

Weekly Notebook

Mechanics

Second
Semester
2024



Mechanics $\rightarrow$ Statics

ch 2.2 :-

Scalar! - $F = 30\text{ N} / F = 40\text{ N}$ (مقدار، مقدار)

vector! - $\vec{F} = (3\hat{i} + 4\hat{j})\text{ N}$ (مقدار، اتجاه)

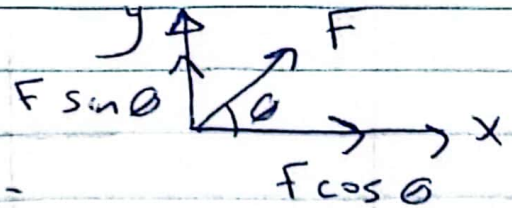
Components! -

$\rightarrow$ any Force could be replaced by two component :-

$$* F_x = F \cos \theta$$

$$* F_y = F \sin \theta = N$$

Scalar

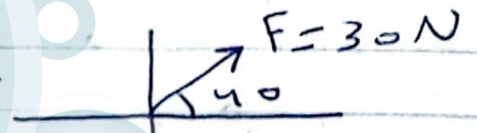


vector expression:-

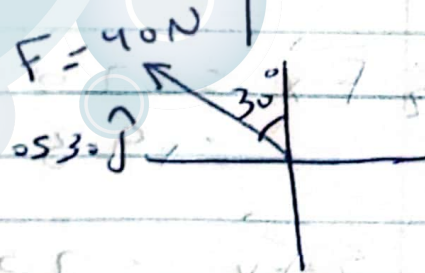
$$\vec{F} = (F_x \hat{i} + F_y \hat{j})\text{ N}$$

Find the vector expression of the force! -

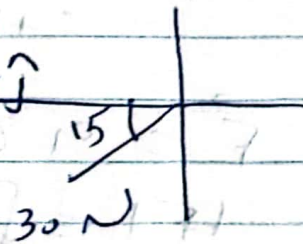
$$1) \vec{F} = 30 (\cos 40^\circ \hat{i} + \sin 40^\circ \hat{j})$$
$$= 38.3 \hat{i} + 32.1 \hat{j}$$



$$2) \vec{F} = -40 \sin 30^\circ \hat{i} + 40 \cos 30^\circ \hat{j}$$
$$= -20 \hat{i} + 34.6 \hat{j}$$



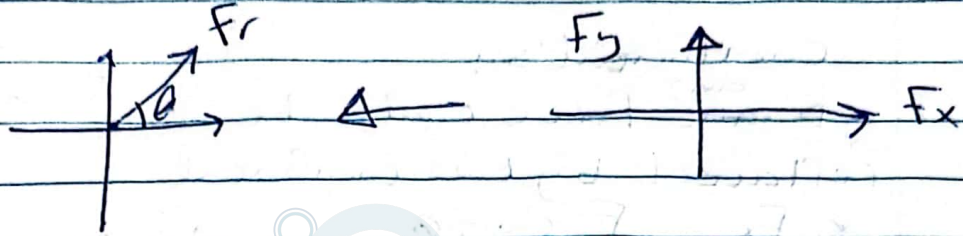
$$3) \vec{F} = -30 \cos 15^\circ \hat{i} + 30 \sin 15^\circ \hat{j}$$
$$= -28.9 \hat{i} + 7.76 \hat{j}$$



Resultant Force :-

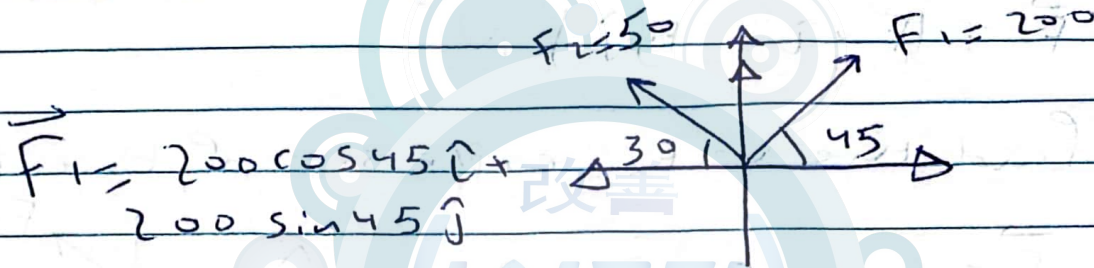
$$F_r = |F| = \sqrt{F_x^2 + F_y^2}$$

$$\tan \theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$



ex:-

Determine the magnitude and the direction of the resultant force :-



$$\vec{F}_1 = 200 \cos 45^\circ \hat{i} + 200 \sin 45^\circ \hat{j}$$

$$\vec{F}_2 = -50 \cos 30^\circ \hat{i} + 50 \sin 30^\circ \hat{j}$$

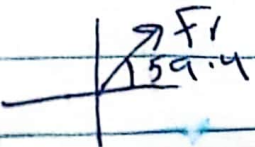
$$\sum \vec{F}_x = 200 \cos 45^\circ \hat{i} - 50 \cos 30^\circ \hat{i} \\ = +98.1 \hat{i}$$

$$\sum \vec{F}_y = 200 \sin 45^\circ \hat{j} + 50 \sin 30^\circ \hat{j} \\ = +166.4 \hat{j}$$

$$\vec{F} = 98.1 \hat{i} + 166.4 \hat{j}$$

$$|F| = \sqrt{98.1^2 + 166.4^2} = 193.1 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{166.4}{98.1} \right) = 59.4^\circ$$



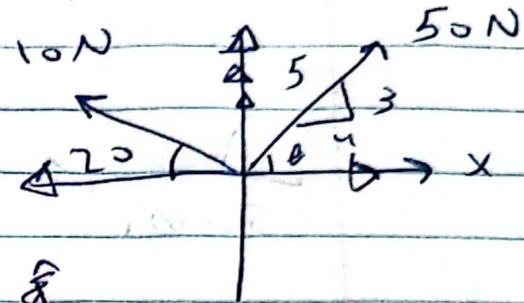
ex 2: Determine the magnitude and the direction of the resultant force :-

$$\sum F_x = 50 \times \frac{4}{5} - 10 \cos 20^\circ$$

$$= 30.16 \text{ N}$$

$$\sum F_y = 50 \times \frac{3}{5} + 10 \sin 20^\circ$$

$$= 33.4 \text{ N}$$

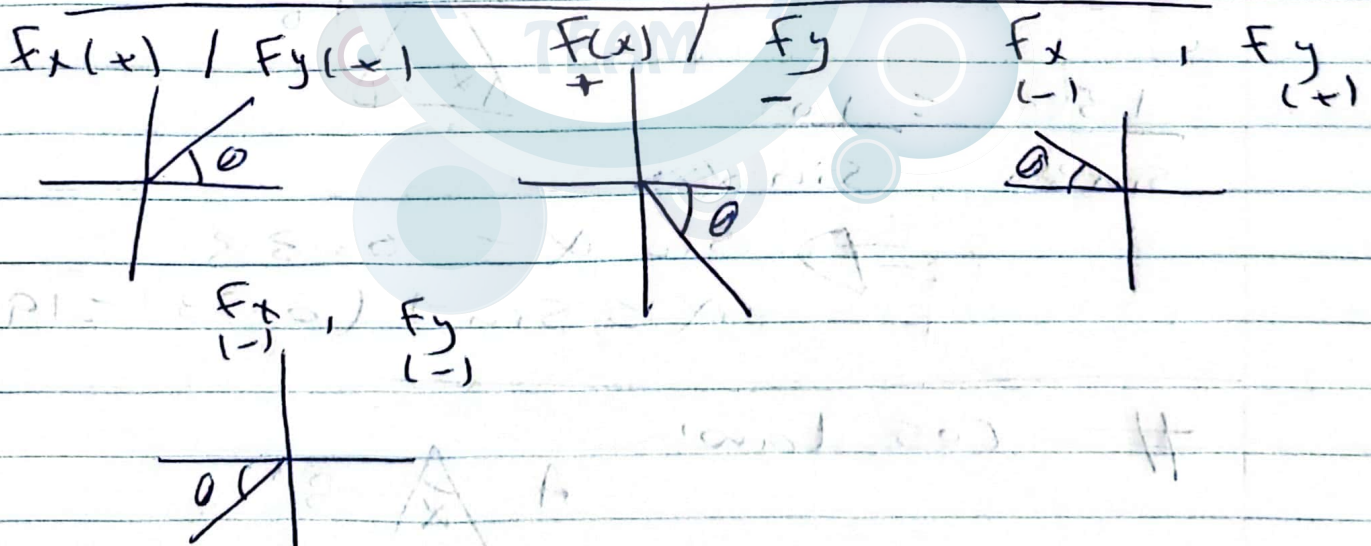
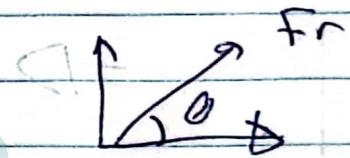


$$\Rightarrow \vec{F} = 30.16 \hat{i} + 33.4 \hat{j}$$

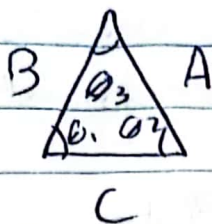
$$|\vec{F}| = \sqrt{30.16^2 + 33.4^2} = 45.3$$

$$\theta = \tan^{-1} \left(\frac{33.4}{30.16} \right)$$

$$= 47.5^\circ$$



Sin Law :-

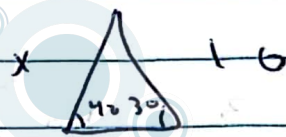


$$\frac{A}{\sin \theta_1} = \frac{B}{\sin \theta_2} = \frac{C}{\sin \theta_3}$$

ex :-

1)

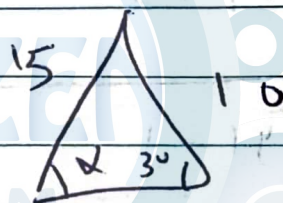
$$\frac{10}{\sin 40} = \frac{x}{\sin 30}$$



$$\Rightarrow x = 7.77$$

2)

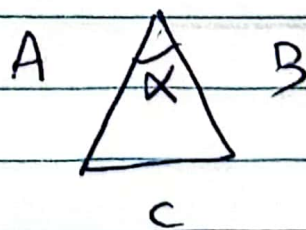
$$\frac{15}{\sin 30} = \frac{10}{\sin x}$$



$$\Rightarrow \sin x = 0.33$$

$$x = \sin^{-1}(0.33) = 19.47$$

Cos Law :-



$$C = \sqrt{A^2 + B^2 - (2AB \cos x)}$$

$$c \times 60 = x \triangle 10$$


$$x = \sqrt{10^2 + 15^2 - (2 \times 10 \times 15 \cos 30)}$$

$$x =$$

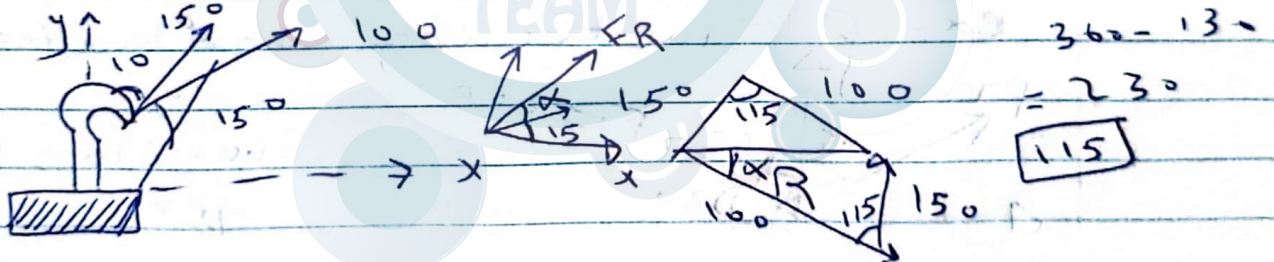
✱ Parallel Law:-



كل زاويتين متقابلتين متساويتان

مجموع زوايا المثلث 360°

Determine the magnitude and the direction of the Resultant force:-



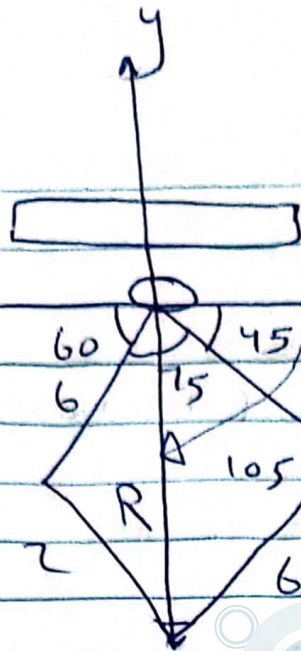
$$R = \sqrt{150^2 + 100^2 - (2 \times 100 \times 150 \cos 115)} = 212.5$$

$$\Rightarrow \frac{212.5}{\sin 115} = \frac{150}{\sin \alpha} \Rightarrow \sin \alpha =$$

$$\alpha = 39.7$$

$$\theta = \alpha + 15^\circ = 54.7$$

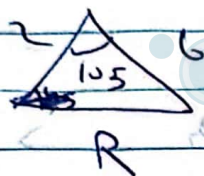
ex:-



Determine the magnitude and the direction of the resultant force $\vec{F}$ clockwise from the x-axis.

$$360 - 150 = 210$$

$$= 105$$



$$R = \sqrt{6^2 + 2^2 - (2 \times 6 \times \cos 105)}$$

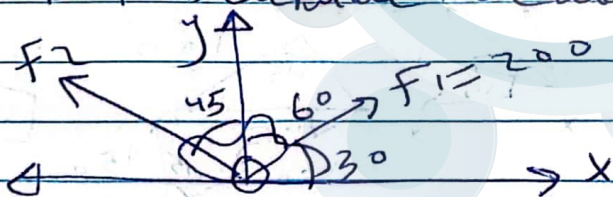
$$R = 6.79 \text{ N}$$

$$\frac{6.79}{\sin 105} = \frac{6}{\sin \alpha} \Rightarrow \sin \alpha =$$

$$\alpha = 58.59$$

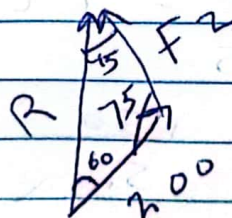
$$\theta = \alpha + 45^\circ = 103.59^\circ$$

ex: Determine the magnitude of the resultant force



$$360 - 210 = 150$$

$$75^\circ$$



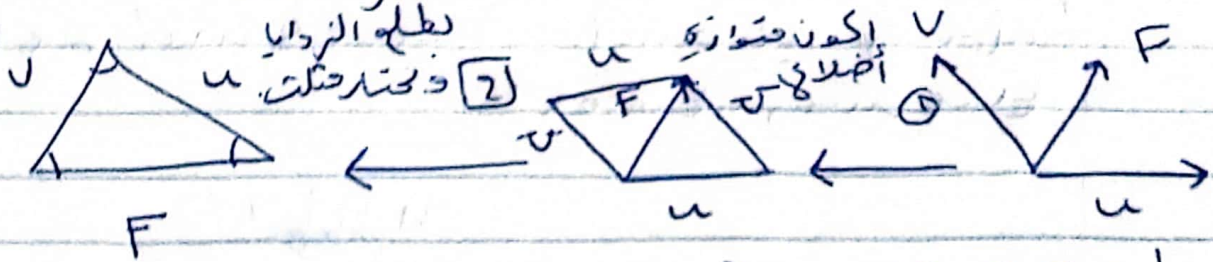
$$\frac{F_2}{\sin 60} = \frac{200}{\sin 45}$$

$$F_2 = 245 \text{ N}$$

$$R = \sqrt{200^2 + 245^2 - (2 \times 200 \times 245 \times \cos 75)}$$

$$= 273.2 \text{ N}$$

Non-Rectangular components :-

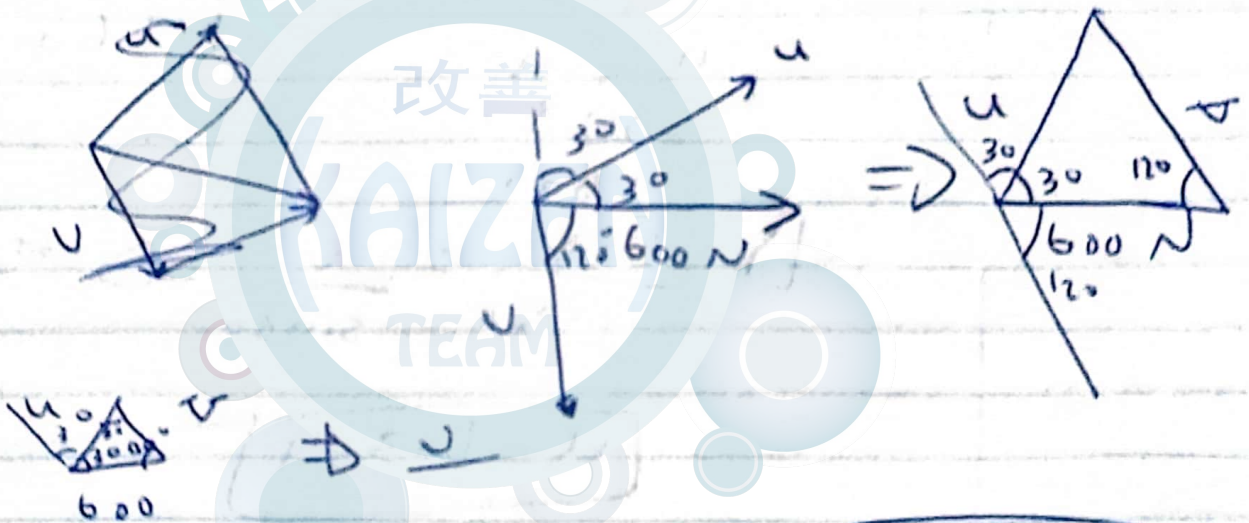


Sin law

cos law

بمقدار قوة وبشكل واحد قيمتها
على الـ axis 2 - axis 1
شبه مبادئ

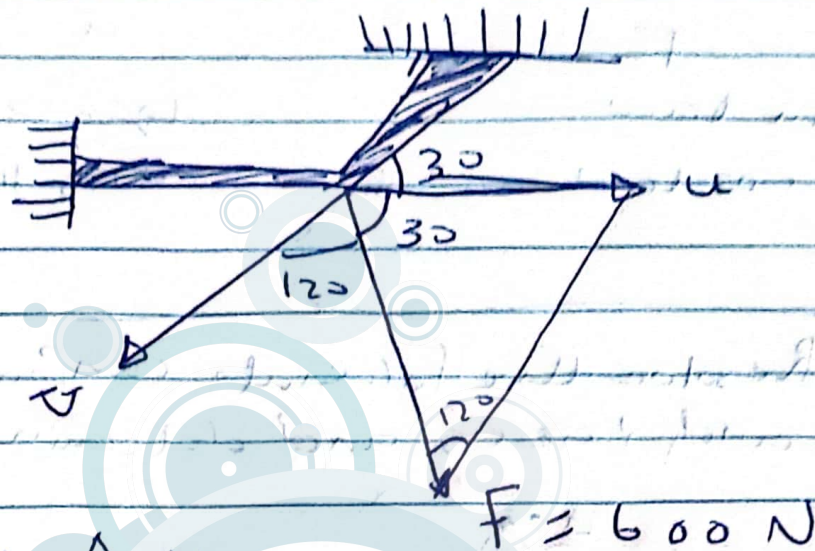
Resolve the force into components acting along u and v axes and determine the magnitudes



$$\Rightarrow \frac{600}{\sin 30} = \frac{u}{\sin 30} \Rightarrow \boxed{u = 600}$$

$$\frac{600}{\sin 30} = \frac{v}{\sin 120} \Rightarrow \boxed{v = 103.9}$$

Resolve the force into components acting along u and v axes and determine the magnitudes!



Using the sine rule in the triangle formed by the force and its components:

$$\frac{600}{\sin 30^\circ} = \frac{u}{\sin 120^\circ}$$

$$\Rightarrow u = \frac{600 \sin 120^\circ}{\sin 30^\circ}$$

$$\Rightarrow u = 1039.2$$

The Force F as a cartesian vectors :-
 $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

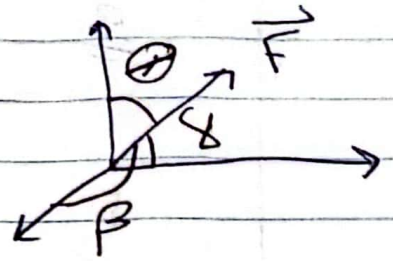
Resultant force :-

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_x = F \cos \beta = N$$

$$F_y = F \cos \gamma = N$$

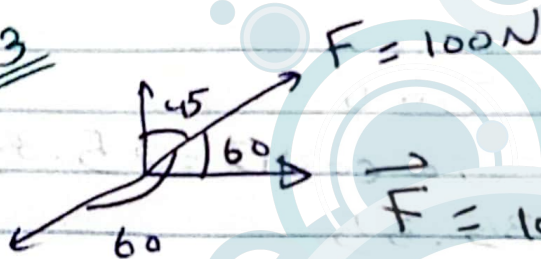
$$F_z = F \cos \theta = N$$



Case 1 :-

Express the force as a cartesian vector :-

3



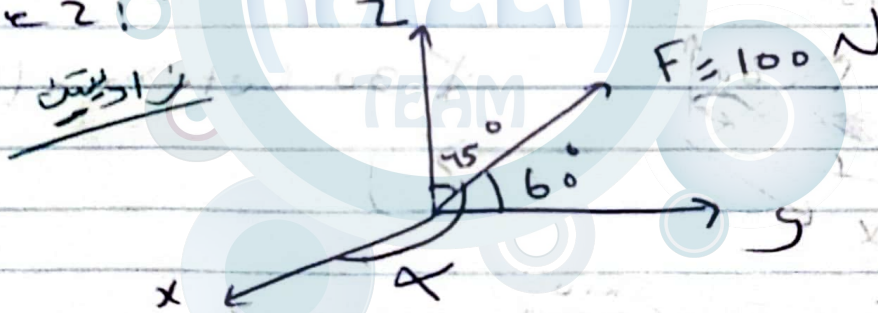
$$F = 100 \text{ N}$$

$$\vec{F} = 100 \cos 60^\circ \hat{i} + 100 \cos 60^\circ \hat{j} + 100 \cos 45^\circ \hat{k}$$

$$\vec{F} = 50 \hat{i} + 50 \hat{j} + 70.7 \hat{k}$$

Case 2 :-

A:



$$\cos^2 \alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

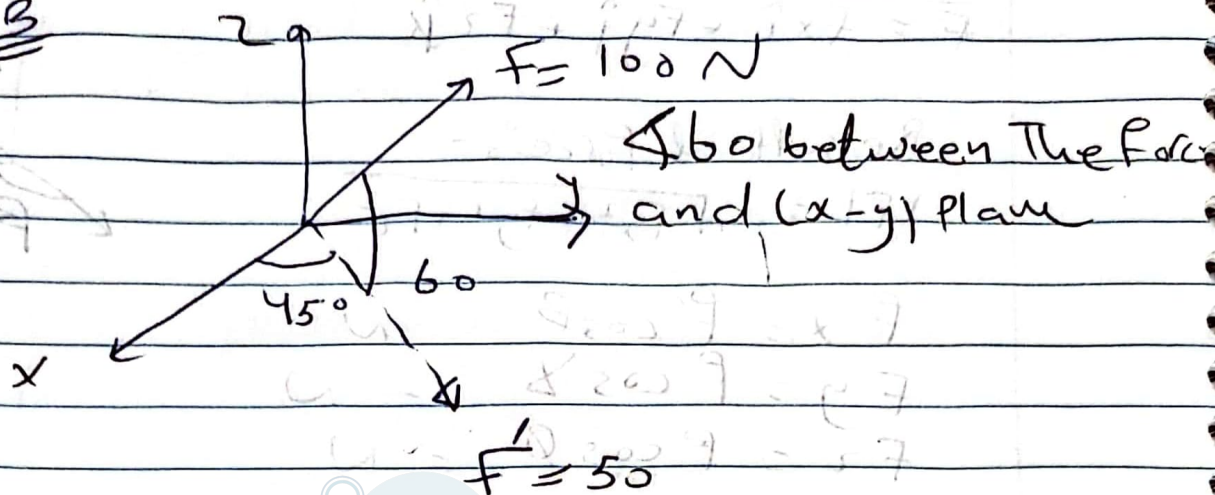
$$\Rightarrow \cos^2 \alpha = \frac{1}{4} \Rightarrow \cos \alpha = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3}$$

$$\vec{F} = 50 \hat{i} + 50 \hat{j} + 70.7 \hat{k}$$

Case 2 :-

B



$$F_1 = 100 \cos 60 = 50$$

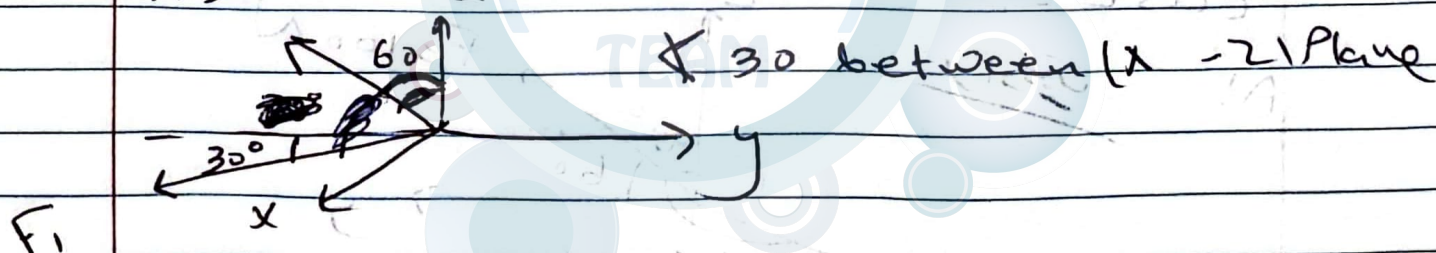
$$F_2 = 100 \sin 60 = 86.6$$

$$F_x = F_1 \cos 45 = 50 \cos 45 = 35.35$$

$$F_y = F_1 \sin 45 = 35.35$$

$$\vec{F} = (35.35\hat{i} + 35.35\hat{j} + 86.6\hat{k})\text{ N}$$

$F = 50\text{ N}$



$$F_1 = 50 \sin 60 = 43.3$$

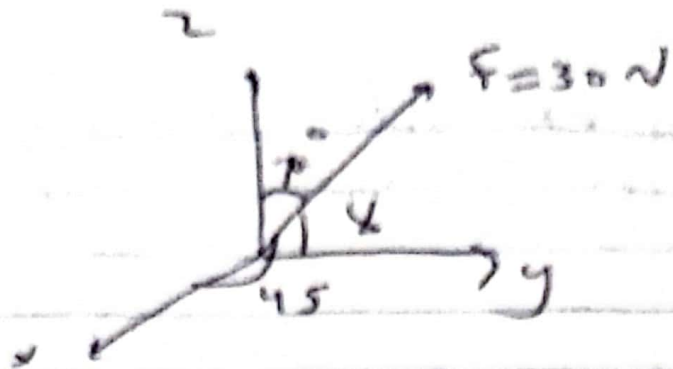
$$F_2 = 50 \cos 60 = 25$$

$$F_y = 50 \cos 30 = 43.3$$

$$F_x = 50 \sin 30 = 25$$

$$\vec{F} = (25\hat{i} - 43.3\hat{j} + 25\hat{k})\text{ N}$$

ex 31



$$\cos^2 45 + \cos^2 70 + \cos^2 \theta = 1$$

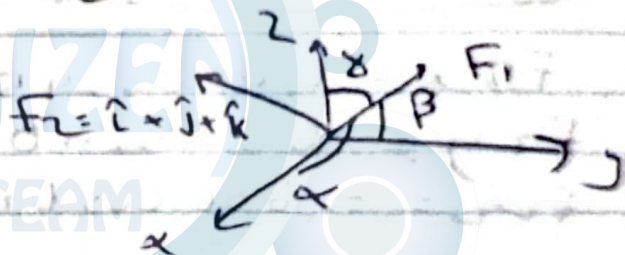
$$\Rightarrow \cos^2 \theta =$$

$$\cos \theta =$$

$$\theta =$$

$$\vec{F} = 30 \cos 45 \hat{j} + 30 \cos \theta \hat{j} + 30 \cos 70 \hat{k} \text{ N}$$

magnitude & direction of resultant force & unit vector.



