Chapter 2: Basic Laws

Lecture#1

Reference:

Fundamentals of
Electric Circuits
Charles K. Alexander | Matthew N. O. Sadiku

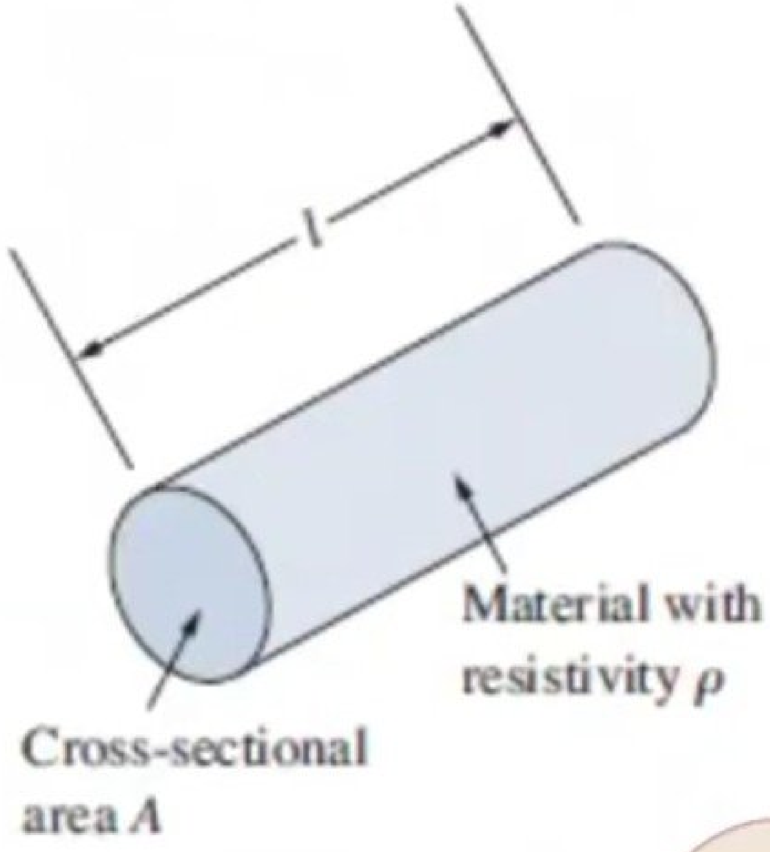
- This chapter will introduce Ohm's law: a central concept in electric circuits.
- Resistors will be discussed in more detail.
- Circuit topology and the voltage and current laws will be introduced.
- Finally, meters for measuring voltage, current, and resistivity will be presented.

Ohm's Law

❑ Resistance:

- ✓ **Def:** *a material property to resist the flow of the electric current.*
- ✓ **Symbol**: R
- ✓ **Unit**: Ohm, Ω
- ✓ In mathematical form (as measured in the Lab):

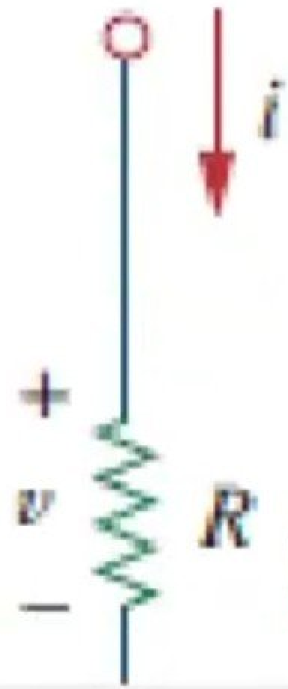
$$R = \rho \frac{\ell}{A}$$



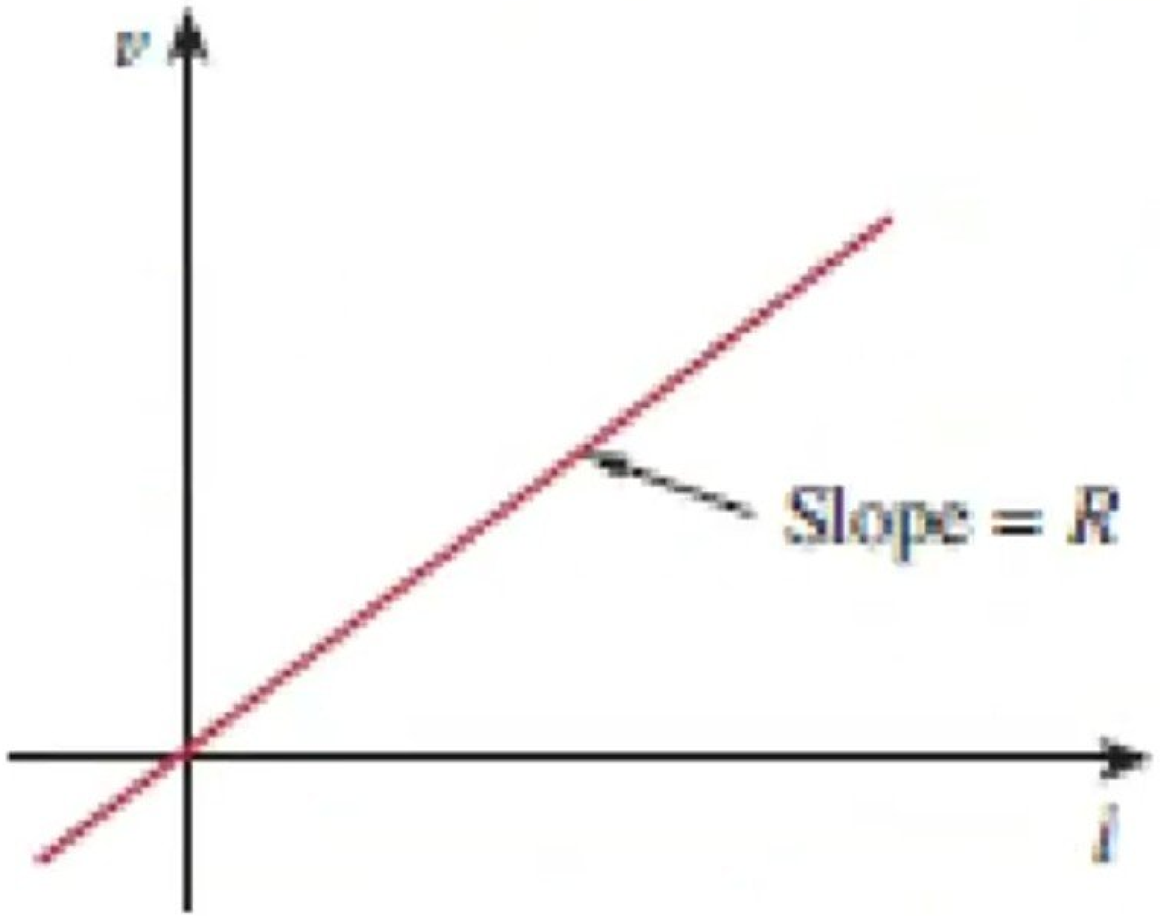
Ohm's Law

❑ The Resistor:

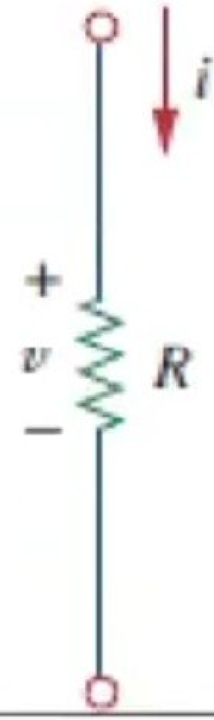
- **Def:** *The circuit element used to model the current-resisting behavior of a material.*
- The resistor has a resistance *R*.
- It is the basic passive element in a CKT.
- Use passive sign convention.



Resistors that obey Ohm's law are linear.



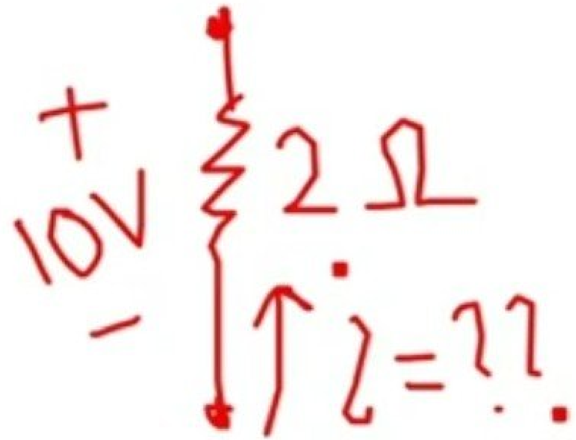
$$v = Ri$$

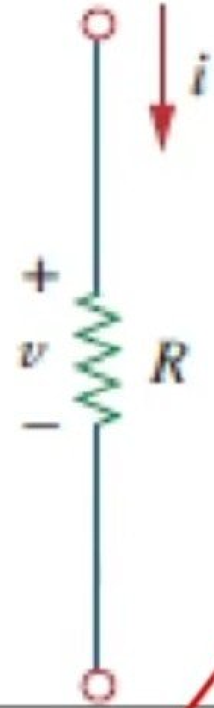


Passive sign convention

✓ **Def**: the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v \propto i$$





✓ Ohm defined the constant of proportionality for a resistor to be the resistance, R, which is a material property.

✓ Mathematical form:

$$v = Ri$$

Passive sign convention

❑Ohm's Law:

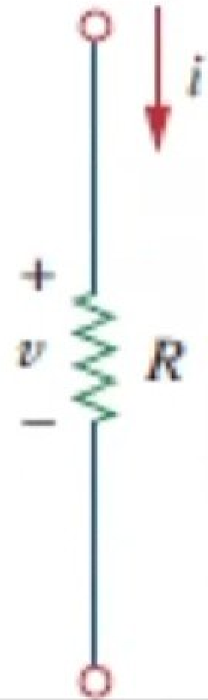
- **Note:** Ohm's law implies that:

$$R = \frac{v}{i}$$ and $$i = \frac{v}{R}$$

and

$$1\ \Omega = 1\ \text{V/A}$$

- **Note:** To apply Ohm's law, the direction of current i and the polarity of voltage v must conform with the passive sign convention.



Passive sign convention

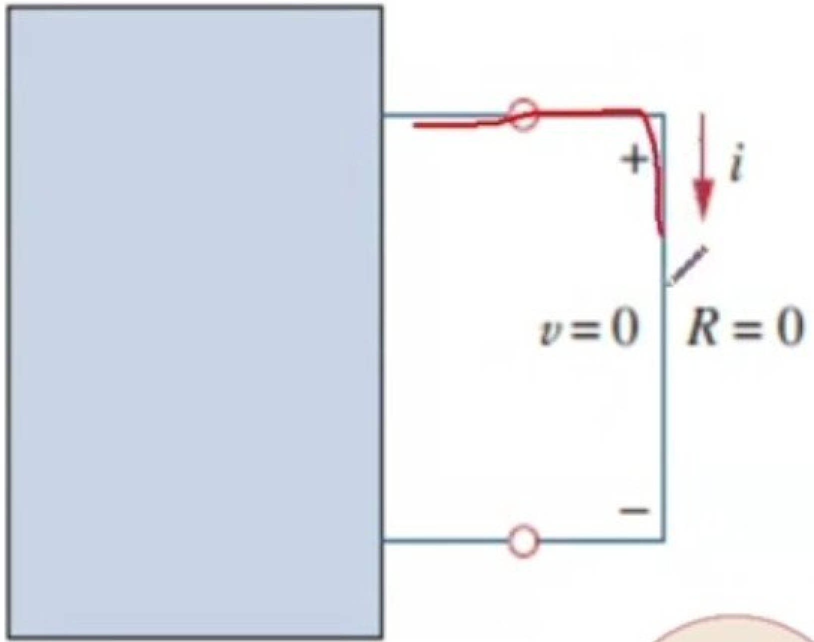
Ohm's Law

□ Short Circuit (S.C):

- **S.C:** is a circuit element with resistance approaching zero.
- This implies that:

$$v = Ri = (0)i = 0$$

but the current could be anything.



