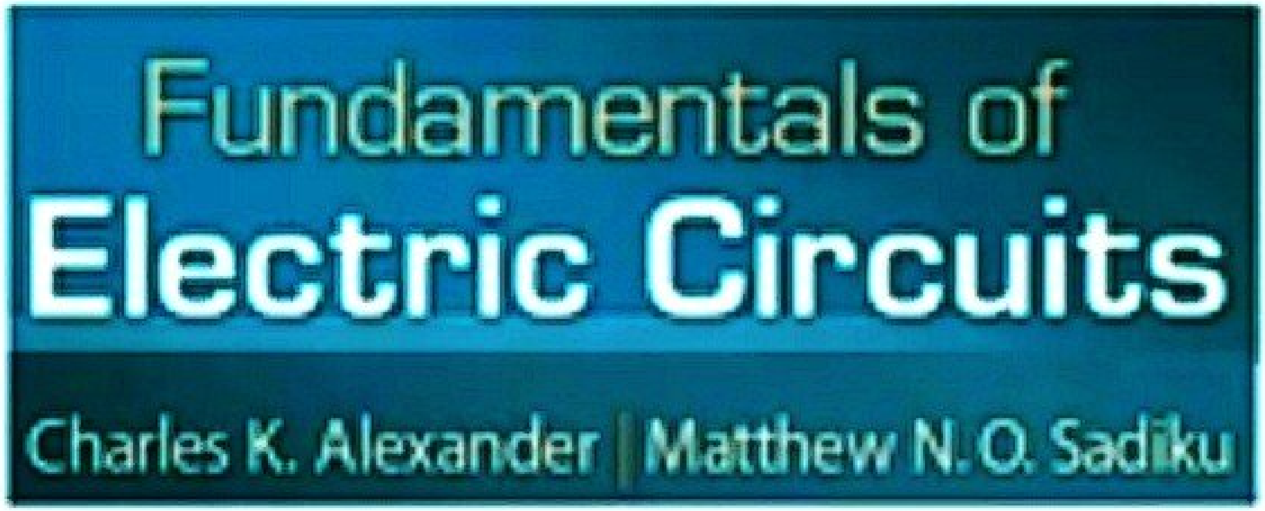
Chapter 3: Methods of Analysis

Lecture#1

Reference:

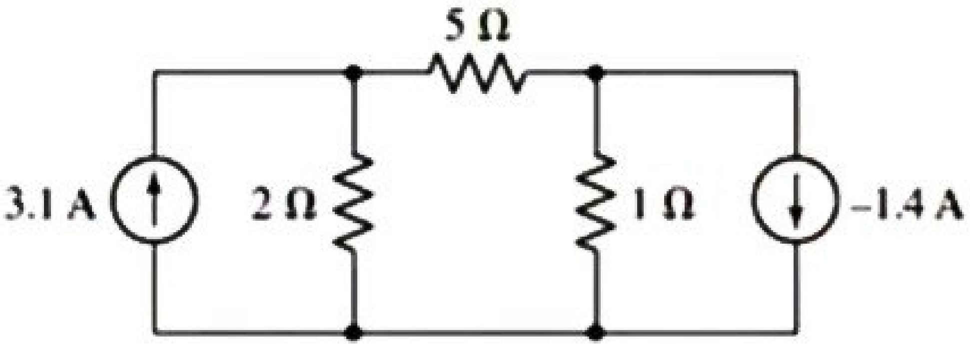


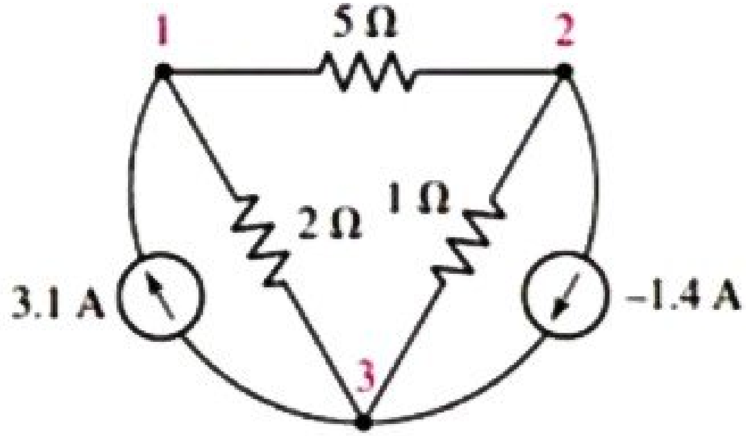
- **In *nodal analysis*, we are interested in finding the <u>node voltages.</u>**
- **We will consider two scenarios:**
  - **Nodal analysis for CKTs with no voltage sources.**
  - **Nodal Analysis for CKTs with voltage sources.**