$$\vec{F}_1 = F_{1x} \hat{i} + F_{1y} \hat{j} + F_{1z} \hat{k}$$

$$\vec{F}_2 = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 = \hat{i} + \hat{j} + \hat{i}$$

$$|\vec{F}_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$$

unit vector:-

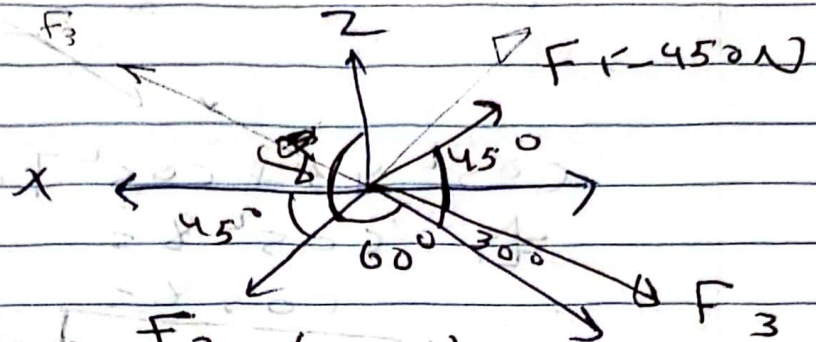
$$\frac{F_{Rx}}{|\vec{F}_R|} \hat{i} + \frac{F_{Ry}}{|\vec{F}_R|} \hat{j} + \frac{F_{Rz}}{|\vec{F}_R|} \hat{k}$$

$$\alpha = \cos^{-1} \left(\frac{F_{1x}}{|\vec{F}_1|} \right)$$

$$\beta = \cos^{-1} \left(\frac{F_{1z}}{|\vec{F}_1|} \right)$$

$$\theta = \cos^{-1} \left(\frac{F_{1y}}{|\vec{F}_1|} \right)$$

ex: Determine the magnitude and direction of angle of the resultant force!



30 (y-z) Plane $F_2 = 600 \text{ N}$

$$F_3 = F_1 \cos 45 = 450 \cos 45 = 318.2$$

$$F_{1y} = F_3 \cos 30 = 275.5$$

$$F_{1z} = 450 \sin 45 = 318.2$$

$$F_{1x} = 318.2 \sin 30 = -159$$

$$\vec{F}_1 = -159\hat{i} + 275.5\hat{j} + 318.2\hat{k}$$

$$F_{2x} = 600 \cos 45 = 424.26 \text{ N}$$

$$F_{2y} = 600 \sin 60 = 519.6 \text{ N}$$

$$F_{2z} = -600 \cos 60 = -300 \text{ N}$$

$$\cos^2 45 + \cos^2 60 + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\vec{F}_2 = (424.26\hat{i} + 519.6\hat{j} - 300\hat{k}) \text{ N}$$

$$\vec{F}_R = 265.26\hat{i} + 575.5\hat{j} + 18.2\hat{k}$$

$$|\vec{F}_R| = 634 \text{ N}$$

$$\alpha = \cos^{-1} \left(\frac{265.26}{634} \right) = 65.3^\circ$$

$$\beta = \cos^{-1} \left(\frac{575.5}{634} \right) = 28.4^\circ$$

Problem 2-62

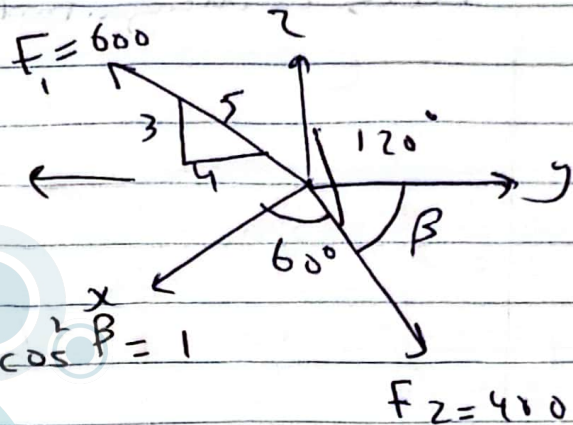
Determine the magnitude and the direction of resultant force.

(x-z) plane

Sol:-

$$\cos^2 60 + \cos^2 120 + \cos^2 \beta = 1$$

$$\Rightarrow \beta = 45^\circ$$



$$F_{2x} = 400 \cos 45 = 282.8 \text{ N}$$

$$F_{2y} = 400 \cos 120 = -200 \text{ N}$$

$$F_{2z} = 400 \cos 120 = -200 \text{ N}$$

$$\vec{F}_2 = (282.8 \hat{i} - 200 \hat{j} - 200 \hat{k}) \text{ N}$$

$$\vec{F}_1 = 600 \times \frac{4}{5} \hat{i} + 600 \times \frac{3}{5} \hat{k}$$

$$= (480 \hat{i} + 360 \hat{k}) \text{ N}$$

$$\vec{F}_R = 680 \hat{i} + 282.8 \hat{j} + 160 \hat{k}$$

$$|\vec{F}_R| = 754 \text{ N}$$

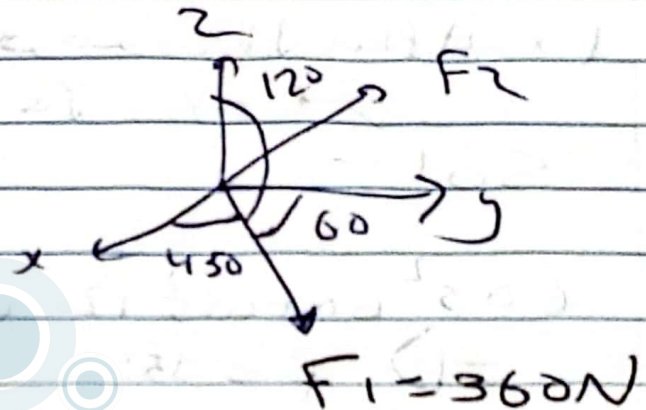
$$\alpha = \frac{680 \hat{i} + 282.8 \hat{j} + 160 \hat{k}}{754}$$

$$\alpha = 25.5^\circ$$

$$\beta = 68.0^\circ$$

$$\gamma = 77.7^\circ$$

ex. Specify the magnitude of F_2 and its coordinate direction Angle so that the resultant force F_R acting along positive y-axis and has a magnitude 800 N.



$$F_R = F_1 + F_2$$

$$800\hat{j} = 360 \cos 45 + 360 \cos 60 + F_2$$

$$800\hat{j} = \cancel{254.5\hat{i}} + 254.5\hat{i} + 180\hat{j} - 180\hat{i} + F_2$$

$$\vec{F}_2 = (-254.5\hat{i} + 620\hat{j} + 180\hat{k}) \text{ N}$$

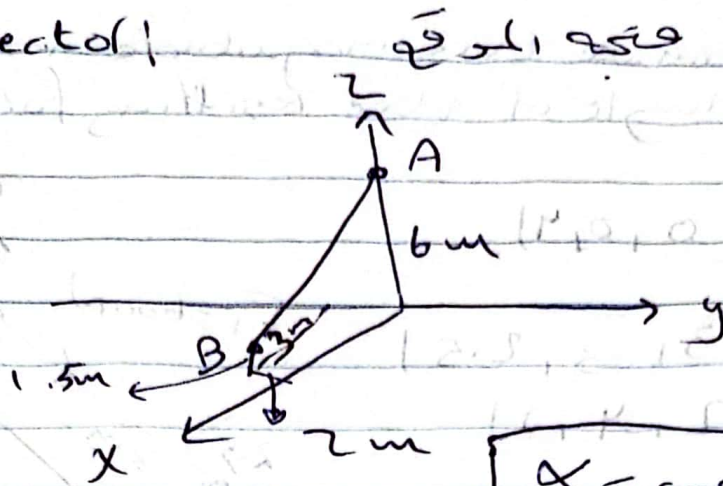
$$|F_2| = 639.9 \text{ N}$$

$$(1) \alpha = \cos^{-1} \left(\frac{-254.5}{639.9} \right) =$$

$$(y) \beta = \cos^{-1} \left(\frac{620}{639.9} \right) =$$

$$(2) \gamma = \cos^{-1} \left(\frac{180}{639.9} \right) =$$

Position vector



$$\vec{A} (0, 0, 6)$$

$$\vec{B} (3, -2, +1.5)$$

$$\vec{r}_{AB} = \vec{B} - \vec{A} = (3, -2, -7.5)$$

$$|\vec{r}| = \sqrt{9 + 4 + 7.5^2}$$

$$\alpha = \cos^{-1} \left(\frac{1.50}{3.50} \right) = 64.1^\circ$$

$$\beta = \cos^{-1} \left(\frac{-1.00}{3.50} \right) = 107^\circ$$

$$\gamma = \cos^{-1} \left(\frac{-3.00}{3.50} \right) = 149^\circ$$

Force vector = $F\hat{u}$

ex) Represent this 350N force acting on the Point A as a Cartesian vector and determine it's direction

Angle:-

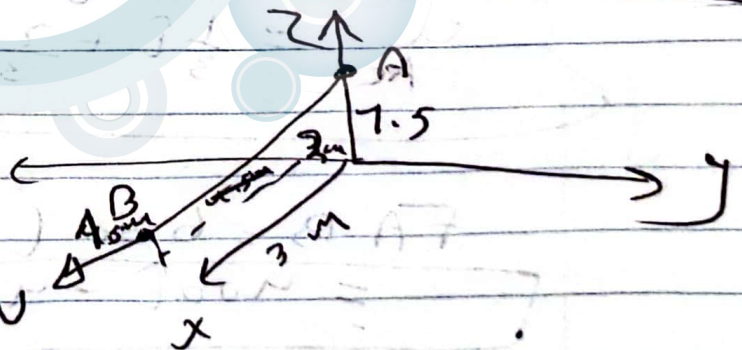
$$A (0, 0, 7.5)$$

$$B (3, -2, 1.5)$$

$$\vec{r}_{AB} = 3\hat{i} - 2\hat{j} - 6\hat{k}$$

$$|\vec{r}|_{AB} = \sqrt{9 + 4 + 36} = 7$$

350N



$$\vec{u}_{AB} = \frac{3\hat{i} - 2\hat{j} - 6\hat{k}}{7} = 0.428\hat{i} - 0.285\hat{j} - 0.857\hat{k}$$

$$\vec{F}_{AB} = 350 (0.428\hat{i} - 0.285\hat{j} - 0.857\hat{k}) = 150\hat{i} - 100\hat{j} - 300\hat{k}$$

Determine the magnitude and the direction of Angle of the Resulting Force Acting on A.

$$A(0, 0, 4)$$

$$B(3, -3, 2.5)$$

$$C(2, 4, 0)$$

$$\vec{r}_{AB} = 3\hat{i} - 3\hat{j} - 1.5\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{9+9+1.5^2}$$

$$u_{AB} = \frac{3\hat{i} - 3\hat{j} - 1.5\hat{k}}{\sqrt{9+9+1.5^2}} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

$$\vec{r}_{AC} = 2\hat{i} + 4\hat{j} - 4\hat{k}$$

$$|\vec{r}_{AC}| = \sqrt{4+16+16} = 6$$

$$u_{AC} = \frac{2\hat{i} + 4\hat{j} - 4\hat{k}}{6}$$

$$u_{AC} = \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

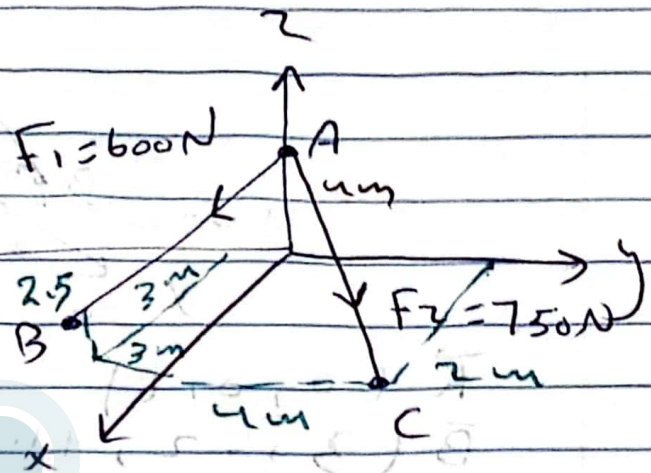
$$F_{AB} = 600 \left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \right) = 400\hat{i} - 400\hat{j} - 200\hat{k}$$

$$F_{AC} = 750 \left(\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k} \right) = 250\hat{i} + 500\hat{j} - 500\hat{k}$$

$$F_R = (650\hat{i} + 100\hat{j} - 700\hat{k}) \text{ N}$$

$$|F_R| = \sqrt{650^2 + 100^2 + 700^2} = 950 \text{ N}$$

$$\alpha = \cos^{-1} \left(\frac{650}{950} \right) = 47.4^\circ \quad \beta = \cos^{-1} \left(\frac{950}{1000} \right) = 84^\circ$$



Determine the magnitude and the direction angles of the resultant force.

$$A(0, 2, 4)$$

$$B(0, 0, 0)$$

$$C(4, 8, 0)$$

$$\vec{r}_{AB} = -2\hat{j} - 4\hat{k}$$

$$|\vec{r}| = \sqrt{4 + 16} = \sqrt{20}$$

$$\vec{u}_{AB} = \frac{-2\hat{j} - 4\hat{k}}{\sqrt{20}} = -0.447\hat{j} - 0.89\hat{k}$$

$$\vec{r}_{AC} = 4\hat{i} + 6\hat{j} - 4\hat{k}$$

$$|\vec{r}| = \sqrt{16 + 36 + 16} = \sqrt{72}$$

$$\vec{u}_{AC} = \frac{4\hat{i} + 6\hat{j} - 4\hat{k}}{\sqrt{72}} = 0.485\hat{i} + 0.72\hat{j} - 0.485\hat{k}$$

$$\vec{F}_B = 600(-0.447\hat{j} - 0.89\hat{k}) = -268.33\hat{j} - 536.6\hat{k}$$

$$\vec{F}_C = 500(0.485\hat{i} + 0.72\hat{j} - 0.485\hat{k}) = 242.54\hat{i} + 363.8\hat{j} - 242.5\hat{k}$$

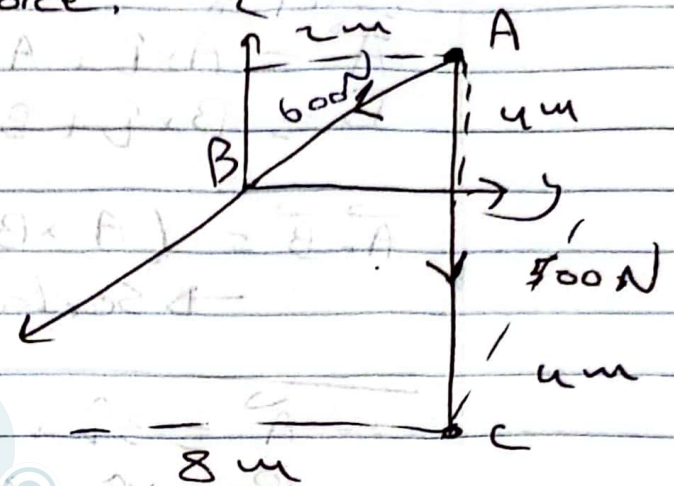
$$\vec{F}_R = (242.54\hat{i} + 95.47\hat{j} - 779.2\hat{k}) \text{ N}$$

$$|\vec{F}_R| = 822 \text{ N}$$

$$\alpha = \cos^{-1}\left(\frac{242.54}{822}\right) = 72.8^\circ$$

$$\beta = \cos^{-1}\left(\frac{95.47}{822}\right) = 83.3^\circ$$

$$\gamma = \cos^{-1}\left(\frac{-779.2}{822}\right) = 162^\circ$$



* Dot Product

2 defs -

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

→ Scalar

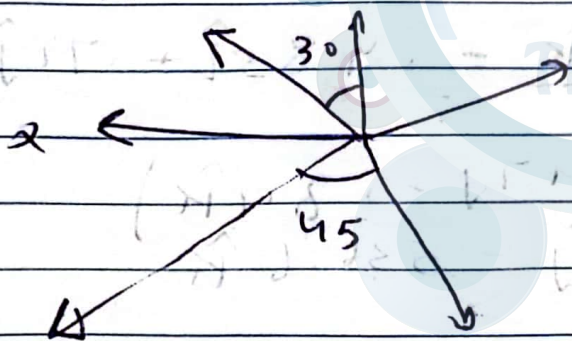
$$\vec{A} = 3\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$$

$$\text{Find } \vec{A} \cdot \vec{B} = 12 + 6 - 2 = \boxed{16}$$

Scalar

$$\vec{F}_1 = 400 \text{ N}$$



$$\vec{F}_2 = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$+ 400 \sin 30^\circ \sin 45^\circ \hat{i}$$

$$\vec{F}_1 = 400 \cos 30^\circ \hat{k} + 400 \sin 30^\circ \cos 45^\circ \hat{j}$$

$$\vec{F}_3 = 400 \sin 30^\circ$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

↳ Angle between them

Find Angle between 2 vector:

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

$$\vec{A} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{B} = \hat{i} + \hat{j} - 4\hat{k}$$

Find the angle between them

$$\theta = \cos^{-1} \left(\frac{2+1-20}{\sqrt{34} \sqrt{18}} \right) = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Projection (of $\vec{a}$ on $\vec{b}$)

Projection $\vec{F}_1$ Parallel $\vec{F}_2$

$$F_{||} = \vec{F}_1 \cdot \hat{u}_{F_2} \Rightarrow \text{Scalar}$$

$$\vec{F}_{||} = F_{||} \hat{u}_{F_2} \Rightarrow \text{Vector}$$

Perpendicular

$$F_{\perp} = \sqrt{F_1^2 - F_{||}^2}$$

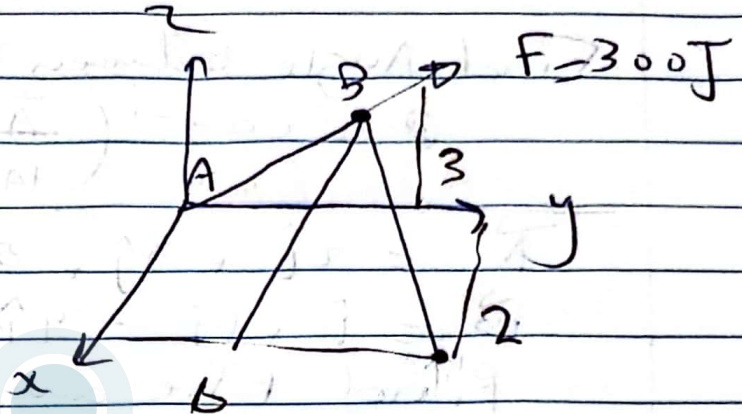
$$\vec{F}_{\perp} = \vec{F}_1 - \vec{F}_{||}$$

Ex:- Find component $\vec{F} = 300\hat{j}$ parallel and perpendicular of AB

$$A(0, 0, 0)$$

$$B(2, 6, 3)$$

$$F_{\parallel} = F \cdot \vec{u}_{AB}$$



$$= 300\hat{j} \cdot (0.286\hat{i} + 0.85\hat{j} + 0.429\hat{k})$$

$$= 257.1 \text{ N}$$

$$\vec{r}_{AB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$|\vec{r}| = \sqrt{4 + 36 + 9} = 7$$

$$\vec{u}_{ab} = \frac{(2\hat{i} + 6\hat{j} + 3\hat{k})}{7} = 0.286\hat{i} + 0.85\hat{j} + 0.429\hat{k}$$

$$\vec{F}_{\parallel} = 257.1 (0.286\hat{i} + 0.85\hat{j} + 0.429\hat{k})$$

$$= 73.5\hat{i} + 220\hat{j} + 110\hat{k}$$

$$F_{\perp} = \sqrt{300^2 - 257.1^2} = 155$$

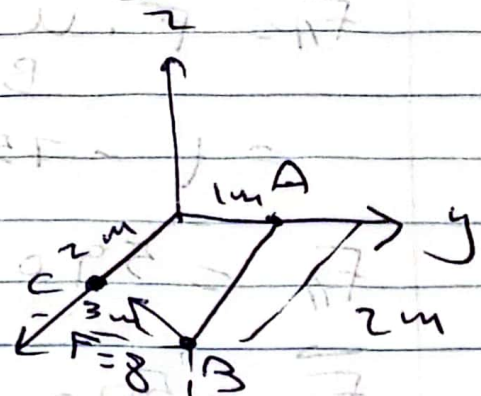
$$\vec{F}_{\perp} = 300\hat{j} - (73.5\hat{i} + 220\hat{j} + 110\hat{k})$$

Determine the Angle between $\vec{F}$ and the segment $\overline{BA}$ and the Parallel Force and the Perpendicular Force on the segment BA .

$$A(0, 1, 0)$$

$$B(2, 3, -1)$$

$$C(2, 0, 0)$$



$$\theta = \cos^{-1} \left(\frac{\vec{BC} \cdot \vec{BA}}{|\vec{BC}| |\vec{BA}|} \right)$$

$$= \cos^{-1} \left(\frac{-4 + 6 + 1}{\sqrt{14} \cdot 3} \right) = \cos^{-1} \left(\frac{3}{3\sqrt{10}} \right) = 42.5^\circ$$

$$\vec{BC} = -2\hat{i} - 3\hat{j} + 1\hat{k}$$

$$\vec{BA} = -2\hat{i} - 2\hat{j} + 1\hat{k}$$

$$|\vec{BC}| = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\vec{BA}| = \sqrt{4 + 4 + 1} = 3$$

$$F_{\parallel} = F \cdot u_{BA}$$

$$\vec{F}_{BC} = 800 \cdot \vec{u}_{BC}$$

$$\vec{u}_{BC} = \frac{-3\hat{j} + 1\hat{k}}{\sqrt{10}}$$

$$\Rightarrow \vec{F}_{BC} = 800 \left(\frac{-3\hat{j} + 1\hat{k}}{\sqrt{10}} \right)$$

$$= -758.9\hat{j} + 253\hat{k}$$

$$\vec{u}_{BA} = \frac{-2\hat{i} - 3\hat{j} + 1\hat{k}}{\sqrt{14}} = -0.66\hat{i} - 0.66\hat{j} + 0.33\hat{k}$$

$$F_{||} = F \cdot u_{BA}$$

$$= | -758.9\hat{j} + 253\hat{k} | \cdot \vec{u}_{AB}$$

$$F_{||} = 590$$

$$\vec{F}_{||} = 590 \cdot u_{BA}$$

$$= 590 (-0.66\hat{i} - 0.66\hat{j} + 0.33\hat{k}) \text{ N}$$

$$F_{\perp} = \sqrt{8002 - 590^2} =$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{||}$$

$$= -758.9\hat{j} + 253\hat{k} - \vec{F}_{||}$$

The End of chapter (2)

