Engineering mechanics

These notes were prepared to help students better understand the course. However, please note that they are not sufficient on their own. It is strongly recommended to practice the suggested questions provided by the instructor to fully grasp the material and prepare well for the exam.
There might be some mistakes in the notes. If you find any, feel free to contact me at the number below. I will review and update them if needed.
I also have the solution for the required Textbook question. If you're interested, feel free to reach out.
& My number: [0770693750]

Mechanics

chapter 1 (statics)

1.1 Mechanics

* Mechanics is branch of physical sciences that is concerned with the state of rest or motion of bodies that are subjected the action of forces, Three branches:-

1) rigid-body Mechanics
2) deformable-body mechanics
3) fluid Mechanics

we will study rigid-body mechanics since it is a basic requirement for the study of the mechanics of fluids, furthermore, rigid-body mechanics is essential for the design and analysis of many types of structural members, many types of structured members, mechanical components or electrical devices encountered in engineering

* Rigid-body mechanics is divided into two areas:-

① statics: deals with the equilibrium of bodies, that is those that are either at rest or move with a constant velocity

② Dynamics: concerned with the accelerated motion of bodies

\# we can consider statics as a special case of dynamics, in which the acceleration is zero, however statics deserves separate treatment in engineering education since many objects are designed with the intention that they remain in equilibrium

1.2 fundamental concepts (Basic quantities)

⇒ length: used to locate the position of a point in space & thereby describe the size of a physical system. once a standard unit (SI units for example) of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.

⇒ Time: concieved as a succession of events. ALthough the principles of statics are time independent, this quantity plays an important role in the study of dynamics

⇒ mass: is a measure of a quantity of matter that is used to compare the action of one body with that of another. this property manifests itself as a gravitational attraction between two bodies and provides a measure of the resistance of metter to a Change in velocity

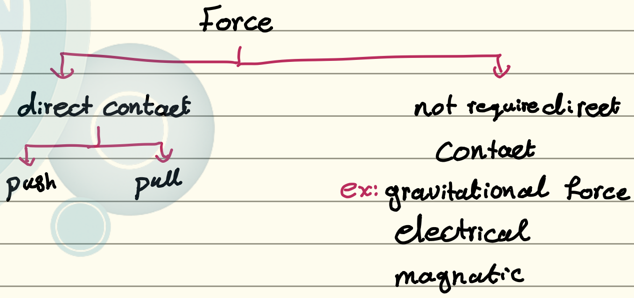
مقاومة الجسم للتغير في سرعته راح تكون أكبر كل ما زادت كتلته

⇒ Force: in general, force is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contanct between the bodies, such as a person pushing on wall, or it can occur through a distance when the bodies are physically separated

examples of the letter type include gravitational, electrical, and magnatic forces (بتأثر الأجسام ببعض بهاي القوى وهم بعاد عن بعض)

⇒ force is completely characterized by its magnitude, direction, and point of application

\# so that



Idealization (modeling):

* Models or idealizations are used in order to simplify application of the theory. Here we will consider three important idealizations:-

⇒ particle : A particle has a mass, but a size that can be neglected (dago). for example the size of the earth can be modeled as a particle when studing its orbital motion. when a body is idealized as a particle, the principles of mechanics reduce to a rather simplified from since the geometry of the body will not be involved in the analysis of the problem

⇒ **Rigid body** : A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after appling a load

⇐ الRigid body ما بصير الو deformation under the load

why is this important? لأنو ما بطّر أدخل ال parameters تبعت ال material بالموضوع (ما راح يهمني من شو المادة مصنوعة)
فإذا ما اطريت أدخل ال properties تبعت المادة سواء mechanical properties / physical properties ⇒ بصير الحل أسهل

\# This model is important because the body's shape does not change when a load is applied, and so we do not have to consider the type of material from which the body is made.

\# In most cases the actual deformation occurring in structures, machines, mechanism and the like and relatively small, and the rigid-body assumption is suitable for analysis.

⇒ **Concentrated force** : A concentrated force represents the effect of a loading which is assumed to act at a point on a body. we can represent a load by concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body. An example would be the contact force between wheel & the ground.

Newton's three Laws of motion

* engineering mechanics is formulated on the basis of Newton's three Laws of motion :-

① First Law: a particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force

② second Law : A particle acted upon by an unbalanced force (F) experiences an accelaration (a) that has the same direction as the force and a magnitude that is directly proportional to the force. if (F) is applied to a particle of mass (m). this Law may be expressed mathmatically as:-

$$\boxed{F = m \cdot a}$$

↓

unit $N = \frac{Kg.m}{s^2}$

③ Third Law : the mutual forces of action & reaction between 2 particles are equal, opposite & coolinear

على استقامة (خط) واحد

Action - reaction Law

* shorltly after formulating his three Laws of motion, Newton postulated a Law governing the gravitational attraction between any two particles:-

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

when:-

F = Force of gravitation between two particles

G = universal constant of gravitation, according to experimental evidance

$$\boxed{G = 66.73 \times 10^{-12} \frac{m^3}{Kg.s^2}}$$

m_1, m_2 : mass of each of two particles

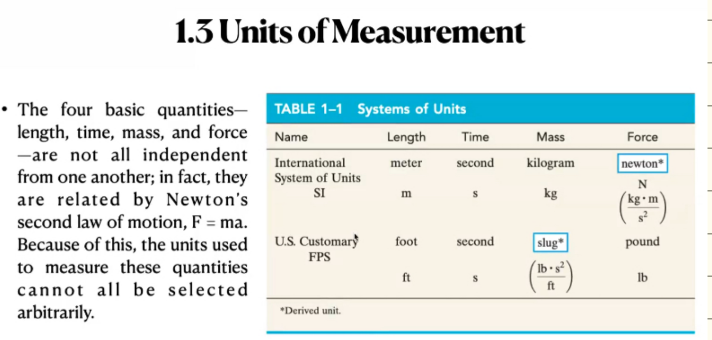
r : distance between the two particles

In the case of a particle located at or near the surface of the earth, the only gravitational force having sizable magnitude is that between the earth and the particle. Consequently, this force, termed the weight, will be the only gravitational force considered in our study of mechanics

$$\boxed{F = G\frac{m_1 m_2}{r^2}} \longrightarrow \boxed{W = G\frac{m M_e}{r^2}}$$

letting $g = \frac{G M_e}{r^2}$ yields

$$\boxed{W = m \cdot g}$$



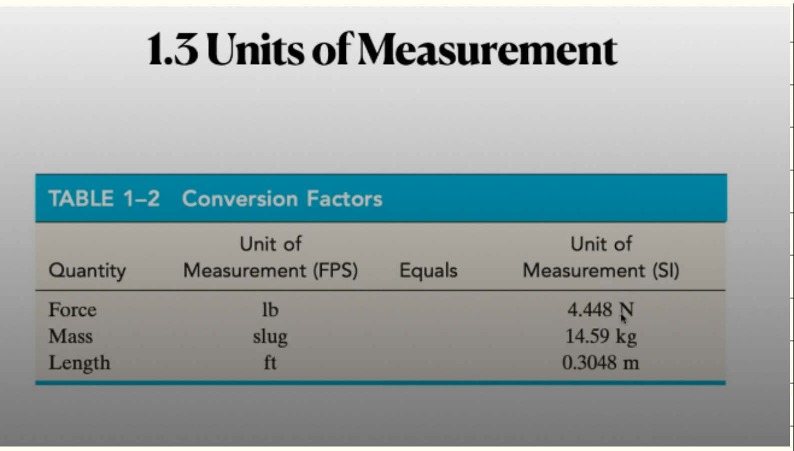
1.3 Units of Measurement

- The four basic quantities—length, time, mass, and force—are not all independent from one another; in fact, they are related by Newton's second law of motion, F = ma. Because of this, the units used to measure these quantities cannot all be selected arbitrarily.

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.



1.3 Units of Measurement

TABLE 1–2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

these numbers will be given in the exam

note:-

$G = 10^4$ $\quad$ $p = 10^{-12}$

$M = 10^6$ $\quad$ $m = 10^{-3}$

$K = 10^3$ $\quad$ $\mu = 10^{-6}$

↳ small letter $\quad$ $n = 10^{-9}$

$\mathring{A} = 10^{-10}$ → Angstrom (we use it to measure length)

because the capital letter for Kilvin

Example 1 :-

convert 50 Km/hr ⟶ ( ) $\frac{m}{s}$

$$50\frac{Km}{hr} \times \frac{1hr}{3600S} \times \frac{10^3 m}{1Kg}$$

$$= \frac{50\times10^3}{3600} = 13.88$$

Example: An object with a mass of (500g) is subjected to an acceleration of $(10\frac{cm}{s^2})$, what is the force acting on it. (N)

$500g \rightarrow 500\times10^{-3} Kg = 0.5Kg$

$$10\frac{cm}{s^2} \times \frac{10^{-2}}{1cm} = 0.1\frac{m}{s^2}$$

$$F = m.a = 0.5\times0.1 = 0.05 N\left(\frac{Kg.m}{s^2}\right)$$

Example: if we have to convert

$100\frac{m}{s} \Rightarrow ft/s$

$$100\frac{m}{s} \times \frac{1Ft}{0.3048m} = 328.083 Ft/s$$

Chapter 2

2.1 scalars and vectors

many physical quantities in engineering mechanics are measured using either scalars or vectors

⇒ Scalar: is any positive or negative physical quantity that can be completely specified by its magnitude (ex: length, time, mass)

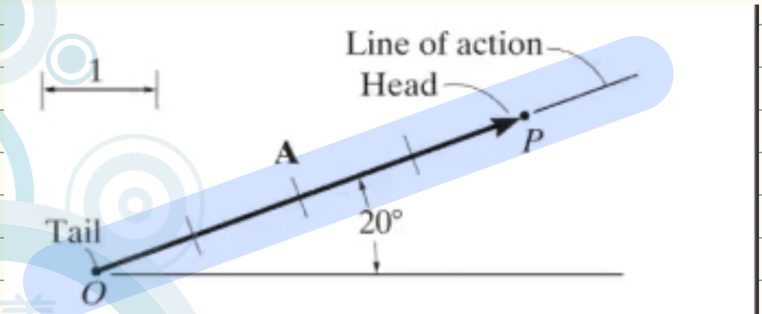
⇒ vector: is any physical quantity that requires both a magnitude & a direction for its complete description (ex: force, position, moment)

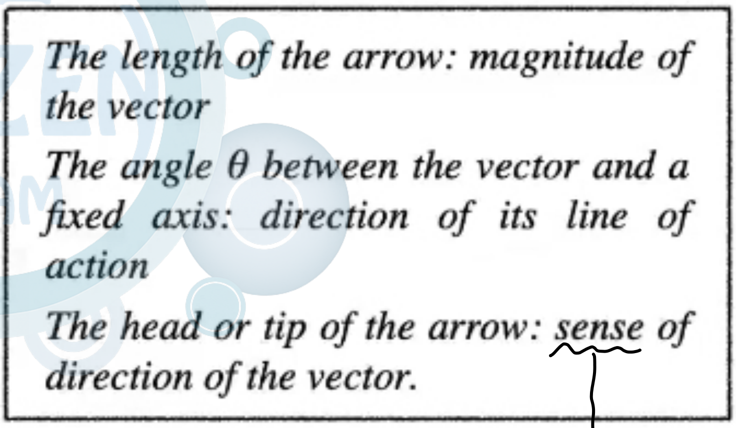
↳ it is a vector when we reference it to a place

A (Bolded): vector quantity

A = vector magnitude

for the magnitude we will use |A|





The length of the arrow: magnitude of the vector

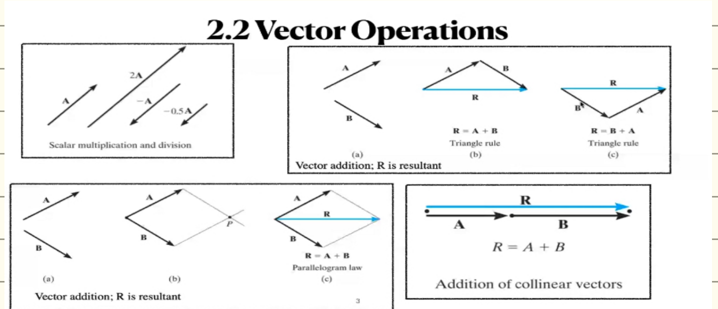
The angle θ between the vector and a fixed axis: direction of its line of action

The head or tip of the arrow: sense of direction of the vector.

اتجاه ال vector محدد عندي بس هل هو يعتبر طالع للأعلى
ولا متجه للأسفل؟ وبنعرفها من خلال ال head of the vector
الي هو ال point (p) إفرض للـ x وين الجهة ال positive ووين ال negative

Line of action : الي محطوط عليه highlighter بالصورة
وهاد الخط بقدر أمده لـ (+∞) أو (−∞)

2.2 vector operations



2.2 Vector Operations

Scalar multiplication and division

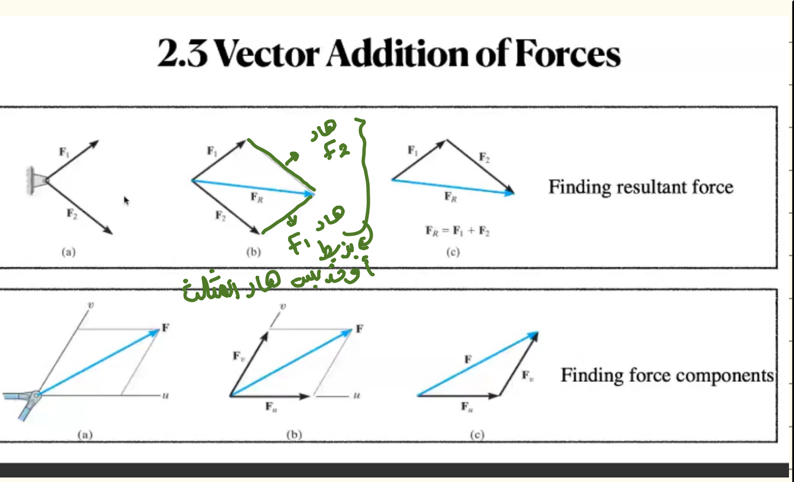
(a) (b) Triangle rule (c) Triangle rule

Vector addition; R is resultant

(a) (b) (c) Parallelogram law

Vector addition; R is resultant

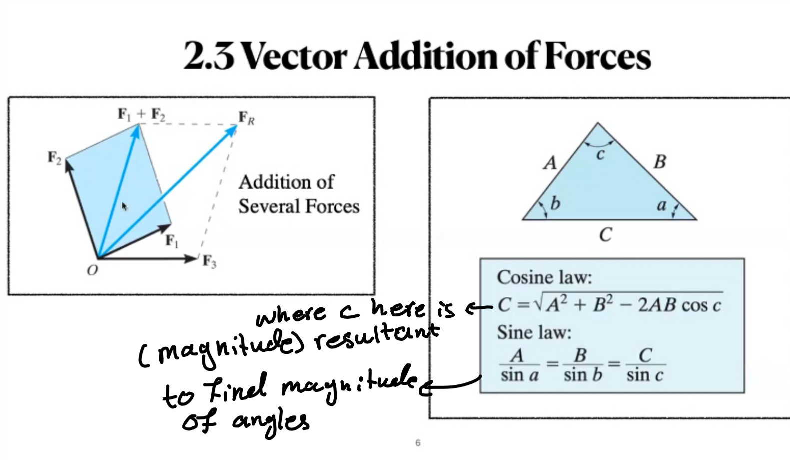
Addition of collinear vectors



2.3 Vector Addition of Forces

Finding resultant force

Finding force components



2.3 Vector Addition of Forces

Addition of Several Forces

Cosine law:
$C=\sqrt{A^2+B^2-2AB\cos c}$ ← where c here is (magnitude) resultant

Sine law:
$\frac{A}{\sin a}=\frac{B}{\sin b}=\frac{C}{\sin c}$ → to find magnitude of angles

#resultant force: هي وين الاتجاه النهائي الي راح يتحرك لعنو الجسم الي مطبقين عليه ال forces

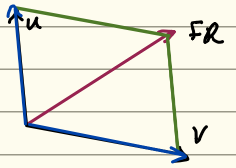
how to find it?

from the end of (F_1) we draw a line which is parallel to (F_2)
from the end of (F_2) we draw a line which is parallel to (F_1)
then we draw a line from the beginingof both vectors to the point of intersection between the 2 lines that we already drawn.
then we have the resultant force vector.
& this method is called parrallelogram method

 # finding force components

↳ & this when we already have a resultant force & we want to find the components for it

example



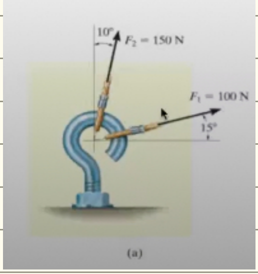
⇒ we draw a line from the head of (Fr) which is parallel to (v) until it intersects (u) & then we draw a line from the head of (Fr) which is parallel to (u) until it intersects the v line (the green lines)
⇒ then the Fu component from the tail of (u) until the intersection of the two lines
& the Fv component from the tail of (v) until the intersection of the both line (as shown)
the blue lines

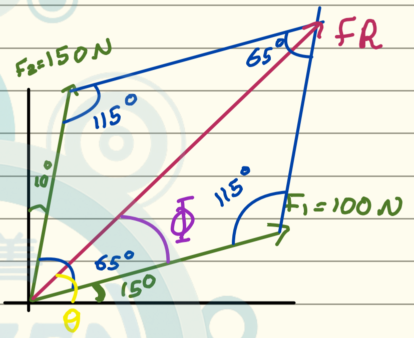
ومش شرط تطلع المركبات نفس طول المتجهات الأصلية زي ما طلعوا بالرسمة

هون عندي 3 forces ⇐ بجمع parallelogram وبجيب ال resultant لأول 2 بعدين برجع بعمل parallelogram وبجيبلهم ال resultant مع الثالثة

Example (2.1/page23)

find the magnitude & direction of the resultant force





 # note: الرسمة مرسومة بالقياسات الصحيحة للزوايا

now we have to calculate FR & the angle

to find the angle of FR

$\Rightarrow 90-10-15=65°$

 # note: the sum of the interior angles of any quadrilateral is 360 degrees

so $360-(2\times65)=\frac{230}{2}=115°$ for each angle

we will use the lower triangle

$F_R=\sqrt{(100)^2+(150)^2-(2\times100\times150\times\cos115)}$ → in degree مش Radians

$F_R=212.6\ N$ ⇒ the magnitude now finding the angle

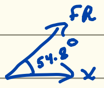
to find θ we must find Φ

$\theta=\Phi+15$

using sin law

$\frac{150}{\sin\Phi}=\frac{212.6}{\sin(115°)}$ $\Phi=39.8$ so $\theta=15+39.8$
$=54.8°$

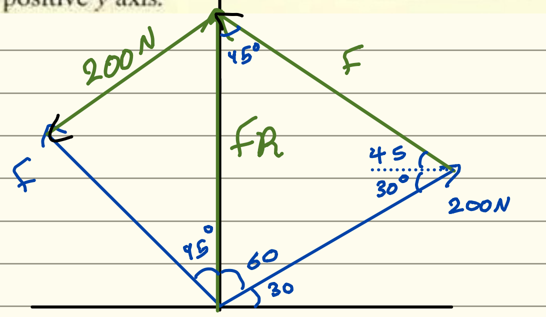
so

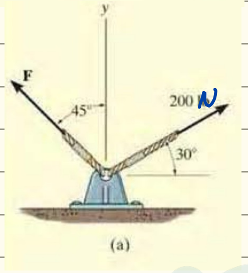
$\overline{F_R} = 212.6$ N at 

#note
F_R : magnitude
$\overline{F_R}$: vector

Example (2.3 / page 25)

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.





(a)

$$\frac{F}{\sin 60} = \frac{200}{\sin 45}$$

$F = 244.9$ N

$$F_R = \sqrt{(244.4)^2 + (200)^2 - (2 \times 244.4 \times 200 \times \cos 75}$$

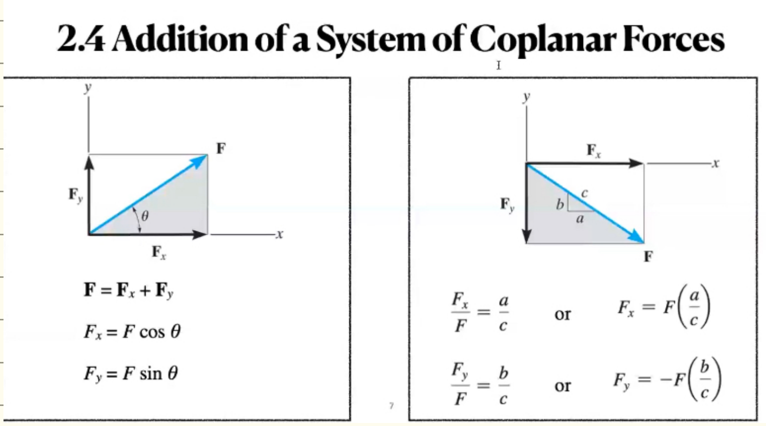
$F_R = 273.2$ N

or using sine law

$$\frac{200}{\sin 45} = \frac{F_R}{\sin 75}$$

$F_R = 273.2$ N

2.4 Addition of a system of coplanar forces



2.4 Addition of a System of Coplanar Forces

$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$

$F_x = F \cos \theta$

$F_y = F \sin \theta$

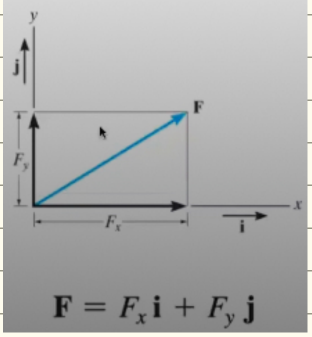
$\frac{F_x}{F} = \frac{a}{c}$ or $F_x = F\left(\frac{a}{c}\right)$

$\frac{F_y}{F} = \frac{b}{c}$ or $F_y = -F\left(\frac{b}{c}\right)$

coplanar ⇒ في نفس ال plane
so F_x, F_y are rectangular components

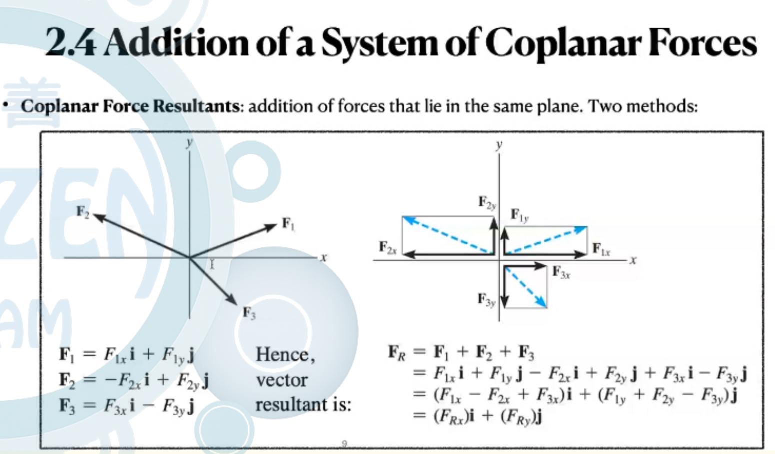
cartesian vector notation: it is also possible to represent the x and y components of a force in terms of cartesian unit vectors i and j. They are called unit vectors because they have a dimenssionless magnitude of (1), and so they can be used to designate the directions of the x and y axes, respectively. ال Magnitude تبعتها 1 و dimention ما الها.

since the magnitude of each component of F is always a positive quantity, which is represented by the (positive) Scalers Fx and Fy, then we can express F as a cartesian vector



$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$

coplanar force resultant: addition of forces that lie in the same plane. two methods



2.4 Addition of a System of Coplanar Forces

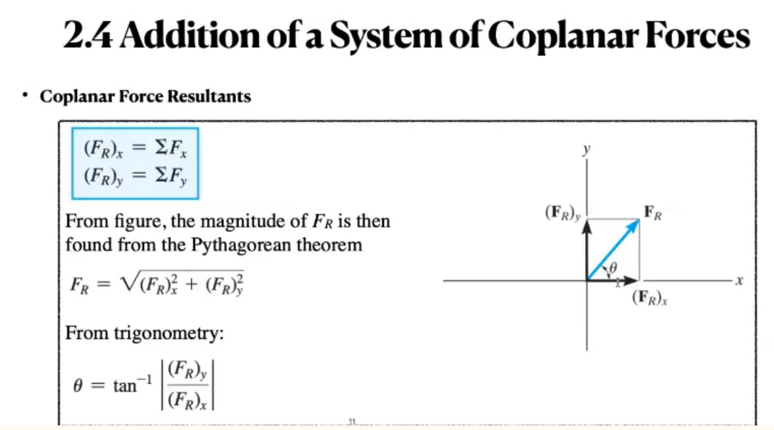
- **Coplanar Force Resultants**: addition of forces that lie in the same plane. Two methods:

$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$
$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$
$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$

Hence, vector resultant is:

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

لـ هاد لو أنا بدي أجيب ال resultant لهم



2.4 Addition of a System of Coplanar Forces

- **Coplanar Force Resultants**

$(F_R)_x = \Sigma F_x$
$(F_R)_y = \Sigma F_y$

From figure, the magnitude of F_R is then found from the Pythagorean theorem

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

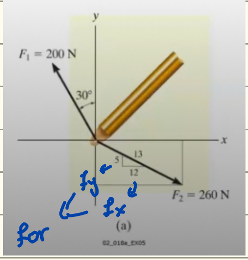
From trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

بلاحظ بقدر أجيب ال resultant من خلال التحليل

Example (2.5)

*determine (F_x & F_y) of F_1 acting on the boom & exprise each force as a cartesian vector



مشانها باتجاه السالب

$F_{1x} = -200 \sin 30$

$= -100\,N = 100\,N \leftarrow$

$F_{1y} = 200 \cos 30 = 173\,N$

or $173\,N \uparrow$

$F_{2x} = 260 \times \frac{12}{13}$

$= 240\ N \rightarrow$

$F_{2y} = -260\left(\frac{5}{13}\right)$

$= -100\,N$

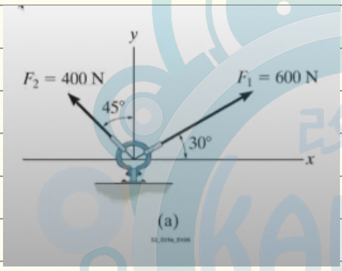
So now we have

$\vec{F_1} = (-100i + 173j)\,N$

$\vec{F_2} = (240i - 100j)\,N$

Example (2.6)

determine the magnitude & direction of the resultant force.



$F_{1x} = 600 \times \cos 30$

$= 514.615$

$F_{1y} = 600 \times \sin 30$

$= 300$

$F_{2x} = -400 \times \sin 45$

$= -282.842$

$F_{2y} = 400 \times \cos 45$

$= 282.842$

$\Sigma F_x = 236.8$

$\Sigma F_y = 582.8$ $\qquad F_R = 236.8i + 582.8j$

$F_R = \sqrt{(F_x)^2 + (F_y)^2}$

$F_R = 629\ N$

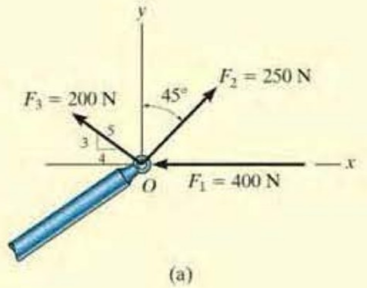
$\theta = \tan^{-1}\left(\frac{582.8}{236.8}\right)$

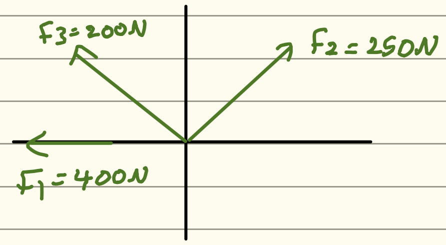
$\theta = 67.9°$

$\overline{F_R} = 629\ N \angle 67.9°$

Example 2.7

The end of the boom *O* in Fig. 2–20*a* is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.





