$\frac{5(4)}{2}$

Question 1: (7 points)

1. Number of possible tours in the undirected complete graph G with 5 vertices is :
   a) 60
   b) 30
   c) 24
   d) 12
   e) 6

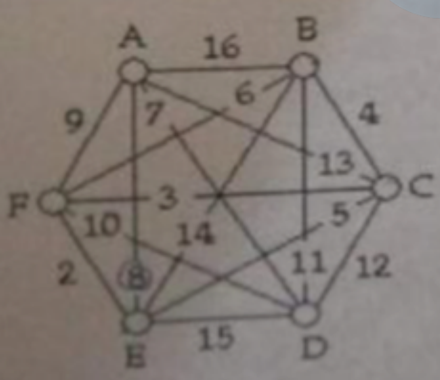
2. Christofides' heuristic is an effective practical heuristic that has the best-known worst-case performance bound for the traveling salesman problem on complete networks satisfying the triangle-inequality
   a) True
   b) False

3. For complete graphs with positive arcs, always the cost of the optimal MST is less than or equal to the cost of optimal TSP
   a) True
   b) False

4. Christofides heuristic will produce a walk with the same total cost regardless of which is node is selected in the initialization.
   a) True
   b) False

5. Given a set of nodes N and a set of arcs representing a spanning tree T, the number of nodes with odd degree with respect to the arc set T is even.
   a) True
   b) False

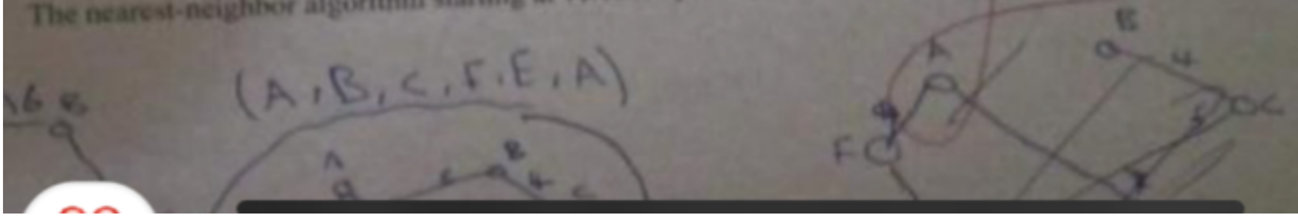
6. A delivery truck must deliver packages to 6 different store locations (A, B, C, D, E, and F). The trip must start and end at A. The graph below shows the distances (in miles) between locations. We want to minimize the total distance traveled.



(F,E) → (F,E) | (A,D) → (A,D)
(F,C) → (E,C) | (A,E) → (E,D)
(C,B) → (E,B) | (E,F) → (F,D)
(E,A) → (A,B) | (E,B) → (D,B)
(A,B) | (C,B) → (D,C)

The nearest-neighbor algorithm starting at vertex A yields the Tour T=[A,D,C,B,F,E,A], cost is 39

(A, B, C, F, E, A)



Dijkstra doesn't terminate w/ -ve values ⇒ False

Bellman doesn't terminate indirect ⇒ true

shortest path tree = MST ⇒ False

always MST + arc = cycle ⇒ False

supply chain broader than logistics ⇒ true

degree of a node in a complete graph ⇒ n -1

complete graph weights are multiplied by a constant ⇒ shortest path changes, MST doesn't

MST , last arc added had a weight of 3 ⇒ rest of arcs not added have weight ≥ 3

ruskal/prim

why is going more expensive than returning ⇒ because of backhauls/empty

Question 2: Suppose that you are working as a consultant for Sears on home appliance delivery logistics. After developing a travel time network model of their Atlanta delivery region, you arrive at the job site for your first day at work. Joan, one of Sears' dispatchers, says she's about to route a truck from their warehouse through a set of 5 customers and then back to their warehouse, and she wants to know whether she's chosen the tour that minimizes travel time. Her proposed tour will take 2.81 hours. To answer this question, you realize that you'd like to solve a TSP problem but you don't have enough time. Suppose that instead you do the following:

Step 1: You use your network model to extract the minimum travel times between the depot and each of the customers in the delivery set, and also to find the minimum travel times between the customers. This data creates a complete undirected graph, G, connecting all of the customers and the depot with costs defined as travel times.