- **Note:** in practice, a short circuit is usually a connecting wire assumed to be a perfect conductor.

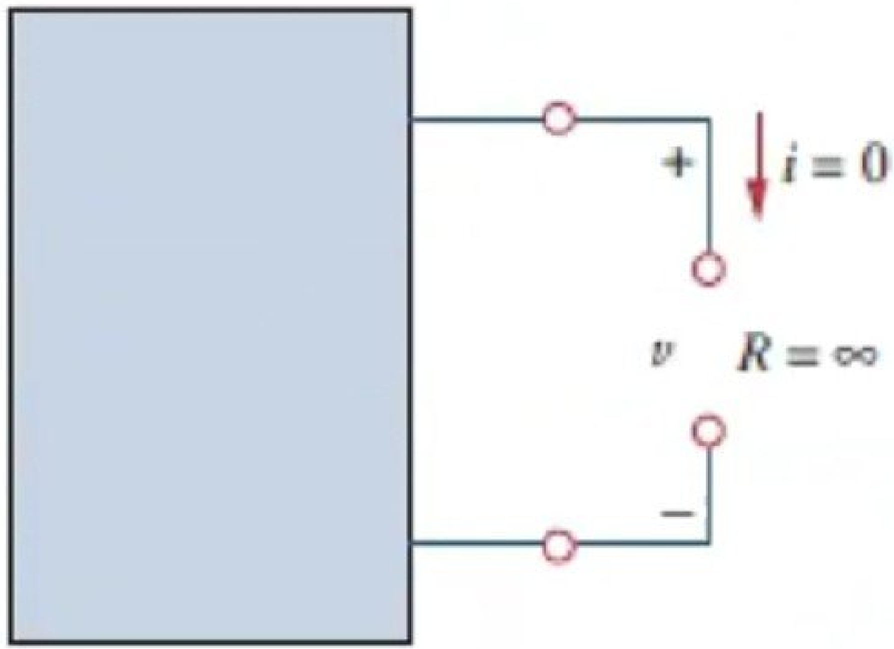
Ohm's Law

❑ Open Circuit (O.C):

- **O.C:** is a circuit element with resistance approaching ∞.
- This implies that:

$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$

but the voltage could be anything.



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Ohm's Law

❑ The Conductance G :

- It is the reciprocal of resistance R.
- It is the ability of an element to conduct electric current.
- it is measured in mhos ℧ or siemens (S).

$$G = \frac{1}{R} = \frac{i}{v}$$

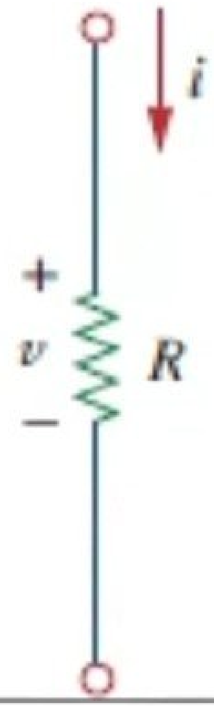
$$1\ \text{S} = 1\ \mho = 1\ \text{A/V}$$

Ohm's Law

❑ Power Dissipated by a Resistor:

$$p = vi = i^2R = \frac{v^2}{R}$$

$$p = vi = v^2G = \frac{i^2}{G}$$



Passive sign convention

Ohm's Law

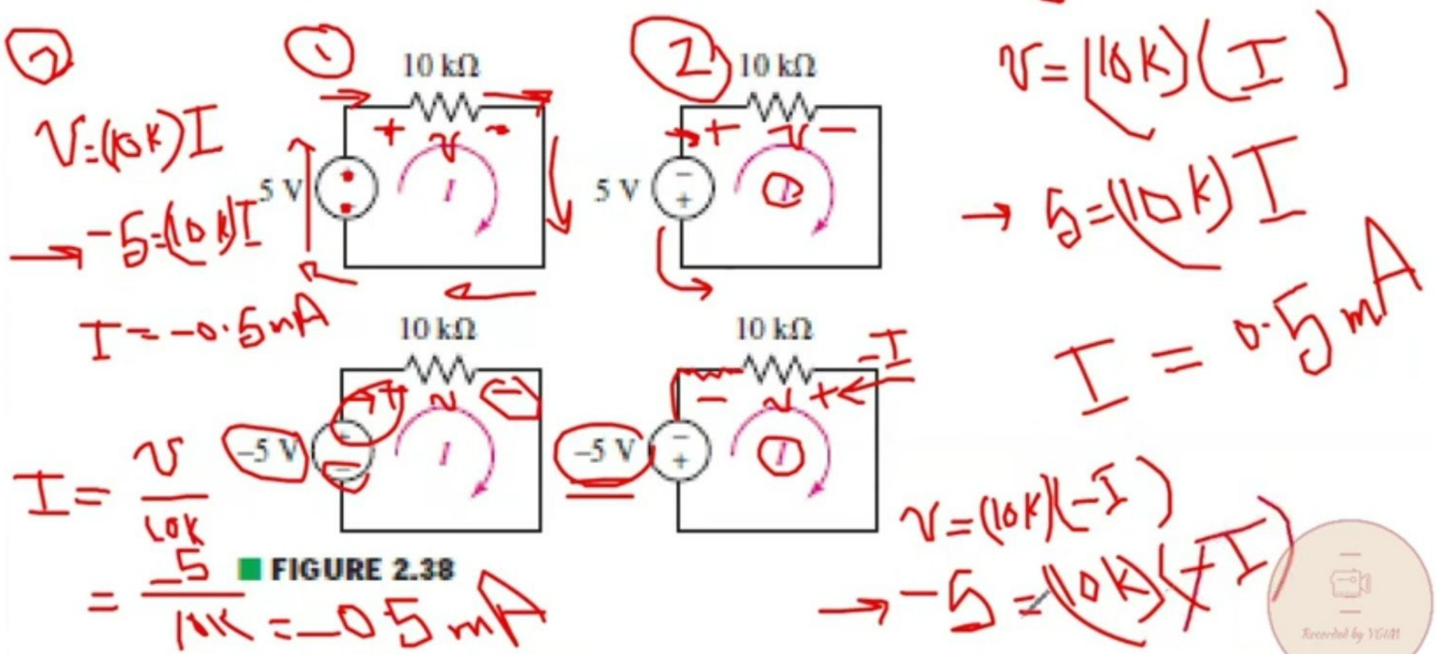
❑ Power Dissipated by a Resistor:

❖ <u>Note</u>:

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since *R* and *G* are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

Ohm's Law

48. For each of the circuits in Fig. 2.38, find the current I and compute the power absorbed by the resistor.



■ FIGURE 2.38

Ref: Hayt, William Hart, Jack Ellsworth Kemmerly, and Steven M...

Ohm's Law

Example 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.

Answer: 30 V, 100 μS, 90 mW.

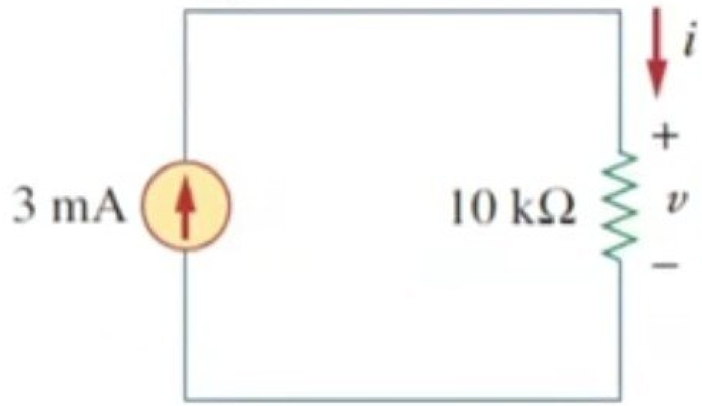


Figure 2.9
For Practice Prob. 2.2

Ohm's Law

48. For each of the circuits in Fig. 2.38, find the current I and compute the power absorbed by the resistor.

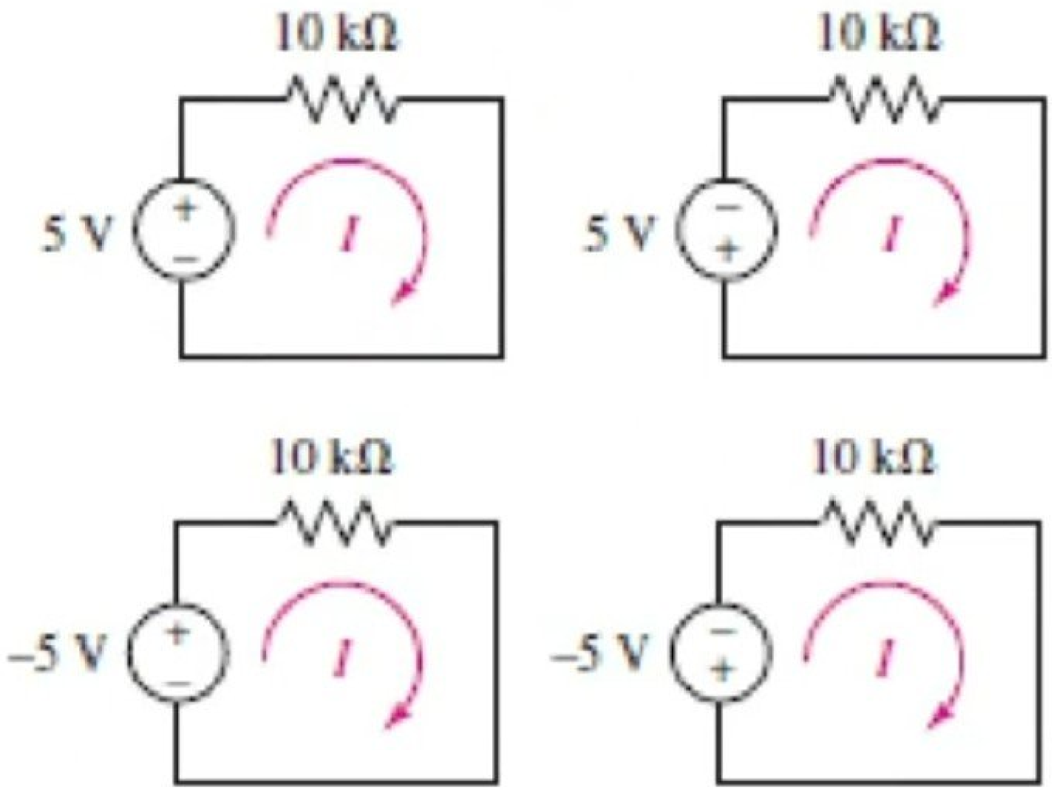
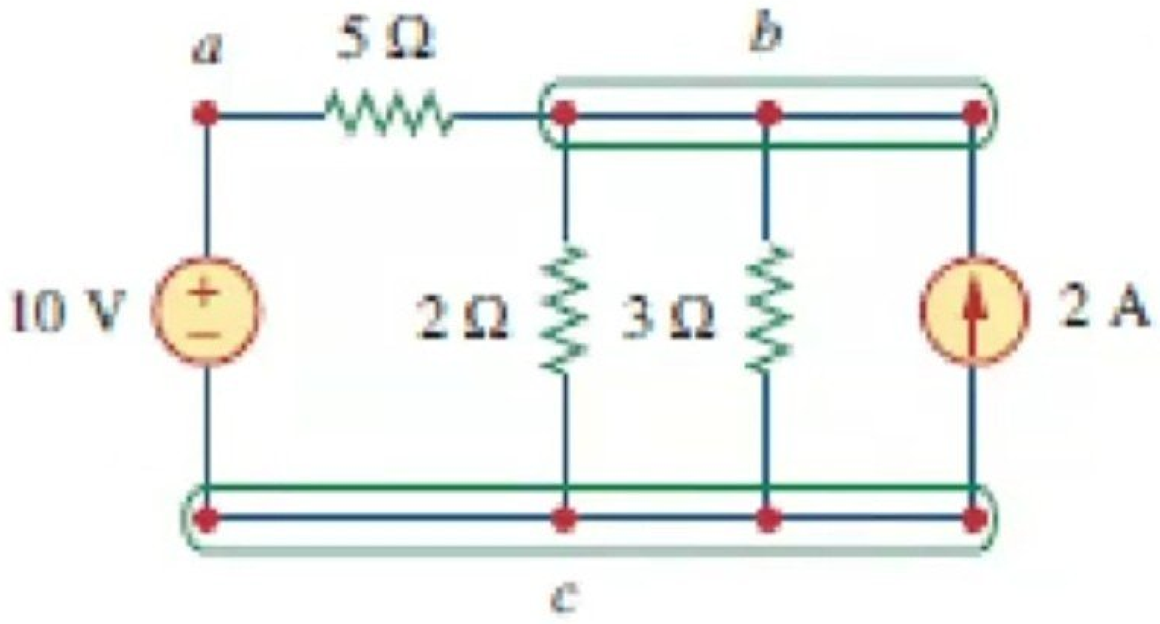