$\Sigma F_x = -400 + -\left(200 \times \frac{4}{5}\right) + 250 \sin 45$

$\Sigma F_x = -383.2\ N$

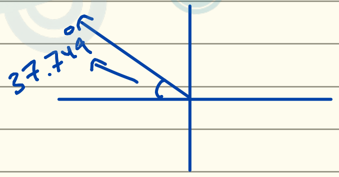
$\Sigma F_y = 250 \cos 45 + \left(200 \times \frac{3}{5}\right)$

$= 246.7\ N$

$F_R = \sqrt{(F_x)^2 + (F_y)^2}$

$= 485\ N$

$\theta = \tan^{-1}\left(\frac{246.7}{383.2}\right) = -37.744$

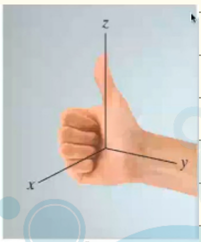


2.5 cartesian vectors

* Cartesian vectors : used to simplify the solving of problems in three dimensions

→ Right handed coordinates system : used to develop the theory of vector algebra that follows

→ rectangular components of a vector : A vector (A) may have one, two, or three rectangular components along x, y, z coordinate axes, depending on how the vector is orianted relative to the axes

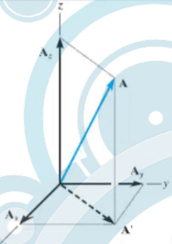


$\bar{A} = \bar{A}' + A_z$

then $\bar{A}' = A_x + A_y$

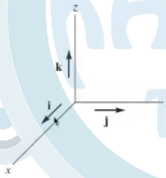
Hence $\bar{A} = A_x + A_y + A_z$

* دائمًا لمن نحل 3D راح نمثل ال vectors على شكل Cartesian vectors

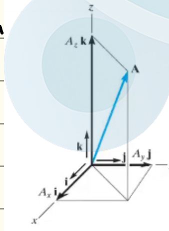


* Cartesian vectors : used to simplify the solving of problems in three dimensions

* Cartesian unit vector : in three dimensions the set of cartisian unit vectors i, j, K, is used to designate directions of x, y, z, axes, respectively

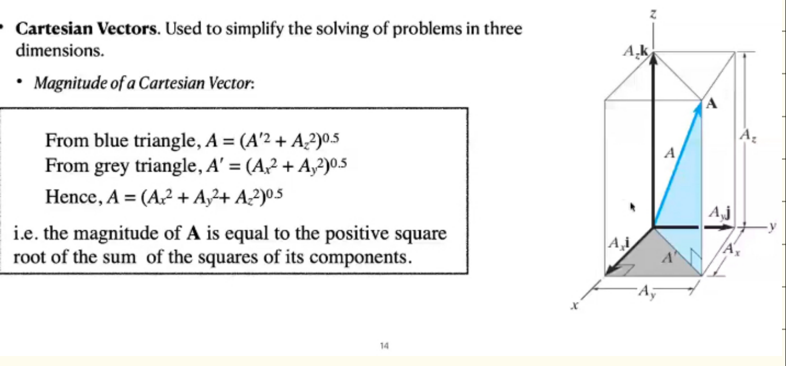


* Cartesian vector representation : $\bar{A}$ in Cartesian vector from can be written as:



$$\bar{A} = \underbrace{A_x i}_{\text{magnitude(scaler)} \times \text{vector} = \text{vector}} + A_y j + A_z K$$

remember that A_x, A_y, A_z are magnitudes and i, j, K are directions



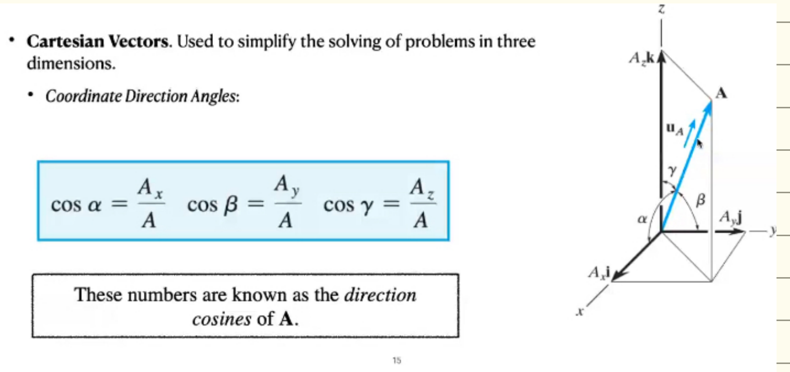
- **Cartesian Vectors**. Used to simplify the solving of problems in three dimensions.
  - *Magnitude of a Cartesian Vector:*

From blue triangle, $A = (A'^2 + A_z^2)^{0.5}$
From grey triangle, $A' = (A_x^2 + A_y^2)^{0.5}$
Hence, $A = (A_x^2 + A_y^2 + A_z^2)^{0.5}$

i.e. the magnitude of **A** is equal to the positive square root of the sum of the squares of its components.

so that $\boxed{A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$

* coordinate direction angles :-



- **Cartesian Vectors**. Used to simplify the solving of problems in three dimensions.
  - *Coordinate Direction Angles:*

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

These numbers are known as the *direction cosines* of **A**.

So that $A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

$\alpha, \beta, \gamma \Rightarrow$ coordinate direction angles

$\cos\alpha, \cos\beta, \cos\gamma \Rightarrow$ direction cosines

$$\boxed{u_A = \frac{\overset{\text{vector A}}{\bar{A}}}{\underset{\text{magnitude of A}}{A}} = \frac{A_x}{A} i + \frac{A_y}{A} j + \frac{A_z}{A} k}$$

u_A: unit vector of A

u_A : will have magnitude of one and be dimenssionles provided $\bar{A}$ is divided by it's magnitude

$$\boxed{u_A = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K}$$

& $\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$

α : between the vector & positive x-axis & the same for β & γ

$\bar{A} = \bar{u}_A \times A$

$$\boxed{\bar{A} = A\cos\alpha\, i + A\cos\beta\, j + A\cos\gamma\, K}$$

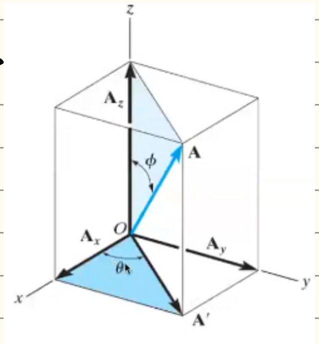
إذا حكالي الزاوية بين ال vector وال negative x-axis هي

so ⇐ 60°

$\alpha = 180 - 60 = 120°$

so that $\alpha = 120°$

* Transverse & azimuth Angles : Sometimes the direction of A can be be specified using two angles namely, a transverse angle(θ) and an azimuth angle Φ



* By applying trigonometry to the light blue & grey triagles, we get

$$\boxed{\bar{A} = A\sin\Phi\cos\theta\, i + A\cos\Phi\cos\theta\, j + A\cos\Phi\, K}$$

2.6 Addition of Cartesian vectors

*the addition (or substraction) of two or more vectors is greatly simplified if the vectors are expressed in terms of their cartesian components.

if $A = A_x i + A_y j + A_z k$ and $B = B_x i + B_y j + B_z k$

then

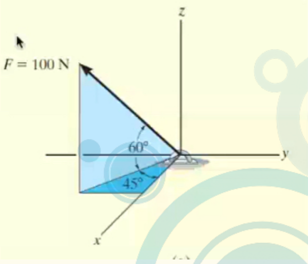
$$R = A + B = (A_x + B_x) i + (A_y + B_y) j + (A_z + B_z) k$$

so that the general form

$$\boxed{F_R = \Sigma F = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k}$$

#Important

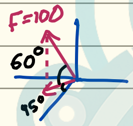
Example 2.8 (page 48)



represent vector F in the cartesian form:-

be careful : y component will be negative

before solving the question are these angles coordinate direction angles or transverse



the second case

so that

we have the angle 60 but it is not between the vector & the positive z axis, the angle $\Phi = 30$

$F_z = 100 \times \sin 60$

$= 86.6 N$

now we will going to find F' to find F_x & F_y

$F' = 100 \cos 60$

$= 50 N$

$F_x = 50 \cos 45$

$F_y = -50 \sin 45$ ⇒ negative عكس ال (y)

$= 35.4 N$

$\overline{F} = 35.4 i - 35.4 j + 86.6 k$

if we want to find the magnitude

$F = \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2}$

$= 100$ → so my solution is correct

if we want to change it into direction cosine angles

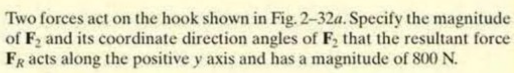
$\cos \alpha = \frac{F_x}{F} = \frac{35.4}{100}$

so that $\alpha = \cos^{-1}\left(\frac{35.4}{100}\right) \approx 69.3$

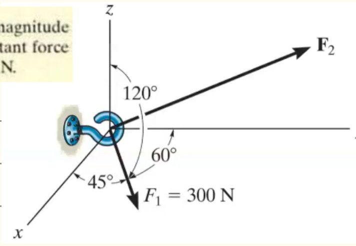
$\beta = \cos^{-1}\left(\frac{-35.4}{100}\right)$

$\gamma = \cos^{-1}\left(\frac{86.6}{100}\right)$

Example (2.9)



Two forces act on the hook shown in Fig. 2–32*a*. Specify the magnitude of **F**$_2$ and its coordinate direction angles of **F**$_2$ that the resultant force **F**$_R$ acts along the positive *y* axis and has a magnitude of 800 N.



first of all in which method we can solve this Question

we will use direction cosine angle

⇒ notice that the angle between F_1 & z axis is 120 so it's component has to be negative

هون هو معطيني كل ال angles تبعت F_1

$\cos \beta = \frac{F_y}{F_1}$

$F_{1y} = \cos \beta \times F_1$

$= \cos 60 \times 300$

$= +150$ → to the positive y axis

$F_{1x} = \cos \alpha \times F_1$

$= 300 \times \cos 45$

$= 212.132 N$

$F_{1z} = -300 \cos 120$

$-150 N$

$F_1 = 212 i + 150 j - 150 k$

$\Sigma F_y = F_{1y} + F_{2y}$

$800 = 150 + F_{2y}$

$-150 \quad -150$

$F_{2y} = 650 N$

if the resultant force has a magnitude only in the positi

y axis so F_{Rx} & $F_{Rz} = 0$

so $\Sigma F_x = 0$

$F_{1x} + F_{2x} = 0$

$212 + F_{2x} = 0$

$F_{2x} = -212 N$

& $\Sigma F_z = 0$

$F_{1z} + F_{2z} = 0$

$-150 + F_{2z} = 0$

$F_{2z} = 150 N$

so that

$F_2 = (-212 i + 650 j + 150 k) N$

$F_2 = \sqrt{(F_{2x})^2 + (F_{2y})^2 + (F_{2z})^2} \approx 700 N$

so $F_2 = 700N$

& $F_2 = (-212i + 650j + 150k)N$

So $\cos\alpha = \frac{-212}{700}$ $\alpha = 108^\circ$

$\cos\beta = \frac{650}{700}$ $\beta = 21.8^\circ$

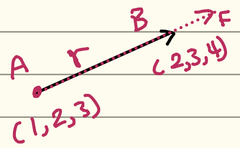
$\cos\gamma = \frac{150}{700}$ $\gamma = 77.6^\circ$

2.7 position vectors

* used to formulate a cartesian force vector directed between two points in space

\# position vector: has magnitude → m, mm, ft

unit vector → i, j, k : unitless



فبطل ينفع نستخدم الطرقة البسيطة اي كنا نستعملها قبل هيك

لانو قبل كان عنا

$\overline{F_A} = \overline{u_A} \times F$

↳ magnitude = 1

so it does not effect the F_A

but we can not say that

$\overline{F} = \overline{r} \times F$

↳ because r has unit

(for example m) → $F = N.m$ → which is wrong

& F : N

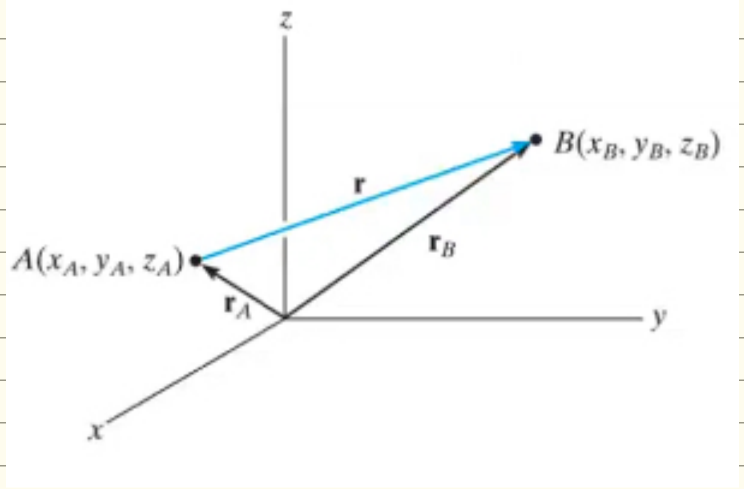
so we have to find the unit vector of

$\vec{r} \Rightarrow$ unit vector (u)

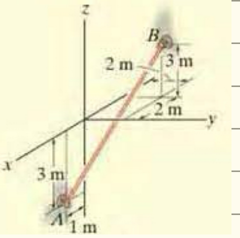
so if we want to find r ⇒ which is AB

$r = r_B - r_A = (x_B i + y_B j + z_B k) - (x_A i + y_A j + z_A k)$

or $\boxed{r = (x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k}$



An elastic rubber band is attached to points *A* and *B* as shown in Fig. 2–37*a*. Determine its length and its direction measured from *A* toward *B*.



$B = (-2i + 2j + 3k)m$

$A = (1i + 0j - 3k)m$

so $AB = (-2-1)i + (2-0)j + (3--3)k$

$AB = -3i + 2j + 6k$

so $\overline{r} = -3i + 2j + 6k$ m

$r = \sqrt{(-3)^2 + (2)^2 + (6)^2}$

$r = 7$ m

to find the direction we have to find (u)

$u = \frac{-3}{7} i + \frac{2}{7} j + \frac{6}{7} k$ m

$\alpha = \cos^{-1}(\frac{-3}{7})$ $\alpha = 115.4^\circ$

$\beta = \cos^{-1}(\frac{2}{7})$ $\beta = 73.4^\circ$

$\gamma = \cos^{-1}(\frac{6}{7})$ $\gamma = 31^\circ$

2.8 force vector directed along a line

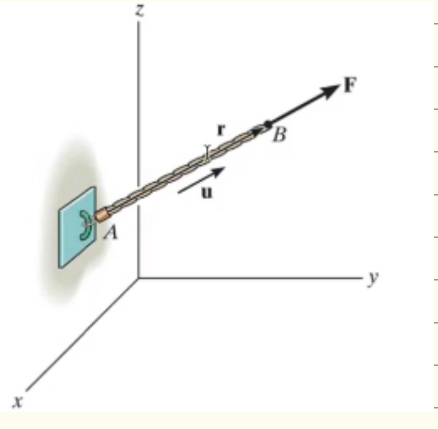
* Quite often in three dimentional statics problems, the direction of a force is specified by two points through which its line of action passes

\# such a situation is shown in figure, where the force (F) is directed along the cord (AB)

\# we can formulate (F) as a cartesian vector (r) directed from point A to point B on the cord

\# This common direction is specified by the unit vector $(u = \frac{\overline{r}}{r})$ Hence

$$\boxed{\overline{F} = Fu = F\left(\frac{\overline{r}}{r}\right) = F\left(\frac{(x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)}$$



ل هاد الحكي بطبقو

لما يكون عنا force

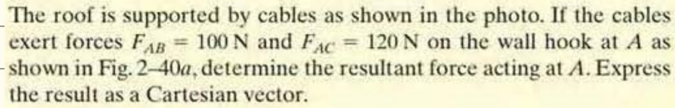
مش طالعة من ال (0,0,0)

point

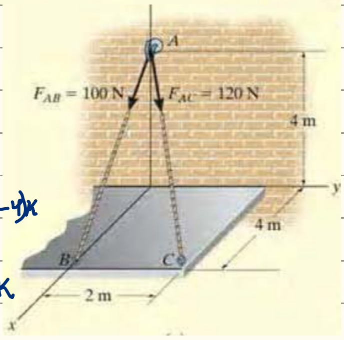
فنجيبلها ال u وبجزبها بال

magnitude تبع ال force

Example 2.12 page (62)



The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A as shown in Fig. 2–40a, determine the resultant force acting at A. Express the result as a Cartesian vector.



A(0, 0, 4)
B(4, 0, 0)
C(4, 2, 0)

so $AB = (4-0)i + (0-0)j + (0-4)k$
$AB = 4i + 0j - 4k$
$Ac = (4-0)i + (2-0)j + (0-4)k$
$Ac = 4i + 2j - 4k$

$r\overline{AB} = (4i + 0j - 4k)m$
$r\overline{Ac} = (4i + 2j - 4k)m$

$AB = \sqrt{4^2 + 4^2} = \sqrt{32} = 5.66$

$AC = \sqrt{16+4+16} = \sqrt{36} = 6$

$\overline{u}AB = \frac{4i}{5.66} - \frac{4k}{5.66}$ ⇒ & this is unitless

$\overline{F}AB = FAB \times uAB$
$= \left(100 \times \left(\frac{4i}{5.66} - \frac{4k}{5.66}\right)\right) n$
$= 70.7i - 70.7k$

$FAB = (70.7i - 70.7k)N$

$\overline{F}ac = Fac \times uac$

$uAC = \frac{4}{6}i + \frac{2}{6}j - \frac{4}{6}k$

$\overline{F}ac = 120\left(\frac{4}{6}i + \frac{2}{6}j - \frac{4}{6}k\right)$

$\overline{F}ac = (80i + 40j - 80k)N$

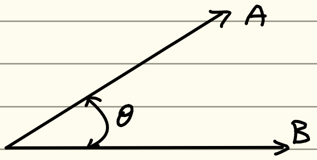
so that $FR = FAB + FAc$
$FR = (70.7i - 70.7k) + (80i + 40j - 80k)$

$FR = 150.7i + 40j - 150.7k$

2.9 Dot product (scaler product) ⇒ كلو الناتج من ضربهم بكون scaler

* Dot product: used to find the angle between two lines or components of a force parallel and perpendicular to a line

$$\boxed{\overline{A}.\overline{B} = A.B.\cos\theta}$$



where $\overline{A}, \overline{B}$ are the vectors
A & B are the magnitude of each vector
θ is the angle Between them

Laws of operation

1. commutative Law A.B = B.A
2. Multiplication of a Scaler: $a(AB) = (aA).B = A.(aB)$
3. Distributive Law: $A(B+D) = (A.B) + (A.D)$

cartesian vector formulation

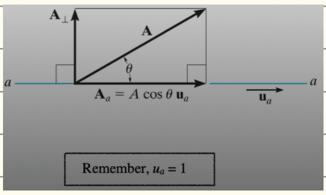
$$\boxed{A.B = A_x B_x + A_y B_y + A_z B_z}$$

Applications

the angle between two lines

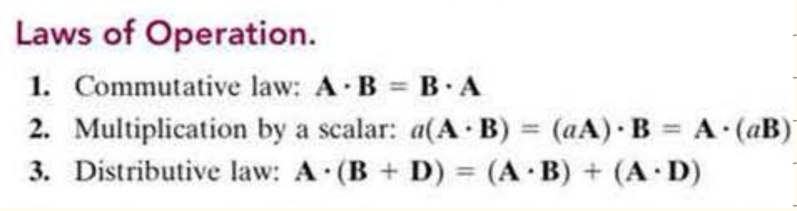
$$\theta = \cos^{-1}\left(\frac{\overline{A}.\overline{B}}{AB}\right) \quad 0° \le \theta \le 180°$$

⇒ the component of a vector parallel & perpendicular to a line :-



$A_a = A\cos\theta = \overline{A}.u_A$

$\overline{A_a} = A_a u_a$
$A\perp = A\sin a$
or
$A\perp = \sqrt{A^2 - A_a^2}$ where $A_a = A\cos\theta\, u_a$



Laws of Operation.

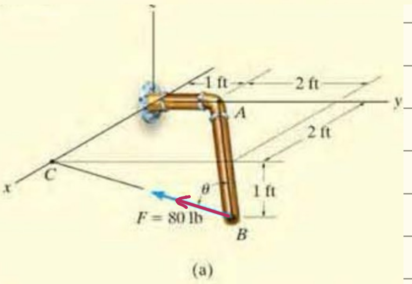
1. Commutative law: **A** · **B** = **B** · **A**
2. Multiplication by a scalar: $a(\mathbf{A}\cdot\mathbf{B}) = (a\mathbf{A})\cdot\mathbf{B} = \mathbf{A}\cdot(a\mathbf{B})$
3. Distributive law: $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$

Example (2.18)

we will assume (ft → m), (Ib → N)

The pipe in Fig. 2–46a is subjected to the force of $F = 80$ lb. Determine the angle θ between **F** and the pipe segment BA and the projection of **F** along this segment.

$A(0,1,0)$
$B(2,3,-1)$
$C(2,0,0)$



using $\bar{A}.\bar{B} = AB\cos\theta$

$\overline{rBA} = (0-2)i + (1-3)j + (0--1)K$
$\overline{rBC} = (2-2)i + (0-3)j + (0--1)K$

$\overline{rBA} = -2i - 2j + 1K$
$\overline{rBC} = 0i - 3j + 1K$

بنجيب الvector هن الرأس الي بين ال 2vectors الي محصورة بينهم الزاوية الى الطرفين

$BA = \sqrt{4+4+1} = \sqrt{9} = 3\ m$
$BC = \sqrt{9+1} = \sqrt{10} = 3.16\ m$

so now using

$A.B = A\ B\ \cos\theta$

$\theta = \cos^{-1}\left(\frac{\bar{A}.\bar{B}}{AB}\right)$

$\theta = \cos^{-1}\left(\frac{7}{3 \times 3.16}\right) \Rightarrow \theta = 42.4$

$\downarrow$
$A.B = (-2\times0) + (-2\times-3) + (1\times1) =$
$= 0 + 6 + 1 = 7$

So the angle is $\theta = 42.4^\circ$

$FAB = F \times \cos\theta$
$= 80 \times \cos 42.4$
$= 59\ N$

له هاي الطريقة السهلة لو كانت معي الزاوية

طيب لو ما كانت معي الزاوية؟ يا برجع بجيبها عن طريقة الطريقة الي فوقه أو :-

$\bar{A}.\bar{B} = \underset{\sim}{A}B\,\underset{\sim}{\cos\theta}$

لو أنا بدي Acosθ معناها بدي الB تكون ⊥ حق لا B=1؟ لما تكون ال B unit vector

so if I want to find $\overline{FAB}$

$\bar{A}.\bar{B} = A\underline{B}\cos\theta$ بس أنا بدي هاي الB=1

$\overline{FAB}.\overline{u}_{AB} = \overline{F}AB\cos\theta\ \underset{\sim}{uAB} \rightarrow =1$

$\overline{FAB}.\overline{u}AB = \overline{F}AB\cos\theta$

طبعًا أنا الvector له

الي عشري BA مش AB

Continued
$\Longrightarrow$

$\overline{r}BA = (-2i - 2j + 1K)m$

$rBA = \sqrt{4+4+1} = \sqrt{9} = 3$

$\overline{u}BA = \frac{-2}{3}i - \frac{2}{3}j + \frac{1}{3}K$

بدنا نطلع ال vector الي عالرسمة بالزهري

$\overline{u}_{BC} = \frac{\overline{r}_{BC}}{r_{BC}} = \frac{-3}{\sqrt{10}}j + \frac{1}{\sqrt{10}}K$

to change it from only magnitude to vector form

$\overline{F}_{BC} = 80\left(\frac{-3}{\sqrt{10}}j + \frac{1}{\sqrt{10}}\right)K$

$\overline{F}_{BC} = (-75.89\,j + 25.3\,K)\,N$

$\overline{F}_{BA}.u_{BA} = (-75.8\,j + 25.3K).\left(\frac{-3}{\sqrt{10}}j + \frac{1}{\sqrt{10}}K\right)$

$= 59\ N$

$F_{BC} \Rightarrow$ الي هي ال80 تاعوني

وأنا بدي أحطها على ال BA

Summery

$\ast\ \overline{F}_{BA} = F_{BA}.u_{BA}$ $\frac{\overline{r}_{BA}}{r_{BA}}$

$F_{80} \times \frac{r_{BC}}{r_{BC}} = \overline{u}_{BC} \times F_{80}$

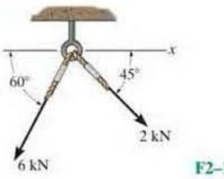
ال vector الي بوخذ عليه ال projection هو الي بدي أخليه unit vector

suggested problems of chapter2

fundamental problems

2-1

F2-1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



F2-1

$$\Sigma F_x = (2\times10^3)\cos45 - 6\times10^3\cos60$$

$$\Sigma F_x = -1585.8\ N$$

$$\Sigma F_y = -2\times10^3\sin45 - 6\times10^3\sin60$$

$$= -6610.37N$$

$$F_R = \sqrt{(-1585.8)^2 + (6610.37)^2}$$

$$\boxed{F_R = 6798\ N}$$

$$\theta = \tan^{-1}\left(\frac{6610.37}{1585.8}\right)$$

$$\theta = 76.51^\circ$$

↳ in the third Q

so that

$$\theta = 180 + 76.51$$

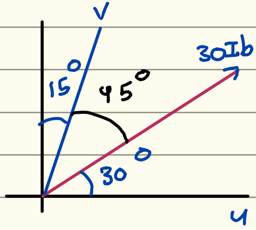
$$\boxed{\theta = 256.51}$$

2-4

H.w

$$F_u = 30\times\cos30 = 25.98$$

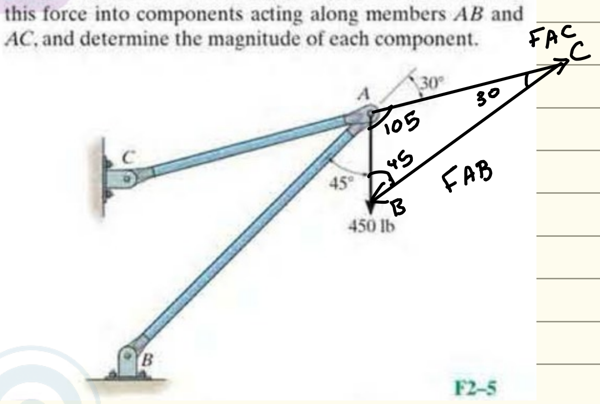
$$F_v = 30\times\cos45 = 21.21$$



the angle between the Force and (u) axis is 30 & the angle between the force & v axis is 45

2.5

F2-5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



F2-5

$$\frac{F_{AC}}{\sin45} = \frac{F}{\sin30} = \frac{F_{AB}}{\sin105}$$

$$\frac{F_{ac}}{\sin45} = \frac{450Ib}{\sin30}$$

$$F_{ac} = \frac{450\times\sin45}{\sin30}$$

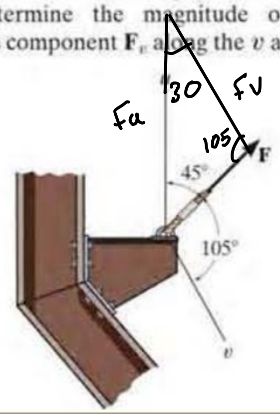
$$\boxed{F_{ac} = 636.3961\ Ib}$$

$$\frac{F_{aB}}{\sin105} = \frac{450Ib}{\sin30}$$

$$F_{aB} = \frac{450\times\sin105}{\sin30} \Rightarrow \boxed{F_{aB} = 869.33\ Ib}$$

2.6

F2–6. If force **F** is to have a component along the *u* axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the *v* axis.



F2–6

$$\frac{Fu}{\sin 105} = \frac{F}{\sin 30}$$

$$\frac{6 \times 10^3}{\sin 105} = \frac{F}{\sin 30}$$

$$\boxed{F = 3105.8285 \, N}$$

$$\frac{Fv}{\sin 45} = \frac{Fu}{\sin 105}$$

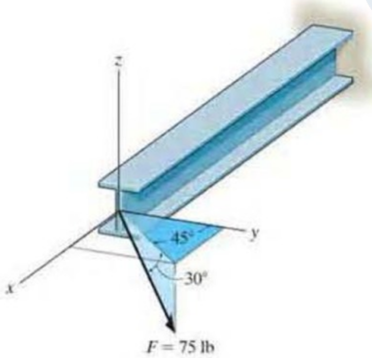
$$\frac{Fv}{\sin 45} = \frac{6 \times 10^3}{\sin 105}$$

$$Fv = \frac{6 \times 10^3 \times \sin 45}{\sin 105}$$

$$\boxed{Fv = 4392.3048 \, N}$$

2.13

F2–13. Determine its coordinate direction angles of the force.



F2–13

$Fz = -75Ib \sin 30 = -37.5 Ib$

$F_{xy} = 75Ib \times \cos 30 = 64.952 Ib$

$Fx = 64.952 \times \sin 45 = 45.43 Ib$

$Fy = 64.952 \times \cos 45 = 45.93 Ib$

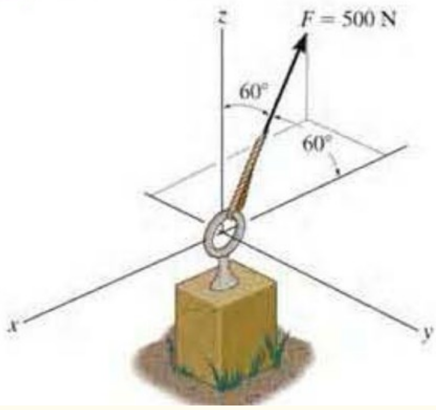
↳ so that we can use this to final :-

$\alpha = \cos^{-1}\left(\frac{45.43}{75}\right)$, $\beta = \cos^{-1}\left(\frac{45.93}{75}\right)$, $\gamma = \cos^{-1}\left(\frac{-37.5}{75}\right)$

$\boxed{\alpha = 52.238^\circ}$, $\boxed{\beta = 52.24^\circ}$, $\boxed{\gamma = 120^\circ}$

2.14

F2–14. Express the force as a Cartesian vector.



the angle between x-axis & the force is 60 α
the angle between z axis & the force is 60 γ
to find the angle between y-axis & the force β

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$(\cos 60)^2 + \cos^2 B + (\cos 60)^2 = 1$$

$$\sqrt{\cos^2 B} = \sqrt{\frac{1}{2}}$$

$$\cos B = \frac{1}{\sqrt{2}}$$

$$\beta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\beta = 45^\circ$$

so that :-

$Fx = 500 \times \cos 60$
$= 250 i \; N$
$Fy = 500 \times \cos 45$
$= 353.554 \; N \hat{j}$

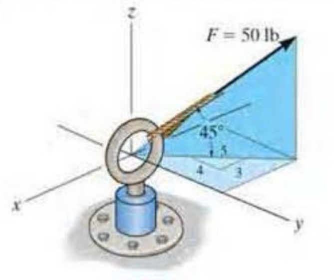
$Fz = 500 \times \cos 60$
$= -250 \hat{k} \; N$

So that the cartesian vector is

$$F = (250 i + 353.334 j + 250 \hat{k}) N$$

2.16

F2–16. Express the force as a Cartesian vector.



F2–16

$F_z = 50 \times \sin 45 = 35.35\ N$

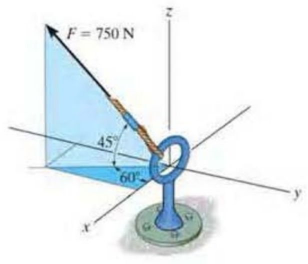
$F_{xy} = 50 \times \cos 45 = 35.35\ N$

$F_x = 35.35 \times \frac{-3}{5} = -21.21$

$F_y = 35.35 \times \frac{4}{5} = 28.28\ N$

2.17

F2–17. Express the force as a Cartesian vector.



F2–17

$F_z = 750 \times \sin 45 = 530.33\ N$

$F_{xy} = 750 \times \cos 45 = 530.33\ N$

$F_x = 530.33 \times \cos 60$

$= 265\ N$

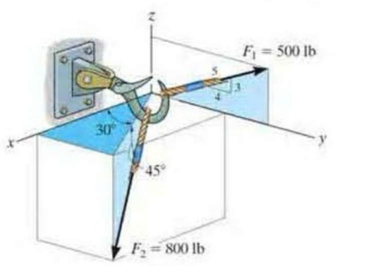
$F_y = 530.33 \times \sin 60$

$= 454.28$

$F = (265\hat{i} + 459\ \hat{j} + 530\ \hat{k})\ N$

2.18

Determine the resultant force acting on the hook.



