Weekly Notebook

Logistics

Second Semester 2024

lecture one: 27.2.2024

*talking about logistics means being interested in two topics (transportation/ wherehousing).

*solving logistics:

- optimal solutions: short time and easy to get the answer.
- heuristic solutions: when the optimal is hard to get and/or need a long time.
  - ↳ considered as a good solution that may be the optimal one but we never know it is the optimal.

?What is supply chain (NetWork)? (SC)

System of organizations and processes that interact to convert raw inputs into products or services for customers. (raw → final product)

flow of		between	
	goods		Supply
	services		Manufacturing
	information		Distribution
	mony		End customer

⤳ logistics is a part of supply chain management.

[Supply network organization]

1) Within your firm.

A- Design and production departments.

B- Logistics / traffic management / transportation departments.

C- Sales and marketing departments.

D- Finance department.

2) other firms.

A. Suppliers.

B- 〃' suppliers (and so on!)

C- Customers

D. 〃' Customers (and so on!)

E- logistics service providers (carriers/3PLs/...)

⤳ logistic service and nonadded value operations it is a decision to provide (how to transport? / When? / What? / ...).

↳ that's why it has to cost a little.

?Importance:

*) Efficient supply network is key to bottom line profit of most firms

?What is logistics? (CSCMP definition)

a part of supply chain management that planes, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption to meet customers' requirements.

*flow = transportation *storage = wherehousing.

⟹ Business logistics:

1) Inbound logistics: supplier ⇝ manufacturing company.

2) Materials management: within the manufacturing company.

3) Physical distribution: manufacturing company ⇝ costumer.

? achieving desired objective while

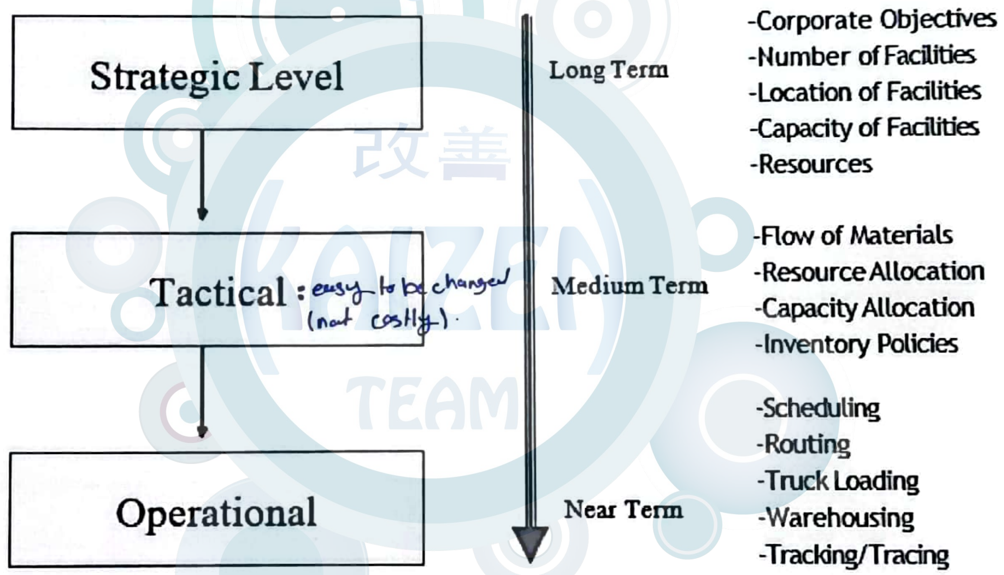
A) Effictiveness: delevering what is committed (price/quality/response time/flexibility).

B) Efficiency: using resources in the best way.

*إذا اهتمينا بواحدة منهم رح نكون مضطرين ناخد من التانية يعني بينهم زي كأنها علاقة عكسية.

...plans, implements and controls...

3 levels of decision-making



From Logistics to SCM

CSCMP definition

- "Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies."

Key Ideas

- Extends concept beyond firm's suppliers and customers
- Stresses partnerships and collaboration

Supply Chain Management Activities

All activities required to plan and operate a supply network, including:

- Relationship management
- Finance
- Product marketing
- Facility design
- Demand forecasting
- Inventory control
- Production planning and scheduling
- Logistics resource scheduling
- Transportation management
- Network design

College of Business

Industrial Facilities Layout and Design, Production Planning and Inventory Control

Challenges in Managing

- Two main challenges
  - Uncertainty
  - Global optimization
- Difficult for one organization
- More difficult for many
- Successful supply chain management can create sustainable competitive advantage
  - Cost
  - Quality
  - Flexibility
  - Response time

lec 2: 29.2.2024.

→ Truck examples: ranked by 2003 revenue:

1) Truckload firms: Schneider National.

2) Less-than-truckload firms: Yellow.

3) Household goods firms: North American.

(البريد) 4) Parcel transports: United Parcel Service.

* top three: United Parcel Service
  FedEX Ground.
  Schneider National

⇒ Full truck load: (FTL) dedicated to one costumer, so wheter you fill or not space/capacity dedicated or it may be owned by one customer.

⇒ Less than truck load: (LTL) the space inside the truck is divided potenally among multiple customers.

⇒ Household goods firms: احنا منسوق وما منعبي فيها الحمولة بس في شروط على محتوياتها مع الشركة ملزم

* Trucking Industry Growth:

- ↑ importance.
- Freight transport revenue share.
- continuing growth.

* Equipment types in fleets:

1) Single-unit trucks: Delivery vans / tank trucks / dump trucks / cement mixers.

2) Tractor-semitrailer combination: often 18-wheelers

3) Multi-trailer combinations: STAA doubles / Longer combination vehicles.

⇒ vehicles combination: conventional / longer (slide 6)

⇒ Single-unit slide 7

⇒ tractor-semitrailer slide 8

⇒ multi-trailer slide 9

* Intermodalism: moving frieght by two or more mode of transportations. (إنترموديل)

↳ charatarstics:

1- economics: can be slower / origin and destination drayage / huge cost savings in drivers pay.

2- long-haul trips: hub-to-hub trips in LTL and package express trucking.

3- TOFC and COFC: equipment investment and management vs price.

⇒ TOFC: trailer on flat car (less cost) 11

⇒ COFC: container on flat car. 12

*Trends:

1) Value-added: tracking and tracking / online quotes / service requests.

2) Dedicated contract carriage: replacement for private carriage: outsourced.

3) Time-sensitive transportation: Expedited options: Fast / time-definite delivery: reliable.

4) 3PL services: warehousing / distribution management / handling.

↳ ~~3~~ third party logistics: doesn't transport only provides logistics services.

*Truckload operations:

A- Direct origin → destination service: NO → equipment changes.
→ intermediate terminals required

B- General vs. specialized: refrigerated / automobile transports / grain carriers.

C- Planning issues: → Supply-demand balancing (backhauls / empty repositioning)
→ Driver issues (relays / home stays).

⇒ backhauls: Freight move in the direction from destination to origin in the direction of light or second demand

⇒ empty repositioning: move back to the origin without anything inside the trailer.

*TL operating problem:

• Given a load request: origin and destination / pickup time / # of trailers request.

• decide: driver & trailer combination to assign.

• Why ~~it~~ difficult?: short term load visibility / loaded flow imbalance. (16-21)

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lec3: 3.3.2024:

*Less than truck load operations:

A- Small shipments: cube (volume) is usually constraint vehicle packing.

B- Hub-and-spoke networks: consolidation and rerouting role of terminals.

C- Planning issues: Network design / routing and scheduling problems.

⇒ Hub: main Spoke: Satellite (23)

→ Service network design and operation:

How to consolidate flows of shipments over a network of distribution centers to minimize transportation and handling costs while maintaining level of service:

↳ Design service network (the pairs of terminals between which ~~are~~ direct service is offered) or
↳ Operate a load plan / guidelines for each terminals indicating where shipments headed for destination d should be loaded to next. both!

⟹ ~~Pb~~ Fundamental model: min cost paths

A- Given a load must be moved A → B which is the least "cost" truck route over the $\overset{\text{road}}{\wedge}$ system?

B- Trucking uses:

- Path finding.
- Standard mileage for driver pay
- Standard mileage for rate negotiation
- Generation of input to routing and scheduling models.

* What is cost? (application-dependent).

1) Distance: certain costs, driver pay, fuel costs, equipment wear-and-tear depend directly on distance travelled.

2) Time: - Customer service considerations often warrant min travel time paths.

- "Consumer" uses of min cost path (Google maps) usually use (travel time minimization) objective.

3) Dollar costs.

end of topic 2.