FIGURE 2.38

Network Topology

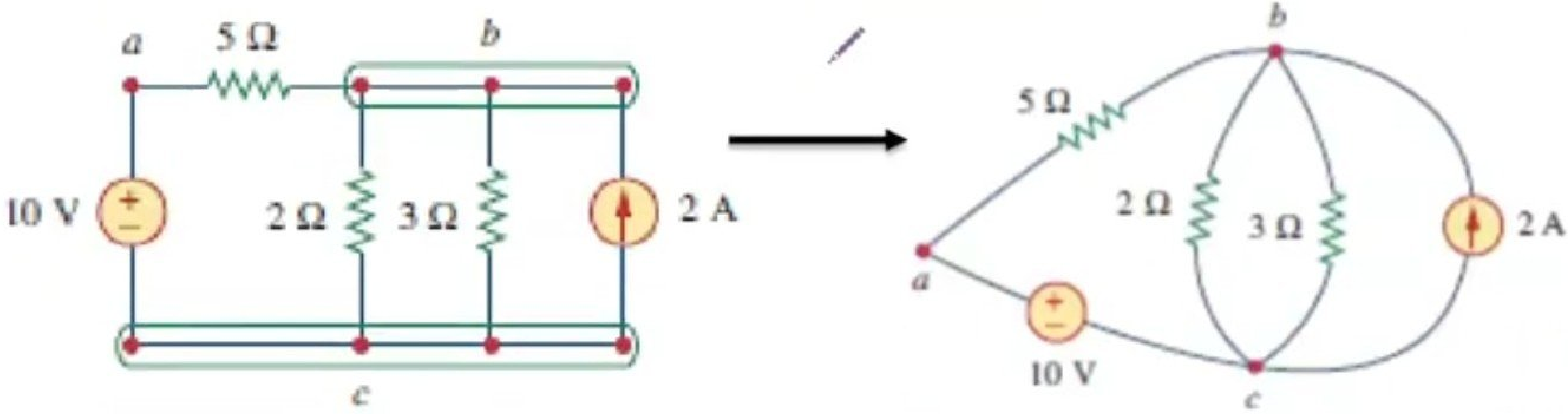
- **Branch**: *represents a single element such as a voltage source or a resistor (two-terminal element.)*

Note: a branch can represent elements in series.



- **Node**: *the point of connection between two or more branches.*

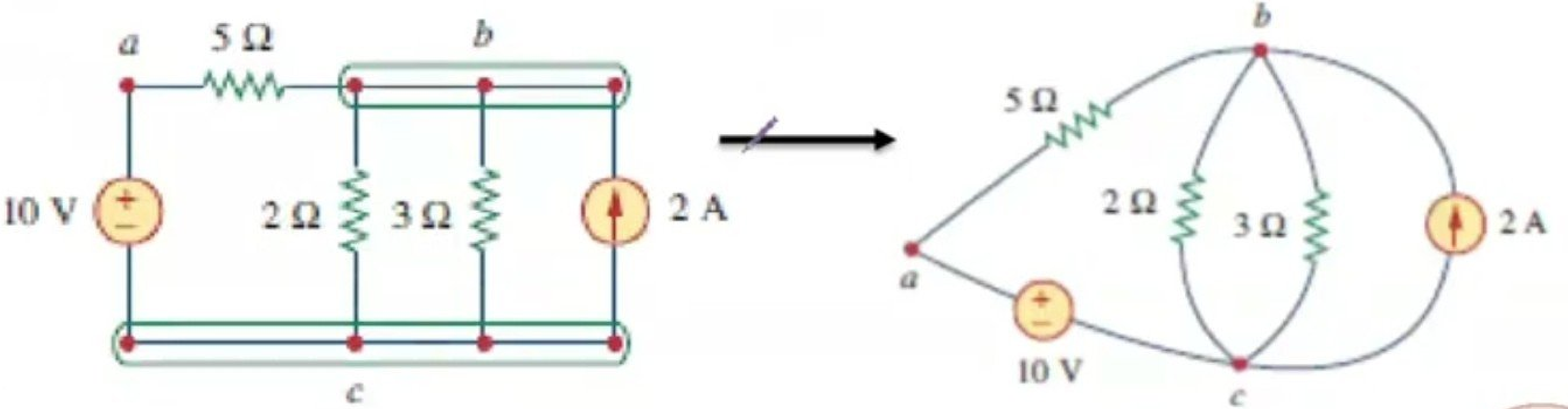
- Note: usually indicated by a dot in a CKT.



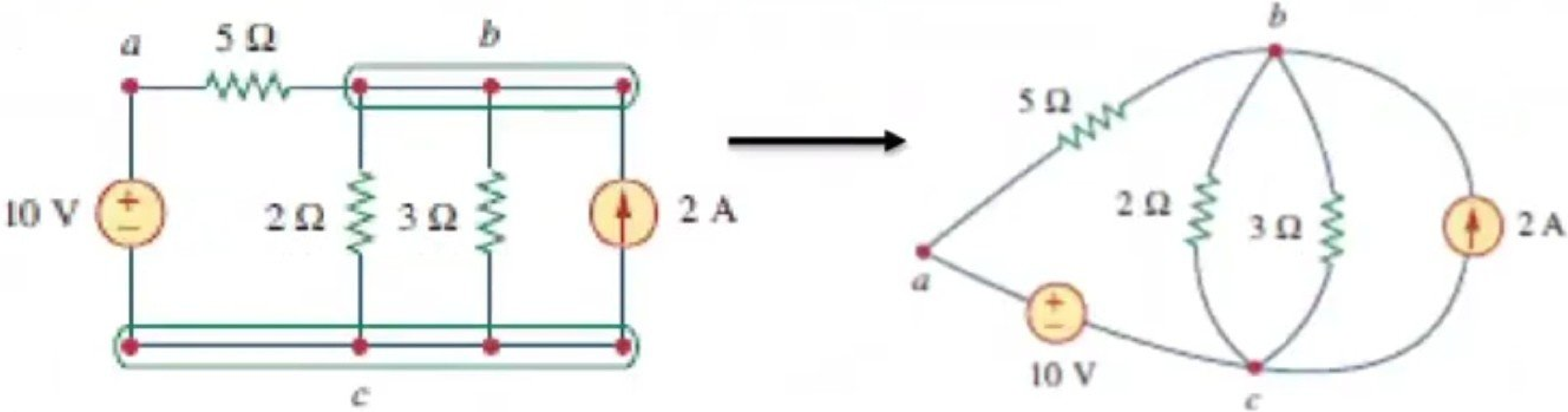
- **The CKT has 3 nodes.**

Network Topology

- **Path**: *a set of nodes and element passed.*
-



Network Topology

- **Loop**: *is ant closed path in a CKT (path from a node and back to the same node).*
- 

- **The CKT has 6 loops.**

Network Topology

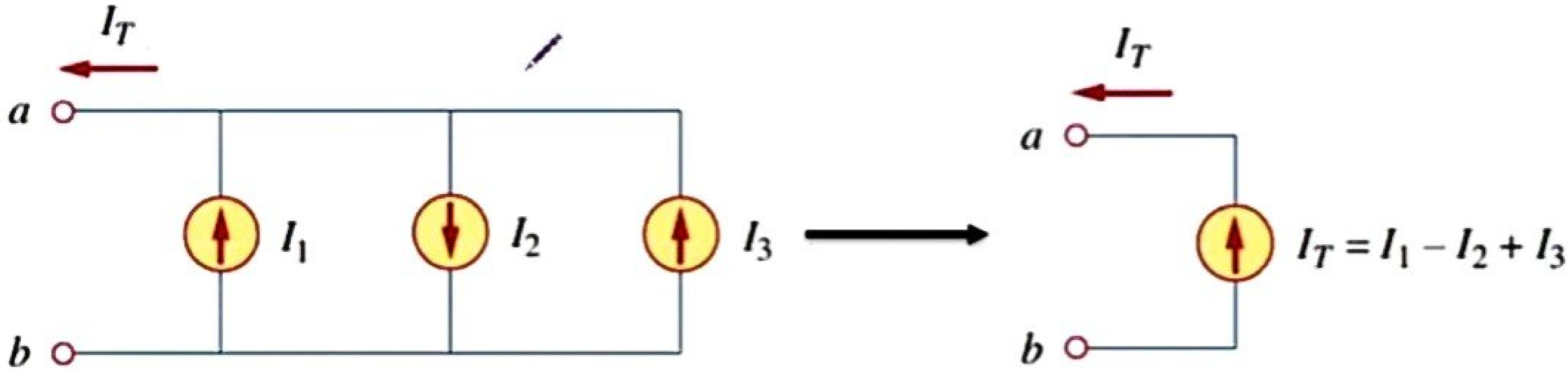
- **Elements in Series**: *exclusively share a single node and consequently carry the same current.*

- **Elements in Parallel**: *are connected to the same two nodes and consequently have the same voltage across them.*

- *Ref: Hayt, William Hart, Jack Ellsworth Kemmerly, and Steven M. Durbin. Engineering circuit analysis. New York: McGraw-Hill, 2002.*

Kirchhoff's current law (KCL).

- **<u>Application of KCL</u>:** combining current sources in parallel.



PRACTICE

3.1 Count the number of branches and nodes in the circuit in Fig. 3.4. If $i_x = 3$ A and the 18 V source delivers 8 A of current, what is the value of R_A? (*Hint:* You need Ohm's law as well as KCL.)