F2–18

$F_{1x} = 0i$ Ib

$F_{1y} = 500 \times \frac{4}{5} = 400j$ Ib

$F_{1z} = 500 \times \frac{3}{5} = 300\hat{k}$ Ib

$F_1 = (0 + 400j - 300\hat{k})$ Ib

$F_2 = 800$ Ib

$F_{2z} = 800 \times \sin 45 \times -1$

$= -565.69$

$F_{2xy} = 800 \times \cos 45 \times -1$

$= -565.69$

$F_{2x} = -565.69 \times \cos 30$

$= -489.89$ Ib i

$F_{2y} = -565.69 \times \sin 30$

$= -282.845$

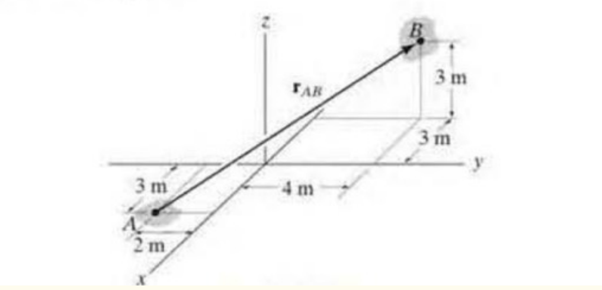
$F_2 = -489i - 282.85j - 565.69\hat{k}$

$F_R = F_1 + F_2 = 400j - 300\hat{k} - 489i - 282j - 565.69\hat{k}$

$-489\hat{i} + 118\hat{j} - 865.96\hat{k}$

2.19

F2–19. Express the position vector $\mathbf{r}_{AB}$ in Cartesian vector form, then determine its magnitude and coordinate direction angles.



$A(-3, +2, 0) \quad B=(-3, +4, 3)$

$rAB = (-3-3)\hat{i} + (+2++4)\hat{j} + (0+3)\hat{K}$

$rAB = \{-6i + 6j + 3K\} m$

$\overline{rAB} = \sqrt{6^2 + 6^2 + 3^2}$

$= \sqrt{36+36+9}$

$= \sqrt{81}$

$= 9$

$rAB = -\frac{6i}{9} + \frac{6j}{9} + \frac{3K}{9}$

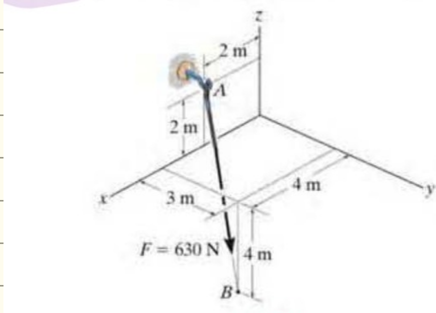
$rAB = -\frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}K$

$\alpha = \cos^{-1}\left(\frac{-2}{3}\right) \quad \beta = \cos^{-1}\left(\frac{+2}{3}\right) \quad \gamma = \cos^{-1}\left(\frac{1}{3}\right)$

$\alpha = 131.81^\circ \quad \beta = 48.19 \quad \gamma = 70.53^\circ$

2.21

F2–21. Express the force as a Cartesian vector.



F2–21

$A = (2, 0, 2) \quad B = (4, 3, -4)$

$rAB = (4-2)i + (3-0)j + (-4-2)\hat{K}$

$rAB = \{2i + 3j - 6\hat{K}\} m$

$(rAB) = \sqrt{4+9+36}$

$= \sqrt{49}$

$= 7 m$

$uAB = \left(\frac{2}{7}i + \frac{3}{7}j - \frac{6}{7}K\right) m$

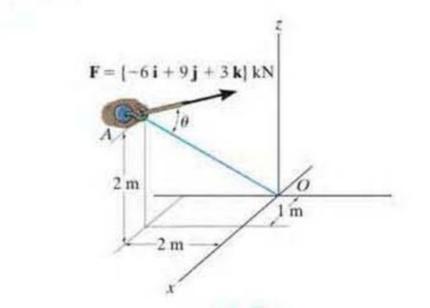
$\overrightarrow{FAB} = FAB . uAB$

$= 630 \times \left(\frac{2}{7}i + \frac{3}{7}j - \frac{6}{7}\hat{K}\right)$

$= 180i + 270j - 540\hat{k}$

2.25

F2–25. Determine the angle θ between the force and the line AO.



$A = (1, -2, 2)$

$O = (0, 0, 0)$

$rAO = (-1, 2, -2) m$

$|rAO| = \sqrt{(1)^2 + (2)^2 + (2)^2} = \sqrt{9} = 3$

$uAO = \left(\frac{-1}{3} + \frac{2}{3} - \frac{2}{3}\right) m$

$uAO = \frac{-1}{3}i + \frac{2}{3}i - \frac{2}{3}K$

$F = (-6i + 9j + 3K) KN$

$|F| = \sqrt{126} = 11.2249$

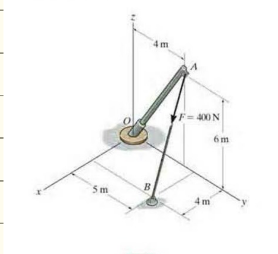
$uF = -0.534522i + 0.8j + 0.26726\hat{K}$

$\theta = \cos^{-1}(uF . uAO)$

$\theta = 57.77^\circ$

2.29

F2–29. Find the magnitude of the projected component of the force along the pipe.



F2–29

$A = (0, 4, 6)$

$B = (4, 5, 0)$

$rAB = 4i + 1j - 6K$

$|rAB| = \sqrt{16+1+36} = \sqrt{53} = 7.28$

$uAB = 0.54945\, i + 0.1373626 j - 0.82418$

$\overline{FAB} = FAB\ uAB$

$\overline{FAB} = 219.78 i + 54.94504 j - 324.672 K$

$rAO = 0i - 4j - 6K \qquad uAO = 0$

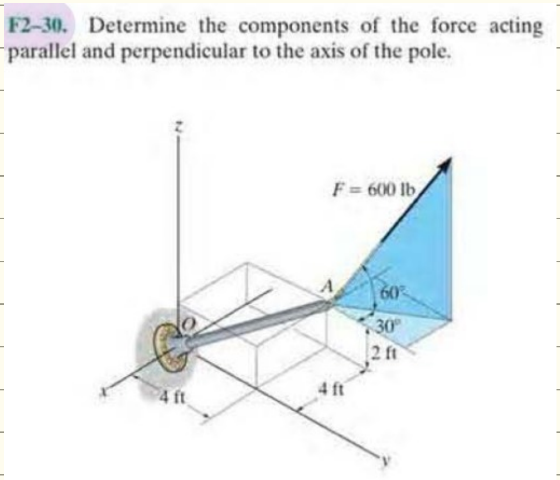
$|rAO| = \sqrt{16+36} = \sqrt{52} = 7.2111 = 2\sqrt{13}$

$\overrightarrow{Fproj} = F \times UAO$

$= (219.78 \times 0) + (54.945 \times -0.5547) + (-324.672 \times -0.832)$

$\overrightarrow{Fproj} = -30.477 + 274.287 = 243.81 \approx 244 N$

2.30



F2–30. Determine the components of the force acting parallel and perpendicular to the axis of the pole.

$O(0,0,0)$

$A(+4, +4, 2)$

$r_{OA}\ (+4, +4, 2)$

$|r_{OA}| = \sqrt{16+16+4} = \sqrt{36} = 6$

$u_{OA} = +\frac{4}{6} i + \frac{4}{6} j + \frac{2}{6} \hat{k}$

$= +\frac{2}{3} i + \frac{2}{3} j + \frac{1}{3} \hat{k}$

now to find F components

F=600Ib

$F_z = 600 \times \cos 30 = 519.615$ Ib

$F_{xy} = 600 \times \cos 60 = 300$ Ib

$F_x = 300 \times \cos 60 = 150$ Ib

$F_y = 300 \times \sin 60 = 259.81$ Ib

$\vec{F} = (150 i + 259.81 j + 519\ k)$ Ib

to find the parallel component

$F_{\parallel} = (\vec{F} \cdot \vec{u}_{OA}) \cdot \vec{u}_{OA}$

to find the magnitude   to find the parallel force

$F_{\parallel} = 100 + 172.9 + 173.2 = 446.1$ Ib

then the magnitude of force that is parallel to OA is:-

$|F_{\parallel}| = 446.1$

to find the vector

$F_{\parallel} = 446.1 \cdot \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$F_{\parallel} = 297.4 i + 297.4 j + 148.7 \hat{k}$

to find F perpendiculer

$F_{\perp} = F - F_{\parallel}$

$\vec{F} = 150 i + 259.81 j + 519\ k$

$F_{\parallel} = 297.4 i + 297.4 j + 148.7 k$

$F_{\perp} = (150 - 297.4) i + (259.81 - 297.4) j + (519 - 148.7) k$

$F_{\perp} = -147.4 - 37.54 j + 370.3 \hat{k}$

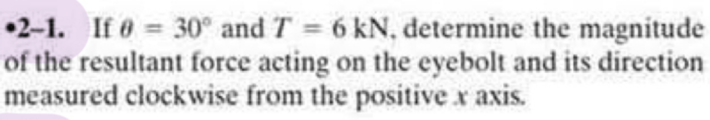
$F_{per} = \sqrt{(147.4)^2 + (37.54)^2 + (370.3)^2}$

$= 400.91 \approx 401$ Ib

هون الأسئلة ما حددت الزوايا مع أي محور
فافترضناها مع ال positive x-axis

Problems

2.1

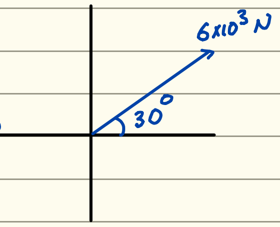


•2–1. If $\theta = 30°$ and $T = 6$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.

$T_x = T.\cos\theta$

$= 6\times10^3 \times \cos 30 = 5196.15$

$T_y = T.\sin\theta = 6\times10^3 \sin 30 = 3000$



to make sure

$T = \sqrt{(T_x)^2 + (T_y)^2}$

$= \sqrt{(5196.2)^2 + (3000)^2} \approx 6000 N$

so our answer is correct

to find the angle we will use

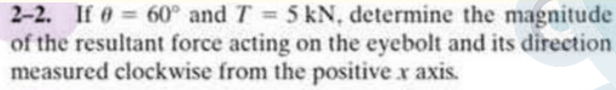
$\theta = \tan^{-1}\left(\frac{T_y}{T_x}\right) = \tan^{-1}\left(\frac{3000}{5196.2}\right) = 30$

so $\theta = 30$

but the Question needs the angle cw

$\theta_{cw} = 360 - 30 = \boxed{330^\circ}$

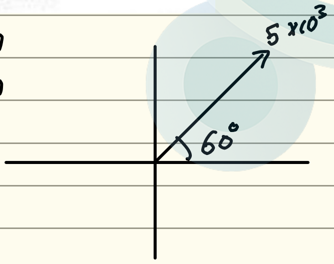
2.2



2–2. If $\theta = 60°$ and $T = 5$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.

$F_x = 5\times10^3 \times \cos 60 = 2500 N$

$F_y = 5\times10^3 \times \sin 60 = 4330.12 N$

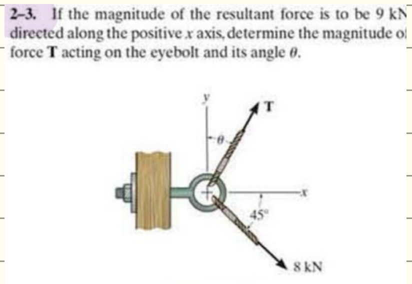


$F = \sqrt{F_x + F_y} \approx 5\times10^3 N$

$\theta = \tan^{-1}\left(\frac{4330.12}{2500}\right) \approx 60°$

the angle cw $= 360 - 60 = \boxed{300°}$

2.3



2–3. If the magnitude of the resultant force is to be 9 kN directed along the positive x axis, determine the magnitude o force **T** acting on the eyebolt and its angle θ.

$F_R = 9\times10^3 \rightarrow$ x-axis

$F_{Ry} = 0$

$F_R = 9\times10^3 i + 0 j$

$F_{Ry} = 0$

$T_y - 8\times10^3 \times \sin 45 = 0$

$T_y = 5656.85 N$

$F_{Rx} = 9\times10^3$

$T_x + 8\times10^3 \times \cos 45 = 9\times10^3$

$T_x = 3343.15$

$T = \sqrt{T_x^2 + T_y^2} = \sqrt{(5656.85)^2 + (3343.15)^2}$

$\boxed{T = 6570.88}$

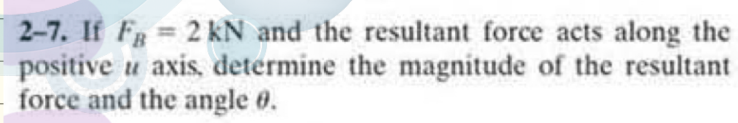
$\theta' = \tan^{-1}\left(\frac{T_y}{T_x}\right) = \tan^{-1}\left(\frac{5656.85}{3343.15}\right) = 59.42°$

& this is the angle between T and the positive x direction

$\theta = 90 - \theta' = 30.583$

$\boxed{\theta = 30.583}$

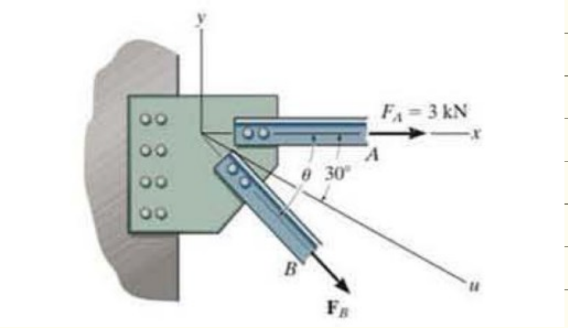
2.7



2–7. If $F_B = 2$ kN and the resultant force acts along the positive u axis, determine the magnitude of the resultant force and the angle θ.

2.8

***2–8.** If the resultant force is required to act along the positive *u* axis and have a magnitude of 5 kN, determine the required magnitude of $\mathbf{F}_B$ and its direction θ.



$Fu = FR$

$Fu = 5 \times 10^3$

$FRx = 5 \times 10^3 \times \cos 30 = 4330.13\, N$

$FRy = 5 \times 10^3 \times \sin 30 = 2500\, N$

then $FBx + FAx = FRx$

$FBx + 3 \times 10^3 = 4330.13\, N \Rightarrow FBx = 4330.13 - 3000$

$FBx = 1330.13$

& $FBy + FAy = FRy$

$FBy + 0 = 2500$

so using the equations

$FBy = 2500$

$FBx = 1330.13$

then

$FB = \sqrt{(FBy)^2 + (FBx)^2}$

$= \sqrt{(2500)^2 + (1330.13)^2} = \boxed{2831.83\ N}$

& the angle of FB is

$\theta FB = \tan^{-1}\left(\frac{FBy}{FBx}\right) = \tan^{-1}\left(\frac{2500}{1330.13}\right)$

$\theta \approx 62^\circ.8$

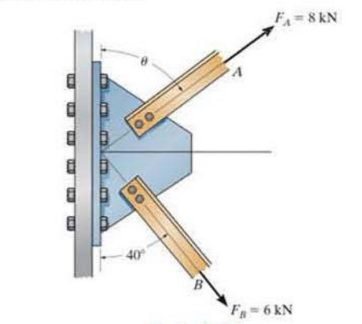
↳ from x axis

So if we want to find the angle between u axis and the FB force

$62.8 - 30 \approx \boxed{32^\circ}$

2.10

2–10. Determine the angle of θ for connecting member *A* to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



the resultant force is directed horizontally to the right ⇒ so $fRy = 0$

$FRy = 0$

$FAy - FBy = 0$

$FAy = FBy$

$FBy = 6 \times 10^3 \times \cos 40 = \bar{}4546.27\, N$ ↱ because it is directed to the negative y-axis

so $FAy = 4596.27\, N$

to find the angle we must find FAx

$FA = 8 \times 10^3$

$FAy = 4596.27$

$FA = 8 \times 10^3 \times \cos\theta$

$4596.27 = 8 \times 10^3 \times \cos\theta$

$\theta = \cos^{-1}\left(\frac{4596.27}{8 \times 10^3}\right)$

$\boxed{\theta = 54.933^\circ \rightarrow \text{we find } \theta}$

to find the magnitude of the resultant force:-

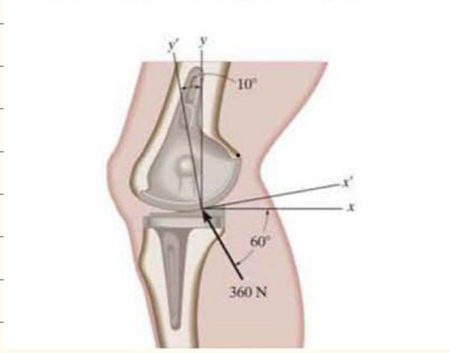
$FR = FAx + FBx \rightarrow$ because it is directed to the positive x axis

$FR = 8 \times 10^3 \times \sin 54.933 + 6 \times 10^3 \times \sin 40$

$\boxed{FR = 10404.57\, N}$

2.13

•2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x' and y axes.



x' is perpendicular on y'

so this means that the angle between x' & x is 10°

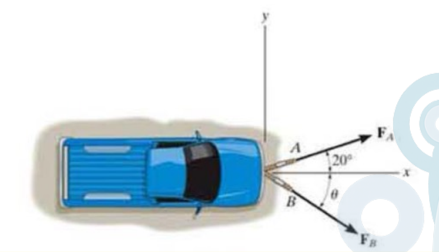
the angle between x' and the force 60+10=70

$\vec{F}_{x'} = 360 \times \cos 70^\circ = 123.13$ N

$\vec{F}_{y'} = 360 \times \sin 70 = 338.24$ N

2.19

2–19. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 20° from the x axis as shown.



$FR = 950i$

$FRy = FAy + FBy =$

$FAy = FA \times \sin 20$

$= 0.342 FA$

$FBy = FB \times \sin\theta$

so that $0.342FA + FB\sin\theta = 0$

$$FB = \frac{-FA(0.342)}{\sin\theta}$$

$FRx = 950$

$FAx + FBx = 950$

$$FA \cdot \cos(20) + \left(\frac{-FA(0.342)}{\sin\theta}\right) \times \cos\theta = 950$$

$FA\ (0.9397) - 0.342FA \cot\theta = 950$

$$FA\left(0.9397 - 0.342\frac{\cos\theta}{\sin\theta}\right) = 950$$

to find fb min the angle must be 90

$\theta = 90$

$FA\ (0.9397 - 0) = 950$

$0.9397FA = 950$

$FA = 1010.96$

now using this to find FB

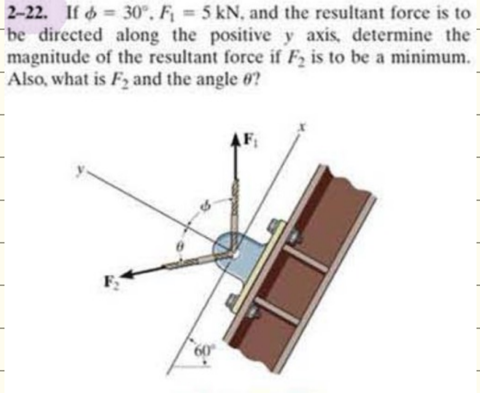
$$FB = \frac{-FA(0.342)}{\sin 90}$$

$$FB = \frac{-1010.96(0.342)}{1}$$

$FB = -345.74832$

↳ the negative sign for the negative direction

2.22



2–22. If $\phi = 30°$, $F_1 = 5$ kN, and the resultant force is to be directed along the positive y axis, determine the magnitude of the resultant force if F_2 is to be a minimum. Also, what is F_2 and the angle θ?

$F_{1x} = 5 \times 10^3 \times \sin 30 = 2500$ N

$F_{1y} = 5 \times 10^3 \times \cos 30 = 4330.127$ N

ΣF is directed along positive y-axis

so that

$\Sigma F_x = 0$

$F_{1x} + F_{2x} = 0$

$2500 - F_2 \sin\theta = 0$

$$\boxed{F_2 = \frac{2500}{\sin\theta}}$$

to minimize F_2 we have to get the maximum value for $\sin\theta$ & the maximum value when $\theta = 90$

so $\theta = 90$, $\sin 90 = 1$

$F_2 = \frac{2500}{\sin 90} \Rightarrow F_2 = 2500$ N

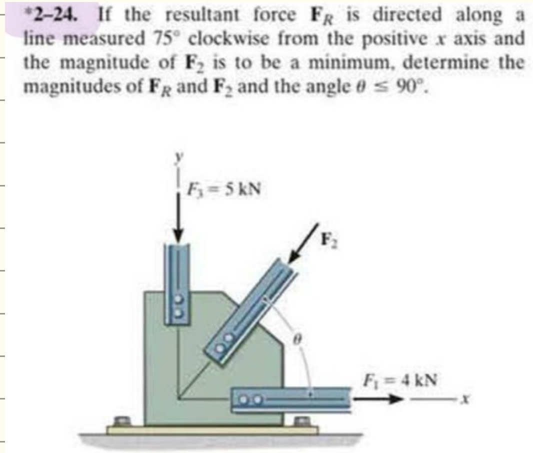
$F_{1y} + F_{2y} = F_{Ry}$

$4330.127 + F_{2y} = F_{Ry}$ $\qquad F_{Ry} = 4330.127$

$4330.127 + 0 = F_{Ry} \Rightarrow$

we already said that the angle is $\theta = 90$ between y-axis & the force, so it is fully directed to xaxis so the $F_{2y} = 0$

2.24



***2–24.** If the resultant force $\mathbf{F}_R$ is directed along a line measured 75° clockwise from the positive x axis and the magnitude of $\mathbf{F}_2$ is to be a minimum, determine the magnitudes of $\mathbf{F}_R$ and $\mathbf{F}_2$ and the angle $\theta \le 90°$.

to let F_2 be min it has to be perpendicu
to the resultant force

$\theta = 90 - 75 = 15$

we can find the resultant for F_1, F_2

$F' = \sqrt{4^2 + 5^2}$

$= \sqrt{16 + 25} = \sqrt{41} = 6.4$ KN

$\tan\alpha = \frac{5}{4}$

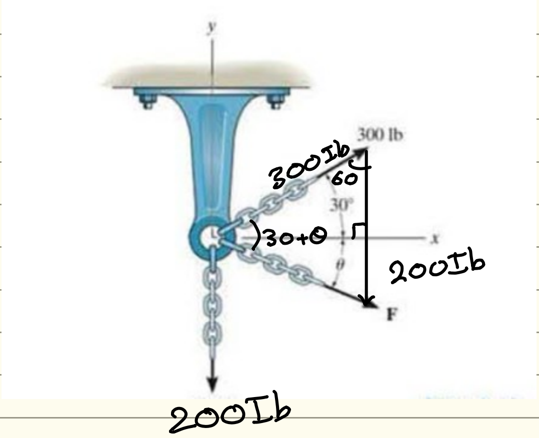
$\alpha = \tan^{-1}\left(\frac{5}{4}\right)$

$\alpha = 51.34°$

$F_2 = 6403$ N $\cdot \cos(15 + 51.34) = 2569.57$ N

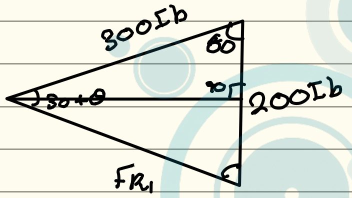
$F_R = 6403 \cdot \sin(15 + 51.34) = 5864.78$ N

2–30. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.



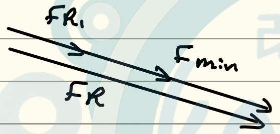
$$FR_1 = \sqrt{(200^2) + (300)^2 - 2(200)(300)\cos 60}$$

$FR_1 = 264.575\ \text{Ib}$



$$\frac{\sin(30+\theta)}{200} = \frac{\sin(60)}{264.575}$$

$$\sin(30+\theta) = \frac{200 \times \sin 60}{264.575}$$



$\sin(30+\theta) = 0.654654$

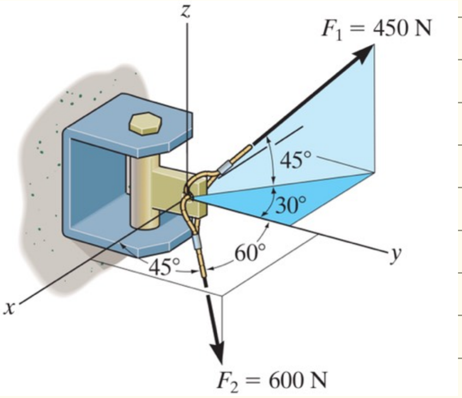
$30 + \theta = 40.9$
$-30 \qquad -30$

$\theta = 10.9^{\circ}$

$FR = FR_1 + F$
$500 = 264.575 + F$
$F = 235.425\ \text{Ib}$

2–59. Determine the coordinate angle γ for $\mathbf{F}_2$ and then express each force acting on the bracket as a Cartesian vector.

***2–60.** Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



to find angle γ for F_2

$\cos^2 45 + \cos^2 60 + \cos^2 \gamma = 1$

$\cos^2 \gamma = \frac{1}{4}$

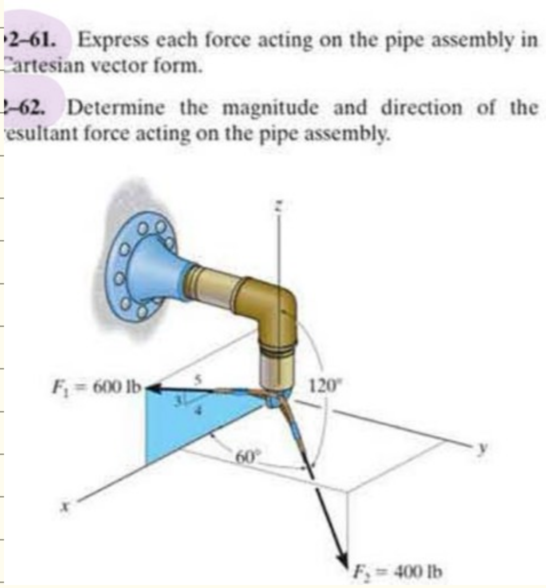
$\cos \gamma = \pm\frac{1}{2} \Rightarrow \gamma = 60^{\circ}$ ✗
$\gamma = 120^{\circ}$ ✓

Finding the resultant force:-

$F_1 z = 450 \times \sin 45$
$= 318.2\ N$
$F_1 xy = 450 \times \cos 45$
$F_1 xy = 318.2\ N$
$F_1 y = 318.2 \times \cos 30$
$= 275.568$
$F_1 x = 318.2 \times \sin 30$
$= 159.1$
$F_2 = 600 N$
$F_2 z = 600 \times \cos 120$
$= -300 N$
$F_2 x = 424.264 N$
$F_2 y = 600 \times \cos 60$
$= 300 N$

$F_2 = (424.264\, i + 300 j - 300 \hat{k})\, N$
$F_1 = (159.1\, i + 275.568 j + 318.2 \hat{k})\, N$

$FR = F_1 + F_2$
$= 583.364 i + 575.568 j + 18.2 \hat{k}$



2–61. Express each force acting on the pipe assembly in Cartesian vector form.

–62. Determine the magnitude and direction of the esultant force acting on the pipe assembly.

$F_{1x} = 600 \times \frac{4}{5} = 480$ Ib
$F_{1z} = 600 \times \frac{3}{5} = 360$ Ib
$F_{1y} = 0$

$F_1 = (480 i + 0j + 360K)$ Ib

to find B
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$(\cos 120)^2 + (\cos B)^2 + (\cos 60)^2 = 1$
$(\cos B)^2 = \frac{1}{2}$
$\cos B = \frac{\pm 1}{\sqrt{2}} \Rightarrow B = \cos^{-1}\frac{1}{\sqrt{2}} \Rightarrow B = 45, ✓$
$B = 135$ ✗

$F_{2z} = 400 \times \cos 120$
$F_{2z} = -200$ Ib
$F_{2x} = 400 \times \cos 60$
$= 200$ Ib
$F_{2y} = 400 \times \cos 45$
$= 282.843$ Ib
$F_2 = (200 i + 282.843 j - 200 \hat{k})$ Ib

to find FR
$\overrightarrow{FR} = (F_1 + F_2 = 680 i + 282.843 j + 160 K)$ Ib

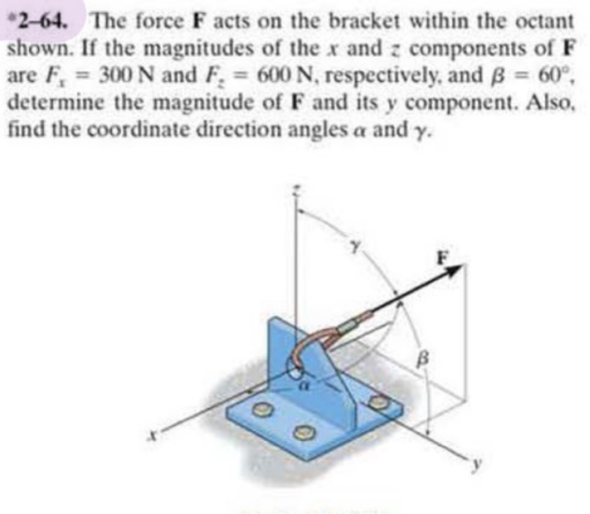
$|FR| = \sqrt{(680)^2 + (282.84)^2 + (160)^2}$

$FR = 753.64$ Ib ≈ 754 Ib

$\alpha = \cos^{-1}\left(\frac{680}{753.64}\right) = 25.54$

$\beta = \cos^{-1}\left(\frac{282.84}{753.64}\right) = 67.457$

$\gamma = \cos^{-1}\left(\frac{160}{753.64}\right) = 77.743$



***2–64.** The force **F** acts on the bracket within the octant shown. If the magnitudes of the x and z components of **F** are $F_x = 300$ N and $F_z = 600$ N, respectively, and $\beta = 60°$, determine the magnitude of **F** and its y component. Also, find the coordinate direction angles α and γ.