CU3: Equilibrium of Articles &

الاشياء (equilibrium) : الاتزان

مجموع القوى المؤثرة على الجسم = 0

$$\sum F_x = 0$$

$$\sum F_y = 0$$

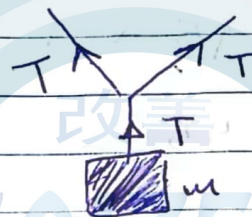
مجموع القوى الأبعاد له صفة : Particle

Types of connection:-

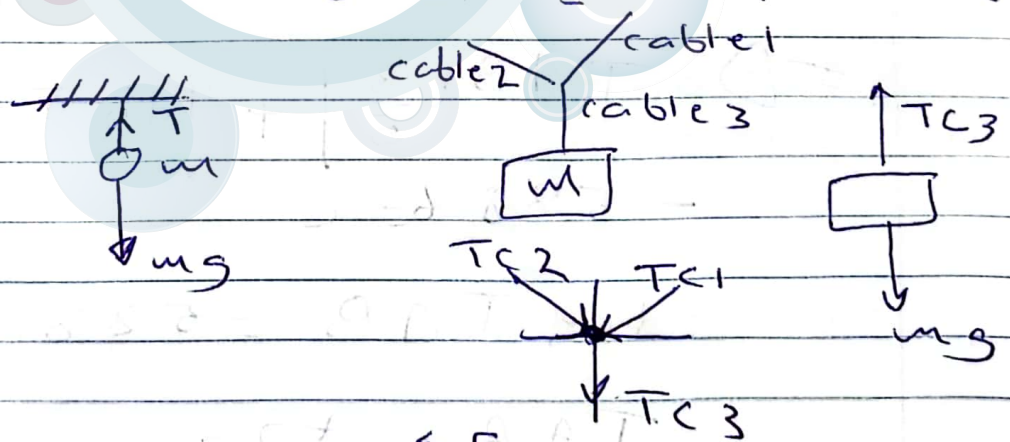
1) Spring زنبرك



2) cables



Free body diagram (f.b.d) الجسم الحر :
 ينفصل الجسم عن المحيط به :
 ينفصل الجسم عن المحيط به :
 ينفصل الجسم عن المحيط به :

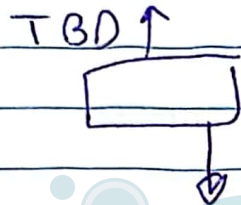
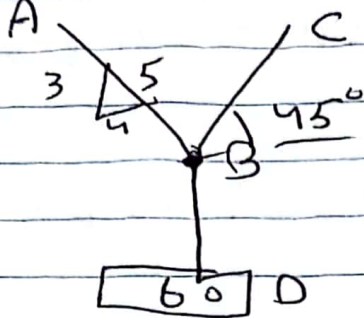


2 equations

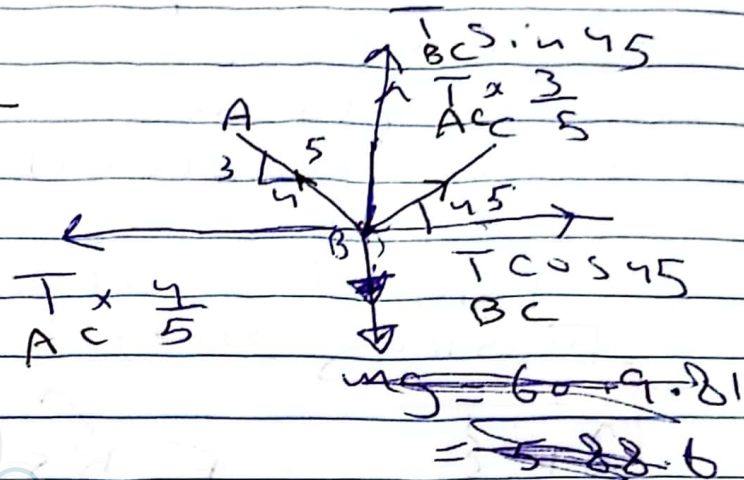
$$\sum F_x = 0$$

$$\sum F_y = 0$$

1) Determine the tension cable AB and BC



$$mg = 60 \times 9.81 = 588.6$$



$$T_{BD} = 588.6$$

$$\text{--- 1) } T_1 \sin 45 + T \times \frac{3}{5} = 588.6$$

$$T_1 \cos 45 = T \times \frac{4}{5} \quad \text{--- 2)}$$

$$0.707 T_{AB} = 0.8 T_{BC}$$

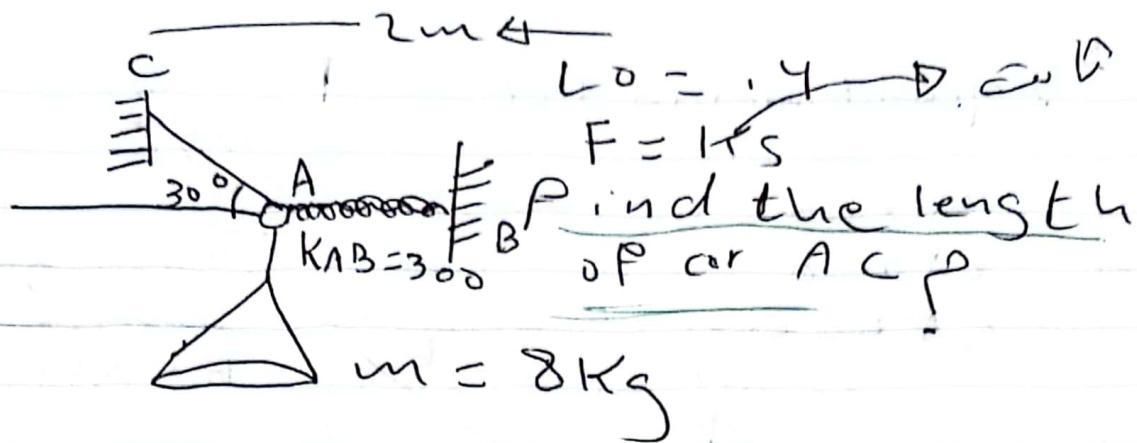
$$T_{BC} = 1.13 T_{AB}$$

$$\Rightarrow T_{AB} \left(\frac{3}{5} \right) + 1.13 T_{AB} \sin 45 - 588.6 = 0$$

$$1.4 T_{AB} = 588.6$$

$$T_{AB} = 420$$

$$T_{BC} = 475.4$$



$$2 = AB + AC \cos 30$$

$$\sum F_y = 0 \Rightarrow$$

$$T = 78.5$$

$$mg = 8 \times 9.81 = 78.5$$

$$\sum F_y = 0 \Rightarrow$$

$$TAC \sin 30 = 78.5$$

$$AC = 157 \text{ N}$$

$$\sum F_x = 0 \Rightarrow TAC \cos 30 = TAB$$

$$157 \cos 30 = TAB$$

$$TAB = 135.9$$

$$F = kx \Rightarrow 135.9 = 300.5$$

$$x = 0.453$$

$$x = L - L_0 = 0.453 = L - 0.4$$

$$\Rightarrow L = 0.853$$

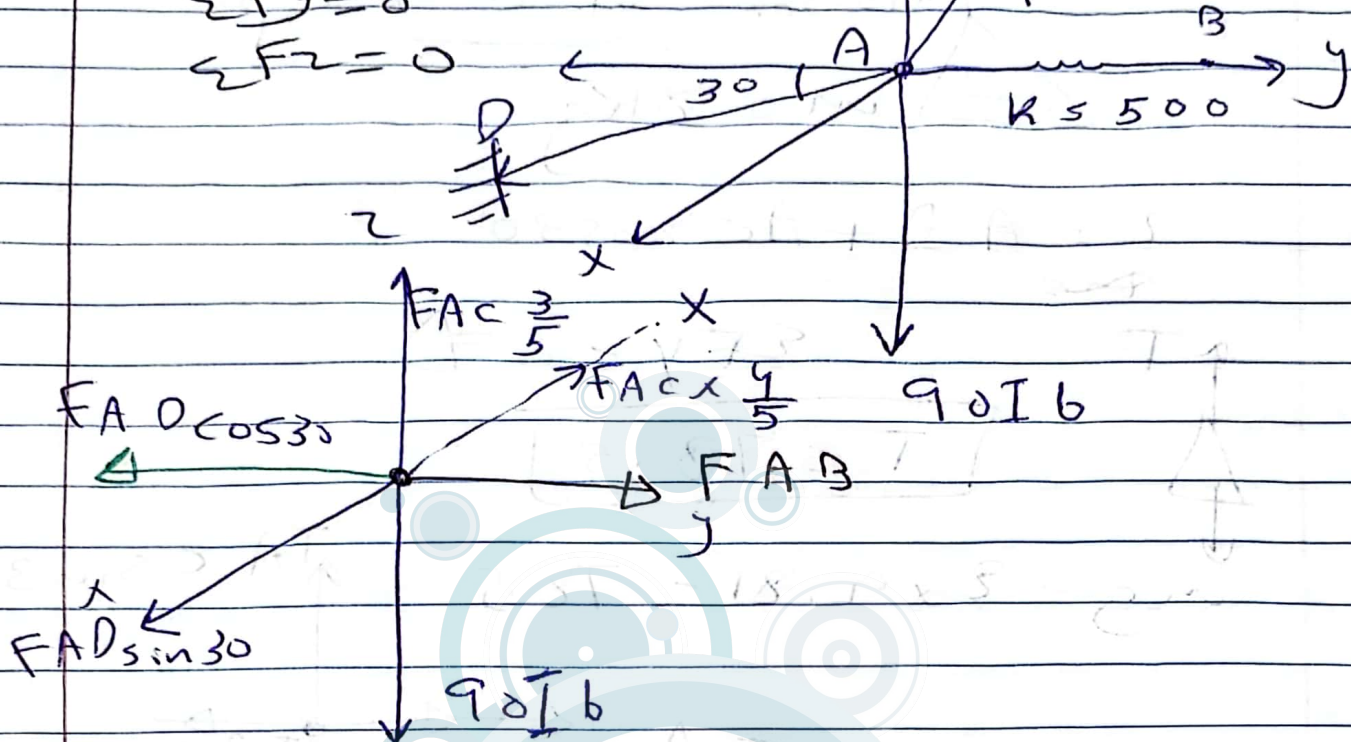
$$2 = 0.853 + AC \cos 30$$

equilibrium in 3 d

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$



$$\sum F_x = 0 \Rightarrow F_A \cos \frac{4}{5} - F_A D \sin 30^\circ \quad (\uparrow)$$

$$\sum F_y = 0 \Rightarrow F_A \sin 30^\circ = F_A D \cos 30^\circ \quad (2)$$

$$\sum F_2 = 0 \Rightarrow F_{AC} \times \frac{3}{5} = 90 \text{ N} \quad \dots (3)$$

$\Rightarrow \boxed{FAC \leq 150}$

$$30 \times 4 = FAD \sin 30$$

FAO = 240

$$F_{AB} = 1240 \cos 30^\circ$$

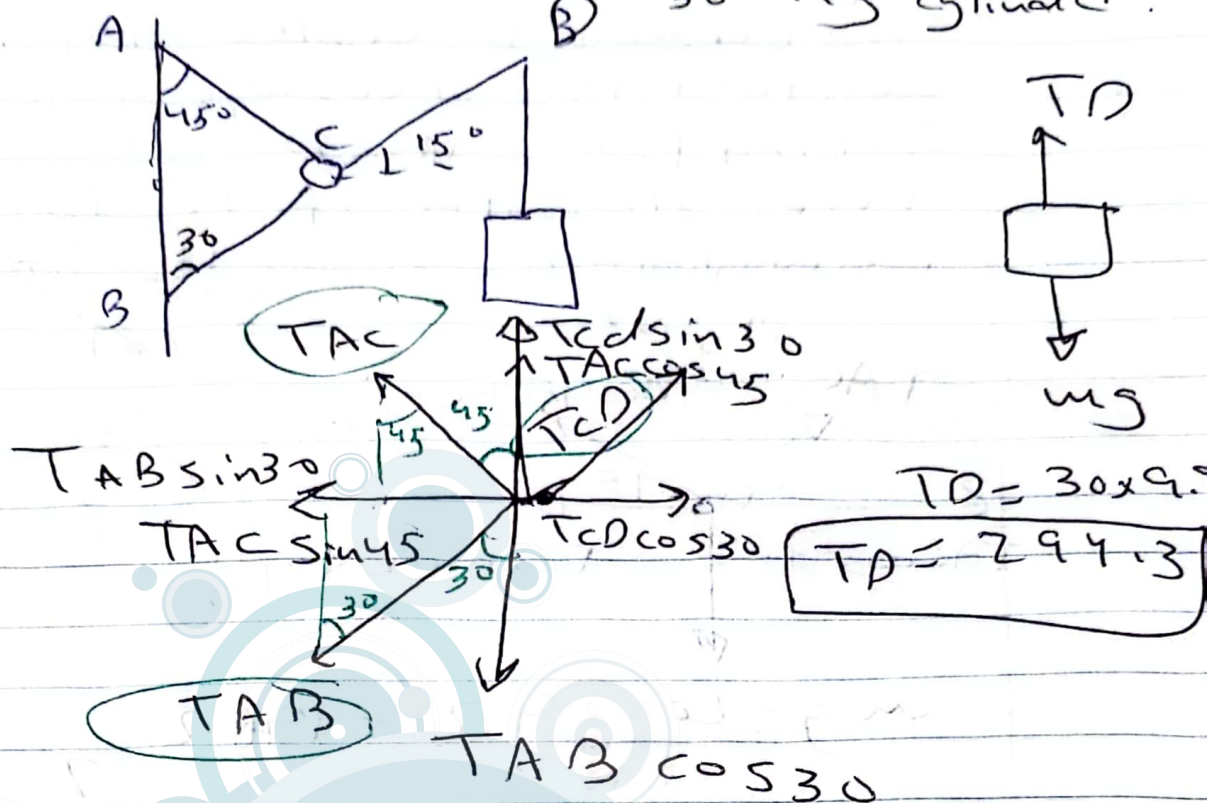
$$FA_3 = 207.84$$

$$\frac{F_A}{B} = K S$$

$$207.8455005$$

$$S = 0.415$$

Determine the tensions in cables A c and B c caused by the weight of B 30-kg glider.



$$\Rightarrow \sum F_x = 0 \Rightarrow T_{CD} \cos 30 = T_{AB} \sin 30 + T_{AC} \sin 45$$

$$254.87 = 0.5 T_{AB} + 0.7 T_{AC} \quad (1)$$

$$\Rightarrow \sum F_y = 0$$

$$T_{CD} \sin 30 + T_{AC} \cos 45 = T_{AB} \cos 30$$

$$147.15 + 0.7 T_{AC} = 0.866 T_{AB}$$

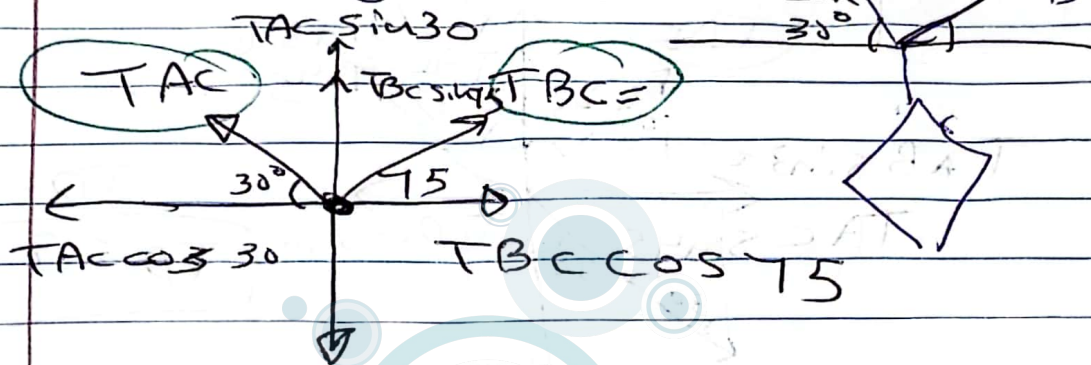
$$\Rightarrow 254.87 = 0.5 T_{AB} + 0.866 T_{AB} - 147.15$$

$$402.02 = 1.366 T_{AB}$$

$$T_{AB} = 294.30$$

$$T_{AC} = 153.88$$

Determine the Force P required to maintain the 200-kg engine in the position for which $\theta = 30^\circ$. The diameter of the pulley at B is negligible.



$$mg = 200 \times 9.81 = 1962$$

$$x = 1.033$$

$$\frac{1.033}{\sin 30^\circ} = \frac{2}{\sin \alpha}$$

$$\Rightarrow \alpha = 75^\circ$$

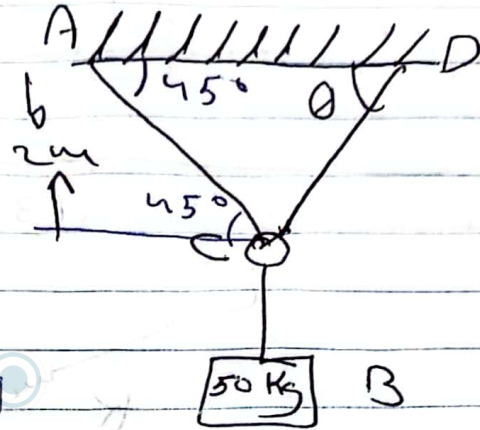
$$\sum F_y = 0 \Rightarrow TAC \sin 30^\circ = 1962$$

$$\Rightarrow TAC = 3924$$

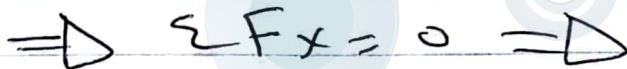
$$\Rightarrow \sum F_x = 0 \Rightarrow TBC \cos 75^\circ = TAC \cos 30^\circ$$

$$TBC = 13129.9$$

What length of cable CD for $\theta = 45^\circ$ →
The equilibrium and what the
tension force in cable CD.



$$\Rightarrow T_{BC} = 490.5$$



$$490.5 \cos 45 = T \cos \theta \dots (1)$$

$$\sum F_y = 0 = \Delta$$

$$TCD = \underline{143.66}$$

siner

$$\boxed{L = 346.83} \quad \cos \theta$$

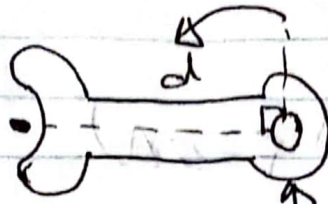
$$T_{CD} = 375.37$$

$$T_{CD} = \frac{173.66}{\sin \theta} = \frac{346.83}{\cos \theta}$$

$$\Rightarrow \tan \theta = \frac{143.66}{346.83}$$

Chapter 4:- Moment (الزم)

مقدار دوران القوى حول محور



$$M = F \cdot d$$

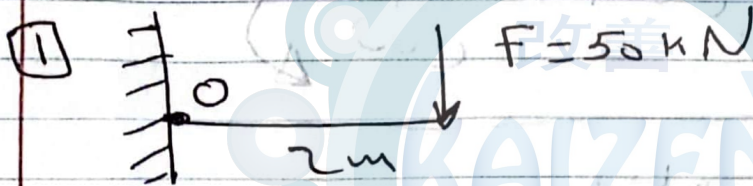
d: المسافة العمودية بين النقطة ونقطة

عمل القوة

عكس عقارب الساعة - ccw

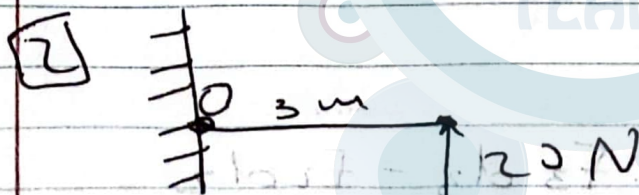
عكس عقارب الساعة + ccw

الأصابع باتجاه اليمين force
الكف باتجاه اليمين d



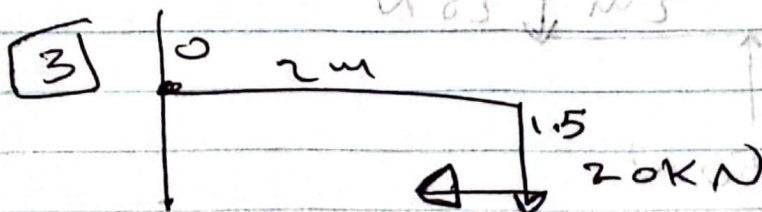
$$M = F \cdot d = 50 \times 2 = -100 \text{ kN} \cdot \text{m}$$

c.w



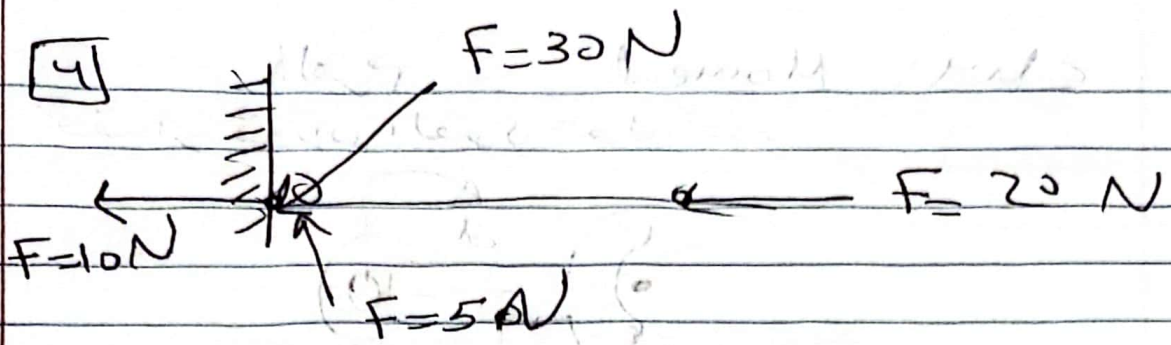
$$M = F \cdot d = 20 \cdot 3 = 60 \text{ N} \cdot \text{m}$$

c.c.w

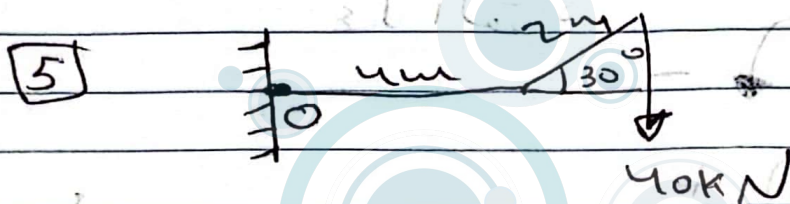


$$M = 20 \times 1.5 = 30 \text{ N} \cdot \text{m}$$

c.w



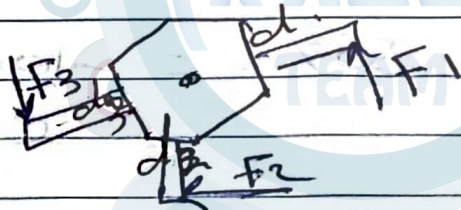
$$M_o = \text{zero}$$



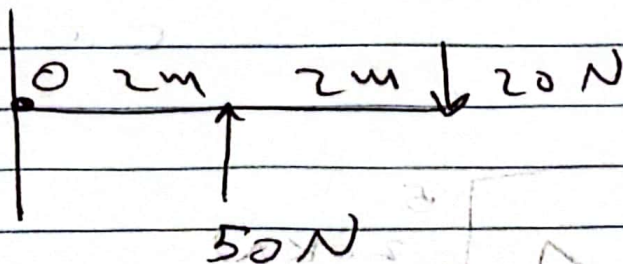
$$M = 40(4 + 2 \cos 30)$$

$$= 229 \text{ N}\cdot\text{m}$$

c.w



$$M_R = F_1 d_1 + F_3 d_3 - F_2 d_2$$

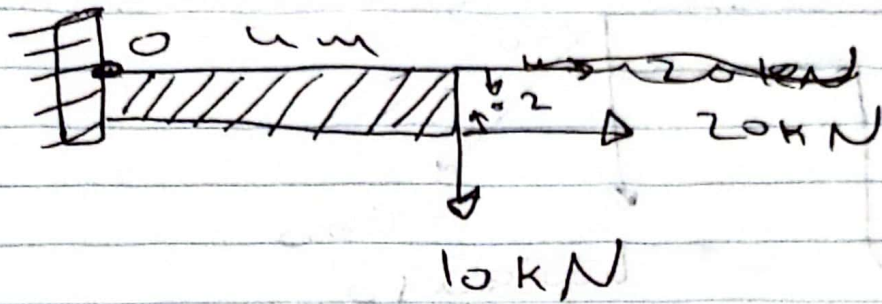


$$M_R = 50 \times 2 - 20 \times 4$$

$$= 100 - 80 = 20 \text{ N}\cdot\text{m}$$

c.w

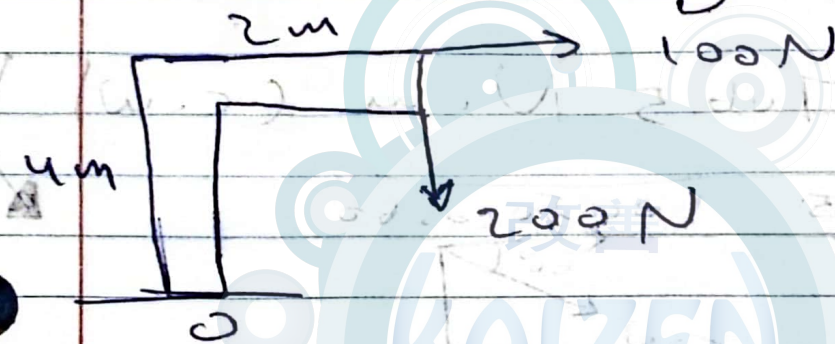
إذا كان صوابه الـ عزم (حالة)
الجواب يكون C.W