Steps to Determine Node Voltages

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

- **Note: both CKTs are the same.**

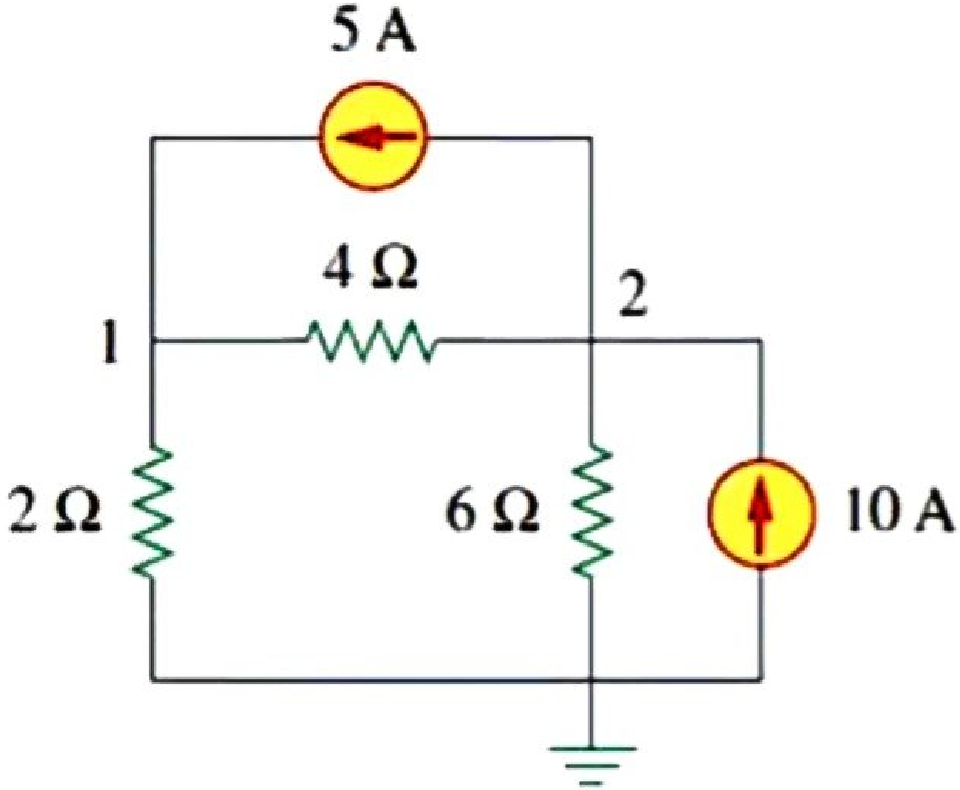




Nodal Analysis

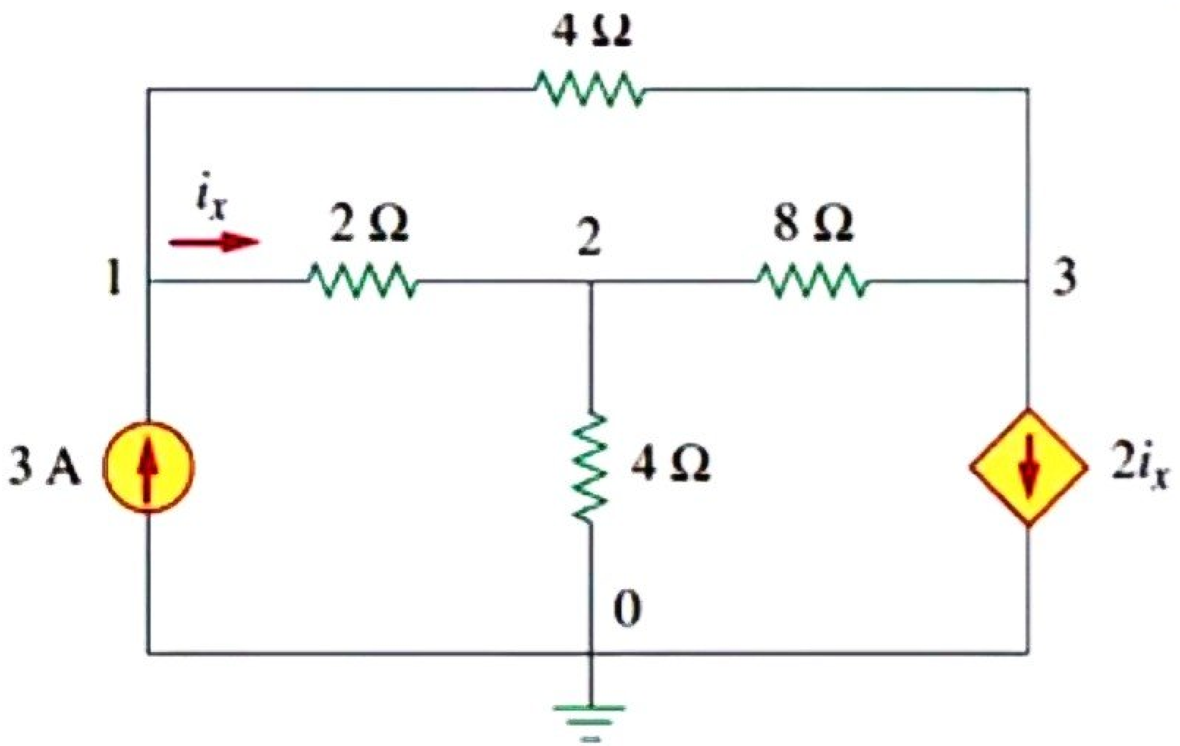
Example 3.1

Calculate the node voltages in the circuit shown



Example 3.2

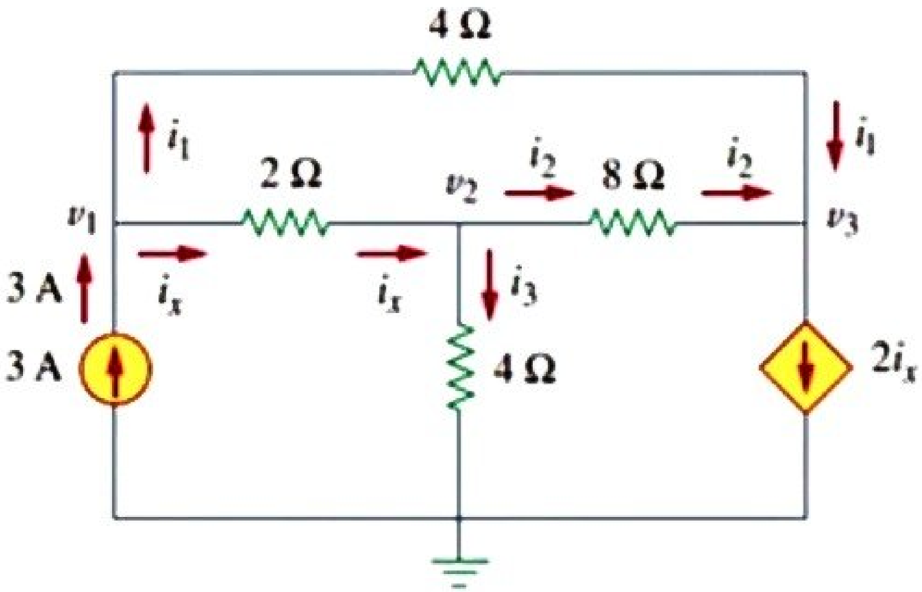
Determine the voltages at the nodes



Nodal Analysis

Example 3.2

Solution:



node 1, $3v_1 - 2v_2 - v_3 = 12$ — (1)

node 2, $-4v_1 + 7v_2 - v_3 = 0$ — (2)

node 3, $2v_1 - 3v_2 + v_3 = 0$ — (3)