Probs. 2–63/64

$F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$

$F = \sqrt{(300)^2 + (F_y)^2 + (600)^2}$

$F^2 = \left(\sqrt{(F_y)^2 + 450000}\right)^2$

$F^2 = (F_y)^2 + 450000$

$F_y = F \times \cos 60 = 0.5 F$

$F^2 = (0.5F)^2 + 450000$
$F^2 = 0.25F^2 + 450000$
$0.75F^2 = 450000$

$F^2 = 600000$
$F = 774.5966$ N

$F_y = 774.596 \times \cos 60$
$F_y = 387.29$ N

$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(-\frac{300}{774.6}\right)$

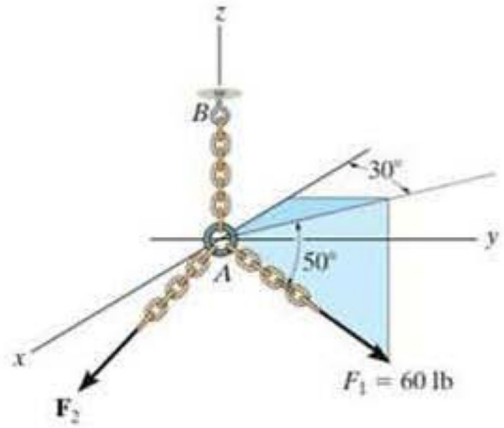
$\alpha = 113$

$\gamma = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.6}\right)$

$\gamma = 39.23°$

•2–65. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.

2–66. Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.



$F_1 = 60 Ib$

$F_{1}z = -60 \times \sin 50$

$F_{1z} = -45.46 Ib$

$F_{1xy} = 60 \times \cos 50$

$= 38.567 Ib$

$F_{1x} = -38.567 \times \cos 30$

$= -33.4 Ib$

$F_{1y} = 38.567 \times \sin 30$

$= 14.2835 Ib$

$F_1 = -33.4i + 14.2835j - 45.46K$

$F_R = F_1 + F_2$

$0i + 0j - 100K = (-33.4 + F_{2x})i + (14.2835 + F_{2y})j + (-45.46K + F_2 Z)$

$0 = -33.4 + F_2 x$

$F_2 x = 33.4 Ib$

$0 = 19.2835 + F_2 y$

$F_2 y = -19.2835 Ib$

$-100 = -45.46 + F_2 Z$

$F_2 Z = -54.04 Ib$

so $F_2 = 33.4i - 19.2835j - 54.04\hat{K}$

$|F_2| = \sqrt{(33.4)^2 + (19.2835)^2 + (54.04)^2}$

$|F_2| = 66.34 \approx 66.4 Ib$

to find coordinate direction angles

$\alpha = \cos^{-1}\left(\frac{33.4}{66.34}\right) \Rightarrow \alpha = 59.746^\circ$

بس تجيبي الزوايا عوضي السالب

$\beta = \cos^{-1}\left(\frac{-19.2835}{66.39}\right)$

$\beta = 106.88^\circ \approx 107^\circ$

$\gamma = \cos^{-1}\left(\frac{-54.04}{66.39}\right)$

$\gamma = 144.486^\circ \approx 144^\circ$

2-66

to find coordinate direction angles

$u_{F_1} = -\cos 50 \times \cos 30 i + \cos 50 \sin 30 j - \sin 50 \hat{K}$

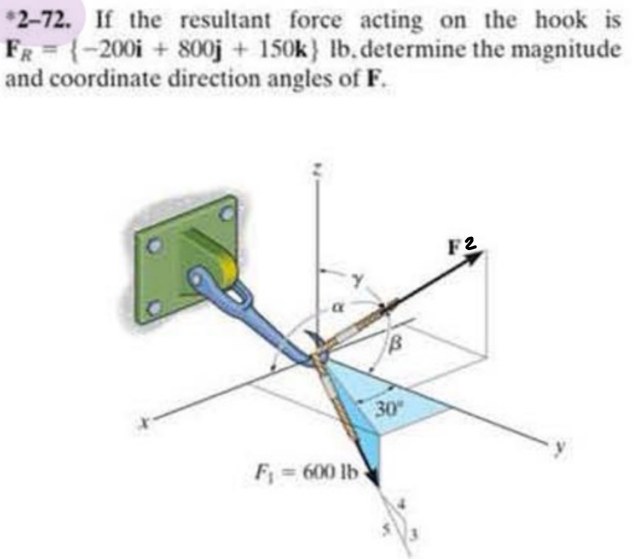
$u_{F_1} = -0.55667 + 0.32134 - 0.766\hat{K}$

$\alpha = \cos^{-1}(-0.55667) \Rightarrow \alpha = 123.826^\circ \approx 124^\circ$

$\beta = \cos^{-1}(0.32139) \Rightarrow \beta = 71.2529^\circ \approx 71.3^\circ$

$\gamma = \cos^{-1}(-0.766) \Rightarrow \gamma = 139.996^\circ = 140^\circ$

2-72


***2–72.** If the resultant force acting on the hook is $F_R = \{-200i + 800j + 150k\}$ lb, determine the magnitude and coordinate direction angles of F.

$F_1z = -600 \times \frac{3}{5}$

$= -360$

$F_1xy = 600 \times \frac{4}{5}$

$= 480$

$F_1x = 480 \times \sin 30$

$= 240$

$F_1y = 480 \times \cos 30$

$= 415.692$

$F_1 = 240i + 415.692j - 360\hat{k}$

$FR = -200i + 800j + 150K$

$FR = F_1 + F_2$

$-200i + 800j + 150K = (240 + F_2x)i + (415.692 + F_2y)j + (-360 + F_2z)K$

$-200 = 240 + F_2x$

$F_2x = -440$

$415.692 + F_2y = 800$

$F_2y = 800 - 415.692$

$F_2y = 384.31$ Ib

$-360 + F_2z = 150$

$F_2z = 510$

$F_2 = -440i + 384.31j + 510\hat{k}$

$|F_2| = \sqrt{(-440)^2 + (384.31)^2 + (510)^2}$

$F_2 = 775.5$ Ib

$\alpha = \cos^{-1}\left(\frac{-440}{775.5}\right)$

$\alpha = 124.5674$

$\alpha \approx 125°$

$\beta = \cos^{-1}\left(\frac{384.31}{775.5}\right)$

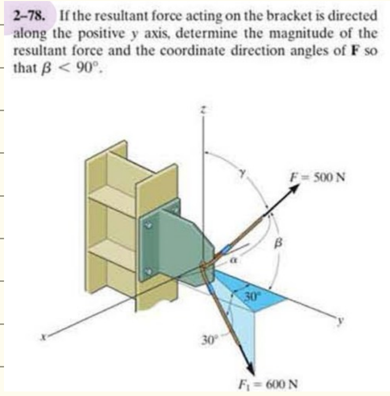
$\beta = 60.3°$

$\gamma = \cos^{-1}\left(\frac{510}{775.5}\right)$

$\gamma = 48.88°$

$\approx 49°$

2-78


2–78. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of F so that $\beta < 90°$.

$F_1z = -600 \times \sin 30$

$= -300$

$F_1xy = 600 \times \cos 30$

$= 514.615$ Ib

$F_1x = 519.615 \times \sin 30$

$= 254.8$ Ib

$F_1y = 519.615 \times \cos 30$

$= 449.99 = 500$

$FR = 0i + FRyj + 0\hat{k}$

$FR = F_1 + F_2$

$0i + FRyj + 0K = (259.8 + F\cos\alpha) + (500 + F\cos\beta) + (-300 + F\cos\gamma)$

$0 = 259.8 + F\cos\alpha$

$F\cos\alpha = -259.8$

$Fx = -259.8$

$-300 + F\cos\gamma = 0$

$F\cos\gamma = 300$

$Fz = 300$

$F = \sqrt{Fx^2 + Fy^2 + (Fz)^2}$

$500 = \sqrt{(-259.8)^2 + Fy^2 + (300)^2}$

$(500)^2 = \left(\sqrt{157496 + Fy^2}\right)^2$

$250000 = 157496 + Fy^2$

$92504 = Fy^2$

$Fy = \pm 304.144 \Rightarrow Fy = +304.144$ Ib

لانو عطانا ال β اقل من $\boxed{90}$

to find coordinate direction angles

$\alpha = \cos^{-1}\left(\frac{-259.8}{500}\right) \Rightarrow \alpha = 121.305°$

$\beta = \cos^{-1}\left(\frac{304.144}{500}\right) \Rightarrow \beta = 52.634°$

$\gamma = \cos^{-1}\left(\frac{300}{500}\right) \Rightarrow \gamma = 53.13°$

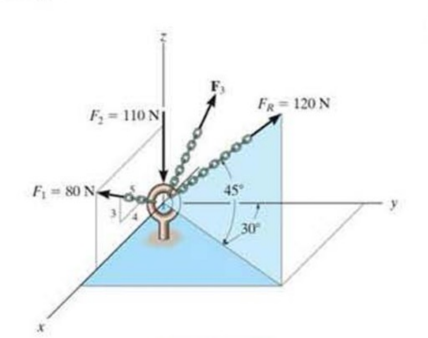
$F_R = F + F_1$

$F_R = F_1 = 259.8i + 450k - 300k +$
$-259.8i + 304.144j + 300\hat{k}$

$F_R = 0i + 754.144j + 0\hat{k}$

2.84

*2–84. Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



$u_{F1} = \frac{4}{5}i + 0j + \frac{3}{5}K$

For F_1 $\quad \alpha = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow \alpha = 36.87°$

$\beta = \cos^{-1}(0) \Rightarrow \beta = 90°$

$\gamma = \cos^{-1}\left(\frac{3}{5}\right) \Rightarrow \gamma = 53.13°$

For F_R

$u_{FR} = \cos 45 \sin 30 i + \cos 45 \cos 30 j + \sin 45 K$

$u_{FR} = 0.35355 i + \frac{\sqrt{6}}{4} j + \frac{1}{\sqrt{2}} K$

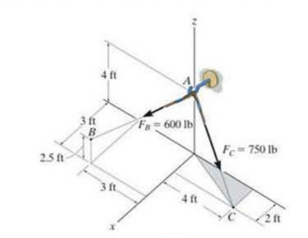
$\alpha = \cos^{-1}(0.35355) \Rightarrow \alpha = 69.3°$

$\beta = \cos^{-1}\left(\frac{\sqrt{6}}{4}\right) \Rightarrow \beta = 52.24°$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \gamma = 45°$

2.89

•2–89. Determine the magnitude and coordinate direction angles of the resultant force acting at A.



$A = (0, 0, 4)$
$B = (3, -3, 2.5)$
$C = (2, 4, 0)$

$\vec{r_{AB}} = (3, -3, -1.5)$ m
$r_{AC} = (2, 4, -4)$ m

$|r_{AB}| = \sqrt{9+9+(1.5)^2} = 4.5$ m

$|r_{AC}| = \sqrt{4+16+16} = 6$ m

$u_{rAB} = \frac{3}{4.5} i - \frac{3}{4.5} j - \frac{1.5}{4.5} \hat{K}$

$u_{rAC} = \frac{1}{3} i + \frac{2}{3} j - \frac{2}{3} \hat{K}$

$\vec{F_{AB}} = F_B \times u_{rAB}$
$= 600 \times \left(\frac{3}{4.5} i - \frac{3}{4.5} j - \frac{1.5}{4.5} K\right)$
$= (400i - 400j - 200k)$ Ib

$\vec{F_{AC}} = F_C \times u_{AC}$
$F_{AC} = 750 \times \left(\frac{1}{3} i + \frac{2}{3} j - \frac{2}{3} \hat{k}\right)$

$F_{AC} = (250i + 500j - 500\hat{k})$ Ib

$F_R = F_{AB} + F_{AC}$
$400i - 400j - 200K + 250i + 500j - 500K$

$F_R = 650 + 100j - 700K$

$|F_R| = \sqrt{(650)^2 + (100)^2 + (-700)^2}$

$F_R = 960.4686$ Ib

$\alpha = \cos^{-1}\left(\frac{F_{Rx}}{F_R}\right) = \cos^{-1}\left(\frac{650}{960.47}\right)$

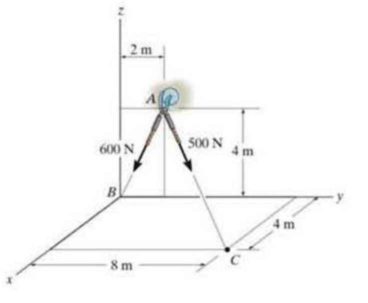
$\alpha = 47.41$

$\beta = \cos^{-1}\left(\frac{100}{960.47}\right) \Rightarrow \beta = 84°$

$\gamma = \cos^{-1}\left(\frac{-700}{960.47}\right) \Rightarrow \gamma = 136.78 \simeq 137°$

2-90

2-90. Determine the magnitude and coordinate direction angles of the resultant force.



A(0, 2, 4)
B(0,0,0)
C(4,8, 0)

$r_{AB} = 0i - 2j - 4k$
$r_{AC} = 4i + 6j - 4k$

$|r_{AB}| = \sqrt{0+4+16} = \sqrt{20} = 4.47$

$|r_{AC}| = \sqrt{16+36+16} = 8.246$

$u_{AB} = \frac{0}{4.47} i - \frac{2}{4.47} j - \frac{4}{4.47} k$

$u_{AB} = 0i - 0.4474j - 0.8948 \hat{k}$

$u_{AC} = \frac{4}{8.246} i + \frac{6}{8.246} j - \frac{4}{8.246} k$

$u_{AC} = 0.485i + 0.7276j - 0.485k$

$\vec{F}_{AB} = F_{AB} \times u_{AB}$
$= 600 \times (-0.4474j - 0.8948\hat{k})$
$= -268.44j - 538.8k$

$\vec{F}_{AC} = F_{AC} \times u_{AC}$
$= 500 \times (0.485i + 0.7276j - 0.485k)$
$= 242.5i + 363.8j - 242.5\hat{k}$

$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC}$

$-268.44j - 538.8k + 242.5i + 363.8j - 242.5\hat{k}$

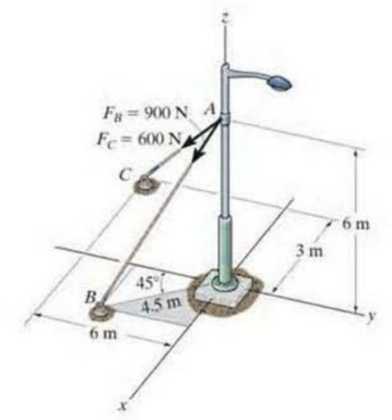
$F_R = 242.5i + 95.36j - 781.3k$

$|F_R| = \; . - - -$

وبعدين بنحسب الزوايا .

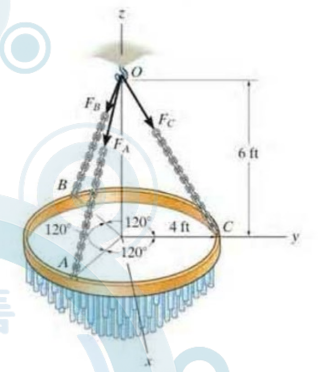
2-91

2-91. Determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



بنفس الطريقة

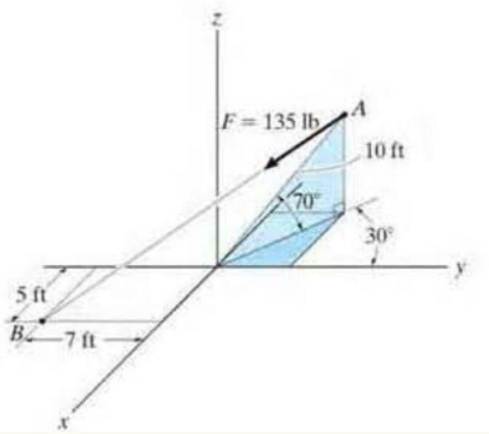
2-94. The chandelier is supported by three chains which are concurrent at point *O*. If the resultant force at *O* has a magnitude of 130 lb and is directed along the negative *z* axis, determine the force in each chain.



ب هم يطلعو يساوو بعضهن و بنفس الطريقة

2. 95

2–95. Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



ال 10 هو طول ال vector من A لـ B

$A = 10\cos70\sin30 i + 10\cos70\cos30 j + 10\sin70 \hat{k}$

$A = -1.71 i + 2.962 j + 9.397 K$

$B = (5, -7, 0)$

$B = 5i - 7j + 0K$

so $r_{AB} = (6.71 i - 4.462 j + 9.397 K) m$

$|r_{AB}| = \sqrt{(6.71)^2 + (-9.962)^2 + (9.397)^2}$

$|r_{AB}| = 15.25$ Ft

$u_{AB} = \frac{6.71}{15.25} + \frac{-9.962}{15.25} + \frac{9.397}{15.25}$

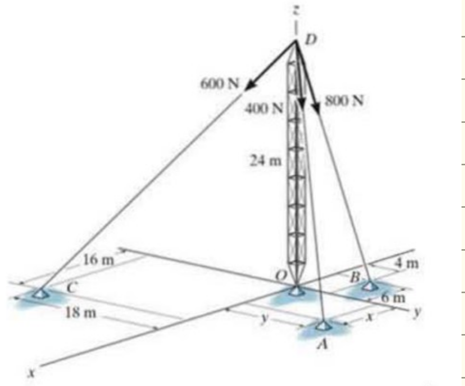
$u_{AB} = 0.44 i - 0.65326 j - 0.61619672 K$

$\alpha = \cos^{-1} 0.44 \Rightarrow \alpha = 63.896°$

$\beta = \cos^{-1} 0.65326 \Rightarrow \beta = 130.78° \approx 131°$

$\gamma = \cos^{-1} -0.616196 \Rightarrow \gamma = 128.0389$

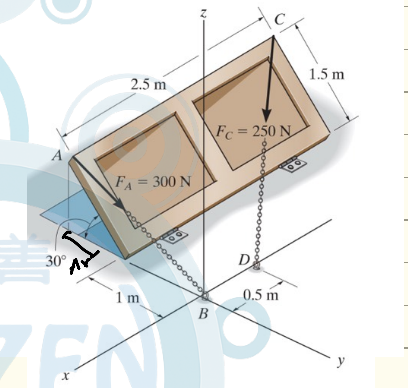
***2–96.** The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



نفس طريقة الحل الي قبل

2– 97

•2–97. The door is held opened by means of two chains. If the tension in *AB* and *CD* is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



$\sin 30 = \frac{Az}{1.5}$

$Az = 0.75 m$

$\cos 30 = \frac{Ax}{1.5}$

$Ay = -1.3 \mp 1$

$Ay = -2.3$

$Ax = 0$

so that

$A(0, -2.3, 0.75)$

$B(0, 0, 0)$

$D(-0.5, 0, 0)$

$C(-2.5, -2.3, 0.75)$

$r_{AB} = 0i + 2.3j - 0.75\hat{k}$

$r_{CD} = 2i + 2.3j - 0.75K$

وبعدين نفس الحل التقليدي

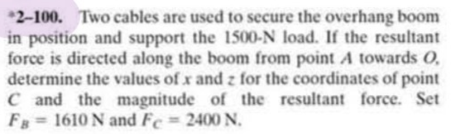
for point C

$Cx = -2.5$

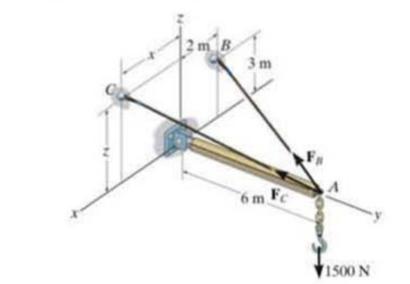
$Cy = Ay = -2.3$

$Cz = Az = 0.75$

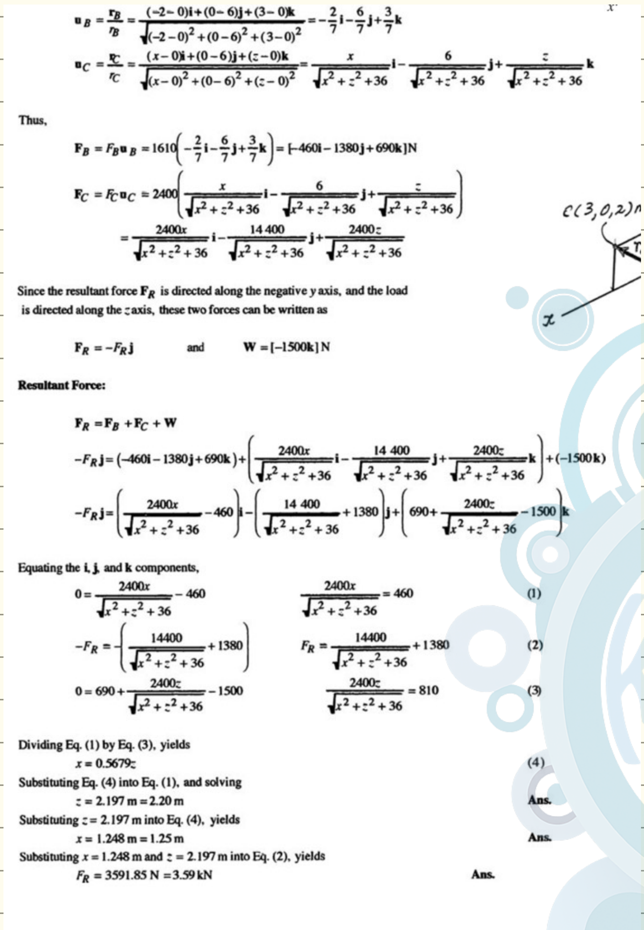
2.100



*2–100. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



هي حلو



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}$$

C(3,0,2)

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\,\text{N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k})+\left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$$z = 2.197\text{ m} = 2.20\text{ m} \qquad \textbf{Ans.}$$

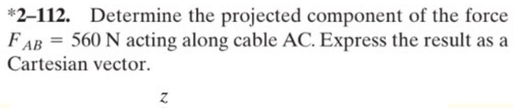
Substituting $z = 2.197$ m into Eq. (4), yields

$$x = 1.248\text{ m} = 1.25\text{ m} \qquad \textbf{Ans.}$$

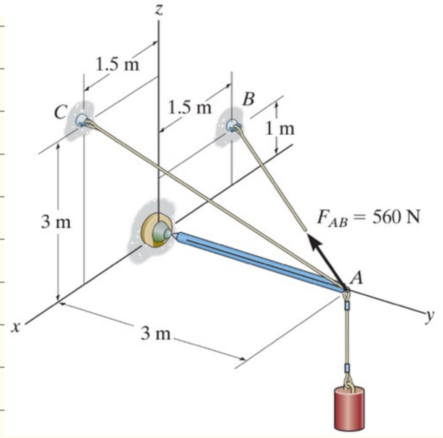
Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$$F_R = 3591.85\text{ N} = 3.59\text{ kN} \qquad \textbf{Ans.}$$

2.112



*2–112. Determine the projected component of the force $F_{AB} = 560$ N acting along cable AC. Express the result as a Cartesian vector.



A (0,3,0)

B (-1.5,0,1)

C(1.5,0,3)

$rAB = -1.5i - 3j + 1K$

$rAC = 1.5i - 3j + 3K$

$|rAB| = \sqrt{(-1.5)^2 + (-3)^2 + 1} = 3.5$

$|rAC| = \sqrt{(1.5)^2 + (-3)^2 + 3^2} = 4.5$

$uAB = \frac{-1.5}{3.5} i + \frac{-3}{3.5} j + \frac{1K}{3.5}$

$uAB = 0.42857i - 0.857143j + 0.2857K$

$uAC = \frac{1.5i}{4.5} - \frac{3j}{4.5} + \frac{3K}{4.5}$

$uAC = 0.33i - 0.66j + 0.666K$

$\overrightarrow{FAB} = FAB \times uAB$

$560 \times (0.42857i - 0.857143j + 0.2857K)$

$\overrightarrow{FAB} = -240i - 480j + 160K$

$(FAB)AC = FAB \cdot uAc$

$(-240i - 480j + 160K) \cdot \left(\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}K\right)$

$-80 + 320 + 106.66$

$= 346.667\,N$

to express it as cartesian vector

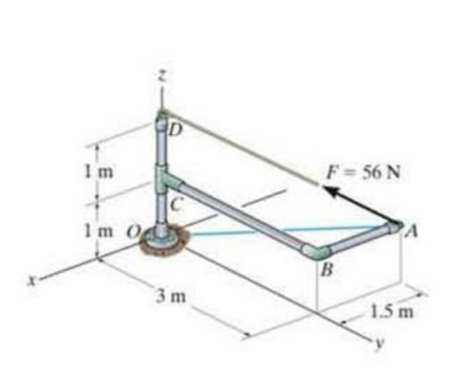
$(\overrightarrow{FAB})AC = (FAB)AC \times uAC$

$346.667 \times \left(\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}K\right)$

$= 115i - 231.11j + 231.11$

2. 113

•**2–113.** Determine the magnitudes of the components of force $F = 56$ N acting along and perpendicular to line AO.



$A\ (-1.5, 3, 1)$
$D\ (0, 0, 2)$
$C\ (0, 0, 1)$
$O\ (0, 0, 0)$
$B\ (0, 3, 1)$

$r_{AD} = 1.5\,i - 3j + 1K$

$r_{AO} = 1.5i - 3j - 1K$

$|r_{AD}| = \sqrt{(1.5)^2 + 3^2 + 1^2} = 3.5$

$u_{AD} = \frac{1.5}{3.5}i - \frac{3}{3.5}j + \frac{1}{3.5}K$

$u_{AD} = 0.42857\,i - 0.8571\,j + 0.285714\,K$

$|r_{AO}| = \sqrt{(1.5)^2 + (3)^2 + 1^2} = 3.5$

$u_{AO} = 0.42857\ i - 0.8571j - 0.2857K$

$\vec{F} = F \times u_{AD} = 56 \times (0.42857i - 0.8571j + 0.2857K)$

$\vec{F} = 24i - 48j + 16K$

هيك جبنا ال vector وال

لـ عوض cartesian coordinates

$(F_{AO})_{parallel} = F \cdot u_{AO}$

$(24i - 48j + 16K) \cdot (\frac{3}{7}i - \frac{6}{7}j - \frac{2}{7}K)$

$(\frac{3}{7} \times 24) + (-48 \times -\frac{6}{7}) + (16 \times -\frac{2}{7})$

$10.2857 + 41.142857 + -4.57143$
$= 46.857127$
$\approx 46.9\ N$

فيثاغورس

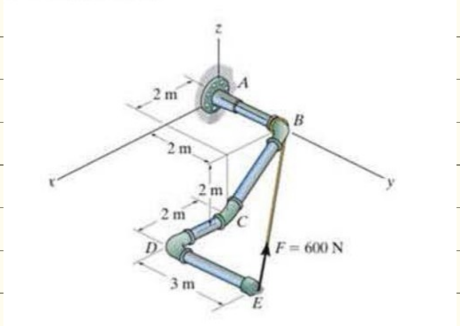
$(F_{AO})_{perpend} = \sqrt{F^2 - (F_{AO})_{paralel}}$

$= \sqrt{(56)^2 - (46.9)^2}$

$= 30.6\ N$

2. 115

2–115. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



$A(0, 0, 0)$
$B(0, 2, 0)$
$C(2, 2, -2)$
$D(4, 2, -2)$
$E(4, 5, -2)$

$r_{EB} = -4i - 3j + 2K$

$r_{ED} = 0i - 3j + 0K$

$|r_{EB}| = \sqrt{16 + 9 + 4} = 5.385$

$r_{ED} = \sqrt{9} = 3 \Rightarrow u_{ED} = \frac{-3j}{3} = -j$

$u_{EB} = \frac{-4}{5.385}i + \frac{-3}{5.385}j + \frac{2}{5.385}K$

$u_{EB} = -0.7428i - 0.5571j + 0.3714K$

$\vec{F}_{EB} = F_{EB} \times u_{EB}$

$= 600 \times (-0.7428i - 0.5571j + 0.3714K)$

$[445.68i - 334.26j + 222.84K]\ N$

$(F_{ED})_{par} = \vec{F}_{EB} \times u_{ED}$

$(445.68i - 334.26j + 222.84K)(-j)$

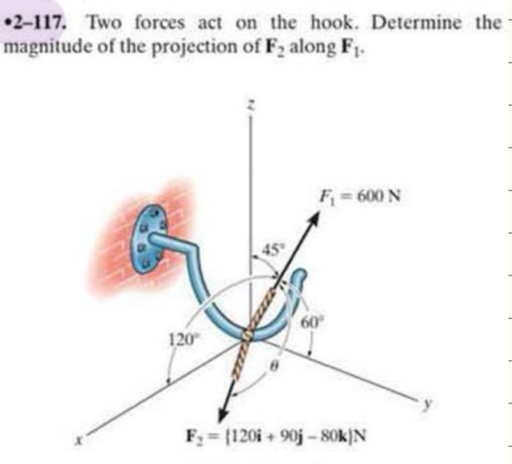
$0 + 334.26j + 0 = 334.2$
$\approx 334\ N$

$F_{ED\,per} = \sqrt{F^2 - F_{parallel}}$

$F_{EDper} = \sqrt{(600)^2 - (334)^2}$

$F_{ED\,per} = 498.44157\ N$
$\approx 498\ N$

2.117



•2–117. Two forces act on the hook. Determine the magnitude of the projection of $\mathbf{F}_2$ along $\mathbf{F}_1$.