$$M_R = -10 \times 4 + 20 \times 0.2$$

$$= -40 + 4 = -36 \text{ N.m}$$

⇒ +36 C.W

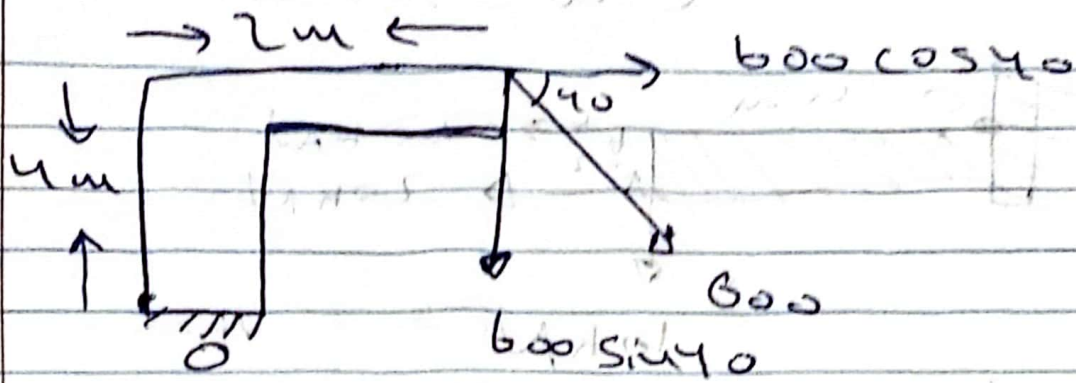


$$M_R = -200 \times 2 + -100 \times 4$$

$$= -400 - 400$$

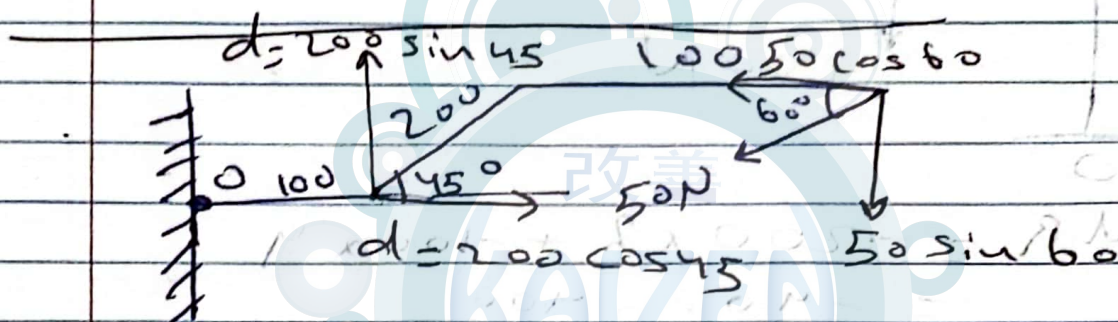
$$= -800 \text{ N.m}$$



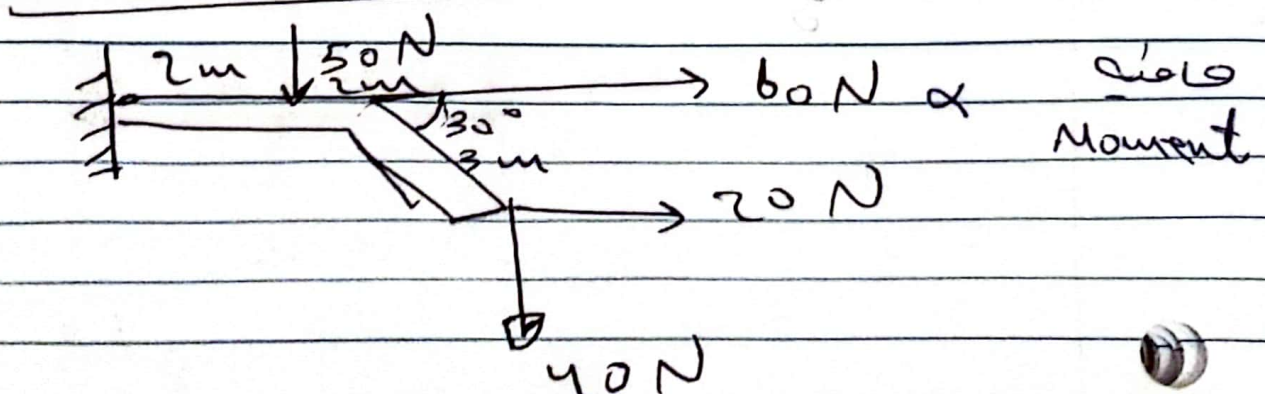


$$M_R = -600 \cos 40 \times 4 + 600 \sin 40 \times 2$$

$$= 706.5 \text{ N.m (c.w.)}$$

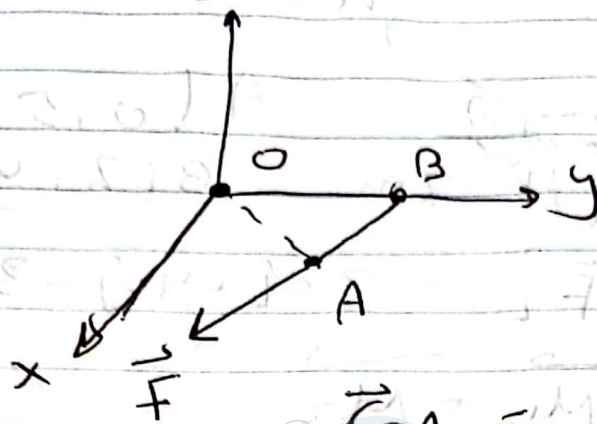


$$M_R = 50 \cos 60 \times 200 \sin 45 + 50 \sin 60 \times 200 \cos 45$$



$$= 20 \times 3 \sin 30 - 40 \times 2 \cos 30$$

Moment about point in 3d.



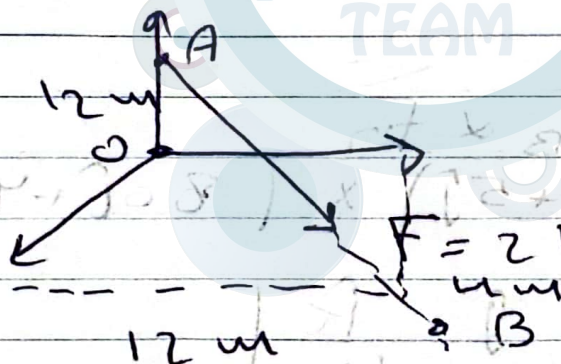
$$\vec{M}_O = \vec{r}_{OA} \times \vec{F} \quad \text{cross product.}$$

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

$$= \begin{vmatrix} x\hat{i} - y\hat{j} + z\hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= -16.5\hat{i} + 5.5\hat{j}$$

Determine the moment about Point O.



$$A(0, 0, 12)$$

$$B(4, 12, 0)$$

$$O(0, 0, 0)$$

$$F = 2 \text{ kN}$$

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

$$\vec{r}_{OB} = 4\hat{i} + 12\hat{j}$$

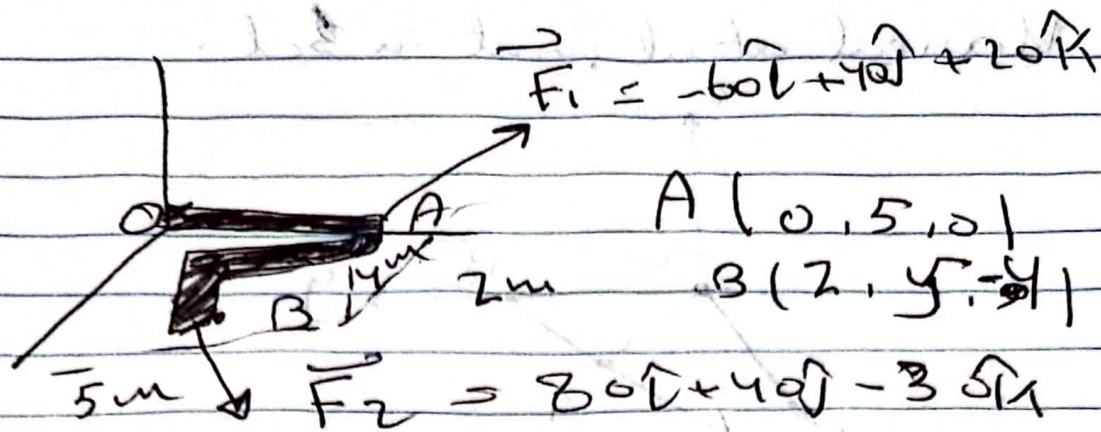
$$\vec{r}_{OA} = -12\hat{k}$$

$$\vec{F} = 2\hat{k}$$

$$= \begin{vmatrix} 4 & 12 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 2 \left(\frac{r_{AB}}{|r_{AB}|} \right)$$

$$0.458\hat{i} + 1.376\hat{j} - 1.376\hat{k} = \frac{2(4\hat{i} + 12\hat{j} - 12\hat{k})}{\sqrt{16 + 144 + 144}}$$



$$A(0, 5, 0)$$

$$B(2, 5, -4)$$

$$\vec{M}_R = \vec{M}_1 + \vec{M}_2$$

$$\vec{M}_1 = \vec{r}_{OA} \times \vec{F}_1$$

$$= 5\hat{j} \times [-60\hat{i} + 40\hat{j} + 20\hat{k}]$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix}$$

$$M_1 = 100\hat{i} + 300\hat{k}$$

$$\vec{M}_2 = \vec{r}_{OB} \times \vec{F}_2$$

$$= (2\hat{i} + 5\hat{j}) \times [80\hat{i} + 40\hat{j} - 30\hat{k}]$$

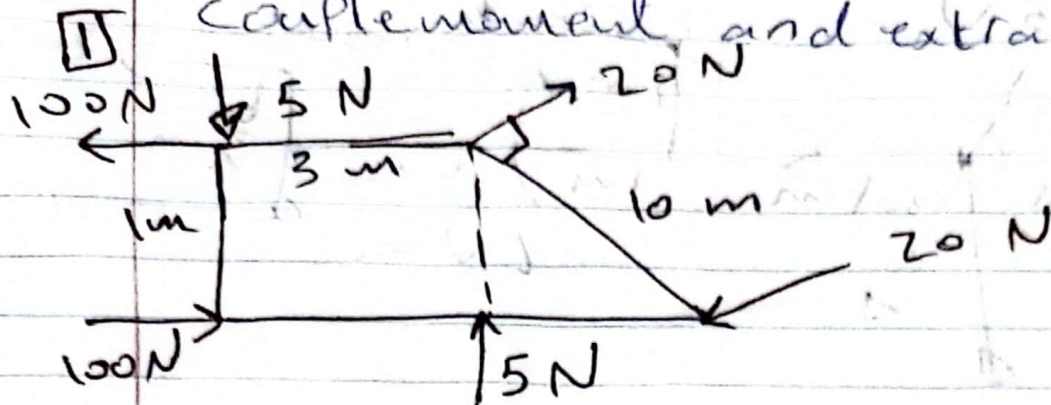
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -4 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= -150\hat{i} + 60\hat{j} + 180\hat{k}$$

$$= -70\hat{i} - 40\hat{j} - 240\hat{k}$$

حساب مومنت و اقتران مومنت

Couple moment and extra examples.

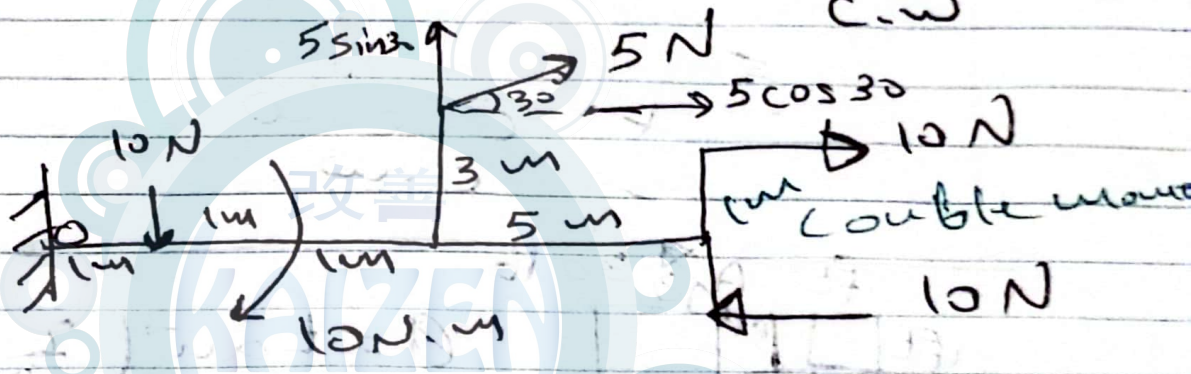


$$M = (100 \times 1) + (5 \times 3) + - (20 \times 10)$$

$$= 100 + 15 - 200 = -85 \text{ N.m}$$

C.W

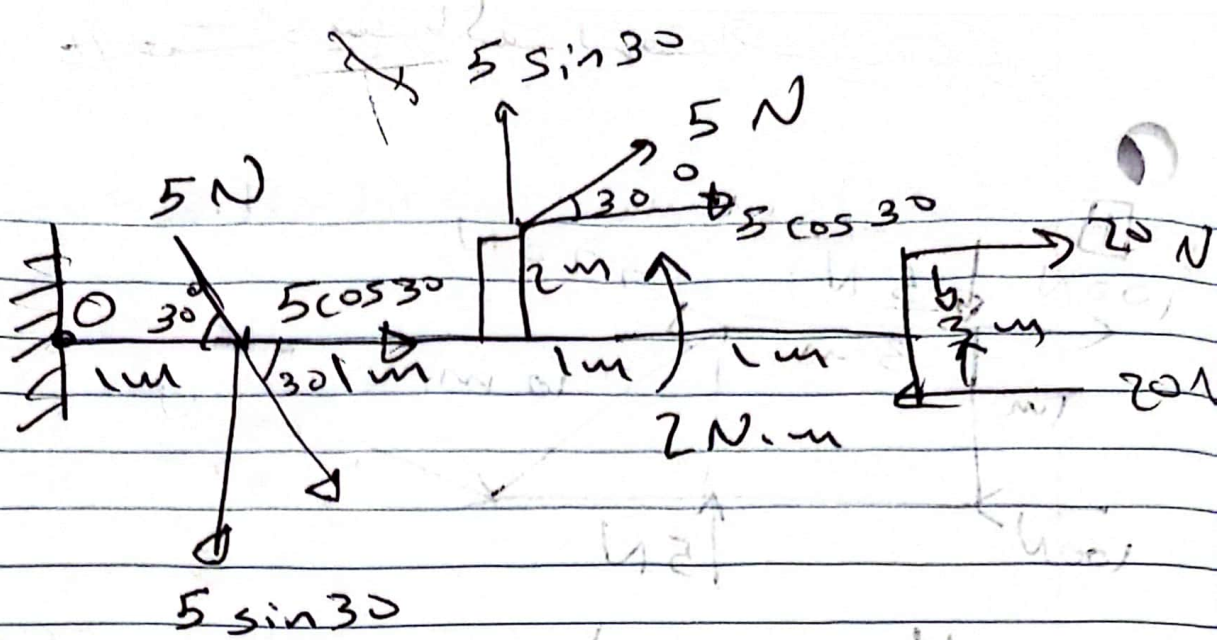
2



$$M = - (10 \times 1) - (10 \times 3) - (5 \cos 30 \times 3) + (5 \sin 30 \times 3)$$

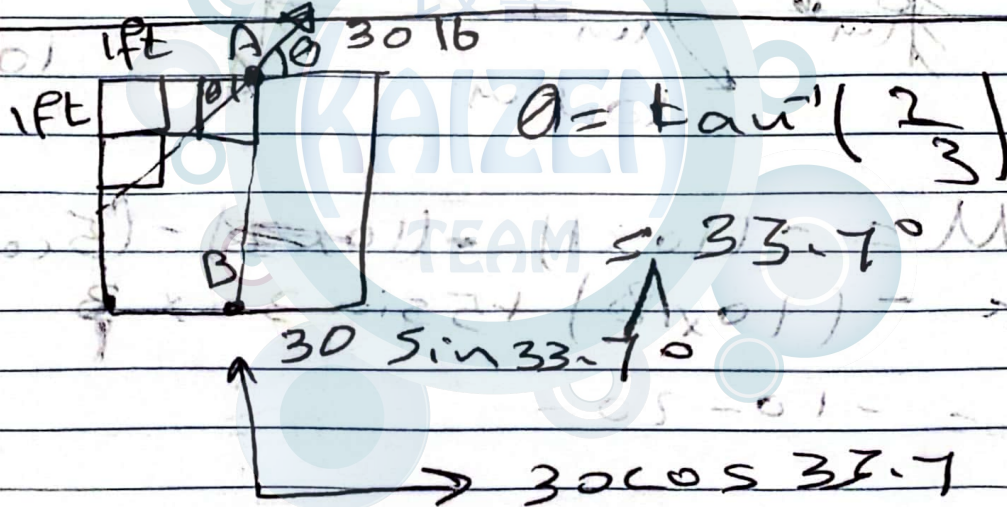
$$= -10 - 30 - 15 + 7.5 = -47.5 \text{ N.m}$$

مومنت و اقتران مومنت

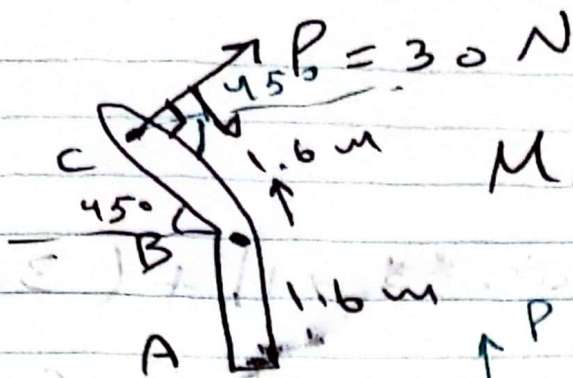


$$\sum M = 0$$

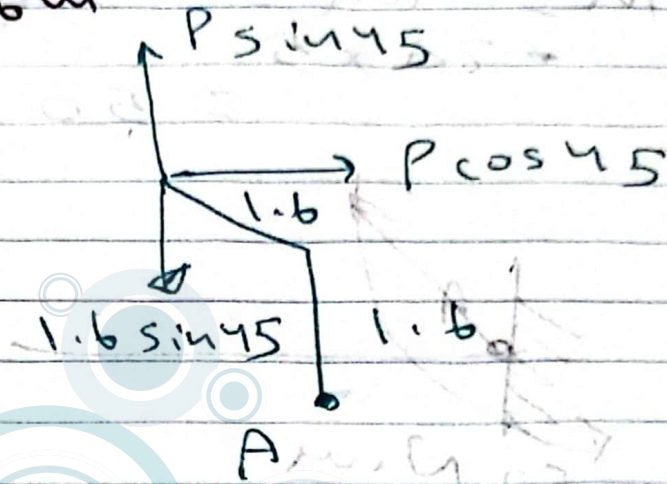
$$-5 \sin 30^\circ \times 1 - 5 \cos 30^\circ \times 2 + 2 \times 2 - 20 \times 3 = 0$$



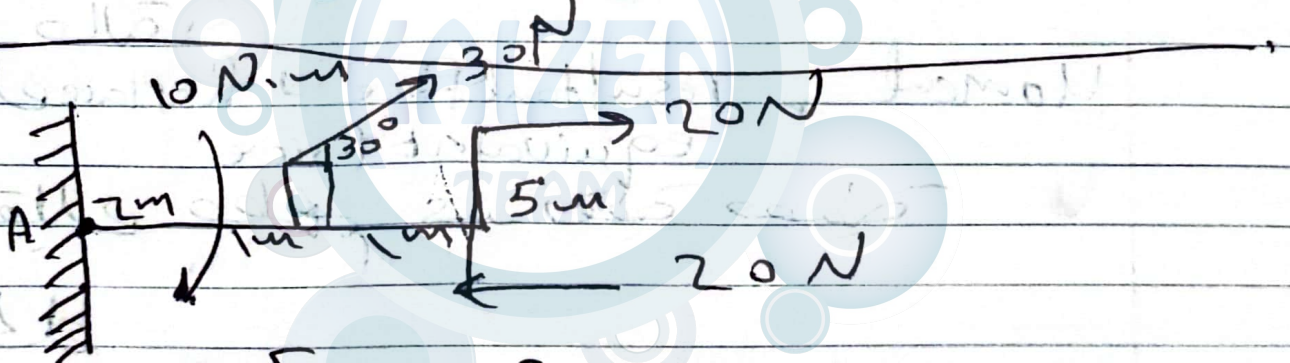
$$M_c = 30 \cos 33.7^\circ \times 4 = 99.8 \text{ N}\cdot\text{m}$$



$$M = 30 \times 1.6 = \text{C.W}$$



$$M = -30 \cos 45 \times (1.6 \times 1.6 \sin 45) = 81.9 \text{ Nm C.W}$$



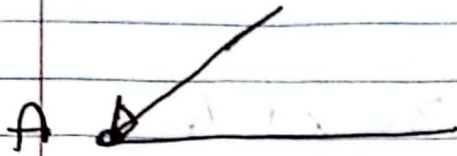
$$\sum F_x = 30 \cos 30 = 25.98$$

$$\sum F_y = 30 \sin 30 = 15 \text{ N}$$

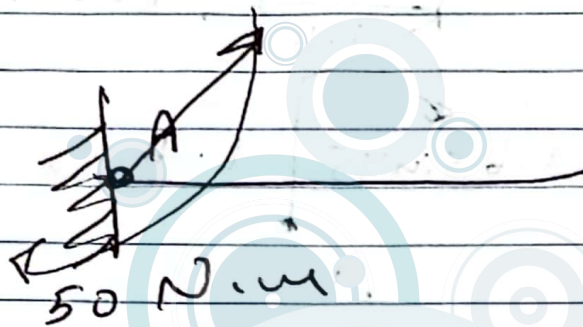
$$|F| = \sqrt{15^2 + 25.98^2}$$

$$\tan \theta = 0.577$$

$$\theta = \tan^{-1} \left(\frac{15}{25.98} \right) = 29.98$$



$$M = -10(30 \cos 30^\circ \times 1) + (30 \sin 30^\circ \times 3) = -20 \times 5 = 50 \text{ N.m (C.W)}$$

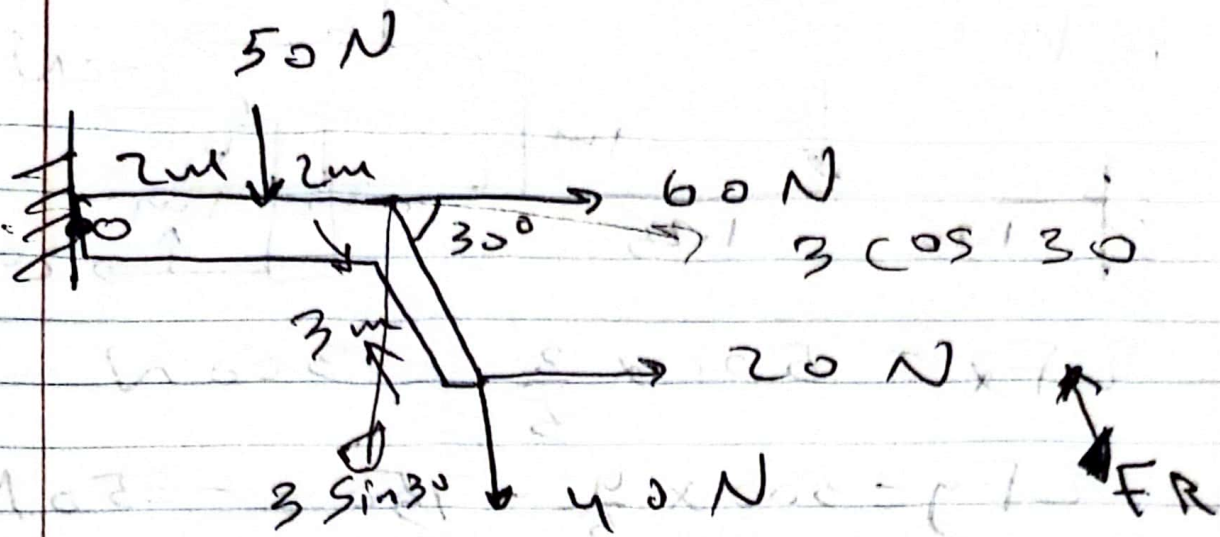


الآن القوة الأهم مفعولة في نقطة
لنقلها * Replace *

أول حالة -
نقلها Replace Resultant Force
equivalent force
ونقلها بطلب من نقطة معينة