Topic 3: Transportation rates and costs:

* economic of scale: marginal cost decreases as quantity shipped increases.
  - shipper: the one who needs the service of transportation.
  - carrier: the one who provides the service of transportation.

* Rate structures of common carriers:

A) Related to shipment size.

B) Related to distance.

C) Related to demand.

D) Related to product shipped.

E) Class rates.

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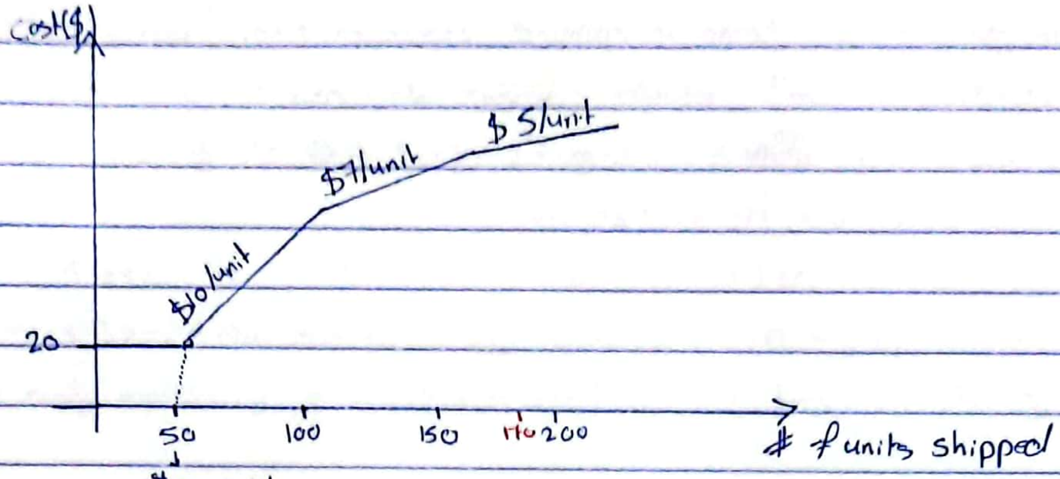
lec 4: 5.3.2024

A) most typically size is measured in units of weight or mass.

- Rates are quoted in $ per cwt (100 lbs).
- Usually several categories or "rate breaks" based on several weight cutoffs.

1) For LTL or LCL shipments, min total rate for quantities below a min threshold, then several weight categories ~~w~~ with different rates. →



$\boxed{\text{* total cost (y) = slope * \# } (\Delta x) \text{ units for each interval.}}$

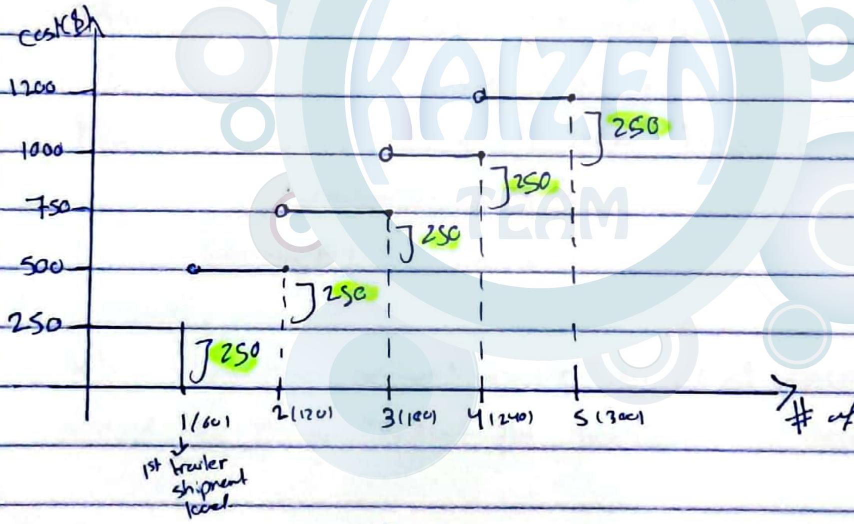
→ total cost for shipping 170 units

ماضربناه بـ Δx لانه ← Fixed cost

$= 20 + 10*50 + 7*50 + 5*20$

$= \$1070$

2) For FTL or FCL, rate only depends on equipment size ordered

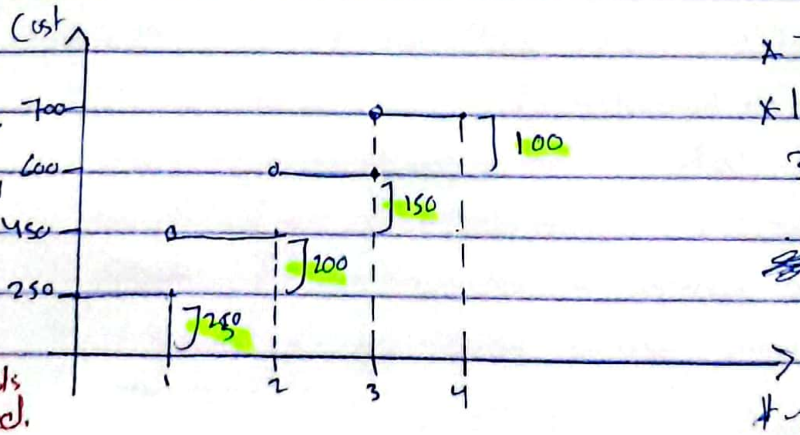


* Trailer capacity = 60 units
* cost/load = $250

Cost for shipping 200 units = $1000 (according to the interval) (Fixed)

3) Incentive rates: for loads utilizing multiple ~~vehicles~~ vehicles or even trainload rates in the case of unit trains.

Why? terminal costs are reduced.



* Trailer capacity = 60 units
* Incentives: 1st load = $250 2nd = 200 3rd = 150 4th = 100

← المسافات بين الخطوط بتظل ثابتة

→ related to freight throughput
not cube.

4) Time-volume rates: encourages shippers to send more regularly in effort by carriers to ensure regular flow of business.
مثال: شركة عندها 5 ~~trailers~~ trucks وأنا بدي 20 رح أعطيك في كل يوم 5 لمدة أربع
أيام لكن كل يوم بسعر أقل من اليوم السابق:
5 → 200 then 5 → 170 then 5 → 160 then 5 → 150
← هيك الشركة بتصير عندها حل لمشكلة الـ short-term load visibility لأنه صار عندهم
خطة لـ 4 أيام لقدام وما رح يرجعوا الـ trucks فاضيين يعني no empty repositioning

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B) 1) Uniform rates: rate independent of distance (eg USPS Priority Mail Why? Most of the cost is in handling). (البريد)

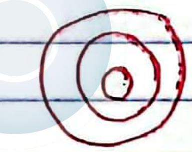
2) Proportional rates: Fixed rate + variable rate per distance. (no taper, but still includes EOS. why?) Truckload rates often like this. (الشحن)

3) Tapered rates: Increase with distance but at a decreasing rate. Especially in air transportation.

4) Blanket rates: constant rates for certain intervals of distance, usually to match competitors rates. Grain, coal, and lumber. UPS rates.

→ \$10 / mile for the first 50 miles | (نفس فكرة الخصم)
\$ 7 / mile " " second 50 "

→ like the zip code (within blanket or region) like diameter.
(50 - 100) miles → \$ 100
(100 → 200) " → \$ 150



C) Often, carriers adjust rates to what shippers are willing/can afford to pay: "What the market will bear".

D) Products are grouped into a classification for the ~~pur~~ purpose of determining rates. Classes depend on the factors of density, stowability, ease of handling and liability:

- Weight/volume.
- Value/weight or value/volume.
- Liability to loss, damage, or theft
- Risk of hazardous materials.
- Security of container or packaging.
- Expense of and care in handling.
  ↳ loading and unloading.
- Rates of analogous articles, competitive products
- Likelihood of injury/damage to other freight, to personnel, to vehicles and equipment

Cost characteristics by mode:

1) Fixed costs: right-of-way, vehicles and other equipment, terminals, other overhead
2) Variable costs: fuel, maintenance (oil, tires, service), drivers.
3) Terminals.

lec 5: 7.3.2024

Continued:

A) Rail	B) Motor truck	C) Water	D) Air
• High fixed cost: infrastructure (railbed, train control system (signals) switching yard, terminals, locomotives, rolling stock). due to overhead • Small variable cost: fuel, crews, efficiency of steel wheels on rail. • High terminal-costs: load/unload, ~~transp~~ transshipment, yard switching to assemble and unassemble trains.	lowest fixed costs: trucks, trailers and terminals. • High variable costs: drivers, fuel, oil, tires, maintenance. Fuel costs include fuel taxes. • Lower terminal costs than rail.	• Medium-high fixed costs: expensive vehicles and terminals, no right-of-way infrastructure. • Variable costs low: low speed high capacity vehicles. • Terminal costs lower than rail, higher than motor trucks. harbor fees and loading/unloading costs.	• High fixed costs: very expensive vehicls, terminal facilities. • Medium to high variable costs: high fuel costs, relatively low capacity, decrease with distance due to take-off and landing fuel inefficiencies, labor, maintenance • Terminal costs high: airport landing fees, storage, space leasing, load/unload.