$F_2=(120i+90j-80k)\ N$

$F_1= 600\times\cos120 i+ 600\times\cos 60 + 600\times\cos45k$

$F_1= -300i+300j+424.26\hat{k}$

$u_{F_1}= \cos120i + \cos60j+\cos45k$

$u_{F_1}= -\frac{1}{2}i+\frac{1}{2}j+\frac{1}{\sqrt{2}}k$

$(F_2)_{F_1} = F_2\times u_{F_1}$

$(120i+90j-80k)\times(-\frac{1}{2}i+\frac{1}{2}j+\frac{1}{\sqrt{2}}k)$

$-60+45+-56.5685$

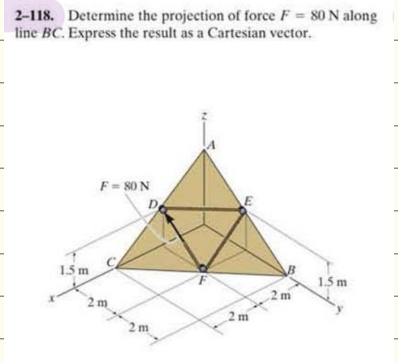
$=-71.5685$

the magnitude $=|\text{proj}\, F_2| = |-71.5685|$

$=71.57\ N$

$=71.6\ N$

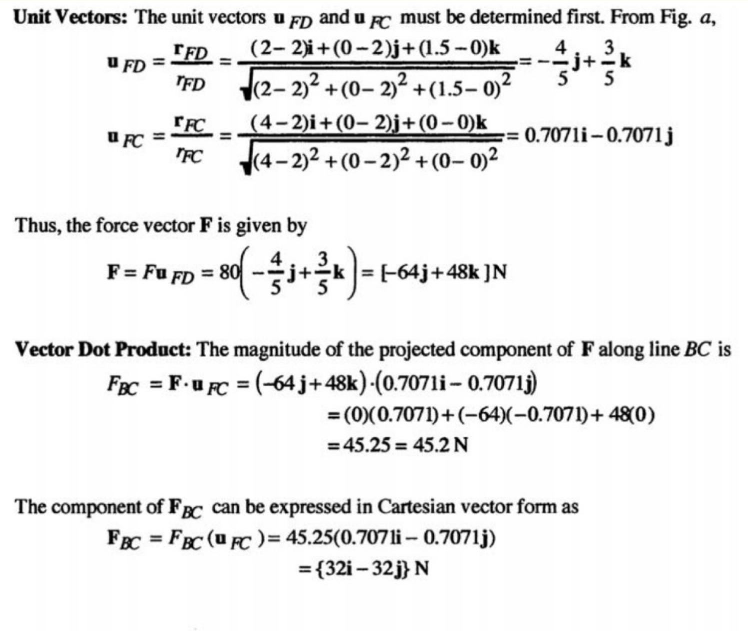
2_118



2–118. Determine the projection of force $F = 80$ N along line BC. Express the result as a Cartesian vector.

نفس الفكرة بس في ملاحظة
أكثر بالسؤال إنو ال FD
ما الها بدو جابلك ال BC
فبنعمل (FC)

حل السؤال



Unit Vectors: The unit vectors $\mathbf{u}_{FD}$ and $\mathbf{u}_{FC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{FD}=\frac{\mathbf{r}_{FD}}{r_{FD}}=\frac{(2-2)\mathbf{i}+(0-2)\mathbf{j}+(1.5-0)\mathbf{k}}{\sqrt{(2-2)^2+(0-2)^2+(1.5-0)^2}}=-\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC}=\frac{\mathbf{r}_{FC}}{r_{FC}}=\frac{(4-2)\mathbf{i}+(0-2)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(4-2)^2+(0-2)^2+(0-0)^2}}=0.7071\mathbf{i}-0.7071\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F}=F\mathbf{u}_{FD}=80\left(-\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}\right)=[-64\mathbf{j}+48\mathbf{k}]\text{N}$$

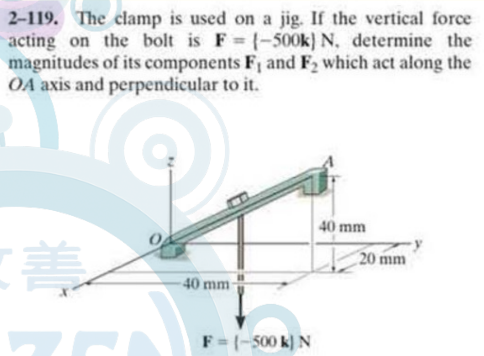
Vector Dot Product: The magnitude of the projected component of **F** along line *BC* is

$$F_{BC}=\mathbf{F}\cdot\mathbf{u}_{FC}=(-64\mathbf{j}+48\mathbf{k})\cdot(0.7071\mathbf{i}-0.7071\mathbf{j})$$
$$=(0)(0.7071)+(-64)(-0.7071)+48(0)$$
$$=45.25=45.2\text{ N}$$

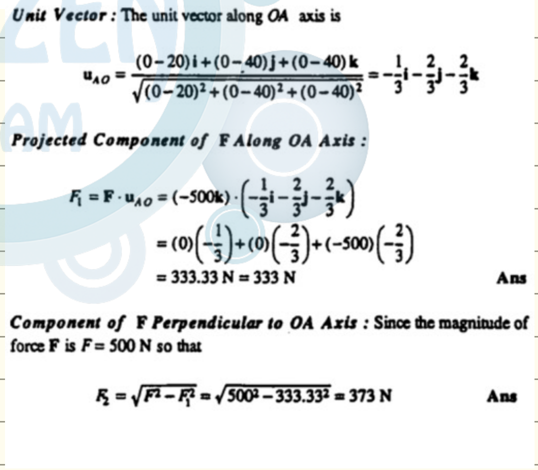
The component of $\mathbf{F}_{BC}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_{BC}=F_{BC}(\mathbf{u}_{FC})=45.25(0.7071\mathbf{i}-0.7071\mathbf{j})$$
$$=\{32\mathbf{i}-32\mathbf{j}\}\text{ N}$$

2- 119



2–119. The clamp is used on a jig. If the vertical force acting on the bolt is $\mathbf{F} = \{-500\mathbf{k}\}$ N, determine the magnitudes of its components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the *OA* axis and perpendicular to it.



Unit Vector: The unit vector along *OA* axis is

$$\mathbf{u}_{AO}=\frac{(0-20)\mathbf{i}+(0-40)\mathbf{j}+(0-40)\mathbf{k}}{\sqrt{(0-20)^2+(0-40)^2+(0-40)^2}}=-\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

Projected Component of F Along OA Axis:

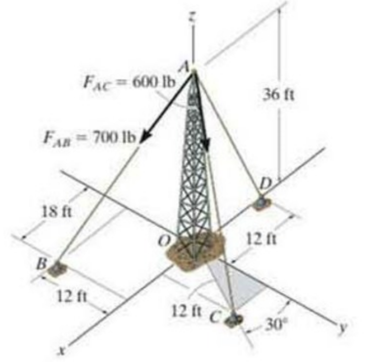
$$F_1=\mathbf{F}\cdot\mathbf{u}_{AO}=(-500\mathbf{k})\cdot\left(-\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)$$
$$=(0)\left(-\frac{1}{3}\right)+(0)\left(-\frac{2}{3}\right)+(-500)\left(-\frac{2}{3}\right)$$
$$=333.33\text{ N}=333\text{ N}\qquad\textbf{Ans}$$

Component of F Perpendicular to OA Axis: Since the magnitude of force **F** is $F = 500$ N so that

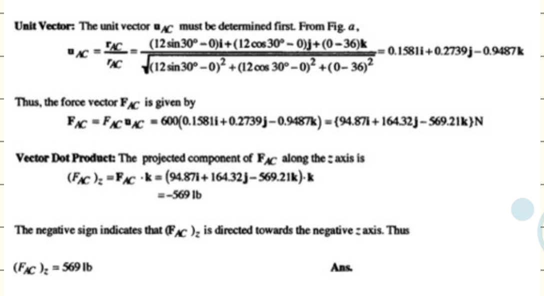
$$F_2=\sqrt{F^2-F_1^2}=\sqrt{500^2-333.33^2}=373\text{ N}\qquad\textbf{Ans}$$

2-121

•2–121. Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



نفس الأفكار.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. a,

$$\mathbf{u}_{AC}=\frac{\mathbf{r}_{AC}}{r_{AC}}=\frac{(12\sin 30°-0)\mathbf{i}+(12\cos 30°-0)\mathbf{j}+(0-36)\mathbf{k}}{\sqrt{(12\sin 30°-0)^2+(12\cos 30°-0)^2+(0-36)^2}}=0.1581\mathbf{i}+0.2739\mathbf{j}-0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC}=F_{AC}\mathbf{u}_{AC}=600(0.1581\mathbf{i}+0.2739\mathbf{j}-0.9487\mathbf{k})=\{94.87\mathbf{i}+164.32\mathbf{j}-569.21\mathbf{k}\}\text{N}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

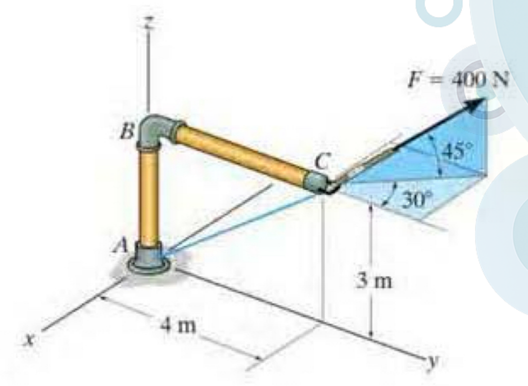
$$(F_{AC})_z=\mathbf{F}_{AC}\cdot\mathbf{k}=(94.87\mathbf{i}+164.32\mathbf{j}-569.21\mathbf{k})\cdot\mathbf{k}$$
$$=-569\text{ lb}$$

The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$(F_{AC})_z = 569$ lb **Ans.**

2_123

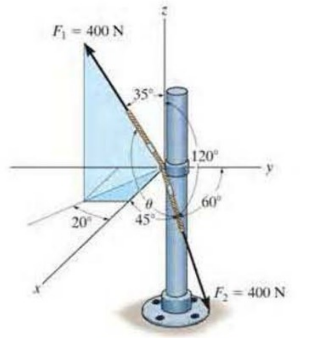
2–123. Determine the magnitudes of the components of force F = 400 N acting parallel and perpendicular to segment BC of the pipe assembly.



نفس الافكار.

2-127

2–127. Determine the angle θ between the two cables attached to the post.

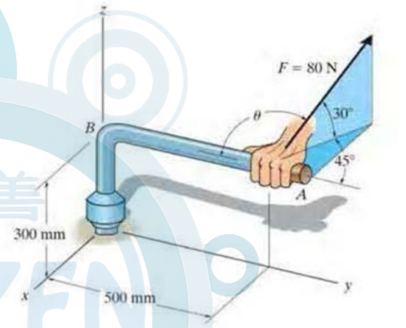


بنجيب $(u_{F_1} \cdot u_{F_2})$ حاصل ضربهم وبنجيب

$\theta = \cos^{-1}(u_{F_1} \cdot u_{F_2})$

2-128

***2–128.** A force of F = 80 N is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB.



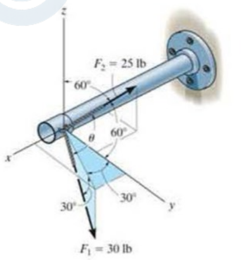
نفس الاشي بنجيب ال unit vector للتنين

بنضربهم ببعض و بنوخذ ال cos

same same

2-134

2–134. Determine the angle θ between the two cables attached to the pipe.



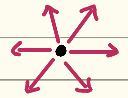
the end of chapter 2

chapter 3:

\# particles:- → mass
→ no size ⇒ مقارنة باشي ثاني

chap3

↳ when we applied a force on this particle



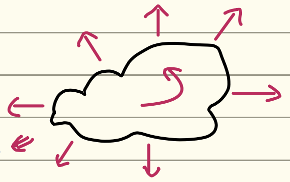
⇒ بتحرك

\- translation عهادل body بيصير عني

\- no rotation ⇒ ما بصير الها دوران

\# Rigid bodies → mass
→ size

chapt 4



لما نطبق force عليه بصير عني

يا :-

\- translation

\- rotation ⇒ M = (moment) الزخم بهمنا لانو راح ندرس ال

\# Equilibrium:

case 1

in statics: equilibrium in statics mean that $\Sigma F = 0$, & the particle is not moving (at rest)

case 2

in dynamics: equilibrium in dynamics mean, $\Sigma F = 0$ so the particle is moving in constant speed (moving & constant velocity)

$F = m.a$ ⇒ when $a = 0$ this system is under equillibrium

$\Sigma F = 0$ ↳ & we will use this to find:-

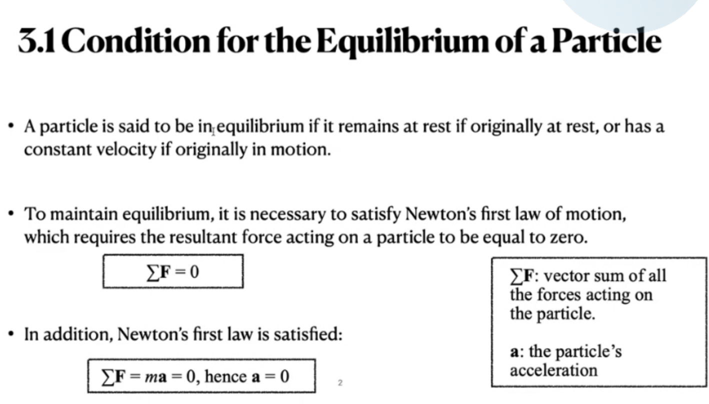
$F_{Rx} = 0$

$F_{Ry} = 0$

$F_{Rz} = 0$

1. Distances
2. lengths
3. angles
4. forces

we call these equations of equillibrium



3.1 Condition for the Equilibrium of a Particle

- A particle is said to be in equilibrium if it remains at rest if originally at rest, or has a constant velocity if originally in motion.
- To maintain equilibrium, it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on a particle to be equal to zero.

$$\boxed{\sum F = 0}$$

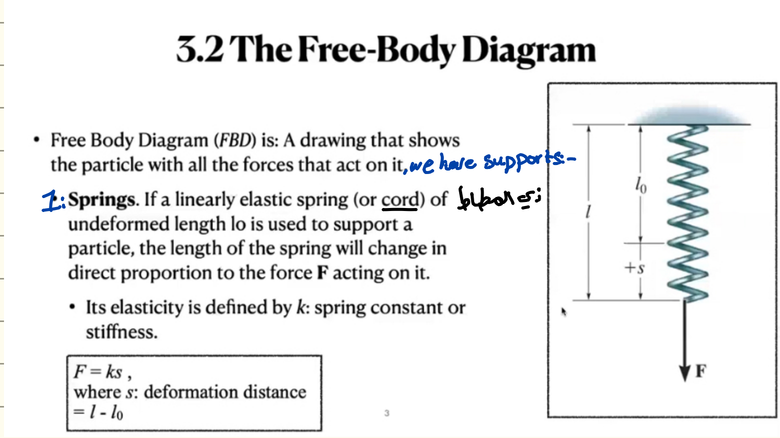
- In addition, Newton's first law is satisfied:

$$\boxed{\sum F = ma = 0, \text{ hence } a = 0}$$

$\sum F$: vector sum of all the forces acting on the particle.

a: the particle's acceleration

2



3.2 The Free-Body Diagram

- Free Body Diagram (FBD) is: A drawing that shows the particle with all the forces that act on it, we have supports:-

1: **Springs.** If a linearly elastic spring (or cord) of ذي المطاط undeformed length lo is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it.

- Its elasticity is defined by *k*: spring constant or stiffness.

$F = ks$,
where s: deformation distance
$= l - l_0$

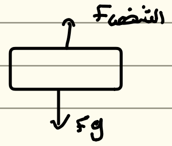
3

\# Free body Diagram

تجريده ↑

isolate the body from it's surroundings

\# مثلًا لما الشخص بحمل صندوقه الصندوقه عليه جاذبية وقوة للأعلى بتمنعو من السقوط ⇐ هاي ي القوة تبعت الشخص هي supporter



\# بس طبعًا هاد مش اشي منطقي بالحياه العملية لانو مارح أضل بدي أمسك عالطول أشخاص وعشان هيك بنحتاج supports أول نوع منهم هو ال springs

1. Springs

l : (final length) الطول بعد الاستطالة

بزبط نسميها l_F

l_0 : الطول الأصلي

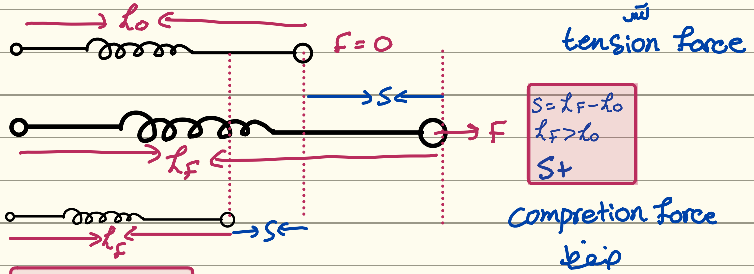
$S : l - l_0$ الفرقة بين الطول الأصلي وال (Final length)

↳ stretching مقدار ال

So the relation between (S) & (F) is direct proportional

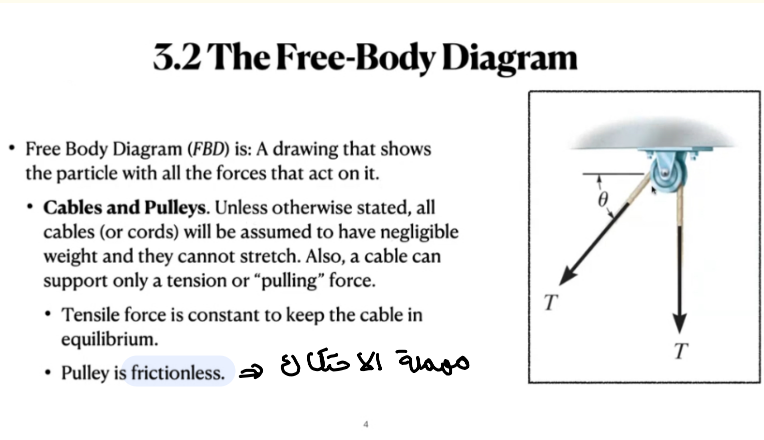
K : stiffness ⇒ كل ما كانت ال K أعلى صار بقاوم الاستطالة أكثر

\# let consider that we have spring



$S = l_F - l_0$
$l_F > l_0$
S+

$S = l_F - l_0$
$S = -$ (value)
S−



3.2 The Free-Body Diagram

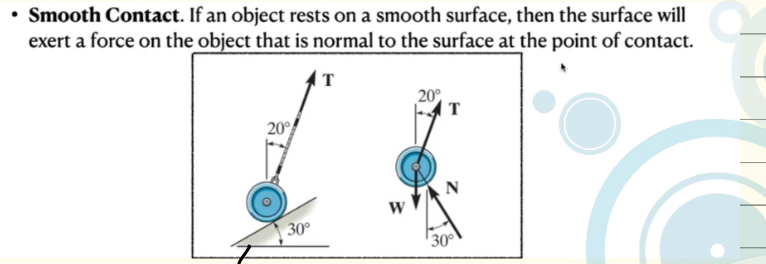
- Free Body Diagram (*FBD*) is: A drawing that shows the particle with all the forces that act on it.
  - **Cables and Pulleys.** Unless otherwise stated, all cables (or cords) will be assumed to have negligible weight and they cannot stretch. Also, a cable can support only a tension or "pulling" force.
    - Tensile force is constant to keep the cable in equilibrium.
    - Pulley is frictionless. ⇒ مهملة الاحتكاك

2. Cables and pulleys :- (حبل)

tension force بس بنطبق عليه compression force الحبل ما بنطبق عليه

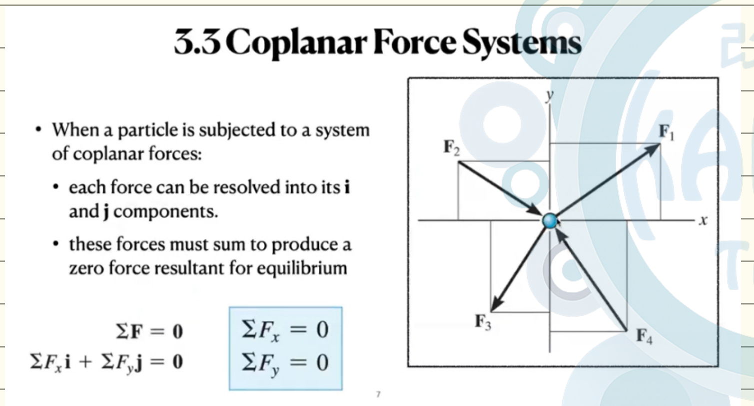
↳ (tensile)

3. Smooth contact



- **Smooth Contact.** If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.

↳ friction مهمل برضو بنعتبرال



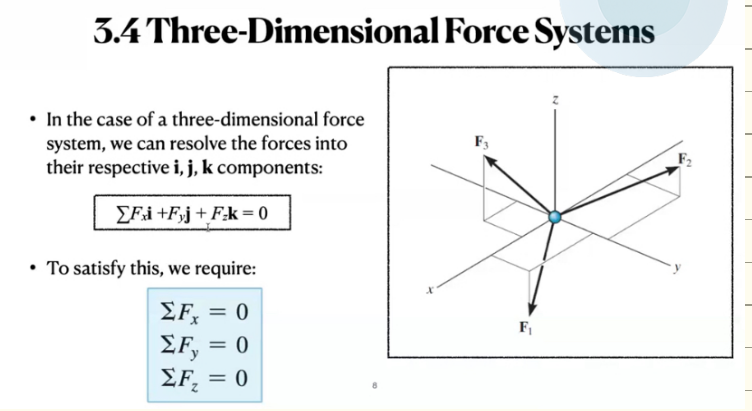
3.3 Coplanar Force Systems

- When a particle is subjected to a system of coplanar forces:
  - each force can be resolved into its **i** and **j** components.
  - these forces must sum to produce a zero force resultant for equilibrium

$$\sum \mathbf{F} = \mathbf{0}$$

$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} = \mathbf{0}$$

$$\boxed{\begin{aligned}\sum F_x &= 0\\ \sum F_y &= 0\end{aligned}}$$



3.4 Three-Dimensional Force Systems

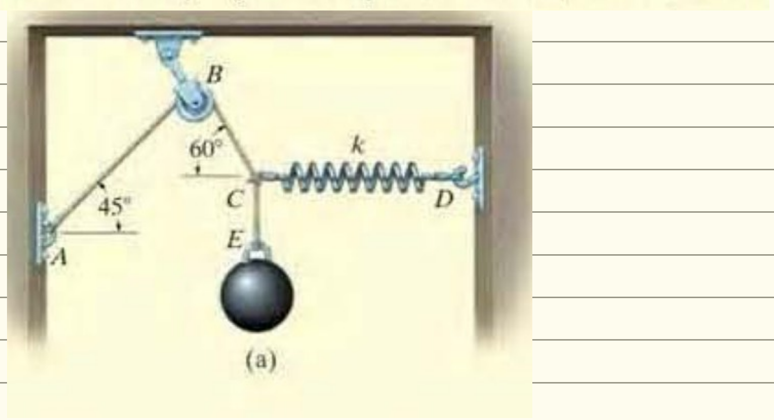
- In the case of a three-dimensional force system, we can resolve the forces into their respective **i, j, k** components:

$$\boxed{\sum F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = 0}$$

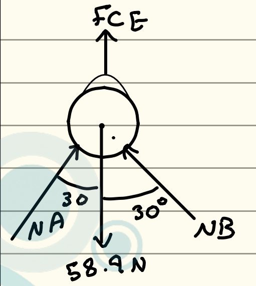
- To satisfy this, we require:

$$\boxed{\begin{aligned}\sum F_x &= 0\\ \sum F_y &= 0\\ \sum F_z &= 0\end{aligned}}$$

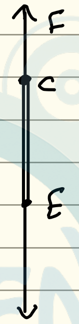
* Example 3.1 page (88)



The sphere in Fig. 3–3*a* has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord *CE*, and the knot at *C*.



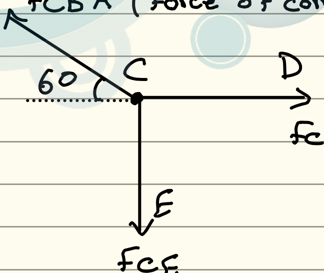
free body diagram for the sphere



F (force of knot acting on the cord

⇒ free body diagram for the cord

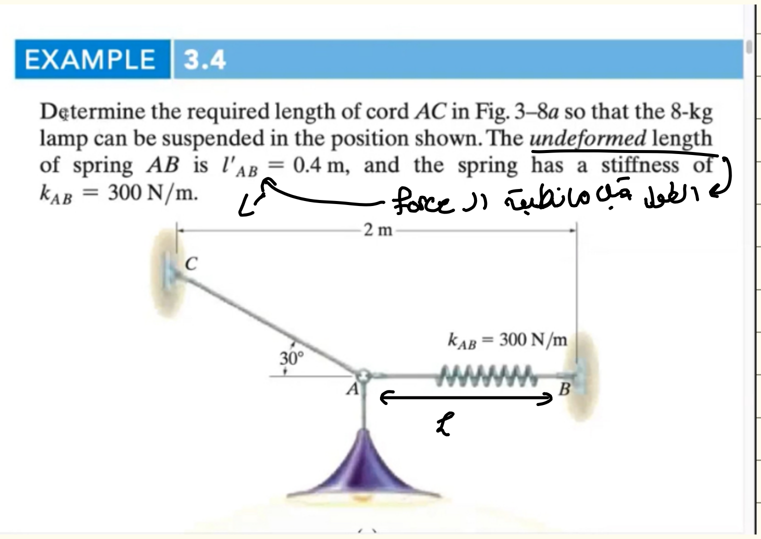
FCE (force of sphere acting on cord CE)



FCBA (force of cord CBA acting on knot)

FCD (force of spring acting on the knot)

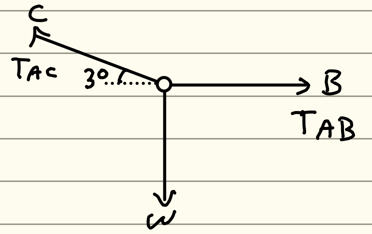
FCE (force of cord CE acting on knot)



EXAMPLE 3.4

Determine the required length of cord *AC* in Fig. 3–8*a* so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring *AB* is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.

الطول قبل ما نطبق ال force



$\Sigma F_x = \Sigma F_y = 0$

$\Sigma F_x = 0$

$T_B - T_C \cos 30 = 0$ --- ①

$\Sigma F_y = 0$

$T_C \sin 30 - w = 0$ --- ②

$T_C \sin 30 - mg = 0$

$T_C \sin 30 = 78.4$

$T_C = \frac{78.4}{\sin 30}$

$T_C = 156.8\,N$

$\approx 157\,N$

$T_B - T_C \cos 30 = 0$

$T_B - 157 \cos 30 = 0$

$T_B = 135.96$

$\approx 136\,N = F_s$

$T_B = KS$

$\frac{136}{300} = \frac{300S}{300}$

$S = 0.453\,m$

$S = L_f - L_s$

$0.453 = \ell_f - 0.4$

$+0.4 \qquad +0.4$

$\boxed{\ell_f = 0.853\,m}$

$d = L_f + \ell_{ACx}$

$2 = 0.853 + \ell_{ACx}$

$-0.853 \qquad -0.853$

$1.147 = \ell_{ACx}$

$\frac{1.147}{\cos 30} = \frac{\ell_{ACx} \cos 30}{\cos 30}$

$\boxed{\ell_{AC} = 1.3244\,m}$ #

Steps to solve problems:-

1. draw free body diagram
2. use ur equtions of equillibrium

* بال 3D-space ⇐ الفرق إني بدي أستعمل ال cartesian vectors
أجيب ال r لكل واحد وبعدين أقسمها عال |r| وبعدين أضربو بقيمة
ال force الي باتجاهها بالتالي بصير عندي كل ال forces على شكل
cartesian vectors

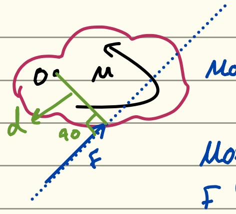
* إذا كان في force كاملة بس عال x-axis ما بجيبلها r لانو هي
كاملة عال x بضربها ب (i)
ونفس الاشي في حالة ال z وال y

the end of chapter3

Chapter4: force system resultants

ch2 → particles {mass, no size ⇒ 1.translation
ch3 2. no rotation

ch4 → rigid bodies {mass, size ⇒ 1. translation
ch5 2. rotation

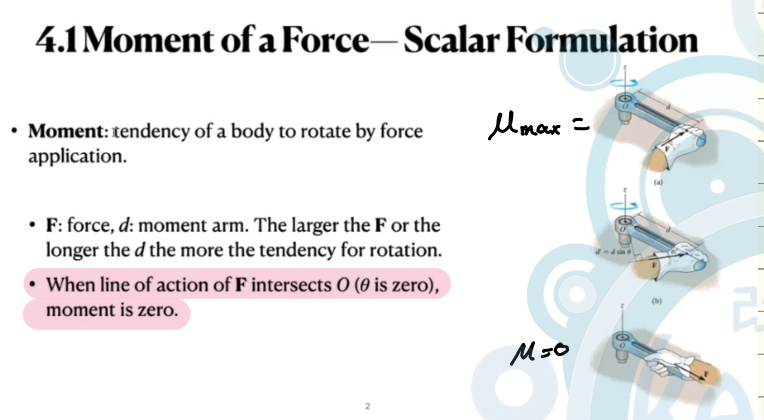


Moment ⇒ $M_{scaler} = F \times d$
↳ moment arm
↳ perpindicular distance

Moment is directrictly proportional to F & d

$[M] = N.m$

⇒ if the rotation is ccw ⇒ M+
⇒ if the rotation is cw ⇒ M−

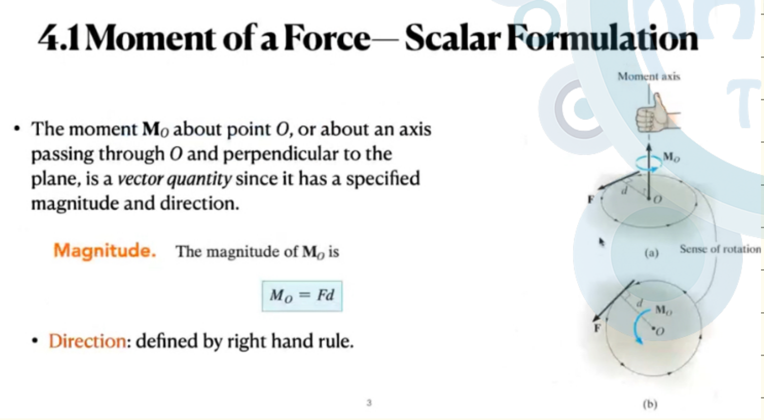


4.1 Moment of a Force— Scalar Formulation

- **Moment**: tendency of a body to rotate by force application.
  - **F**: force, *d*: moment arm. The larger the **F** or the longer the *d* the more the tendency for rotation.
  - When line of action of **F** intersects *O* (*θ* is zero), moment is zero.