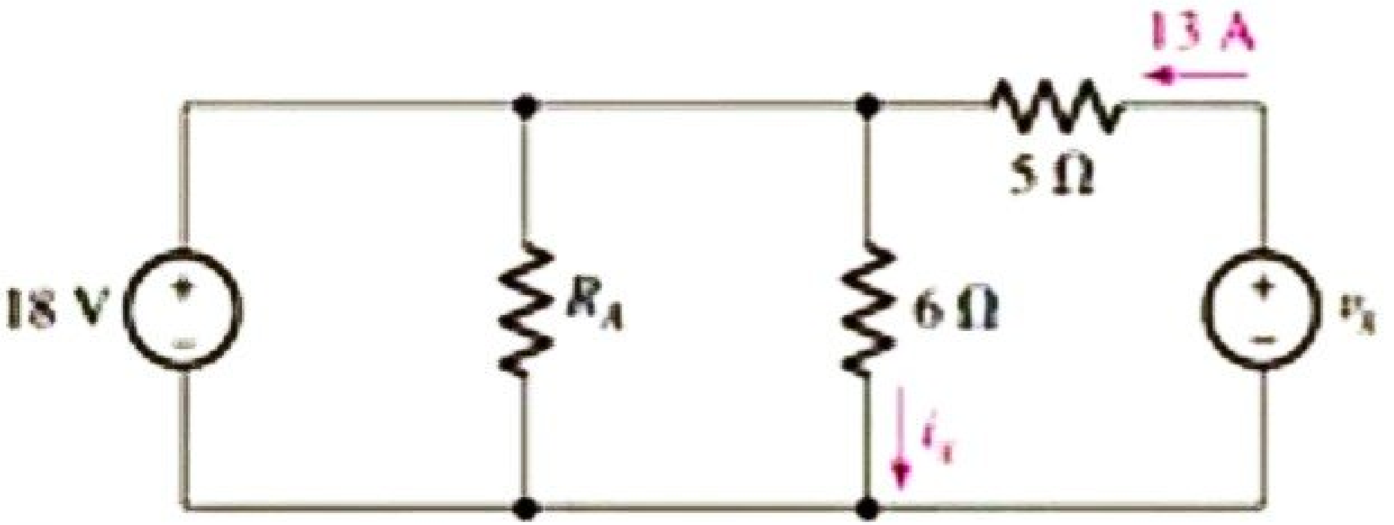


FIGURE 3.4

Ans: 5 branches, 3 nodes, 1Ω.

Kirchhoff's current law (KCL).

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

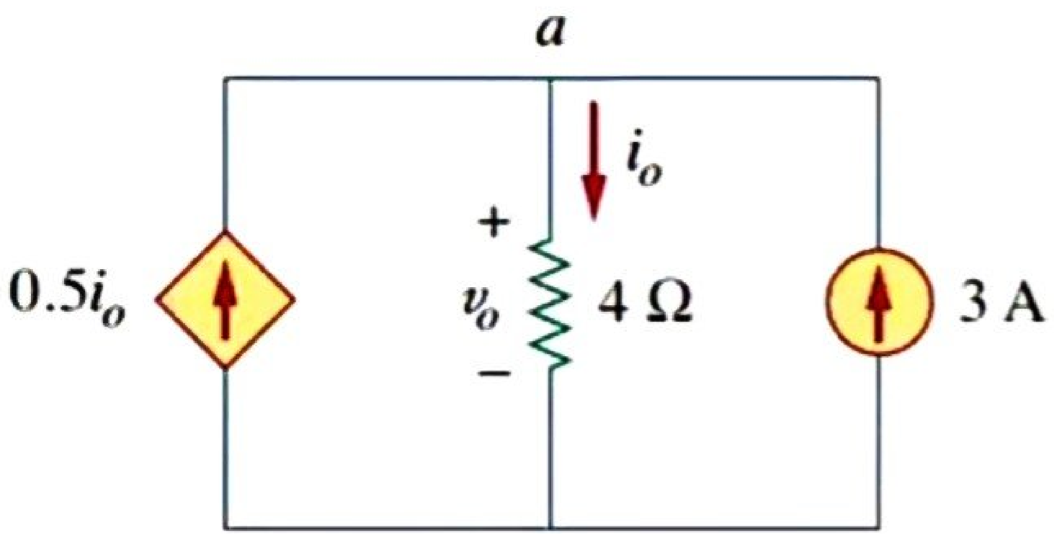


Figure 2.25
For Example 2.7.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

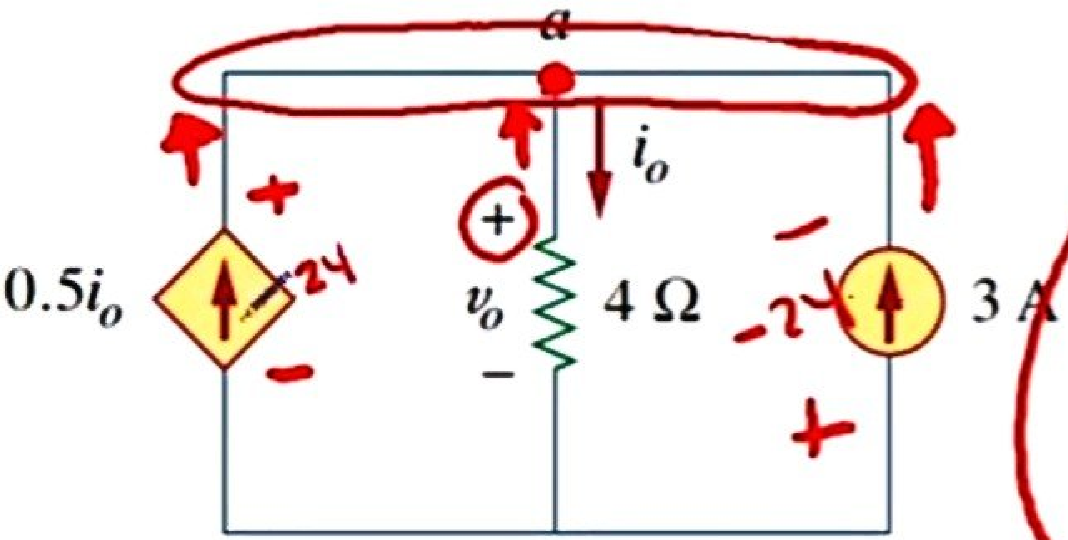


Figure 2.25
For Example 2.7.

$v_o = 4 i_o$ — (1)

$0.5 i_o - i_o + 3 = 0$

$\Rightarrow i_o = 6\text{ A}$ ✓

$v_o = (4)(6) = 24\text{ V}$ ✓

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 12 V, 6 A.

<u>HW</u>: also find the power absorbed by each element.

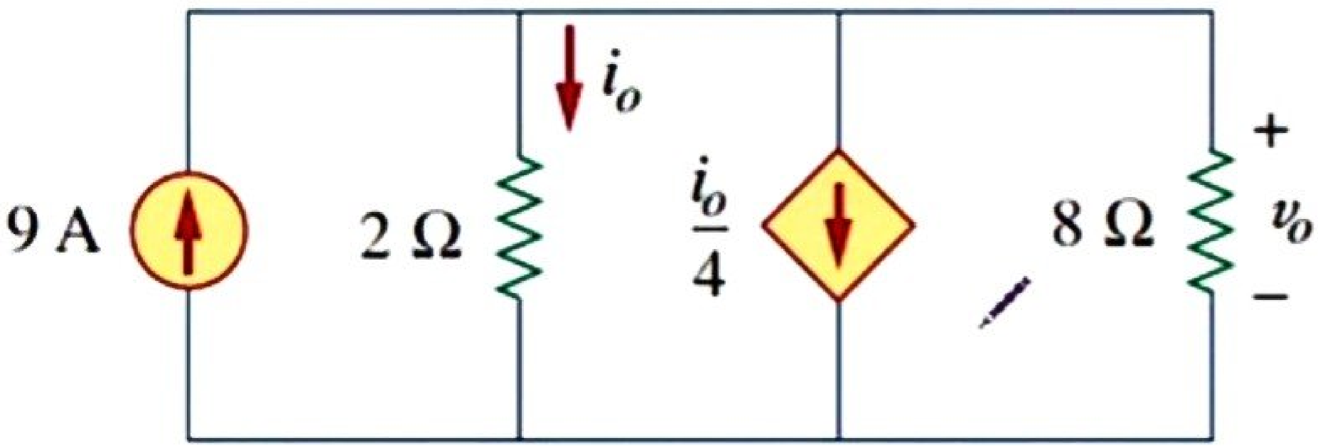


Figure 2.26
For Practice Prob. 2.7.

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 12 V, 6 A.

HW: also find the power absorbed by each element.

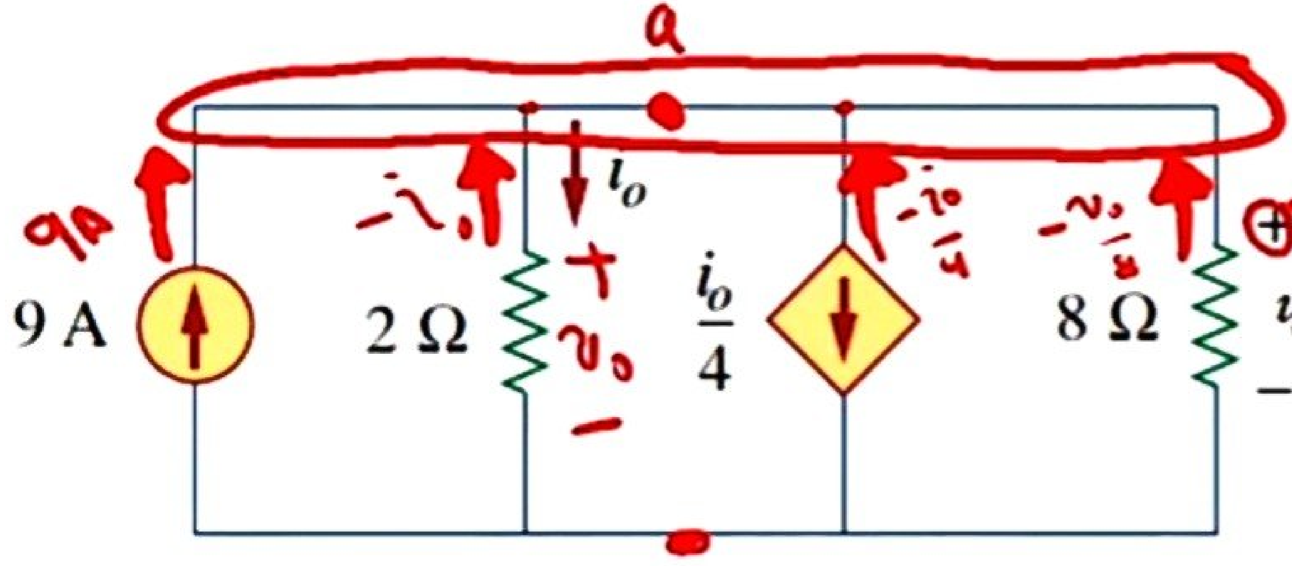


Figure 2.26
For Practice Prob. 2.7.

Kirchhoff's current law (KCL).

- <u>Note</u>: In the last two examples are CKTs are called <u>single-node CKT,</u> in which all elements are in parallel and connected between two nodes.
- For such CKTs it is better to apply KCL.

- Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.

- Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887).

- These laws are formally known as:

✓ **Kirchhoff's current law (KCL).**

✓ **Kirchhoff's voltage law (KVL**

Kirchhoff's voltage law (KVL).

