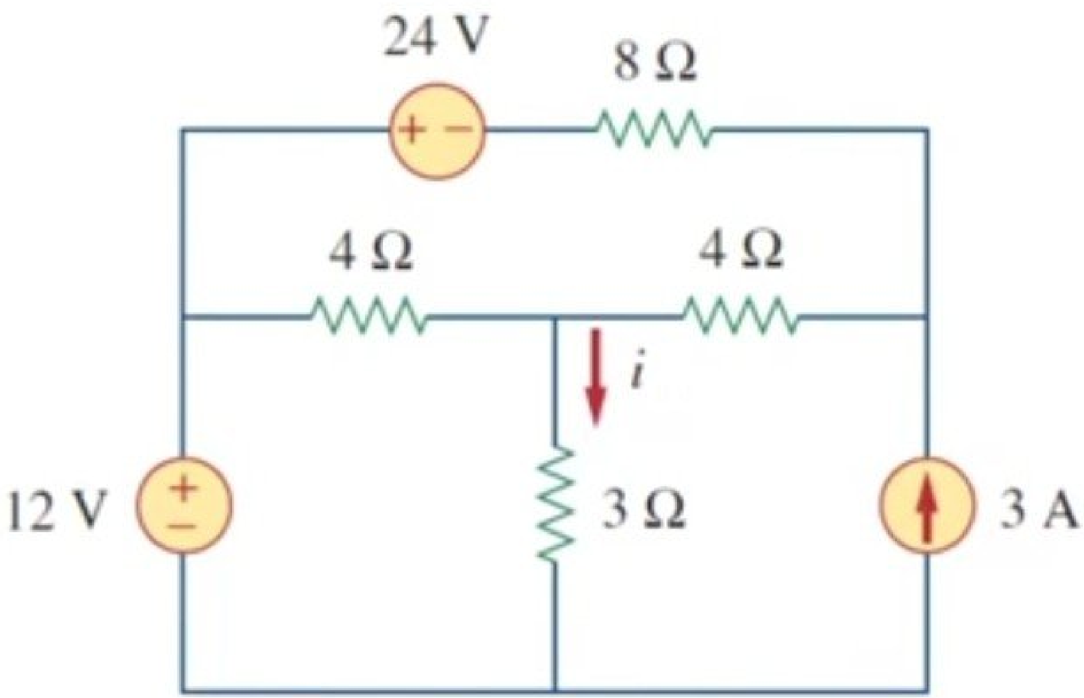


- To kill (turn off) an independent current source, replace it by O.C.



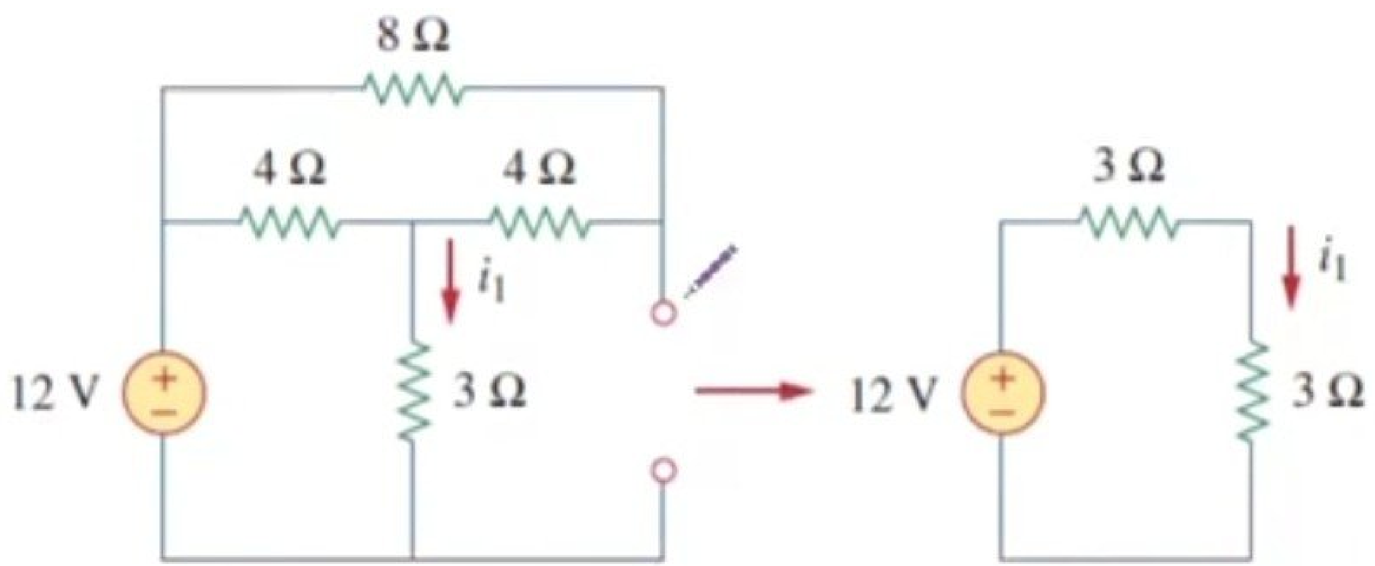
Example 4.5

use the superposition theorem to find i.



Superposition

Example 4.5 **Solution:**

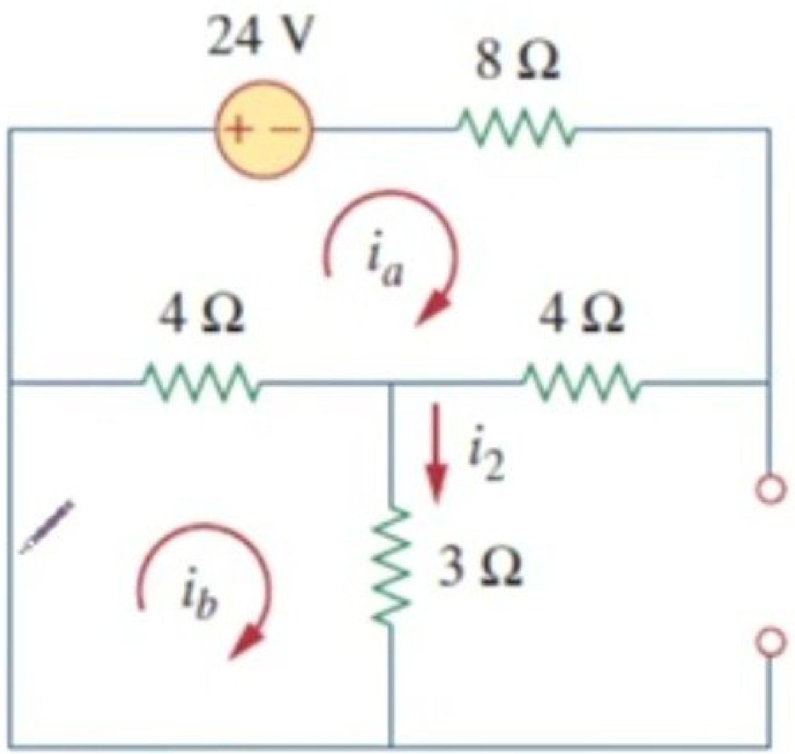


$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{12}{6} = 2\text{ A}$$

Example 4.5

Solution:



mesh analysis

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6$$

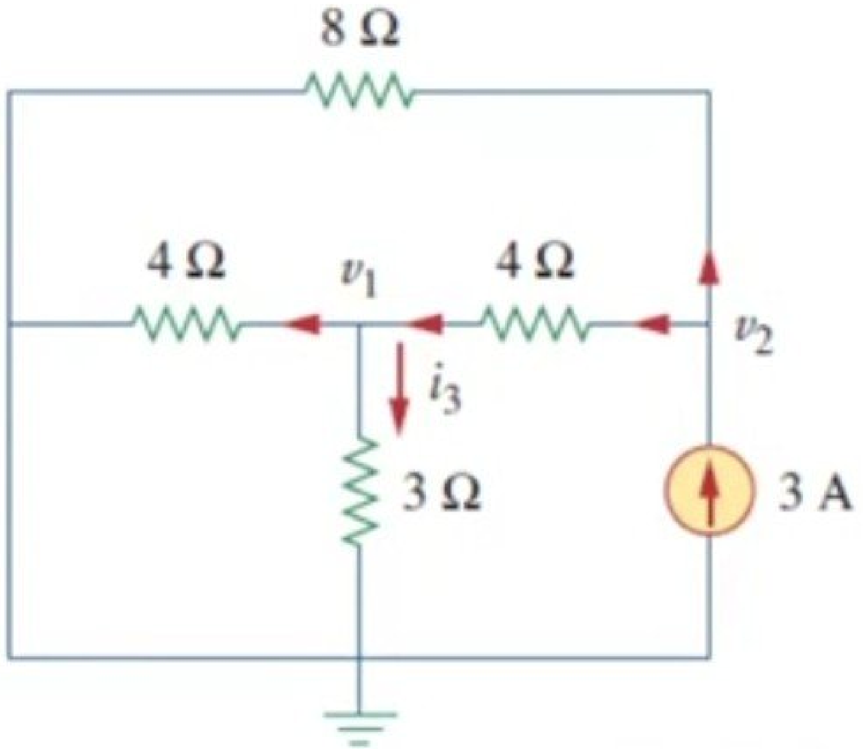
$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b$$

$$i_2 = i_b = -1$$

Superposition

Example 4.5

Solution:



nodal analysis

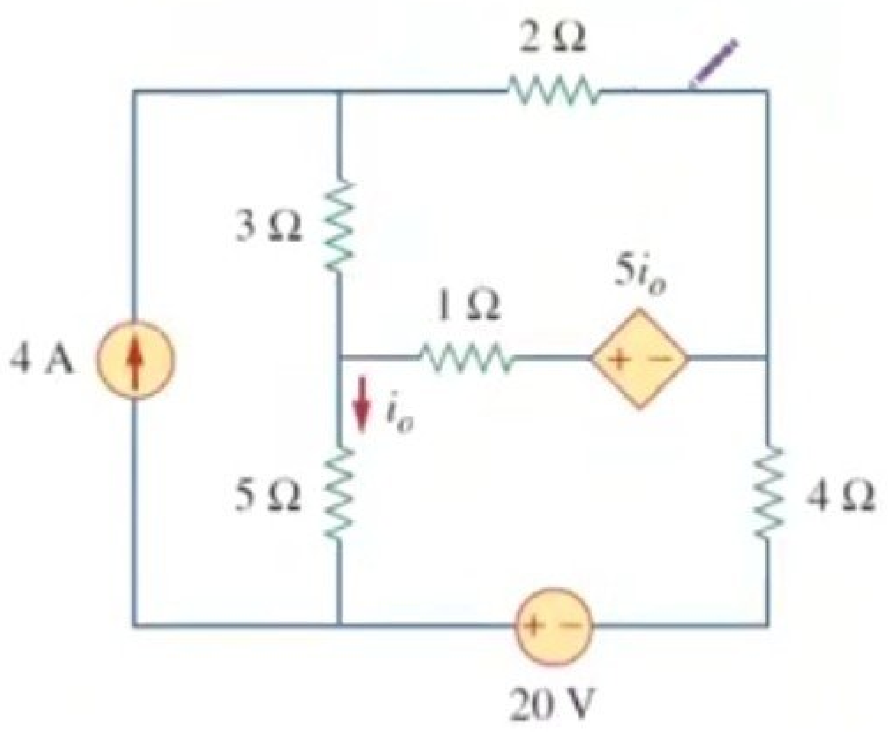
$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1$$