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

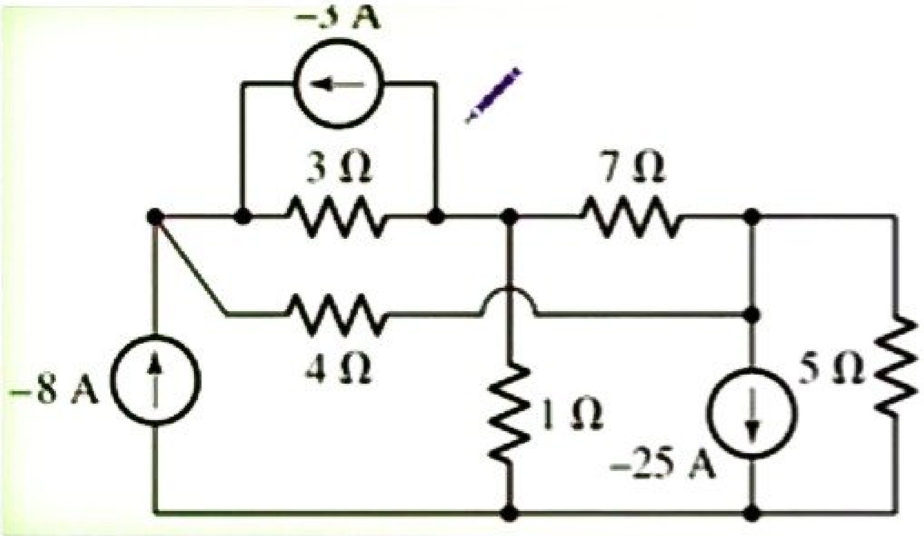
s, $v_1 = 4.8$ V, $v_2 = 2.4$ V, and $v_3 = -2.4$ V,

