

Control

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2 parts in each page

part (1)

part (2)

Control

Laplace part (1) :

$f(t) \rightarrow$ find Laplace : $\mathcal{L} f(t) = F(s)$

“ “ inverse : $\mathcal{L}^{-1} F(s) = f(t)$

3 cases for partial fraction :

- unrepeated factor $\Rightarrow$ فنكسي المقام منفرد $(s)(s+2)(s+3)$.
- repeated factor $\Rightarrow$ قوس الة قوة بالمقام $(s+8)^2 / (s)^2$
- Complex $\Rightarrow$ عبارة تربيعية لا تقبل بالمقام $(s^2 + 5s + 3)$.

Ex : $\frac{(s^2)(s+2)(s+3)(s^2+2s+5)}{(s)(s+2)(s+4)}$
repeated unrepeated complex

① unrepeated : $\frac{8(s+3)(s+8)}{(s)(s+2)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+4}$
 $\mathcal{L}^{-1}(F(s)) \rightarrow f(t)$
 $\left. \frac{8(s+3)(s+8)}{(s+2)(s+4)} \right|_{s=0} = 24$, $\begin{matrix} k_2 = -12 \\ k_3 = -4 \end{matrix}$

$f(t) = 24 - 12e^{-2t} - 4e^{-4t}$

Laplace part (2) : Repeated Roots.

Ex : $\frac{8(s+1)}{(s+2)^2} = \frac{k_1}{s+2} + \frac{k_2}{(s+2)^2}$, $\Rightarrow$ بياي الحالة باستخدام توحيد المقامات
 $8(s+1) = k(s+2) + k$
 $\begin{matrix} s=0 \Rightarrow 8 = 2k_1 + k_2 \\ s=-2 \Rightarrow -8 = k_2 \end{matrix} \Rightarrow \begin{matrix} k_1 = 8 \\ k_2 = -8 \end{matrix}$
ans : $8e^{-2t} - 8te^{-2t} = f(t)$.

② Complex : سنحل باآمال مربع

Ex : $\frac{k_1s + k_2}{s^2 + 2s + 5} = \frac{k_1s + k_2 + k_1 - k_1}{s^2 + 2s + 1 + 4} = \frac{k_1(s+1) + (k_2 - k_1)}{(s+1)^2 + 2^2}$
 $\frac{k_1(s+1)}{(s+1)^2 + 2^2} + \frac{k_2 - k_1}{(s+1)^2 + 2^2} = k_1 \cos 2t \cdot e^{-t} + \left(\frac{k_2 - k_1}{2}\right) \sin 2t \cdot e^{-t}$

Laplace part (3) :

Important case : find $(\mathcal{L})$ then $(\mathcal{L}^{-1}) : x(s) \rightarrow x(t)$

$\ddot{x} + 3\dot{x} + 2x = 5 \sin(t)$, Apply using Laplace to find $x(t)$.

$\mathcal{L}(\ddot{x} + 3\dot{x} + 2x) = \mathcal{L}(5 \sin(t))$ from table $= 5\left(\frac{1}{s^2+1}\right)$

$\ddot{x} = s^2 X(s) - s x(0) - \dot{x}(0)$
 $\dot{x} = s X(s) - x(0)$
 $x = X(s)$
 $\begin{matrix} x(0) = 1 \\ \dot{x}(0) = 0 \end{matrix}$

$(s^2 X(s) - s x(0) - \dot{x}(0)) + 3(s X(s) - x(0)) + 2(X(s))$
 $X(s)(s^2 + 3s + 2) - s - 3 = \frac{5}{s^2 + 1}$

$X(s) = \frac{5}{(s^2+1)(s^2+3s+2)} + \frac{(s+3)}{(s^2+3s+2)}$ Partial fraction the (f^-) .
Part (A) Part (B)

part (A) : $\frac{5}{(s^2+1)(s+2)(s+1)} = \frac{c_1s + c_2}{s^2+1} + \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$