$$i_3 = \frac{v_1}{3} = 1\ \text{A}$$

Example 4.4

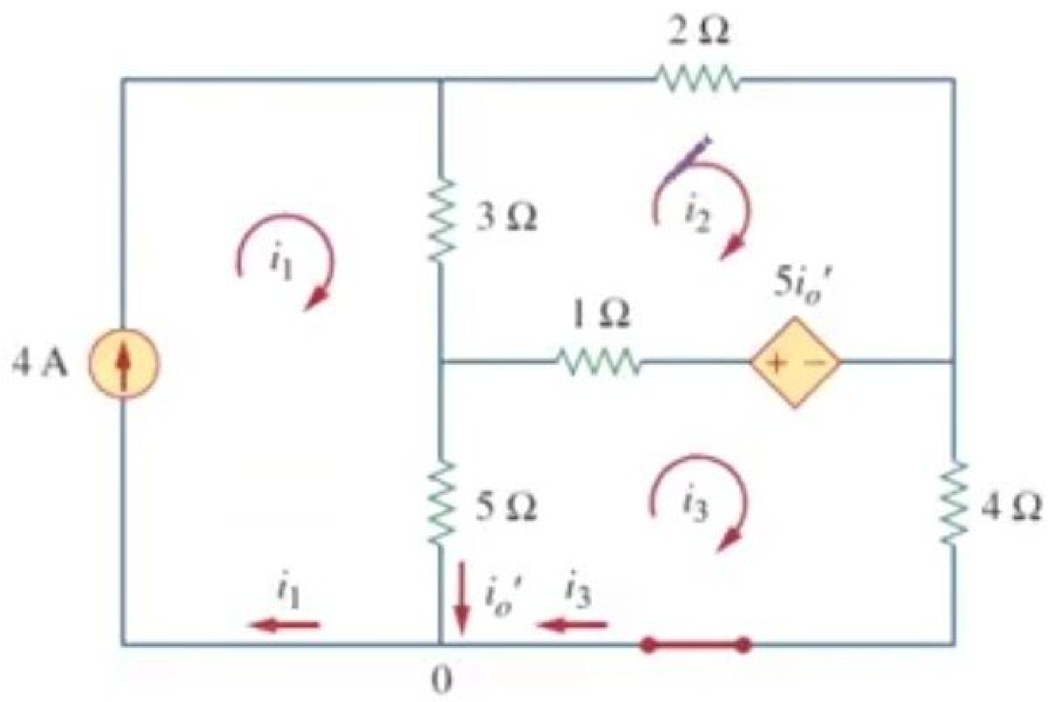
Find i_o in the circuit using superposition.



Example 4.4 **Solution:**

$$i_o = i'_o + i''_o$$

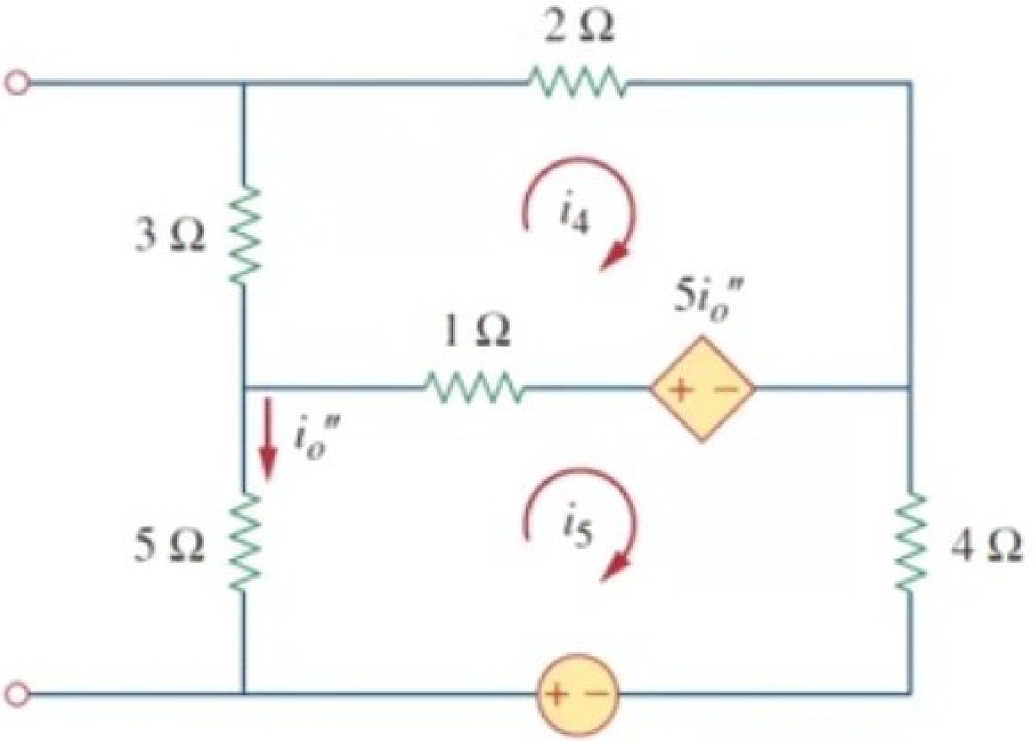
$$i'_o = \frac{52}{17}\ \text{A}$$



Superposition

Example 4.4 **Solution:**

$$i_o'' = -\frac{60}{17}\text{ A}$$

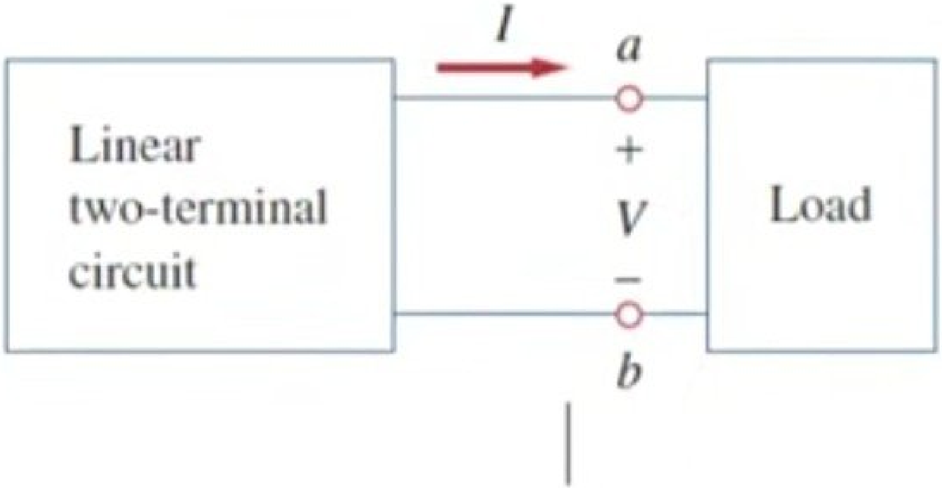


Thevenin's Theorem

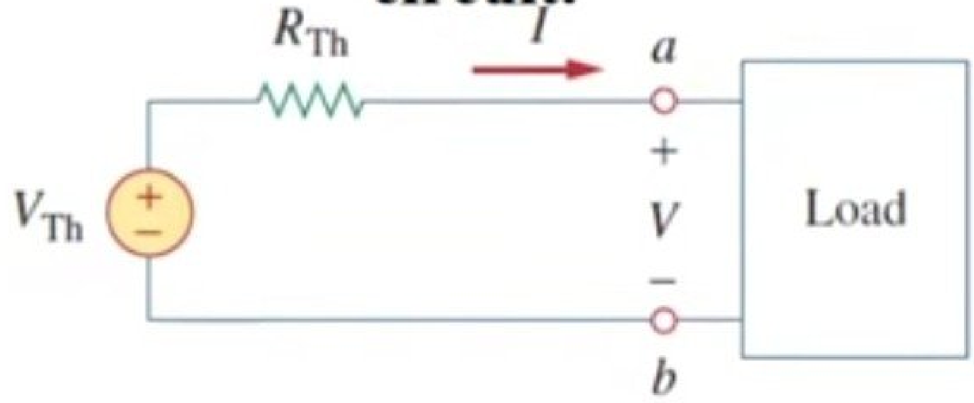
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thevenin's Theorem

The original circuit



The Thevenin equivalent circuit.



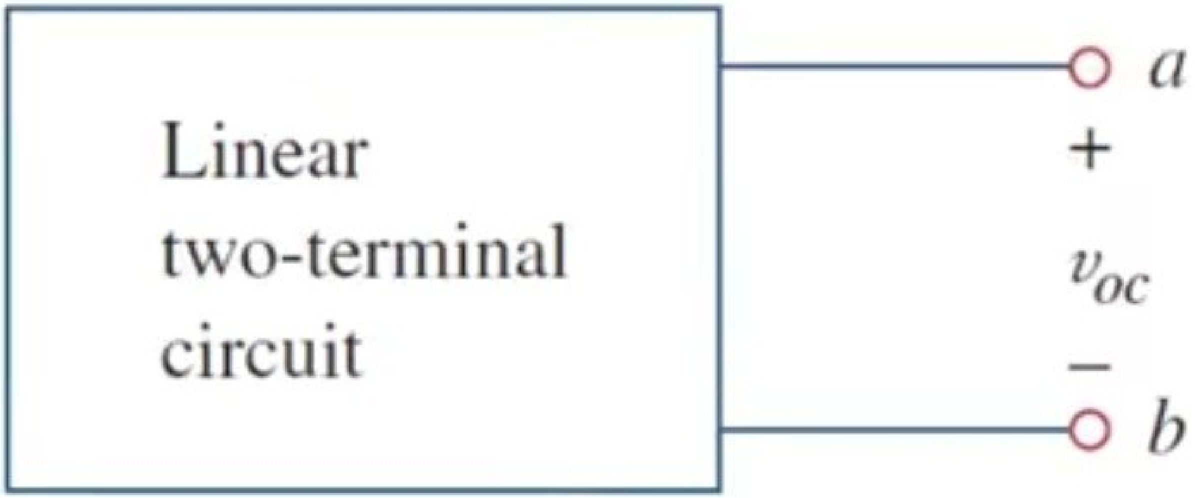
Find the Thevenon equivalent seen

- How to find V_{TH} and R_{TH}?? See next

Thevenin's Theorem

❑To find v_{TH}.