Step 2: Suppose now that you find the minimum cost tree on G that excludes exactly one node, which in this case is node 3. Let $1TREE_3^*$ be this tree, with cost $C(1TREE_3^*) = \sum_{(i,j)\in 1TREE_3^*} c_{ij}$ equal to 2.29 hours. Suppose that you then print out the costs of the arcs adjacent to node 3 (in no particular order) and they turn out to be: (0.21, 1.54, 0.79, 0.31).

Without any additional computation, you run to Joan to tell her that she's already found the optimal tour! Are you correct, or have you made a grave mistake? If you think Joan has found the optimal TSP tour, explain precisely why using your knowledge of trees, minimum spanning trees, and TSP tours. Otherwise, explain precisely why not. (1.5 points)

LB = 2.29 + 0.21 + 0.31 = 2.81 hr

b) Suppose that later in your consulting assignment with Sears, you use Christofides' [illegible] to solve a TSP problem, and the resulting tour T has a cost of C(T) = 45. Working on the same problem, your co-worker claims to have found a tour BS with a cost C(BS) = 29. You turn to your co-worker, and tell her that she has made a mistake. Explain (0.5 points)

H

c(chris) $\leq$ 1.5 opt.

opt. $\geq \frac{\text{chris}}{1.5} \geq 30$

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Problem 2: (30 points)

1. Table 1 gives the travel costs c_{ij} between vertices i and j in a network G. the distances are symmetric, that is, $c_{ij} = c_{ji}$ for all nodes i and j.

From\To	A	B	C	D	E	F
A	0	18	85	57	84	66
B	18	0	71	53	65	49
C	85	71	0	47	57	68
D	57	53	47	0	89	83
E	84	65	57	89	0	24
F	66	49	68	83	24	0

Table 1: Travel costs c_{ij}

a. In the table below, identify the node to insert during iterations 1 and 2 of the Farthest Insertion heuristic for constructing a TSP tour. Do not perform the insertions.

Step	Node to insert	
Init	D, E	
Iter 1	A	D-A-E
Iter 2	C	D-A-C-E

Iter 1

	D	E
A	57	84
B	53	65
C	47	57
F	83	24

Pick A

Iter 2

	A	D	E
B	18	53	65
C	85	47	57
F	66	83	24

Pick C

$C_{DA}+C_{AE}-C_{DE}=52$

D-A-E

D-A: $C_{DC}+C_{CA}-C_{DA}=75$

A-E: $C_{AC}+C_{CE}-C_{AE}=58$

b. Suppose that during some iteration of an insertion heuristic to find a TSP tour on G, the current tour is T = {B, C, D, B} with total cost 171. Calculate the insertion cost associated with inserting a between D and B. show your work.

B - C - D - B → B - C - D - A - B

$C_{DA}+C_{AB}-C_{DB}=57+18-53=22$

The total cost C({B,C,D,A,B}) = 171 + 22 = 193

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c. Suppose you want to calculate a lower bound on the cost of the optimal TSP tour that visits every node in this network. One valid lower bound is the cost of the minimum spanning tree on the network. Using an appropriate algorithm that we discussed in class, identify the minimum spanning tree on the network and **draw it on the graph provided** in Figure 1. The travel cost matrix is provided again for your reference. Do not assume the graph is drawn to scale.