$5 = (c_1s + c_2)(s+2)(s+1) + k_1(s^2+1)(s+2) + k_2(s^2+1)(s+1)$

① $s = -2 : 5 = k_2(5)(-1) \Rightarrow k_2 = -1$

② $s = 0 : 5 = c_2(2) + k_1(2) + \frac{-1}{2} \Rightarrow 3 = c_2 + k_1$

③ $s = 1 : 5 = (c_1 + c_2)(6) + k_1(6) + \frac{-1}{4} \Rightarrow 3 = 2(c_1 + c_2) + 2k_1$

④ $s = -1 : 5 = k_1(2) \Rightarrow k_1 = 5/2$
 $\begin{matrix} c_2 = 1/2 \\ c_1 = -3/2 \end{matrix}$

Part (B) : $\frac{(s+3)}{(s^2+3s+2)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$

$(s+3) = k_1(s+2) + k_2(s+1)$

① $s = -2 : 1 = -k_2 \Rightarrow k_2 = -1$

② $s = -1 : 2 = k_1 \Rightarrow k_1 = 2$

the final answer :

$\left(\frac{-3}{2}\right)\frac{s + (\frac{1}{2})}{s^2 + 1} + \left(\frac{5}{2}\right)\frac{1}{(s+1)} + \frac{-2}{(s+2)} + \frac{2}{(s+1)} + \frac{-1}{(s+2)}$

$\left(\frac{q}{2}\right)e^{-t} - (2)e^{-2t} + \mathcal{L}^{-1}\left[\frac{-\left(\frac{3}{2}\right)s + \frac{1}{2}}{s^2 + 1} + \frac{1/2}{s^2 + 1}\right]$
 $= \left(\frac{q}{2}\right)e^{-t} - (2)e^{-2t} + \left(\frac{-3}{2}\right)\cos t + \left(\frac{1}{2}\right)\sin t = x(t)$

Laplace part (4):

① final value theorem (FVT), ② Initial value theorem (IVT).

$F(s) \rightarrow$ find $\begin{cases} \text{FVT} \\ \text{IVT} \end{cases}$

$F(s) \cdot s \rightarrow \lim_{s \rightarrow 0} \begin{cases} s \rightarrow 0 \text{ (final)} \\ s \rightarrow \infty \text{ (initial)} \end{cases}$

Ex: $F(s) = \frac{s+3}{(s)(s+2)}$, find the (FVT) then (IVT).

$\lim_{s \rightarrow 0} \left(\cancel{s} \cdot \frac{s+3}{\cancel{s}(s+2)} \right) = 3/2$, $\lim_{s \rightarrow \infty} \left(\cancel{s} \cdot \frac{s+3}{\cancel{s}(s+2)} \right) = \frac{\infty}{\infty} = 1$

The system start from (1) and goes to (3/2).

Matlab code: write the matlab code

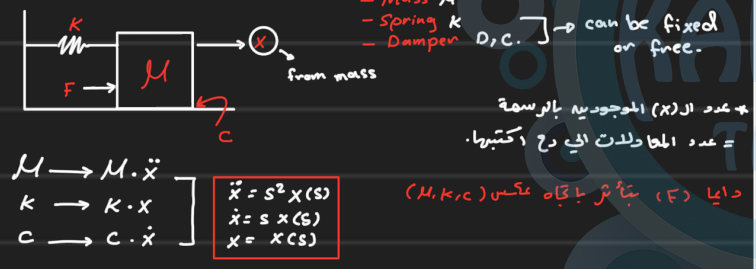
Ex: $f(t) = t e^{-4t} \Rightarrow L$:

`syms t s`
`Laplace(t*exp(-4*t)) = -4/(s+1)^2` $\rightarrow$ case (1).

Ex: $F(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)} \Rightarrow L^-$:

`syms t s`
`ilaplace(.....)`

Mechanical system:



*] Dampen $\rightarrow$ fixed: effected by only one (x)
free: 4 9 more than one (x).