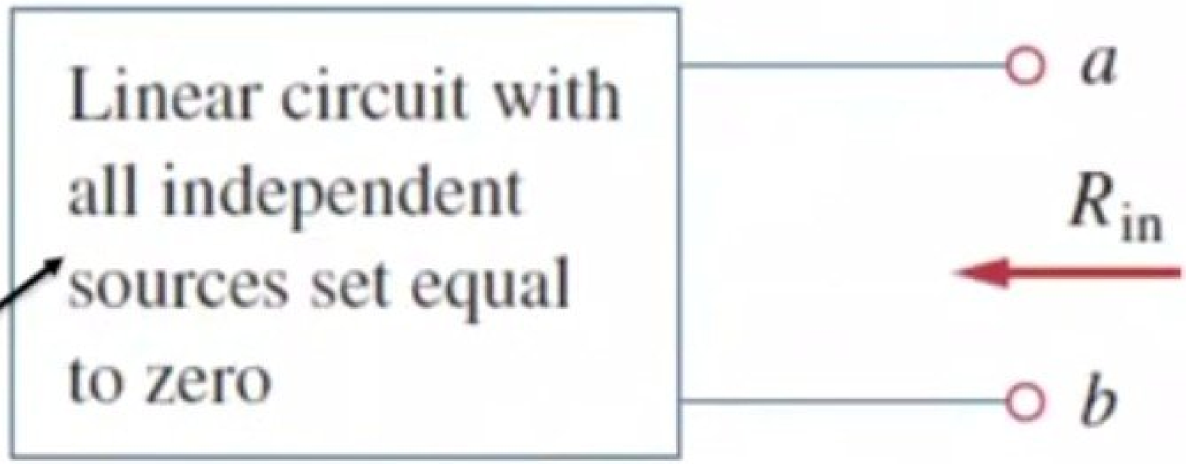
Make the terminal $a - b$ open-circuited (by removing the load), then find $v_{o.c}$.



$$V_{Th} = v_{oc}$$

❑There are cases to find R_{TH}:

■ **CASE 1** If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b



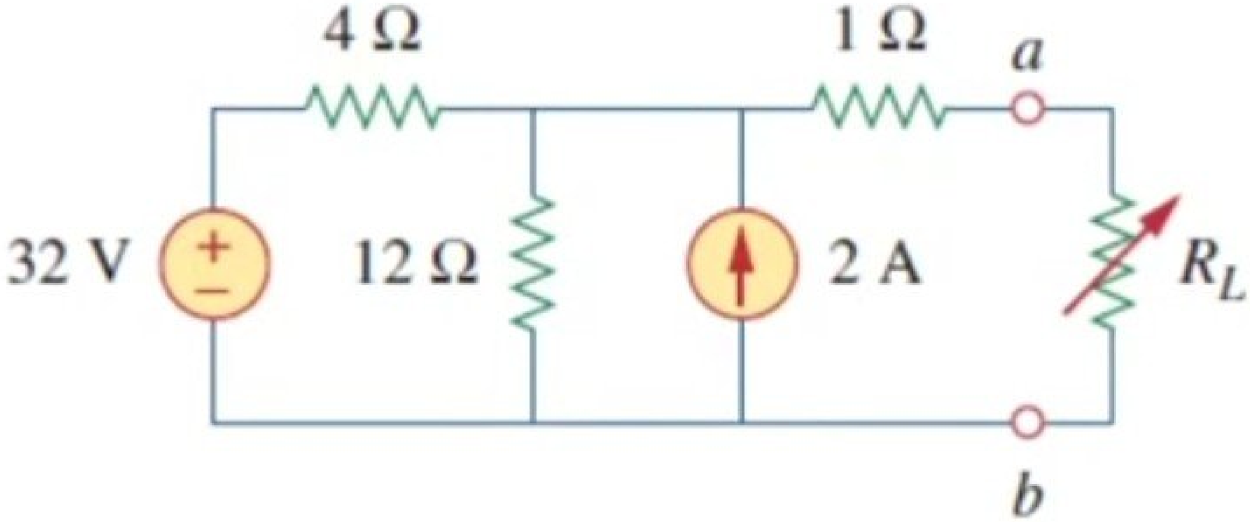
The circuit has no dependent sources. So after killing all independent sources the elements left are just passive elements

$$R_{Th} = R_{in}$$

Thevenin’s Theorem

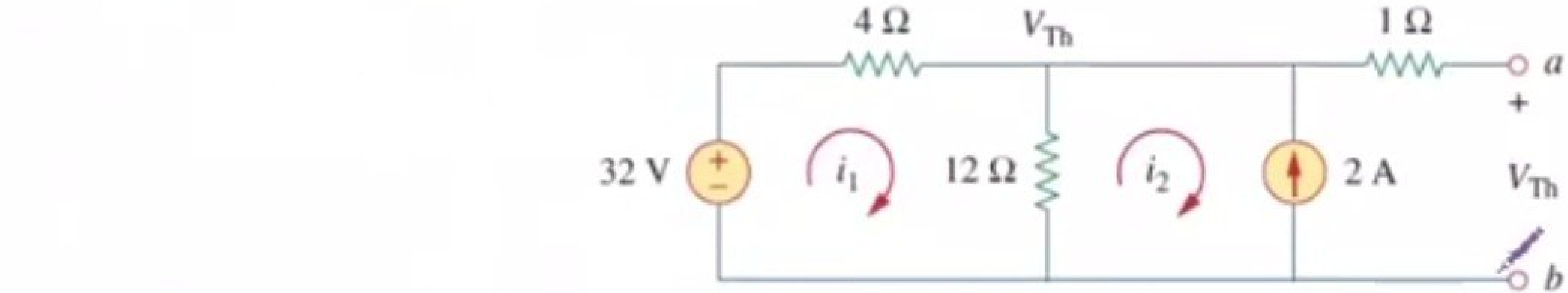
Example 4.8

Find the Thevenin equivalent circuit of the circuit shown
Then find the current through $R_L = 6$,



Thevenin's Theorem

Example 4.8 **Solution:**



$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

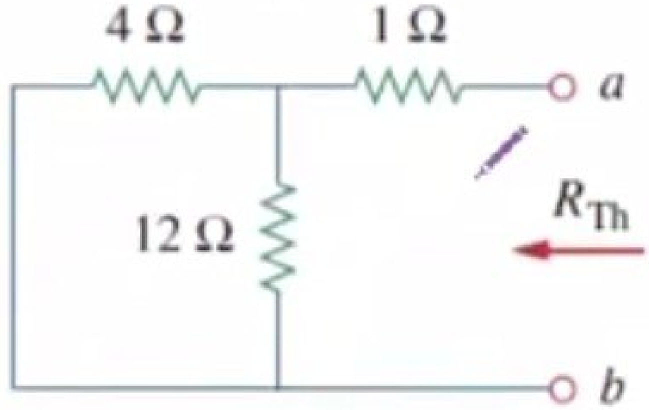
Solving for i_1, we get $i_1 = 0.5$ A.

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

Thevenin's Theorem

Example 4.8 **Solution:**

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

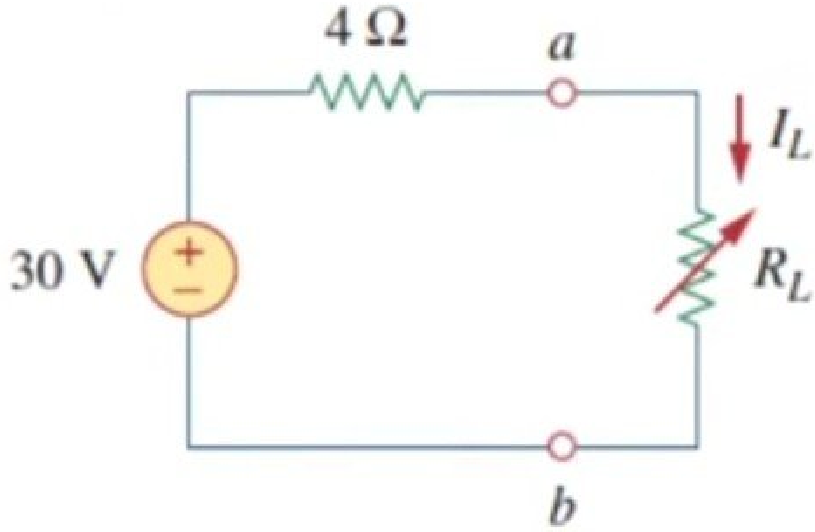


Thevenin's Theorem

Example 4.8 **Solution:**

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

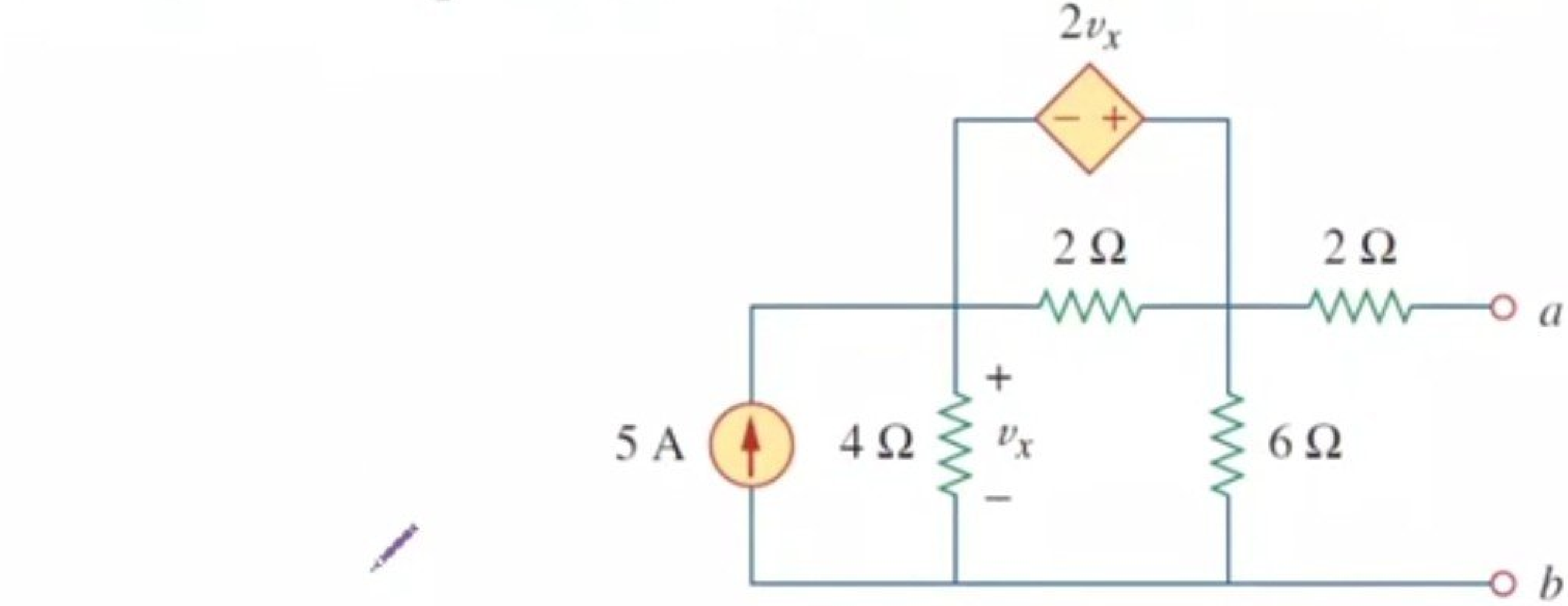
$$I_L = \frac{30}{10} = 3\ A$$



Thevenin's Theorem

Example 4.9

Find the Thevenin equivalent of the circuit at terminals *a*-*b*.



