Control Quizzes

Jana Househ

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s^2+1)}$$

Solution : $\dfrac{1}{s(s^2+1)} = \dfrac{k_1}{s} + \dfrac{k_2 s + k_3}{s^2+1}$

$1 = k_1(s^2+1) + (k_2 s + k_3)(s)$

$s=0 \rightarrow k_1 = 1$, $s=1$: $1 = 2 + (k_2+k_3)$.

$s=2$: $1 = 5 + (2k_2 + k_3)$ (2)

$k_2 = \cancel{-3}\ -1$ $\quad k_3 = \cancel{2}\ 0$.

$1 - \cos t$.

Name: ID:

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s+1)^2}$$

$\dfrac{1}{s(s+1)^2} = \dfrac{k_1}{s} + \dfrac{k_2}{s+1} + \dfrac{k_3}{(s+1)^2}$

$1 = k(s+1)^2 + k_2(s)(s+1) + k_3(s)$.

$s=0$: $1 = 4k_1 \Rightarrow k_1 = 1/4$.

$s=-1$: $1 = -1k_3 \Rightarrow k_3 = -1$

$s=1$: $1 = 1 + 2k_2 + -1 \quad k_2 = \frac{1}{2}$

$\Rightarrow \frac{1}{4} + \frac{1}{2}e^{-t} + te^{-t}$

Name: ID:

Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{s(s^2+4s+13)}$$

$\dfrac{k_1}{s} + \dfrac{k_2 s + k_3}{(s+2)^2+3^2} = \dfrac{1}{s(s^2+4s+13)}$

$\rightarrow \dfrac{k_2(s+2)}{\cdots} + \dfrac{k_3 - 2k_2}{\cdots}$

$k_1((s+2)^2+3^2) + (k_2 s + k_3)(s) = 1$.

$s=0$: $1 = k_1(13) \rightarrow k_1 = 1/13$

$s=1$: $1 = \frac{18}{13} + (k_2+k_3)$. $\quad k_2 = -1/13$

$s=-1$: $1 = \frac{10}{13} + k_2 - k_3 \quad k_3 = -4/13$

$\dfrac{1}{13} + \dfrac{-1}{13} e^{-2t} \cos 3t + \dfrac{-5}{39} e^{-2t} \sin(3t)$.

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Q1) Find the inverse laplace transform using partial fractions for

$$C(s) = \frac{1}{(s+3)(s^2+2s+10)}$$

$\dfrac{k_1}{s+3} + \dfrac{k_2 s + k_3}{(s+1)^2+3^2} \rightarrow \dfrac{k_2(s+1)}{\cdots} + \dfrac{k_3 - k_2}{\cdots}$

$1 = k_1(s^2+2s+10) + (k_2 s + k_3)(s+3)$.

$s=-3$: $1 = 13k_1 \longrightarrow k_1 = 1/13$

$s=0$: $1 = \frac{10}{13} + 3k_3 \rightarrow k_3 = 1/13$

$s=1$: $1 = 1 + 4k_2 + \frac{4}{13} \rightarrow k_2 = -4/13$

$\dfrac{1}{13} e^{-3t} + \dfrac{-4}{13} e^{-t} \cos 3t + \dfrac{5}{39} e^{-t} \sin 3t$.

Find the solution of the following differential equation

$$\dot{X} + 2X + X = 1, \qquad \textit{Initial conditions zero}$$

$sX(s) - \cancel{\ddot{X}(0)} + 2X(s) + X(s) = 1/s$

$X(s)(s+2+1) = 1/s$

$X(s) = \dfrac{1}{(s+3)(s)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+3}$.

$1 = k_1(s+3) + k_2(s)$ $\quad s=0$: $k_1 = \frac{1}{3}$ $\quad s=-3$: $k_2 = -\frac{1}{3}$.

$\frac{1}{3} - \frac{1}{3} e^{-3t}$ $\quad$ #.

Find C(t)

$$C(s) = \frac{5}{(s+1)(s+3)}$$

$5 = k_1(s+3) + k_2(s+1)$ $\quad s=-1 \quad k_1 = 5/2$ $\quad s=-3 \quad k_2 = -5/2$

$\frac{5}{2} e^{-t} + \frac{-5}{2} e^{-3t}$

Write the MATLAB code to find the inverse laplace for

$$C(s) = \frac{5}{(s+1)(s+3)(s^2+2s+3)}$$

Type equation here.

```
syms t s
ilaplace(5/((s+1)*(s+3)*((s^2)+2*s+3))).
```

Find the final value for

$$C(s) = \frac{s+3}{s^2+2s+5}$$

$F(s) = \dfrac{s+3}{s^2+2s+5}$.

final $t \rightarrow \infty \Rightarrow s \rightarrow 0$

$\lim_{s \to 0} (s)\left(\dfrac{s+3}{s^2+2s+5}\right) = 0 * \dfrac{3}{5} = 0$.

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Find the final value for

$$C(s) = \frac{s+1}{s(s+1)}$$

$\dfrac{k_1}{s} + \dfrac{k_2}{s+1} \Rightarrow s+1 = k_1(s+1) + k_2 s$.

$s=0$: $1 = k_1$

$s=1$: $2 = 2 + k_2 = k_2 = 0$

$c(t) = 1$.

$\lim_{s \to 0} \cancel{s} \cdot \dfrac{(s+1)}{(s+1)\cancel{(s)}} = \dfrac{1}{1} = 1$.

Quiz 1 Industrial control Prof M. Barghash

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Find the initial value for

$$c(s) = \frac{s+2}{s(s+5)}$$

$$s \cdot F(s) = (\cancel{s}) \cdot \frac{(s+2)}{(\cancel{s})(s+5)}$$, initial value $t=0 \longrightarrow s=\infty$

$$\lim_{s \to \infty} \frac{s+2}{s+5} = \frac{\infty}{\infty} = 1.$$

Name: ID:

Find the final value for the following function using final value theorem

$$C(s) = \frac{s+3}{s(s+1)^2}$$

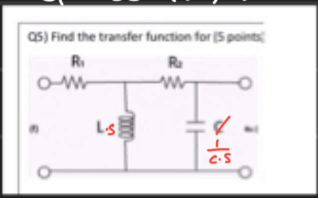
$$\lim_{s \to 0} \cancel{s} \cdot \frac{s+3}{\cancel{s}(s+1)^2} = \frac{3}{1} = 3$$

Find c(t) the time response for

$$C(s) = \frac{s+2}{s(s+1)^2}$$

$$\frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{(s+1)^2} \quad ----$$

Quize (2):



$$V_o = I * \frac{1}{C\cdot S} \quad \dots (1)$$

$$Z_{eq} = \left(\frac{\left(\frac{1}{C\cdot S} + R_2\right) * L\cdot S}{\left(\frac{1}{C\cdot S} + R_2\right) + L\cdot S} \right) + R_1$$

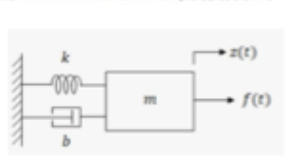
$$\frac{C\cdot S}{C\cdot S} * \frac{\frac{1+R_2C\cdot S}{C\cdot S} * L\cdot S}{\left(\frac{1+R_2C\cdot S}{C\cdot S} + L\cdot S\right)} + R_1 = \left(\frac{(1+R_2CS)*L\cdot S}{1+R_2C\cdot S + LCS^2}\right) + R_1$$

Zeq

$$V_{in} = I * Z_{eq}$$

=> the transfer function.

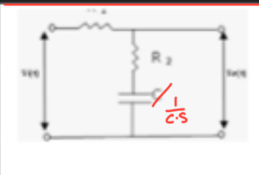
$$\frac{\cancel{I}}{C\cdot S * \cancel{I}\left(\frac{(1+R_2CS)*L\cdot S}{(1+R_2CS+LCS^2)} + R_1\right)}$$



$$F(t) = M\ddot{x} + b\dot{x} + kx$$

$$Ms^2X(S) + bsX(S) + kX(S)$$

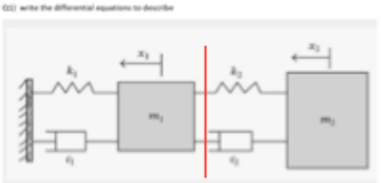
$$\frac{X(S)}{F(x)} = \frac{1}{Ms^2 + bs + k}$$



$$V_o = I * Z_{R_2C} = I\left(R_2 + \frac{1}{C\cdot S}\right)$$

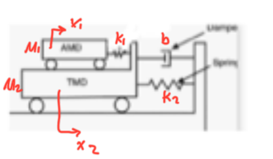
$$Z_{eq} = R_2 + \frac{1}{C\cdot S} + R_1 = \left(\frac{1 + R_1CS + R_2C\cdot S}{C\cdot S}\right)$$

$$\left(\frac{V_o}{V_{in}} = \frac{\left(R_2 + \frac{1}{C\cdot S}\right)}{R_2 + R_1 + \frac{1}{C\cdot S}}\right)$$



$$0 = M_1\ddot{x} + c_1\dot{x}_1 + kx_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2)$$

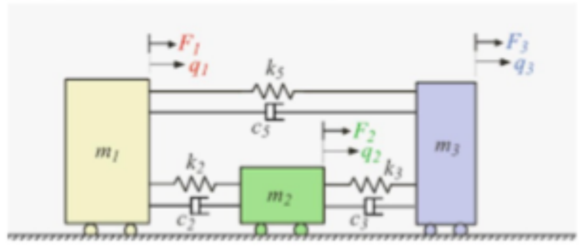
$$0 = M_2\ddot{x} + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)$$



$$0 = M_1\ddot{x}_1 + k_1(x_1 - x_2).$$

$$0 = M_2\ddot{x}_2 + b\dot{x}_2 + k_2x_2 + k_1(x_2 - x_1).$$

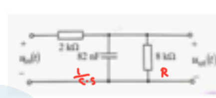
Write the differential equaions describing the following system



$$F_1 = M\ddot{x}_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + k_5(x_1 - x_3) + c_5(\dot{x}_1 - \dot{x}_3).$$

$$F_2 = M_2\ddot{x}_2 + k_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_3) + c_3(\dot{x}_2 - \dot{x}_3).$$

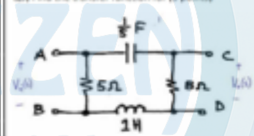
$$F_3 = M_3\ddot{x}_3 + k_3(x_3 - x_2) + c_3(\dot{x}_3 - \dot{x}_2) + k_5(x_3 - x_1) + c_5(\dot{x}_3 - \dot{x}_1)$$



$$V_o = I * \left(\frac{R_1 * \frac{1}{C\cdot S}}{R_1 + \frac{1}{C\cdot S}}\right) = \left(\frac{8000}{6.56 * 10^{-4}s + 1}\right)$$

$$V_{in} = I\left(Z_{RC} + Z_{R_2}\right) = \left(\frac{1000 + 1.3s}{6.56 * 10^{-4}s + 1}\right)$$

$$\frac{V_o}{V_{in}} = \frac{8000}{1.3s + 1000}.$$



$$V_o = I * 8.$$

$$Z_{eq(1)} = 8 + \frac{5}{s} + 1\cdot S = \left(\frac{s^2 + 8s + 5}{s}\right) \qquad \left(\frac{40 + \frac{25}{s} + 5s}{8 + \frac{5}{s} + s + 5}\right)$$

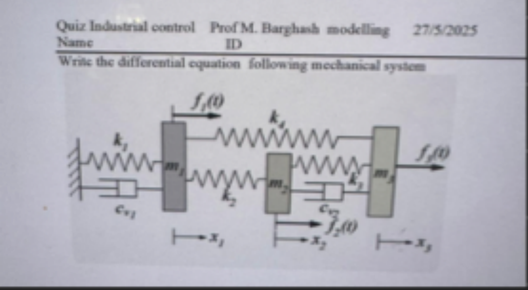
$$Z_{eq} = \frac{\left(\frac{s^2 + 8s + 5}{\cancel{s}}\right) * 5}{\frac{s^2 + 8s + 5 + 5s}{\cancel{s}}} = \frac{(s^2 + 8s + 5)5}{s^2 + 8s + 5 + 5s.}$$

$$\frac{8(s^2 + 8s + 5 + 5s)}{5(s^2 + 8s + 5)} \Rightarrow \qquad \frac{8}{5}\left(\frac{8 + \frac{5}{s} + s + 5}{(8 + \frac{5}{s} + s)}\right) *$$

$$\frac{8}{5} + \frac{\cancel{5}}{(8 + \frac{5}{s} + s)} * \frac{8}{\cancel{5}}$$

$$\frac{8}{5}\left(\frac{s^2 + 8s + 5}{s^2 + 8s + 5} + \frac{5s}{s^2 + 8s + 5}\right)$$

$$\frac{8}{5} + \frac{8}{\cancel{5}} * \frac{\cancel{5}}{s + 8 + \frac{5}{s}} = \left(\frac{8}{5} + \frac{8}{s + \frac{5}{s} + 8.}\right)$$