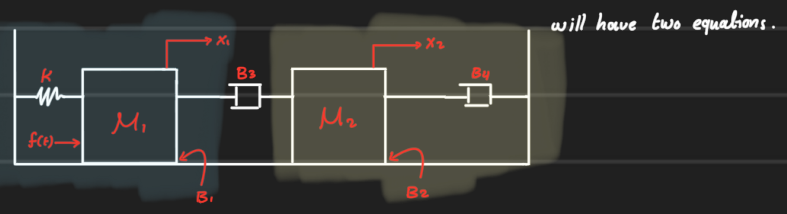
find the transfer function
 $= \frac{\text{output}}{\text{input}} = \frac{x(s)}{F(s)}$

the solution: (one (x) means one equation)

$F = M\ddot{x} + C\dot{x} + Kx \Rightarrow Ms^2x(s) + Cs x(s) + Kx(s)$

$F(s) = x(s) (Ms^2 + Cs + K) \Rightarrow \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + Cs + K}$

Anothe case:



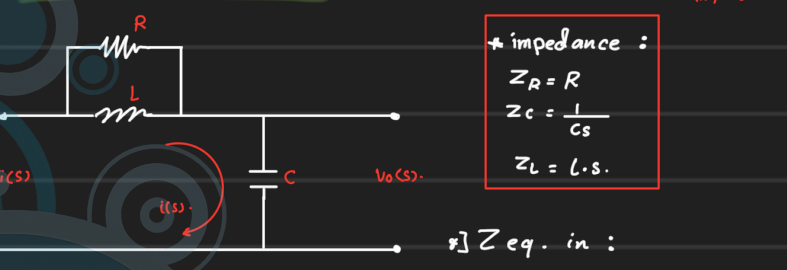
equation (1) for (x_1): $F = M_1 \ddot{x}_1 + K_1 x_1 + B_1 \dot{x}_1 + B_3 (\dot{x}_1 - \dot{x}_2)$

" (2) " (x_2): $0 = M_2 \ddot{x}_2 + B_3 (\dot{x}_2 - \dot{x}_1) + B_4 \dot{x}_2 + B_2 x_2$

$F(s) = M_1 s^2 x_1(s) + K_1 x_1(s) + B_1 s x_1(s) + B_3 s (x_1(s) - x_2(s))$

$0 = M_2 s^2 x_2(s) + B_3 s (x_2(s) - x_1(s)) + B_4 s x_2(s) + B_2 x_2(s)$

Electrical system:



Step (1): find the impedance for each element.

Step (2): the relationship between (V_o & V_i) with the element.

Step (3): transfer equation.

* V_o is the voltage of the element that parallel to it.

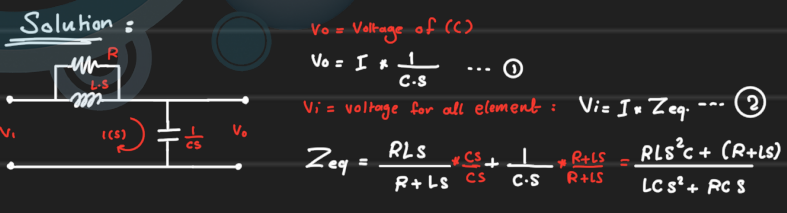
* V_i is the voltage of all elements.

Series: $Z_{eq} = Z_R + Z_C + Z_L$

Parallel: $\frac{1}{Z_{eq}} = \frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L}$

in case of two element:

$(Z_{eq} = \frac{Z_1 + Z_2}{Z_1 + Z_2})$

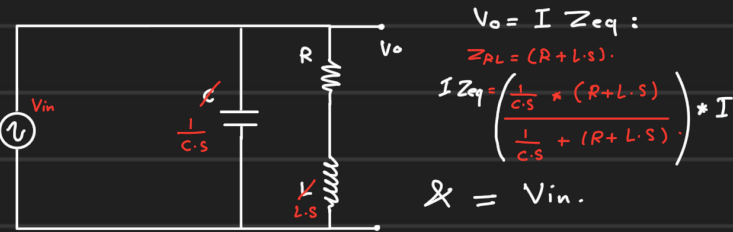


$T.f = \frac{V_o}{V_i} = \frac{I * (\frac{1}{C \cdot s})}{I * (\frac{RLs}{R+Ls} + \frac{1}{C \cdot s})} = \frac{1}{C \cdot s} * \frac{1}{(\frac{RLs}{R+Ls} + \frac{1}{C \cdot s})}$