$M_{max} =$

$M = 0$



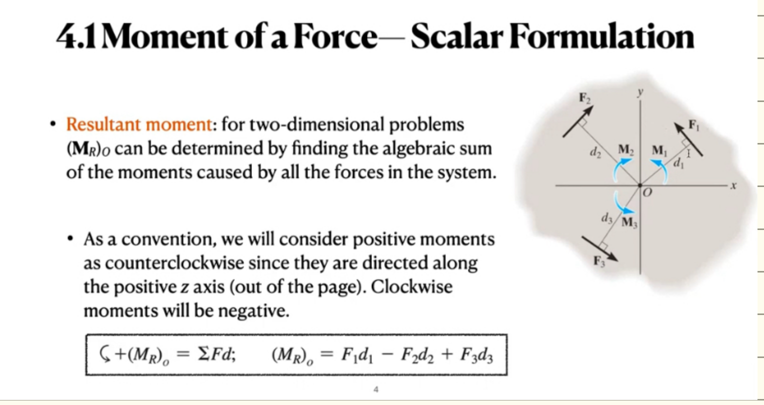
4.1 Moment of a Force— Scalar Formulation

- The moment $\mathbf{M}_O$ about point *O*, or about an axis passing through *O* and perpendicular to the plane, is a *vector quantity* since it has a specified magnitude and direction.

Magnitude. The magnitude of $\mathbf{M}_O$ is

$$\boxed{M_O = Fd}$$

- Direction: defined by right hand rule.

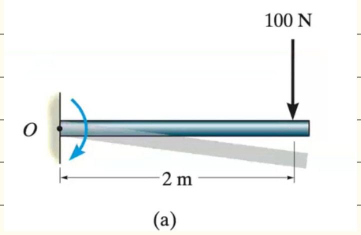


4.1 Moment of a Force— Scalar Formulation

- Resultant moment: for two-dimensional problems $(\mathbf{M}_R)_O$ can be determined by finding the algebraic sum of the moments caused by all the forces in the system.
  - As a convention, we will consider positive moments as counterclockwise since they are directed along the positive *z* axis (out of the page). Clockwise moments will be negative.

$$\boxed{\circlearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1d_1 - F_2d_2 + F_3d_3}$$

Example :-

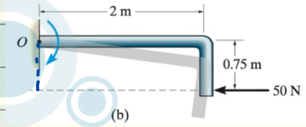


(a)

find the moment around point O

$M = Fd\sin\theta$
$= 100 \times 2 \times \sin 90$
$= 200$ N.m(−)
$= -200$ N.m
or 200 cw N.m

example:-



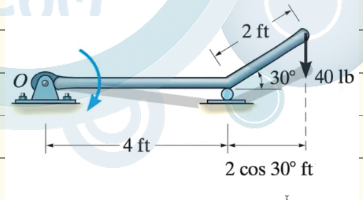
(b)

find the moment around point O

line of action بنوقف اسقاط ال O العمودي على
For the force

$M = 50 \times 0.75$
$= 37.5$ N.m cw
$= -37$ N.m

example :-

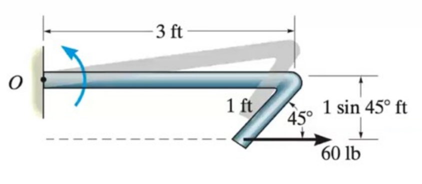


(c)

$d = 4 + 2\cos 30$
$d = 5.732$

$M = F.d$
$= 40.(5.732)$
$= 229.28$ Ib.ft cw
$= -229.28$ Ib.ft

example:-



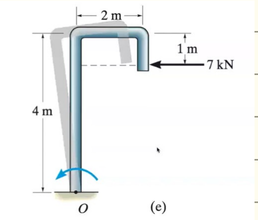
$\perp$ line = sin45 = 0.7071

$M = f.d$

$= 60 \times 0.7071 = 42.4264$ Ib.ft ccw

$= 42.4264$ Ib.ft

Example:-



$d = 4 - 1 = 3$

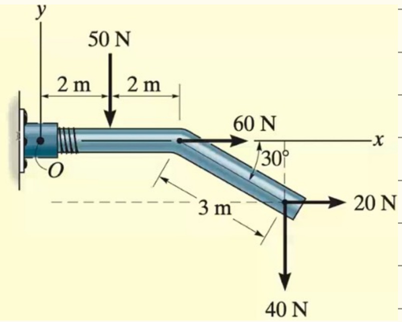
$M = 7KN \times 3$

$= 21$ KN.m ccw

$= 21 \times 10^3$ N.m CCW

$\overset{+}{\curvearrowleft} 21 \times 10^3$ N.m

example: find the resultant moment around point(O).



sol:

$f = 50 \Rightarrow M_1 = 50 \times 2 = -100$ N.m

$F = 60 \Rightarrow M_2 = 0$ because line of action of the force passes through O

$F = 20 \Rightarrow M = 20 \times 3 \sin 30 = +30$ N.m

$F = 40 \Rightarrow M = 40 \times (2 + 2 + 3 \cos 30)$

$= 40 \times (6.6)$

$= -263.923$ N.m

$M_{RO} = -100 + 30 - 263.923$

$= -334$ N.m

until now we only use moment as scalar not vector quantity but sometimes we need to use moment as vector

لحد هلا حكينا عن ال 2D-moment

$M = F \times d$

بدل $\underline{d}$

3d $\quad \bar{M} = \underbrace{\bar{r} \times \bar{F}}$ ← الترتيب مهم

where $\bar{M}$: vector moment

$\vec{r}$: position vector (unit: m, cm) ⇒ in vector form

$\bar{F}$: force

* cross product: we use it to find M in vector form

$$\boxed{\vec{r} \times \vec{F} \neq \vec{F} \times r}$$

Consider that we have

$\vec{r} = (1i + 2j - 1K)$ m

$\vec{F} = (2i - j + K)$ N

$$\vec{M} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{K} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} \begin{matrix} \\ \rightarrow r \text{ لازم هون} \\ \rightarrow F \text{ لازم هون} \end{matrix}$$

$\vec{M} = (2 - 1)i - (1 + 2)j + (-1 - 4)K$

$\bar{M} = (1i - 3j - 5K)$ N.m

$|\bar{M}| = \sqrt{(1)^2 + (3)^2 + 5^2}$

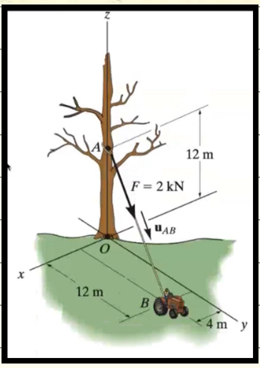
$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$ N.m

$|\bar{M}| = 5.916$ N.m

Example 4.3

Determine the moment produced by the force **F** in Fig. 4–14*a* about point *O*. Express the result as a Cartesian vector.



sol

O(0,0,0)
B(4, 12,0)
A(0,0,12)

$r_{AB} = (4i + 12j - 12K)m$

$|r_{AB}| = \sqrt{304} = 17.4355\ m$

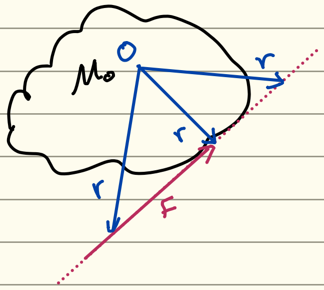
$$u_{AB} = \frac{4}{17.43}i + \frac{12}{17.43}j - \frac{12}{17.43}K$$

$u_{AB} = 0.3i + 0.6884j - 0.68847\hat{K}$

$\vec{F} = F.u_{AB}$

$F_{AB} = 2\times10^3(0.3i + 0.6884j - 0.68847K)$

$F_{AB} = (458.8i + 1376.8j - 1376.44K)N$



r: Any position vector from point o to Any point along the line of action of (f)

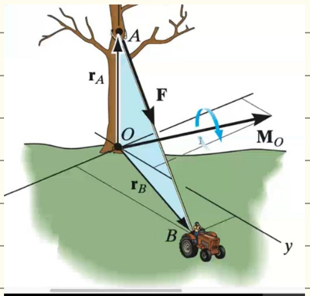
so for r we can use $\overline{r_{OA}}$, $\overline{r_{OB}}$

$\overline{r_{OA}} = (0i + 0j + 12K)m$ → easier to use
$\overline{r_{OB}} = (4i + 12j + 0K)m$

$$\overline{M} = \begin{vmatrix} \oplus & \ominus & \oplus \\ i & j & K \\ 0 & 0 & 12 \\ 458.8 & 137.6 & -137.6 \end{vmatrix}$$

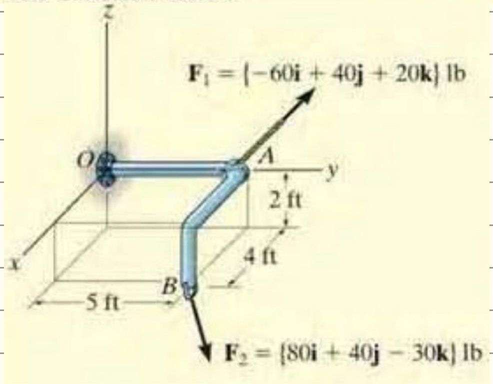
$\overline{M} = (0 - 1651.2)i - (0 - 5505.6)j + 0K$

$\vec{M} = -1651.2i + 5505.6j$



example 4.4

Two forces act on the rod shown in Fig. 4–15*a*. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



sol:

O (0,0,0)
A(0, 5,0)
B(4, 5,-2)

$F_1 = (-60i + 40j + 20K) Ib$
$F_2 = (80i + 40j - 30K) Ib$

$r_{OA} = 0i + 5j + 0K$

$M_1 = \vec{r_1} \times \vec{F_1}$

$$\overline{M_1} = \begin{vmatrix} \oplus i & \ominus j & \oplus K \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix}$$

$\overline{M_1} = (100 - 0)i + 0j + (0 - -300)K$

$\overline{M_1} = 100i - 0j + 300K$

$r_{OB} = 4i + 5j - 2K$

$\vec{M_2} = \vec{r_2} \times \vec{F_2}$

$$\overline{M_2} = \begin{vmatrix} \oplus & \ominus & \oplus \\ i & j & K \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

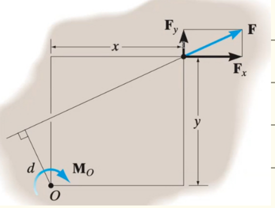
$\overline{M_2} = (-150 - -80)i - (-120 - -160)j + (160 - 400)K$

$\overline{M_2} = -70i - 40j - 240K$

$M_1 + M_2 = (100 - 70)i + (0 - 40)j + (+300 - 240)K$

$= 30i - 40j + 60K$

4.4 principle of Moments (varignon's theorem)

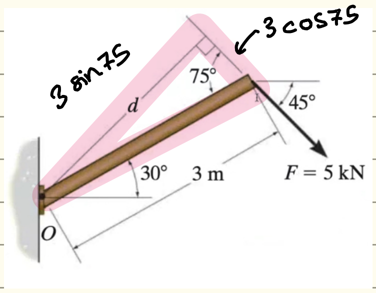


$$\boxed{M_F = M_{F_x} + M_{F_y}}$$

$$M_O = -F_x y + F_y x$$

we will tring not to break F into it's component unless it's very hard to find it

Example:-



<u>Sol</u> : $M = F.d$

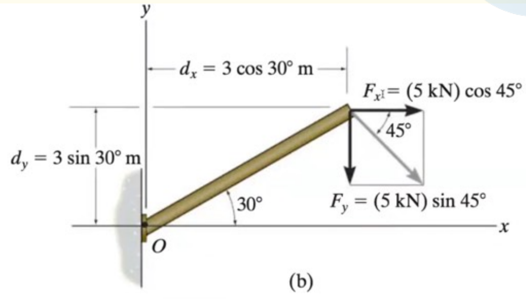
try to Find d without break F into it's components

it's possible to find (d) using the hilighted triangle

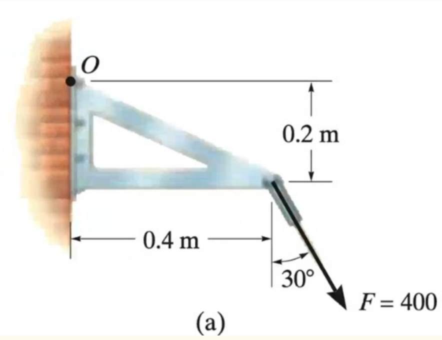
$$d = 3\sin 75 = 2.4\ m$$

$$M = 5\times10^3 \times 2.9$$
$$= 14.488\ KN.m\ cw$$
$$= -14.488$$

but if we break it into it's components



Example:-



$$F_y = 400\times\cos 30 = 346.41\ N$$

$$F_x = 400\times\sin 30 = 200\ N$$

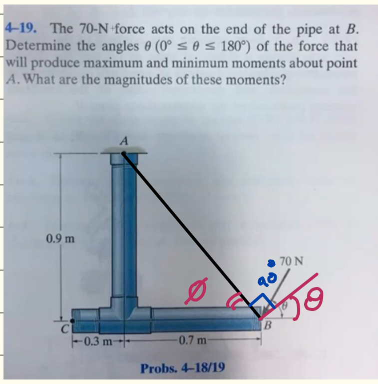
$$M_y = 346.41 \times 0.4$$
$$= -138.564\ N.m$$

$$M_x = 200\times0.2$$
$$= +40\ N.m$$

$$M_O = M_x + M_y$$
$$M_O = -48.564\ N.m$$
$$\text{or } 98.564 \circlearrowright$$

* find moment as cartesian vector

$$M = -(98.6\ \hat{k})\ N.m$$



* 4–19. The 70-N force acts on the end of the pipe at B. Determine the angles θ ($0° \leq \theta \leq 180°$) of the force that will produce maximum and minimum moments about point A. What are the magnitudes of these moments?

Probs. 4–18/19

الزاوية بين القوة والـ d

$M = F.d \sin 90$ ↑

when $\theta = 90$, then the moment is in it's maximum value

when $\theta = 0$, the the moment is in it's minimum

$Ø = \tan^{-1}\left(\frac{0.9}{0.7}\right)$

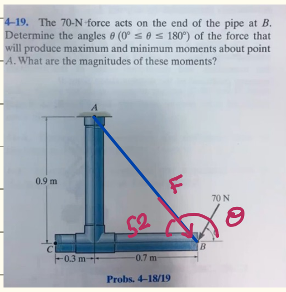
$Ø = 52$

$\theta = 180 - 90 - 52$

$= 37.87°$

So when $\theta = 37.87° \Rightarrow$ max

* the Moment in it's min value when the line of action of the force parallel



4–19. The 70-N force acts on the end of the pipe at B. Determine the angles θ ($0° \leq \theta \leq 180°$) of the force that will produce maximum and minimum moments about point A. What are the magnitudes of these moments?

Probs. 4–18/19

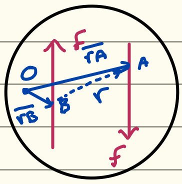
$\theta = 180 - 52$

$= 128°$

↓

when the moment in it's min value $= 0$

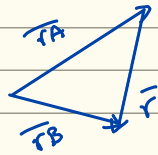
* moment of a couple



F: Rotation

no translation

↳ Fcouple $F_1 = F_2$



$\circlearrowleft + M_o = \Sigma M$

$= \vec{r}_B \times \bar{F} + \bar{r}_A \times -\bar{F}$

$= \underset{\downarrow}{(r_B - r_A)} \times \bar{F}$

لازم بنفس الترتيب

$r_A + r = r_B$

$r = r_B - r_A$

$= \underbrace{\bar{r} \times \bar{F}}_{\downarrow}$

رجعنا لنفس القانون شو يعني؟ يعني اي ال moment الي نعملوه هاي الforces

$M = \bar{r} \times \bar{F}$

the 2point يربط position vector ↙ ↘ أي وحدة من هدول القوتين

line of action

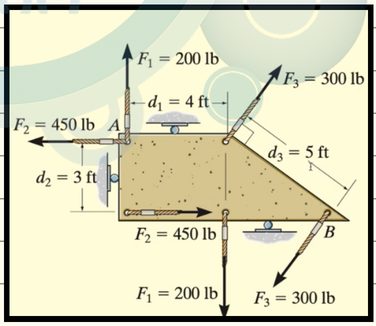
& this is called free vector

يعني ال Moment الي حعملو هدول ال 2 forces ما رح بتغير باختلاف النقطة راح يظل نفس

* قبل هيك حكينا انو ال r هي أي vector بلش من O على ال line of action of the force ⇐ هاد الحكي صح بال 3D بس بال 2D المسافة بينهم هي أقصر مسافة عمودية عليهم

بال 3-D ⇐ هي $\vec{r}$ بين ال 2 forces

* Example 4.10



find the moment that acting on the plate?

ما حكالي حولين أي (point) فبفترض وحدة وبحسب عليها الباقي

↳ in this Question we can find the moment for each Force & then taking the sum of all of them

or we can use couple moment to find the resultant moment

sol :-

⇒ $M_3 = 300 \times 5$

$= -1500$ Ib.ft

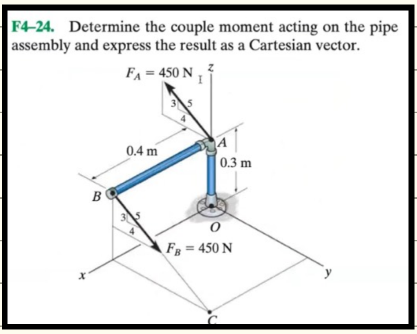
⇒ $M_1 = 200 \times 4 = -800$ Ib.ft

⇒ $M_2 = 450 \times 3 = +1350$ Ib.ft

$M_R = M_1 + M_2 + M_3 = -1500 - 800 + 1350$

$= -950$ Ib.ft

F- 4-24



F4–24. Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.

Sol :-

A(0,0,0.3) $\vec{M} = \vec{r} \times F$

B(0.4, 0, 0.3)

$\vec{r}_{AB} = (0.4i + 0j + 0K)$ m

we are going from poin A to B then the head of this vector will be the inetical point for FB, then we will use with $\vec{r}_{AB} \Rightarrow FB$

finding F_B components

$F_{Bx} = 0$

$F_{By} = 450 \times \frac{4}{5}$

$= 360$ N

$F_{Bz} = 450 \times \frac{3}{5}$

$= 270$ N

$F_B = 0i + 360j - 270K$

$$M = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & K \\ 0.4 & 0 & 0 \\ 0 & 360 & -270 \end{vmatrix}$$

$0i + 108j + (0 - 0.4 \times 360)K$

$M = (108j + 144K)$ N.m

to check our answer

$|\vec{M}| = \sqrt{(108)^2 + (144)^2}$

$|\vec{M}| = 180$

using only scaler form

$M = 450 \times 0.4$

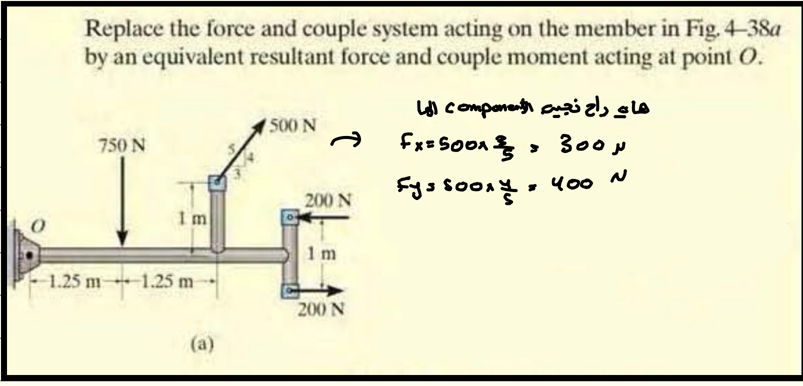
$= 180$ N.m

then our answer is correct.

simplification of a fource & couple system

راح نبسط كل الـforces & moments اي بأثرو على body معين بدنا نخليهم (1 force & couple moment)

Example 4.15:-



Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

هاي راح نجيب الـcomponents

$\rightarrow$ $F_x = 500 \times \frac{3}{5} = 300$ N

$F_y = 500 \times \frac{4}{5} = 400$ N

Step 1: finding resultant force that acting on x-axis & y-axis & then finding it as a magnitude

$\xrightarrow{+} F_{Rx} = 200 - 200 + (500 \times \frac{3}{5})$

$F_{Rx} = 300$

$\rightarrow F_{Ry} = -750 + (500 \times \frac{4}{5}) = -350$

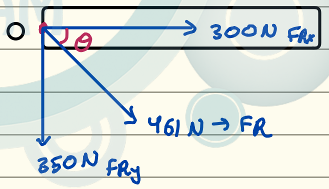
$F_R = 300i - 350j$

$F_R = \sqrt{(300)^2 + (-350)^2}$

$F_R = 460.97$ N

≈ 461 N

now



$\theta = \tan^{-1}\left(\frac{350}{300}\right)$

$\theta = 49.4°$

Step 2: finding moment with

$\overset{+}{\curvearrowright} (M_R)_O = \Sigma M_O + \Sigma M$, where ΣM_O: Moments due to forces

ΣM: Moments due to existing couples

CW يعني sign (-) ال

القوة الي حللناها

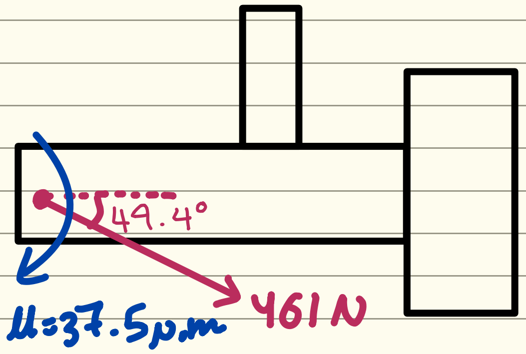
couple moment

$= (-750 \times 1.25) + (-300 \times 1) + (+400 \times 2.5) + (200 \times 1) = -37.5$ N.m

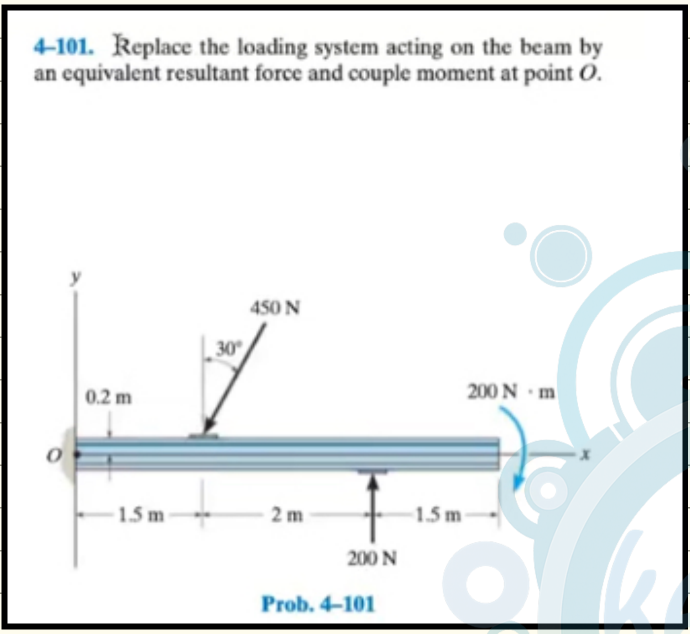
$= 37.5$ N.m CW

continued

Step 3: redrawing



\# F(4-101)



4-101. Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.

Prob. 4-101

Sol

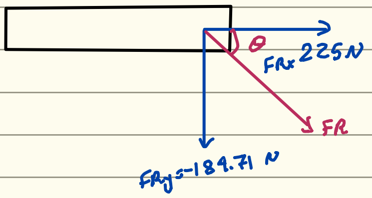
step 1:-

$F_x = 450 \times \sin 30 = 225\ N$

$F_y = -450 \times \cos 30 + 200 = -189.71$

$F_R = 225\ i + -189.51\ j$

$F_R = \sqrt{(225)^2 + (189.71)^2} = 294.3\ N$



$\theta = \tan^{-1}\left(\frac{189.71}{225}\right)$

$\theta = 40.1°$

continued ⇒

Step 2: Moment

$\circlearrowleft + \ M_R = \Sigma M_O + \Sigma M$

$(450 \times \sin 30 \times 0.2) + (-450 \cos 30 \times 1.5) + (200 \times 3.5)$

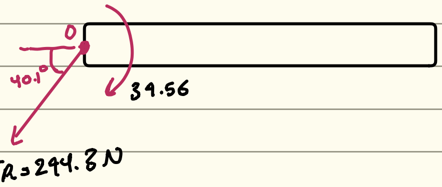
-200

$= -39.567\ N.m$

or 39.57 N.m cw

اي هو كاتبلي اياها ومحددلي اتجاهها cw

Step 3:-



* Further simplification of a force & couple system

we can reduce this system into single resultant force when line of action of FR and $(M_R)_O$ are perpedicular to each other

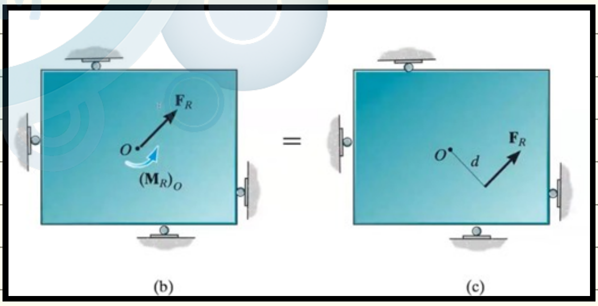
$M = F_R \times d$

$d = \frac{M_R}{F_R}$

* if line of action of the forces intersect each other then we can reduce them into one resultant force

* if line of action of the forces are not intersecting each other then we can reduce them into one resultant force but it should have rotational effect

↳ we need to further simplify that

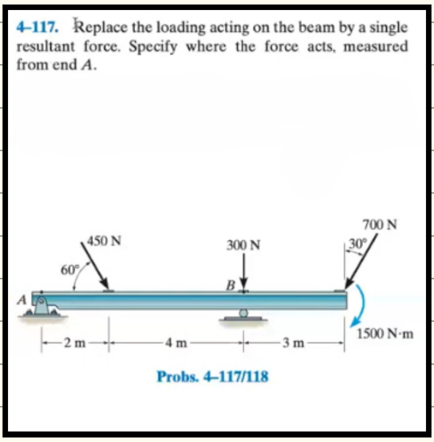


From both force & moment that effect point (O)

into one force that has distance (d) from point (O) & this force will have rotational effect

\# بهاي ال locase راح نشيل ال vectors

* problem 4-117


4-117. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.

Probs. 4-117/118

step1:-

$F_{Rx} = (450 \times \cos 60) + (-700 \times \sin 30) = -125 \quad N$

$F_{Ry} = (-450 \times \sin 60) + (-300) + (-700 \times \cos 30) = -1295.95 \ N$

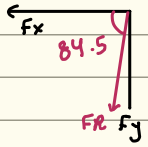
$F_R = (125 i - 1295.93 j) \ N$

$F_R = \sqrt{(125)^2 + (1295.93)^2} = 1301.94$

$\theta = \tan^{-1}\left(\frac{1295.93}{125}\right)$

$\theta = 84.49°$

then



step2 (Moment):-

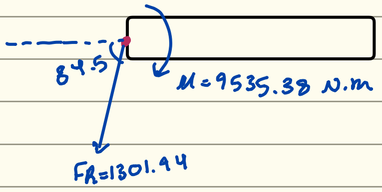
$+\circlearrowleft M = \Sigma M_o + \Sigma M$

$(-450 \times \sin 60 \times 2) + (-300 \times 6) + (-700 \times \cos 30 \times 9) - 1500$

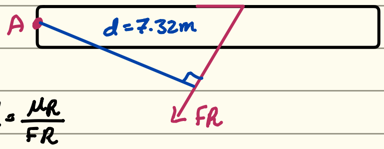
$= -9535.38 \ N.m$

$= 9535.38 \ N.m \ CW$

Step 3:-



Step 4:- change the position of FR so it has the rotation effect & distance(d) away from O



$d = \frac{M_R}{F_R}$

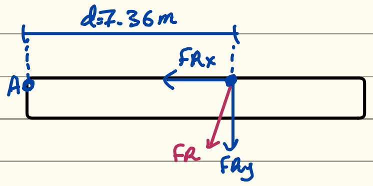
$= \frac{9535.38}{1301.9} = 7.32 \ m$

or we can find the perpendicular distance between beam & Fy, remember that the only component that causes moment around O is Fy →

So,

$d = \frac{M_R}{F_y}$

$= 7.36 \ m$



& here d is the perpendicular distance from point A to the Force (Fy) that causes rotation

4.9: Reduction of a simple distributed loading

instead of a single force at one point, sometimes loads are spread out over an area or length

& this spread-out force is called a distributed load

It's usually drawn as a set of arrows across a beam

& It can vary with position (w(x)) or be uniform

So our target is to replace this messy distributed load with one single equivalent force (FR) to make solving problems much easier

step1: magnitude of resultant force

$F_R = \int_L w(x)\, dx$ = Area under the load diagram

⇒ interpretation: the total force is just the area under distributed load curve

Sometimes the shape is uniform so we can use the following to find the area:-

→ rectangular ⇒ $F_R = w \times L$

⇒ triangular → $F_R = \frac{1}{2} \times \text{base} \times \text{height}$

⇒ trapezoid ⇒ $F_R = \frac{1}{2} \times (w_1 + w_2) \times L$

Step2: location of resultant force

$\bar{x} = \frac{\int_L x\, w(x)\, dx}{\int_L w(x)\, dx}$ = $\frac{\text{first moment or area}}{\text{total area}}$

* the resultant force acts at the centroid (center of gravity) of the load area

* Shortcut rules for common shapes:-

→ rectangular → acts at the center

⇒ triangle → acts at 1/3 from the base (the heavier side)

⇒ trapezoid → can split into rectangle + triangle, then combine

chapter 12

only translation ↑

\# Dynamics : accelerated motion of a particle / Body

↳ translation
↳ rotation

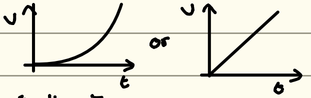
↳ Kinematics (motion) → velocity, acceleration, position

↳ Kinetics (force): what causing the motion

in chapter 12 we will discuss Kinematics of a particle.