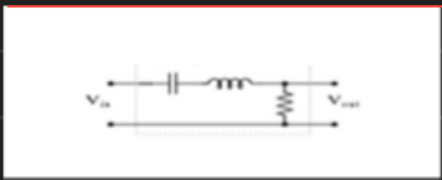


Quiz Industrial control Prof M. Barghash modelling 27/5/2025
Name ID
Write the differential equation following mechanical system

$$M_1\ddot{x}_1 + C_{v1}(\dot{x}_1) + K_1x_1 + K_2(x_2-x_1) + K_4(x_3-x_1) = F_1(s).$$

$$M_2\ddot{x}_2 + C_{v2}(\dot{x}_2-\dot{x}_3) + K_2(x_2-x_1) + K_3(x_2-x_3) = F_2(s).$$

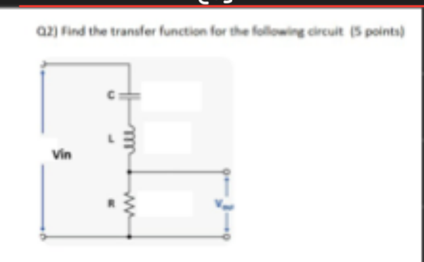
$$M_3\ddot{x}_3 + C_{v2}(\dot{x}_3-\dot{x}_2) + K_3(x_3-x_2) + K_4(x_3-x_1).$$



$$V_o = I\cdot R$$

$$Z_{eq} = R + \frac{1}{C\cdot S} + L\cdot S.$$

$$\frac{V_o}{V_o} = \frac{R}{R+\frac{1}{C\cdot S}+L\cdot S}.$$



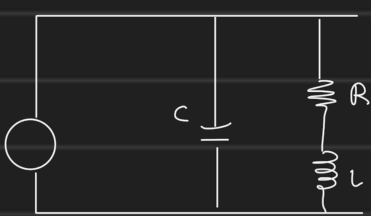
Q2) Find the transfer function for the following circuit (5 points)

$$V_o = IR$$

$$Z_{eq} = R + \frac{1}{C\cdot S} + L\cdot S$$

$$V_{in} = I\left(R+\frac{1}{C\cdot S}+L\cdot S\right).$$

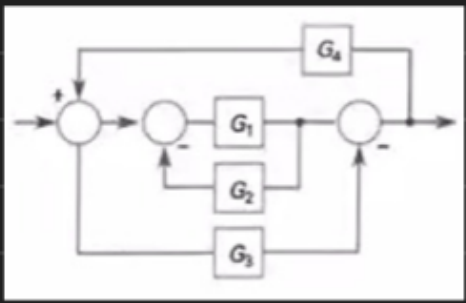
$$\frac{V_o}{V_{in}} = \frac{R}{\left(R+\frac{1}{C\cdot S}+L\cdot S\right)}.$$



$$T.F = \frac{R+L\cdot S}{\frac{\frac{(R+LS)}{C\cdot S}}{R+LS+\frac{1}{C\cdot S}}}$$

$$Z_{eq} = \frac{\frac{(R+LS)}{\cancel{CS}}}{\left(R+LS+\frac{1}{C S}\right)\cancel{CS}} \qquad \left(\frac{R+LS}{RCS+CLS^2+1}\right)$$

Quize (3) Block diagram.



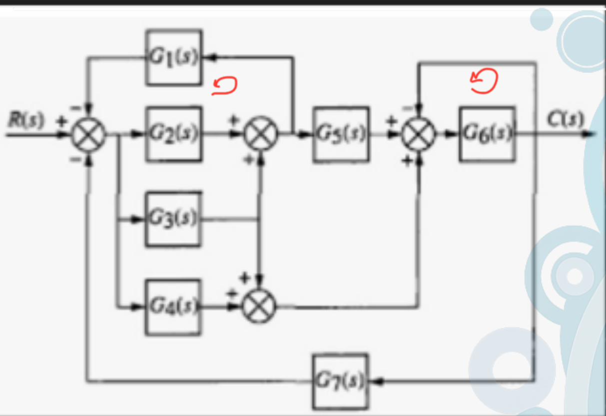
Paths ① P_1 G ② P_2 $-G_3$

Δ ① $\Delta_1 = 1$ ② $\Delta_2 = 1-(-G_1G_2)$

loops $[-G_1G_2,\ G_1G_4,\ -G_3G_4]$

nontouching loops ⟹ $G_1G_2G_3G_4$

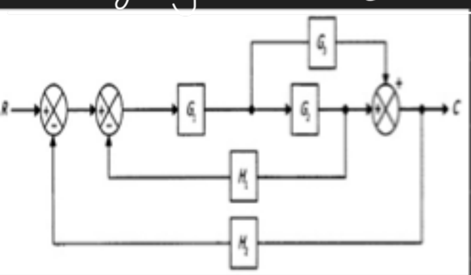
$$T.F = \frac{G_1 - G_3(1+G_1G_2)}{1+G_1G_2+G_3G_4-G_1G_4+G_1G_2G_3G_4}$$



Pathes $[G_2G_5G_6,\ G_3G_5G_6,\ G_4G_6,\ G_3G_6]$ $\Delta = 1$

loops $[-G_1G_2,\ -G_6,\ -G_2G_5G_6G_7,\ -G_3G_5G_6G_7,$
$-G_4G_6G_7,\ -G_3G_6G_7]$

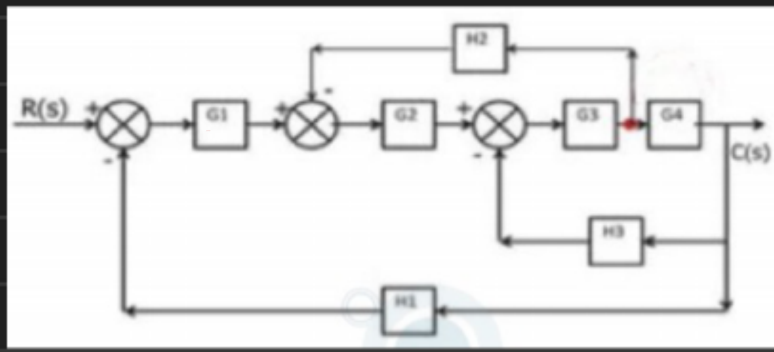
nontoughing $G_1G_2G_6$



no nontoching loops.

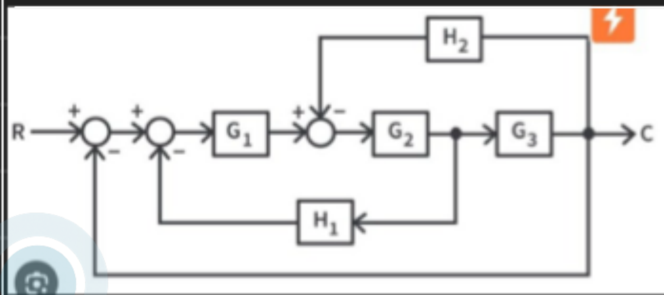
Pathes $[G_1G_2,\ G_1G_3]$ $\Delta=1$

loops $[-G_1G_2H_1,\ -G_1G_2H_2,\ -G_1G_3H_2]$



Pathes $[G_1G_2G_3G_4]$ $\Delta=1$

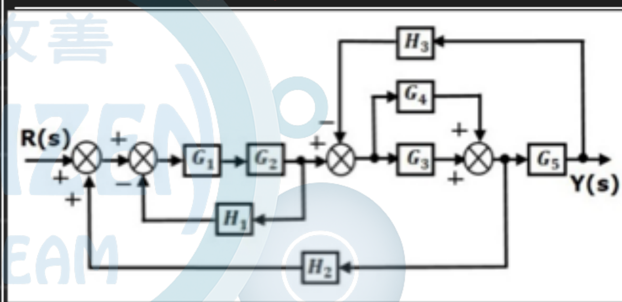
loops $[-G_2G_3H_2,\ -G_3G_4H_3,\ -G_1G_2G_3G_4H_1]$



Pathes $[G_1G_2G_3]$ $\Delta=1$

loops $[-G_1G_2H_1,\ -G_1G_2G_3,\ -G_2G_3H_2]$

$$\Delta P = \frac{G_1G_2G_3}{1+G_1G_2H_1+G_1G_2G_3+G_2G_3H_2}.$$

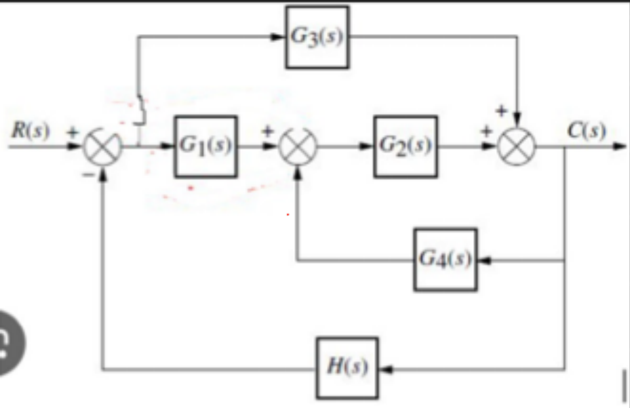


Pathes $[G_1G_2G_3G_5,\ G_1G_2G_4G_5]$.

loops $[-G_1G_2H_1,\ G_1G_2G_3H_2,\ G_1G_2G_4H_2,\ -G_3G_5H_3,$
$-G_4G_5H_3]$.

nontouching loops $[G_1G_2H_1G_3G_5H_3,\ G_1G_2H_1G_4G_5H_3]$

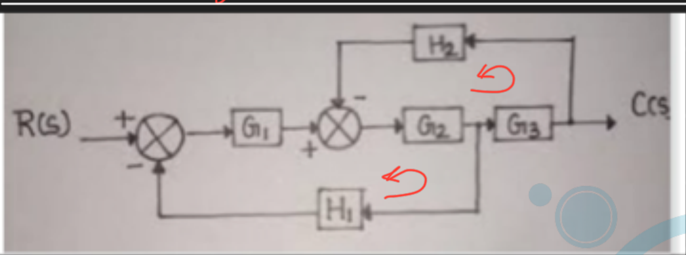
$1 - \Sigma\text{loops} + \text{non}.$



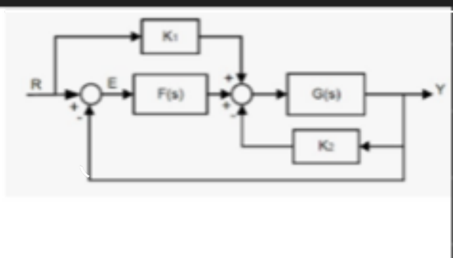
Pathes $[G_1 G_2 \,,\, G_3]$ $\Delta_1 = 1$, $\Delta_2 = 1$.

Loops $[G_2 G_4 \,,\, -G_1 G_2 H \,,\, -G_3 H]$

no nontouching loops.



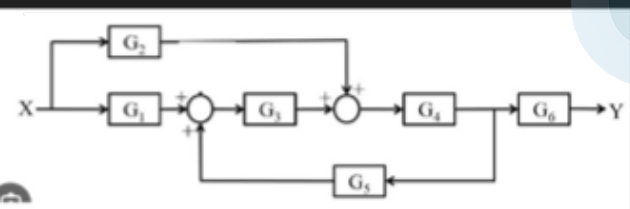
pathes $[G_1 G_2 G_3]$, loops $[-G_2 G_3 H_2 \,,\, -G_1 G_2 G_3 H_1]$



Mason rule=(K1*G+F*G)/(1+GH+FGH)

Pathes $[K_1 G \,,\, FG]$.