Thevenin's Theorem

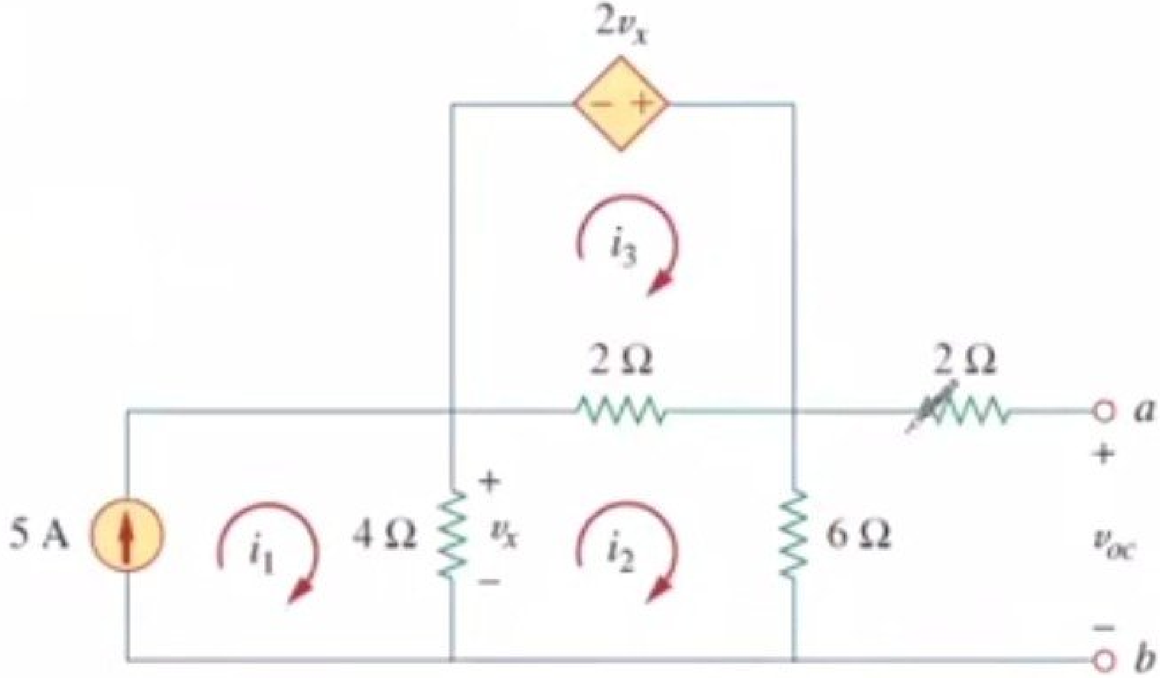
Example 4.9 **Solution:**

Find the Thevenin voltage. Analyze the circuit using mesh or nodal. Try nodal:

Supernode: $\frac{v_1}{4} + \frac{v_2}{6} = 5$
Voltage source gives:
$3v_1 - v_2 = 0$. Solving the two equations gives:

$$v_1 = \frac{20}{3}$$
$$v_2 = 20.$$

So $v_{th} = 20$. Or by mesh analysis: $V_{Th} = v_{oc} = 6i_2 = 20\text{ V}$

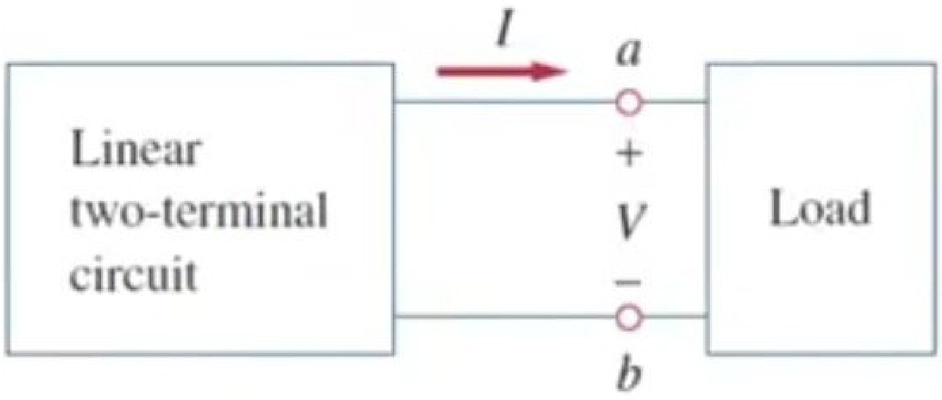


Norton's Theorem

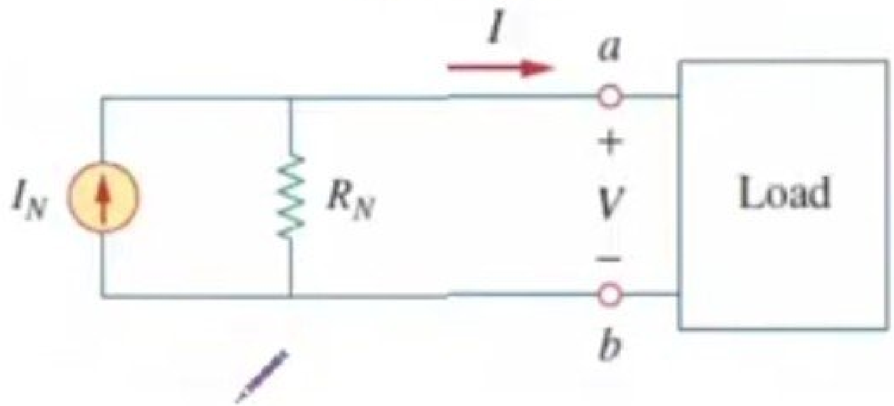
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Norton's Theorem

The original circuit



The Norton equivalent circuit.

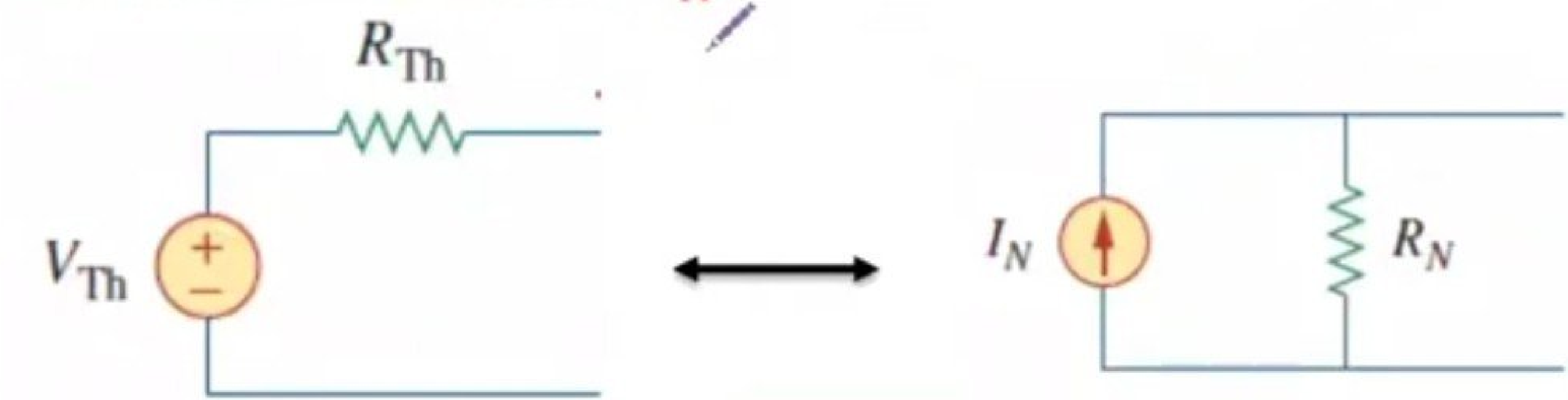


Find the Norton equivalent for the linear two terminal circuit

- How to find I_N and R_N?? See next…

Norton's Theorem

- By source transformation we can exchange between Thevenin and Norton I_N.



$$R_N = R_{Th}$$

$$I_N = \frac{V_{Th}}{R_{Th}}$$

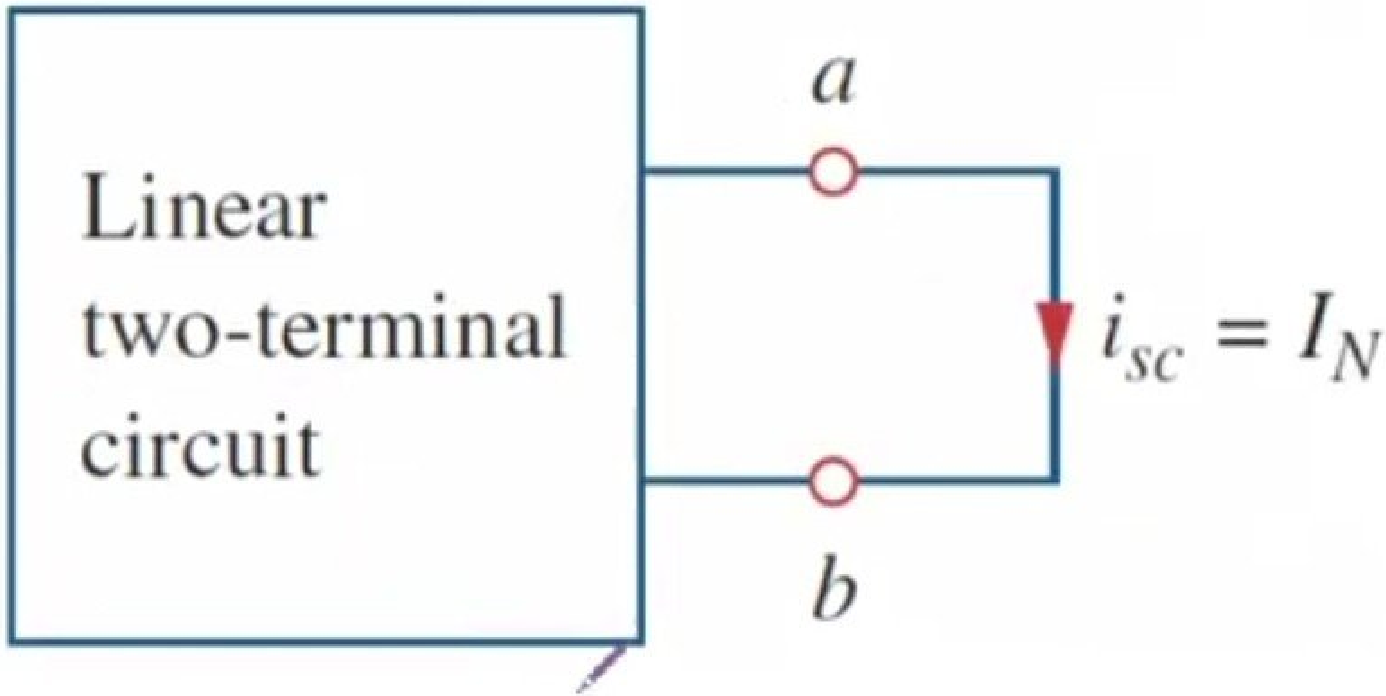
- From this we can also conclude that: $R_{Th} = \frac{v_{oc}}{i} = R_N$

Norton's Theorem

- We find R_N in the same way we find R_{TH} (*see the previous lecture*)

$$R_N = R_{\text{Th}}$$

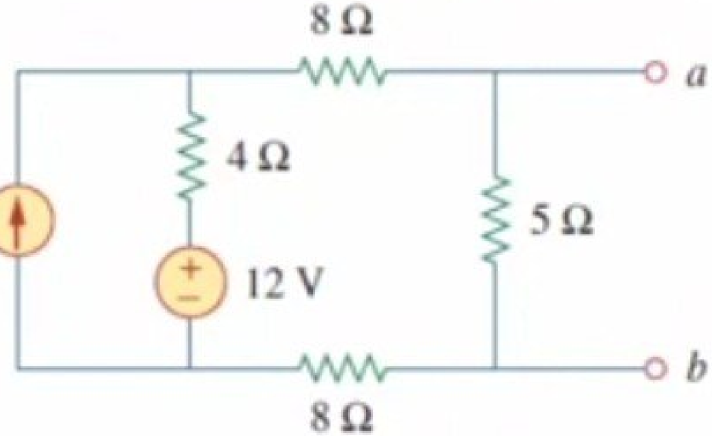
- **To find I_N.**
- It is the S.C current flowing from terminal a to b.



$$I_N = i_{sc}$$

Norton's Theorem

Example 4.11 Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.