From\To	A	B	C	D	E	F
A	0	~~18~~	~~85~~	~~57~~	~~84~~	~~66~~
B	18	0	~~71~~	~~53~~	~~65~~	~~49~~
C	85	71	0	47	~~57~~	~~68~~
D	57	53	47	0	~~89~~	~~83~~
E	84	65	57	89	0	24
F	66	49	68	83	24	0

18, 24, 47, 49, 53, 57, 57
65, 66, 68, 71, 83, 84
85, 89

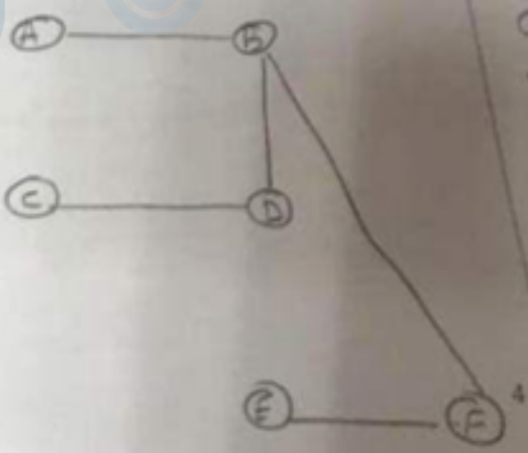
Figure 1: Graph for Problem 1-c

* kruskal's to be used

Edges
A - B 18
A - C 85
A - D 57
A - E 84
A - F 66
B - C 71
B - D 53
B - E 65
B - F 49
C - D 47
C - E 57
C - F 68
D - E 89
D - F 83
E - F 24

Sorting LIST
A-B ✓
E-F ✓
C-D ✓
B-F ✓
B-D ✓ → n-1 edges ⇒ stop
A-D ×
C-E ×
B-E ×
A-F ×
C-F ×
B-C × → A-C ×
D-F × D-E ×
A-E ×

TREE
{(A-B), (E-F), (C-D), (B-F), (B-D)}



-0

Problem 3: (50 points)

a. A truck stationed at the depot (location 0) is to serve the demand of sales points 1 through 7, depicted in Figure 1, using a single tour. The distances between the depot and the sales points are given in Table 2. The network is symmetric. In this problem, you will use Christofides heuristic to construct an initial tour through the depot and seven sales points.

	0	1	2	3	4	5	6	7
0	-	~~27.9~~	54.6	42	56.5	37	30.9	34.1
1	-	-	67.2	25.6	28.8	48.4	57.4	21.6
2	-	-	-	60.5	95.8	18.8	60.4	52.1
3	-	-	-	-	39.4	43.1	$\boxed{70.2}$	12.2
4	-	-	-	-	-	~~77.2~~	~~84.5~~	44.4
5	-	-	-	-	-	-	51.6	34.0
6	-	-	-	-	-	-	-	59.3
7	-	-	-	-	-	-	-	-

30.9
34

34
60.4

4→7 = 44.4
4→3 = 39.4

Table 2: Distances between depot and sales points

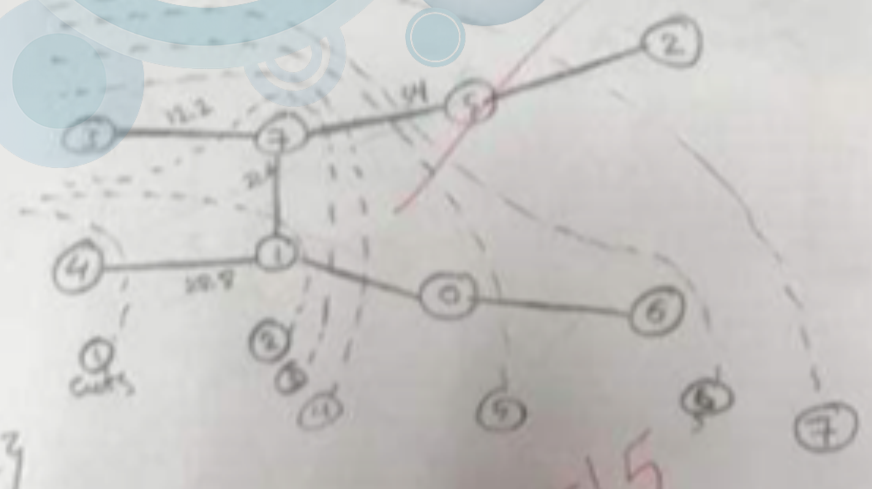
Figure 2: Geographic Locations of Depot and Sales Points