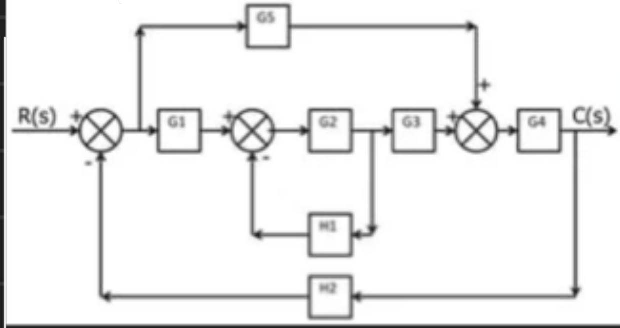
loops $[FG \,,\, GK$



Pathes $[G_1 G_3 G_4 G_6 \,,\, G_2 G_4 G_6]$

Loops $[G_3 G_4 G_5]$

$$T.F = \frac{G_1 G_3 G_4 G_6 + G_2 G_4 G_6}{1 - G_3 G_4 G_5}$$

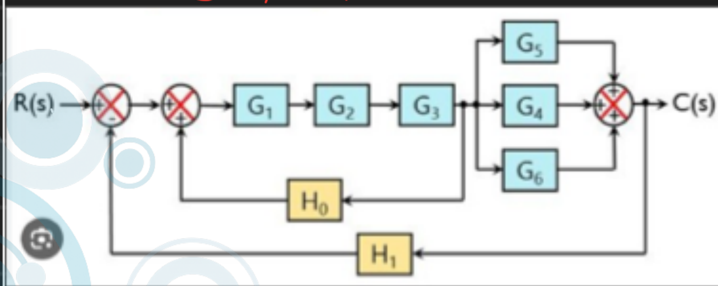


Pathes $[G_1 G_2 G_3 G_4 \,,\, G_5 G_4]$ $\Delta_1 = 1$ $\Delta_2 = 1 + G_2 H_1$

Loops $[-G_2 H_1 \,,\, -G_1 G_2 G_3 G_4 H_2 \,,\, -G_5 G_4 H_2]$

non touching $[G_2 H_1 G_5 G_4 H_2]$.

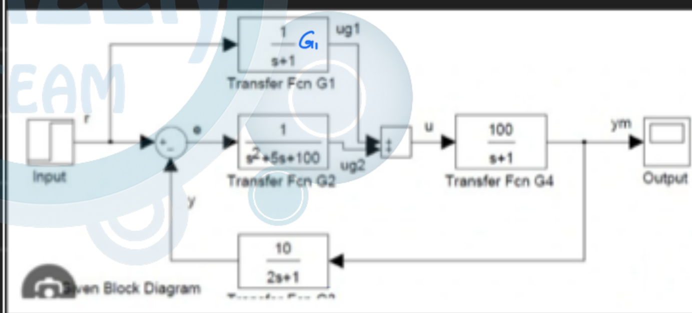
$$T.F = \frac{G_1 G_2 G_3 G_4 - G_5 G_4 (1 + G_2 H_1)}{1 - \sum loops + G_2 H_1 G_5 G_4 H_2}$$



Pathes $[G_1 G_2 G_3 G_5 \,,\, G_1 G_2 G_3 G_4 \,,\, G_1 G_2 G_3 G_6]$ $\Delta = 1$

Loops $[G_1 G_2 G_3 H_0 \,,\, -G_1 G_2 G_3 G_5 H_1 \,,\, -G_1 G_2 G_4 H_1$
$-G_1 G_2 G_6 H_1 .]$

no nontouching loops.

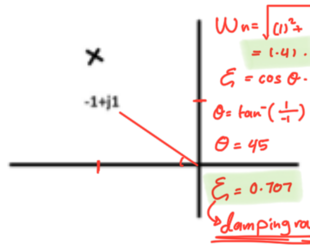


$$T.F = \frac{\frac{100}{(s+1)^2} + \frac{100}{(s^2+5s+100)(s+1)}}{\frac{1001}{(s^2+5s+100)(s+1)(2s+1)}} = \frac{\frac{100(s^2+5s+100) + 100(s+1)}{\cancel{(s^2+5s+100)}(s+1)^{\cancel{2}}}}{\frac{1001}{\cancel{(s^2+5s+100)}\cancel{(s+1)}(2s+1)}}$$

$$\frac{100\left((s^2+5s+100) + (s+1)\right)}{1001} * \frac{(2s+1)}{(s+1)}$$

Quiz (4): first & Second Order.

For the following pole find the settling time



$W_n=\sqrt{(1)^2+(1)^2}=1.41$.

$\xi=\cos\theta$.

$\theta=\tan^{-1}(\frac{1}{1})$

$\theta=45$

$\xi=0.707$ damping ratio

① $T_s=\frac{4}{0.707*1.41}=4.012\,s$.

② $T_p=\frac{\pi}{1.41\sqrt{1-0.707^2}}=3.11\,s$.

③ $MP=e^{\frac{-\pi*0.707}{\sqrt{1-0.707^2}}}*100=4.5\%$

④ $T_r=\frac{\pi-\theta}{1.41\sqrt{1-0.707^2}}=2.34\,s$. $\quad 45*\frac{\pi}{180}=0.78$

For the following system find the rise time, settling time and percent overshoot

$$\frac{C}{R}=\frac{s+3}{s^2+s+4.25}$$

$W_n=\sqrt{4.25}=2.06$, $\xi=\frac{1}{2*2.06}=0.24$

① $T_r \Rightarrow \theta=\cos^{-1}(0.24)=76.113 \Rightarrow T_r=\frac{\pi-76.113*\frac{\pi}{180}}{2.06\sqrt{1-0.24^2}}=0.906\,s$

② $T_p \Rightarrow \frac{\pi}{2.06\sqrt{1-0.24^2}}=1.57$

③ $T_s=\frac{4}{2.06*0.24}=8.09\,s$, ④ $MP=e^{\frac{-\pi*0.24}{\sqrt{1-0.24^2}}}*100=45.99$.

rise time for the following system

$$G(s)=\frac{7}{s^2+14s+40}$$

$(\frac{14}{2})^2>40 \longrightarrow$ first order

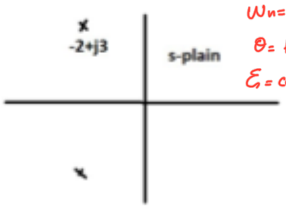
$\frac{7}{(s+10)(s+4)}=k_1e^{-10t}+k_2e^{-4t}$. $\quad t_1=0.4$, $t_1=1$

$*\ \frac{k_2}{s+4} \Longrightarrow \frac{k_2}{s+\frac{1}{\tau}}$

$4=\frac{1}{\tau} \Rightarrow \tau=\frac{1}{4}$, $T_r=2.2*\frac{1}{4}=0.55$

$T_s=4\tau=1$.

For the following system with roots shown in the following s-plane find the rise time, settling time and percent overshoot



$W_n=\sqrt{2^2+3^2}=3.605$

$\theta=\tan^{-1}(\frac{3}{2})=56.309$

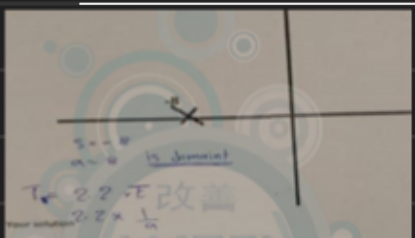
$\xi=\cos(\theta)=0.554$

① $T_s=\frac{4}{3.605*0.554}=2.05\,s$.

② $MP=e^{\frac{-\pi\cdot0.554}{\sqrt{1-0.554^2}}}*100=12.36\%$

③ $T_r=\frac{\pi-(56.309*\frac{\pi}{180})}{3.605\sqrt{1-0.554^2}}=0.719\,s$,

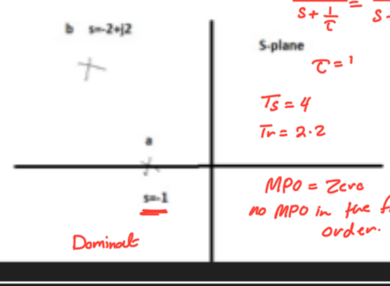
④ $T_p=\frac{\pi}{3.605\sqrt{1-0.554^2}}=1.046\,s$



$\frac{K}{s+8}=\frac{K}{s+\frac{1}{\tau}}$

$\tau=\frac{1}{8}$

Q5) a) For a and b poles in the following fins the associated damping ratio and percent opevershoot (5 points)



$\frac{k}{s+\frac{1}{\tau}}=\frac{k}{s+1}$

$\tau=1$

$T_s=4$

$T_r=2.2$

MPO = Zero

no MPO in the first order.

$W=\sqrt{2^2+2^2}=\sqrt{8}$

$\xi=$

For the following system find the rise time, settling time and percent overshoot

$$\frac{C}{R}=\frac{s+3}{s^2+4s+13}$$ complex

$W_n=\sqrt{13}=3.605$

$\xi=\frac{4}{2*3.605}=0.554$

$\theta=\cos^{-1}(0.554)$

① $T_s=\frac{4}{3.605*0.554}=2.05\,s$.

③ $T_r=\frac{\pi-(56.309*\frac{\pi}{180})}{3.605\sqrt{1-0.554^2}}=0.719\,s$,

② $MP=e^{\frac{-\pi\cdot0.554}{\sqrt{1-0.554^2}}}*100=12.36\%$

④ $T_p=\frac{\pi}{3.605\sqrt{1-0.554^2}}=1.046\,s$

For the following system find the rise time, settling time and percent overshoot

$$\frac{C}{R}=\frac{s+3}{s^2+9s+18}$$ $\sqrt{18}=$

$\frac{s+3}{(s+3)(s+6)}=k_1e^{-3t}+k_2e^{-6t}$

Dominant

$\frac{k_1}{s+\frac{1}{\tau}}=\tau=\frac{1}{3}=0.33$, $T_s=1.33\,s$, $T_r=0.733$.

Q1) for the following pole what is the percent overshoot

$$G(s)=\frac{k}{s^2+2s+6}$$ complex

$W_n=\sqrt{6}=2.45$

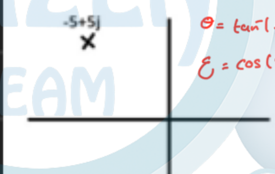
$\xi=\frac{2}{2*2.45}=0.408$

$\theta=\cos^{-1}(0.408)=65.9$

$T_s=\frac{4}{2.45*0.408}=4s$, $MPO=e^{\frac{-\pi*0.408}{\sqrt{1-0.408^2}}}*100=24.56\%$

$T_p=\frac{\pi}{2.45\sqrt{1-0.408^2}}=1.433\,s$, $T_r=\frac{\pi-(65.9*\frac{\pi}{180})}{2.45\sqrt{1-0.408^2}}=0.9088\,s$

Q1) for the following pole what is the peak time Tp



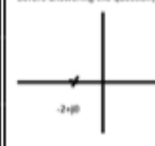
$W_n=\sqrt{5^2+5^2}=7.07$

$\theta=\tan^{-1}(\frac{5}{5})=45$

$\xi=\cos(45)=0.707$

Q1)find the percent overshoot and settling time related to pole in the figure

(note: remember this pole is first order not second order , initially draw the expected time response before answering the question)



Real root $\frac{k}{s+2}=\frac{k}{s+\frac{1}{\tau}}$

$\tau=\frac{1}{2}=0.5$

$T_s=4*0.5=2$

$T_r=2.2*0.5=1.1$

Quiz (5) Steady state error :

Q1) find the K value to achieve a steady state error (E<0.1) for a unit step input

$G(s) = \frac{K(s+1)}{(s+5)(s+7)}$

$e_{ss} = \frac{1}{1+K_p} < 0.1$, $K_p = \lim_{s\to 0} G(s)$

① N=0

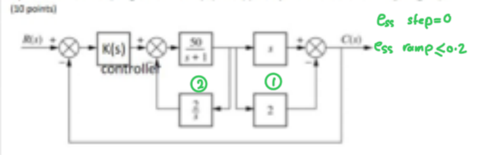
$K_p = \frac{K}{35} \Rightarrow e_{ss} = \frac{1}{\frac{35}{35}+\frac{K}{35}} = \frac{35}{35+K} < 0.1$

$350 < 35+K \Rightarrow K > 315$

② N ≥ 1 e=0 *

$K_p = \lim_{s\to 0} \frac{K}{s} * \left(\frac{s+1}{(s+5)(s+7)}\right) = \infty$

Q2) for the following system design the steady state error for a step input to be zero and for a ramp input to be less than 2 (design the controller)? (note K(s) is any function of s to be suitably designed) (10 points)



e_{ss} step=0

e_{ss} ramp<0.2

① Parallel (s-2)

② feed back $\left(\frac{\frac{50}{s+1}}{1+\frac{100}{s(s+1)}}\right)$

$\frac{50s}{s^2+s+100}$

① & ② series ⇒ $\left[\frac{50s(s-2)}{s^2+s+100} = G(s)\right]$.