$\frac{1}{\frac{RLCs^2}{R+Ls} + 1} = T.f$

Example of Electrical systems:

* find the transfer function:



then the transfer function

$$\frac{V_o}{V_{in}} = \frac{I \cdot Z_{eq}}{I \cdot Z_{eq}} = 1$$

because they have the same element.

Lec (6) transfer functions.

Example (2) $\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + 2}$

complex root & sys. without complex is slow

$$s^2 = 2 \rightarrow s = \sqrt{2} j$$

we care about dominant

Sinusoidal behavior.

Example (3) $\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + s + 2}$

$$(0.5) = \frac{1}{2} (s \text{ coefficient}) \rightarrow e^{-at} = e^{-0.5t}$$

in the imaginary it's frequency.

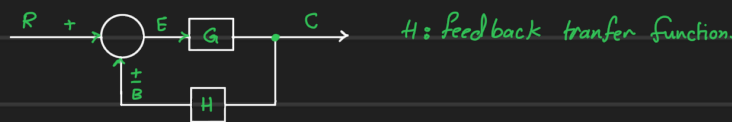
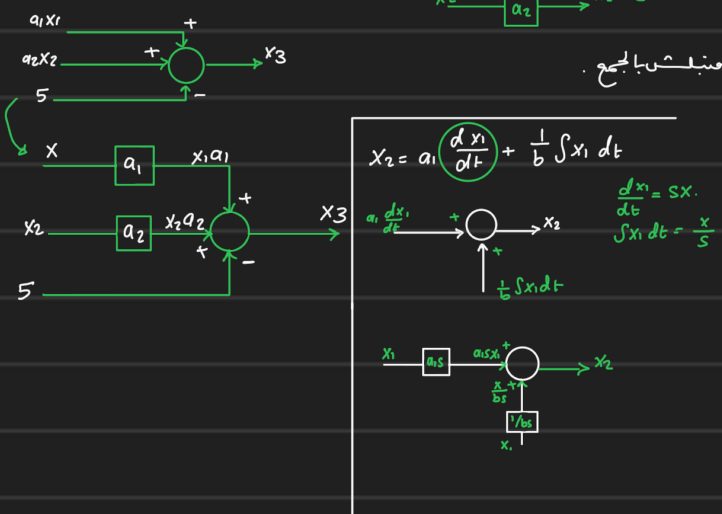
$$\sin(1.4t) \text{ or } \cos(1.4t)$$

if I have a Real Root $\rightarrow$ will be in x-axis with no vibration.

* Block diagram:



Example: $x_3 = a_1 x_1 + a_2 x_2 - 5$



① $\frac{C}{R}$: closed-loop = $\frac{G}{1 \pm GH}$ (مقلب الاشارة)

② $\frac{E}{R}$: error ratio = $\frac{1}{1 \pm GH}$ (المقلب)