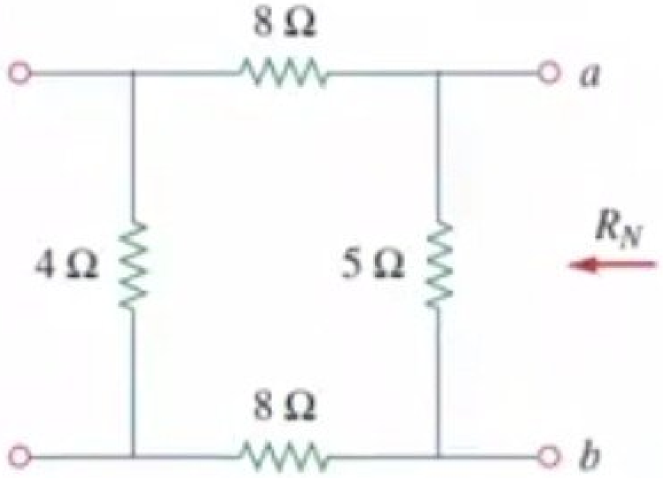
Solution:

Norton's Theorem

Example 4.11

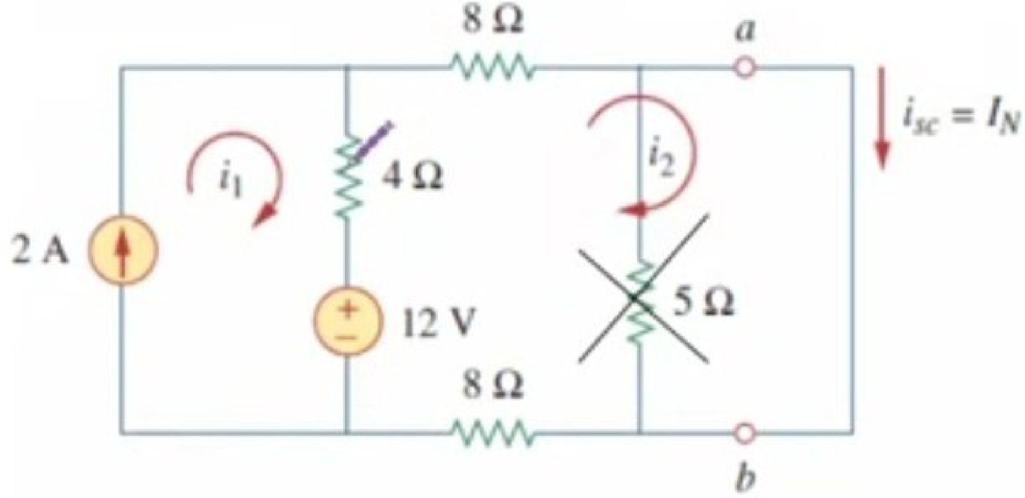
Solution:

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$



Example 4.11

Solution:



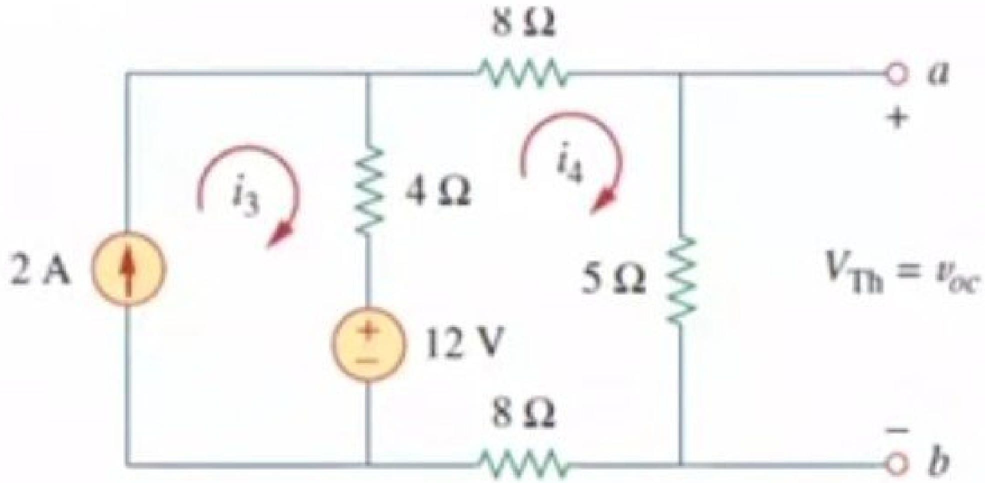
$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$

$i_2 = 1\text{ A} = i_{sc} = I_N$

So, the Norton's equivalent circuit is:

Norton's Theorem

- For the previous example find the Thevenin equivalent circuits?
- Solution: *two approaches:*
- *Approach 1: find* $V_{Th} = v_{oc}$



$$i_3 = 2 \text{ A}$$

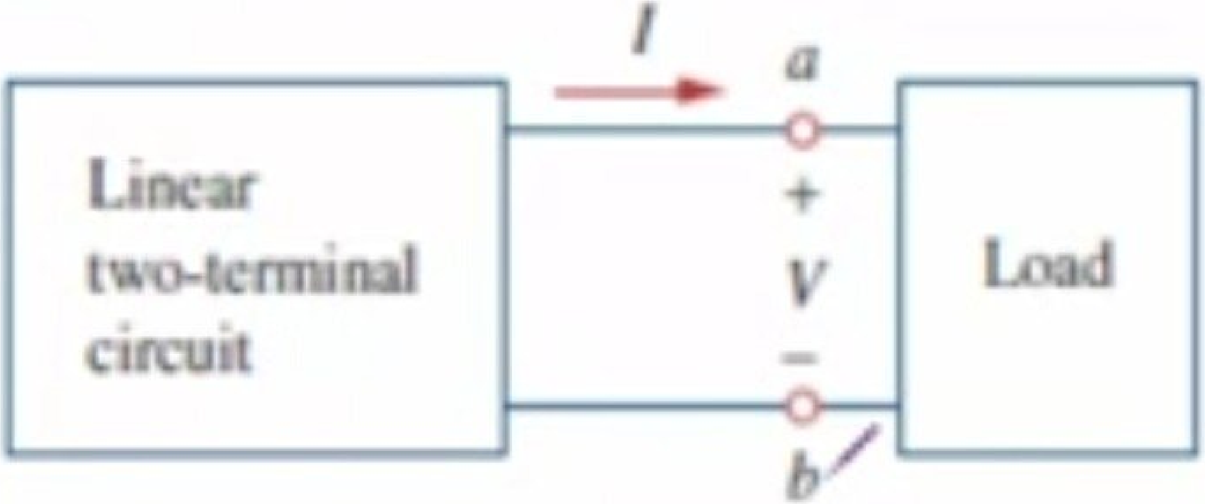
$$25i_4 - 4i_3 - 12 = 0 \quad \rightarrow \quad i_4 = 0.8 \text{ A}$$

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

- *Approach 2: use source transformation:*

Maximum Power Transfer

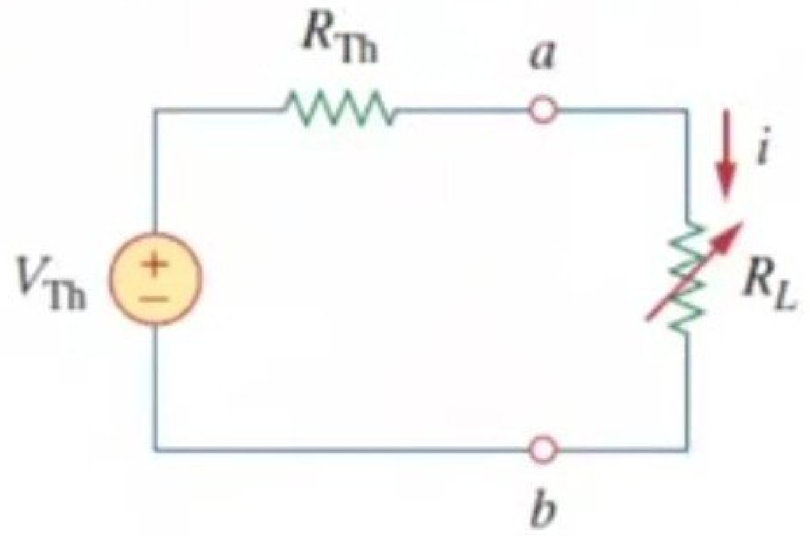
- In many practical situations, it is desirable to design the circuit such that the power delivered to the load is maximize.



Maximum Power Transfer

- The *Theveinin equivalent* is useful in finding the maximum power a linear circuit can deliver to a load.
- Assume the load has variable resistor.
- The power delivered to the load is:

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

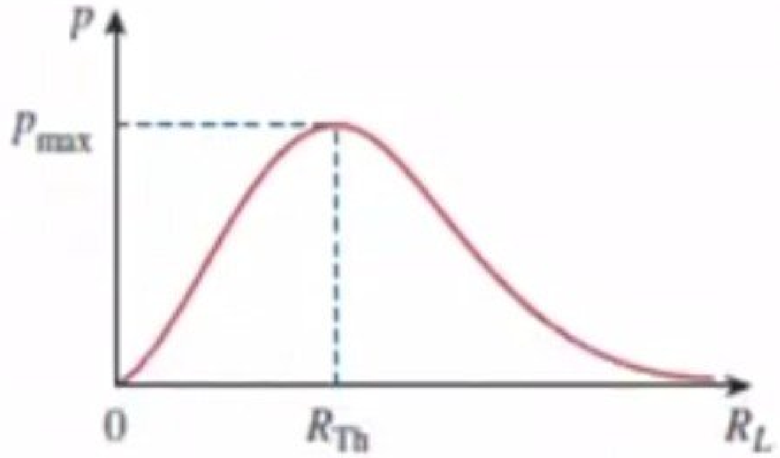


Maximum Power Transfer

➤ Now find R_L such that p is maximum:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right]$$