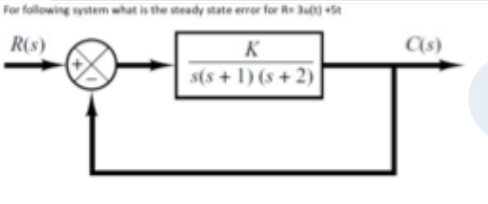
e_{ss} step $=0 \Rightarrow \frac{1}{1+k_p}$, $K_p = \lim_{s\to 0} \frac{K}{s^2} * \frac{50s(s-2)}{s^2+s+100} = \infty$

for ramp :

$e_{ss} = \frac{1}{k_v} < 2$, $k_v > \frac{1}{2} = \lim_{s\to 0} s \cdot \left(\frac{K}{s^2}\right) * \frac{50s(s-2)}{s^2+s+100} = \frac{-100K}{100} = \frac{1}{2}$

$K = \frac{-1}{2}$, $e_{ss} = 0 \Rightarrow k_v = \infty$ when $\frac{K}{s^3}$

For following system what is the steady state error for R= 3u(t) +5t



$e_{ss} = \frac{3}{1+k_p} + \frac{5}{k_v}$

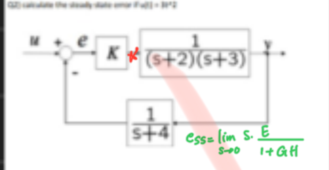
$K_p = \lim_{s\to 0} G(s) = \frac{K}{0} = \infty$

$K_v = \lim_{s\to 0} s \cdot G(s) = \frac{K}{2} \Rightarrow e_{ss} = \frac{3}{\infty} + \frac{10}{K} = \frac{10}{K}$.

* design k to achive 0.1 ess in ramp :

$e_{ss} = \frac{1}{K_v} = 0.1 \Rightarrow K_v = 10 = \frac{K}{2} \Rightarrow K = 20$ *

Q2) calculate the steady state error if u(t) = 3t^2

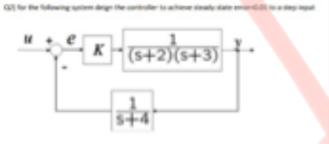


$e_{ss} = \lim_{s\to 0} s \cdot \frac{E}{1+GH}$

$E = \frac{6}{s^3}$, $1+GH = 1+\frac{K}{(s+2)(s+3)(s+4)}$

$\lim_{s\to 0} s \cdot \frac{6}{s^3} \cdot \frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}} = \infty$.

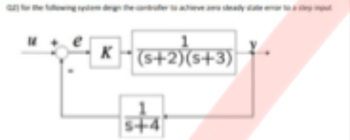
Q2) for the following system design the controller to achieve steady state error 0.01 to a step input



$e_{ss} = \lim s \cdot \frac{1}{s} \cdot \left[\frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}}\right]$ ↗ $1+\frac{K}{24}$

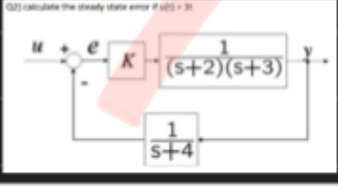
$\frac{24}{24+K} = 0.01 = 2376$

Q2) for the following system design the controller to achieve zero steady state error to a step input



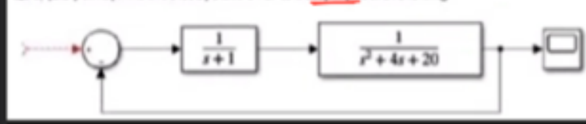
$e_{ss} = \frac{1}{1+k_p} = 0$, $k_p = \infty \Rightarrow K = \frac{1}{s}$

Q2) calculate the steady state error if u(t) = 3t



$e_{ss} = \lim \frac{3}{s^2} * \frac{1}{1+\frac{K}{(s+2)(s+3)(s+4)}} * s = \infty$

Q2 a) (2.5 points) Find the steady state error to a step input the following



$e_{ss} = \frac{1}{1+k_p}$, $k_p = \lim_{s\to 0} G(s)$, $G(s) = \frac{1}{s+1} * \frac{1}{s^2+4s+20} = \frac{1}{(s+1)(s^2+4s+20)}$

$K_p = \lim_{s\to 0} G(s) = \frac{1}{1*20} = \frac{1}{20} \Rightarrow e_{ss} = \frac{1}{1+\frac{1}{20}} = \frac{20}{21}$ error of step unit #

or

(design a $\overbrace{\text{controler}}^{k}$) to achive $e_{ss}=1$ when the input $\left(\frac{1}{s^2}\right)$ → ramp

$e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s\to 0} G(s) \cdot s$, $G(s) = \frac{1}{(s+1)(s^2+4s+20)}$

$K_v = \lim_{s\to 0} G(s) \cdot s * \frac{K}{s}$ ramp $= \left[K_v = \frac{K}{20}\right] \Rightarrow e_{ss} = \frac{1}{\frac{K}{20}} = 1$, $k = \frac{20}{s}$

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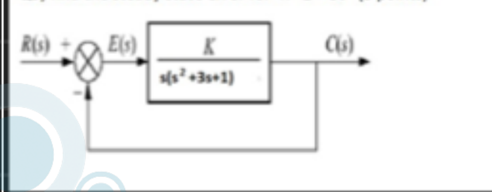
Q5) find the steady state error for R= 2 + 3t (5 points)

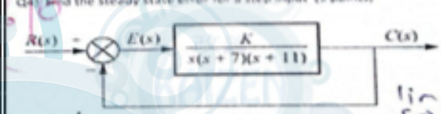


$\frac{2}{1+k_p} + \frac{3}{k_v}$

$K_p = \lim_{s\to 0} G(s) = \frac{K}{0} = \infty$

$k_v = \lim_{s\to 0} s \cdot G(s) = \frac{K}{1} \Rightarrow e_{ss} = 3/K$.

Q4) find the steady state error for a step input (5 points)

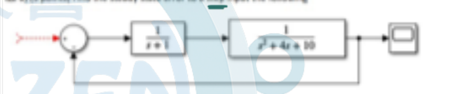


$e_{ss} = \frac{1}{1+k_p}$ $e_{ss} = 0$.

$K_p = \lim_{s\to 0} G(s) = \infty$

or N = 1 : Zero error

Q2 a) (5 points) Find the steady state error to a step input the following



$G(s) = \frac{1}{(s+1)(s^2+4s+10)}$

$K_p = \lim_{s\to 0} G(s) = \frac{1}{10}$, $e_{ss} = \frac{10}{11}$

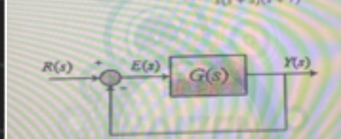
For following system add a controller to achieve 0.1 steady state error for a step input

$e_{ss} = \frac{1}{1+K_p} = 0.1$ $K_p = \lim_{s\to 0} k \cdot G(s) = \frac{K}{2}$

$10 = 1+\frac{K}{2} = K = 18$

Q1) find the steady state error for R=(4-3t)u(t)

$G(s) = \frac{K(s+1)}{s(s+5)(s+7)}$

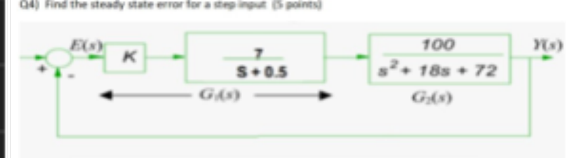


$\frac{4}{1+k_p} - \frac{3}{k_v}$, $k_p = \infty$

$K_v = \frac{K}{35}$

$e_{ss} = \frac{35*3}{K} = \frac{105}{K}$

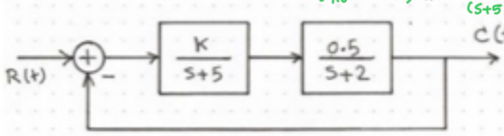
Q4) Find the steady state error for a step input (5 points)



$G(s) = \frac{700K}{(s+\frac{1}{2})(s^2+18s+72)}$

$k_p = \lim_{s\to 0} G(s) = \frac{350}{9}K$ $e = \frac{9}{9+350K}$

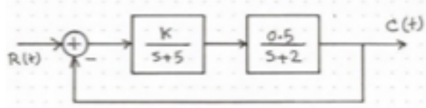
Q2) calculate the steady state error if R(t) = 3t $\rightarrow \left[\frac{3}{K_v}\right]$, $G(s)=\frac{0.5K}{(s+5)(s+2)}$



$K_v = \lim_{s\to 0} s\,G(s) = 0$

$e_{ss} = \infty$

Q2) design the controller to achieve steady state error of 0 to a step input R(s) =1/s

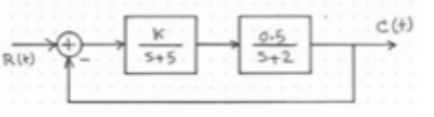


$e_{ss} = \frac{1}{1+K_p} \Rightarrow K_p = \infty$

if $k = \frac{1}{s}$

then $K_p = \lim_{s\to 0} \frac{0.5}{s(s+5)(s+2)}$

Q2) design the controller to achieve steady state error of 0 to a ramp input R(s) =1/s^2

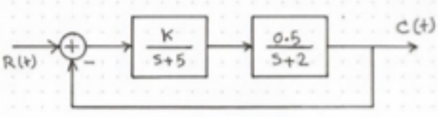


$e_{ss} = \frac{1}{K_v}$

$K_v = \lim s \cdot G(s)$

$k = \frac{1}{s^2}$

Q2) calculate the steady state error if R(t) = 5u(t) (u(t) is step input)



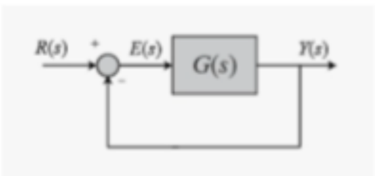
$e_{ss} = \frac{5}{1+K_p}$, $K_p = \frac{0.5k}{10}$

$K_p = \frac{k}{20} \Rightarrow e_{ss} = \frac{100}{20+K}$

An industrial conveyor belt system uses a **proportional controller** with gain K to regulate speed. The dynamics of the system (plant) are modeled by the open-loop transfer function:

$$G(s) = \frac{1}{s(2s+3)}$$

$e_{ss} = \frac{1}{K_v} = 0.05$



$K_v = \lim_{s\to 0} s \cdot G(s) \cdot k$

$K_v = \frac{K}{3}$ $\quad \frac{3}{K} = 0.05$

$K = 60$

Add a suitable controller to have the steady state error for a ramp input R(t)=t , to be equal to 0.05

Find the steady state error for the following system for an input of R(t) =5+3t

$e_{ss} = \frac{5}{1+K_p} + \frac{3}{K_v} = \frac{5}{\infty} + \frac{3}{5} = 0.6$

A unity feedback system has the open-loop transfer function:

$$G(s) = \frac{10}{s(s+2)}$$

Add a controller for the following system to achieve steady state error 0f less than 0.5 for R(t) =t

$e_{ss} = \frac{1}{K_v} < 2 \Rightarrow K_v > 2$ $\quad K_v = \lim s \cdot G(s) \cdot K$

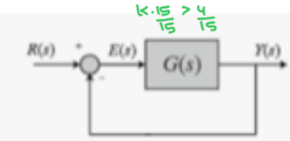
Consider a feedback control system with the following open-loop transfer function:

$K \cdot \frac{5(3)}{2} > 2$

$$G(s) = \frac{5(s+3)}{s(s+2)}$$

$K \cdot \frac{15}{15} > \frac{4}{15}$

$K > \frac{4}{15}$ ✓



An industrial temperature control system uses a proportional controller with gain K. The process dynamics are modeled as:

$K_p = K \cdot \frac{1}{3} > 19$

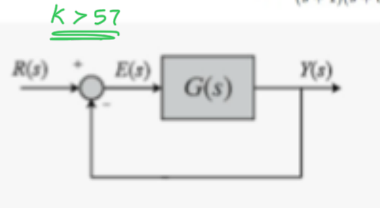
$$G(s) = \frac{1}{(s+1)(s+3)}$$

$e_{ss} = \frac{1}{1+K_p} < 0.05$

$K > 57$

$1 + K_p > 20$

$K_p > 19$

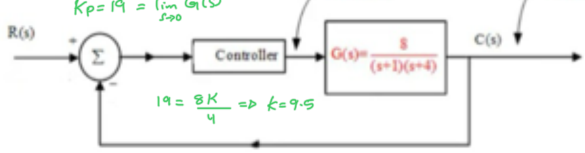


Calculate the value of K that limits the steady-state error to less than **0.05**.