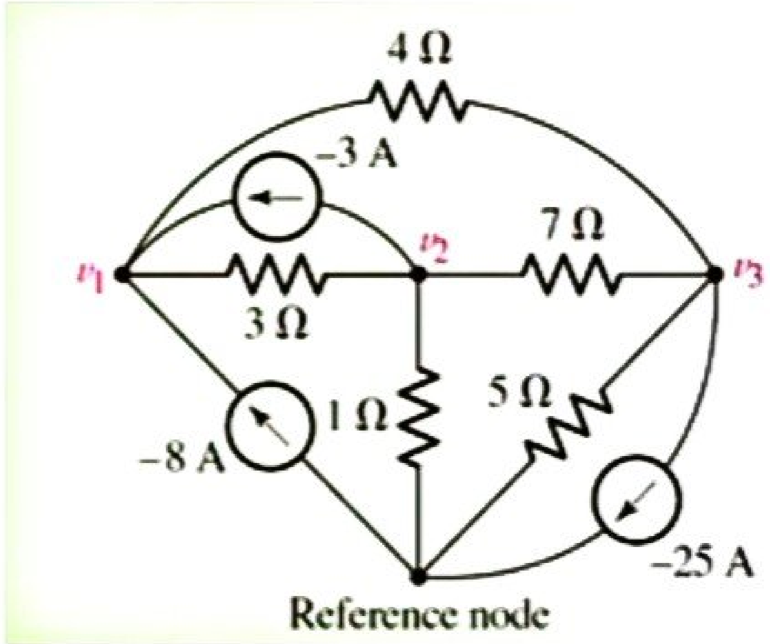
Nodal Analysis

This example is from Hayet

EXAMPLE 4.2

Determine the nodal voltages



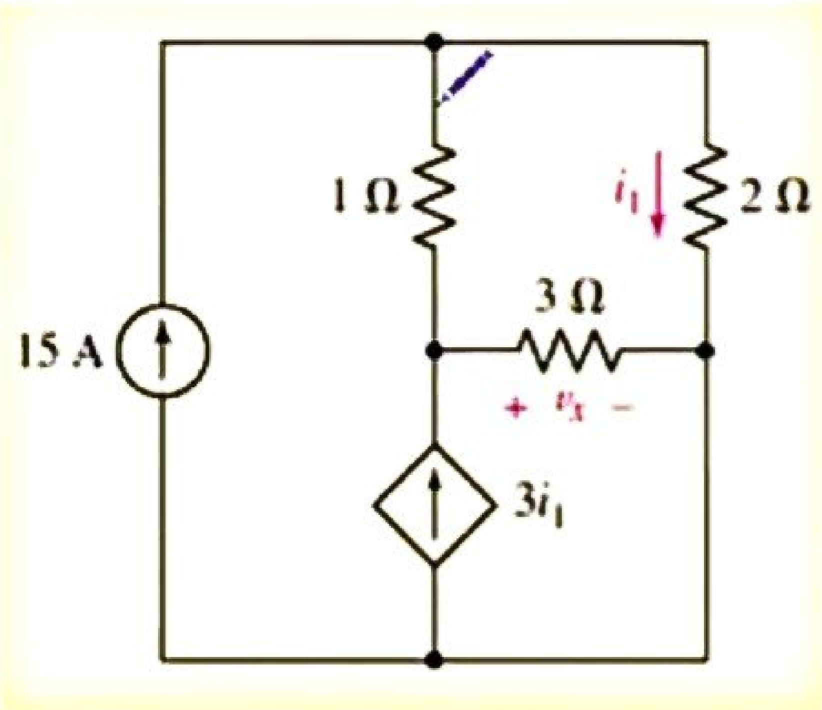


$$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$$

$$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$$

$$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$$

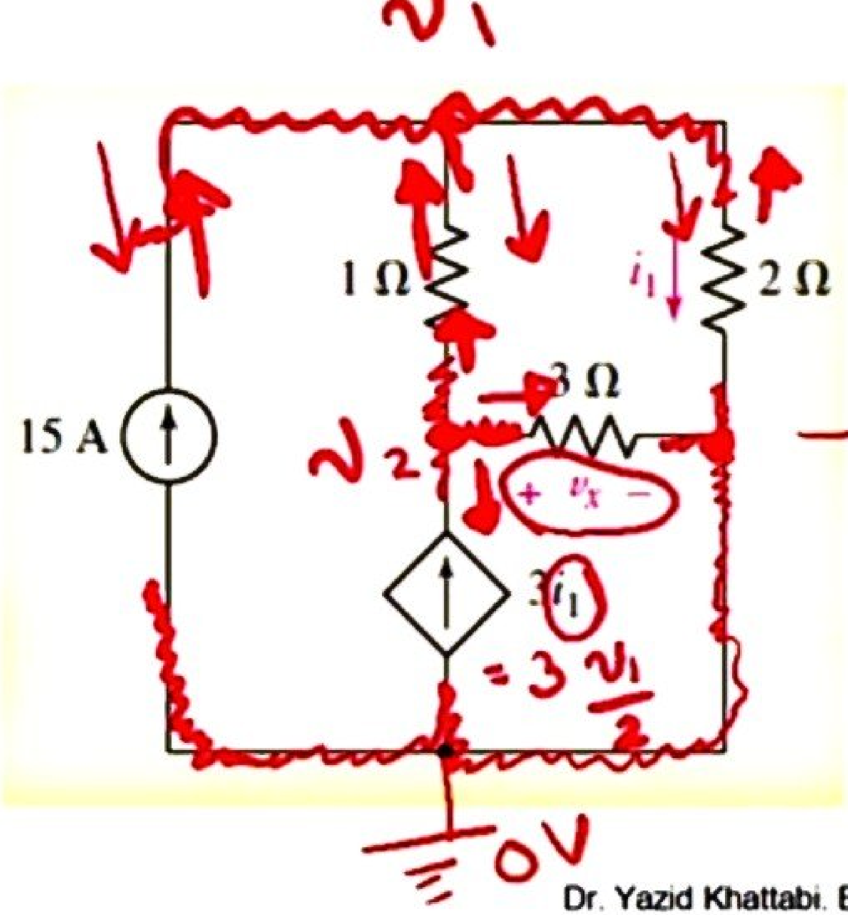
□HW: Find v_x using nodal analysis



☐HW: Find v_x using nodal analysis

$-5 + \frac{v_1 - v_2}{1} + \frac{v_1 - 0}{2} = 0$ ✓

KCL at node #1

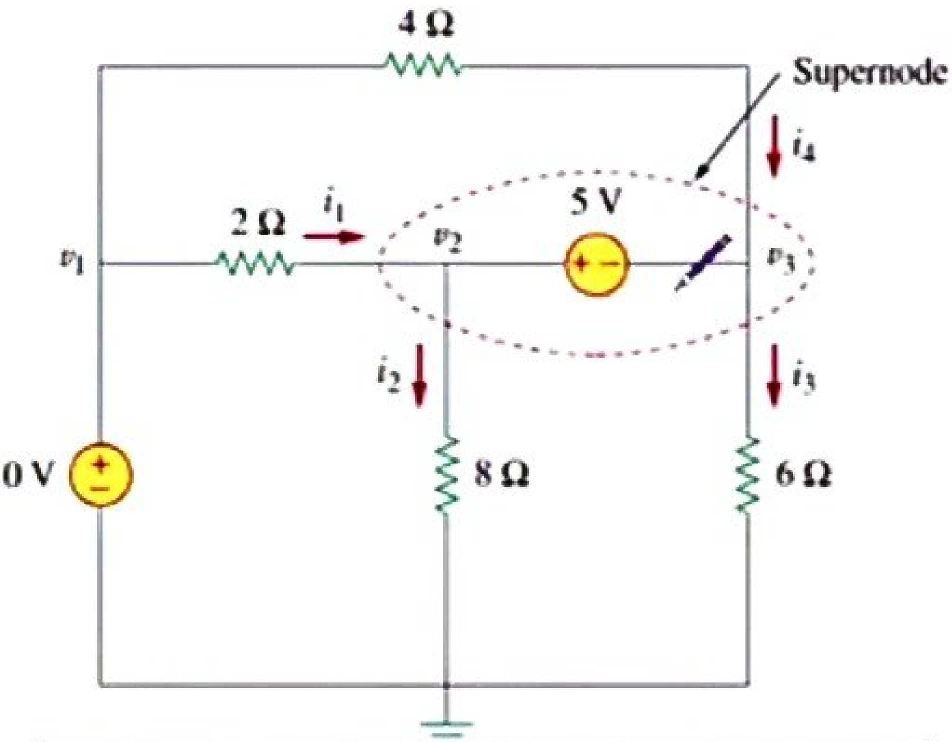


$15 + \frac{v_2 - v_1}{1} + \frac{0 - v_1}{2} = 0$ ✓

$-1.5v_1 + v_2 = -15$ —(1)

$\frac{v_2 - 0}{3} + \frac{v_2 - v_1}{1} + 3\frac{v_1}{2} = 0$

$0.5v_1 + \frac{4}{3}v_2 = 0$ —(2)



Note: A supernode requires the application of both KCL and KVL.