① MST*

TREE = { }

S = {4}

T = {1, 2, 3, 5, 6, 7}



S = {4, 1, 7, 3, 0, 6, 5, 2}

T = { }

TREE = {4, 1, 7, 3, 0, 6, 5, 2}

-15

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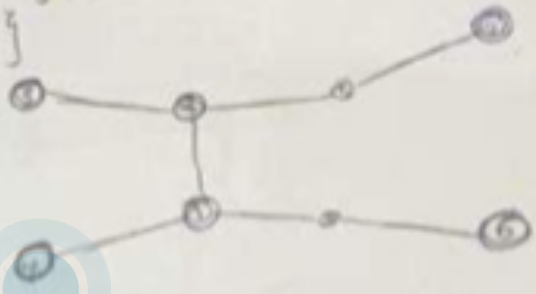
2- S → Set of odd degree node

S = {6, 2, 1, 7, 3, 4}



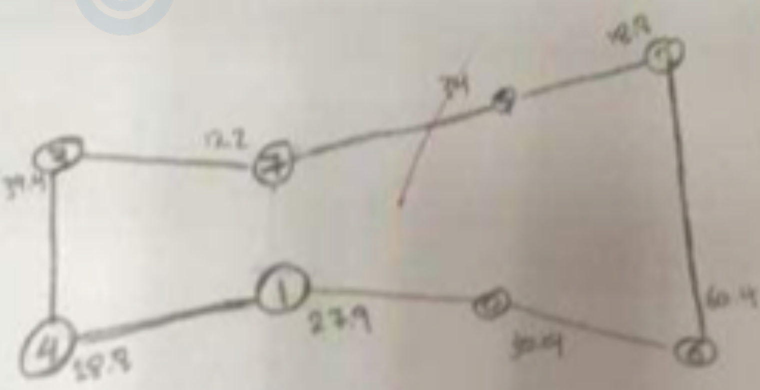
3- Perfect node matching

6-2 = 60.4	1-4 = 28.8	1-7 = 21.6	1-[illegible] = 21.6	6-2 = 60.4
1-7 = 21.6	7-3 = 12.2	6-4 = 70.[illegible]	6-3 = 70.3	7-4 = 44.4
3-4 = 39.4	6-2 = 60.4	3-2 = 60.4	4-2 = 95.8	1-3 = 25.6
121.4	101.4	152.3	187.7	130.4

~~just one~~ again 10 choices

W* = {(1-4), (7-3), (6-2)}

R = {0-1-4-1-7-3-7-5-2-6-0}



T = {0-1-4-3-7-5-2-~~6~~6-0}

As a logistics engineer working for a food production factory, your job entails building vehicle routes that trucks should follow to fulfill retailers' demands. All trucks are located at your company-operated distribution center (DC) to start routing towards assigned customers and then return to the same DC. On a given day, the company must serve eight customers (A, B, C, D, E, F, G, H). Each customer requests a given number of the product units from the DC, as shown below. The trucks you may use to deliver the goods have equal capacities with 65 units each. The minimum distance path between all customers' locations and between DC and all customers' sites is summarized in the last table.

Use the Clarke-wright savings heuristic to generate a set of routes. Record the savings you implement at each iteration. Draw the tours on a copy of the graph provided, report the distances for each tour, total distances traveled by all vehicles, and how many units each vehicle will deliver.

Table 2 Customers demands

Customer	A	B	C	D	E	F	G	H
Demand	16	20	14	29	22	15	28	36

Table 3 Distances between all pairs

To / From	DC	A	B	C	D	E	F	G	H
DC	0	25	16	21	7	24	16	24	28
A		0	14	22	26	34	40	38	53
B			0	24	13	21	27	35	44
C				0	26	41	33	15	39
D					0	17	15	31	32
E						0	25	49	44
F							0	32	20
G								0	30
H									0