Cost classifications:

1) ~~Transportation~~ Holding costs: opportunity cost of capital for time waiting.
2) Handling costs: packing/unpacking "containers" (boxes, bags, pallets) intra-facility movements such as moving into and out of storage.
3) Transportation costs: movements via vehicle, loading/unloading.
4) Facility rent cost: economic 'rent' for facility space, storage infrastructure and maintenance.

Mode choice problem:

* Find optimal mode of transportation.
* Find optimal quantity to be transported (q)
  → usually and the best
  $q \leq$ capacity.

⟹ We will consider 3 inventory costs:

1) Inventory held at the origin.

2) " " " " distination.

3) " " intransit from origin to distination.

⟹ Inventory cost = holding cost + space rent cost.

the rent of space required for storing inventory.

* Common unit used $/year

⟹ Holding cost is usually expressed as function of the value of inventory held and holding cost interest rate.

⟹ Holding cost: represents the potential return that could be generated by investing the inventory value in alternative investment.

* Picked time period for our analysis is <u>year</u>.

Steps for solving:

1) Determine inventory space cost at origin.

2) " " " " " distination

3) " " holding cost at origin

4) " " " " " distintion

5) " pipeline inventory cost (intransit inventory cost).

6) " transportation cost for each mode of transportation

7) " total cost for " " " ".

8) " optimal shipment size (Q) for ...

9) " the most economical mode of transportation.

1)+3) = inventory cost at origin

2)+4) = " " " distination.

In-class Example

Choosing a Transportation Mode

Hypothetical company has:

- 1 plant, producing 1 product
- 1 warehouse, storing the product from which customers supplied directly
- Plant keeps 0 safety stock inventory, warehouse maintains 100 units
- Average customer demand rate at warehouse: 10 units/day
- Plant produces 10 units/day

Suppose we would like to compare two modes of transportation for moving product from the plant to the warehouse. We want to make a mode choice decision that minimizes total cost.

Rail:

- Shipments in railcars containing at most 400 units
- Cost: $1200 / carload
- Transit time: 20 days

Truck:

- Truckloads of at most 100 units
- Cost: $700 / truckload
- Transit time: 3 days Inventory costs

Inventory cost:

Value of items at plant: $500 / unit

Value of items at warehouse: $512 / unit

Inventory holding cost rate: 25% of value per year held

Plant space cost: $20 / unit*year

Warehouse space cost: $25 / unit*year

Goal :

- Our goal is to determine the optimal shipment size q for each mode, and the total cost of the optimally configured system.

Assumptions:

- Assume for now that we will never ship more than a single railcar or a single truckload at a time when minimizing cost

1. What are the annual inventory holding costs at the production plant? (Include the space and holding cost in this calculation.)

2. What are the annual inventory holding costs at the warehouse (Include the space and holding cost in this calculation.)

3. What are the pipeline inventory costs for each mode?

4. What are the transportation costs for each mode?

5. What are the total cost equations for each mode of transportation?

Solution for the example by the doctor:

Factory

safty stock = 0

planed production = 10 units/day

value = $500/unit

holding cost rate = 25% /year

space cost = $20 /unit × year

Ware house

safty stock = 100 (maintained).

demand rate = 10 unit /day.

value = $512/unit

holding cost rate = 25% /year.

space cost = $25/unit × year

OPtion	(unit / load) Capacity	cost $/load	transportation time.
rail	400	1200	20 days
truck	100	700	3 days.

منعمل اعى مزة اجابي.

lec6: 10.3.2024:

Inventory cost analysis: - inventory held at origin

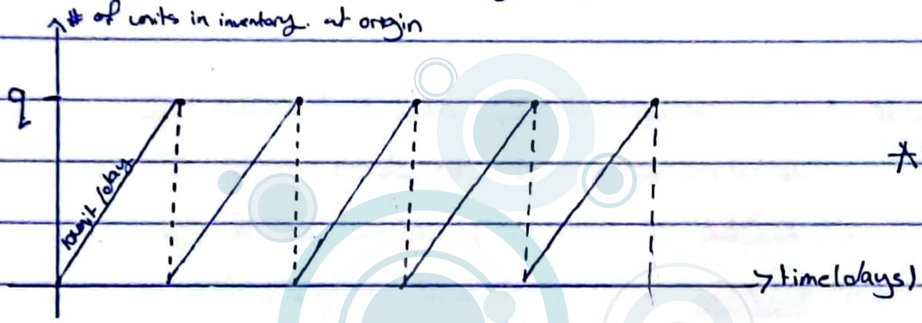
- " " " destination

- pipeline inventory.

back to the example:

assumption: for now we will never ship more than a truck or rail load.

→ the target is ~~is~~ finding q: max accumulation of item at storages.



*no safety stock.

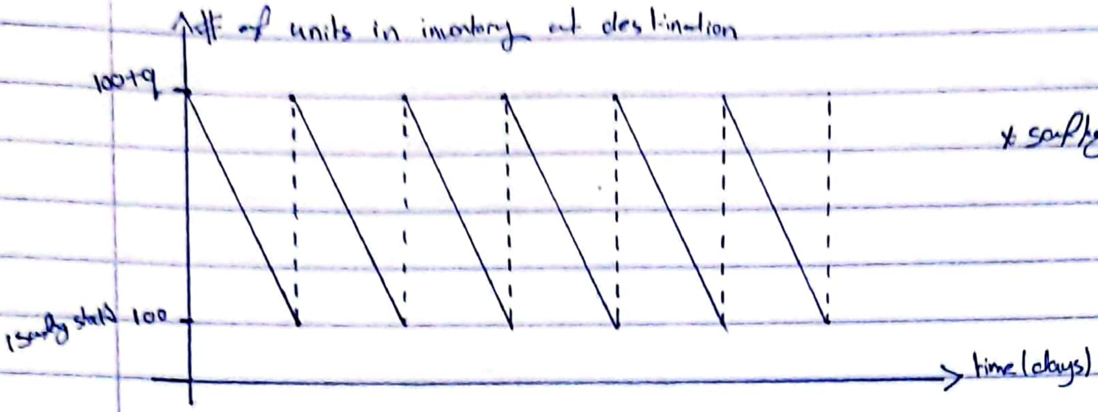
1) Space rent cost at origin = q * space cost

$= q\,(\text{unit}) * \dfrac{\$20}{\text{unit.year}} = \$20q/\text{year}.$

2) Inventory holding cost at origin = inventory holding cost rate * average # of units at inventory * value of item

$= \dfrac{25}{100\,\text{year}} * \dfrac{q}{2}\,\text{unit} * \dfrac{\$500}{\text{unit}} = \$62.5q/\text{year}.$

annual or total inventory cost at origin = 1 + 2 = $\$82.5q/\text{year}.$



* safety stock included.

3) space rent cost at destination = $(q+100) * 25 = \$(25q + 2500)$/year.

4) inventory holding cost at destination = $0.25 * (100 + \frac{q}{2}) * 512 = \$(64q + 12800)$/year

total inventory cost = 3+4 = $(89q + 15300) /year.

5) ⇒ it is an opportunity cost so it is dependent on the ~~shipment size~~ mode of the transportation

⇒ # of units per year = 365 * 10 = 3650

⇒ pipeline inventory cost doesn't depend on the shipment size it only depends on the mode of transportation

* if the value of items intransit weren't given, take the average of origin and destination $\left(\frac{500+512}{2}\right) = \$506$.

Rail:

$\boxed{\text{\# of units/year} * \text{transit time} * \frac{1}{365} * \text{inventory holding cost} * \text{value of items intransit}}$

conversion factor

$3650 * 20 * \frac{1}{365} * 0.25 * 506 = \$25300$/year.