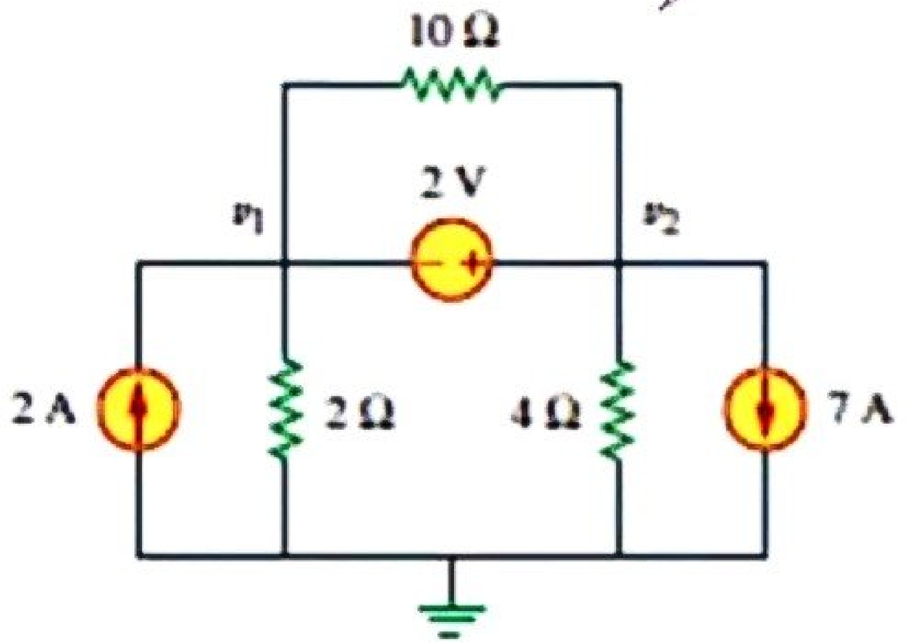
- **Two cases:**

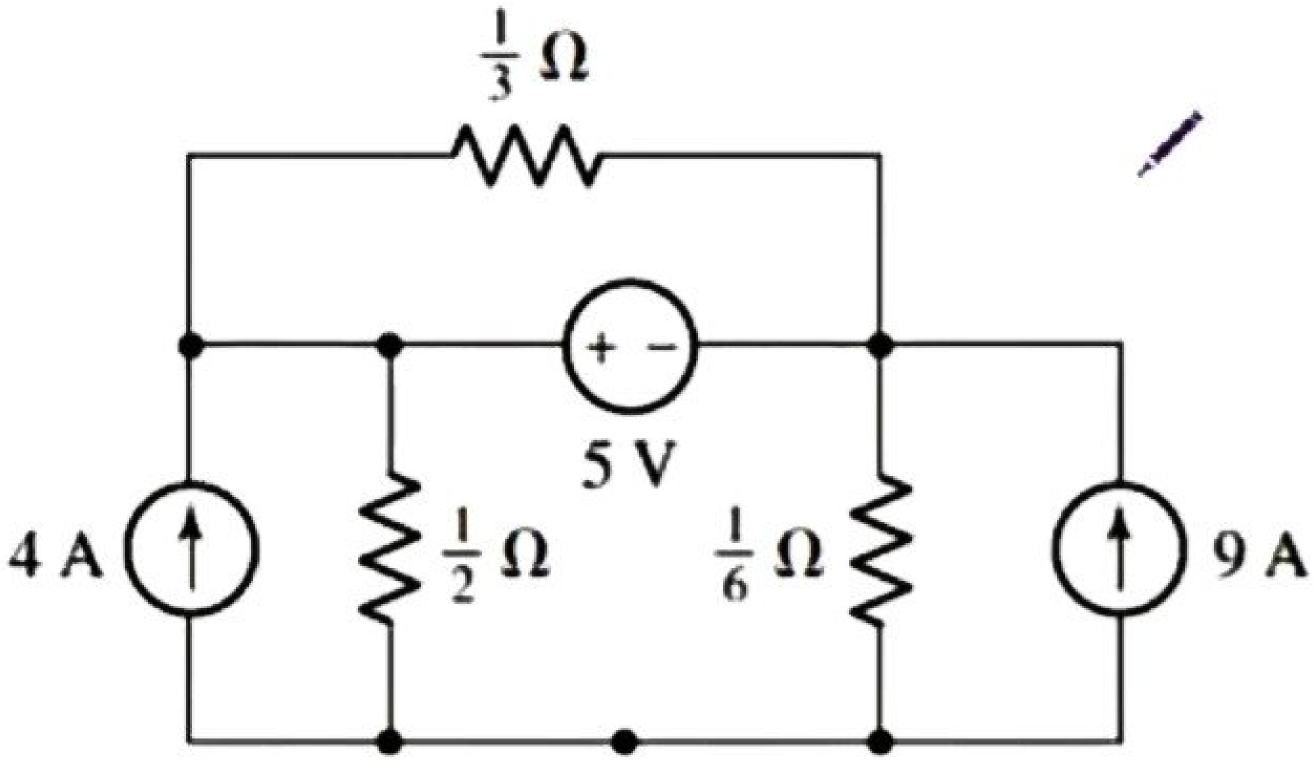
- **Case1**: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source

$$v_1 = 10\text{ V}$$

- **Case2:** If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.



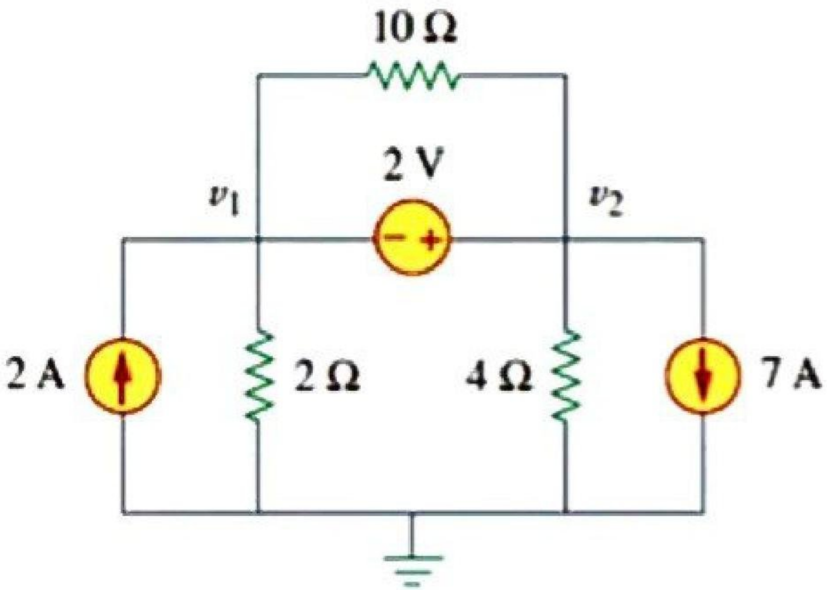


1/3 Ω
+ −
5 V
4 A
1/2 Ω
1/6 Ω
9 A

Example 3.3

find the node voltages.