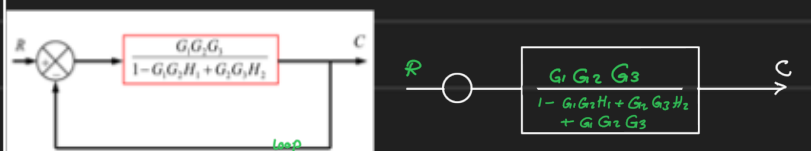
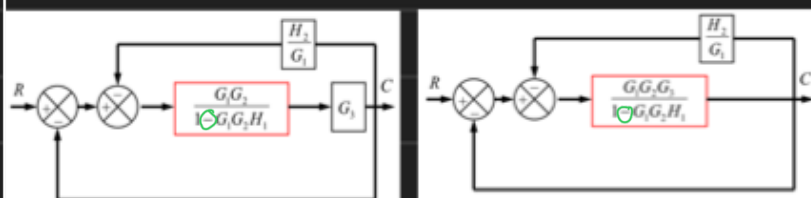
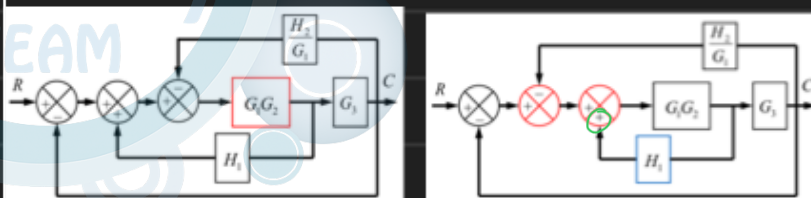
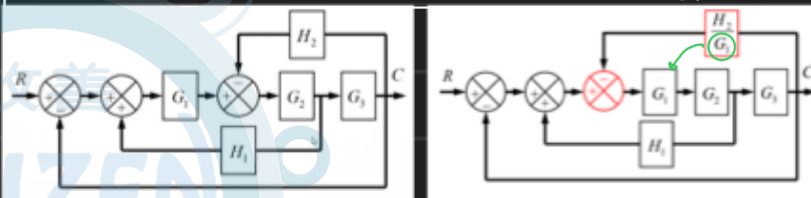
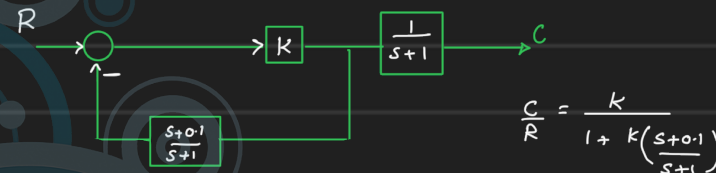
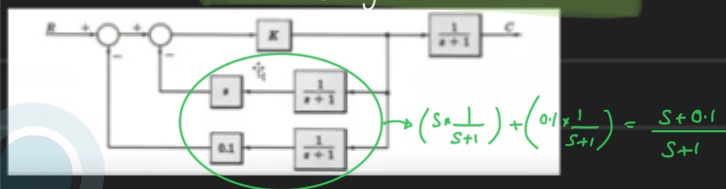
two block in series:

مربعين على التوالي $\leftarrow$ ضربهم

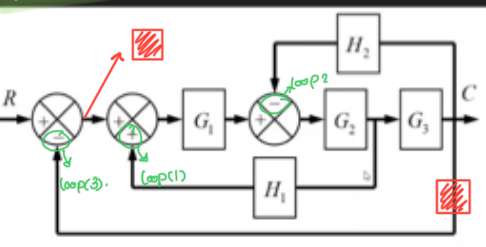
على التوازي $\leftarrow$ جمعهم وطرحهم

Lec (7) T.

Block diagram reduction:



Lec (7)



Solving it using Masons Rule :

$$\frac{C}{R} = \frac{\sum P_i \Delta_i}{\Delta} = \frac{G_1 G_2 G_3 * \Delta_i}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

P_i (path from R to C)
 Δ_i (path from R to C)
 $\Delta = 1 - (\sum \text{loops})$
 $\Delta = 1 - (G_1 G_2 H_1 - G_1 G_2 G_3 - G_2 G_3 H_2)$

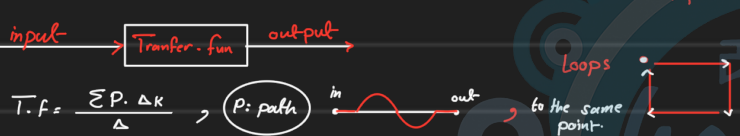
بالعادة نختار block الفاضي عشان استوف
 متو مسترل بحدينه بعوضه (1).