Truck:

$3650 * 3 * \frac{1}{365} * 0.25 * 506 = \$3795$/year.

notes:

- each unit shipped will spent the same # of days intransit.
- if holding cost of modes of transportation were given, add it to the cost/load.
- pipeline holding cost/unit.year = 0.25 * 506 = $126.5/unit.year.

lec 7: 12.3.2024

Step 6: Transportation cost: we will analyze transportation cost for each mode of transportation, regardless of the mode of transportation we first determine # of shipment per year as a function of shipment size (q)

$\# \text{ of shipment} = \frac{3650}{q}$

*always round up q

$\lceil \; \rceil$ round up

$\lfloor \; \rfloor$ round down

(→ pipeline inventory is a constant not a function of q)

Step: Transportation cost depends on → - # of shipments (عدد الرحلات)

- transportation mode.

by rail $\frac{\$1200}{\text{trailer}} * \frac{3650}{q_{rail}}$ unit/year $= \frac{\$\,4\,380\,000}{q}$ /year.

by truck $\frac{\$700}{\text{trucker}} * \frac{3650}{q_{truck}}$ unit/year $= \frac{\$\,2\,555\,000}{q}$ /year.

Step 7) Total cost for rail / truck =

inventory cost at origin (1+2) + inventory cost at destination (3+4) + Pipeline cost (5) + transportation cost (6)

$TC(q)$ for rail $= 82.5q + 15300 + 89q + 25300 + \frac{4380\,000}{q}$

$TC(q)$ for truck $= 82.5 + 15300 + 89q + \cancel{25300}\,3795 + \frac{2\,555\,000}{q}$

8) $\frac{dTC}{dq} = 0$

optimal $q_{rail} \Rightarrow \frac{dTc_{rail}}{dq} = 0 \Rightarrow 82.5 + 89 - \frac{4380000}{q^2} = 0$

$q_{rail} = 180$ # < capacity ✓

$TC_{rail} = \$95415$/year

optimal $q_{truck} = 82.5 + 89 - \frac{2555000}{q^2} = 0 \Rightarrow q_{truck} = 122$ # > capacity

2 truck needed

# of truck needed $= \frac{3650}{100} = 36.5 = 37$ $TC_{truck} = \$81795$/year

9) truck is more economical.

lec 8: 17.3.2024

*Shortest path algorithime (minimum cost path ~~...~~ algorithime):

→ Important notations and definitions:

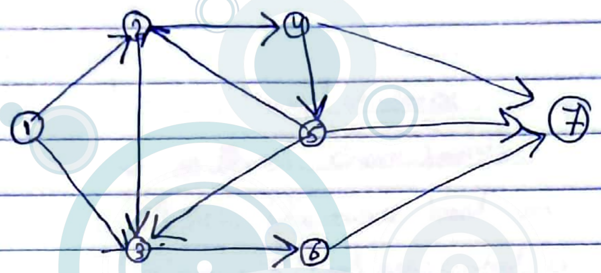
- Graph ≡ Network ≡ G

↳ mathematical structure includes a set of nodes/vertices and a set of edges/arcs connecting nodes.

$G = (N, A)$

N: set of nodes (#)

A: set of arcs (#)



$N = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{(1,2), (1,3), (2,3), (2,4), (3,6), (4,5), (4,7), (5,2), (5,3), (5,7), (6,7)\}$

$m = \boxed{|A|} = 11$

$n = \boxed{|N|} = 7$

↳ size of ....

- types of graphs:

1) Directed graph: consist of set of nodes and set of directed arcs.

→ directed arcs: ordered pair of nodes represented by $(\underset{start}{U}, \underset{end}{V})$ that is directed from U to V.

U → V (one way street).

2) Undirected graph: consist of set of nodes and set of undirected arcs

→ undirected arcs: arcs are undirected or unordered pair of nodes. (U,V) or (V,U)

U → V (two way street)

- Degree: indegree + outdegree.

→ For directed graph:

* indegree: the # of incomming arcs to the node
→ # of arcs for which u is the terminal vertex)
* outdegree: the # of outcomming arcs from the node.
→ # of arcs for which u is the initial vertex).

⇒ Σ for indegrees for all nodes = Σ for outdegrees for all nodes

$\Sigma\ \text{indegrees}(i) = \Sigma\ \text{outdegrees}(i) = |A| = m$

⇒ degree of graph = $2 \times m$.

- Minimum cost path:

↳ $ / time / distance.

arcs may have different characteristics:

1) Capacity of arc U_{ij}: max load can be moved from i to j.
2) Cost C_{ij}: (distance / $ / time) associated with traveling from i to j

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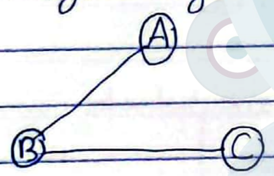
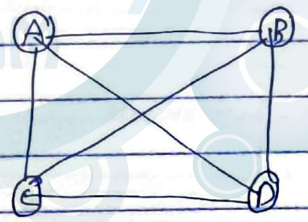
lec9: 19.3.2024

→ Complete graph: simple undirected graph in which pair of noods is connected by a unique arc.

→ graph G is said to be complete if every vertix in G is connected by every node



not connected

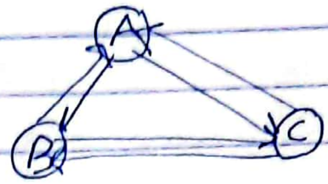


Connected

# of graphs in connected graph with n nodes $= \frac{n(n-1)}{2}$

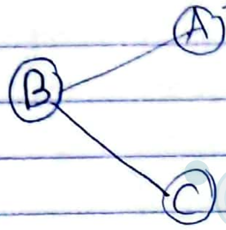
→ Complete diagraph: is a directed graph in which every pair of nodes directed vertices is connected by unique edges one in each direction.

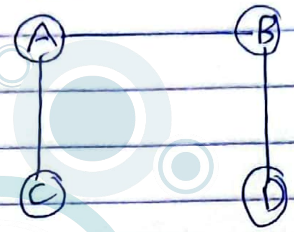
# of arcs in complete digraph $= n(n-1)$
with n nodes



→ Connectivity: we say that nodes i and j are connected, if the graph contain at least one path from i to j

→ A graph is connected if every pair of it's nodes is connected otherwise the graph in <u>unconnected</u> .



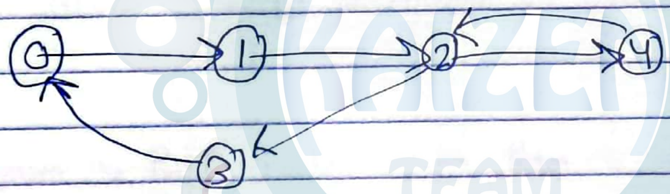


not complete
but connected.

* Every complete graph is connected graph (true ✓)

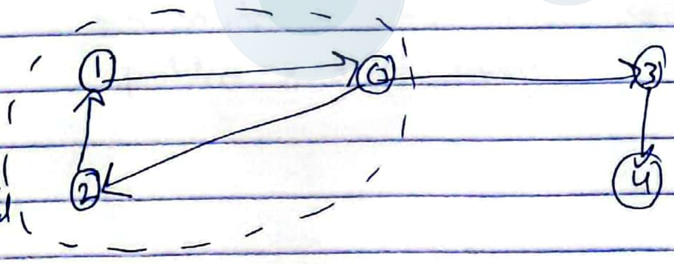
* Strongly connected graph is associated with directed graph where is a directed path link each pair of node

*ماهو ذكي connected
ولا لأ بس ذكي strongly
أولا.



directed → complete
→ strongly

undirected → connected
→ complete.



strongly completed

* overall it is not strongly con.

* We say that a path exist from V_0 to V_n if there is a list of vertices $\{V_0, V_1, \dots, V_n\}$ such that $(V_i, V_{i+1}) \in E$ for all $0 \le i \le n$

⇒ Path: a set of edges of which each edge is connected to the initial vertix of next edges.

$G = (N, A)$
$= (V, A)$

→ path P in G from starting node (s) to distenation node (t) is an ordered sequance of arcs connects (s) to (t).

$P = [(S,i), (i,j), (j,K), (K,L), \dots (V,r), (V,t)]$
$(V_i, V_{i+1}) \in A$

⇒ cost of path $= \Sigma C_{ij} \quad (i,j) \in P$.

lec 10: 21.3.2024.

* Single source shortest path problem:

shortest path (optimal path): among all paths P in G connecting (s) source to (t) distenation, we choose P^* such that $C(P)^* \le C(P)$

$C(P)^*$: ~~real~~ shortest path
$C(P)$: cost of any path

→ aim to find shortest path from a single source node to all other nodes in the graph.

1st method: Dijkstra's algorithm

→ a solution to the single source - shortest path path problem in graph theory.