Q1) if the controller is K, and the input is a step R=1/s what is k for the steady state error to be less than 0.05; no need for a calculator just give the answer in the form of an equation

$e_{ss} = \frac{1}{1+K_p} < 0.05$

$K_p = 19 = \lim_{s\to 0} G(s)$

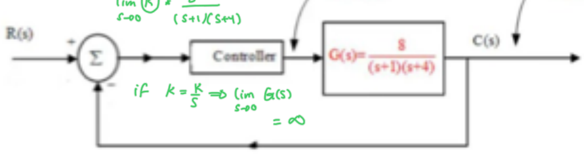


$19 = \frac{8K}{4} \Rightarrow K = 9.5$

Q1) design a controller to achieve zero steady state error to a step input R=1/s for the following system

$\frac{1}{1+K_p} = 0 \Rightarrow K_p = \infty$

$\lim_{s\to 0} (K) \cdot \frac{8}{(s+1)(s+4)}$



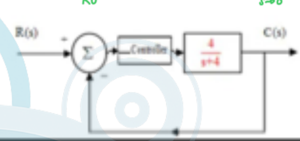
if $K = \frac{K}{s} \Rightarrow \lim_{s\to 0} G(s) = \infty$

Q4) Find the steady state error for a ramp input 1/s^2 (5 points)

$e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s\to 0} s \frac{700K}{s(s^2+18s+72)} = \frac{700K}{72} \Rightarrow e = \frac{72}{700K}$

Q1) design a controller to achieve zero steady state error to a ramp input R=1/s^2 for the following system

$e_{ss} = \frac{1}{K_v} = 0$, $K_v = \infty$, $K_v = \lim_{s\to 0} s \cdot (K) \cdot \frac{4}{s+4}$, if controller $= \frac{K}{s^2}$

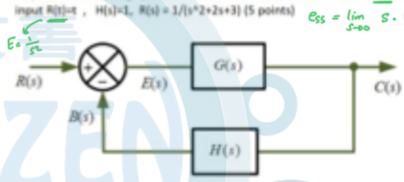


$K_v = \lim_{s\to 0} s \cdot \frac{K}{s^2} \cdot \frac{4}{s+4} = \infty$

Q3) for the following system find the added controller to achieve zero steady state error to a ramp input R(t)=t , H(s)=1, R(s) = 1/(s^2+2s+3) (5 points)

$e_{ss} = \lim_{s\to 0} s \cdot \frac{E}{1+GH} = 0$

$E = \frac{1}{s^2}$



$e_{ss} = \frac{1}{K_v} = 0$

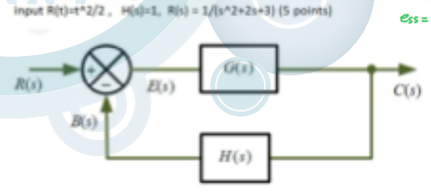
$K_v = \infty = \lim s(K) \frac{1}{s^2+2s+3}$

if $K = \frac{1}{s^2} \Rightarrow$ ✓

Q3) for the following system find the added controller to achieve zero steady state error to a parabolic input R(t)=t^2/2 , H(s)=1, R(s) = 1/(s^2+2s+3) (5 points)

$e_{ss} = \frac{1}{K_a}$

$K_a = \lim_{s\to 0} s^2 \cdot (K) \cdot \frac{1}{s^2+2s+3}$



$K = \frac{K}{s^3} \Rightarrow \infty$

$e = 0$

Quiz (6) Root locus.

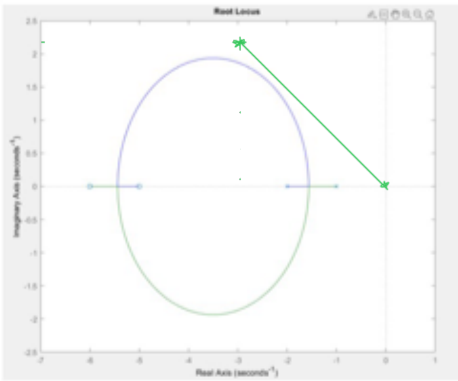
Q2) a) For the following system $G = \frac{(s+5)(s+6)}{(s+1)(s+2)}$, H=1 design the feedback controller k to achieve a damping ratio of > 0.8. (you must use the root locus figure below) (7.5 points)

$\xi > 0.8$ $\quad \tan(\theta) = 0.75$

$\theta = \cos^{-1}(0.8)$ $\quad x = 3$

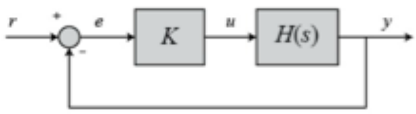
$\theta = 36.87$ $\quad y = 2.25$

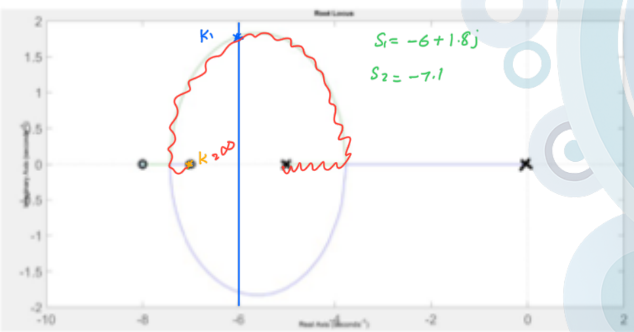
all the points are satisfing (under the line)



$PO\% \leq 25\% \Rightarrow \xi = 0.404$

Q6)What is the) a) For the following system $H(s) = \frac{(s+7)(s+8)}{s(s+5)}$ design the feedback controller k to achieve settling time < 0.666. (you must use the root locus figure below) (5 points)





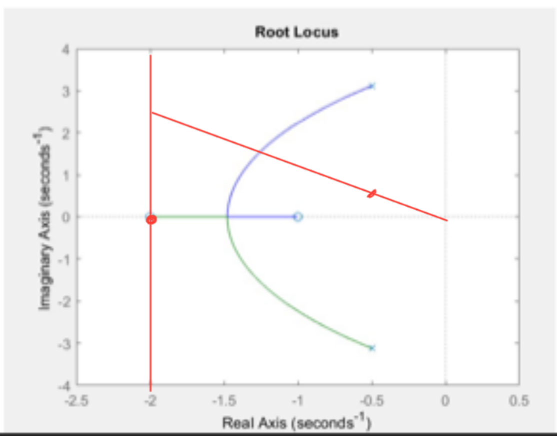
$S_1 = -6 + 1.8j$

$S_2 = -7.1$

For the following system determine K that achieves a damping ratio of > 0.7 and settling time < 2 s

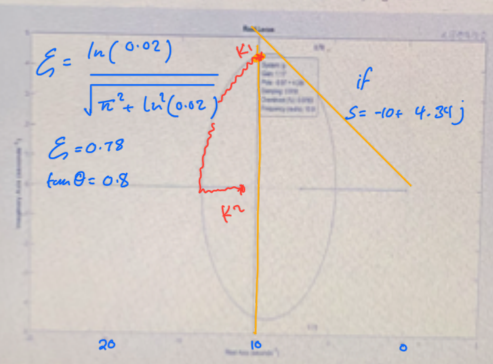
GH(s)= (s+1)(s+2)/((s^2 +s +10))

$\tan = 1.02$



Q2) a)(6 points) design K for the system GH(s)= $\frac{(s+11)(s+20)}{s(s+7)}$ to achieve a settling time < 0.4 and percent overshoot < 2% (use the root locus shown below

-ln(0.02)/(pi^2+ln(0.02)^2)^0.5 =0.7797 which is the damping ratio



$\xi = \frac{\ln(0.02)}{\sqrt{\pi^2 + \ln^2(0.02)}}$

$\xi = 0.78$

$\tan\theta = 0.8$

if $S = -10 + 4.34j$

settling < 0.4

$\frac{4}{\omega\xi} = 0.4$

$\omega\xi = 10$

$K_2 = \infty$

Determine the value of k to achieve settling time less < 2 and damping ratio less the 0.75 for

G= k(s+1)/ (s^2 + 2s+5)

$\xi = 0.75$

$\tan = 0.88$

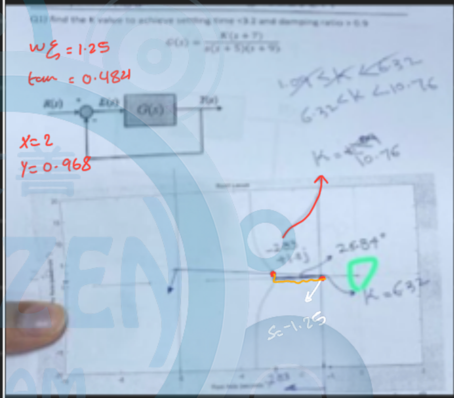
$x_1 = 1$

$y = 0.88$

$S_1 = -2 + 2j \Rightarrow K_1 = 1.8$

$S_2 = -2 \Rightarrow K_2 = 5$

$1.8 < K < 5$



$\omega\xi = 1.25$

$\tan = 0.484$

$x = 2$

$y = 0.968$

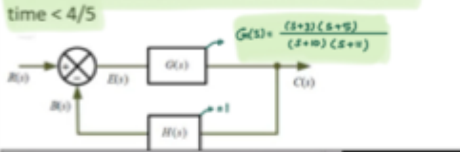
$K = \frac{1}{|GH|}$

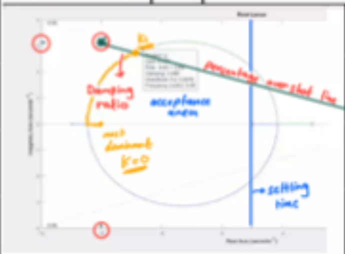
اصفار البسط ⇐ $K = \infty$

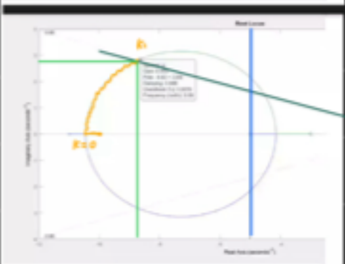
〃 المقام ⇐ $k = 0$

Q5) For the following system design a controller K to achieve a damping ratio of > 0.95 and settling time < 4/5

$G(s) = \frac{(s+3)(s+5)}{(s+10)(s+11)}$







in Matlab :

tan (a cos (0.95)) = 0.3287

Theta = a cos (0.95) * (180 / pi) = 18.1949

اذا مشيت بي على (X) بمقدار (10) رح

امشي على (Y) بمقدار ←

$10 * 0.3287 = 3.287$

settling time : $\frac{4}{6} = -4 \Rightarrow T_s = -5$

(0,0)

the solution $0 < K < K_1$

$S_1 = -8.8 + j2.75$ in matlab

$S = -8.8 + 2.7i$

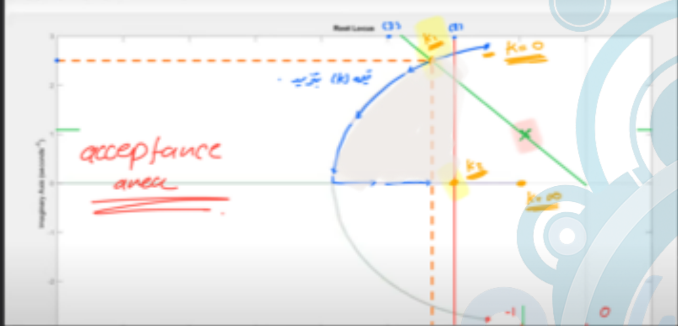
g = (s+3)*(s+5)/((s+10)*(s+11))

$k_1 = abs(1/g) = 0.3451$

Final answer $0 < K < 0.345$

For the following system determine K that achieves a damping ratio of > 0.7 and settling time < 2s

GH(s)= (s+1)/ (s^2 +3s +10)



$\xi > 0.7 \Rightarrow \theta = \cos^{-1}(0.7) = 45.57$ Scale = tan(θ) = 1.02

if $x = 1 \Rightarrow y = 1.02$

settling time < 2 $\Rightarrow \frac{4}{\omega\xi} = 2 \Rightarrow \omega\xi = 2$

$\left.\begin{array}{l} S_1 = -2.3 + j2.45 \\ S_2 = -2 \end{array}\right] \Rightarrow k = \frac{1}{|GH(s)|} = \left|\frac{s^2+3s+10}{(s+1)}\right|$

$K_1 = 1.652$, $K_2 = 8 \Rightarrow K_1 < K < K_2$

$1.652 < K < K_2$.

Quiz (7) Matlab

write MATLAB code using ODE45 to solve the following differential equations

$\ddot{x} + x = \log(t)$, $\dot{x}(0) = 0,\ x(0) = 0$

Plot(y) and plot(x) on same figure for a time interval 0 to 20 seconds.