الحالة -
أولاً $\sum F_x$
ثانياً $\sum F_y$
المagnitude $|F| = \sqrt{F_x^2 + F_y^2}$
ولا نعلم كونه عارف (الزاوية)

أولاً زاوية ال resultant مع محور x
ثانياً ال moment



$$\sum F_x = 60 + 20 = 80 \text{ N}$$

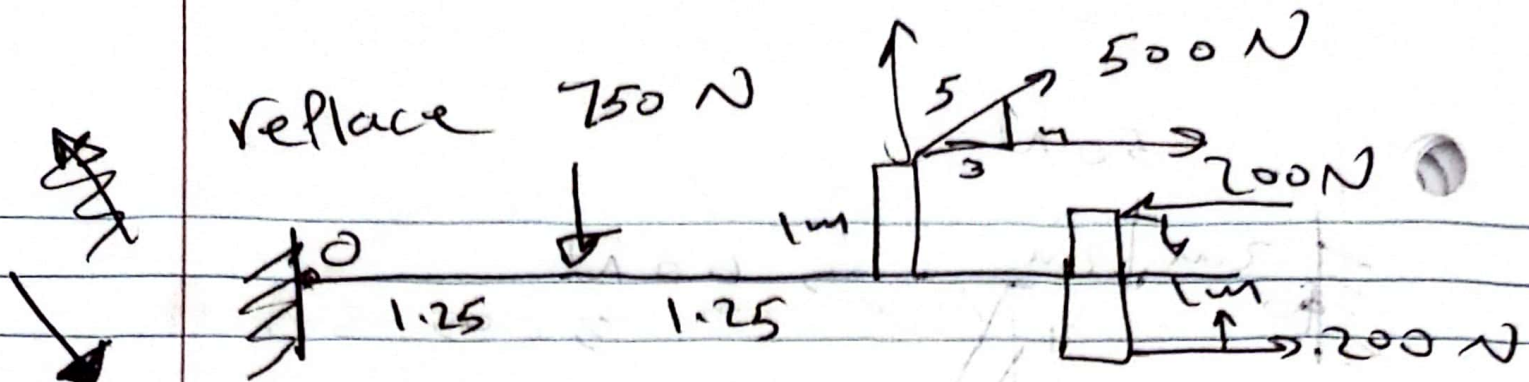
$$\sum F_y = -50 - 40 = -90 \text{ N}$$

$$|F| = \sqrt{80^2 + 90^2} = 120.41 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{90}{80}\right) = 48.3^\circ$$

$$\begin{aligned} M &= -(50 \times 2) - \cancel{60 \times 3} \\ &\quad + (20 \times 3 \sin 30^\circ) - (40 \times (4 + 3 \cos 30^\circ)) \\ &= -334 \text{ N}\cdot\text{m} \quad (\text{c.w}) \end{aligned}$$

$$\frac{M}{L} = \frac{-334}{7} \quad \bigg| \quad \frac{M}{L} = x$$



FR

$$\sum F_x = 500 \times \frac{3}{5} = 300 \text{ N}$$

$$\sum F_y = 500 \times \frac{4}{5} - 750 = -350 \text{ N}$$

$$F_R = \sqrt{300^2 + 350^2} = 460 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{350}{300} \right) = 49.4$$

$$M_o = -750 \times 1.25 - 500 \times \frac{3}{5} \times 1 + 500 \times \frac{4}{5} \times 2.5 + 200 \times 1 = 37.5 \text{ N}\cdot\text{m (c.w.)}$$

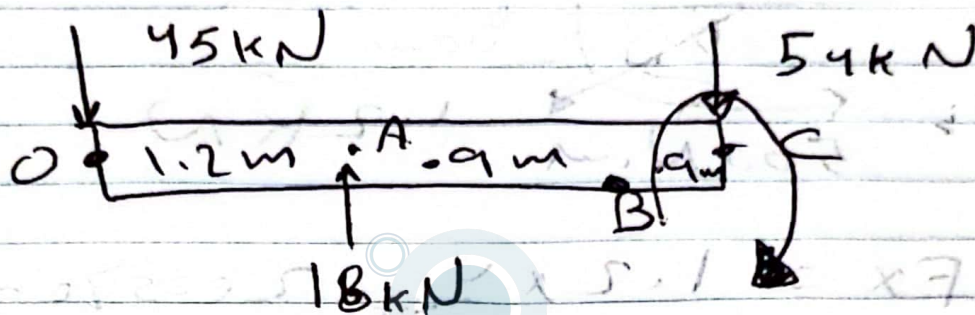
x-intercept (y, x) calcd
y-intercept

$$x = \frac{Mr}{F_y} \quad / \quad y = \frac{Mr}{F_x}$$

replace

← equivalent force

moment / Resultant في تلك النقطة #
zero = moment في نقطة (pass point)



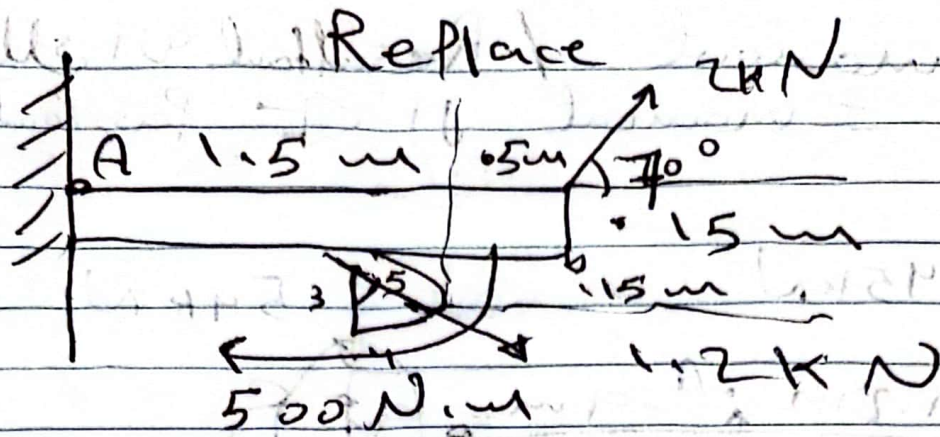
$$\sum M_B = 0 \Rightarrow -54 \times 9 - 18 \times 9 + 45 \times 2.1 =$$

$$M = 19 \text{ kN}\cdot\text{m}$$

$$\sum F_x = 0 \Rightarrow$$

$$\sum F_y = -45 + 18 - 54 = -81 \text{ kN}$$

$$|F| = \sqrt{81} = 9 \text{ kN}$$



$$\sum F_x = 1.2 \times \frac{4}{5} + 2 \cos 70^\circ =$$

$$\sum F_y = 2 \sin 70^\circ - 1.2 \times \frac{3}{5}$$

$$FR = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

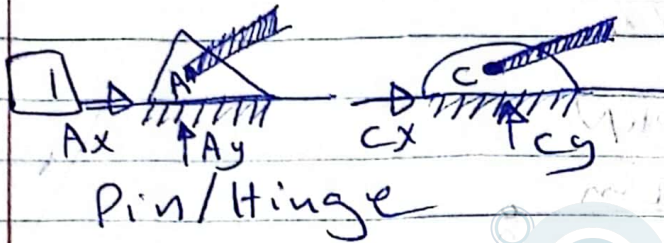
$$= \tan^{-1} \left(\frac{1.7}{1.7} \right) = 45^\circ$$

$$WRP TEAM 6 = 1.71$$

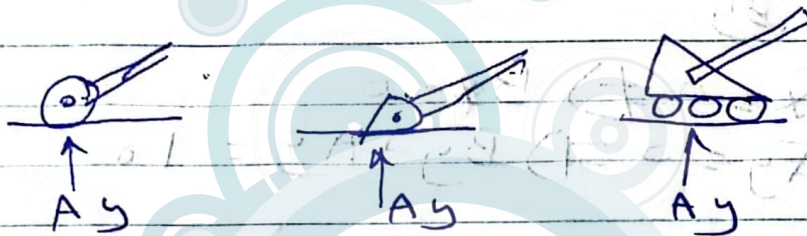
CH5 :- Equilibrium of a rigid bodies.

* Support *

وسيلة ربط على نظام ما
للتحقق وضع الاتزان



[2] Roller / Rocker.



[3] Fixed.

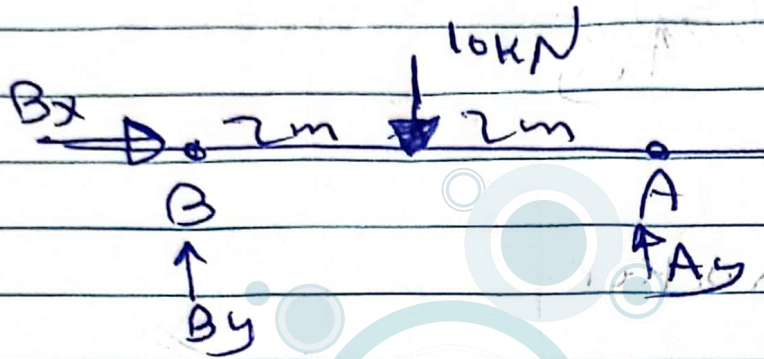
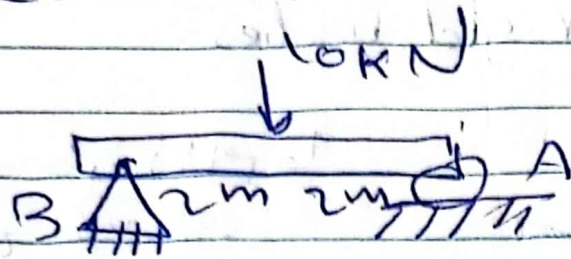
قفل



خطوات الحل :-
F.B.D [1]
← بدل ال Support بال Reaction
← هذه القوة (القوة) ان وجدت
← Distributed load
← concentrated load

[2] طبق معادلات الاتزان
 $\sum F_x = 0, \sum F_y = 0, \sum M_p = 0$

example 57



$$\sum F_x = 0 \Rightarrow B_x = 0$$

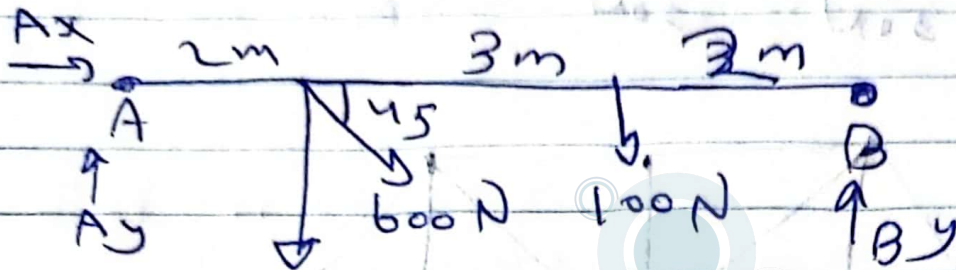
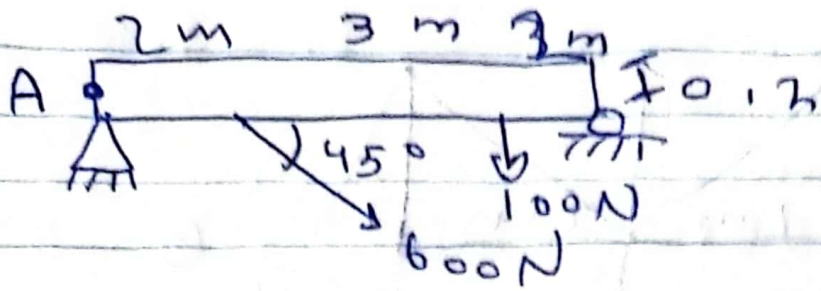
$$\sum F_y = 0 \Rightarrow B_y + A_y = 10$$

$$\sum M_B = 0 \Rightarrow$$

$$-10 \times 2 + A_y \times 4 = 0$$

$$\cancel{3} \cancel{5kN} \Rightarrow A_y = 5kN$$

$$B_y = 5kN$$



$$600 \sin 45 = 424.26 \text{ N}$$

$$\sum F_x = 0 = D$$

$$424.26 + A_x = 0$$

$$A_x = -424.26 \text{ N} \quad \leftarrow$$

$$\sum F_y = 0 \Rightarrow A_y + B_y = 100 + 424.26$$

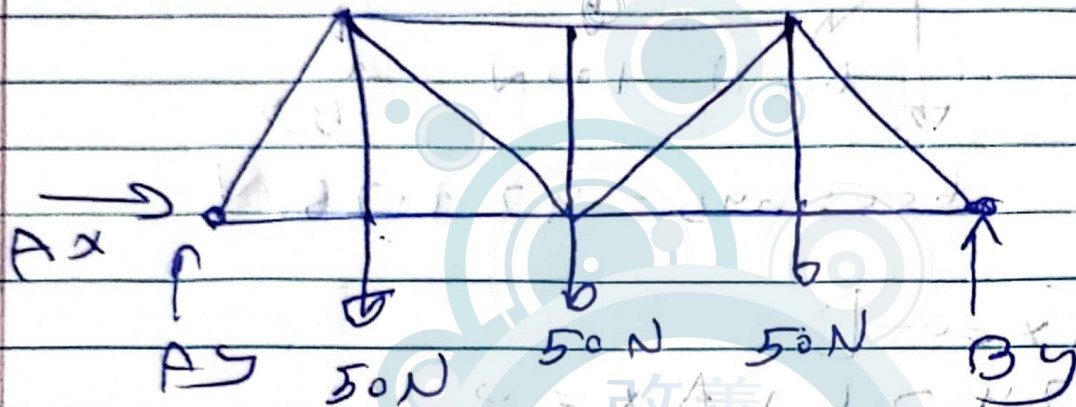
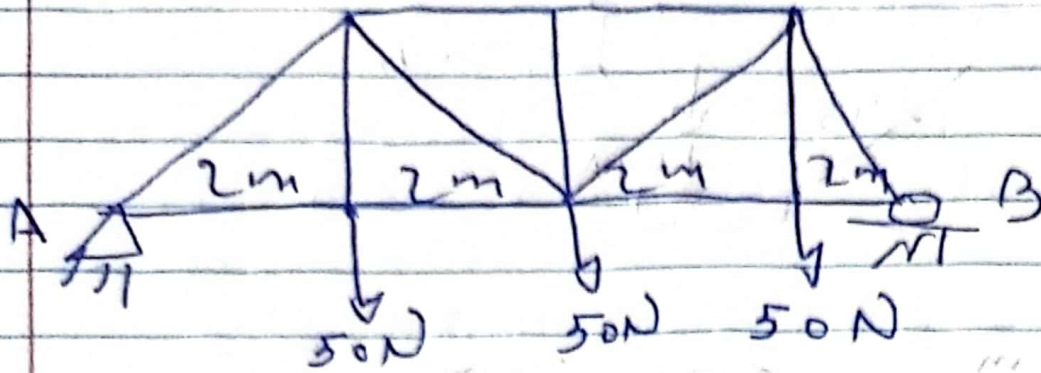
$$A_y + B_y = 524.26 \text{ N}$$

$$\sum M_A = -424.26 \times 2 - 100 \times 5 + B_y \times 8 = 0$$

$$B_y = \frac{449.5 \text{ N}}{204 \text{ N}}$$

$$A_y = 524.26 - 204.5$$

$$319.76 \text{ N}$$



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow$$

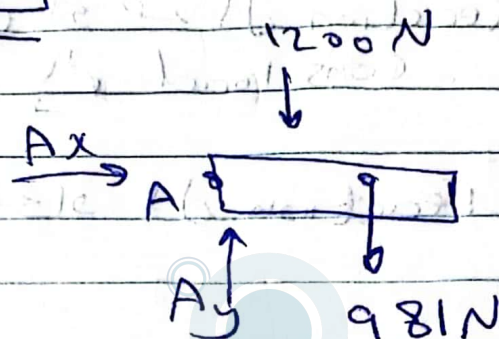
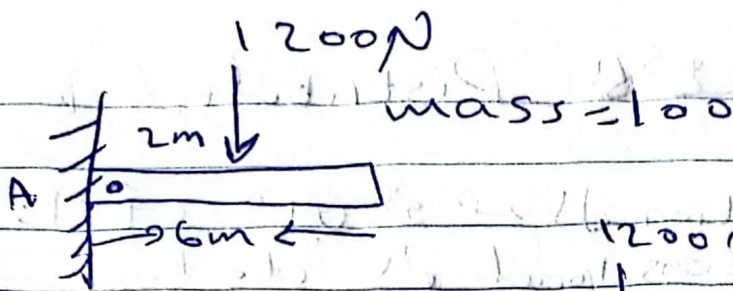
$$A_y + B_y = 150$$

$$\sum M_A = -50 \times 2 - 50 \times 4 - 50 \times 6 + B_y \times 8 = 0$$

$$8B_y = 100 + 200 + 300$$

$$B_y = \frac{600}{8} = 75 \text{ N}$$

$$\boxed{A_y = 75 \text{ N}}$$



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y = 1200 + 981 = 2181 \text{ N}$$

$$\sum M_A = 0 \Rightarrow -1200 \times 2 - 981 \times 3 = 0$$

$$M_A = 5343 \text{ N}\cdot\text{m}$$



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y = 2181 \text{ N}$$

$$\sum M_A = 0 \Rightarrow -1200 \times 2 - 981 \times 3 + M_A = 0$$

$$M_A = 5343 \text{ N}\cdot\text{m}$$

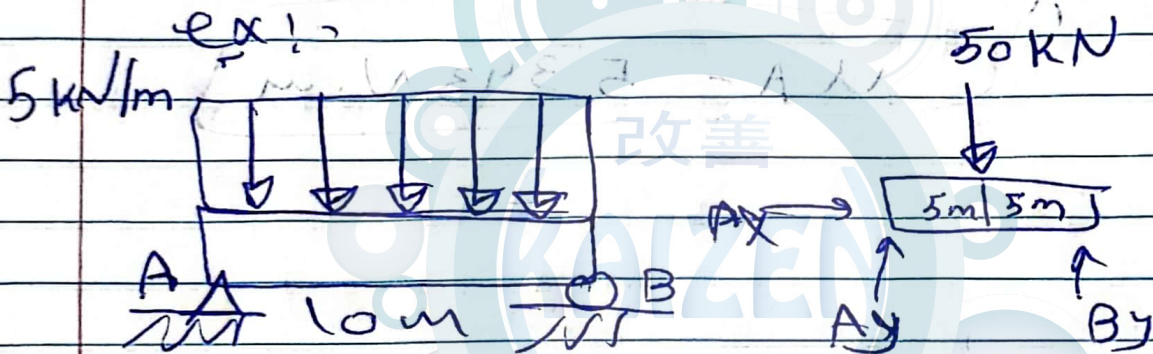
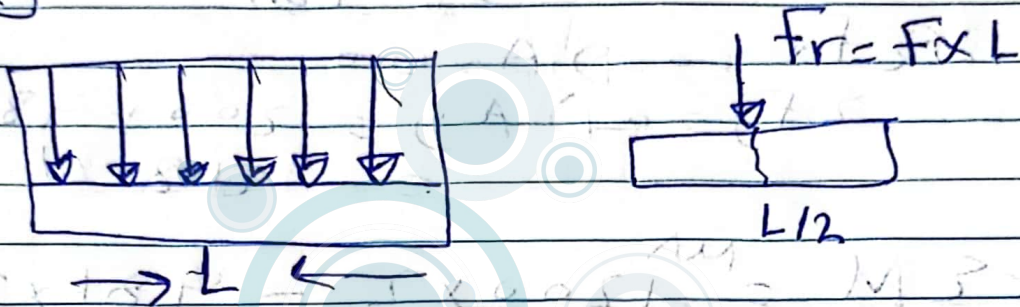
$$M_A = 5343 \text{ N}\cdot\text{m}$$

توزيع الحمل Distributed load

Reaction of the beam to the distributed load
 Reaction of the beam to the distributed load

Reaction of the beam to the distributed load

①



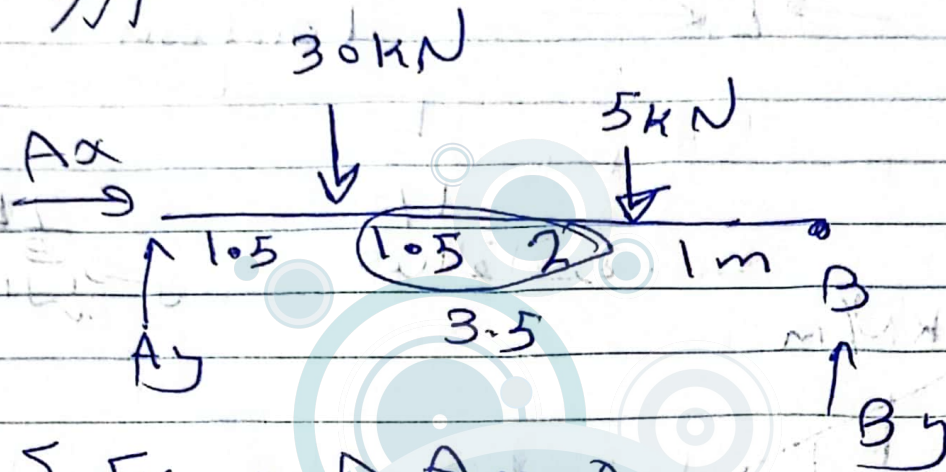
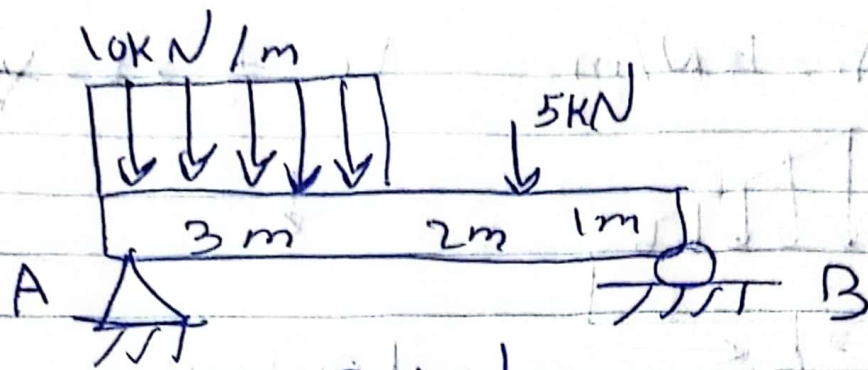
$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$A_y + B_y = 50$$