−4, −6

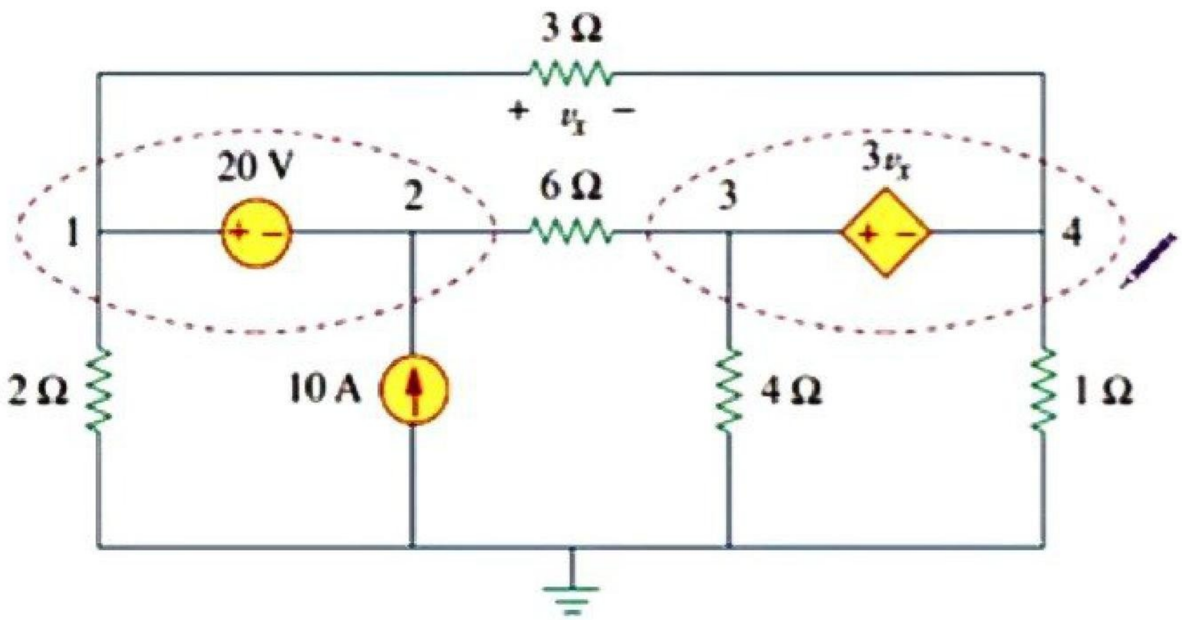


$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

$$v_2 = v_1 + 2$$

$$v_1 = -7.333 \text{ V}$$
$$v_2 = v_1 + 2 = -5.333 \text{ V}$$

Example 3.4



$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

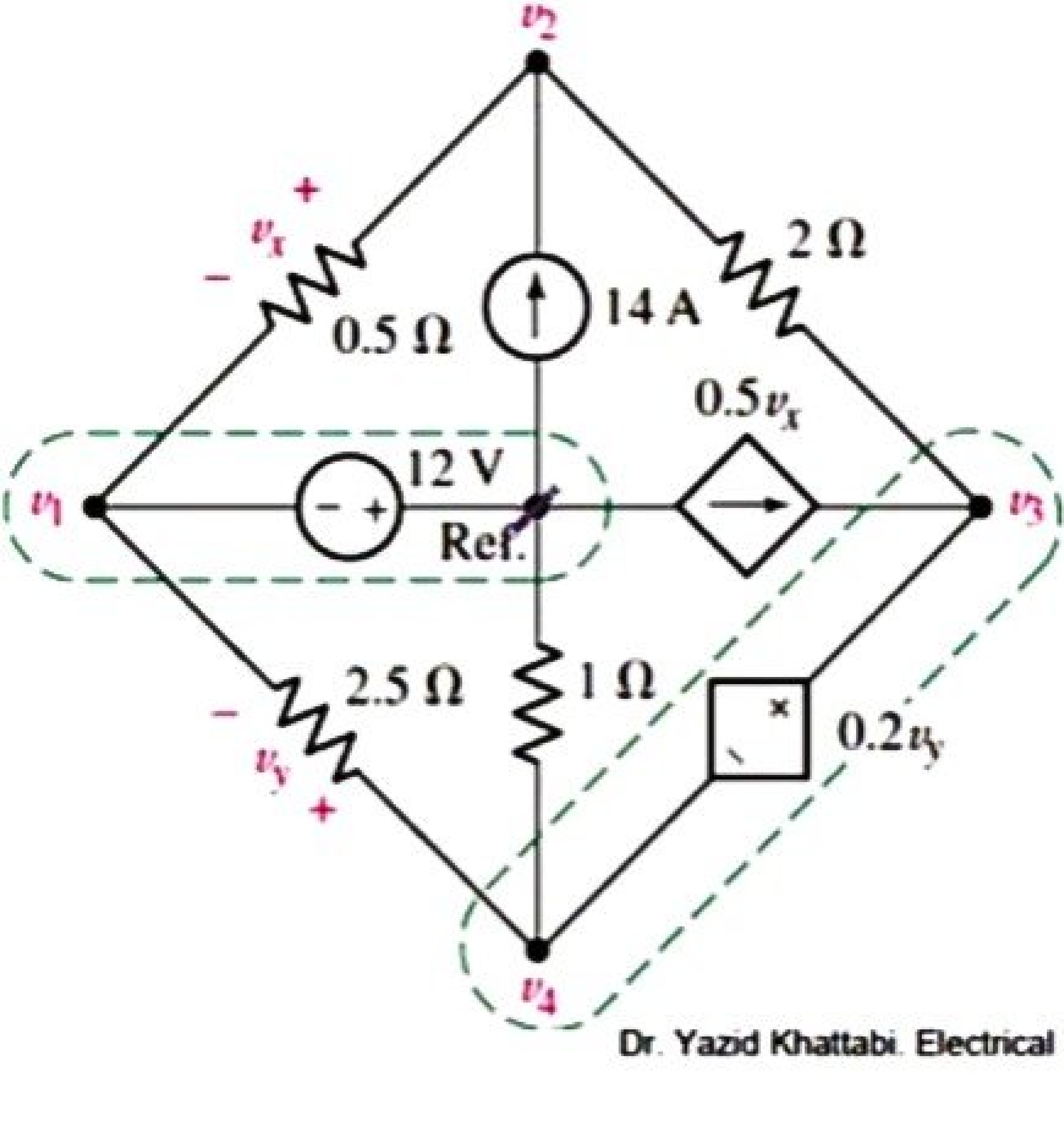
$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

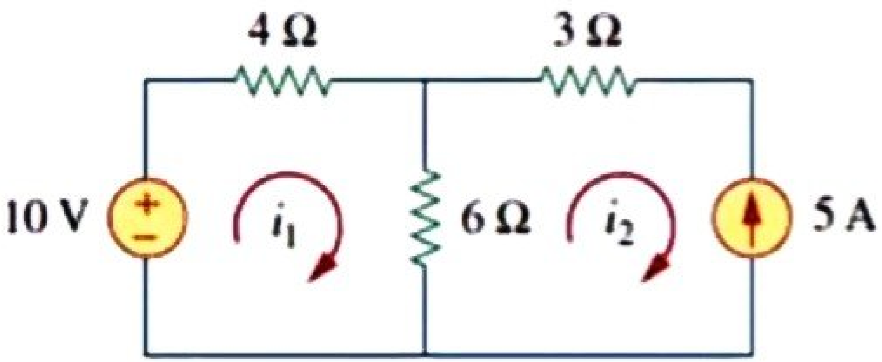
$$3v_1 - v_3 - 2v_4 = 0$$

$$-2v_1 - v_2 + v_3 + 2v_4 = 20$$

HW:

Determine the node-to-reference voltages





- **Two cases:**
  - **<u>Case1</u>:** when a current source exists only in one mesh.