$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} \right] = 0$$



→ $0 = (R_{Th} + R_L - 2R_L) = (R_{Th} - R_L)$

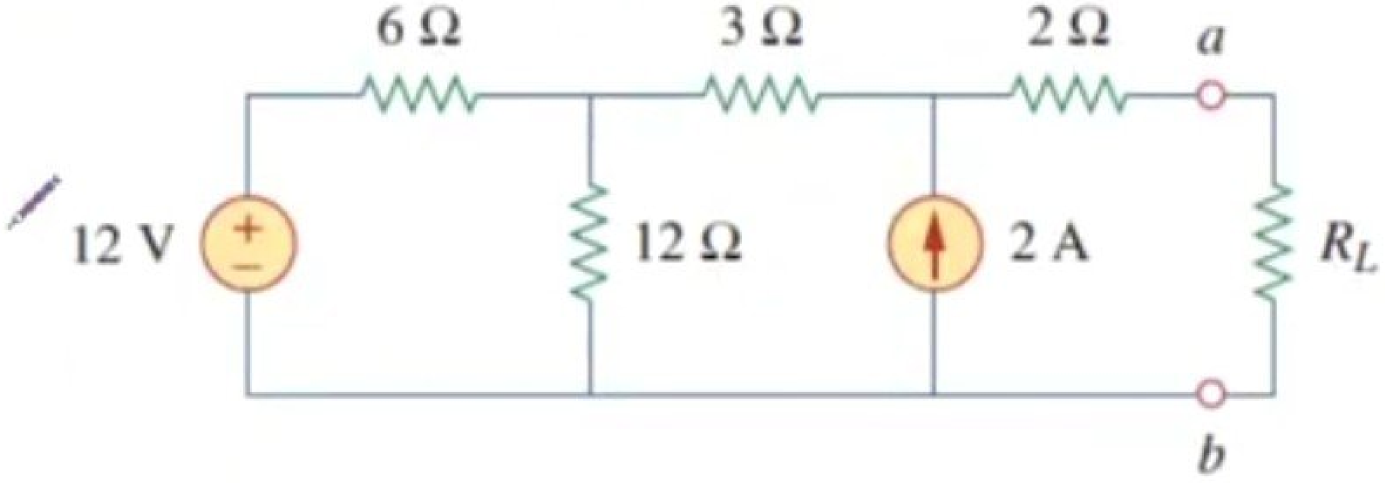
→ $$\boxed{R_L = R_{Th}}$$

Maximum Power Transfer

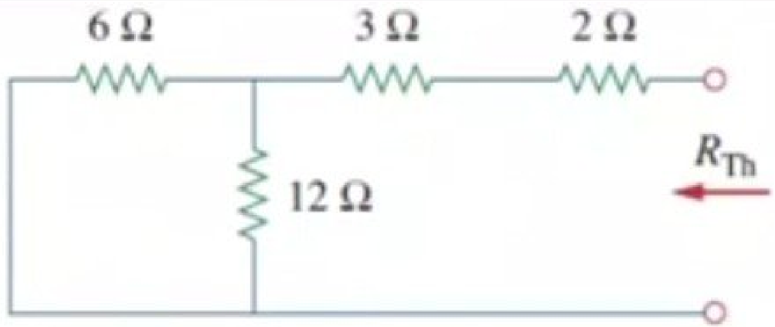
Example 4.13

Find the value of R_L for maximum power transfer in the circuit

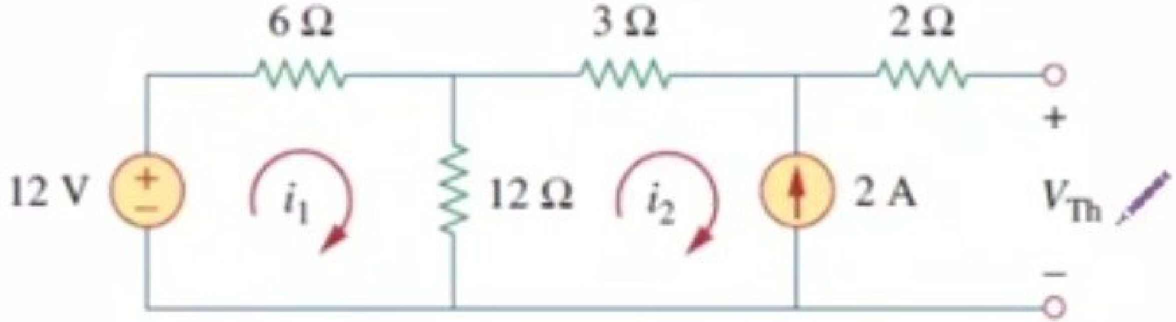
Find the maximum power.



$$R_{\mathrm{Th}} = 2 + 3 + 6 \,\|\, 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$



$$V_{\mathrm{Th}} = 22\ \mathrm{V}$$



For maximum power transfer,

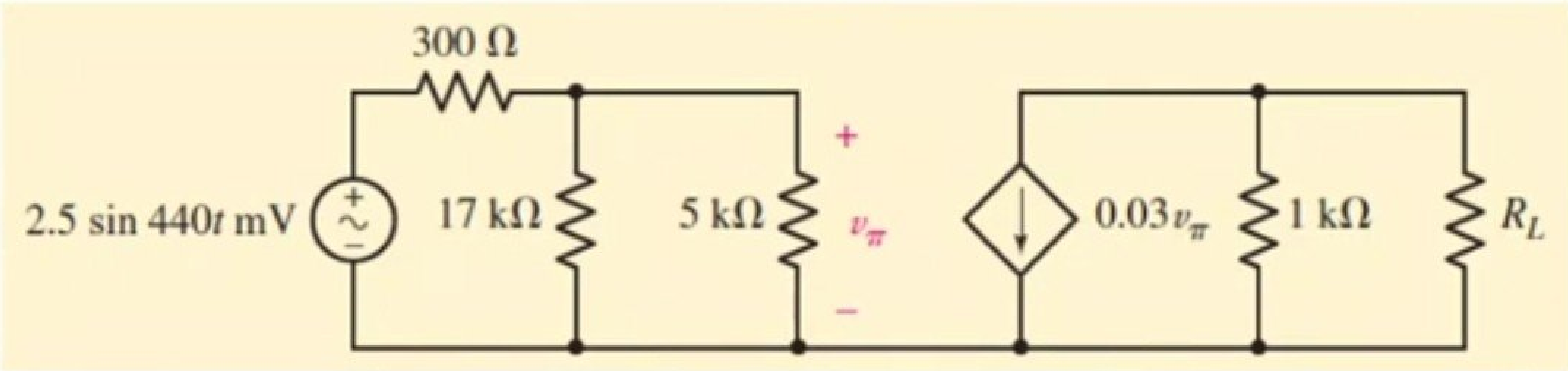
$$R_L = R_{\mathrm{Th}} = 9\ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \mathrm{W}$$

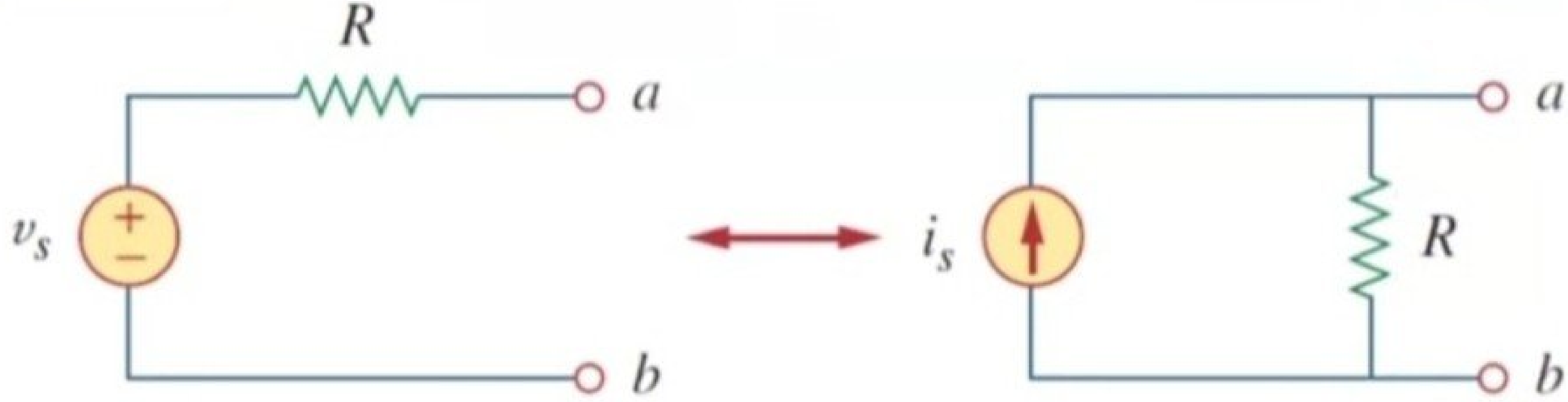
Maximum Power Transfer

EXAMPLE **Choose a load resistance so that maximum power is transferred to it, and calculate the actual power absorbed.**



Source Transformation

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

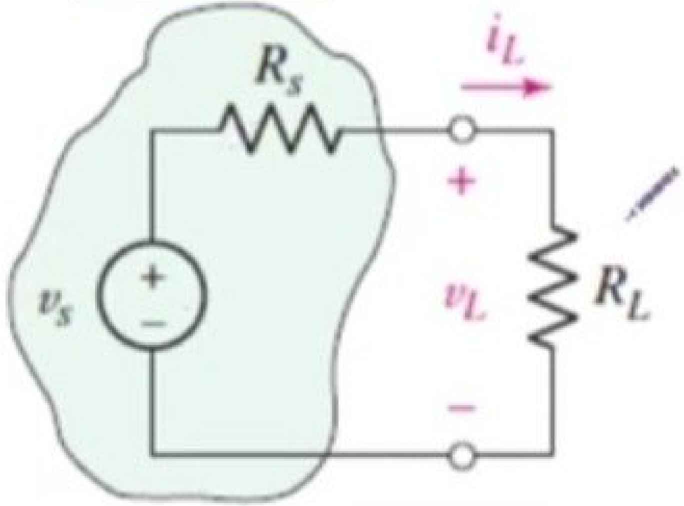


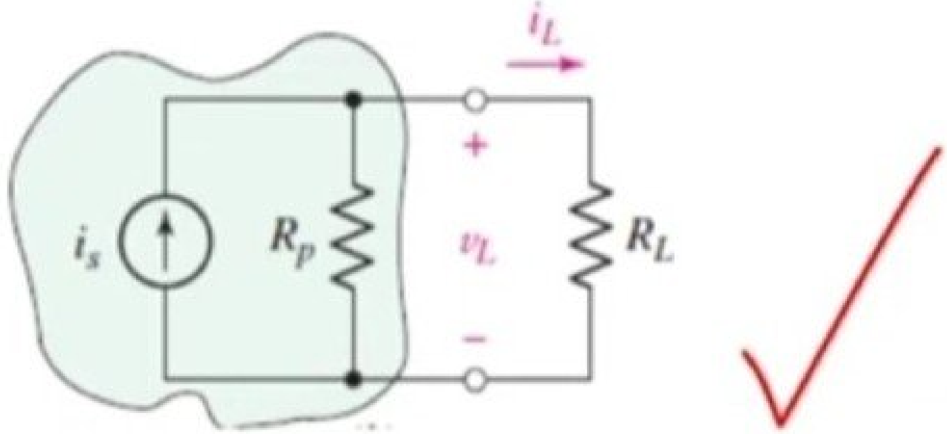
$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

Note: source transformation is not valid for $R = 0$ or $R = \infty$.

Note: the arrow of the current source is directed toward the positive terminal of the voltage source

❑ Proof:





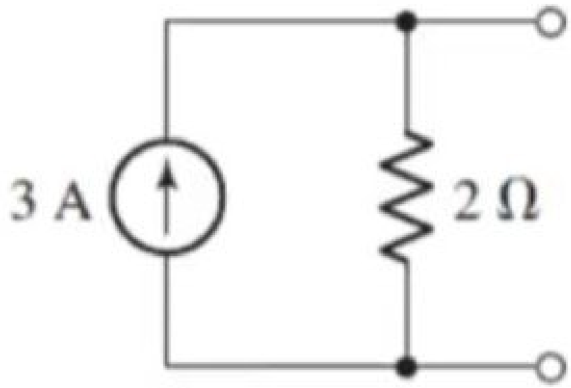
$$v_L = v_s \frac{R_L}{R_s + R_L}$$

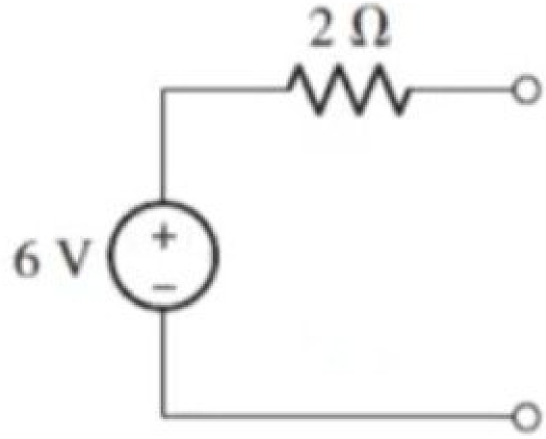
$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) \cdot R_L$$