$$\sum M_A = 0 \Rightarrow -50 \times 5 + B_y \times 10 = 0$$

$$B_y = 25\text{ N}$$

$$A_y = 25\text{ N}$$



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow$$

$$A_y + B_y = 35$$

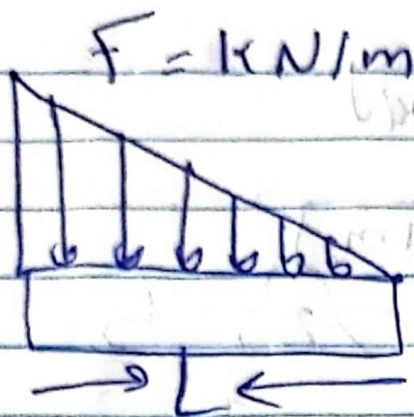
$$\sum M_A = 0 \Rightarrow$$

$$-30 \times 1.5 + 5 \times 5 + B_y \times 6 = 0$$

$$B_y = 11.66 \text{ N}$$

$$A_y = 23.3 \text{ N}$$

①



$$F_R = \frac{1}{2} \times L \times F$$



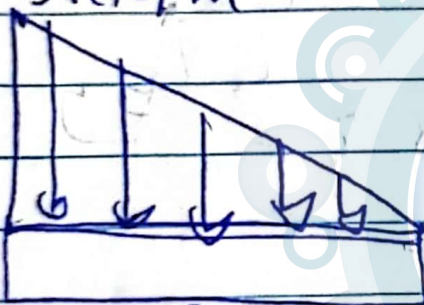
Location



$$L/3$$

$$\frac{2L}{3}$$

3 kN/m



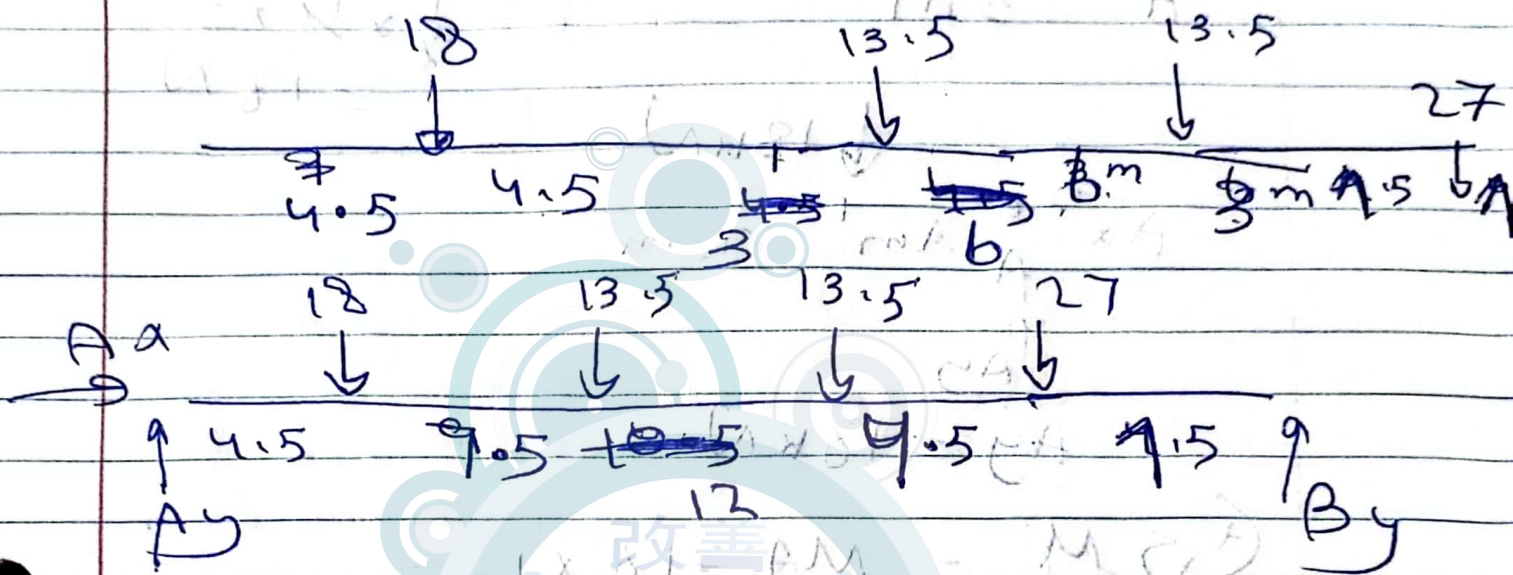
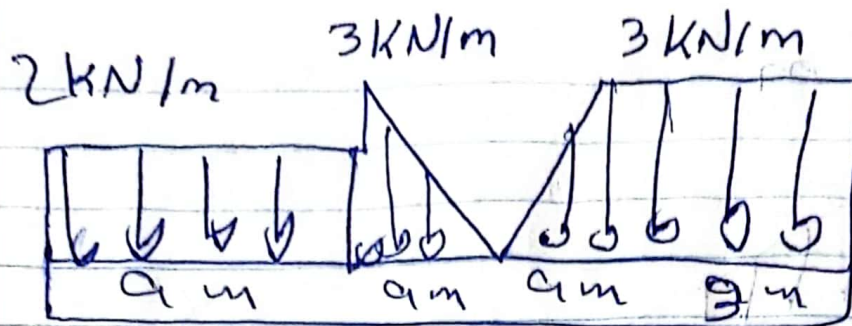
9m

$$F = \frac{27}{2} \text{ kN} = 13.5 \text{ kN}$$

$$X = \frac{1}{3} \times 9 = 3$$

3m 6m

$$F_R = \frac{27}{2} \text{ kN} = 13.5 \text{ kN}$$



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + B_y = 72$$

$$A_y + B_y = 72$$

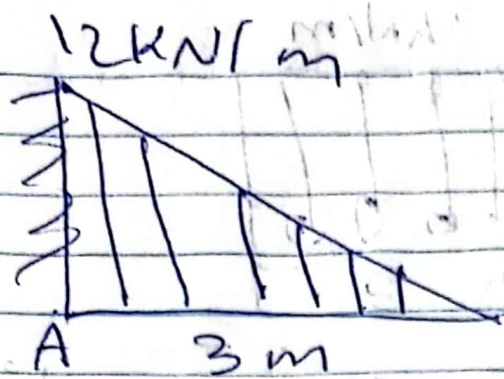
$$\sum M_A = 0$$

$$-18 \times 4.5 - 13.5 \times 13.5 - 13.5 \times 22.5 - 27 \times 31.5 + B_y \times 36 = 0$$

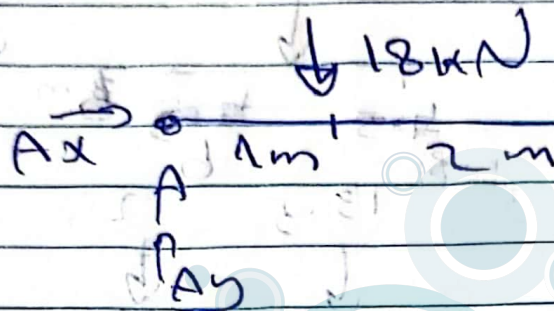
$$27 B_y = 1246.5$$

$$B_y = 46.16 \text{ kN}$$

$$A_y = 25.83 \text{ kN}$$



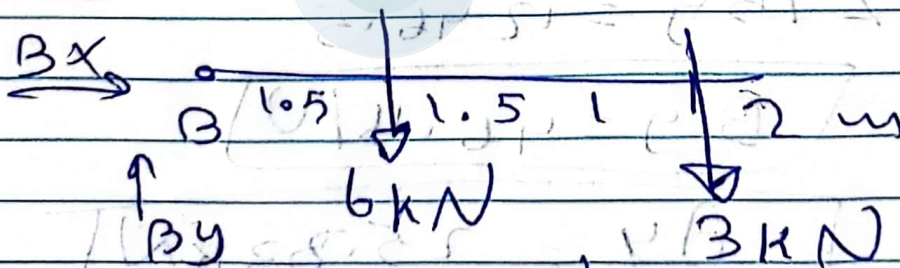
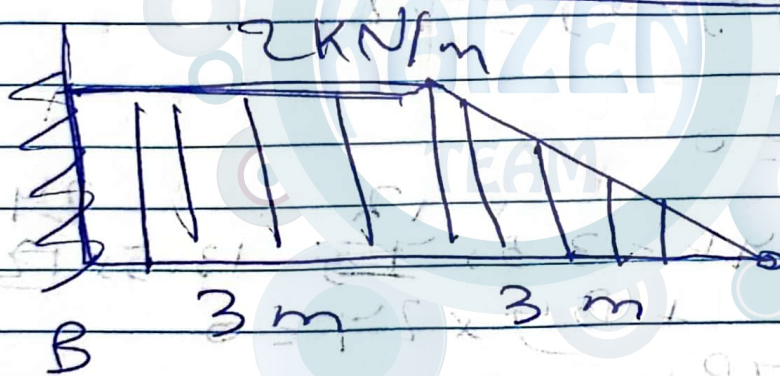
$$F = \frac{1}{2} \times 12 \times 3 = 18 \text{ N}$$



$$A_y = 18 \text{ kN}$$

$$\sum M_A = M_A - 18 \times 1 = 0$$

$$M_A = 18 \text{ N.m}$$

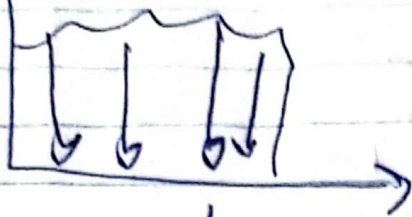


$$B_y = 9 \text{ kN}$$

$$\sum M_B = 0 \Rightarrow M_B - 6 \times 1.5 - 3 \times 4 = 0$$

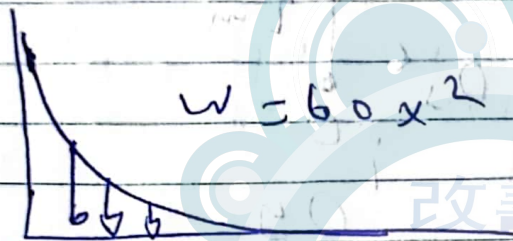
$$M_B = 21 \text{ N.m}$$

3 function load:



$$F_r = \int_0^L F_x \cdot dx$$

$$\text{Location} = \frac{\int F_x \cdot x \, dx}{F_r}$$

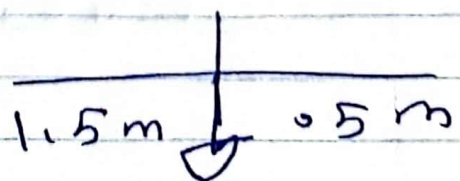
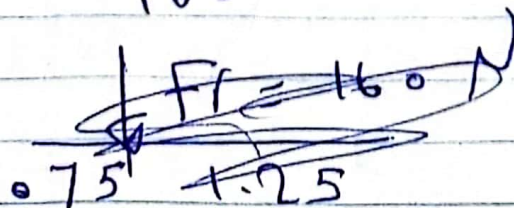


$$F_r = \int_0^2 60x^2 \cdot dx$$

$$= \frac{60x^3}{3} \Big|_0^2 = 20 \times 8 = 160 \text{ N}$$

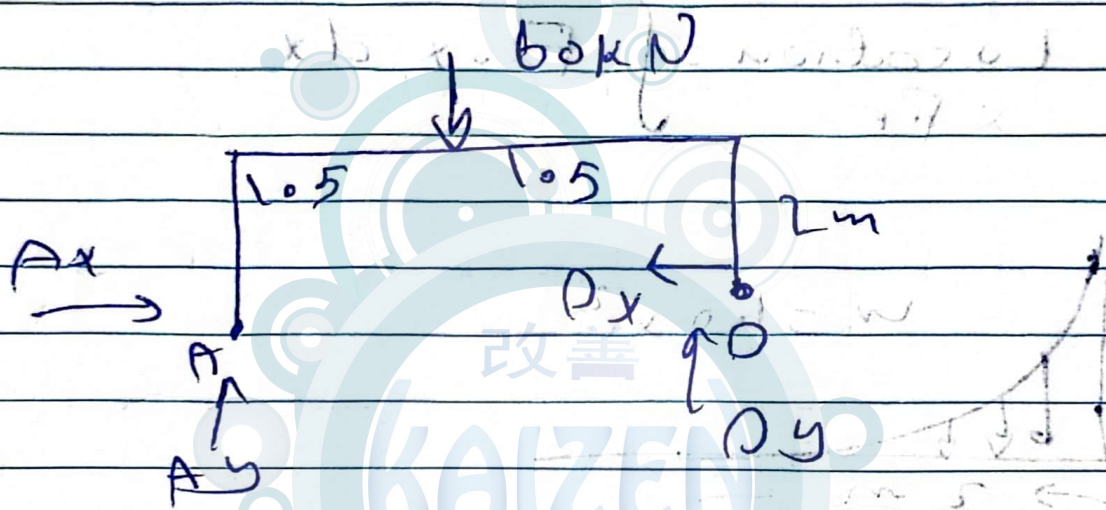
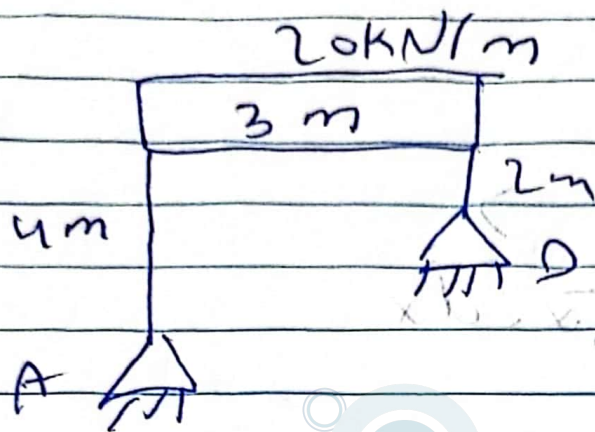
$$\int_0^2 60x^3 \cdot dx = \frac{60x^4}{4} \Big|_0^2 = 15 \times 16 = 240$$

$$= \frac{240}{160} = 1.5$$



$$F_r = 160 \text{ N}$$

Frames analysis

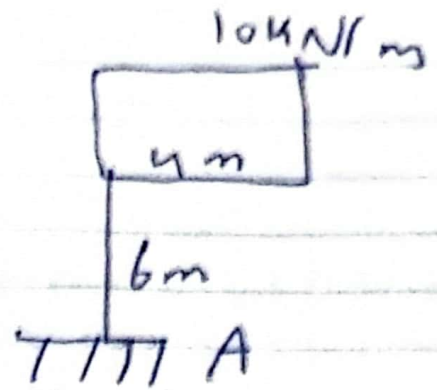
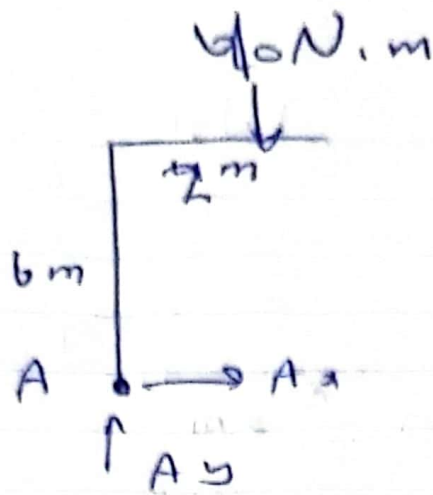


$$\sum F_x = 0 \Rightarrow A_x = D_x$$

$$\sum F_y = 0 \Rightarrow D_y + A_y = 60$$

$$\sum M_A = -60 \times 1.5 + D_y \times 3 = 0$$

$$D_y = 30 \text{ kN}$$

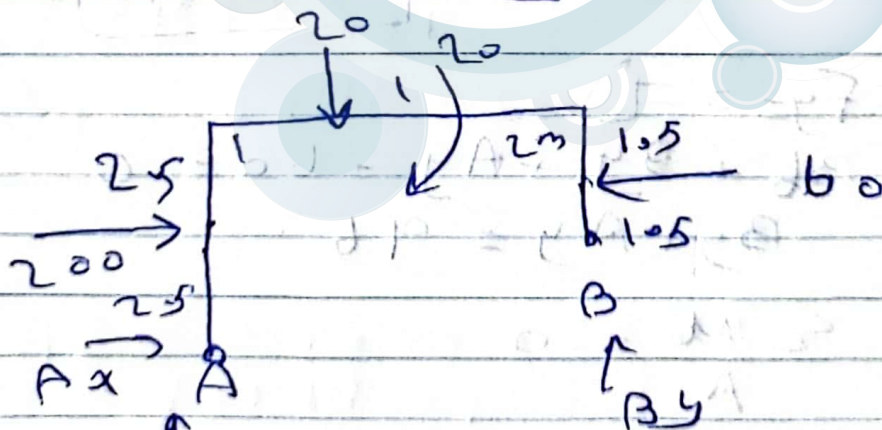
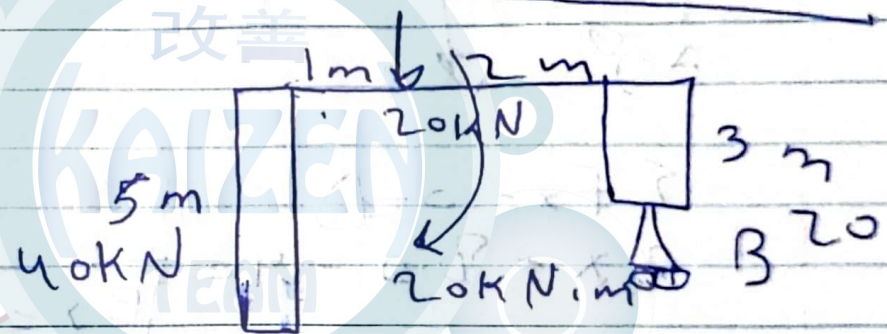


$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y = 40 \text{ N}$$

$$\sum M_A = M_A - 40 \times 2 = 0$$

$$M_A = 80 \text{ N}\cdot\text{m}$$

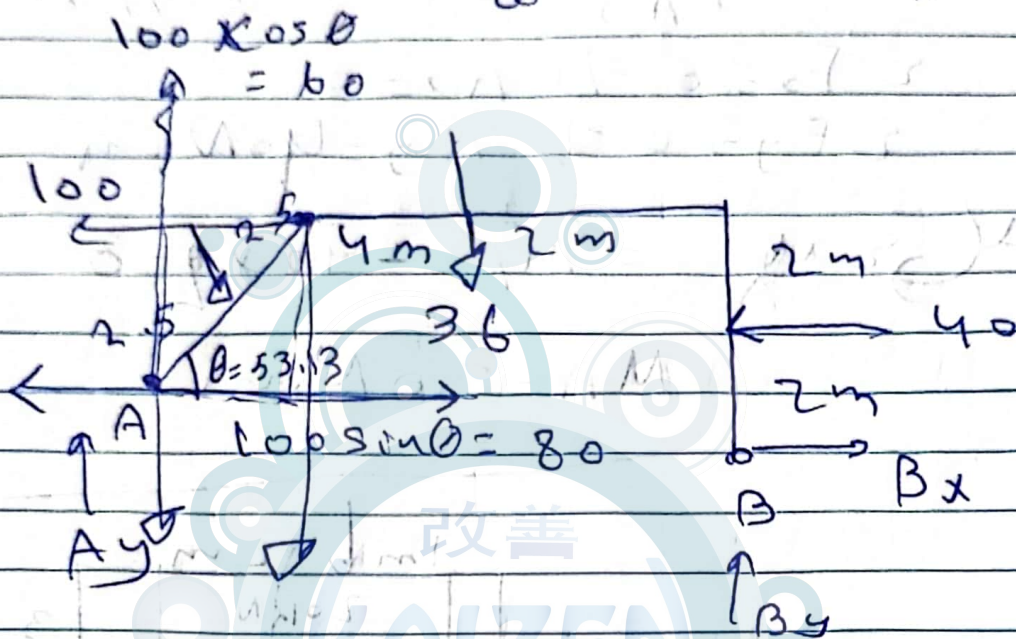
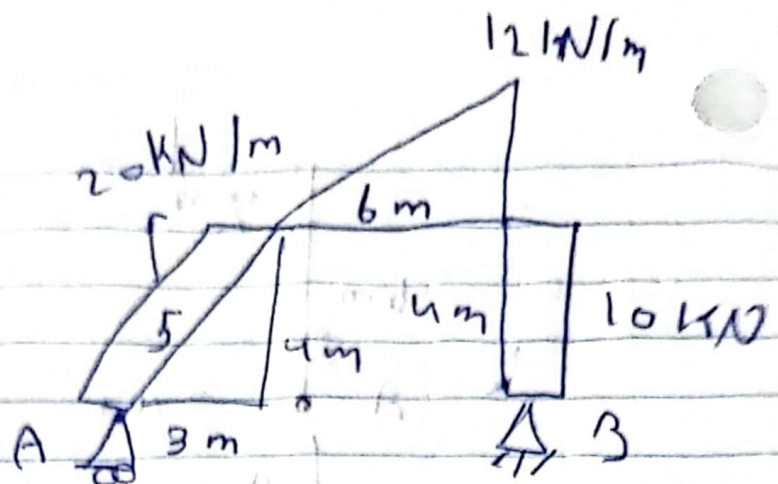


$$\sum F_x = 0 \Rightarrow A_x + 200 = 60$$

$$A_x = 140 \text{ N}$$

$$\sum M_A = -20 \times 1 - 20 + 60(3.5)$$

$$11.5 - 200(2.5) = B_y$$



$$\sum F_x = 0 \Rightarrow$$

$$B_x - 36 - 40 = 0 \quad \boxed{B_x = 40 \text{ N}}$$

$$\sum F_y = 0 \Rightarrow$$

$$-36 + B_y + A_y + 60 = 0$$

$$B_y + A_y = 96$$

$$\sum M = 0$$

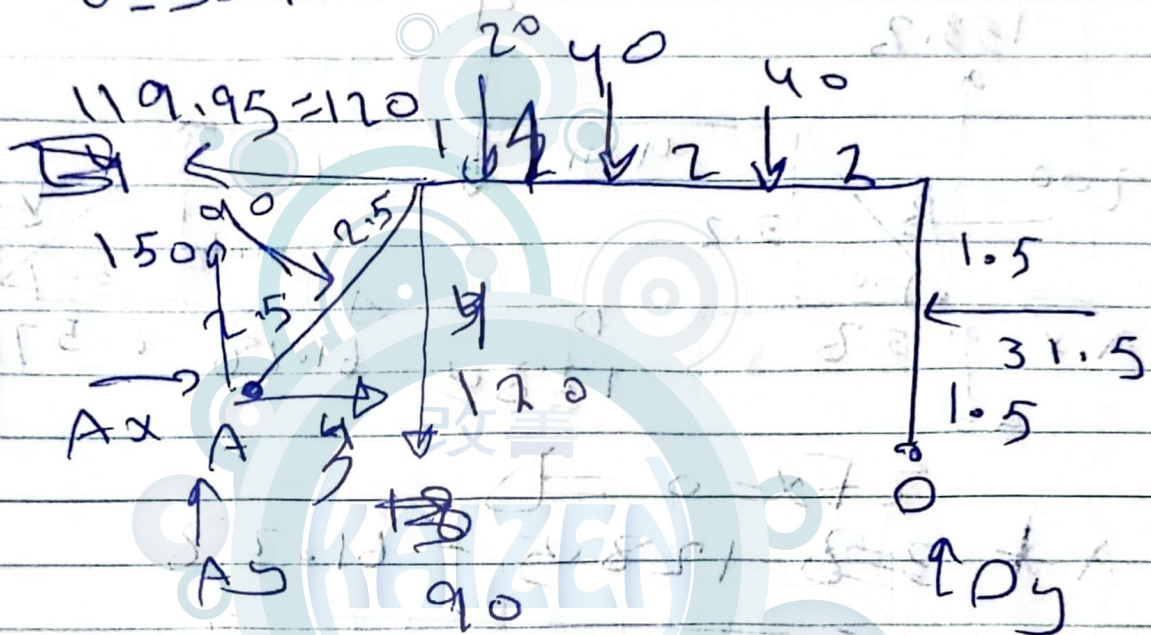
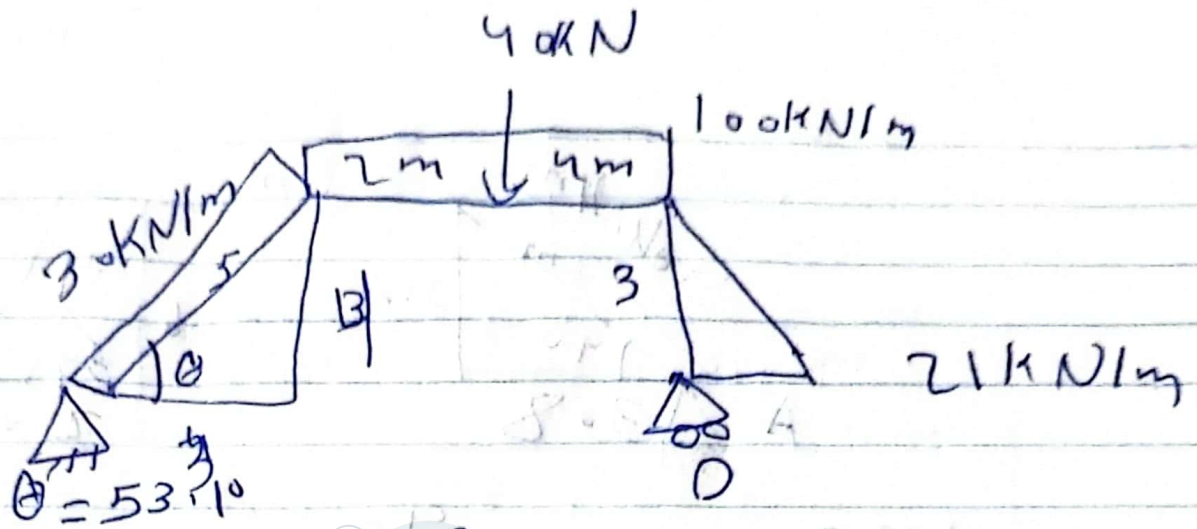
A