❑**KVL:** **the algebraic sum of all voltages around a closed path (or loop) is zero.**

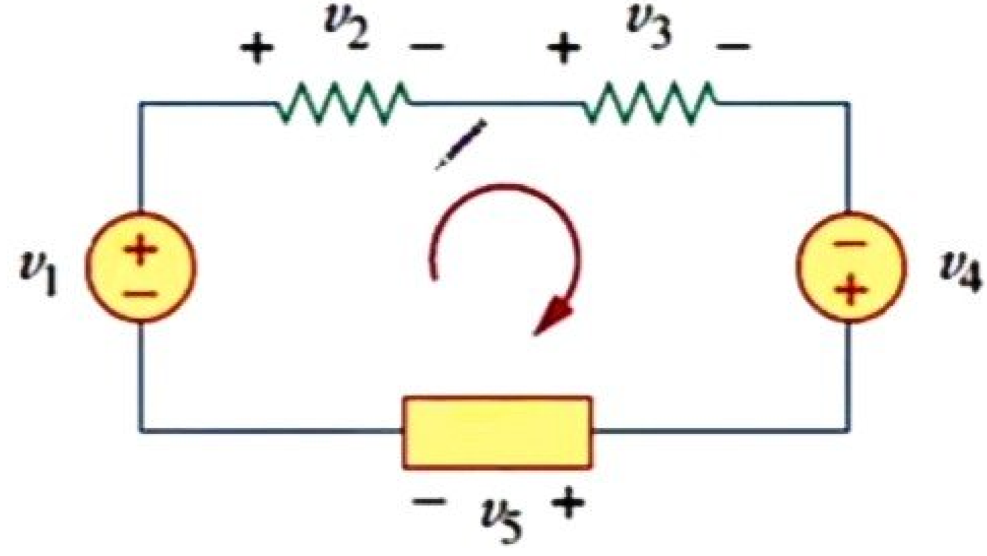
Mathematically:

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

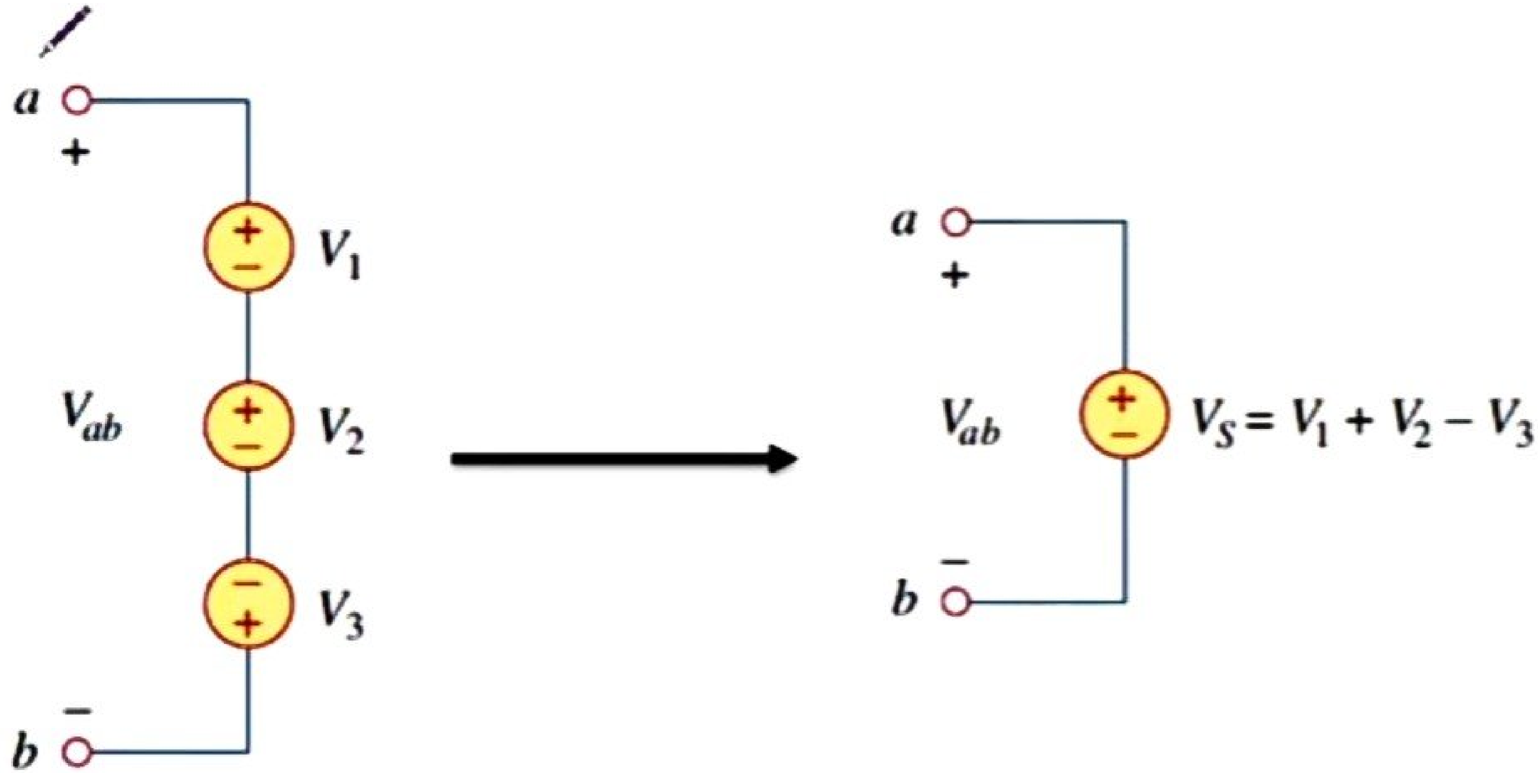
Kirchhoff's voltage law (KVL).

- **KVL:** the algebraic sum of all voltages around a closed path (or loop) is zero.



$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

❑Application on KVL: combining in series voltage sources.



❑ *Note: voltage sources in parallel should have same value.*

Kirchhoff's voltage law (KVL).

In the circuit of Fig. 3.6, find v_x and i_x.

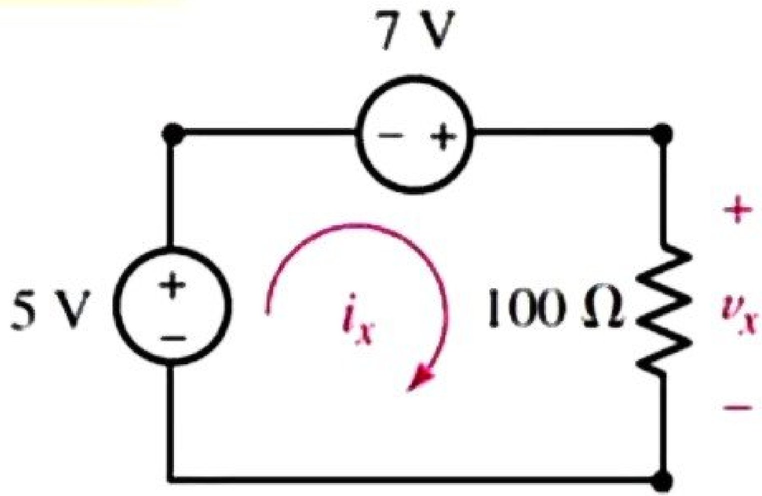
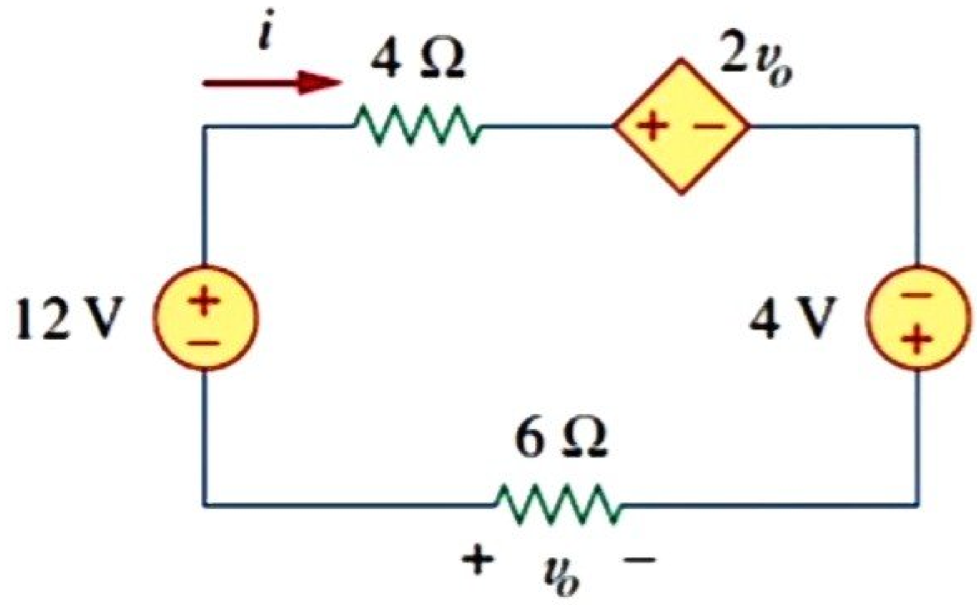


FIGURE 3.6 A simple circuit with two voltage sources and a single resistor.



$$-12 + 4i + 2v_o - 4 + 6i = 0$$

$$v_o = -6i$$

$$i = -8 \text{ A}$$

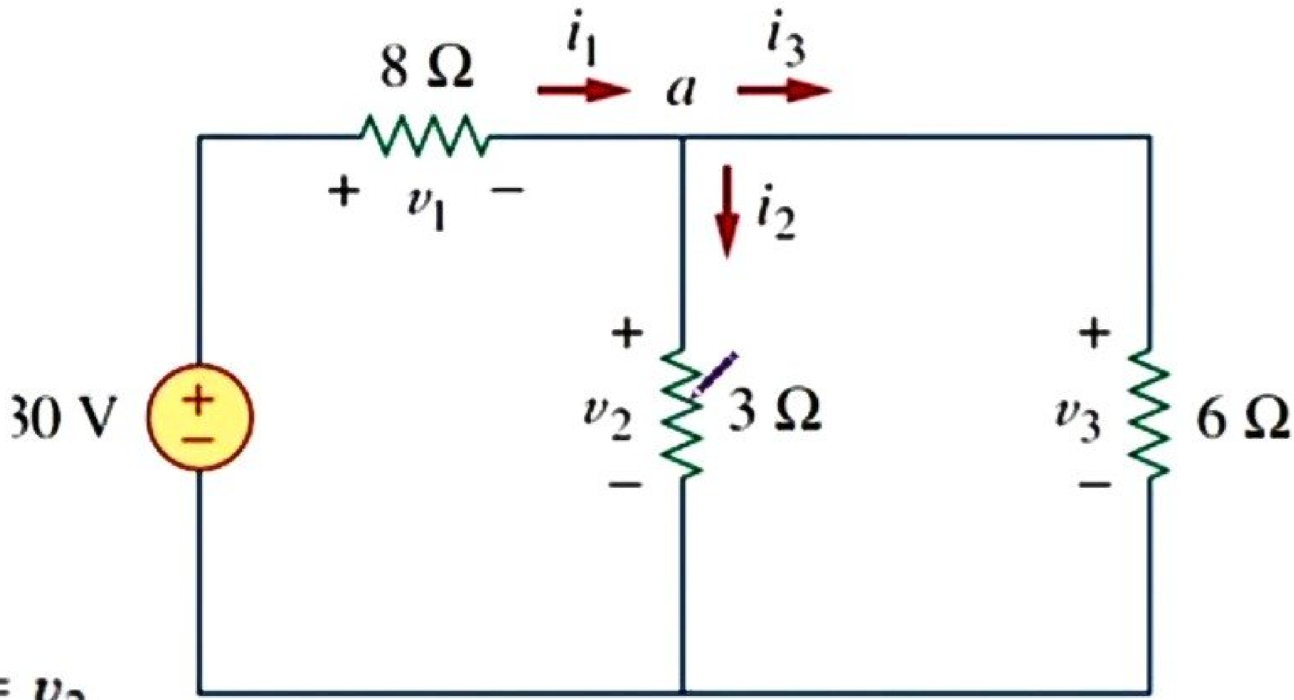
$$v_o = 48 \text{ V}.$$

Kirchhoff's current law (KCL).