The two circuits are electrically equivalent, if

$$R_s = R_p \quad \text{and} \quad v_s = R_p i_s = R_s i_s$$

Example:





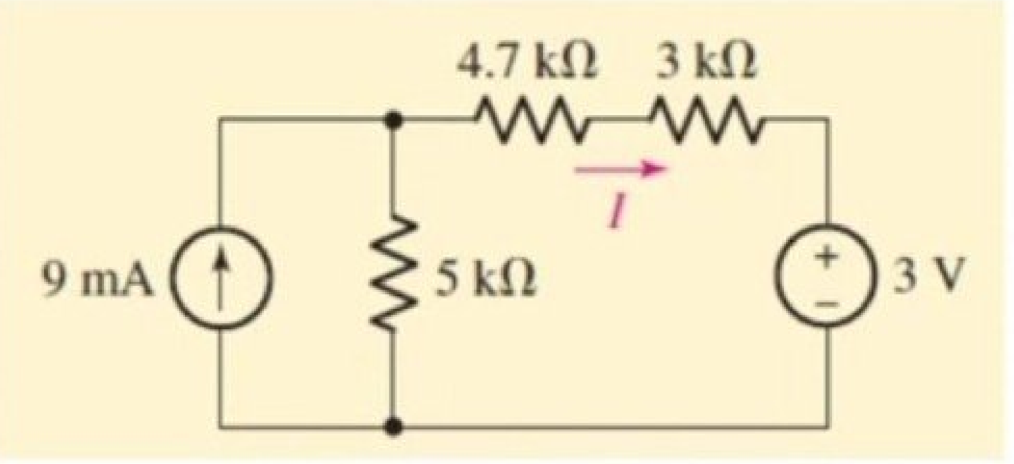
Source Transformation

- ***Source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis.***
- ***Usually we use it to convert the circuit to be <u>single-loop</u> or <u>single-node</u> circuit.***

Source Transformation

EXAMPLE

Compute the current through the 4.7 kΩ resistor in Fig. 5.17*a* after transforming the 9 mA source into an equivalent voltage source.



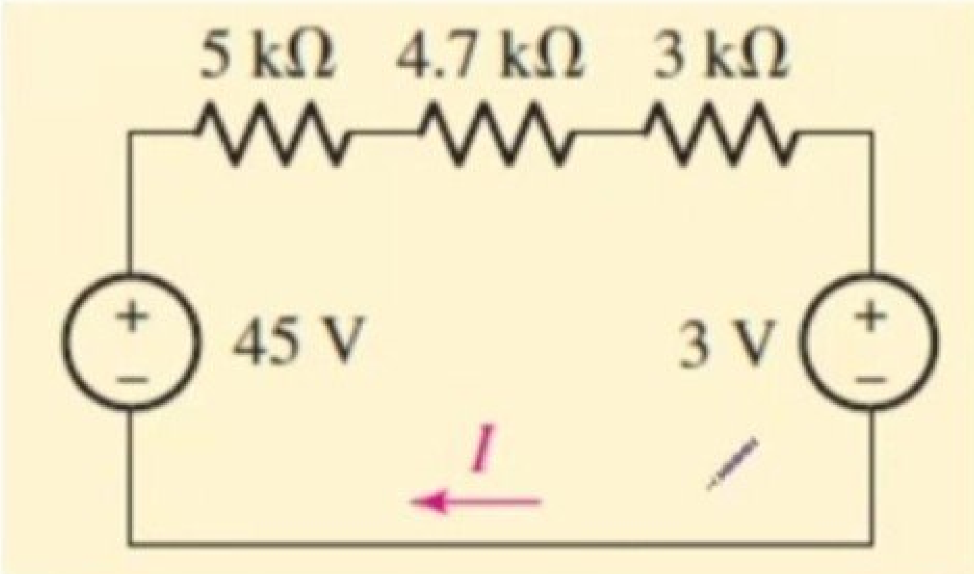
Source Transformation

Solution: by source transformation we have.

By KVL:

$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

→ $I = 3.307$ mA.



Source Transformation

PRACTICE

5.3 For the circuit of Fig. 5.18, compute the current I_X through the 47 kΩ resistor after performing a source transformation on the voltage source.

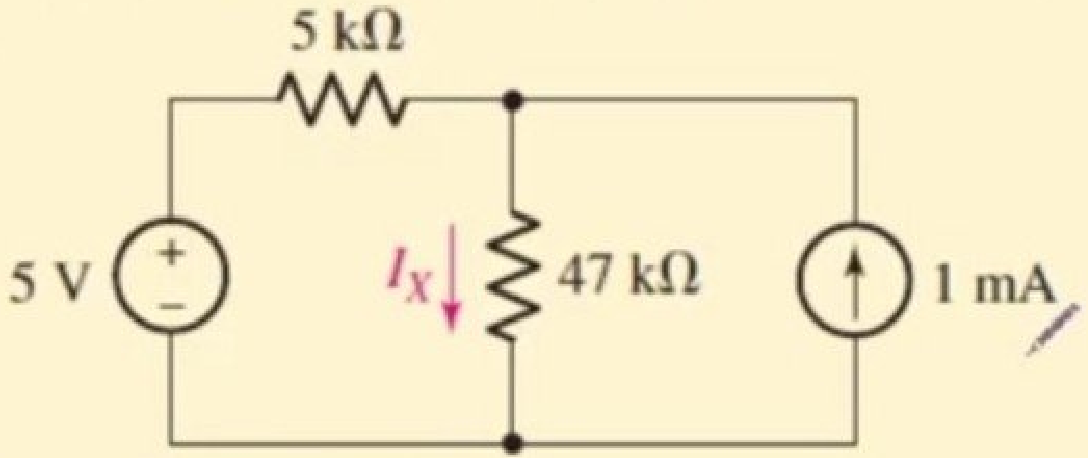


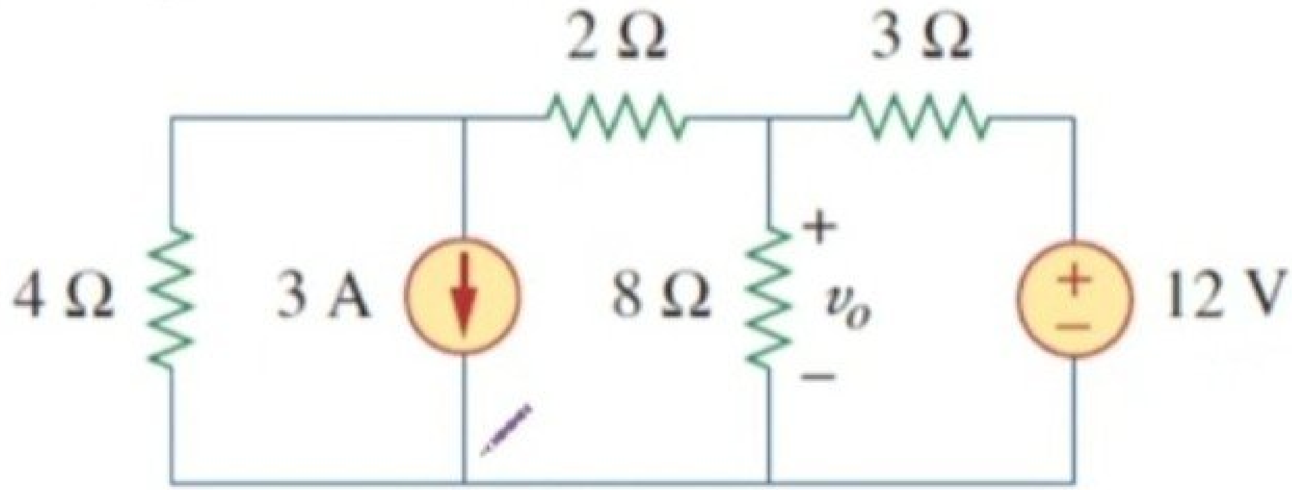
FIGURE 5.18

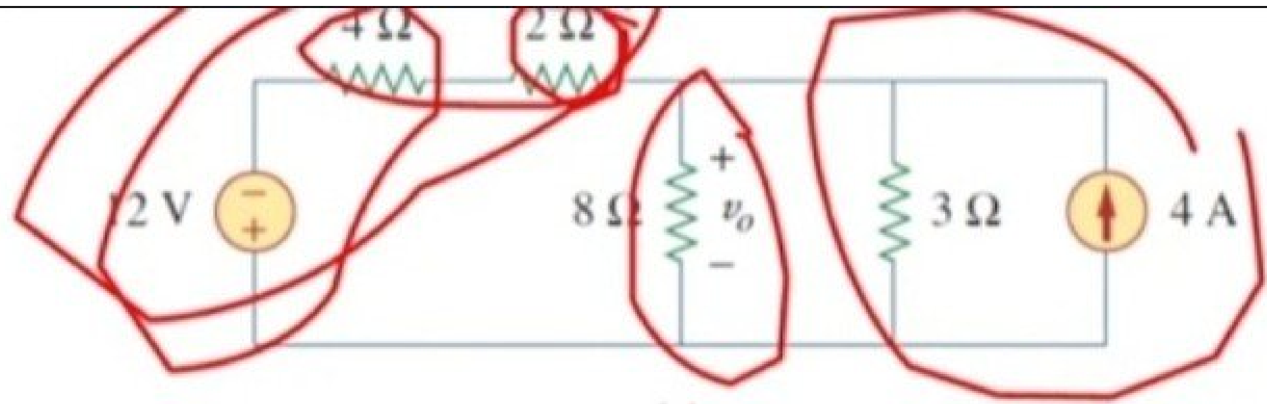
Ans: 192 μA.

Source Transformation

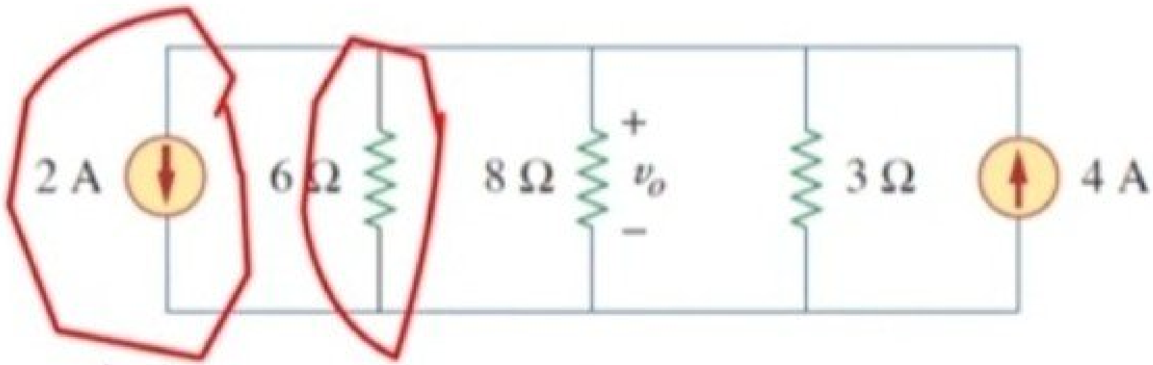
Example 4.6

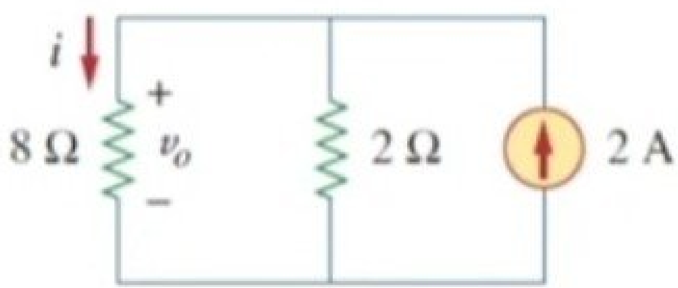
Use source transformation to find v_o in the circuit