~~80 kN~~

$$36(7) - 40(2) + 100(2.5) = 9B_y$$

$$B_y = 46.88$$

$$A_y = 49.11$$

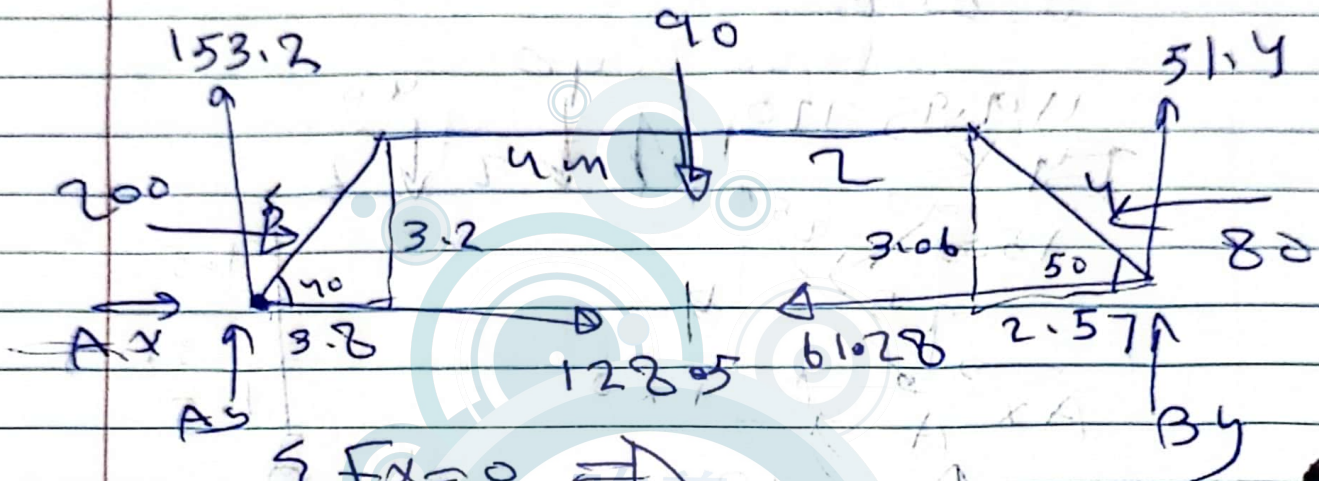
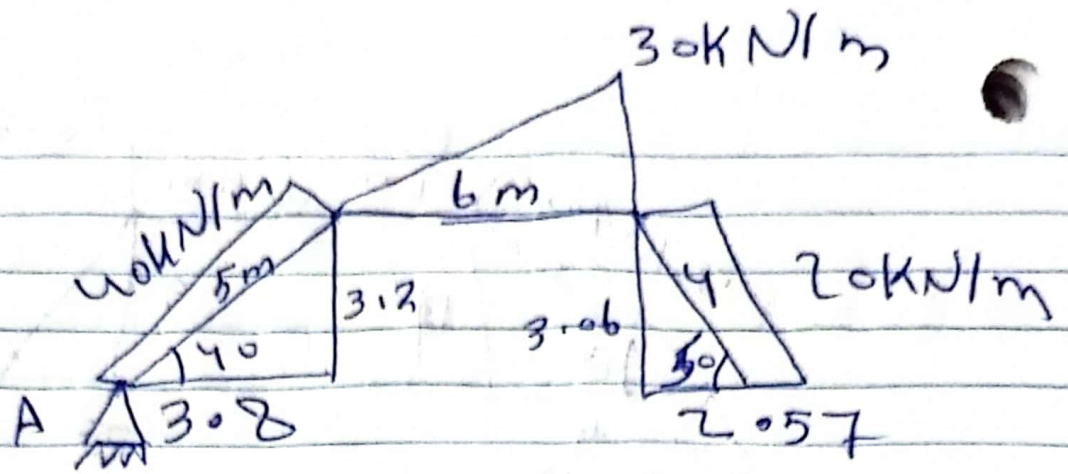


$$\sum F_x = 0 \Rightarrow Ax = 88.5 \Rightarrow 120 - 31.5 = 88.5$$

$$\sum M_A = 0 \Rightarrow -150(2.5) + 20 \times 4 + 40 \times 5 + 40 \times 7 - 31.5 \times 2 = Dy \times 9$$

$$Dy = 96.88$$

$$Ay =$$



$$\sum F_x = 0 \Rightarrow$$

$$A_x - 3.8 \times 128.5 = 61.28$$

$$\Rightarrow \boxed{A_x = 67.22 \text{ N}}$$

$$\sum F_y = 0 \Rightarrow$$

$$A_y \times 153.2 - 90 + 51.4 \times B_y = 0$$

$$A_y + B_y = 114.6$$

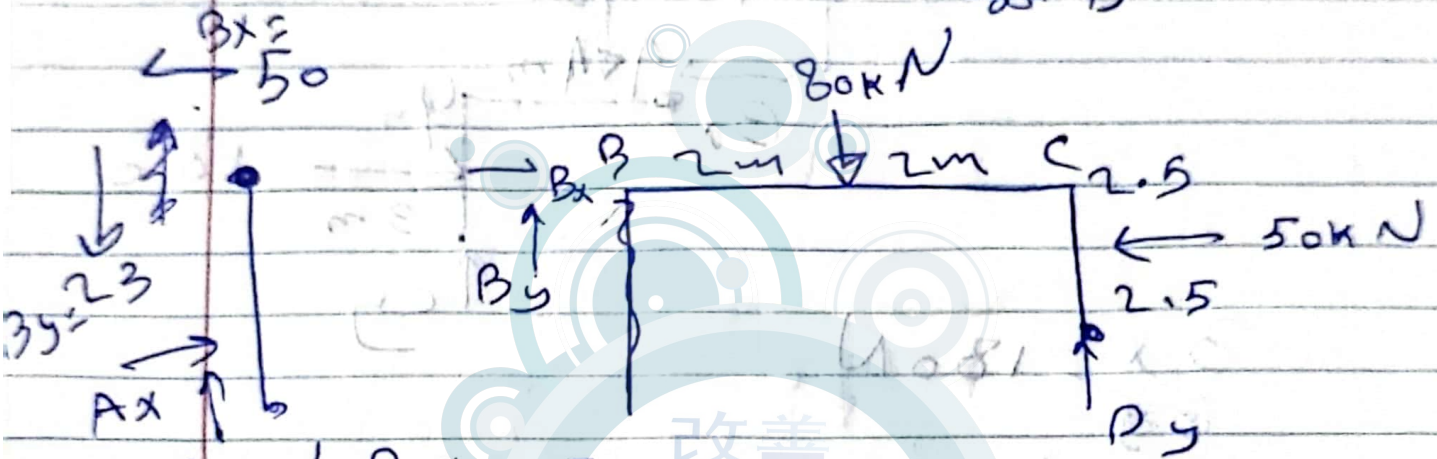
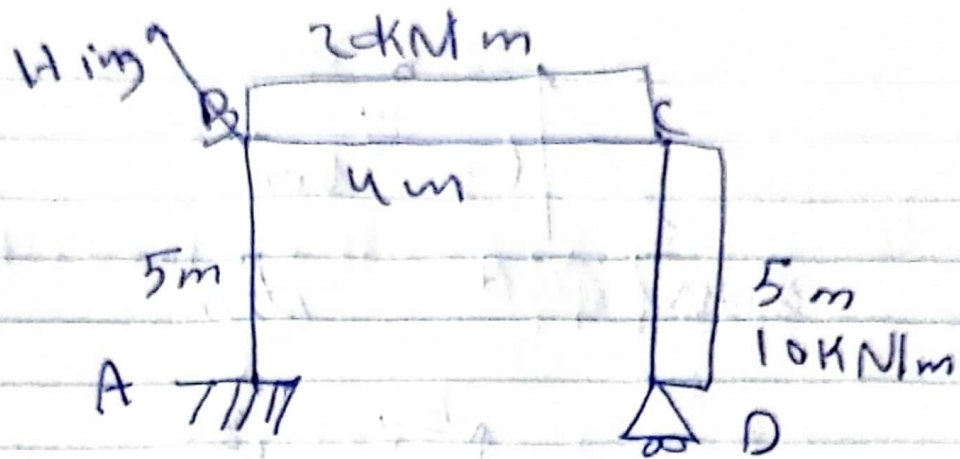
$$\sum M_A = 0$$

$$90 \times 7.8 + 200 \times 2.5 + 51.4 \times$$

$$(3.8 + 12.85) - 61.22 (153.2 + 0.14)$$

$$= B_y (12.37)$$

$$\Rightarrow$$



$$\sum M_B = 0$$

$$80 \times 2 + 80 \times 2.5 - D_y \times 5 = 0$$

$$D_y = 57$$

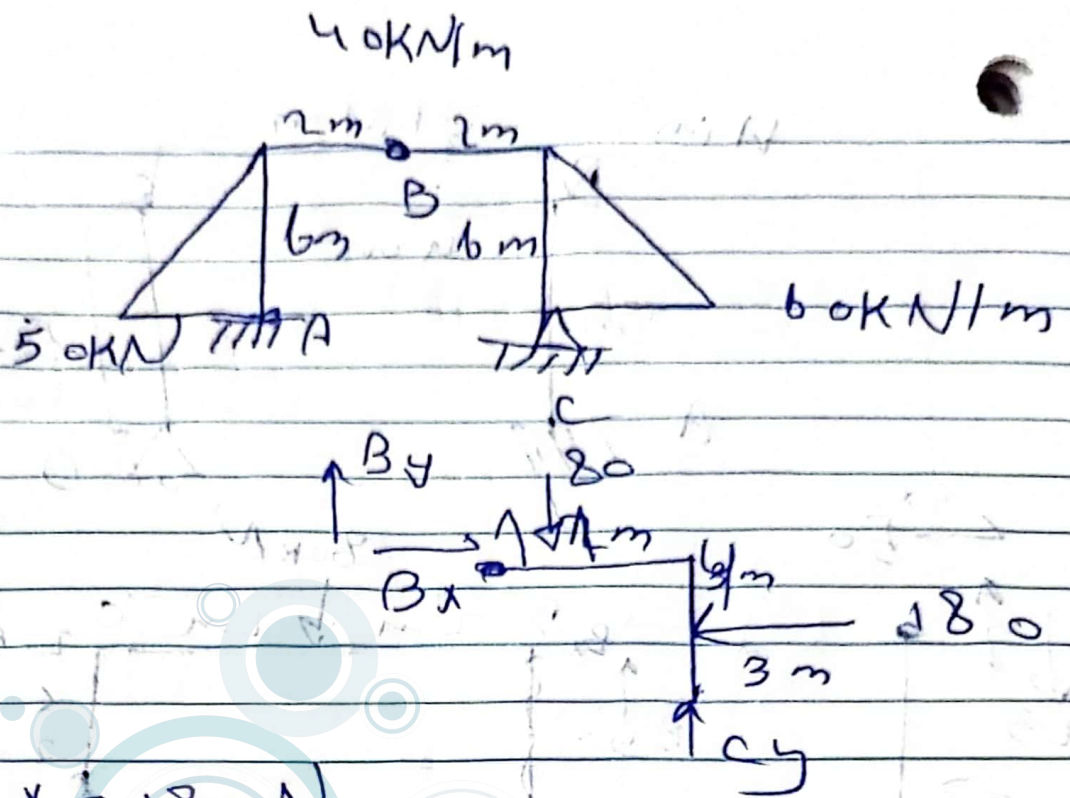
$$B_y = 23$$

$$A_x = 50 \text{ N}$$

$$A_y = 23 \text{ N}$$

$$M_A = 50 \times 5 = 250 \text{ N.m}$$

(C.C.W)



$$B_x = 180 \text{ N}$$

$$+\circlearrowleft \sum M_B = 0$$

$$-80 - 180 \times 4 + C_y \times 2 = 0$$

$$C_y = 400$$

$$B_y = 320 \downarrow$$

$$B_x = 180 \leftarrow$$

$$B_y = 320$$

$$A_y \uparrow \quad A_x \rightarrow$$

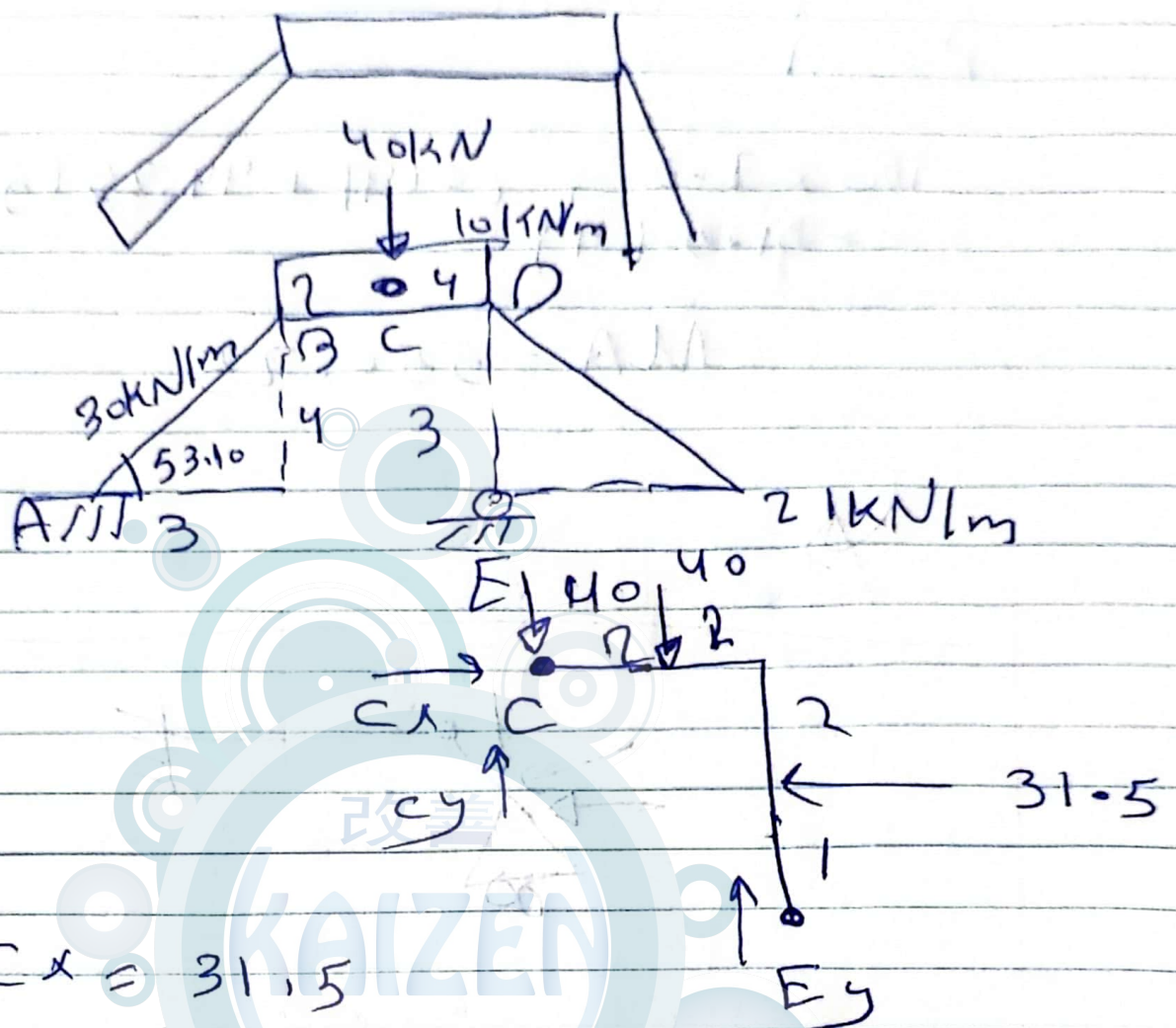
$$\sum F_x = 0 \Rightarrow$$

$$A_x = 30 \text{ N}$$

$$A_y = 240 \text{ N}$$

$$+\circlearrowleft \sum M_A = 0 \Rightarrow$$

$$180(6) + 320(2) - 80(1) - 150(2) = 0 \Rightarrow M_A = 1340$$



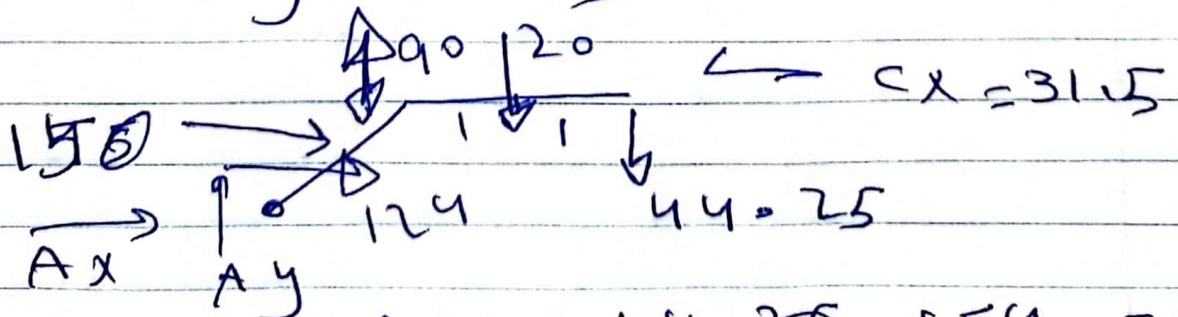
$$C_x = 31.5$$

$$\sum M_C = 0$$

$$40(2) + 31.5(2) - E_y(4)$$

$$E_y = 35.75 \text{ N}$$

$$C_y = 44.25$$



$$A_y = 64.25 \quad 154.25$$

$$A_x = 88.5$$

$$\sum M_A = 0$$

$$150 \times 2.5 + 20(4) + 44.25(5) - 31.5(6.4)$$

$$M_A = 550.25$$



$$\sum M_A = 0$$

$$150 \times 2.5 + 20(4) + 44.25(5) - 31.5(6.4)$$

$$M_A = 550.25$$

$$150 \times 2.5 + 20(4) + 44.25(5) - 31.5(6.4)$$

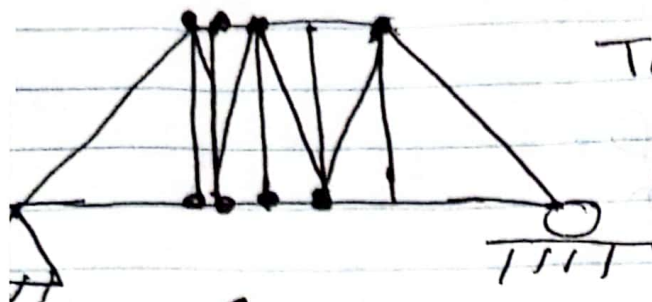
$$M_A = 550.25$$

$$M_A = 550.25$$

$$M_A = 550.25$$

CMB:

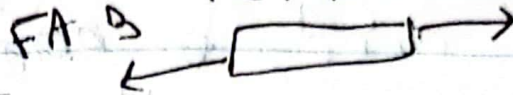
Structural Analysis



Simple

Truss, Two Foremembers

Tension - F_A



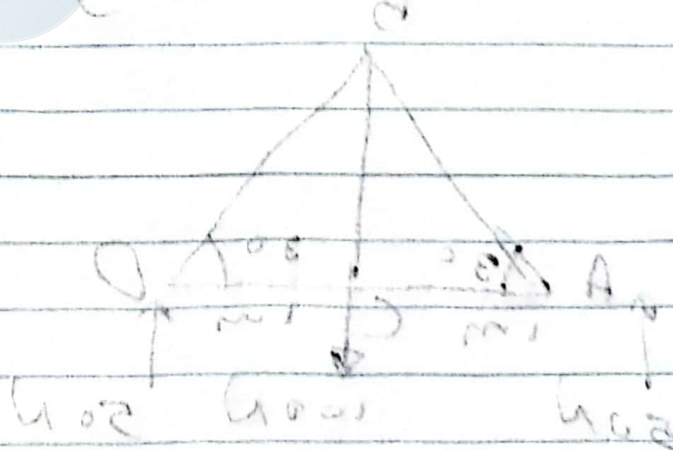
Compression

- 1 - All forces applied at joints only
each member are 2 force member.

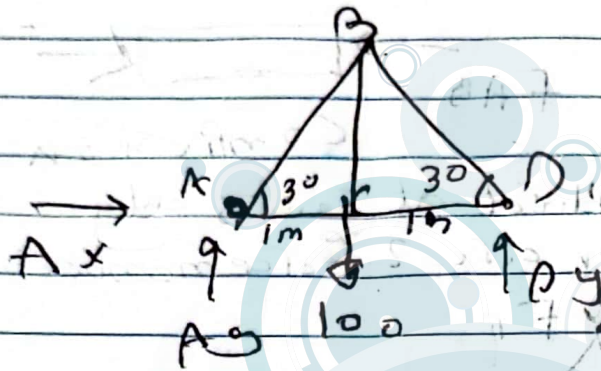
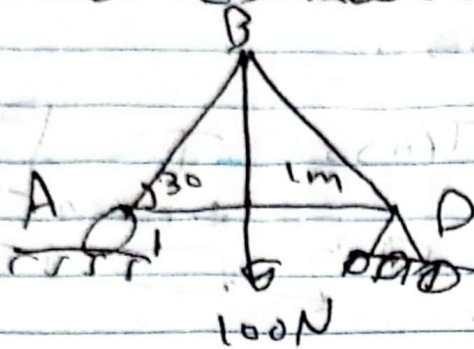


1. Load and reaction are External.
2. Force in member are Internal (T, C)

هذا إلى رعاك



Joint's method,



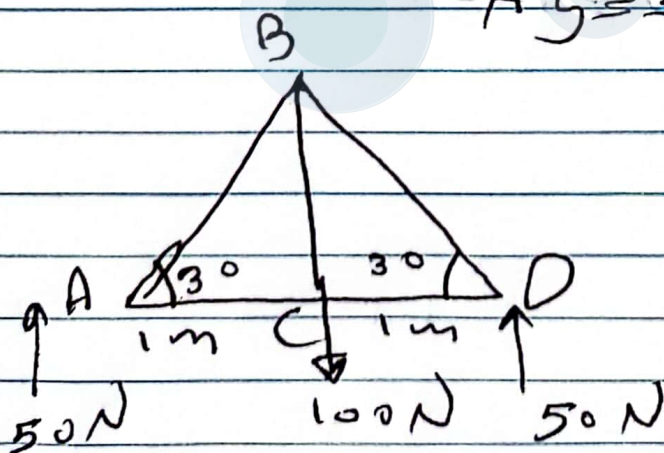
$$\sum F_x = 0 \Rightarrow \boxed{A_x = 0}$$

$$\sum F_y = 0 \Rightarrow A_y + D_y = 100$$

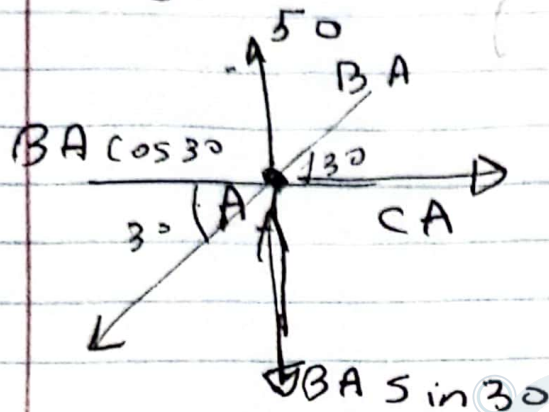
$$\left(\sum M = 0 \Rightarrow \right) -100 \times 1 + D_y \times 2 = 0$$

$$D_y = 50\text{N}$$

$$A_y = 50\text{N}$$



Joint A



$$\sum F_x = 0 \Rightarrow CA = BA \cos 30$$

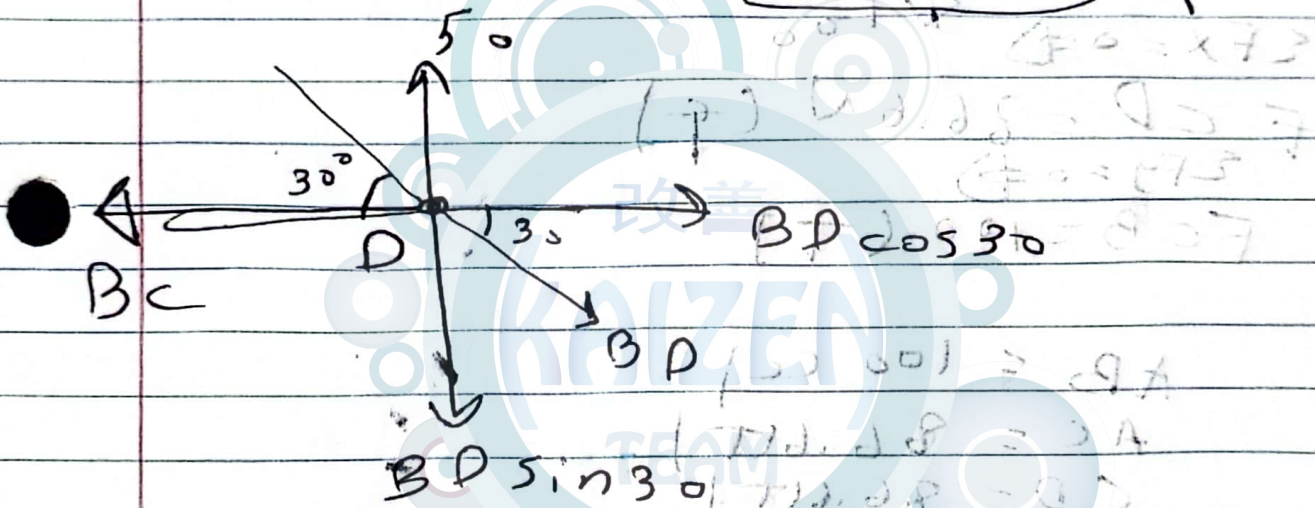
$$\sum F_y = 0 \Rightarrow$$

$$50 = BA \sin 30$$

$$100 = F_{BA} \quad C$$

$$F_{CA} = 100 \cos 30$$

$$CA = 86.6 \text{ N} \quad T$$



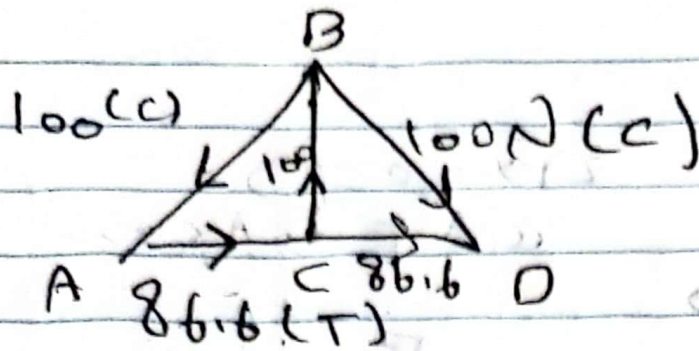
$$\sum F_x = 0 \Rightarrow BC = BD \cos 30 \quad (1)$$