Dijkstra's algorithm

- Requires that $c_{ij} \geq 0$

Initialization:

$$v(j) \leftarrow \begin{cases} 0 & j = s \\ +\infty & \text{otherwise} \end{cases}$$

$$pred(j) \leftarrow \begin{cases} 0 & j = s \\ -1 & \text{otherwise} \end{cases}$$

$$PERM = ;\ \overline{PERM} = N$$

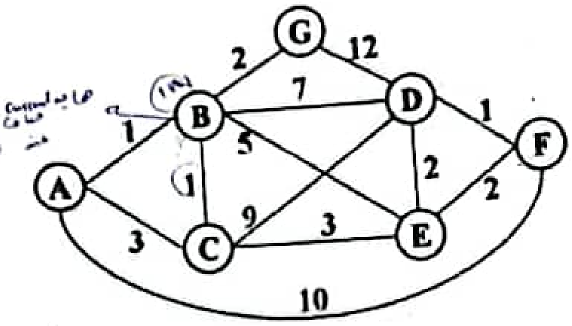
Iterations:

while $\overline{PERM} \neq \emptyset$:

1. Let $p \in \overline{PERM}$ be node for which $v(p) = \min\{v(j), j \in \overline{PERM}\}$
2. $PERM \leftarrow PERM \cup \{p\}$
3. $\overline{PERM} \leftarrow \overline{PERM} \setminus \{p\}$
4. For every arc $(p, j) \in A$, $j \in \overline{PERM}$ leading from node p:
   (a) if $v(p) + c_{pj} < v(j)$, then:
   - $v(j) \leftarrow v(p) + c_{pj}$
   - $pred(j) \leftarrow p$

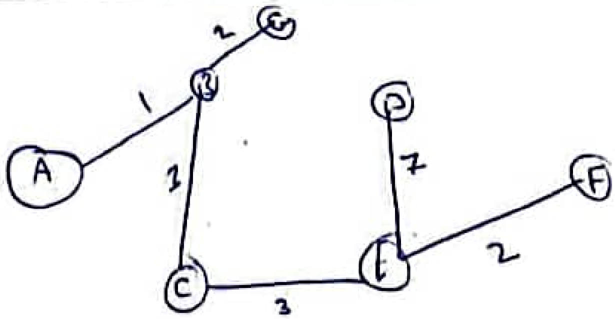
1. Assign to every node a distance value. Set it to zero for our initial node and to infinity for all other nodes.
2. Mark all nodes as unvisited. Set initial node as current.
3. For current node, consider all its unvisited neighbors and calculate their tentative distance (from the initial node). For example, if current node (A) has distance of 6, and an edge connecting it with another node (B) is 2, the distance to B through A will be 6+2=8. If this distance is less than the previously recorded distance (infinity in the beginning, zero for the initial node), overwrite the distance.
4. When we are done considering all neighbors of the current node, mark it as visited. A visited node will not be checked ever again; its distance recorded now is final and minimal.
5. If all nodes have been visited, finish. Otherwise, set the unvisited node with the smallest distance (from the initial node) as the next "current node" and continue from step 3.

Example # 1: Given the graph in the Figure shown below, find the shortest paths to all nodes using the vertex A as the starting source. Use Dijkstra's algorithm.



Iteration	A (source)	B	C	D	E	F	G	P (current node)	PERM (visited node)	$\overline{PERM}$ (unvisited node)
0	0 (0) vi pred	∞ (-1)	∞ (-1)	∞ (-1)	∞ (-1)	∞ (-1)	∞ (-1)	[illegible]	ϕ	N = {A, ...
1	0(0)*	1(A)	3(A)	∞(-1)	∞(-1)	10(A)	∞(-1)	A	A	N/{A}
2	—	1(A)*	2(B)	8(B)	6(B)	10(A)	3(B)	B	A, B	N/{A, B}
3	—	—	2(B)*	8(B)	5(C)	10(A)	3(B)	C	{A, B, C}	N/{A, B, C}
4	—	—	—	8(B)	5(C)	10(A)	3(B)*	G	A, B, C, G	N/{A, B, C, G}
5	—	—	—	7(E)	5(C)*	7(E)	—	E	A, B, C, G, E	{D, F}
6	—	—	—	7(E)*	—	7(E)	—	D	N/{F}	F
7	—	—	—	—	—	7(E)*	—	F	N	ϕ

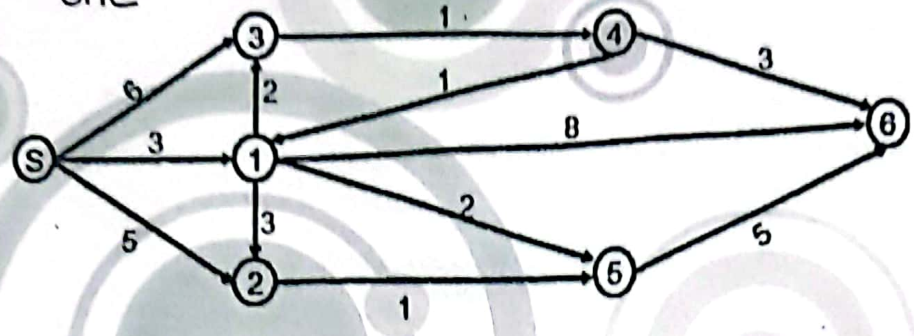
iteration (n)



A — B — C — E — D
1 + 1 + 3 + 2
= 7

(min cost path)

- **Example:**
one

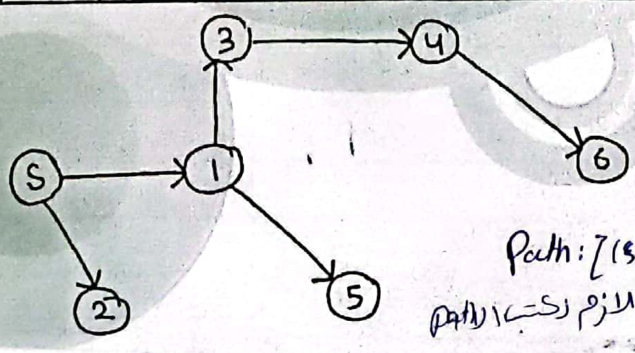


initial (starting) node

$N = \{S, 1, 2, 3, 4, 5, 6\}$.

Iteration	s	1	2	3	4	5	6	P	PERM	PERM
0	0(0)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	—	∅	N
1	0(0)*	3(S)	5(S)	6(S)	∞(-1)	∞(-1)	∞(-1)	S	S	N/{S}
2	—	3(S)*	5(S)	5(1)	∞(-1)	5(1)	11(1)	1	S,1	N/{S,1}
3	—	—	5(S)*	5(1)	∞(-1)	5(1)	11(1)	2	S,1,2	N/{S,1,2}
4	—	—	—	5(1)*	6(3)	5(1)	11(1)	3	S,1,2,3	N/{S,1,2,3}
5	—	—	—	—	6(3)	5(1)*	10(5)	5	S,1,2,3,5	N/{S,1,2,3,5}
6	—	—	—	—	6(3)*	—	9(4)	4	S,1,2,3,5,4	N/{S,1,2,3,5,4}
7	—	—	—	—	—	—	9(4)*	6	N	∅

#



⇒ min cost path tree from the single source (S) to all other nodes.

Path: [(S,1) (1,3) (3,4) (4,6)]

لو طلب الشكل الازم نكتب الpath

Cost of shortest path = S-1-3-4-6
= 3 + 2 + 1 + 3
= 9

lec 11: 24.3.2024

The output of Dijkistra

1) $V_c^*(j)$: cost of shortest path (min cost path) from source to node j

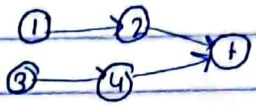
2) $Pred^*(j)$: node preceding j on the shortest path from the source.

3) minimum cost path tree

↳ we call it rooted out tree (one source).

What is the rooted in tree?

given an ending node (t) a tree on G such that a direct path exist from any node to ending node



* الأشياء بتميز الـ Dijkistra عن غيرها: إنه أنا ما بزور الـ node الواحدة مرتين

تستخدم لل directed و indirected

$C_{ij} \geq 0$ دائماً

* dense network (graph): كثير nodes وكثير arcs

* Pros:

1) Achive fastest running times (theoretically and practically) for dense networks. Typical geographic networks are not dense, though.

يعتبرها أحسن لإنها ما برجع لنفس الـ node مرتين.

2) Can be truncated. Each iteration, one node p is given a permanent, optimal minimum cost label and predecessor. Thus if you are interested only in path to some subset of the nodes $T \subset N$ (subset) you can stop the algorithm once $PERM \supseteq T$.

↳ subset or equal to

* Cons:

1) Alternative methods are faster (practically) on sparse networks.

ex الـ bellman أسرع بحالات عدة.

2) Efficient implementations require programming a priority queue (heap).

* in Bellman, the priority is for the 1st node in the list while in Dijkistra there is no specific priority.

lec12: 26.3.2024.