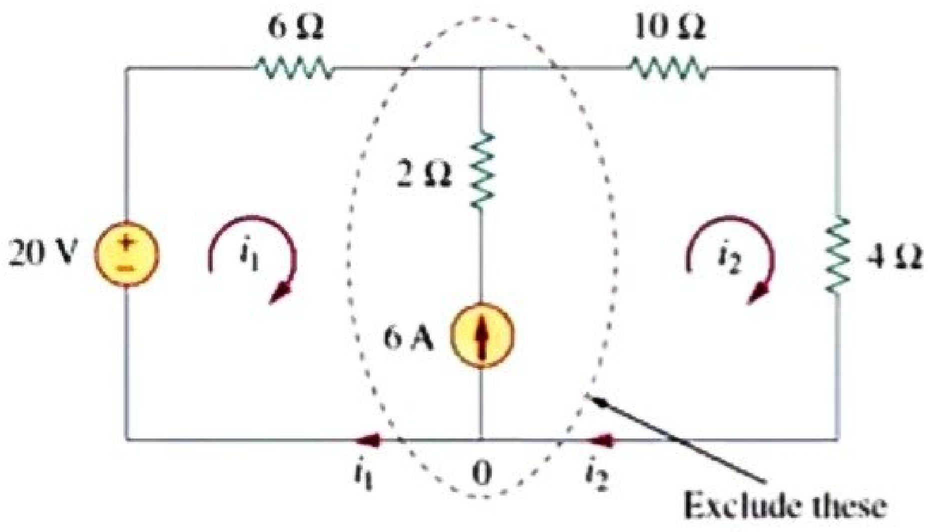
$$i_2 = -5 \text{ A}$$

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

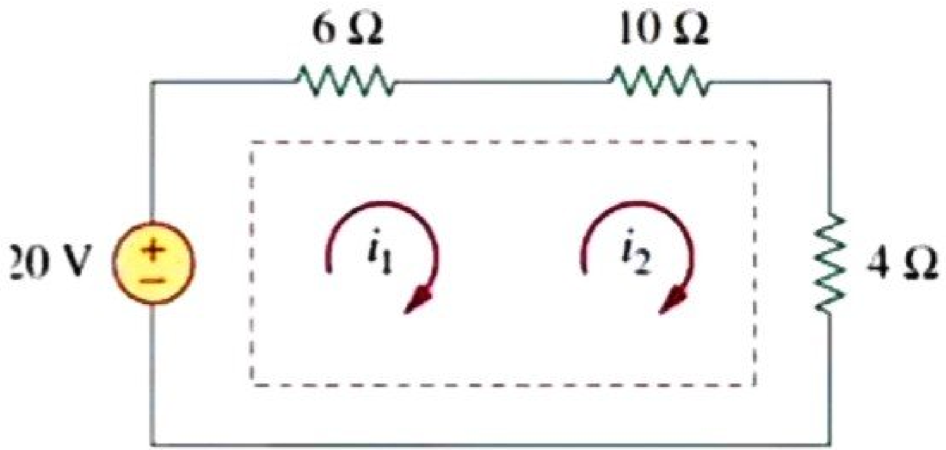
- **<u>Case2:</u>** When a current source exists between two meshes: We create a <u>*supermesh*</u> by excluding the current source and any elements connected in series with it.

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$i_2 = i_1 + 6$$



(a)



meshes 1 and 3

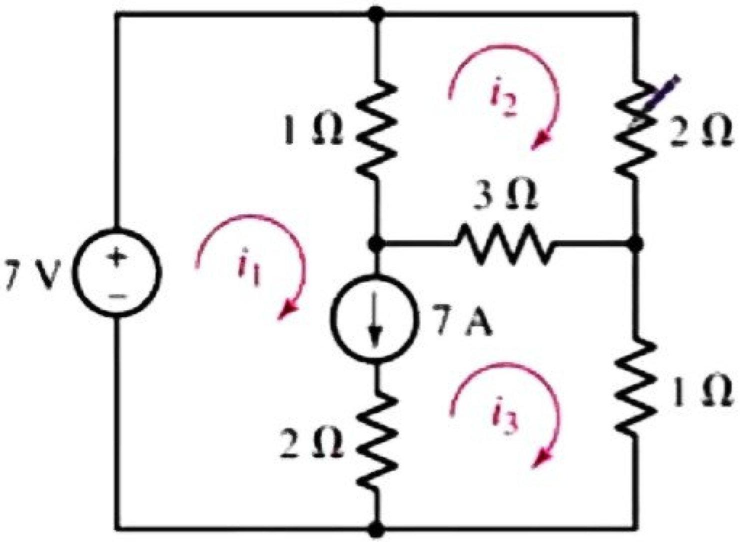
$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

$$\rightarrow \quad i_1 - 4i_2 + 4i_3 = 7$$

mesh 2

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$\rightarrow \quad -i_1 + 6i_2 - 3i_3 = 0$$