$$\sum F_y = 0 \Rightarrow$$

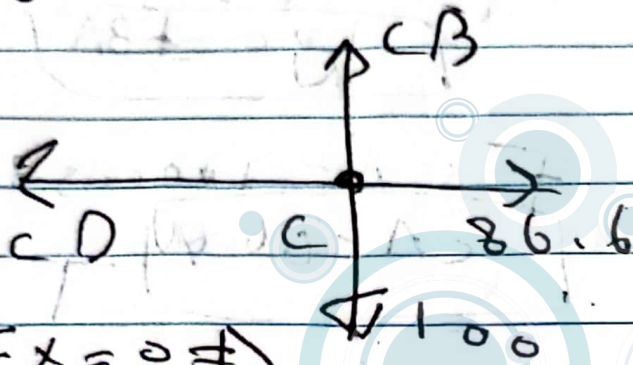
$$50 = BD \sin 30$$

$$100 = BD \quad C$$

$$F_{BC} = 86.6 \text{ N} \quad T$$



Joint C



$$\sum F_x = 0 \Rightarrow$$

$$F_{CD} = 86.6 \text{ N (C)}$$

$$\sum F_y = 0 \Rightarrow$$

$$F_{CB} = 100 \text{ (T)}$$

$$AB = 100 \text{ (C)}$$

$$AC = 86.6 \text{ (T)}$$

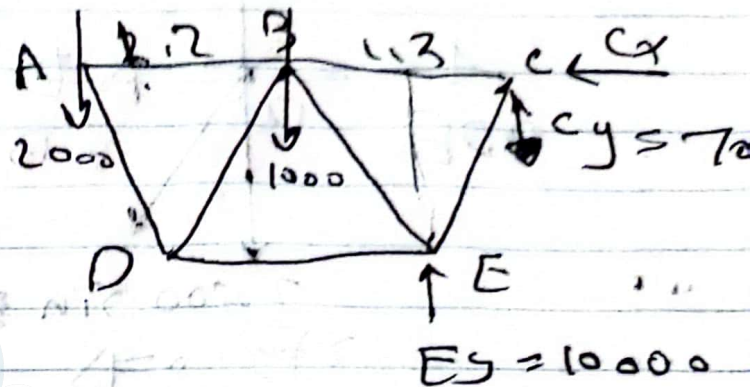
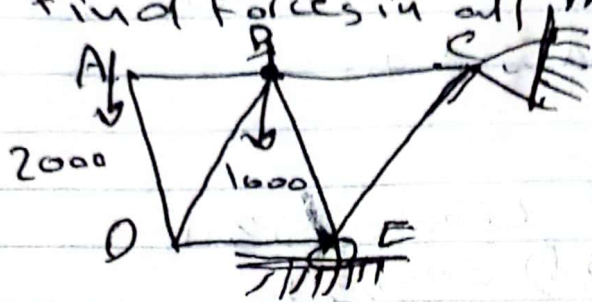
$$CD = 86.6 \text{ (T)}$$

$$CB = 100 \text{ (T)}$$

$$DB = 100 \text{ (C)}$$

$$T \text{ (Tension) } = 86.6$$

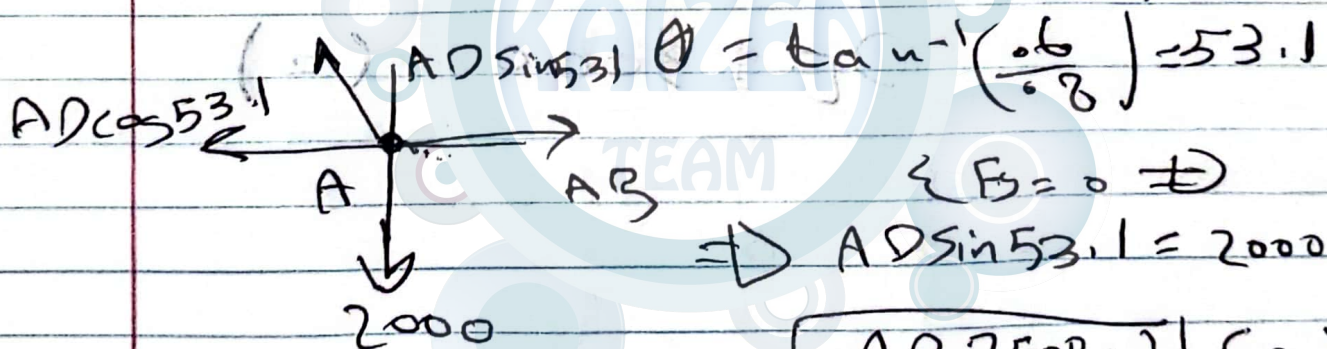
Find Forces in all members.



$$\begin{aligned} \sum M_c &= 0 \Rightarrow 1000 \times 1.2 + 2000 \times 2.4 \\ &- E_y \times 6 = 0 \end{aligned}$$

$$\Rightarrow E_y = 10000$$

$$C_y = 7000 \text{ N} \downarrow$$



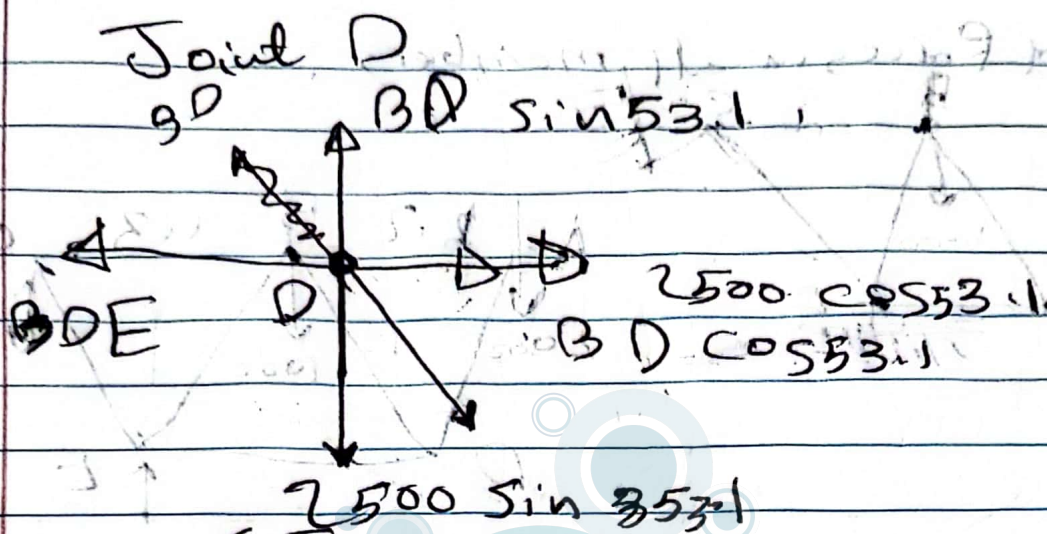
$$\sum F_y = 0 \Rightarrow$$

$$\Rightarrow AD \sin 53.1^\circ = 2000$$

$$AD = 2500 \text{ N} \quad (C)$$

$$\sum F_x = 0 \Rightarrow AB = AD \cos 53.1^\circ$$

$$AB = 1501.5 \text{ N} \quad (T)$$



$$\sum F_y = 0 \Rightarrow$$

$$BD \sin 53.1 = 2500 \sin 53.1$$

$$BD = 2500 \text{ (T)}$$

$$\sum F_x = 0 \Rightarrow$$

$$2500 \cos 53.1 + 2500 \cos 53.1 = DE$$

$$DE = 3000 \text{ (C)}$$

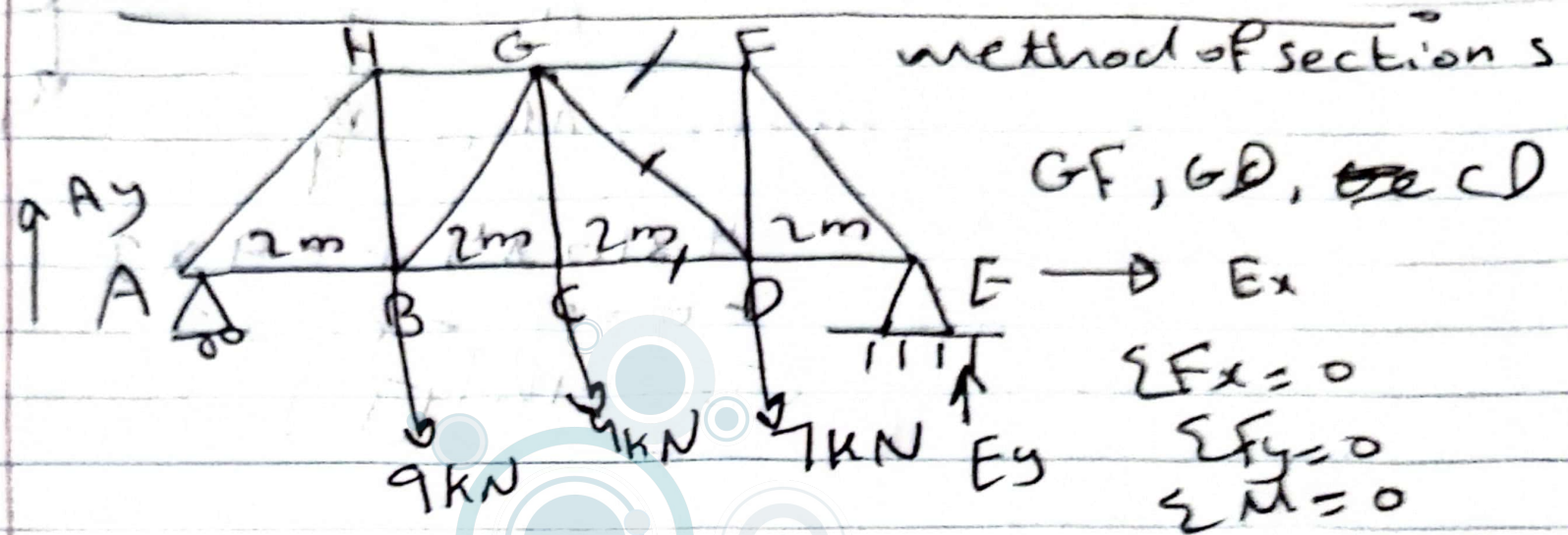
$$CF = 0$$

$$1.82 \times 10^3 = 1.82 \text{ kN}$$

$$(C) \quad 1.82 \text{ kN}$$

$$1.82 \text{ kN} \quad 0.8 \text{ kN} \quad 0.75 \text{ kN}$$

$$(T) \quad 1.82 \text{ kN}$$



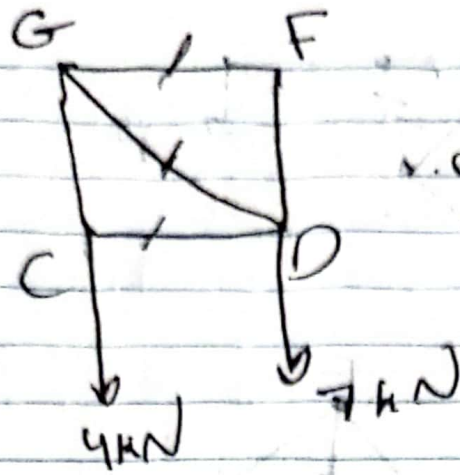
$$\sum F_x = 0 \Rightarrow E_x = 0$$

$$\uparrow + \sum M_A = 0 \Rightarrow$$

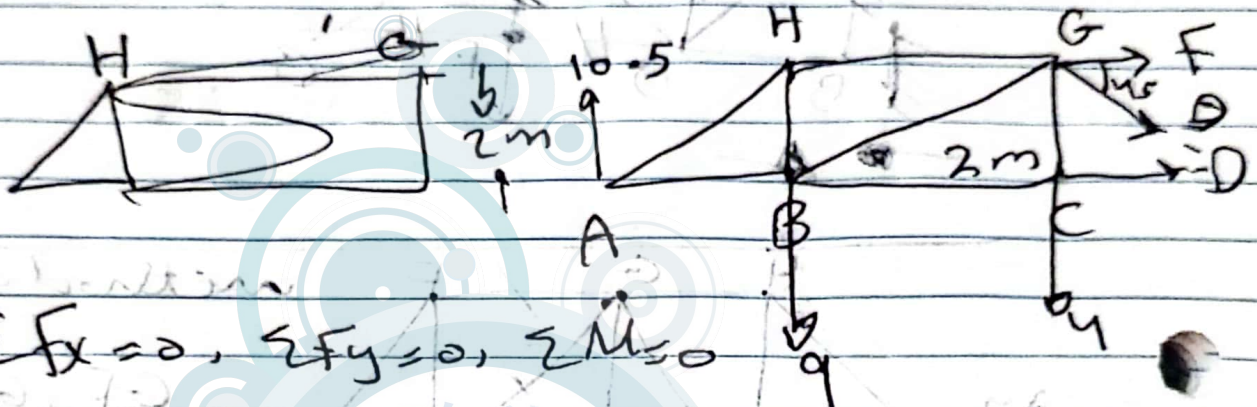
$$9 \times 2 + 4 \times 4 + 7 \times 6 - E_y \times 8 = 0$$

$$E_y = 9.5 N$$

$$A_y = 10.5 N$$



8 زمره 12 اضعه يكون
عدد الجواب (3)

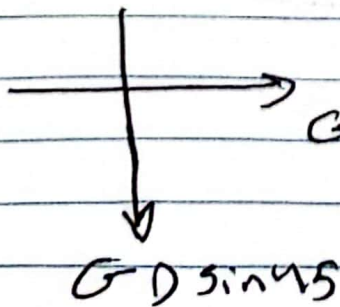


$$\sum F_x = 0, \sum F_y = 0, \sum M = 0$$

$$\begin{aligned} \uparrow + \sum M_G &= 4 \times 10 \times 4 + -9 \times 2 \\ &= -12 - CD \times 2 \\ \boxed{CD = -12} \text{ (T)} \end{aligned}$$

$$\begin{aligned} \uparrow + \sum M_D &= 0 \Rightarrow 10.5 \times 6 - 9 \times 4 - 4 \times 2 \\ &+ GF \times 2 = 0 \\ -2 GF &= 19 \end{aligned}$$

$$\boxed{GF = 9.5 \text{ (C)}}$$



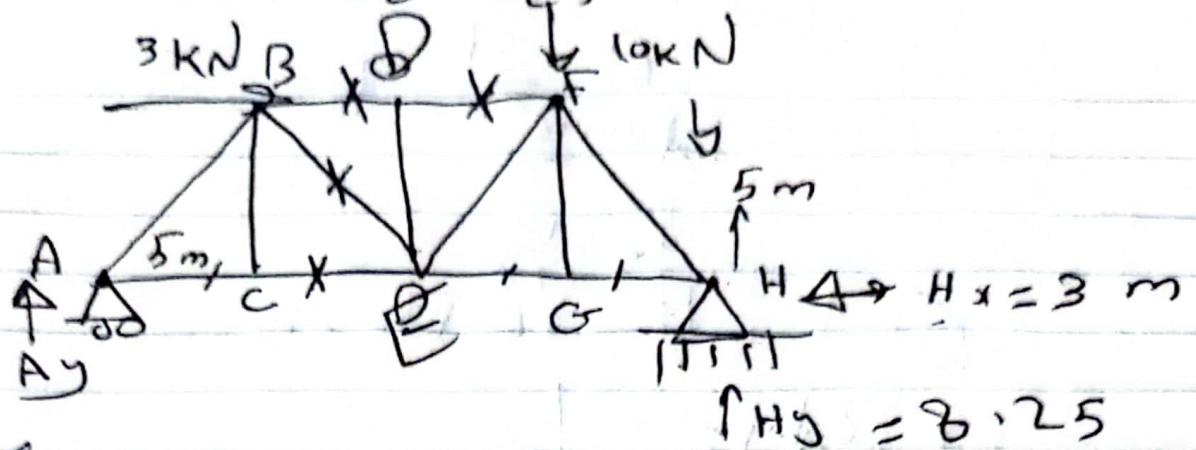
$$\sum F_y = 0 \Rightarrow 10.5 - 9 - 4 - 0.7GD = 0$$

$$10.5 - 9 - 4 - 0.7GD = 0$$

$$-2.5 = 0.7GD$$

$$GD = \frac{25}{7} \text{ (C)}$$

Find the force in members
BD, BE, CE, DF

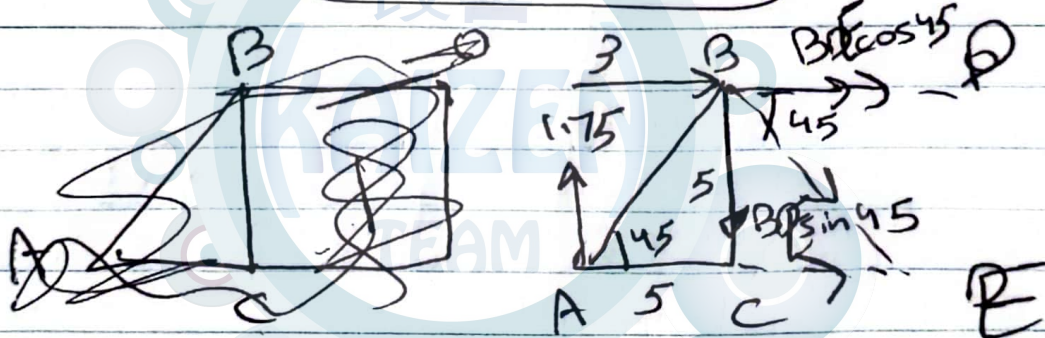


$$+\circlearrowleft \sum M_A = 0 \Rightarrow$$

$$-3 \times 5 + -10 \times 15 + H_y \times 20 = 0$$

$$H_y = 8.25$$

$$A_y = 1.75$$



$$+\circlearrowleft \sum M_B = 0 \Rightarrow$$

$$-1.75 \times 5 + CE \times 5 = 0$$

$$CE = 1.75 \text{ (T)}$$

$$+\circlearrowleft \sum M_E = 0 \Rightarrow$$

$$-3 \times 5 + 1.75 \times 10 - BD \times 5 = 0$$

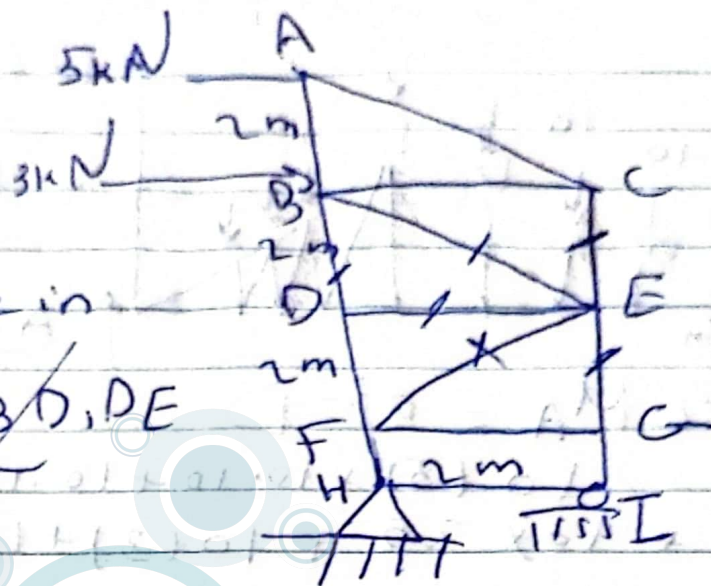
$$BD = 6.5 \text{ N (C)}$$

$$\sum F_y = 0 \Rightarrow$$

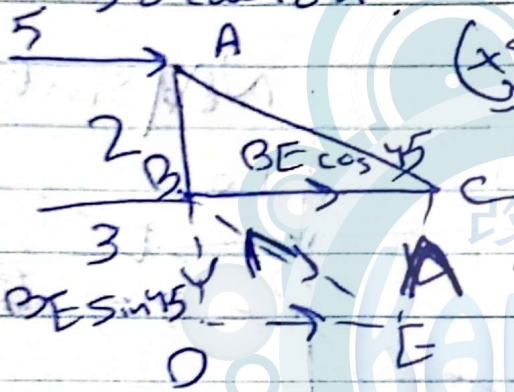
$$1.75 - 0.7 BE = 0 \Rightarrow$$

$$BE = 2.5 \text{ (C)}$$

Find force in
member's
CE, BE, BD, DE
EG



Solution:-



$$(\sum M_B = 0 \Rightarrow)$$

$$-5 \times 2 - CE \times 2 = 0$$

$$CE = 5 (C)$$

$$(\sum M_E = 0 \Rightarrow)$$

$$-5 \times 2 + BD \times 2 - 3 \times 2 = 0$$

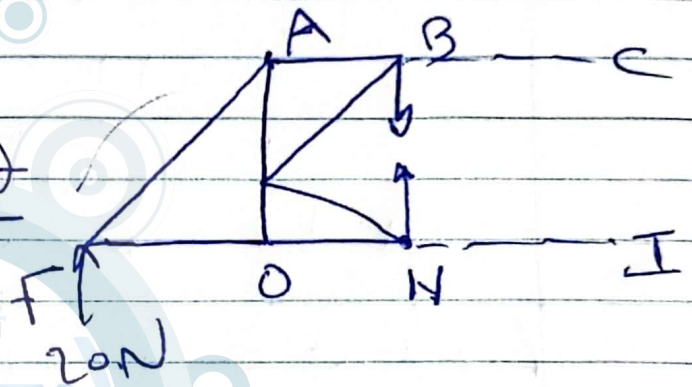
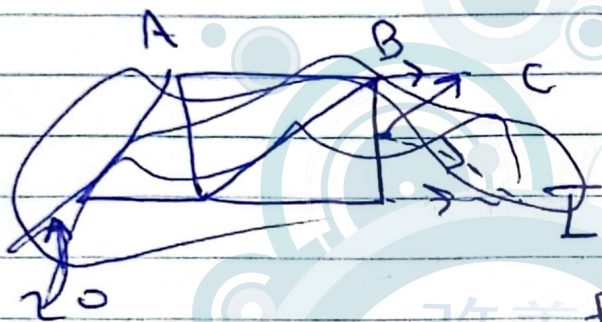
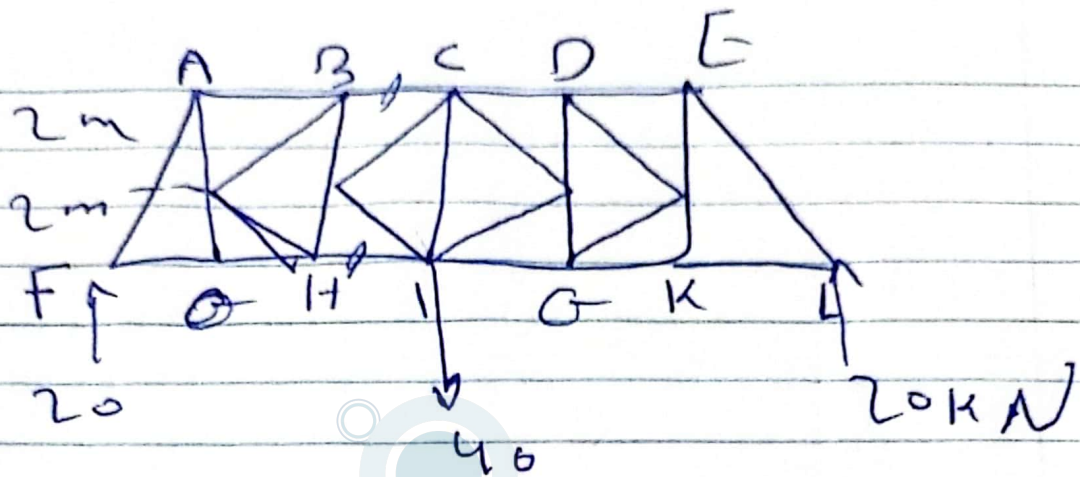
$$21 \times 2 + 0.5 \times 2 \times BD = 16 \Rightarrow BD = 13 (T)$$

$$(\sum F_x = 0 \Rightarrow)$$

$$BE \cos 45 + 5 + 3$$

$$BE = +11.4 (C)$$

Find Force in member's BC, HI.



$$\uparrow + \sum M_H = 0 \Rightarrow +20 \cdot 4 + BC(4) = 0$$

$$BC = 20 (C)$$

$$\uparrow + \sum M_B = 0 \Rightarrow$$

$$20(4) - HI(4) = 0 \Rightarrow$$

$$HI = 20 (T)$$

method of section is used as
method of joints.