ODE45 : Numerical answer :

Convention ⇒ there is a second order

x_1 , x_2 $\ddot{x} + x = \log(t)$

$\dot{x}_1 = x_2$ —① $\dot{x}_2 + x_1 = \log(t)$

$\ddot{x} = \dot{x}_2$ $\dot{x}_2 = \log(t) - x_1$ ... ②

```
function dx = sos(t,x)
[m,n] = size x ;
dx = zeros (m,n) ;
dx(1) = x(2) ;
dx(2) = log10(t) - x(1) ;
```

```
tspan = [0 , 20]
x0 = [0 ; 0] ;
[t x] = ode45 (at sos, tspan, x0)
plot (t , x)
plot (t, x(:,1))
```

Normal first order equation :

$\dot{x} + ax = b \Rightarrow \dot{x} + 2x = b$ $\dot{x} = b - ax$

```
function dx = sos(t,x)
a = 2 ;
b = 4 ;
dx = b - a*x
```

```
tspan = [0 20] ;
x0 = [0]
[t x] = ode45 (at sos, tspan, x0)
plot (t , x)
```

changing parameters :

$\dot{x} + ax = b$ $\dot{x} = b - ax$

```
function dx = sos(t,x,param)
a = param (1)
b = param (2)
dx = b - a*x
```

```
tspan = [0 20] ;
x0 = [0] ;
param [ , ] ;
[t x] = ode45 (at sos, tspan, x0, [ ], param)
plot (t , x)
```

Symbolic :

write MATLAB symbolic code to solve the following differential equations

$\ddot{x}y + y = \log(t)$, $\ddot{y}x + \dot{y} = t$, $\dot{x}(0) = 0,\ \dot{y}(0) = 0, x(0) = 0,\ y(0) = 0$

x(t) , y(t)

```
syms x(t) y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
      , diff(y,t,2)*x + diff(y,t) == t ] ;
dx = diff (x,t) ;
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0 , x(0)==0 , y(0)==0] ;
sol = dsolve (eqn, cond)
```

في حال لغى المعادلة الثانية

```
syms x(t) y(t)
eqn = [diff(x,t,2)*y + y == log10(t)
      , ~~diff(y,t,2)*x + diff(y,t)==t~~ ] ;
dx = diff (x,t) ;
dy = diff (y,t) ;
cond = [dx(0)==0, dy(0)==0 , x(0)==0 , y(0)==0] ;
sol = dsolve (eqn, cond)
```

بس منلغي هاي .

Solve the following differential equation using ODE45 for t from 0 to 5

$$\dddot{W} + \ddot{W} + \dot{W} + 2W = \sin(t) \quad W(0) = 0 \quad \dot{W}(0) = 1 \quad \ddot{W}(0) = 2;$$

$w_2 = \dot{w}_1$ --- 1

$w_3 = \dot{w}_2 = \ddot{w}_1$ --- 2

$\dot{w}_3 = \ddot{w}_2 = \dddot{w}_1$

$\dot{w}_3 + w_3 + w_2 + 2w_1 = \sin t$

$\dot{w}_3 = \sin t - w_3 - w_2 - 2w_1$

```
function dw = sos (t,w)
[m,n]= size(w);
dw = zeros (m,n);
dw(1)= w(2) ;
dw(2)= W(3) ;
dw(3)= sin t - W(3) - W(2) - 2*W(1)
tspan = [0 5];
Wo = [0; 1 ;2];
[t w]= ode45 (@ sos , tspan , Wo)
```

Solve the following differential equation using symbolic

$$\ddot{X} + 3X + 2t = 0 \quad X(0) = 0 \quad \dot{X}(0) = 1;$$

```
syms x(t)
eqn =[diff(X,t,2) + 3*X + 2*t == 0];
dx= diff (x,t) ;
cond = [X(0) ==0 , dx(0)= 1];
sol = dsolve (eqn, cond )
```

Solve the following differential equation using symbolic

$$\dddot{W} + \ddot{W} + \dot{W} + 2W = \sin(t) \quad W(0) = 0 \quad \dot{W}(0) = 1 \quad \ddot{W}(0) = 2;$$

```
syms w(t)
eqn = [diff (w,t,3) + diff (w,t,2) + diff (w,t)
      + 2*w == sin (t)];
dw = diff(w,t) ;
dw2 = diff (w,t,2) ;
cond = [w(0)==0, dw(1)==1, dw2(0) == 2];
sol = dsolve (eqn , cond ).
```

Solve the following differential equation using ODE45

$$\ddot{X} + 3X + 2t = 0 \quad X(0) = 0 \quad \dot{X}(0) = 1;$$

$X_2 = \dot{X}_1$ --- 1 , $\dot{X}_2 = \ddot{X}_1$

$\dot{X}_2 = -2t - 3X$

```
function dx = sos(t,x)
[m,n] = size(X) ;
dx= zeros (m,n);
dx(1)= X(2) ;
dX(2) = -3*X - 2*t
tspan= [0 5];
Xo = [0 ; 1]
[t w] = ode 45(@sos , tspan , Xo);
plot (t,x)
```

$\ddot{x} + 1 = t \qquad x(0)=0 \;,\; \dot{x}(0)=0$

① ODE 45 :

$x_2 = \dot{x}_1 \quad \cdots ① \qquad \dot{x}_2 = \ddot{x}_1 \Rightarrow \dot{x}_2 = t-1 \quad \cdots ②$

```
function dx= sos(t,x)
[m,n]=size(x);
 dx= zeros (m,n);
 dx(1)= x(2) ;
 dx(2) = t-1 ;
```

```
tspan =[0 10];
x0 = [0;0];
[t x]=ode45(@sos,tspan,x0)
plot(t,x)
```

② Symbolic

```
syms x(t)
eqn=[diff(x,t,2)+1 == t];
dx= diff (x,t);
Cond =[x(0)==0, dx(0) ==0];
sol = dsolve (eqn, cond).
```

$\ddot{x} + x = \log(t) \qquad \dot{x}(0)=0 \quad x(0)=0$

Symbolic:

```
syms x(t)
eqn = [ diff (x,t,2)+ x == log10 (t)];
dx= diff (x,t);
cond = [dx(0) ==0 , x(0) == 0];
sol = dsolve (equ , cond )
```

③ Tool box:

Example : $y'' + 6y' + 5y = 4u' + 3u$. ① Transfer function.

$s^2 Y(s) + 6s\,Y(s) + 5Y(s) = 4s\,U(s) + 3U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}$

```
>> num=[4 3];
>> den=[1 6 5];          G=tf([4 3],[1 6 5])
>> G = tf(num,den)
```

$G = \frac{1}{(s+3)(s^2+2)}$ to find r.locus & step response

```
num =[1];
den = conv([1 3],[1 0 2]);
g = tf (num,den)
g = tf(1, con([1 3],[1 0 2])).
rlocus (g)
step (g)
```

$G = \frac{1}{s^2+2s+7}$ to find r.locus & step response

```
num = [1];
den = [1 2 7];
g = tf(num , den);
rlocus (g);
step (g);
```

11/8

Matlab

Control system Toolbox:

Transfer function assuming zero initial condition

Example: $y'' + 6y' + 5y = 4u' + 3u$. ① Transfer function.

$s^2Y(s) + 6sY(s) + 5Y(s) = 4sU(s) + 3U(s) \rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}$.

```
>> num = [4 3];
>> den = [1 6 5];          G=tf([4 3],[1 6 5])
>> G = tf(num,den)
```

② Zero-pole-gain (ZPK)

$\frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5} = \frac{4(s+0.75)}{(s+1)(s+5)}$, G=ZPK([-0.75],[-1,-5],4)

* Normal: s=tf('s')

G=(4*s+3)/((s^2)+6*s+5)

③ State-Space Model (ss)

$\frac{Y}{R} = \frac{4s+3}{s^2+6s+5}$, $\frac{X}{R} = \left(\frac{1}{s^2+6s+5}\right)$, $\frac{Y}{X} = (4s+3)$.

$R = x'' + 6x' + 5x$, $Y = 4x' + 3x$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]R.$$

G=ss([0 1;-5 -6],[0;1],[3,4],0)

Symbolic:

⇒ Solve the diff. equ. $\frac{dy}{dx} = e^{-\frac{1}{x}} \frac{1}{x^2}$:

```
syms y(x)
eqn= diff(y) == exp(-1/x)/x^2
cond= [y(0) == 1];
sol(x)= dsolve(eqn, cond)
```

⇒ system of differential equ.

$\frac{dy}{dt} = z$, $\frac{dz}{dt} = y$.

```
syms y(t) z(t)
eqns=[diff(y,t)==z       diff(z,t)==-y]
sol=dsolve(eqn)
sol.z
sol.y
```

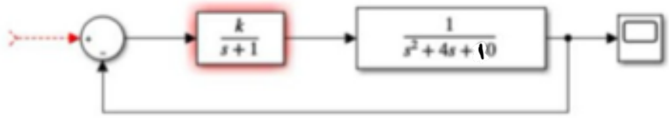
$(1+t^2)\ddot{w} + 2t\dot{w} + 3w = 2$ 2 order

$t_0 = 0$, $t_f = 5$, $w(t_0) = 0$, $\dot{w}(t_0) = 1$

```
function dx= diff(t,x)
[m,n]=size(x)
dx=zeros(m,n)
dx(1)=x(2)
dx(2)=(2-2*t*x(2)-3*x(1))/(1+t^2)
tspan=[0 5]
x0=[0;1]
[t,x]=ode23(@diff, tspan, x0)
plot(t,x(:,1)).
plot(t,x(:,2),t,x(:,2))
```

Quiz (8): Routh hurwtz:

Q3(5 points) Use the Routh Hurwitz array to find the



Solution

s^3	1	14	0
s^2	5	$k+10$	0
s^1	$(60-k)/5$	0	0
s^0	$k+10$	0	0

$60>k>0$

K=......................

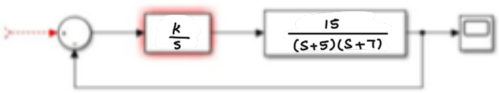
$$\frac{\frac{k}{(s+1)(s^2+4s+10)}}{1+\frac{k}{(s+1)(s^2+4s+10)}} \Rightarrow \frac{k}{(s+1)(s^2+4s+10)+k} = \frac{k}{s^3+5s^2+14s+(k+10)}$$

$\frac{70-(k+10)}{5} = \frac{60-k}{5}$,

$\frac{60-k}{5} > 0 \Rightarrow 60 > k$ / $k+10>0$

$k>-10$ can't take the negative value.

Q3(5 points) Use the Routh Hurwitz array to find the



Solution

s^3	1	35	0
s^2	12	$15k$	0
s^1	A	0	
s^0	$15k$	0	

$28>k>0$

K=......................

$$\frac{\frac{15k}{s(s+5)(s+7)}}{\frac{s(s+5)(s+7)+15k}{s(s+5)(s+7)}} = \frac{15k}{s^3+12s^2+35s+15k}$$

$A = \frac{420-15k}{12}$, $\frac{420-15k}{12} > 0$

$420 > 15k \Rightarrow 28 > k$, $15k>0$

$k>0$

$s^3+s^2+2s+k=0$, what's (k) for stability?

Solution

s^3	1	2	0
s^2	1	k	0
s^1	$2-k$	0	0
s^0	k	0	0

$k>0$

$2>k$

$2>k>0$

Q3(5 points) Use the Routh Hurwitz array to find the

Solution

s^3	1	24	0
s^2	5	$k+20$	0
s^1	A	0	0
s^0	$k+20$	0	0

K=......................

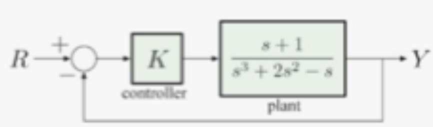
$$\frac{k}{s^3+5s^2+24s+(20+k)}.$$

$A = \frac{120-k-20}{5} = \frac{100-k}{5}$.

$k+20>0 \Rightarrow k>-20 \Rightarrow k>0$ ∴ no(-ve) value.