Tree is subnetwork of G as $T = (N, A_T)$ wher $A_T \subset A$ (subset) such that:

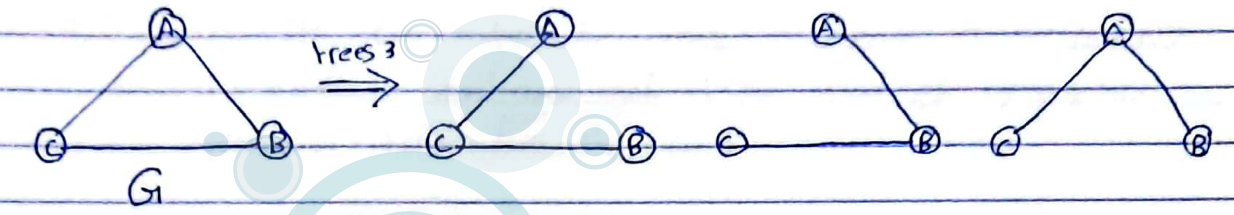
1) A path exist between all pair of nodes in N (as long as I ignore the direction).

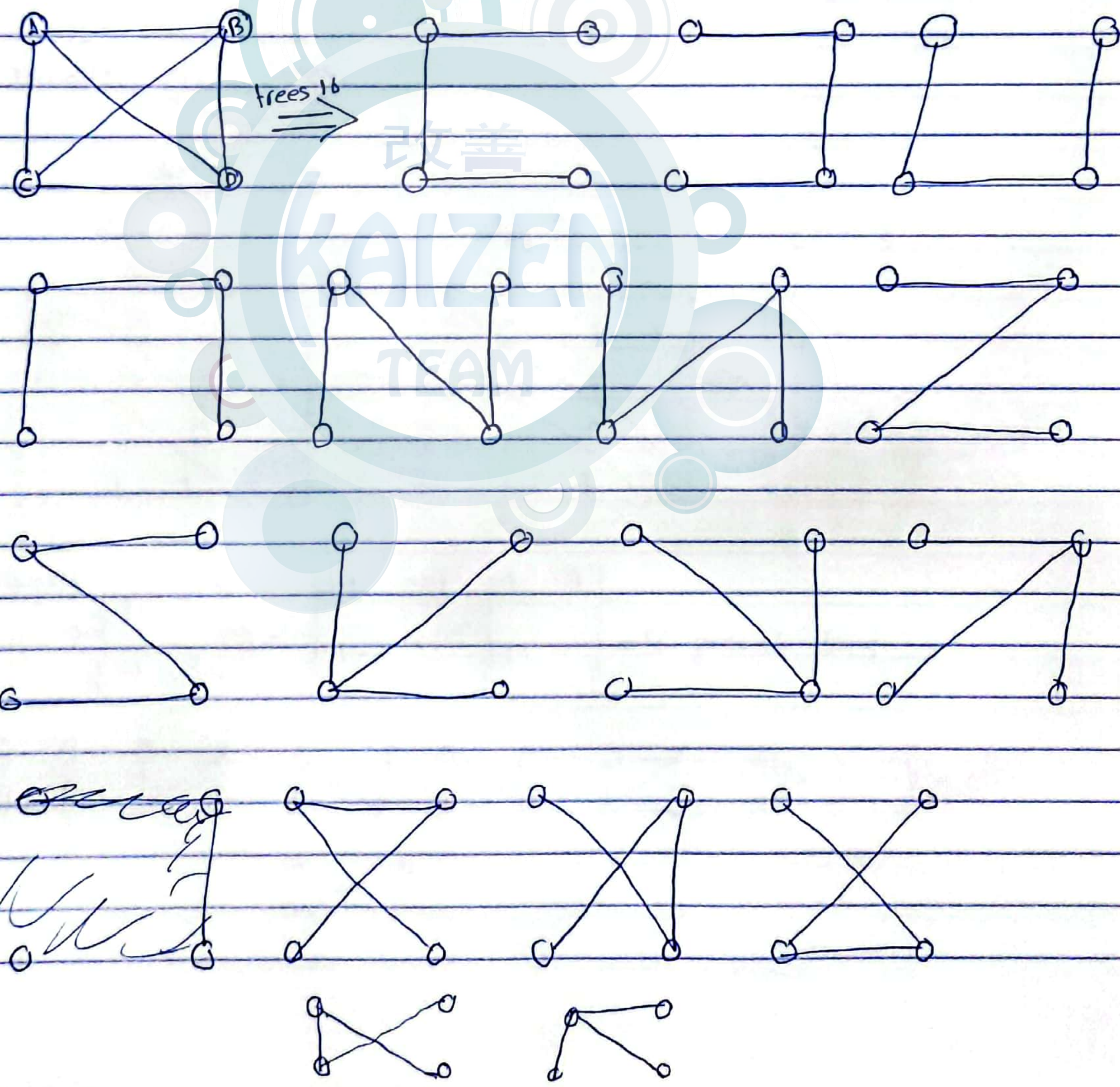
2) $|A_T| = n-1$ $\quad$ $n = |N|$

arcs of tree $\quad$ ↳ # of nodes in the original graph

3) No cycles (loops)

examples:





* For complete graph: # of possible trees (spanning tree) $= n^{n-2}$

* For incomplete graphs we have to follow Kirchoff's method:

<u>step1</u>: Create adjacency matrix for the given graph.

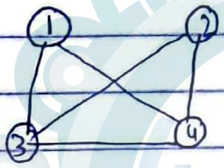
<u>step2</u>: Replace all the diagonal elements with the degrees of the nodes.

<u>step3</u>: Replace all the nondiagonal elements of <u>ones</u> with (-1)

<u>step4</u>: Calculate co-factor for any element in the matrix
$\Rightarrow$ the co-factor that you get is the total # of possible spanning tree for the G.

Note: # of possible trees for incomplete < for complete.

Example:

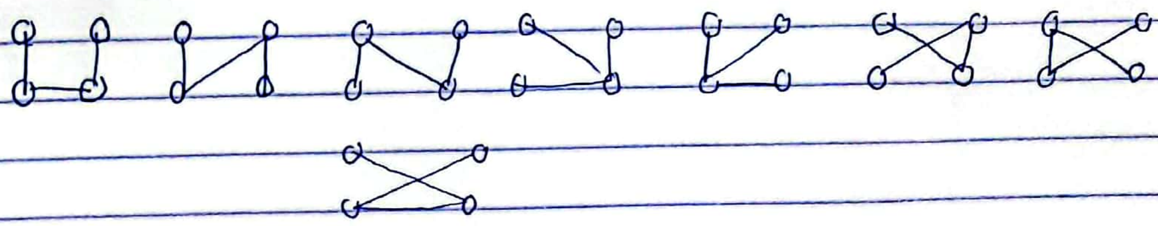


step1 / step2 / step3

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 0 \end{array} \Rightarrow \begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 & -1 & -1 \\ 0 & 2 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

step4.

$$C_{11} = 2\begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} - -1\begin{vmatrix} -1 & -1 \\ -1 & 3 \end{vmatrix} -1\begin{vmatrix} -1 & 3 \\ -1 & -1 \end{vmatrix} = 8 \text{ possible trees.}$$



lec13: 28.3.2024:

ع الورقة أول:

Bellman-Ford algorithm:

Pros:

1) Achieves fastest running times (practically) for sparse networks, especially when implemented with Pape's Modification.
2) Very simple to implement.

Cons:

1) Labels and predecessors not optimall untill full completion of algorithm.
2) Running times may be slow on dense networks due to frequent label updates.

Bellman-Ford algorithm

Sometimes, *label-correcting* algorithm. No requirement of non-negative arc costs. Often more efficient in practice for sparse transportation problems.

Initialization:

$$v(j) \leftarrow \begin{cases} 0 & j = s \\ +\infty & \text{otherwise} \end{cases}$$

$$pred(j) \leftarrow \begin{cases} 0 & j = s \\ -1 & \text{otherwise} \end{cases}$$

$LIST = \{s\}$

Iterations:

while $LIST \neq \emptyset$:

1. Remove node p from top of $LIST$
2. For every arc (p, j) leading from node p:
   (a) if $v(p) + c_{pj} < v(j)$, then:
   - $v(j) \leftarrow v(p) + c_{pj}$
   - $pred(j) \leftarrow p$
   - if $j \notin LIST$, $LIST \leftarrow LIST + \{j\}$

*If $j \notin$ List (if j doesn't belong ~~to~~ the current list)

two cases:

1) if j has never been in the list before.

List $\leftarrow$ List + $\{j\}$

add modified node to the end of the list.

2) if j has been in the list before

List. $\leftarrow$ $\{j\}$ + List.

add the node of the top of the list.