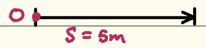
12.2 : Reetilinear Kinematics : contineous motion

straight line

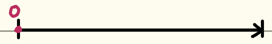


مافي تغير مفاجئ
بالسرعة فلة تتغير ببطئ

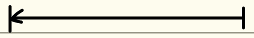
1. position : vector



S = 5m
= +5m
= 5m →



$S_2 = +10m$



$S_3 = -10m$

2. Displacement : vector

$\Delta S = S_f - S_i$

→ if we want to find displacement from S_2 & S_1

$\Delta S = S_f - S_i$

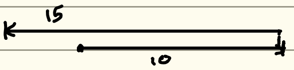
$= S_2 - S_1 = 10 - 5 = +5m$

→ displacement for S_2 & S_3

$\Delta S = S_3 - S_2 = -10 - 10 = -20 m$

3. Distance : +ve scalar

↳ total distance



distance = 10 + 15 = 25m

4. velocity :-

$$V_{avg} = \frac{\Delta S}{\Delta t} , \quad V_{inst} = \boxed{V = \frac{ds}{dt}} \text{ --- ①}$$

↓

this eqn contains 3var (v, S, t), where (v) is velocity

$(V sp)_{avg} = \frac{S_T}{\Delta t}$, where V : speed , S_T : distance travelled

\# what is the difference between speed & velocity?

speed is scalar & velocity is vector

5. acceleration

vector

$$a_{avg} = \frac{\Delta V}{\Delta t} , \quad \Delta V = V_f - V_i$$

$$a_{inst} = \boxed{a = \frac{dv}{dt}} \text{ --- ②} \Rightarrow \text{this eqn connects 3var } (a, v, t)$$

vector

the common variable between inst (v, a) is t , then

$$V = \frac{ds}{dt} , \quad a = \frac{dv}{dt} \Rightarrow dt = \frac{ds}{v} , \quad dt = \frac{dv}{a}$$

this means that these two terms are equal , then

$$\boxed{a \, ds = v \, dv} \text{ --- ③}$$

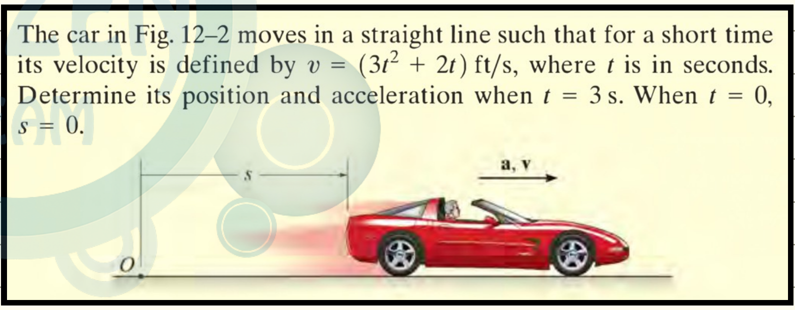
& in all (1,2,3) equations the acceleration is not constant

diff
اشتقاق

$$\begin{tikzcd}
\text{position} \arrow[r, "\text{time}"] \arrow[rr, bend left, "\text{diff}"] & \text{velocity} \arrow[r, "\text{time}"] & \text{acceleration} \arrow[ll, bend left, "\text{integ}"]
\end{tikzcd}$$

تكامل
integ

\# example 12.1



The car in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

$V = 3t^2 + 2t$

to find position we have to integ

$$S = \int_{t_0}^{t} v \, dt \Rightarrow \text{or} \int_0^s ds = \int_0^t (3t^2 + t) \, dt$$

$$S = \int_0^3 3t^2 + 2t = t^3 + t^2 \Big|_0^3 = (27 - 9) = 18 \text{ ft}$$

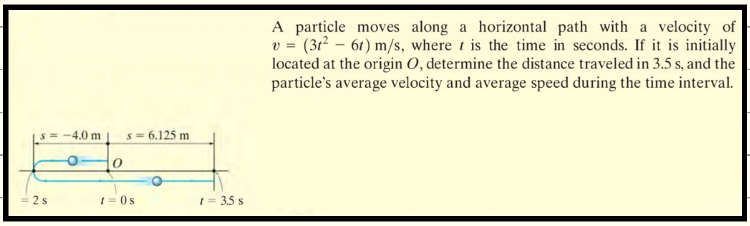
to find acceleration we will diff

$$a = \frac{dv}{dt} = \frac{d}{dt} (3t^2 + 2t)$$

$= 6t + 2$ at $t = 3$

$= 6 \times 3 + 2 = 18 + 2 = 20$ ft/s²

Example 12.5



A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.

① S_T in 3.5 s

we can not find the distance directly but we can find the position

$v = \frac{ds}{dt}$

$ds = v\,dt$

$= (3t^2-6t)dt$

$\int_0^s ds = \int_0^t (3t^2-6t)\, dt$

$s = (t^3-3t^2)\,m$

the behaviour of this particle will change along time (t)

then to test where does this particle stop moving (v=0)

$3t^2-6t=0$

$t(3t-6)=0$

$t=0$ $\quad 3t-6=0$

↳ $t=2$

then $S_T = 4+4+6.125$

$= 14.125\,m$ #

② $V_{avg} = \frac{\Delta S}{\Delta t} = \frac{S_f - S_0}{3.5} = \frac{6.125}{3.5-0} = 1.75\,m/s$

③ $(V_{sp})_{avg} = \frac{S_T}{\Delta t} = \frac{14.125}{3.5} = 4.04\,m/s$

all the previous eqns are used when the acceleration is changed

what if the acceleration is constant?

we can use these eqns only when the acceleration is constant

↳ example: $g = 9.81\,m/s$ ⇒ gravitation due to gravity

& we call it a_c ⇒ constant acceleration

$a = \frac{dv}{dt}$

$\int dv = \int a_c\, dt$

$\int_{v_0}^{v} dv = \int_0^t a_c\, dt$

$v - v_0 = a_c t$

$v = a_c t + v_0$ ---- ①

↳ so this is v when we assume that velocity is constant

$S = f(t)$

$V = \frac{ds}{dt}$

↓

but the acc is constant so we will use

$V = V_0 + a_c t$

$V_0 + a_c t = \frac{ds}{dt}$

$\int_{S_0}^{S} ds = \int_0^t (V_0 + a_c t)\, dt$

$S - S_0 = V_0 t + \frac{a_c t^2}{2}$

$S = S_0 + V_0 t + \frac{1}{2} a_c t^2$ -- ②

function of distance

③ velocity = f(S)

$\int_{S_0}^{S} a_c\, ds = \int_{V_0}^{V} v\, dv$

↳ $v^2 = v_0^2 + 2a_c(S - S_0)$ --- ③

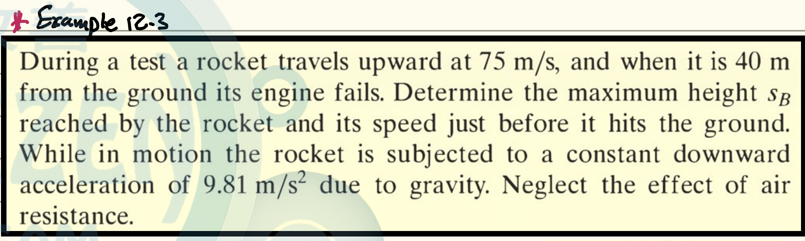
* So we will use these three eqns in order to solve problems when the acceleration is constant

$V_f = V_i + at$

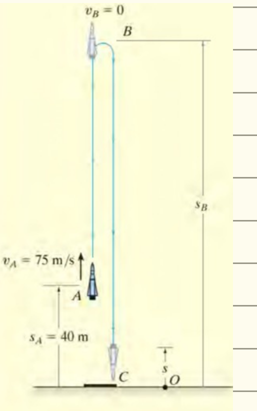
$V_f^2 = V_i^2 + 2a\Delta d$

$\Delta d = V_i t + \frac{1}{2} a t^2$

Example 12.3



During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s² due to gravity. Neglect the effect of air resistance.



Sol

① $U_i = 75\,m/s$

$S_i = 40$

$S_f = ?$

$a = -9.81\,m/s^2$

$V_f = 0$

$V_f^2 = V_i^2 + 2a\Delta S$

$0 = (75)^2 + 2(-9.81) \times (S_B - 40)$

$0 = 5625 - 19.62\,S_B + 784.8$

$S_B = 327\,m$

② using this for the second part

$V_f = ??,\ V_i = 0,\ a = -9.81,\ S_C = 0,\ S_B = 327m$

$V_f^2 = V_i^2 + 2a\Delta S$

$V_f^2 = 0 + 2 \times (-9.81) \times (0 - 327)$

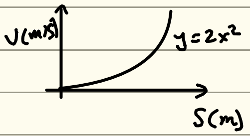
$V_f^2 = 6415.74$

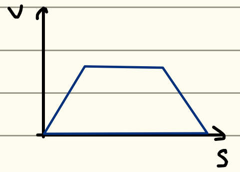
$V_f = \pm 80.1\,m/s$

because the particle is moving downword then

$V_f = (-80.1)\,m/s$

12.3 :Rectilinear Kinemetics: erratic motion





contineous motion

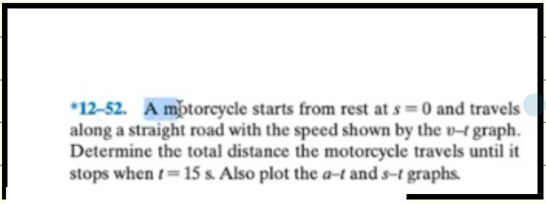
ماني تغيرات مفاجأة
بال velocity

erratic motion
or piecewis motion

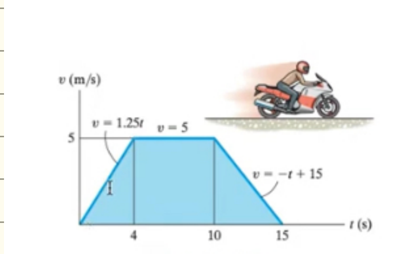
& we will use the same previous eqns

$\boxed{v = \frac{ds}{dt}}$ $\boxed{a = \frac{dv}{dt}}$ $\boxed{a\,ds = v\,dv}$

Example:-



*12–52. A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



Probs. 12–52/53

distance = area under curve

for $0 < t < 4$

$a = \frac{dv}{dt}$, $v = 1.25t$

$a = \frac{d}{dt}(1.25t)$

$\boxed{a = 1.25 \text{ m/s}^2}$ ...①

$v = \frac{ds}{dt}$

$\int ds = \int_0^4 v\,dt$

$S = \int_0^4 1.25t\,dt$

$S = \frac{1.25}{2} t^2 \Big|_0^4$

$S = 0.625 \times (4)^2$

at $t=4$, at $t=0$

$S(4) = 10$ m , $S(0) = 0$

for $4 < t < 10$

$a = \frac{dv}{dt}$

$a = \frac{d}{dt}(5)$

$a = 0$ m/s²

$v = \frac{ds}{dt}$

$\int_{10}^{s} ds = \int_4^t v\,dt$

$S = \int_4^t 5\,dt$

inetial condition

$S = 5t - 10$

at $t=4$

$S = 5(4) - 10 = 10$

at $t=10$

$S = 5 \times 10 - 10$

$= 50 - 10 = 40$m

so $t = 4$, at $t=10$

$S = 10$m , $S = 40$m

for $10 < t < 15$

$a = \frac{dv}{dt}$

$a = \frac{d}{dt}(-t+15)$

$a = -1$

$V = \frac{ds}{dt}$

$\int_{40}^{s} ds = \int v\,dt$

$S = \int_{10}^{t} (-t+15)\,dt$

$S = \frac{-1}{2} t^2 + 15t - 60$

at $(t=10)$

$S = -\frac{1}{2}(100) + 150 - 60$

at $t = 10$s, at $t = 15$

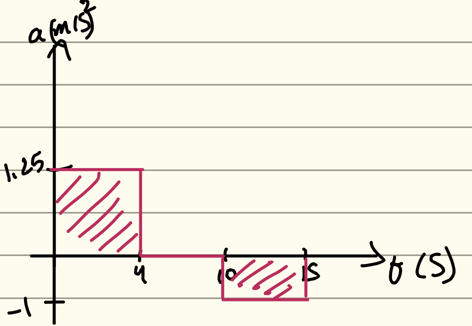
$S = 40$ m , $S = \underset{\sim}{52.5\text{m}}$

↳ this is the total distance travelled by a motorcycle

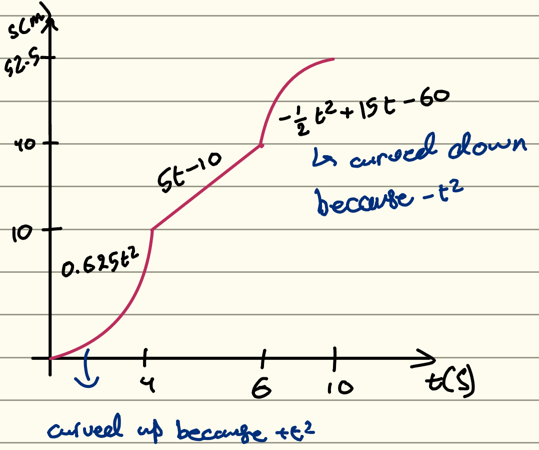
$\int_0^4 1.25t + \int_4^{10} 5 + \int_{10}^{15} -t+15$

$\frac{1.25}{2}t^2 \Big|_0^4 + 5t \Big|_4^{10} + \left(\frac{-t^2}{2} + 15t\right) \Big|_{10}^{15}$

$10 + 30 + 12.5 = 52.5$ ← هاي الطريقة أسهل



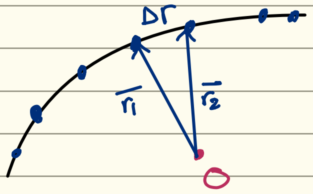
a vs t graph



S vs t graph

Rectilinear motion ⇐ straight line motion لحد هلا حكينا عن ال

12.4 & 12.5: curvilinear motion



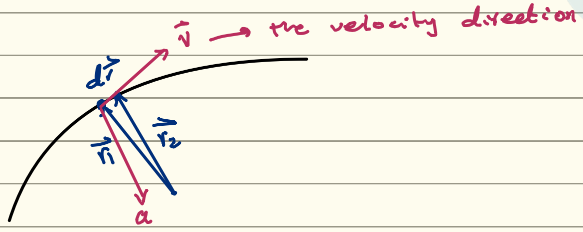
$\vec{r}$: position vector

$\Delta\vec{r}$: change in position (ΔS displacement)

& the Velocity here:-

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{V}_{inst} = \frac{d\vec{r}}{dt} \text{ (tangent)}$$



then the velocity = tangent for the curve (of curvilinear motion) at this point & since the $\Delta r \approx 0$ between r_1 & r_2 we can consider them as one point

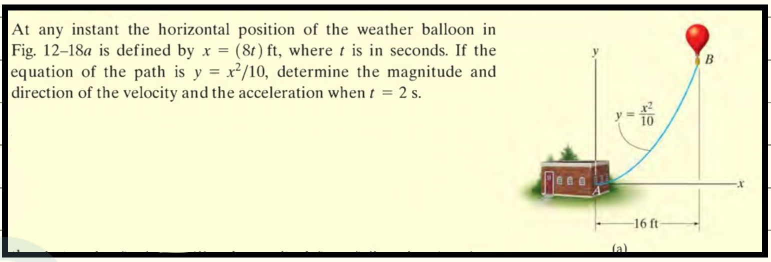
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt}$$ (always to the inside)

\# we can find the values of acceleration & velocity in many ways :-

1. Rectangular components
2. Normal & tangential components
3. cylindrical components will not be covered in this course

\# example 12.4



At any instant the horizontal position of the weather balloon in Fig. 12–18a is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

$$\vec{r} = (x i + y j + z K)\, m$$

$$\vec{v} = d r / dt$$

$$\vec{v} = (\dot{x} i + \dot{y} j + \dot{z} K)\, m/s$$

where $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$, $\dot{z} = \frac{dz}{dt}$

↳ v_x ↳ v_y ↳ v_z

then we also can express v as $\vec{v} = (v_x i + v_y j + v_z K)\, m/s$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= (\dot{v}_x i + \dot{v}_y j + \dot{v}_z K)$$

$$= (a_x i + a_y j + a_z K)\, m/s^2$$

Sol :-

$$\vec{r} = (x i + y j)\, ft$$

$$\vec{r} = \left((2t) i + \left(\frac{x^2}{5}\right) j \right) m$$

but $x = 2t$

$$\vec{r} = \left((2t) i + \left(\frac{4}{5} t^2\right) j \right) m$$

& $\vec{v} = \frac{d\vec{r}}{dt}$

$$\vec{V} = \left(2 i + \left(\frac{8}{5} t\right) j \right) m/s$$

$$\vec{V} = (2 i + (1.6 t) j)\, m/s$$

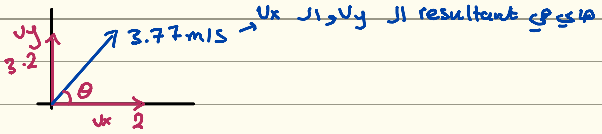
at $t = 2$

$$\vec{V} = (2 i + 3.2 j)\, m/s$$

$|\vec{v}| = \sqrt{2^2+(3.2)^2} =$

$|\vec{N}| = 3.774\, m/s$

$\vec{v} = 2i + 3.2j$



→ Ux ال و Vy ال resultant هاي

$\theta = \tan^{-1}\left(\frac{3.2}{2}\right)$

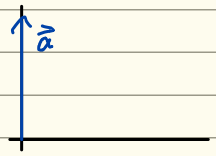
$\theta = 58°$

$\vec{v} = (2i + (1.6t)j)\, m/s$

$\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{a} = (0i + 1.6j)\, m/s^2$

$\vec{a} = \sqrt{0^2+(1.6)^2} = 1.6\, m/s^2$



& $\theta = 90$ or we can say that a is directed along the positive y-axis

* suggested problems of chapter 12

fundimental problems:-

F12–1. Initially, the car travels along a straight road with a speed of 35 m/s. If the brakes are applied and the speed of the car is reduced to 10 m/s in 15 s, determine the constant deceleration of the car.

$V_i = 35$, $V_f = 10$, $t = 15$

constant acceleration

↳ we can use

$V_f = V_i + at$

$10 = 35 + a(15)$

$-35 \quad -35$

$a = -1.66\, m/s^2$

F12–3. A particle travels along a straight line with a velocity of $v = (4t - 3t^2)$ m/s, where t is in seconds. Determine the position of the particle when $t = 4$ s. $s = 0$ when $t = 0$.

$v = \frac{ds}{dt}$

$\int_0^s ds = \int_0^4 v\, dt$

s=0 when t=0

$S = \int_0^4 (4t - 3t^2)\, dt$

$S = \frac{4t^2}{2} - \frac{3t^3}{3} \Big|_0^4$

$S = 2t^2 - t^3 \Big)_0^4$

$S = (2\times16 - 64) - 0$

$S = -32\, m$

F12–4. A particle travels along a straight line with a speed $v = (0.5t^3 - 8t)$ m/s, where t is in seconds. Determine the acceleration of the particle when $t = 2$ s.

$a = \frac{dv}{dt}$

$a = \frac{d}{dt}(0.5t^3 - 8t)$

$a = 1.5t^2 - 8$

when $t = 2s$

$a = (1.5)\times4 - 8$

$a = 6 - 8 = -2\, m/s^2$

F12–5. The position of the particle is given by $s = (2t^2 - 8t + 6)$ m, where t is in seconds. Determine the time when the velocity of the particle is zero, and the total distance traveled by the particle when $t = 3$ s.

$S = (2t^2 - 8t + 6)m$

$v = \frac{ds}{dt} = \frac{d(2t^2 - 8t + 6)}{dt}$

$v = 4t - 8 = 0$

$t = 2$

then we want to find S at t=0,2,3

$S(0) = 6m$, $S(2) = -2m$, $S(3) = 0m$

& we have 2 periods

absolute value ال [illegible]

$0 < t < 2 \Rightarrow \Delta S = S(0) - S(2) = -2-6 = -8m$

$2 < t < 3 \Rightarrow \Delta S = S(3) - S(2) = +2m$

$(\Delta S)_{tot} = 8m + 2m = 10m$

F12–6. A particle travels along a straight line with an acceleration of $a = (10 - 0.2s)\ \mathrm{m/s^2}$, where s is measured in meters. Determine the velocity of the particle when $s = 10$ m if $v = 5$ m/s at $s = 0$.

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \cdot \frac{dv}{ds}$$

So $v\,dv = a\,ds = (10 - 0.2S)\,ds$

$\int v\,dv = \frac{v^2}{2}$ ↳ $\int(10 - 0.2S)\,ds$

$= 10S - 0.1S^2 + c$

then

$\frac{v^2}{2} = 10S - 0.1S^2 + c$

at $S = 0$, $v = 5$, $c = 12.5$

$v^2 = (10S - 0.1S^2 + 12.5)2$

$v^2 = 20S - 0.2S^2 + 25$

at $S = 10$

$v^2 = 200 - (0.2 \times 100) + 25$

$v^2 = 205$

$v = \sqrt{205} = 14.32$ m/s

F12–7. A particle moves along a straight line such that its acceleration is $a = (4t^2 - 2)\ \mathrm{m/s^2}$, where t is in seconds. When $t = 0$, the particle is located 2 m to the left of the origin, and when $t = 2$ s, it is 20 m to the left of the origin. Determine the position of the particle when $t = 4$ s.

$a = (4t^2 - 2)\ \mathrm{m/s^2}$

$S(0) = -2$ m

$S(2) = -20$

$S(4) = ??$

$v = \int a\,dt$

$v = \int 4t^2 - 2$

$v = \frac{4}{3}t^3 - 2t + c_1$

$S = \int \frac{4}{3}t^3 - 2t + c_1$

$S = \frac{1}{3}t^4 - t^2 + c_1 t + c_2$

$S(0) = -2$, $S(2) = -20$

$-2 = c_2$ $c_1 = -\frac{24}{3}$

then

$V = \frac{1}{3}t^4 - t^2 - \frac{24}{3}t - 2$

F12–8. A particle travels along a straight line with a velocity of $v = (20 - 0.05s^2)$ m/s, where s is in meters. Determine the acceleration of the particle at $s = 15$ m.

$a\,ds = v\,dv$

$a = v\frac{dv}{ds}$

$\frac{dv}{ds} = -0.1S$

then

$a = (20 - 0.05S^2)(-0.1S)$

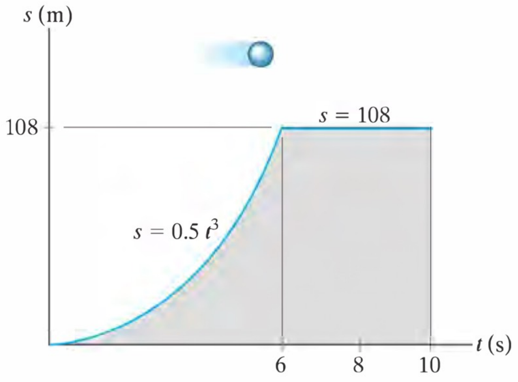
$a = -2S + 0.005S^3$

at $S = 15$ m

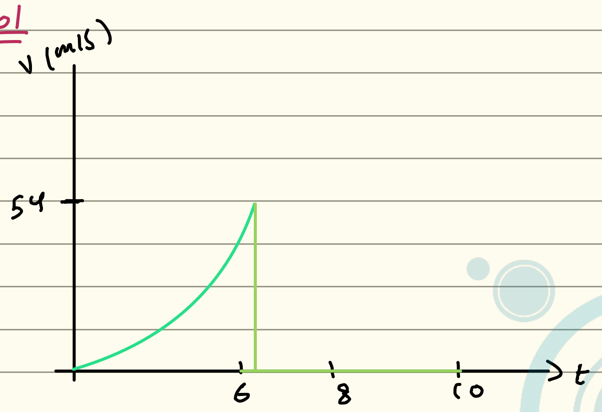
$a = (-2 \times 15) + (0.005 \times 15^3)$

then $a = -13.125\ \mathrm{m/s^2}$

F12–9. The particle travels along a straight track such that its position is described by the $s-t$ graph. Construct the $v-t$ graph for the same time interval.



Sol



when $0<t<6$

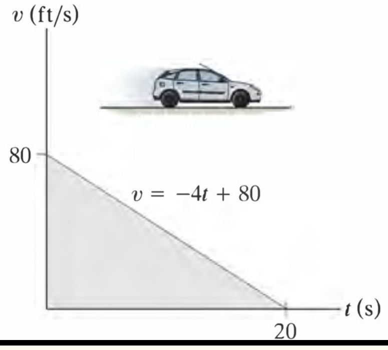
$$v=\frac{ds}{dt}=\frac{d(0.5t^3)}{dt}$$

$$v=1.5t^2$$

when $t=0 \Rightarrow v=0$

when $t=6 \Rightarrow v=54$

F12–10. A van travels along a straight road with a velocity described by the graph. Construct the $s-t$ and $a-t$ graphs during the same period. Take $s=0$ when $t=0$.



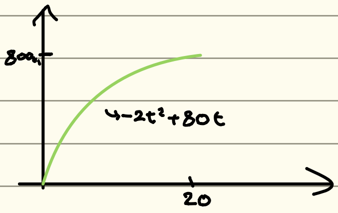
s=0 when t=0

$$v=\frac{ds}{dt}$$

$$\int_0^s ds=\int_0^{20} v\,dt$$

$$S=\int_0^{20} -4t+80 = -2t^2+80t\Big|_0^{20}$$

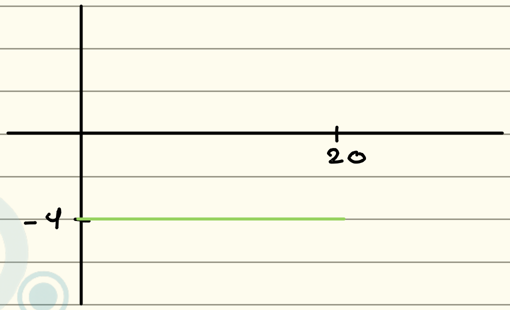
$$=-2(400)+(80\times20)=800\,m$$



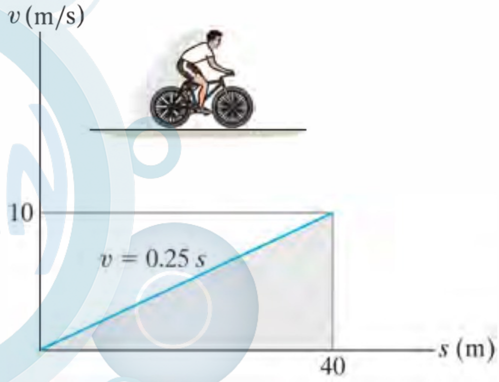
$$a=\frac{dv}{dt}$$

$$a=\frac{d(-4t+80)}{dt}$$

$$a=-4\ m/s^2$$



F12–11. A bicycle travels along a straight road where its velocity is described by the $v-s$ graph. Construct the $a-s$ graph for the same time interval.



$$v=\frac{ds}{dt}$$

$$\int_0^t dt=\int_0^s \frac{ds}{v}$$

$$t=\int_0^s \frac{1}{0.25s}\,ds$$

$$t=\frac{1}{0.25}\ln|0.25\,s|\Big|_0^s$$

$$t=4\ln|0.25s|$$

then

$$t=4\ln|0.25s|$$

$$e^t=e^{\ln|0.25s|^4}$$

$$e^t=(0.25s)^4$$

$$e^t=\frac{1}{256}s^4$$

$$S=\sqrt[4]{256e^t}$$

$$a=\frac{dv}{dt}$$

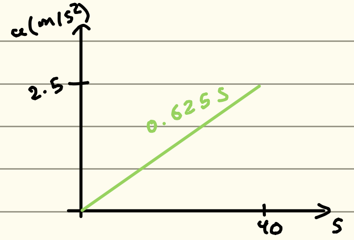
$$a=\frac{dv}{ds}\cdot\frac{ds}{dt}$$

$$a=\frac{dv}{ds}\cdot v$$

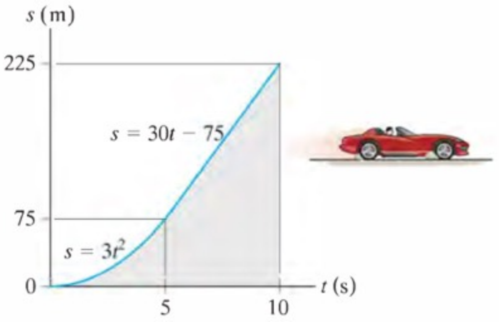
$$a=0.25\times(0.25s)$$

$$\boxed{a=0.0625\,s}$$

a at $s=40 \Rightarrow a=2.5$



F12–12. The sports car travels along a straight road such that its position is described by the graph. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 10$ s.