$100>k \Rightarrow 100>k>0$

Q1) Determine K for stability



$$\frac{k(s+1)}{s^3+2s^2+(k-1)s+k}.$$

s^3	1	$k-1$	0
s^2	2	k	0
s^1	(A)	0	0
s^0	k	0	0

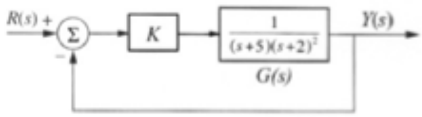
$A = \frac{2k-2-k}{2} = \frac{k-2}{2}$

$\frac{k-2}{2} > 0$

$k>2$
$k>0$

$k>2$

Q1) find k for stability for following system, what is the auxiliary equation and the point of cross with the imaginary axis.



$$\frac{k}{s^3+9s^2+24s+(k+20)}$$

s^3	1	24	0
s^2	9	$k+20$	0
s^1	A	0	0
s^0	$k+20$	0	0

$A = \frac{216-k-20}{9} = \frac{196-k}{9}$

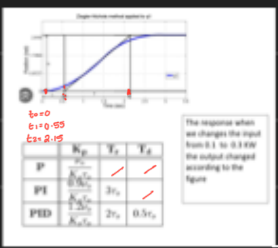
$k+20>0$ $k>0$, no -ve

$196-k>0 \Rightarrow 196>k>0$

* Anxiliary: $9s^2+(196+20) = 9s^2+216=0$

$s^2+24=0 \Rightarrow s = \pm\sqrt{24}\,j$

Quiz (9) PID :

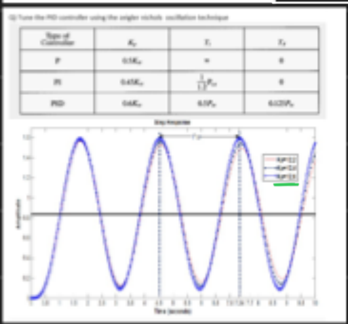


	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

$V_o = 1.6$ s

$T_o = 0.55$ s

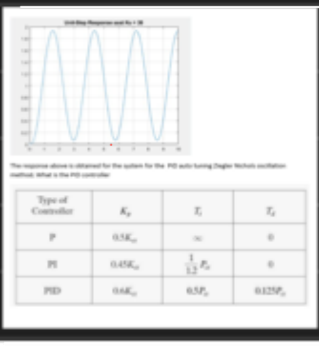
$K_o = \frac{2.75}{0.3-0.1} = 13.75$ s



Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$P_{cr} = 2.84$ s

$K_{cr} = 12.6$ s

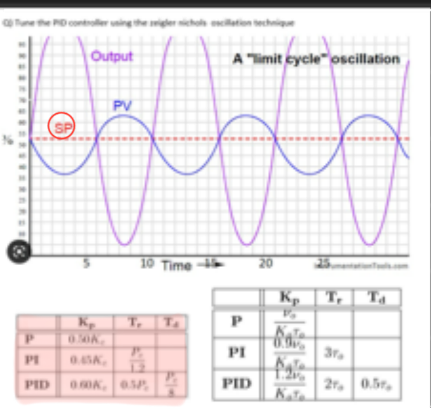


Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$P_{cr} = 5.81 - 2.95 = 3.86$

$K_{cr} = 30$

$K_p = 18$, $T_i = 1.43$ ...



	K_p	T_i	T_d
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

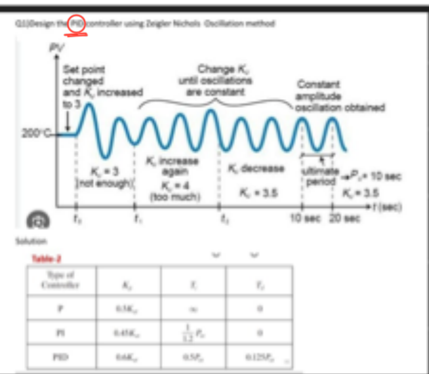
	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

$P_{cr} = 11$

$K_{rc} = 3$

$K_p = 1.8$

$T_r = 5.5$



Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

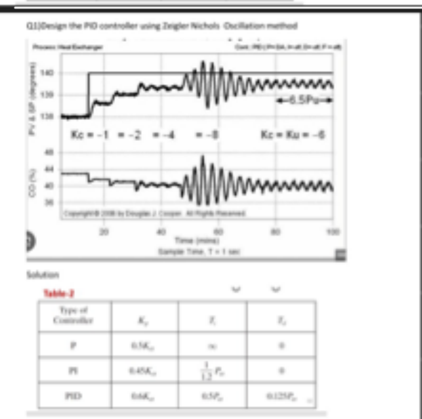
$P_{cr} = 20 - 10 = 10$ s.

$T_d = 1.25$

$T_i = 5$ s

$K_{cr} = 3.5$

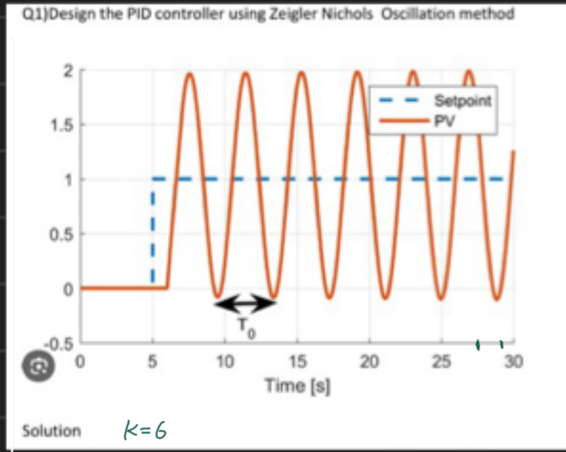
$K_p = 0.6 * 3.5 = 2.1$



Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

$\frac{19}{7} = 2.85$ min $* 6.5 = 18.525$

$\frac{200}{7} = 28.5 * 6.5 = 184.6$

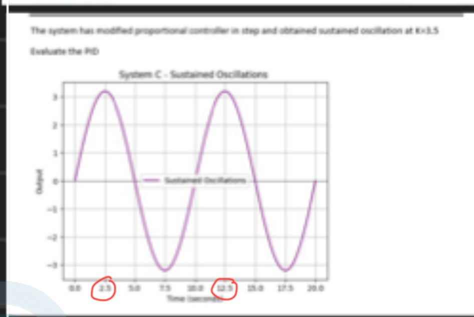


$P_{cr} = 4$, $K_{cr} = 6$

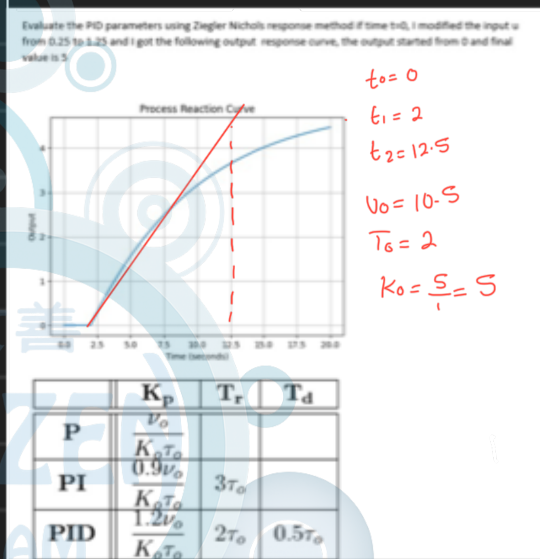
$K_1 = 3.6$

$T_i = 2$

$T_d = 0.5$



$P_{cr} = 10$, $K = 3.5$



	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

$t_0 = 0$

$t_1 = 2$

$t_2 = 12.5$

$V_o = 10.5$

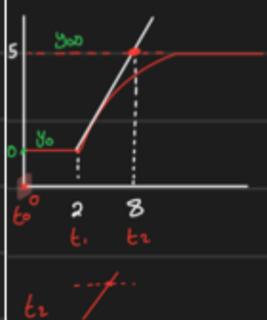
$T_o = 2$

$K_o = \frac{5}{1} = 5$

$K_p = \frac{1.2 * 10.5}{5 * 2} = 1.26$

$T_r = 4$

Example : find the final PID ? $u_\infty = 1.25$, $u_o = 0.25$



① $V_o = t_2 - t_1 = 6$

② $T_o = t_1 - t_o = 2$

③ $K_o = \frac{y_\infty - y_o}{u_\infty - u_o} = \frac{5}{1} = 5$

(,) given

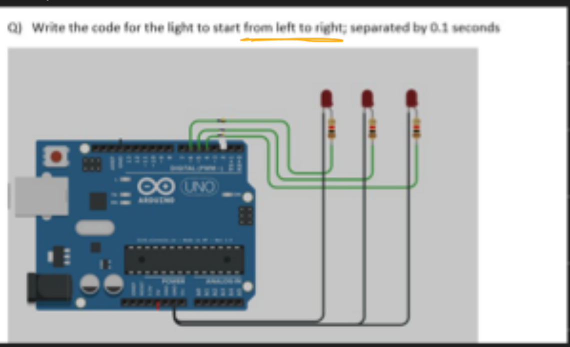
وبعدين على الجدول

PID : $\frac{1.2\,V_o}{k_o\,T_o} = \frac{1.2 * 6}{5 * 2}$

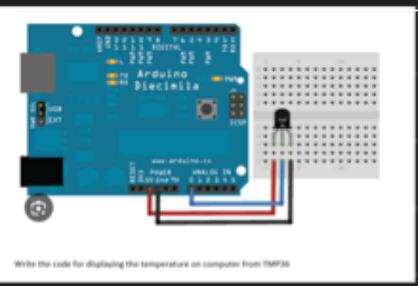
$2T_o = 4$

$0.5\,T_o = 1$

Ard.




```
Const int LED=6, LED2=5, LED3=4 ;
void setup ()
{ Pin Mode (LED1, OUTPUT);
  pin Mode (LED2, OUTPUT);
  Pin Mode (LED3, OUTPUT);   }
void Loop ()
{ digital Write (LED1, HIGH);
    delay (100);
  digital Write (LED2, HIGH);
    delay (100);
  digital Write (LED3, HIGH);   }
```




```
Const int Tpin = 0 ;
Void setup ()
{ Serial.begin (9600);  }
void Loop ()
{ double voltage, Temperature ;
  voltage = get voltage (T pin);
  Temperature = (Voltage - 0.5) * 100 ;
  Serial.print ("T is");
  Serial.println(Temperature);
  delay (1000);  }

douple get voltage (int Tpin)
{ return (analog Read
     (Tpin) * 5/1024 ); }
```

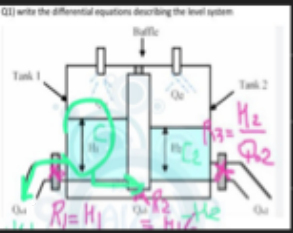
For the LED, Code for the ligthing sequance

right then middle then left
then middle, sepurated by (1s).

```
Const int LED1 = 8 , LED2=9 , LED3=10;
void setup ()
{ pin Mode (LED1, OUTPUT);
  pin Mode (LED2, OUTPUT);
  pin Mode (LED3, OUTPUT);  }

void loop ()
{ digital Write (LED1, HIGH);
  digital Write (LED2, LOW);
  digital Write (LED3, LOW);
  delay (1000);
  digital Write (LED1, LOW);
  digital Write (LED2, HIGH);
  digital Write (LED3, LOW);
  delay (1000);
  digital Write (LED1, LOW);
  digital Write (LED2, LOW);
  digital Write (LED3, HIGH);
  delay (1000);
  digital Write (LED1, LOW);
  digital Write (LED2, HIGH);
  digital Write (LED3, LOW);
  delay (1000);  }
```

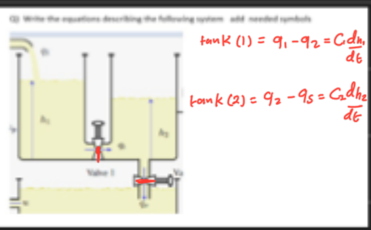
Quiz : Tanks / Modelling :



$$\text{tank }(1) = Q_{i1} - Q_{o1} - Q_{o3} = C\frac{dh}{dt}$$

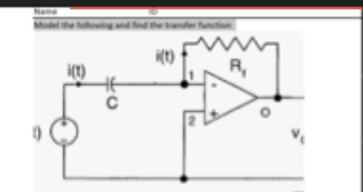
$$\text{tank }(2) = Q_{i2} + Q_{o3} - Q_{o2} = C\frac{dh}{dt}$$

$$R_1 = \frac{H_1}{Q_{o1}}, \quad R_2 = \frac{H_1 - H_2}{Q_{o3}}, \quad R_3 = \frac{H_2}{Q_{o2}}$$

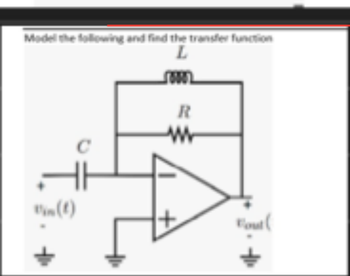


$$R_1 = \frac{h_1 - h_2}{Q_2}$$

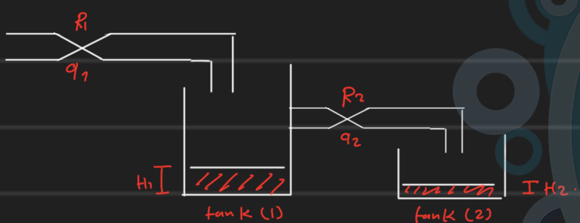
$$R_2 = \frac{h_2}{Q_s}$$



$$T.F = \frac{-Z_2}{Z_1} = -R_f C \cdot S.$$



$$T.F = \frac{-Z_2}{Z_1} = \frac{\frac{R \cdot L \cdot S}{R + L \cdot S}}{\frac{1}{C \cdot S}} = \frac{RLCS^2}{R + L \cdot S}$$