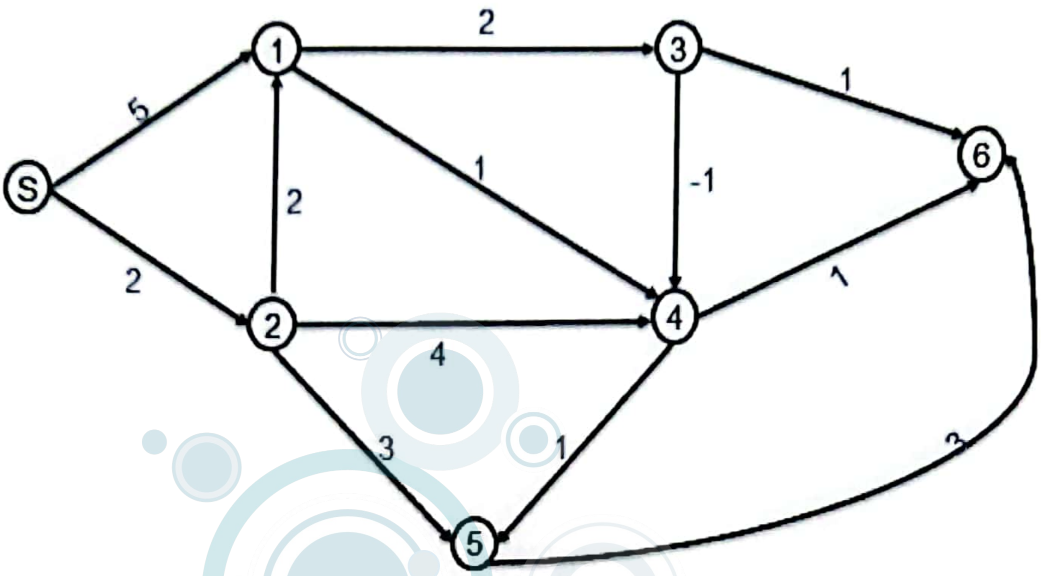
* أهم ميزات Bellman:

*negative is okay

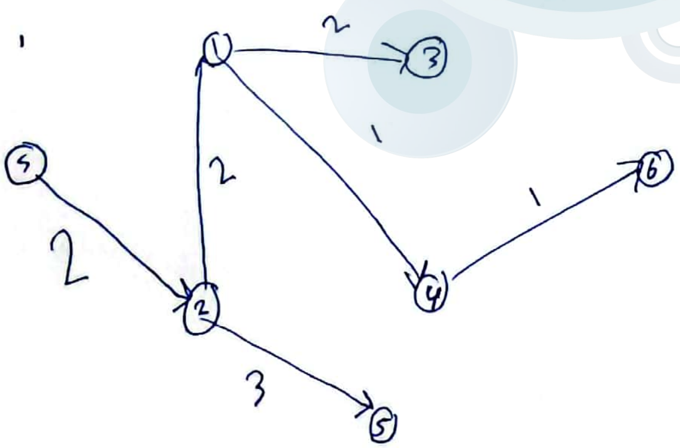
* G should be directed

Example:

Given the graph in the Figure shown below, find the shortest paths to all nodes using the vertex s as the starting source. Use Bellman-Ford algorithm.



Iteration	s	1	2	3	4	5	6	P	LIST
0	0(0)	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$		S
1	0(0)*	5(S)	2(S)	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$	$\infty(-1)$	S	1, 2
2	0(0)	5(S)*	2(S)	7(1)	6(1)	$\infty(-1)$	$\infty(-1)$	1	2, 3, 4
3	0(0)	4(2)	2(S)	7(1)	6(1)	5(2)	$\infty(-1)$	2	1, 3, 4, 5
4	0(0)	4(2)	2(S)	6(1)	5(1)	5(2)	$\infty(-1)$	1	3, 4, 5
5	0(0)	4(2)	2(S)	6(1)	5(1)	5(2)	7(3)	3	4, 5, 6
6	0(0)	4(2)	2(S)	6(1)	5(1)	5(2)	6(4)	4	5, 6
7	0(0)	4(2)	2(S)	6(1)	5(1)	5(2)	6(4)	5	6
8	0(0)	4(2)	2(S)	6(1)	5(1)	5(2)	6(4)	6	$\emptyset$.



* كل ال tree منجيبها من آخر row:

بسأل هاي مين سبقها ال 6 اجاب 4
برسم سهم من 4 لـ 6 وهكذا.

From S to 6

$P = \{(S,2)\,(2,1)\,(1,4)\,(4,6)\}$

cost of the path $= 2 + 2 - 1 + 1 = 6$.

lec15: 2.4 2024

Multi-label algorithm

Initialization:

Initially, the origin node s is given label $(0,0)_1^s$. (The subscript indicates the reference number $k=1$ for this label, and the superscript indicates that this label exists at node s) All other nodes receive no labels. The predecessor for this label, $pred(1)$ is 0.

$LIST = \{(0,0)_1^s\}$ $r=2$

Iterations:

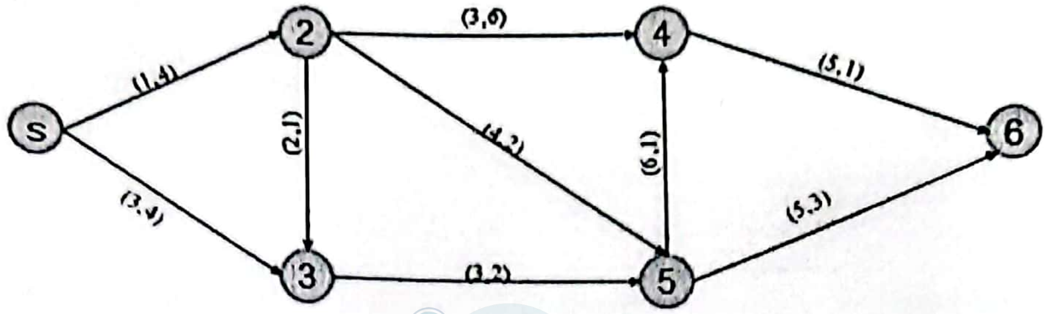
while $LIST \neq \emptyset$:

1. Remove label k from the beginning of the list. Suppose label k has components $(d_k, t_k)_k^p$, so that it exists at node p
2. For every arc (p,j) leading from node p:
   (a) Create a new label r at node j where $(d_r, t_r) = (d_k + d_{pj}, t_k + t_{pj})$
   (b) Define $pred(r) = k$
   (c) Add label r to the end of the $LIST$
   (d) Compare label r to each existing label (d_q, t_q) currently at node j:
      i. If $d_r \geq d_q$ -and- $t_r \geq t_q$, then label r is dominated by (d_q, t_q). Discard the new label, and remove it from $LIST$.
      ii. Else If $d_q \geq d_r$ -and- $t_q \geq t_r$, then label r dominates label q. Discard the label (d_q, t_q), and if necessary remove it from $LIST$.
      iii. Else retain both labels

Stopping criterion:

When no labels remain in $LIST$, this indicates that no remaining label corrections can be found, and all of the paths have been found.

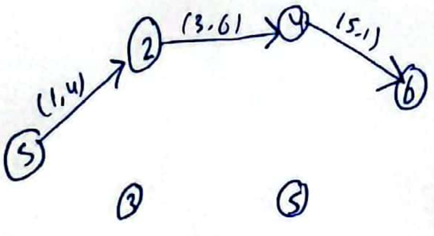
Example: Given the graph in the Figure shown below, find the shortest paths to all nodes using the vertex s as the starting source. Use **Multi-label algorithm**



	S	2	3	4	5	6	LIST
Labels	$(0,0)^{s}_{1}$ Pred(1) = 0	$(1,4)^{2}_{2}$ Pred(2) = 1	$(3,4)^{3}_{3}$ Pred(3) = 1 $(3,5)^{3}_{4}$ X Pred(4) = 2	$(4,10)^{4}_{5}$ Pred(5) = 2 $(11,7)^{4}_{10}$ Pred(10) = 6	$(5,6)^{5}_{6}$ Pred(6) = 2 $(6,6)^{5}_{7}$ X Pred(7) = 3	$(9,11)^{6}_{8}$ Pred(8) = 5 $(10,9)^{6}_{9}$ Pred(9) = 6 $(16,8)^{6}_{11}$ Pred(11) = 10	$(0,0)^{s}_{1}$ X $(1,4)^{2}_{2}$ X $(3,4)^{3}_{3}$ X $(3,5)^{3}_{4}$ X $(4,10)^{4}_{5}$ X $(5,6)^{5}_{6}$ X $(6,6)^{5}_{7}$ X $(9,11)^{6}_{8}$ X $(10,9)^{6}_{9}$ X $(6,6)^{5}_{7}$ X $(11,7)^{4}_{10}$ X $(16,8)^{6}_{11}$