- <u>Note</u>: In the last two examples the CKTs are called <u>single-Loop CKT,</u> in which all elements are in series and carry same current.
- For such CKTs it is better to apply KVL.

Example 2.8

Find currents and voltages in the circuit shown



$$i_1 - i_2 - i_3 = 0$$

$$-30 + 8i_1 + 3i_2 = 0$$

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2$$

$$6i_3 = 3i_2$$

$i_1 = 3$ A, $\quad i_3 = 1$ A, $\quad v_1 = 24$ V, $\quad v_2 = 6$ V, $\quad v_3 = 6$ V

In the circuit of Fig. 3.8 there are eight circuit elements. Find v_{R2} (the voltage across R_2) and the voltage labeled v_x.

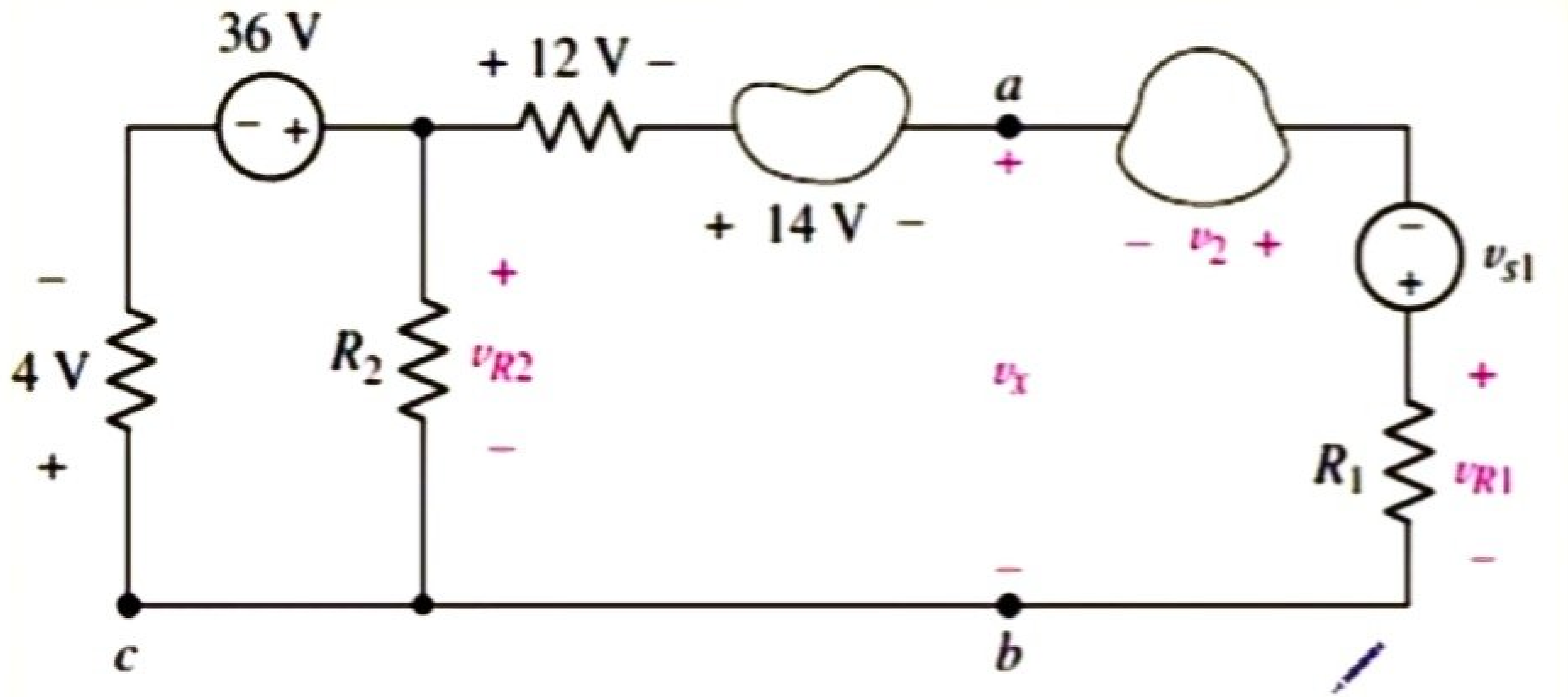


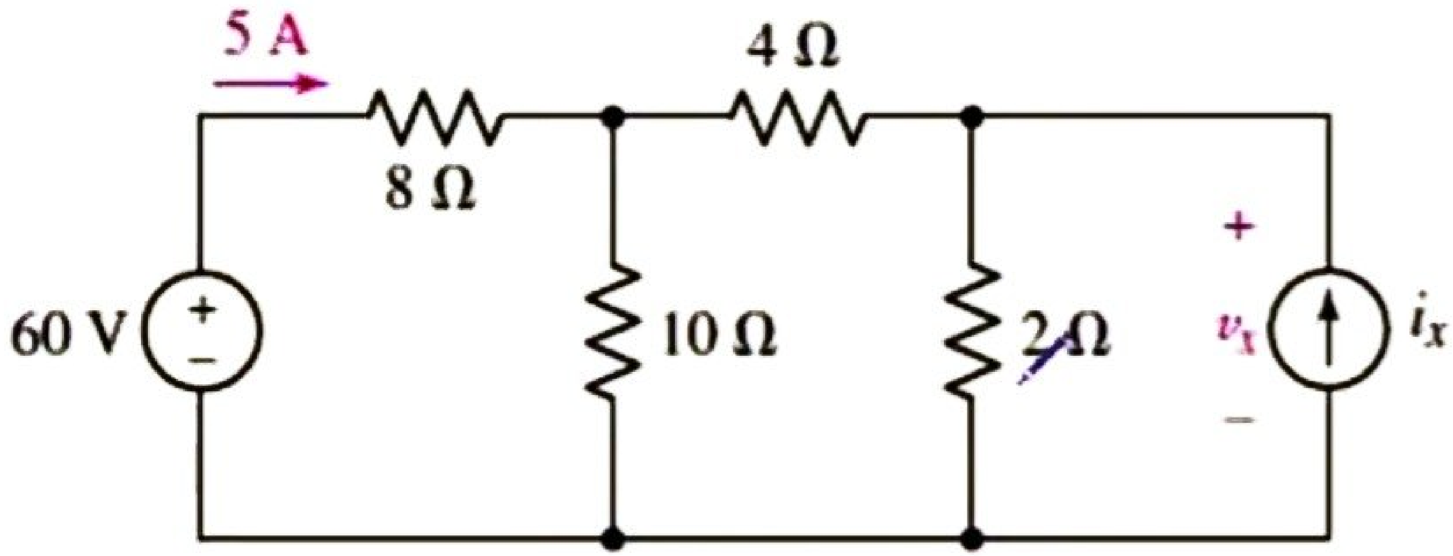
FIGURE 3.8 A circuit with eight elements for which we desire v_{R2} and v_x.

$4 - 36 + v_{R2} = 0$ $\qquad$ $v_{R2} = 32$ V.

$+4 - 36 + 12 + 14 + v_x = 0$ $\qquad$ $v_x = 6$ V

EXAMPLE 3.4

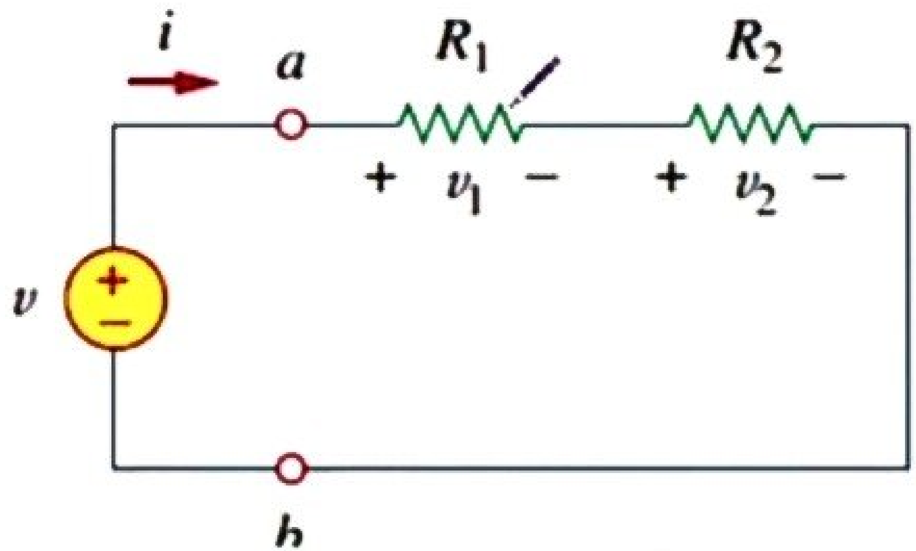
Determine v_x in the circuit of Fig. 3.10*a*.



(*a*)

Series Resistors& Voltage Division

$v_1 = iR_1, \qquad v_2 = iR_2$



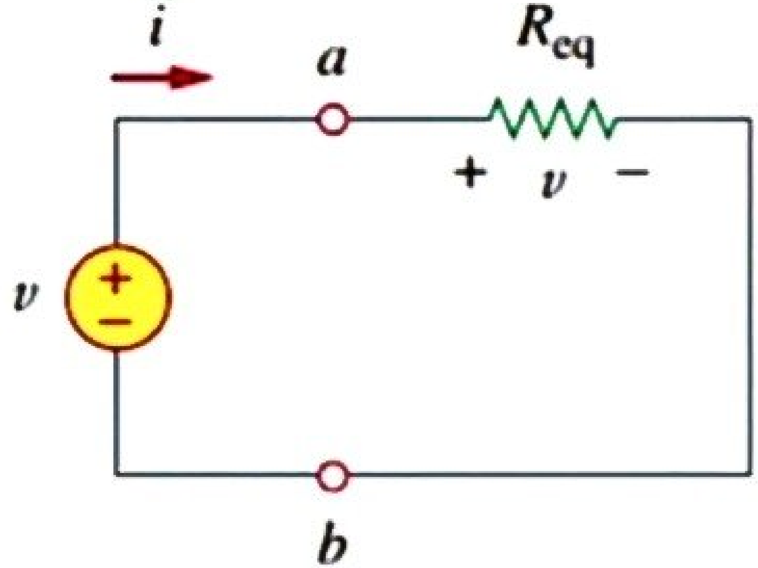
KVL:

$$-v + v_1 + v_2 = 0$$

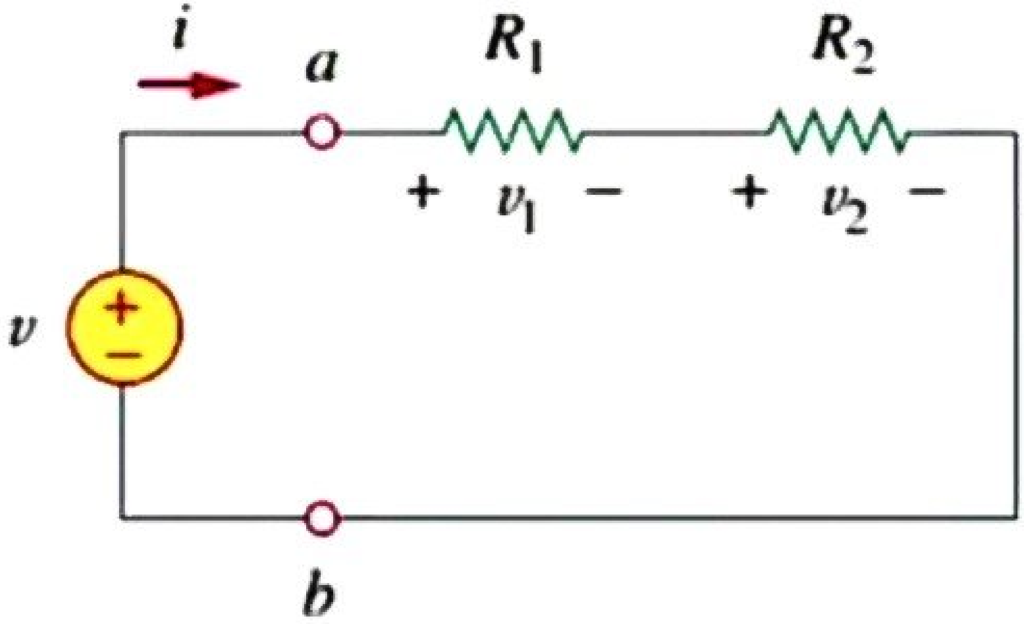
$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq}$$



$$\boxed{R_{eq} = R_1 + R_2}$$



$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Rule:

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

& Current Division

❑ Series resistors in terms of conductance:

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}$$

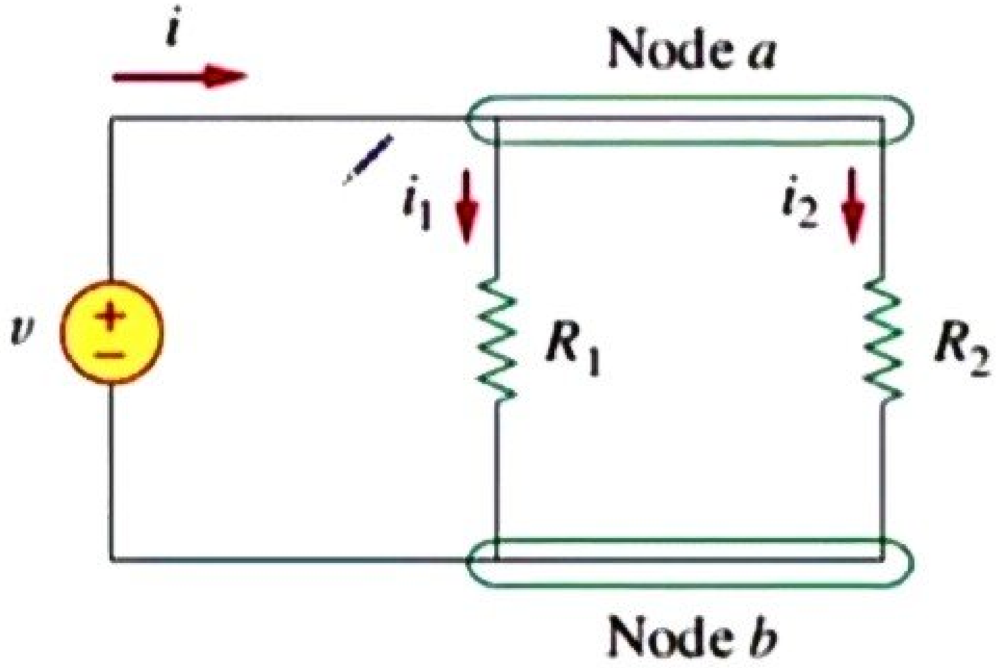
where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, …, $G_N = 1/R_N$.

Parallel Resistors & Current Division

Ohm's law,

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

KCL $\quad i = i_1 + i_2$



$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

& Current Division

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N}$$

Parallel Resistors & Current Division

❑ **Parallel resistors in terms of conductance:**

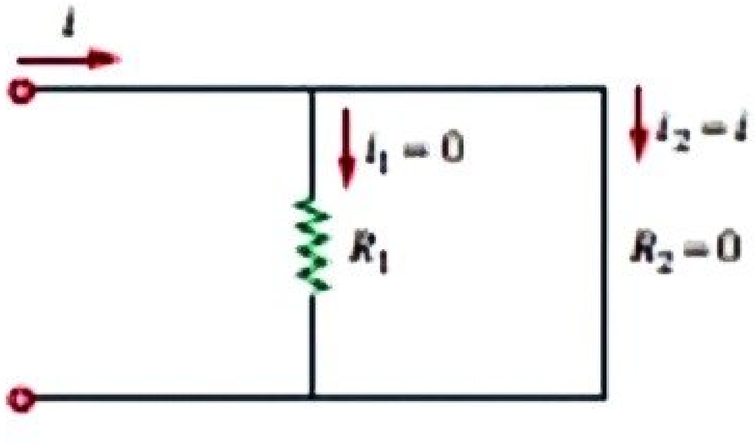
$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, . . . , $G_N = 1/R_N$.

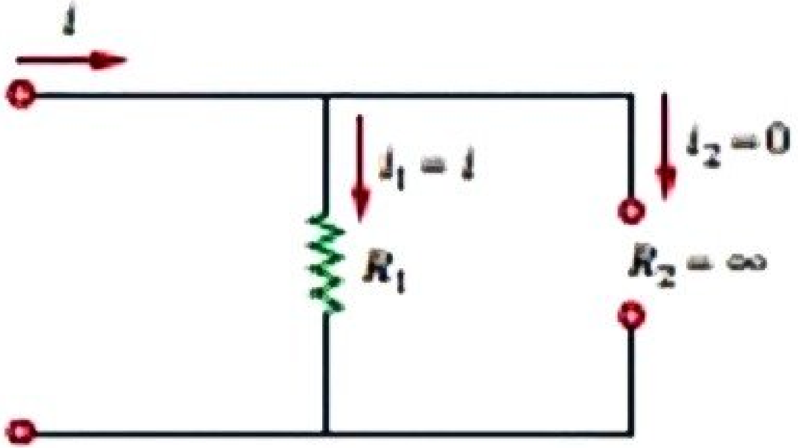
❑ **Current through the nth resistor:**

$$G_n$$

- **Note:**



(a)



Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

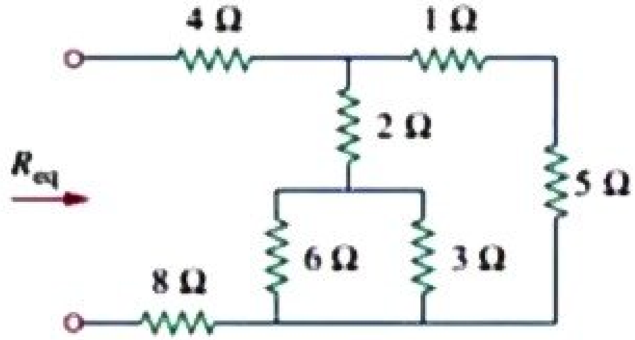


Figure 2.34
For Example 2.9.

$$6\,\Omega \parallel 3\,\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

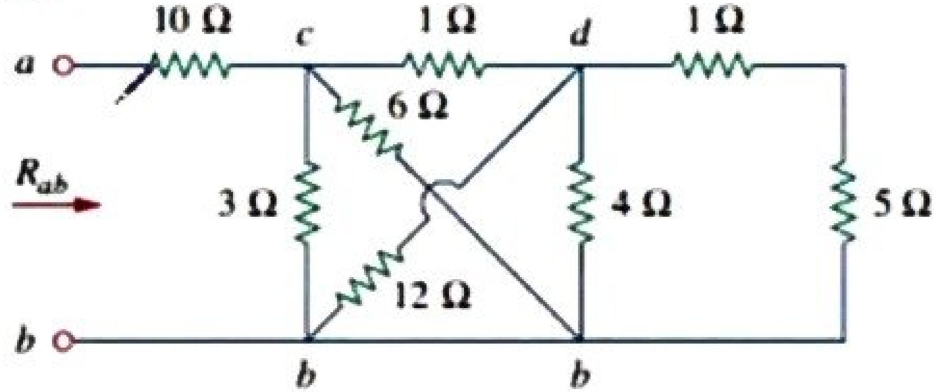
$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit



$$3\,\Omega \parallel 6\,\Omega = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

$$12\,\Omega \parallel 4\,\Omega = \frac{12 \times 4}{12 + 4} = 3\,\Omega$$

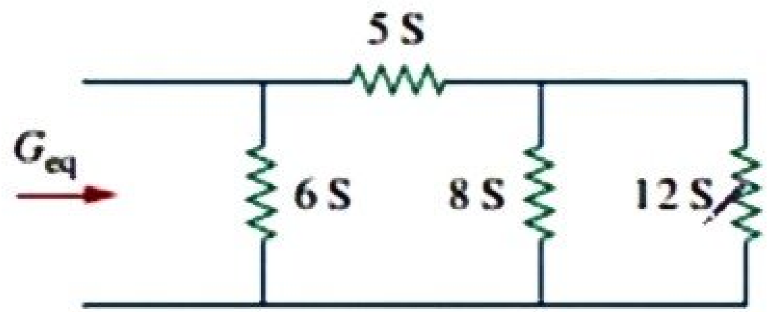
$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

$$2\,\Omega \parallel 3\,\Omega = \frac{2 \times 3}{2 + 3} = 1.2\,\Omega$$

$$R_{ab} = 10 + 1.2 = 11.2\,\Omega$$

Example 2.11

Find the equivalent conductance G_{eq}



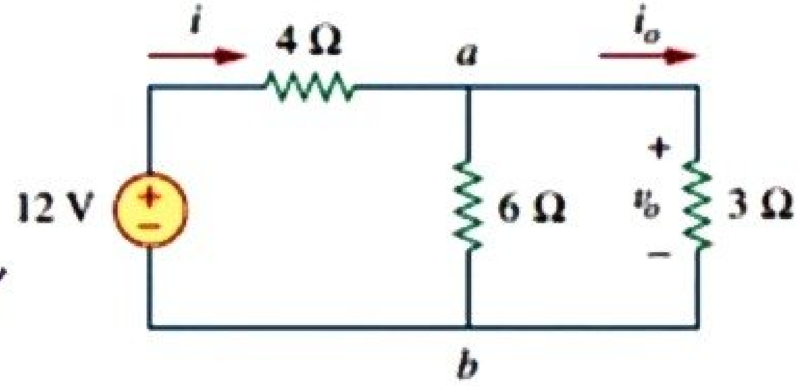
$8\text{ S} + 12\text{ S} = 20\text{ S}$

$\frac{20 \times 5}{20 + 5} = 4\text{ S}$

$G_{eq} = 6 + 4 = 10\text{ S}$

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.