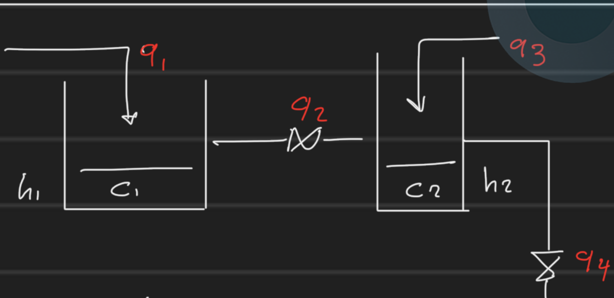


$\boxed{1}$ tank (1) $\Longrightarrow C_1 \frac{dh_1}{dt} = q_1 - q_2$

tank (2) $\Longrightarrow C_2 \frac{dh_2}{dt} = q_2 - 0.$

$\boxed{2}$ $R_1 = \frac{-H_1}{q_1}$

$R_2 = \frac{H_1 - H_2}{q_2}$

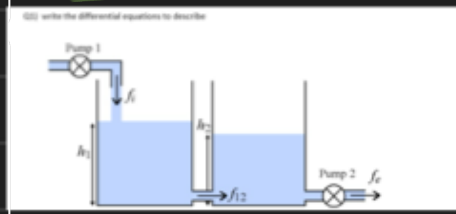


$$\text{tank }(1) = C_1\frac{dh_1}{dt} = q_1 - q_2$$

$$\text{tank }(2) = C_2\frac{dh_2}{dt} = q_2 + q_3 - q_4$$

$$R_1 = \frac{-h_1}{q_1}, \quad R_2 = \frac{h_1 - h_2}{q_2}, \quad R_3 = \frac{-h_2}{q_3}, \quad R_4 = \frac{h_2}{q_4}$$

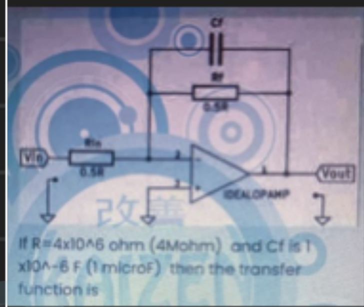
Gear : $\frac{\text{Teeth A}}{\text{Teeth B}} = \frac{\text{Speed (B)}}{\text{Speed (A)}}$ ...



$$\text{tank }(1) = C_1\frac{dh_1}{dt} = f_i - f_{12}$$

$$\text{tank }(2) = C_2\frac{dh_2}{dt} = f_{12} - f_e$$

$$R_1 = \frac{-h_1}{f_i}, \quad R_2 = \frac{h_2}{f_e}.$$

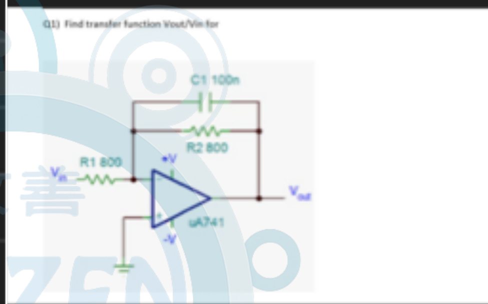


$$Z_2 = \frac{\frac{R_f}{C \cdot S}}{R_f + \frac{1}{C \cdot S}} = \frac{R_f}{R_f C S + 1}$$

$$Z_1 = R_{in}$$

$$T.F = -\left(\frac{\frac{R_f}{R_f \cdot C \cdot S + 1}}{R_{in}}\right).$$

$$T.F = \frac{2*10^6}{(1 + 2*10^6 * 1*10^{-6} * S)\, 2*10^6} = \frac{1}{1 + 2S}.$$



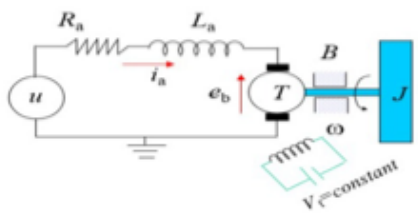
$$Z_2 = \frac{R_2}{R_2 C \cdot S + 1}$$

$$Z_1 = R_1$$

$$T.F = -\frac{\frac{R_2}{R_2 C \cdot S + 1}}{R_1}$$

$$T.F = \frac{800}{(800 * 100 * 10^{-9} S + 1) * 800} = \frac{800}{800 + 0.064 S}.$$

Q5) Figure below is a schematic for an armature controlled dc motor. Find the transfer function w/u where w is the rotational velocity and u is the voltage. (10 points)



$$T_{motor} = J\dot{\omega} + B\omega \quad (\to J\omega S)$$

$$= K_t I \Rightarrow I = T/K_t$$

$$e_b = K_b \omega$$

$$-u + RI + (La \cdot S)I + e_b = 0$$

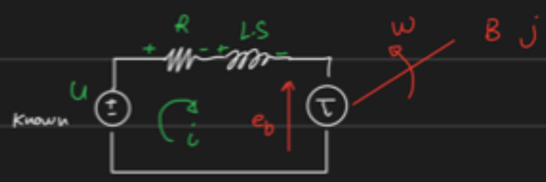
$$U(S) = (R + La \cdot S)I + (Kb \cdot \omega)$$

$$\frac{(R + La \cdot S)(J\omega S + B\omega)}{K_t} + (Kb \cdot \omega)$$

$$u = \omega\left(\frac{La\, J \cdot S^2 + (La \cdot B + RJ)S + BR + K_b K_t}{K_t}\right)$$

$$\frac{\omega}{u} =$$

*] DC motor :



$$-u + RI + LSI + e_b = 0$$

$$T = K_t * I, \quad I = T/K_t, \quad e_b = K_b \omega.$$

$$T = J\dot{\omega} + B\omega$$

$$u = \frac{J\omega s + B\omega}{K_t}(R + LS) + (K_b \omega).$$

$$u = \omega\left(\frac{JLS^2 + (JR + BL)S + BR + K_b K_t}{K_t}\right)$$

$$\frac{\omega}{u} = \frac{K_t}{JLS^2 + (JR + BL)S + BR + K_b K_t}$$

Arduino :

on , off
input , output

Digital.

[0V — 5V
0 — 1024]

Analog.

* Every Arduino "Sketch" has :

```
// variable declarations
const int LED 13 ;
void setup ()
{
 // configuration of pins . etc .
}
void Loop ()
{
// what the program does , in the continuous loop
}
```

Example : blink

```
const int LED = 13 ;
void Setup ()
{
pin Mode (LED, OUTPUT) ;
}
Void Loop ()
{
digital Write (LED, HIGH) ;
delay (1000) ;
digital Write (LED, LOW) ;
delay (1000) ;
}
```

وصلة مع (Pin 13)

OUTPUT
INPUT

5V

delay (1000)
تأخير ثانية واحدة

button

```
void Loop (0)
{
val = digitalRead (Button) ;
if (val == HIGH) {
{
digital Write (LED, HIGH) ;
}
else {
   digital Write (LED, LOW) ;
   }
}
```

بس لازم اكون قبل معرفه

```
Const int Button = ;
pin Mod (Button, INPUT)
```

Morse code

LED will ligth long. ← الاشارة الطويلة

النقطة ← اشارة قصيرة

```
void dot ()
{ blink (200, 200) ; }
void dash ()
{ blink (600, 200) ; }
void letterspace ()
{ delay (400) ; }
void wordspace ()
{ delay (800) ; }
```

فصل

```
void blink (int ontime , int offtime ).
{
 digital Write (LED, HIGH) ;
  delay (ontime) ;
 digital Write (LED, LOW) ;
  delay (offtime) ;
}
```

Perhaps letter functions :

```
void morse_s ()
{ dot () ; dot () ; dot () ; letterspace () ; }
void morse_o ()
{ dash () ; dash () ; dash () ; Letter space () ; }
```

Arduino : Temperature Sensors

مُصمم بطريقة تحسيناً عشان يقيس voltage .

بيستقبل من الاطراف (Zero & 5 V)

تتغير المقاومة فيه (بتنزل مع الحرارة)

```
// We'll use analog input 0 to read Temperature Data const int t
void setup()
 { Serial.begin(9600); }
void loop()
{ float voltage, degreesC, degreesF;
voltage = getVoltage(temperaturePin);
 // Now we'll convert the voltage to degrees Celsius.
// This formula comes from the temperature sensor datasheet:
 degreesC = (voltage - 0.5) * 100.0;
 // Send data from the Arduino to the serial monitor window
Serial.print("voltage: ");
Serial.print(voltage);
Serial.print(" deg C: ");
Serial.println(degreesC);
delay(1000);
// repeat once per second (change as you wish!) }
float getVoltage(int pin)
 { return (analogRead(pin) * 0.004882814); }
// This equation converts the 0 to 1023 value that analogRead()
// returns, into a 0.0 to 5.0 value that is the true voltage
// being read at that pin.
```

```
// Read Temerature Values from NTC Thermistor
 const int temperaturePin = 0;                const int temp = 0
 void setup()
{ Serial.begin(9600); }
 void loop()
 { int temperature = getTemp();
Serial.print("Temperature Value: ");
Serial.print(temperature);
 Serial.println("*C");
delay(1000);
 }
 double getTemp()
{
// Inputs ADC Value from Thermistor and outputs Temperature in Celsius int RawADC =
analogRead(temperaturePin);
long Resistance;
 double Temp;
 // Assuming a 10k Thermistor. Calculation is actually: Resistance = (1024/ADC)
Resistance=((10240000/RawADC) - 10000);
 // Utilizes the Steinhart-Hart Thermistor Equation:
// Temperature in Kelvin = 1 / {A + B[ln(R)] + C[ln(R)]^3}
// where A = 0.001129148, B = 0.000234125 and C = 8.76741E-08 Temp = log(Resistance);
 Temp = 1 / (0.001129148 + (0.000234125 * Temp) + (0.0000000876741 * Temp * Temp *
Temp)); Temp = Temp - 273.15;
// Convert Kelvin to Celsius return Temp;
 // Return the Temperature
 }
```

Lecture 1

Arduino :

digital [10]
on / off → on = 5V , off = 0 V

input : Button 1 0 — Output LED 1 0 :

const int Button = 10; (1) منشيها لا تغير
, LED = 9;

بالنسبة لـ (LED) دايما في resistance

كيف يدخله بالطاولة؟؟

void setup( )

{ pinMode(Button, INPUT);
pinMode(LED, OUTPUT);

في حال كان على (9)
و Button على (10)

{ Void loop( )
{ int X;

X = digitalRead (Button);

if (x==1)
digitalWrite (LED, HIGH);
else
digitalWrite (LED, LOW);
}

1

Digital pins.
بيوخذو قيمة
[0/1].

→ Analog pin

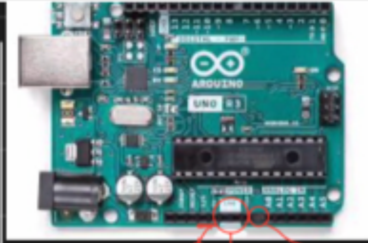
* Analog pin [1024]
using analog Read
analog Write
delay (1000);

2

$$R_{total} = 1000 + RT = \frac{V}{I} = \frac{1024}{\frac{V}{1000}}$$

```
float getTemp (int Tpin)
{ int V= analogRead (Tpin);
  float R=(10240000 /v)-10000
  float logR= log(R);
  return (1/(0.001129148 + 0.000234125 log(R)
       + 8.76741 E8*logR *logR * logR) }
```

− 273.15 → if it's in (C)



5V, Ground, A0

TMP 36 :

الطرف اليمين مع الـ Ground.
اليسار مع (5V)
الوسط مع A0

1

```
Const int Tpin = 0;

Void setup ( )
{ Serial.begin (9600);
}
Void Loop ( )
{ float voltage, temprature;
voltage = get voltage (Tpin);
temperature =(voltage - 0.5)*100;
Serial.print ("T is ");
Serial.println (temperature);
delay (1000);
}
```

0 1024
0V 5V
⟹ 512 → 2.5V
$= \frac{5}{1024}$.

```
float get voltage (int Tpin)
{ return (analog Read (Tpin) * 5/1024)
```

ثابتة

2

NTC Thermistor

A0, V_1, RT, 1000Ω, 5V

voltage div
5V → 1024

```
const int Tpin = 0;

Void setup ( )
{ Serial.begin(9600);
{ Void loop (
{ float Temp;
Temp = get Temp (Tpin);
Serial.Print ("T is ");
Serial.print (Temp);
Serial.println (Temp);
delay (1000);
{
```

① Button by LED:

```
const int Button = 12;
const int LED = 13;
void setup ( )
{ pinMode (Button, INPUT);
  pinMode (LED, OUTPUT); }
void loop ( )
{  val = digitalRead (Button);
   if (val == HIGH)
    { digitalWrite (LED, HIGH); }
   else
    { digitalWrite ( LED, LOW ); }
 }
```

② Blink LED

```
const int LED = 13;
void setup( )
{ pinMode (LED, OUTPUT ); }
  void loop( )
{ digitalWrite(LED, HIGH);
  delay (1000);
  digitalWrite (LED, LOW);
  delay (1000);
}.
```

③ TMP = 36.