* for $0 < t < 5$

$v = \frac{ds}{dt}$

$v = \frac{d(3t^2)}{dt}$

$v = (6t)$ m/s

at $t = 5$

$v = 30$ m/s

$a = \frac{dv}{dt}$

$a = \frac{d(6t)}{dt}$

$a = 6$ m/s^2

at $t = 5$

$a = 6$ m/s^2

* for $5 < t < 10$

$v = \frac{ds}{dt}$

$v = \frac{d(30t - 75)}{dt}$

$v = 30$ m/s

& for $t = 10$

$v = 30$ m/s

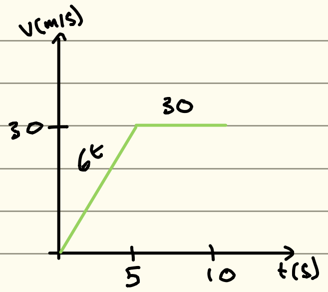
$a = \frac{dv}{dt}$

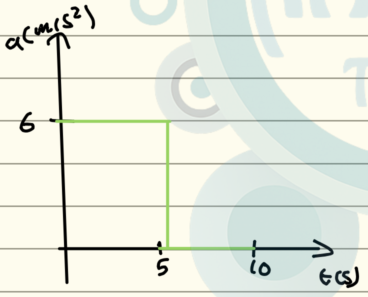
$a = \frac{d(30)}{dt}$

$a = 0$ m/s^2

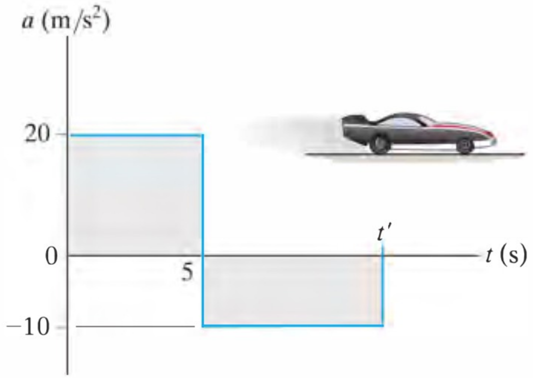
for $t = 10$

$a = 0$ m/s^2





F12–13. The dragster starts from rest and has an acceleration described by the graph. Construct the $v-t$ graph for the time interval $0 \le t \le t'$, where t' is the time for the car to come to rest.



since a is a constant function

* for $0 < t < 5$

$a = 20$

$a = \frac{dv}{dt}$

$\int_0^v dv = \int_0^5 a\, dt$

$v = \int_0^t 20\, dt$

$v = 20t$

then for $t = 5$

$v(5) = 20 \times 5 = 100$ m/s

$v(t) = v(5) + \int_5^t (-10)\, dt = 100 + -10(t-5)$

$= 100 - 10t + 50$

$= 150 - 10t$

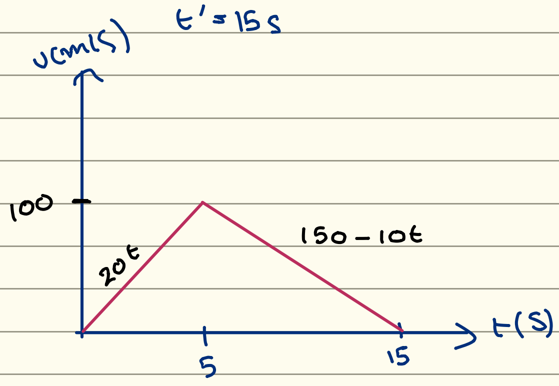
$t' \rightarrow v(t') = 0$

$v(t') = 0$

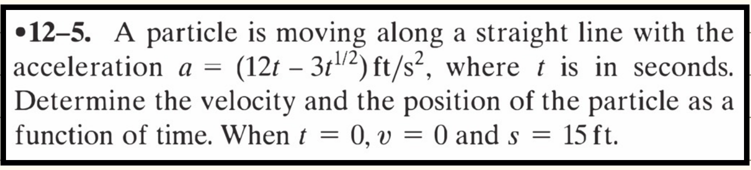
$150 - 10t' = 0$

$\frac{150}{10} = \frac{10t'}{10}$

$t' = 15$ s



problems



•12–5. A particle is moving along a straight line with the acceleration $a = (12t - 3t^{1/2})\ \text{ft/s}^2$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. When $t = 0$, $v = 0$ and $s = 15$ ft.

$V(0)=0$

$S(0)=15$

$a=(12t-3\sqrt{t})\ ft/s^2$

$a=\frac{dv}{dt}$

$\int dv = \int a\, dt$

$v = \int 12t - 3t^{\frac{1}{2}}$

$v = 6t^2 - 2t^{\frac{3}{2}} + C_1$

$v(0)=0$

$0 = C_1$

then

$v = (6t^2 - 2t^{\frac{3}{2}})\ ft/s$

$v = \frac{ds}{dt}$

$\int ds = \int v\, dt$

$s = \int 6t^2 - 2t^{\frac{3}{2}}$

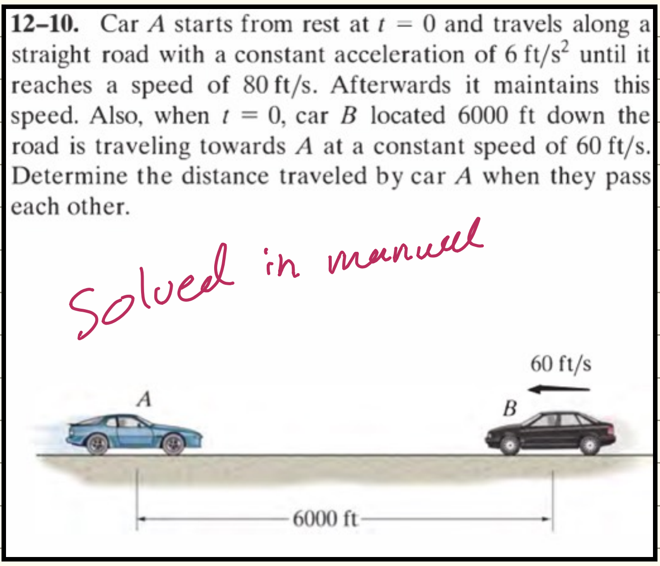
$S = 2t^3 - \frac{4}{5} t^{\frac{5}{2}} + C_2$

$S(0)=15$

$C_2 = 15$

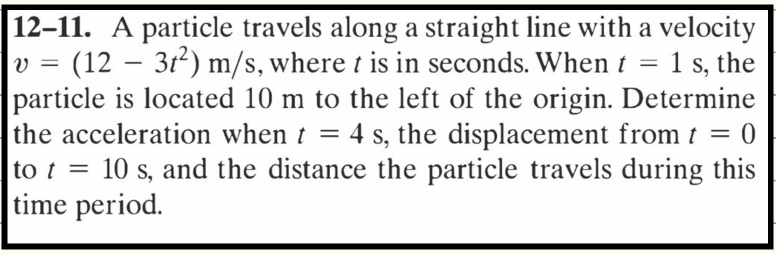
then

$S' = (2t^3 - \frac{4}{5} t^{\frac{5}{2}} + 15)\ ft$



12–10. Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of $6\ \text{ft/s}^2$ until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.

Solved in manual



12–11. A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

$V = 12 - 3t^2$

$S(1) = -10$

$a(4)$??

$\Delta S \quad 0<t<10$

distance ?

$a = \frac{dv}{dt}$

$a = \frac{d(12-3t^2)}{dt}$

$a = -6t\ m/s^2$

$a(4) = -6\times 4 = -24\ m/s^2$

$V = \frac{ds}{dt}$

$\int ds = \int v\, dt$

$S = \int 12 - 3t^2$

$s = 12t - t^3 + C_1$

$S(1) = -10$

$-10 = 12 - 1 + C_1$

$-10 = 11 + C_1$

$-11 \quad -11$

$C_1 = -21$

then $S = (12t - t^3 - 21)\ m$

$S(0) = -21$

$S(10) = -901$

$\Delta S = S(10) - S(0) = -901 + 21 = -880m$

displacement between $0<t<10 = -880m$

$V(t) = 0$

$12 - 3t^2 = 0$

$\frac{3t^2}{3} = \frac{12}{3}$

$t^2 = 4$

$t = \pm 2$

↳ −2 is not possible since t is time

$t = 2$

then

$S(0) = -21$

$S(2) = -5$

$S(10) = -901\ m$

$\Delta S_1 = S(2) - S(0) = -5 - -21 = 16\ m$

$\Delta S_2 = S(10) - S(2) = -901 - -5 = -896$ taking the absolute value

the total distance $(\Delta S_{tot}) = 16 + 896 = 912\ m$

***12–12.** A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

$V_i = 27$ m/s

$a = -6t$ m/s²

$\Delta d = ??$

$a = \frac{dv}{dt}$

$\int dv = \int a\, dt$

$V = \int -6t\ dt$

$V = \frac{-6}{2}t^2 + C_1$

$V = -3t^2 + C_1$

$V(0) = 27$

$\boxed{27 = C_1}$

then $v = -3t^2 + 27$

at $v = 0$

$-3t^2 + 27 = 0$

$\frac{27}{3} = \frac{3t^2}{3}$

$\sqrt{t^2} = \sqrt{9}$

$t = \pm 3$

$t = +3$

distance travelled

$V = \frac{ds}{dt}$

$\int ds = \int v\, dt$

$S = \int_0^t -3t^2 + 27$

$S = -t^3 + 27t$

at $t = 3$

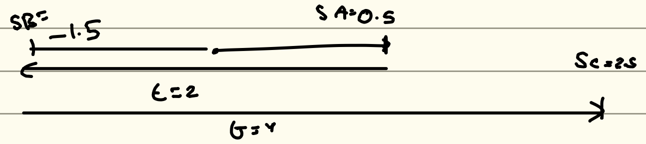
$S = -27 + 27 \times 3$

$= 54$ m

•12–13. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

$S_A = 0.5$ m

$S_B = -1.5$



displacement $= \Delta S = S_C - S_A = 2.5 - 0.5 = 2$ m

$V_{avg} = \frac{\Delta S}{\Delta t} = \frac{2}{6} = \frac{1}{3} = 0.333$ m/s

$(V_{sp})_{avg}$

$S_T = 0.5 + 1.5 + 1.5 + 2.5 = 6$m

$(V_{sp})_{avg} = \frac{6}{6} = 1$ m/s

12–22. A particle moving along a straight line is subjected to a deceleration $a = (-2v^3)\ \text{m/s}^2$, where v is in m/s. If it has a velocity $v = 8$ m/s and a position $s = 10$ m when $t = 0$, determine its velocity and position when $t = 4$ s.

$a = (-2v^3)$ m/s²

$v(0) = 8$ m/s

$S(0) = 10$m

$a\, ds = v\, dv$

$\int_{10\,m}^{s} ds = \int_{8\,m/s}^{v} \frac{v}{a}\, dv$

$S - 10 = \int_{8\,m/s}^{v} \frac{v}{-2v^3}\, dv$

$S - 10 = \int_{8\,m/s}^{v} \frac{-1}{2} \frac{1}{v^2}\, dv$

$S - 10 = \frac{-1}{2} \int_{8\,m/s}^{v} v^{-2}\, dv$

$S - 10 = \frac{+1}{2}\ \frac{1}{v} \Big|_{8\,m/s}^{v}$

$\left(S - 10 = \frac{1}{2v} - \frac{1}{16}\right) 2v$

$2vS - 20v = 1 - \frac{1}{8}v$

$2v\ S - 20v + \frac{1}{8}v = 1$

$v(2S - 20 + \frac{1}{8}) = 1$

$v = \frac{1 \times 8}{2S - \frac{159}{8} \times 8}$

$v = \frac{8}{16S - 159}$

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_{10m}^{s} \frac{16s - 159}{8}\, ds$$

$$t = \frac{1}{8}\left(\frac{16}{2}s^2 - 159s\right)\Big|_{10}^{s}$$

$$t = \left(\frac{1}{8}\right)\left[\left[8s^2 - 159s\right] + 790\right]$$

$$8(t) = \left(\frac{1}{8}\left[8s^2 - 159s + 790\right]\right)8$$

$$\boxed{8t = 8s^2 - 159s + 790}$$

we find this equation to find relation between s & t

when t=4

$$8 \times 4 = 8s^2 - 159s + 790$$

$$8s^2 - 159s + 758 = 0$$

$$s_1 = 11.938\text{ m},\ s_2 = 7.936\text{m}$$

Choosing $s > 10$, then $s = 11.94$

$$v = \frac{8}{16s - 159}$$

$$v = \frac{8}{16(11.94) - 159}$$

$$v = 0.25\text{ m/s}$$

12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)$ m/s^2, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

$$a = -2v \text{ m/s}^2$$

$v = 20$m/s at $s=0$ & $t=0$

قبل هيك كان بدو مني ال velocity & position هلا بدو ال acceleration & velocity

$$a = -2v$$

بدي أعمل relation بين ال (time & acc) & position

$$\frac{dv}{dt} = -2v$$

$$\int_{20m/s}^{v} \frac{dv}{v} = \int_0^t -2\, dt$$

$$\ln v \Big|_{20m/s}^{v} = -2t \Big|_0^t$$

$$\ln v - \ln 20 = -2t$$

$$\overset{e}{\ln}\left(\frac{v}{20}\right) = \overset{e}{-2t}$$

$$20 \times \frac{v}{20} = e^{-2t} \times 20$$

$$v = (20e^{-2t})\text{ m/s}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d(20e^{-2t})}{dt}$$

$$a = -40e^{-2t} \quad \text{m/s}^2$$

$$\int_0^s ds = \int_0^t v\,dt$$

$$s = \int_0^t 20e^{-2t}$$

$$s = -10e^{-2t} + 10$$

$$s = -10(e^{-2t} - 1)$$

or

$$s = 10(1 - e^{-2t})$$

***12–24.** A particle starts from rest and travels along a straight line with an acceleration $a = (30 - 0.2v)$ ft/s^2, where v is in ft/s. Determine the time when the velocity of the particle is $v = 30$ ft/s.

$a = (30 - 0.2v)$ ft/s^2

$v = 30$ ft/s

$$a = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{a}$$

$$t = \int_0^v \frac{dv}{30 - 0.2v}$$

$$t = \int_0^v \frac{dv}{30 - 0.2v}$$

$$t = \frac{1}{-0.2} \ln(30 - 0.2v) \Big|_0^v$$

$$t = -5\left[\ln(30 - 0.2v) - 3.4\right]$$

$$t = -5\ln(30 - 0.2v) + 17$$

$$t = -5\ln(30 - 0.2 \times 30) + 17$$

$t = 1.1097$ S

•12–33. A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 50 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motorcycle at a constant speed of 30 ft/s. Determine the time and the distance traveled by the motorcycle when they pass each other.

when we have constant acceleration first phase

$v(0) = 0$

$a = 6$

$V = 50$

$S(0) = 6000$

when we have constant speed second phase

$V = 30$

$(S_1 = S_2)$ أول شي بنشوف هل التقو ب firstphase ؟ كيف نعرف ؟

أو الفرق بالمسافة بينهم = 0

$V_f = v_i + at$

$50 = 0 + 6t$

$t = 8.333$ S

هلا بدنا نشوف ال distance الي قطعها بهاد الوقت

الدراجة

$(V_f)^2 = (v_i)^2 + 2a\Delta S$

$(50)^2 = 0^2 + 2(6)(S_1 - 0)$

$S_1 = 208.33$ m

هلا بدنا نشوف ال distance الي قطعتها السيارة بهاد الوقت

$$v = \frac{S_2}{t}$$

$S_2 = vt'$ where t' is the time of the first phase for the motorcycle

$S_2 = 30t'$

$S_2 = 30 \times (8.33)$

$S_2 = 250$ m

then did they already met each other in the first phase?

$6000 - 250 - 208.33 = 5541.671$ m

لم remains يعني لسا ما التقو 6 طيب بدنا نشوف كم بدهم

وقت عشان يلتقو

لل car

$S = v_0 t$

$x = 30t''$

$5541.61 - x = 30t''$ ...①

بالنسبة لل motor cycle

$S = v_0 t$

$x = 50t''$ ...②

t'': وقت الانتقال phase الثاني

solving 1 & 2

$x = 50t''$

$5541.61 - 50t'' = 30t''$

$$\frac{5541.61}{80} = \frac{80t''}{80}$$

$t'' = 69.27$ s

$x = 3463.54$ ft

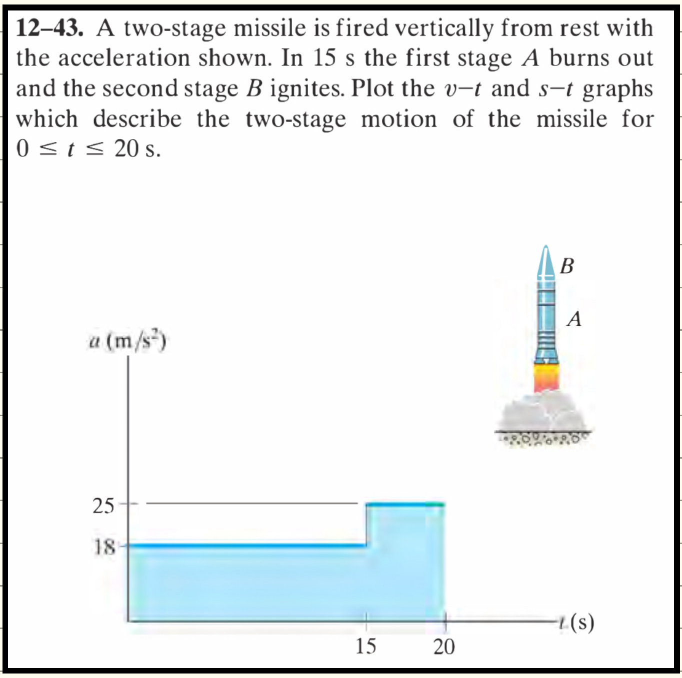
then the time

$T_{tot} = t' + t'' = 8.33 + 69.27 = 77.603$ s

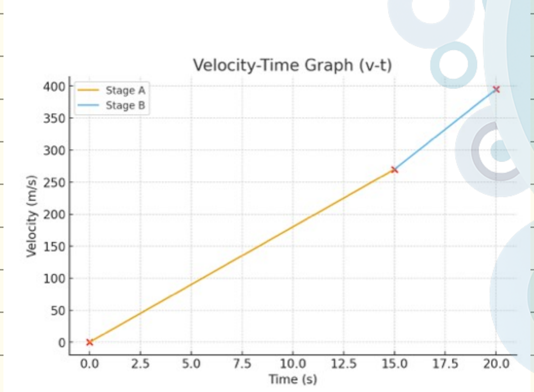
$S_m = 208.33 + 3463.54 = 3671.87$ ft

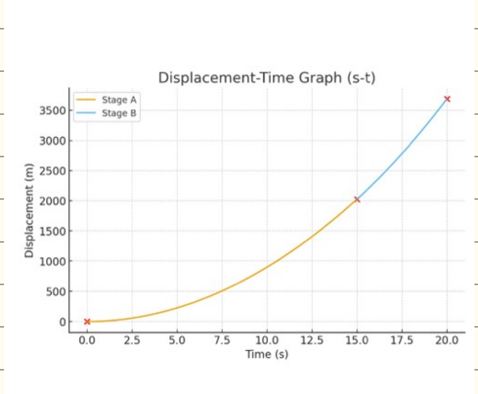
↳ motor المسافة المقطوعة بالنسبة لل

الي راح يلتقو فيها



12–43. A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the two-stage motion of the missile for $0 \leq t \leq 20$ s.





* for $0<t<15$

$v = \int 18 dt$

$v = 18t$

at $t=0 \Rightarrow v=0$

at $t=15 \Rightarrow v=270$

$s = \int v dt$

$s = \int_0^t 18t$

$s = 9t^2$

at $t=0 \Rightarrow s=0$

at $t=15 \Rightarrow s= 2025$

* for $15<t<20$

$V = v(15) + \int_{15}^{t} a\, dt$

$v = 270 + \int_{15}^{t} 25\, dt$

$v = 270 + 25(t-15)$

at $t=20$

$U = 270+25(20-15)$

$U = 395$ m/s

to find s

$S = 2025 + \int_{15}^{t} [270+25(t-15)]\, dt$

$= 2025 + 270t \Big|_{15}^{20} + \frac{25}{2} (t-15)^2 \Big|_{15}^{20}$

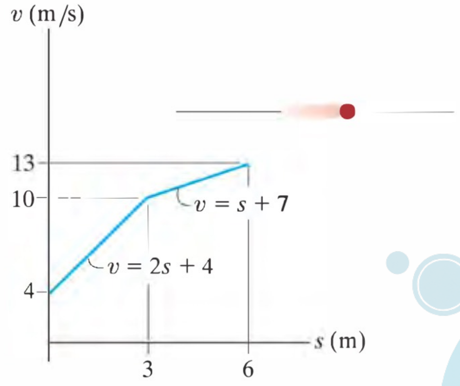
$= 2025 + 270(5) + 12.5 \; (5^2 - 0^2) = 3687.5$

$= 2025 + 1350 + 312.5 =$

12–46. A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the v–t graph and determine the maximum speed of the train.

موجود بالمانيوال

12–47. The particle travels along a straight line with the velocity described by the graph. Construct the a–s graph.



for $0 < t < 3$

$a = \frac{dv}{dt}$

$a = \frac{dv}{ds} \cdot \frac{ds}{dt}$

$a = \frac{dv}{ds} \cdot v$

$a = 2 \cdot (2S + 4)$

$a = (4s + 8)\ m/s^2$

for $3 < t < 6$

$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt}$

$a = \frac{dv}{ds} \cdot v$

$a = 1 . (S + 7)$

$a = (S + 7)\ m/s^2$

at $S = 0$

$a = 8\ m/s^2$

at $S = 3$

$a = 20\ m/s^2$

at $t = 3$

$a = 10\ m/s^2$

at $t = 6$

$a = 13\ m/s^2$

•12–49. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \le t \le 3$ s.

$v = \frac{ds}{dt} = \frac{d(t^3 - 3t^2 + 2t)}{dt}$

$v = 3t^2 - 6t + 2 \Rightarrow$

when v=0 the particle changes it's direction

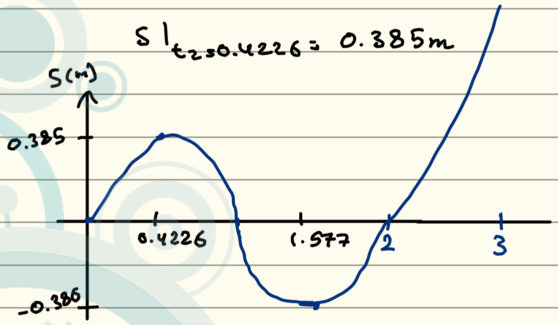
$a = \frac{dv}{dt} = \frac{d(3t^2 - 6t + 2)}{dt} = 6t - 6$

$3t^2 - 6t + 2 = 0$

$t_1 = 1.577$, $t_2 = 0.4226$

then $s\big|_{t_1 = 1.577} = -0.386 m$

$s\big|_{t_2 = 0.4226} = 0.385 m$



التكملة بالمانيوال

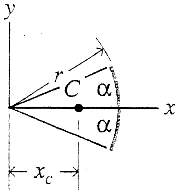
TABLE 5-1 CENTROID LOCATIONS FOR A FEW COMMON LINE SEGMENTS AND AREAS

Circular arc

$$L = 2r\alpha$$

$$x_C = \frac{r \sin \alpha}{\alpha}$$

$$y_C = 0$$

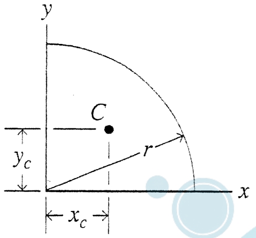


Quarter circular arc

$$L = \frac{\pi r}{2}$$

$$x_C = \frac{2r}{\pi}$$

$$y_C = \frac{2r}{\pi}$$

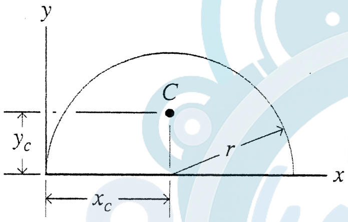


Semicircular arc

$$L = \pi r$$

$$x_C = r$$

$$y_C = \frac{2r}{\pi}$$

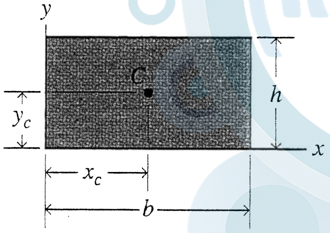


Rectangular area

$$A = bh$$

$$x_C = \frac{b}{2}$$

$$y_C = \frac{h}{2}$$

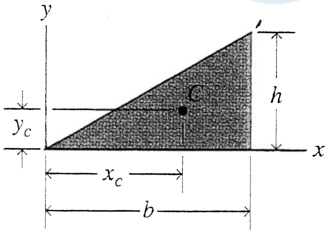


Triangular area

$$A = \frac{bh}{2}$$

$$x_C = \frac{2b}{3}$$

$$y_C = \frac{h}{3}$$

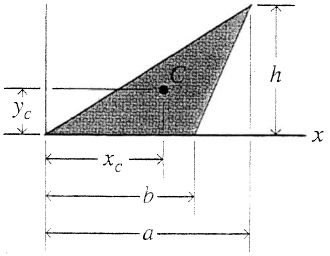


Triangular area

$$A = \frac{bh}{2}$$

$$x_C = \frac{a + b}{3}$$

$$y_C = \frac{h}{3}$$

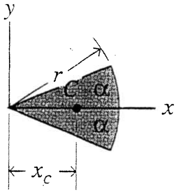


Circular sector

$$A = r^2\alpha$$

$$x_C = \frac{2r \sin \alpha}{3\alpha}$$

$$y_C = 0$$

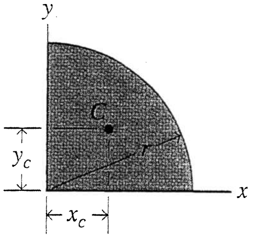


Quadrant of a circle

$$A = \frac{\pi r^2}{4}$$

$$x_C = \frac{4r}{3\pi}$$

$$y_C = \frac{4r}{3\pi}$$

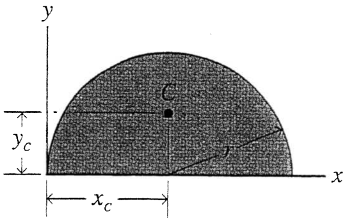


Semicircular area

$$A = \frac{\pi r^2}{2}$$

$$x_C = r$$

$$y_C = \frac{4r}{3\pi}$$

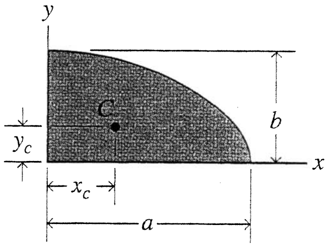


Quadrant of an ellipse

$$A = \frac{\pi ab}{4}$$

$$x_C = \frac{4a}{3\pi}$$

$$y_C = \frac{4b}{3\pi}$$

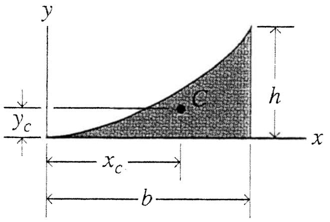


Parabolic spandrel

$$A = \frac{bh}{3}$$

$$x_C = \frac{3b}{4}$$

$$y_C = \frac{3h}{10}$$



Quadrant of a parabola

$$A = \frac{2bh}{3}$$

$$x_C = \frac{5b}{8}$$

$$y_C = \frac{2h}{5}$$

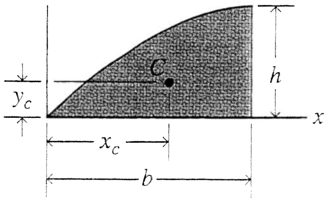


TABLE 5-2 CENTROID LOCATIONS FOR A FEW COMMON VOLUMES

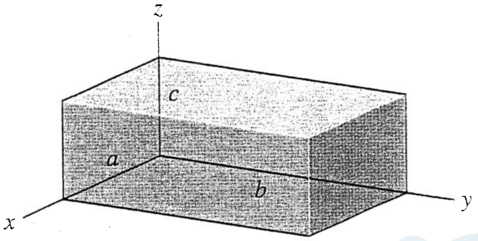
Rectangular parallelepiped

$$V = abc$$

$$x_C = \frac{a}{2}$$

$$y_C = \frac{b}{2}$$

$$z_C = \frac{c}{2}$$



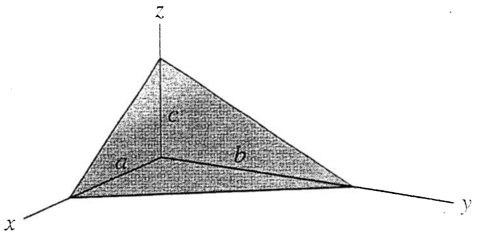
Rectangular tetrahedron

$$V = \frac{abc}{6}$$

$$x_C = \frac{a}{4}$$

$$y_C = \frac{b}{4}$$

$$z_C = \frac{c}{4}$$



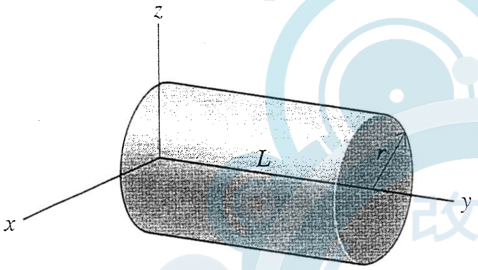
Circular cylinder

$$V = \pi r^2 L$$

$$x_C = 0$$

$$y_C = \frac{L}{2}$$

$$z_C = 0$$



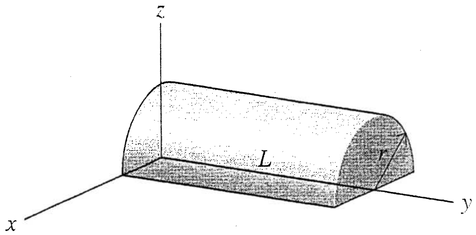
Semicylinder

$$V = \frac{\pi r^2 L}{2}$$

$$x_C = 0$$

$$y_C = \frac{L}{2}$$

$$z_C = \frac{4r}{3\pi}$$



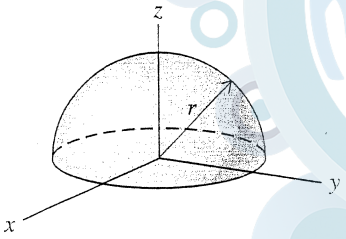
Hemisphere

$$V = \frac{2\pi r^3}{3}$$

$$x_C = 0$$

$$y_C = 0$$

$$z_C = \frac{3r}{8}$$



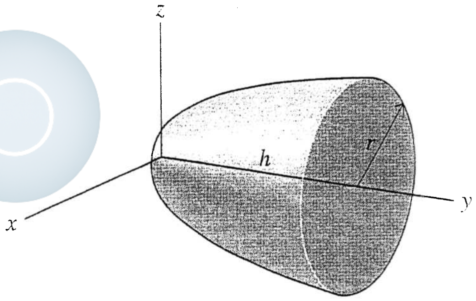
Paraboloid

$$V = \frac{\pi r^2 h}{2}$$

$$x_C = 0$$

$$y_C = \frac{2h}{3}$$

$$z_C = 0$$



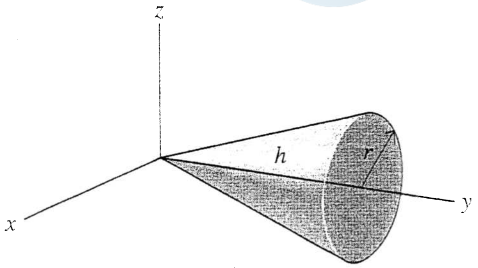
Right circular cone

$$V = \frac{\pi r^2 h}{3}$$

$$x_C = 0$$

$$y_C = \frac{3h}{4}$$

$$z_C = 0$$



Half cone

$$V = \frac{\pi r^2 h}{6}$$

$$x_C = 0$$

$$y_C = \frac{3h}{4}$$

$$z_C = \frac{r}{\pi}$$