(a)

$v_o = 4$ V $\quad i_o = \frac{4}{3}$ A $\quad p_o = 5.333$ W

Practice Problem 2.12

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

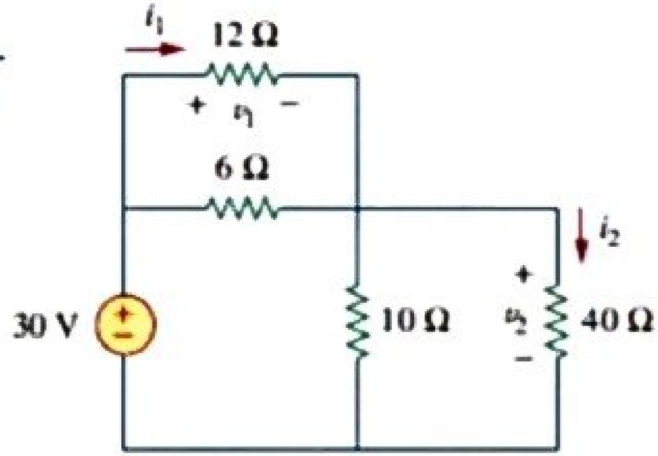
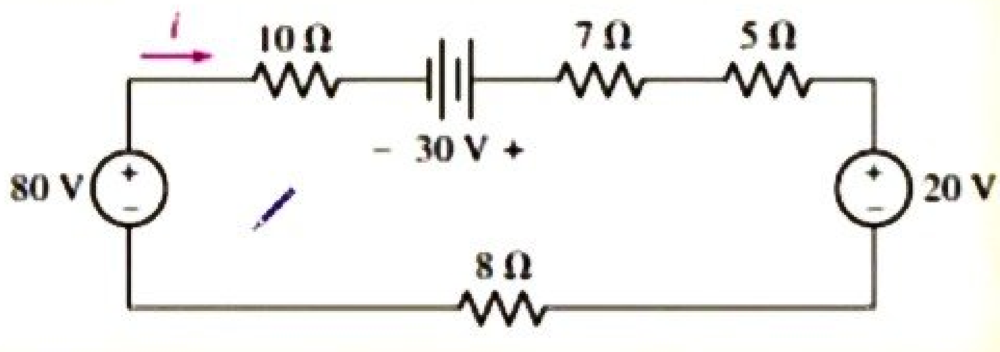


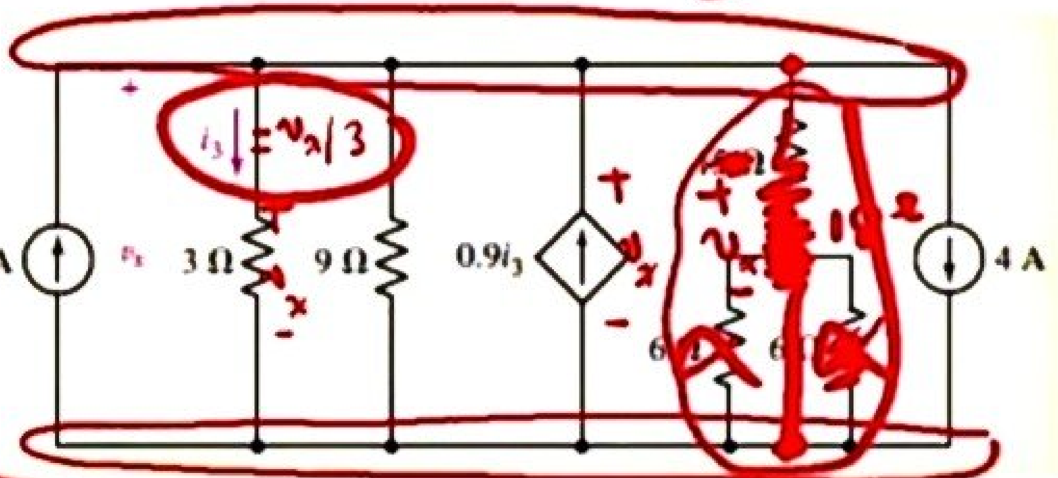
Figure 2.43
For Practice Prob. 2.12.

Use resistance and source combinations to determine the current *i*



Series & Parallel Resistors. Voltage & Current Divisions

Calculate the power and voltage of the dependent source

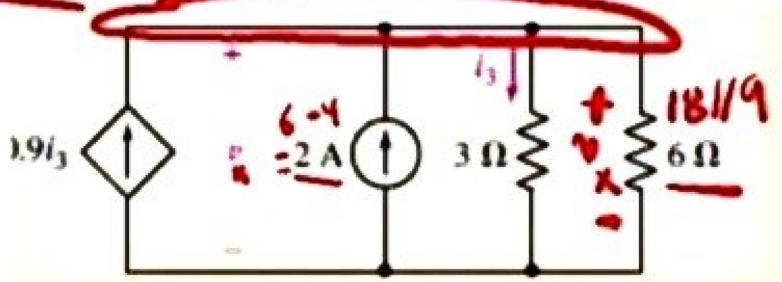


$$-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0$$

$$v = 3i_3$$

$$i_3 = \frac{10}{3}\ \text{A}$$

$$v = 3i_3 = 10\ \text{V}$$



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The Y and Δ networks are said to be *balanced* when

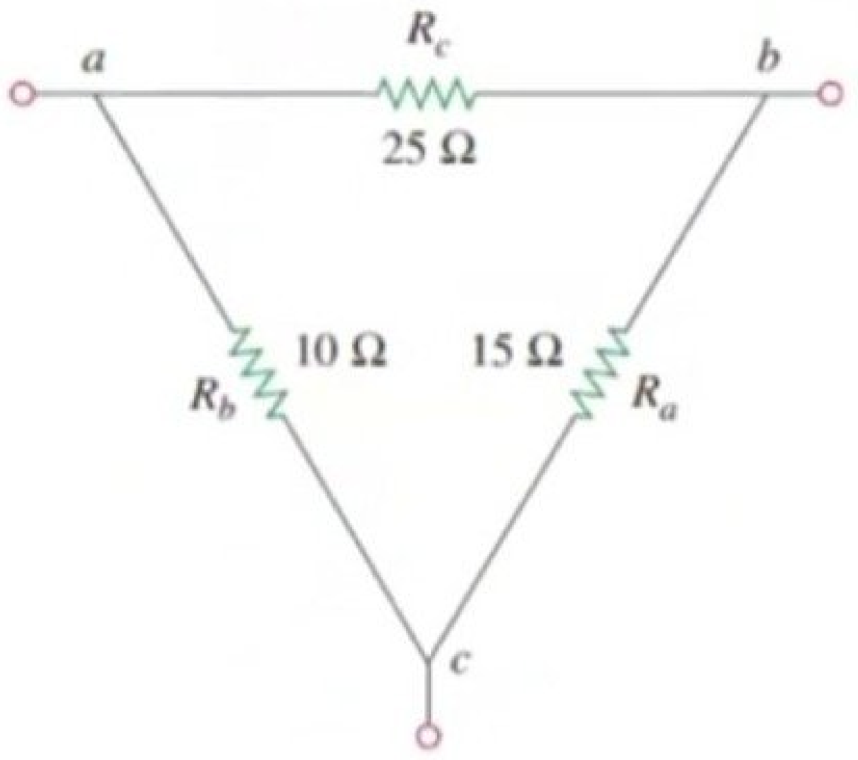
$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y$$

Example 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.



(a)

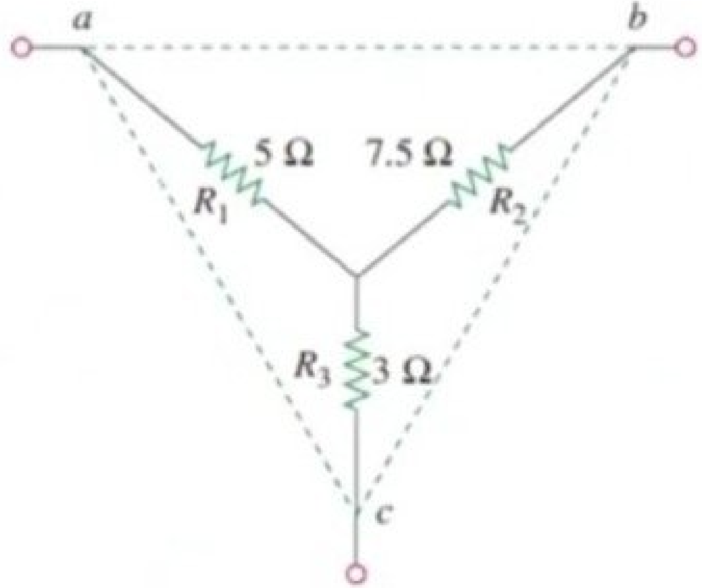
Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$



Practice Problem 2.14

Transform the wye network in Fig. 2.51 to a delta network.

Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

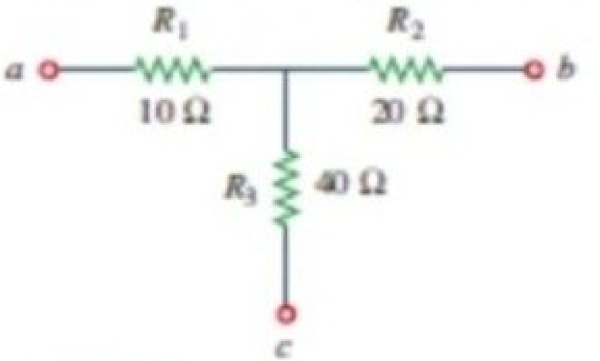


Figure 2.51
For Practice Prob. 2.14.

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

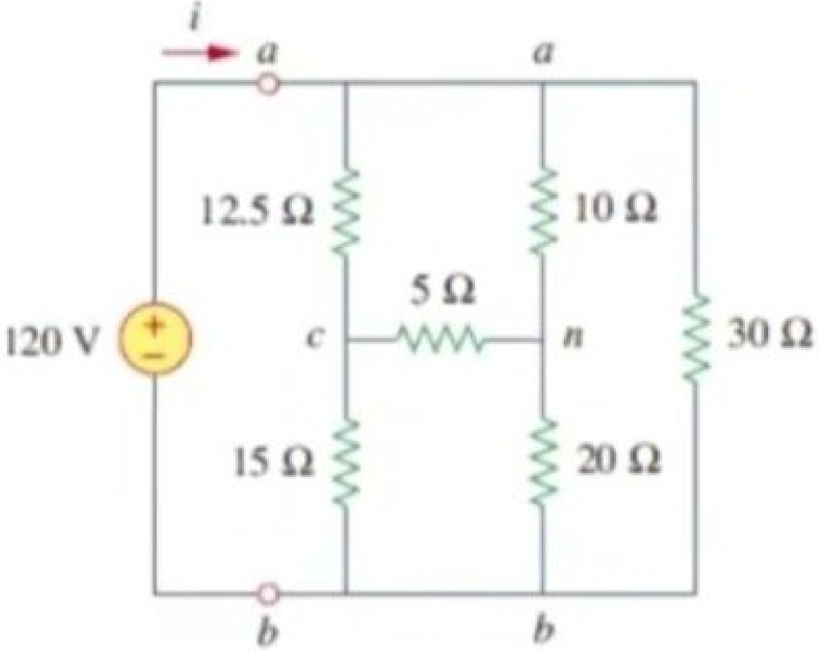
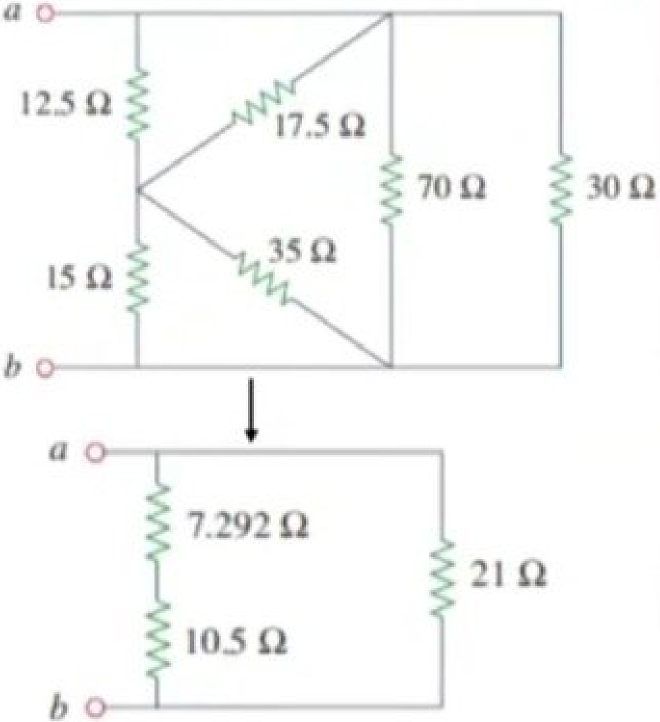


Figure 2.52
For Example 2.15.

- Solution:



$R_1 = 10\,\Omega, \quad R_2 = 20\,\Omega, \quad R_3 = 5\,\Omega$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\,\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\,\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\,\Omega$$

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\,\Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\,\Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\,\Omega$$

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\,\Omega}$$

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\,A}$$