Ø

Path 1



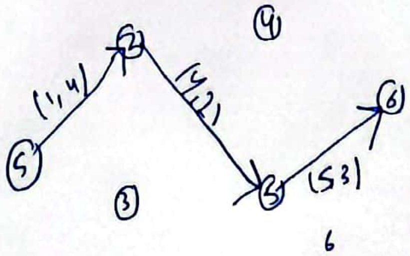
Same as $(9,11)^{6}_{8}$

so correct

1+3+5 = 9

4+6+1 = 11

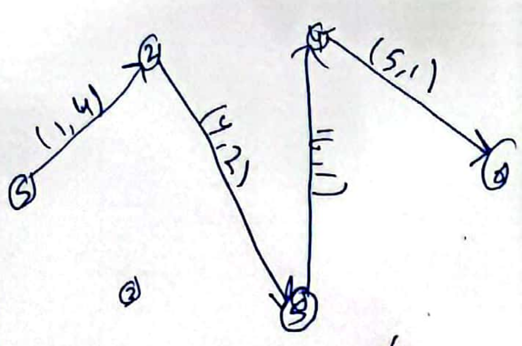
Path 2



Same as $(10,9)^{6}_{9}$

1+4+5 = 10

3+2+4 = 9



Same as $(16,8)^{6}_{11}$

5+6+4+1 = 16

1+1+2+4 = 8.

Minimum Cost Paths

Question .

Suppose you have gone to the grocery store and stocked up with a week of groceries. You would like to make it home in a reasonable amount of time without traveling too far so that your ice cream does not melt. The grocery store will be your starting location, referred to as *s*, and your house will be the final destination, referred to as *t*. The table below shows all connections in the transportation network you will use to travel home (if a connection is not in the table, that arc does not exist in the network). The table also gives the distance and time between any pair of nodes for which a connection exist.

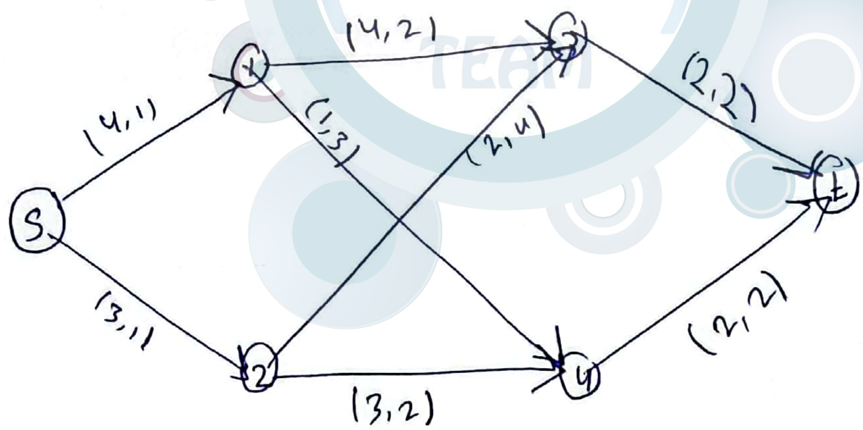
Connection	Distance	Time
(s,1)	4	1
(s,2)	3	1
(1,3)	4	2
(1,4)	1	3
(2,3)	2	4
(2,4)	3	2
(3,t)	2	2
(4,t)	2	2

(a) Draw the network, labeling each node with the appropriate node number $\{s,t,1,2,3,4\}$ and labeling each arc with the cost (d_{ij},t_{ij}) where d_{ij} is the distance of the connection and t_{ij} is the time of the connection.

(b) Use the **multi-label algorithm** to find the minimum cost paths from the grocery store *s* to your home *t*, where the two types of costs being considered are described below. Show your work in the form of a table as we did in class.

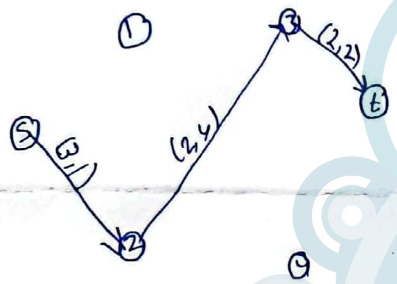
i. Find the minimum distance path from *s* to *t*.

ii. Find the path from *s* to *t* that minimizes the total time.

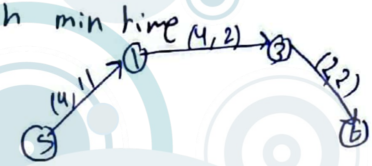


s	1	2	3	4	t	list
$(0,0)^{s}_{1}$	$(4,1)^{1}_{2}$	$(3,1)^{2}_{3}$	$(8,3)^{3}_{4}$	$(5,4)^{4}_{5}$	~~$(10,5)^{t}_{8}$~~	~~$(0,0)^{s}_{1}$~~
Pred(1)=0	Pred(2)=1	Pred(3)=1	Pred(4)=2	Pred(5)=2	Pred(8)=4	~~$(4,1)^{1}_{2}$~~
			$(5,5)^{3}_{6}$	$(6,3)^{4}_{7}$	$(7,6)^{t}_{9}$	~~$(3,1)^{2}_{3}$~~
			Pred(6)=3	Pred(7)=3	Pred(9)=5	~~$(8,3)^{3}_{4}$~~
					~~$(7,7)^{t}_{10}$~~	~~$(5,4)^{4}_{5}$~~
					Pred(10)=6	~~$(5,5)^{3}_{6}$~~
					$(8,5)^{t}_{11}$	~~$(6,3)^{4}_{7}$~~
					Pred(11)=7	~~$(10,5)^{t}_{8}$~~
						~~$(7,6)^{t}_{9}$~~
						~~$(7,7)^{t}_{10}$~~
						~~$(8,5)^{t}_{11}$~~
						$\emptyset$

Path min distance



Path min time



2 4

لو كنت مهتم فقط بال time بدي احسبها بوحدة من ال methods الي قبل

كأنها distance.

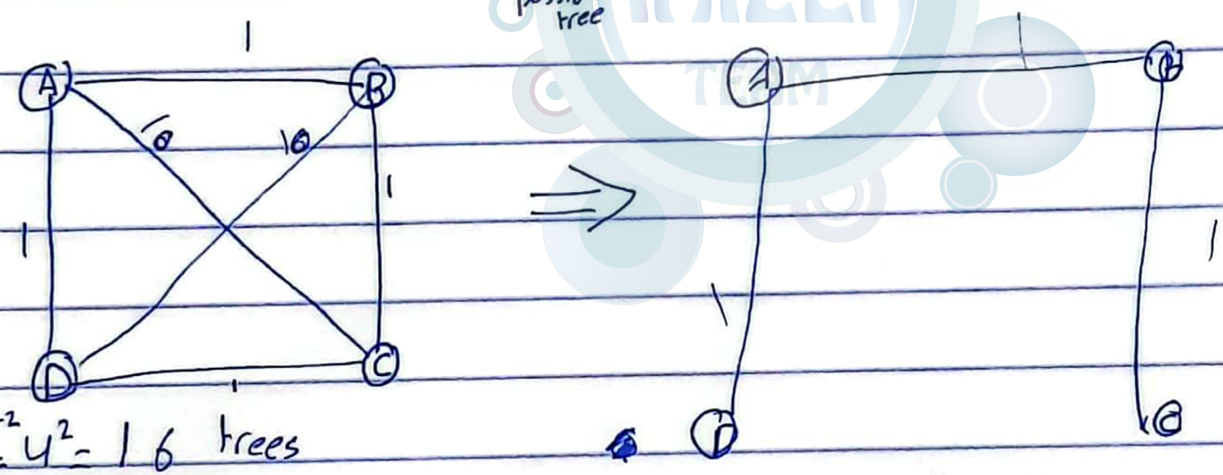
lec16: 7-4.2024

* IN $G(N, A)$ the TREE is a subNetwork that includes all nodes but subset of arcs $\Rightarrow T = (N, A_T)$ such that:

1) a path exist between all pair of nodes
2) $|A_T| = n-1 = |N| - 1$
3) No cycles.

* Minimum spanning tree: denoted by T^* or MST in a networks where each arc has cost C_{ij} in a tree such that total cost of arcs in the tree is minimized

Mathamaticaly $\Rightarrow T^* = \text{argmin } C_{ij}$
among all possible tree $T \in G$



$4^{n-2} = 4^2 = 16$ trees

cost = 3.