(a)



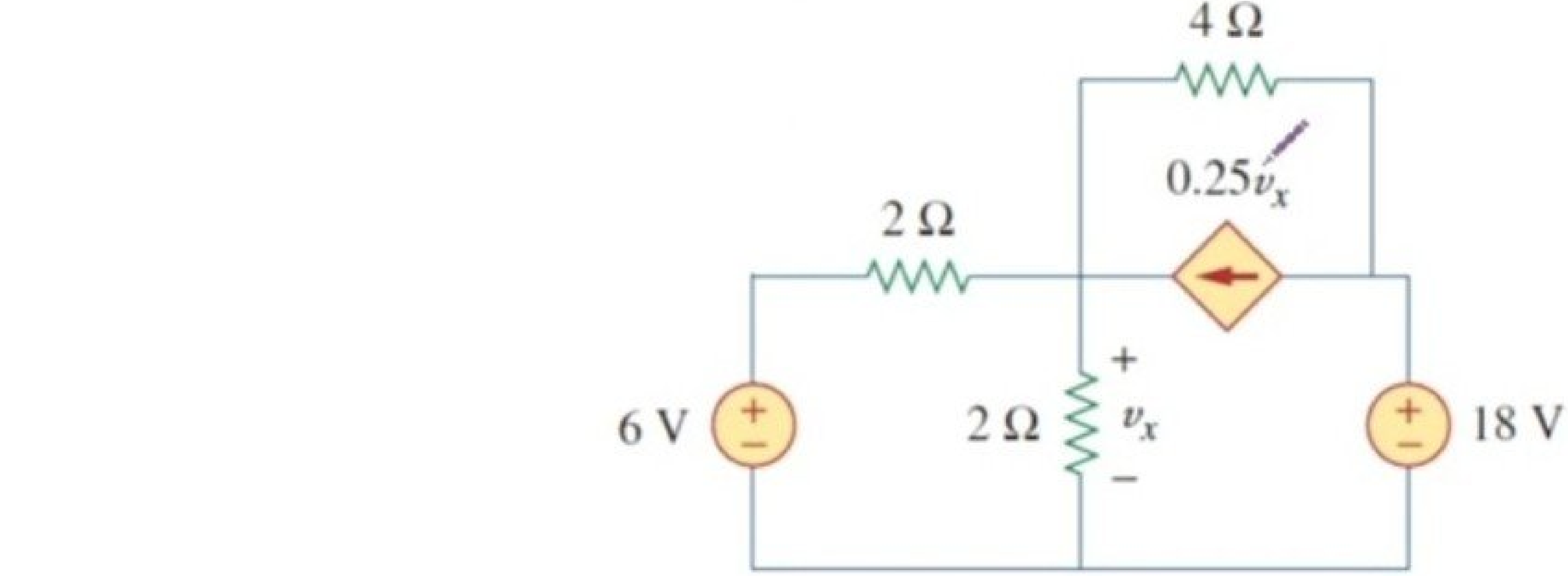


$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

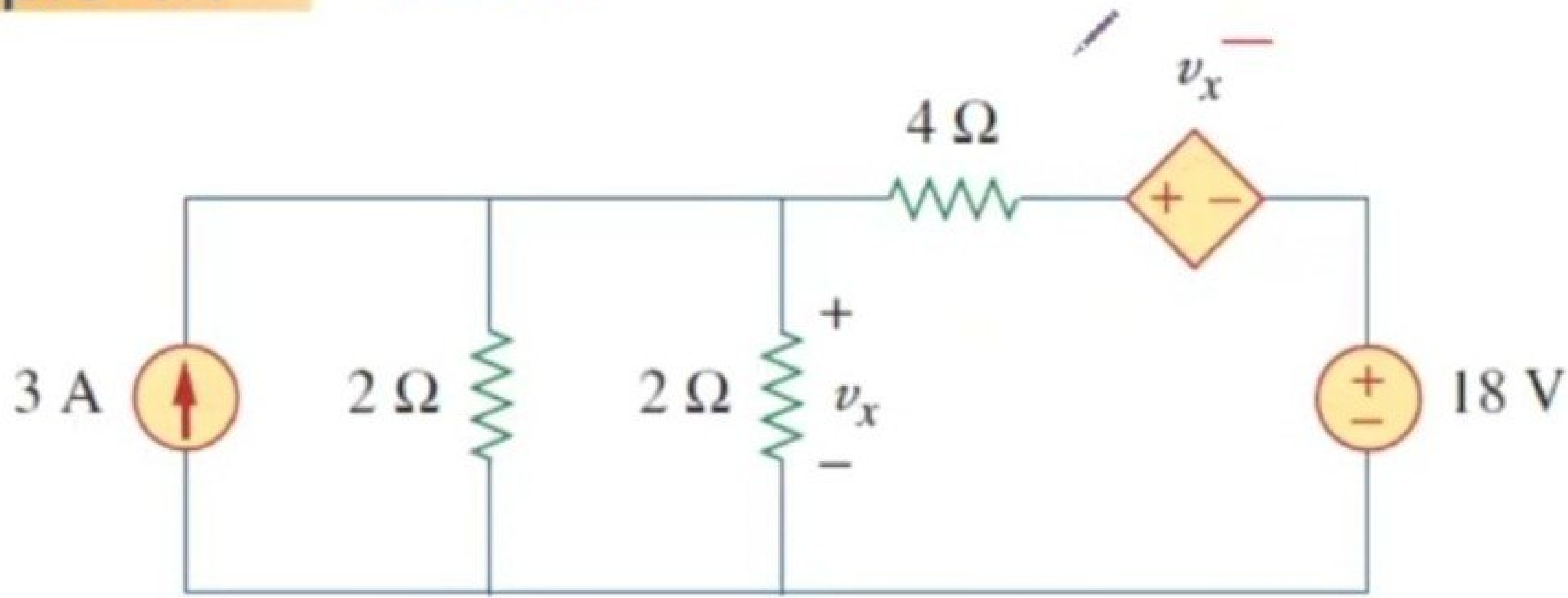
Example 4.7

Find v_x in Fig. 4.20 using source transformation.



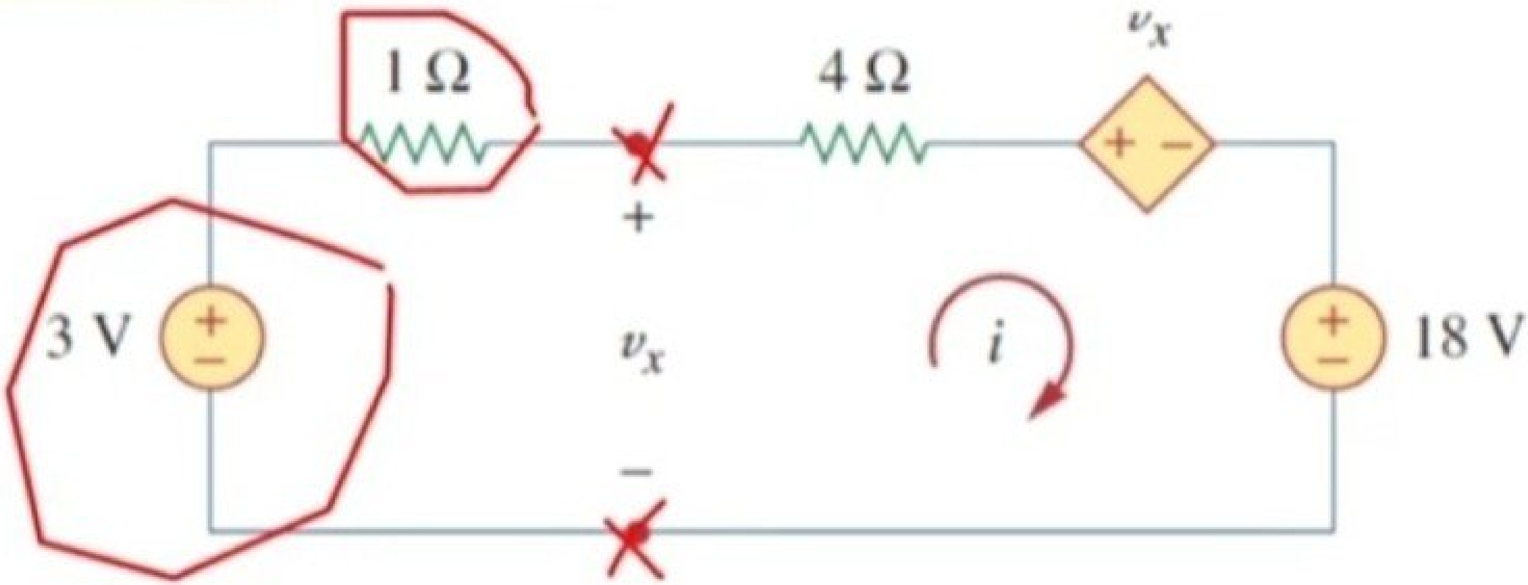
Source Transformation

Example 4.7 **Solution:**



Source Transformation

Example 4.7 **Solution:**



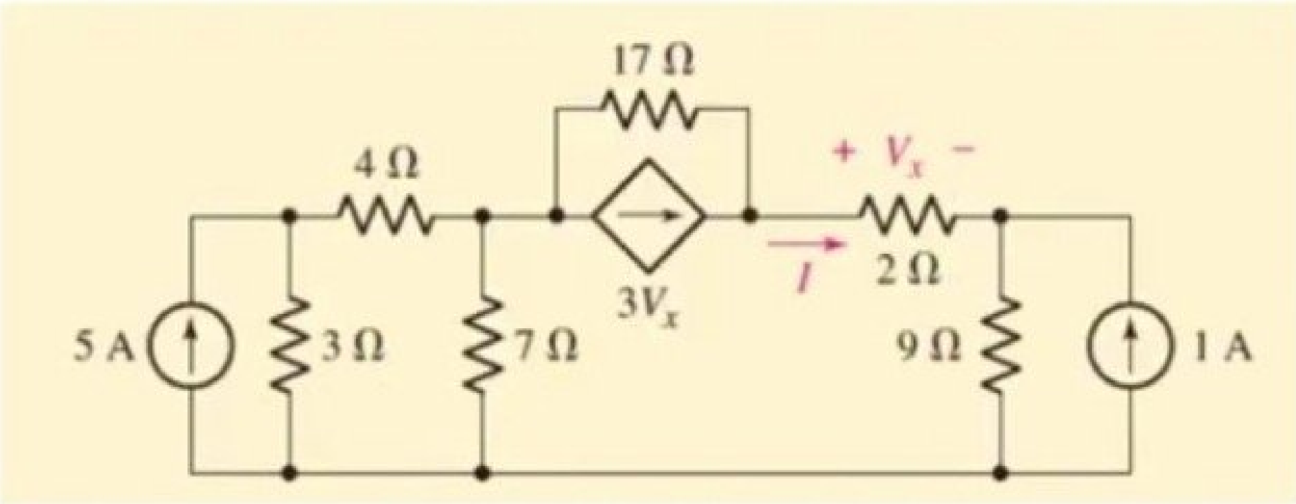
$$-3 + 5i + v_x + 18 = 0$$

$$i = -4.5 \text{ A}$$

$$v_x = 3 - i = 7.5 \text{ V}.$$

$$-3 + 1i + v_x = 0$$

Source Transformation



Convert to