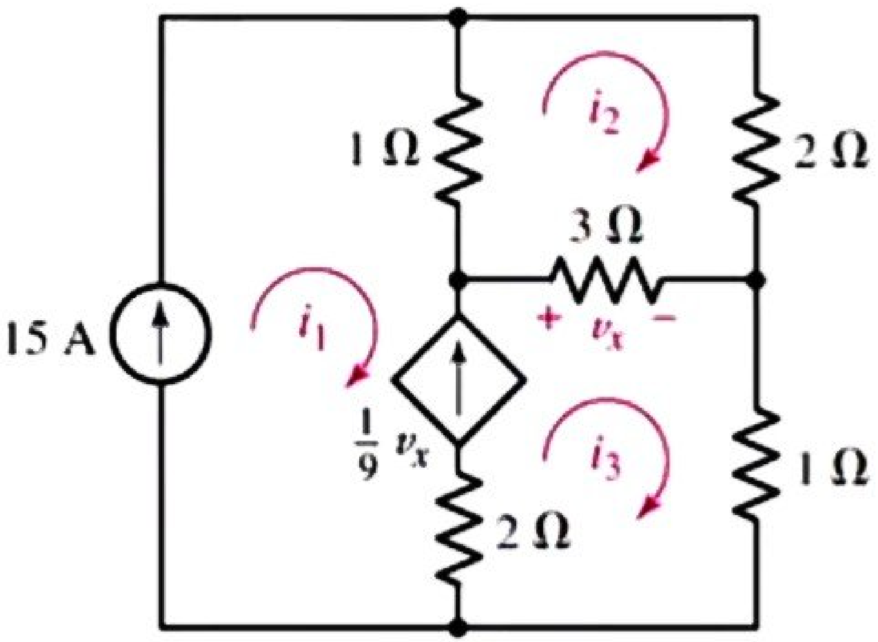
$$i_1 - i_3 = 7$$

$$i_1 = 9 \text{ A}$$
$$i_2 = 2.5 \text{ A}$$
$$i_3 = 2 \text{ A.}$$

mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$\rightarrow \quad 6i_2 - 3i_3 = 15$$



No need for supermesh 1-3 because i_1 is known. So we write

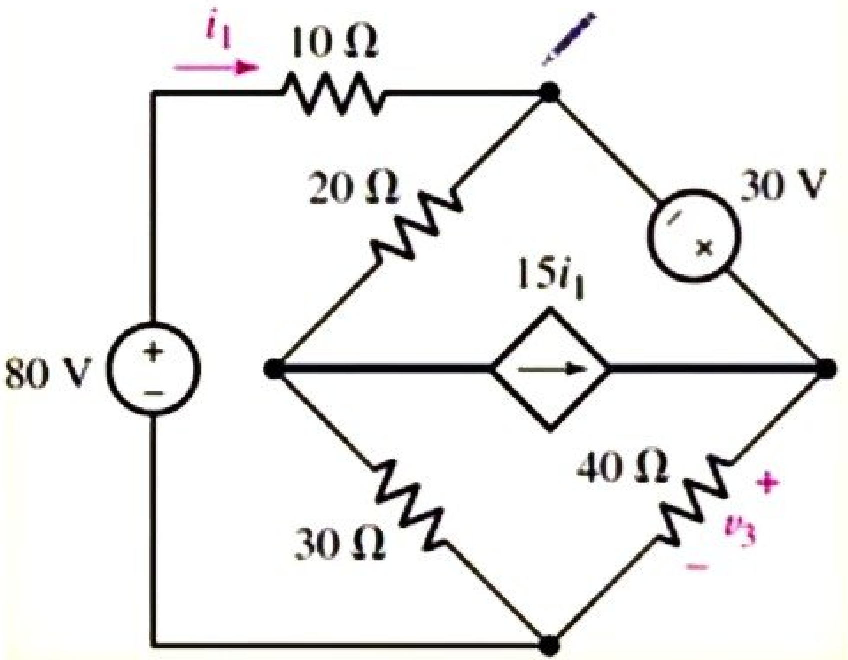
$$\frac{v_x}{9} = i_3 - i_1 = \frac{3(i_3 - i_2)}{9}$$

$$\rightarrow \quad \frac{1}{3} i_2 + \frac{2}{3} i_3 = 15$$

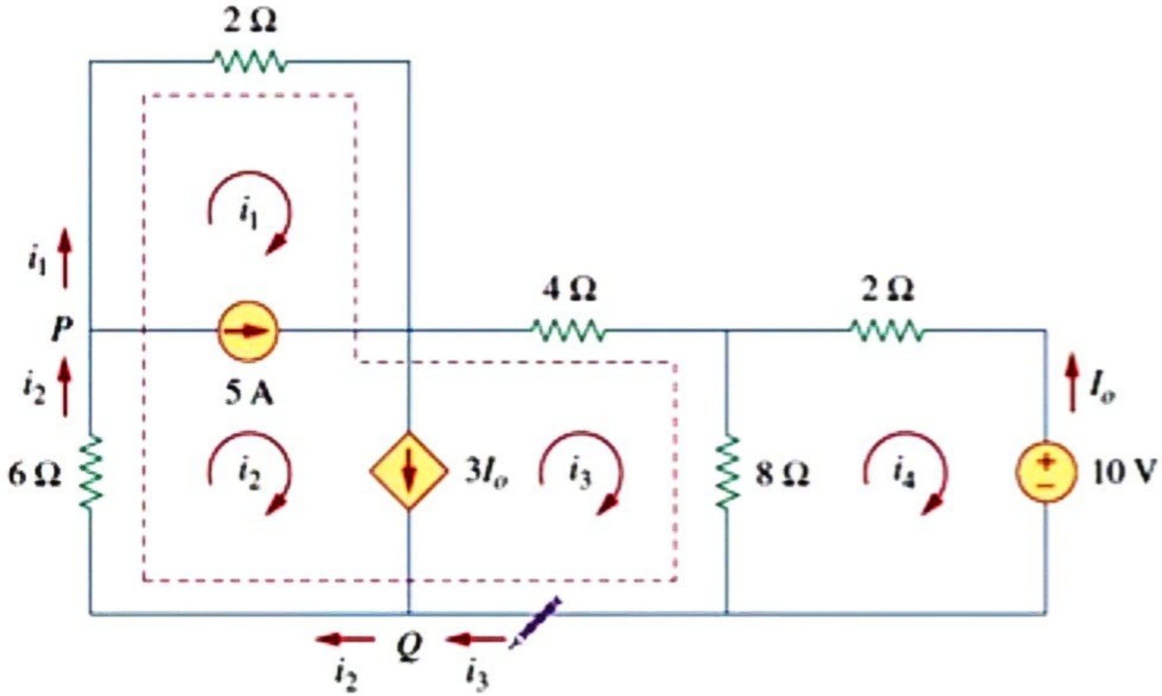
$i_2 = 11$ A and $i_3 = 17$ A;

PRACTICE

4.10 Determine v_3 in the circuit of



Ans: 104.2 V



find i_1 to i_4 using mesh analysis.

Solution:

Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

we apply KCL to node P: $i_2 = i_1 + 5$

we apply KCL to node Q: $i_2 = i_3 + 3I_o$ But $I_o = -i_4$,

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$