Δ_i نفس Δ لكنه لو في استي مسترل مع البطل بطل صفر

$\Delta_i = (1 - 0 - 0 - 0) = 1$

instead of $\rightarrow$

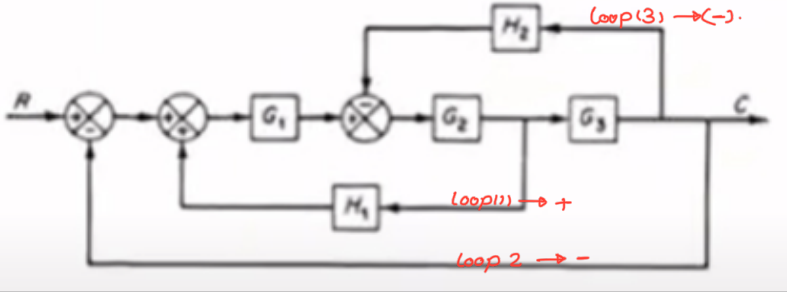
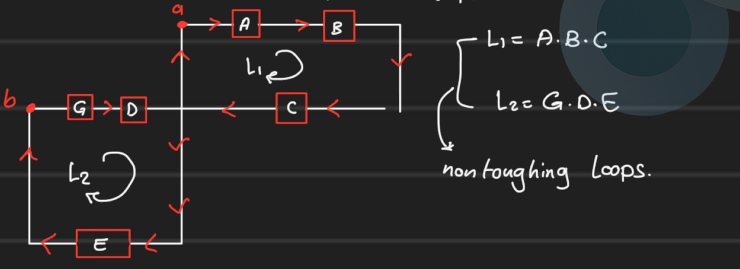
Mason's Rule : to find the transfer function = $\frac{\text{output}}{\text{input}}$



$T.F = \frac{\sum P_i \Delta_i}{\Delta}$, P_i : path in out to the same point.



$\Delta = 1 - (\sum \text{loops}) + 2 \text{ non touching loops} - 3 \text{ non touching loops}$



$P = G_1 G_2 G_3$
 $\Delta_k = (1 - 0 - 0 - 0)$
 $\Delta = 1 - (G_1 G_2 H_1 - G_1 G_2 G_3 - G_2 G_3 H_2)$

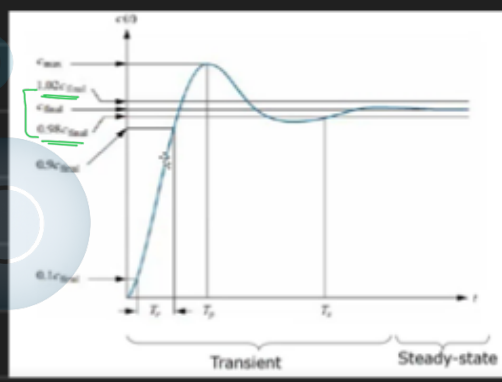
paths من input الى output.
 Δ_k من input الى output.
 $(1 - \sum \text{loop non touching})$
 $(\text{loops من input الى output})$
 Δ من input الى output.

Lec (8)

Steady State Response :

any value over the target value is called overshoot.
 for Example [25-250] if I get 270 then it's overshoot &
 the time from (10% - 90%) : 250 - 25 = 225 (10% - 90%) : Rise time.
 the time when it's stable with (2%) from 250 $\Rightarrow$
 then the sys. reach the stability. (settling)

if it reached 300
 then the overshoot
 $= (\frac{50}{225}) * 100$
 $\approx$ from 98% to 102%
 stability.



1 First Order System using partial fraction.

The fast response effect goes fast
 so we will have the dominant root
 we care about complex root. (best behavior).

First Order is about Real Roots.

$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$, $R(s) = \frac{1}{s}$ (step response).

$C(s) = \frac{1}{\tau s + 1}$, $R(s) = \frac{1}{s(\tau s + 1)} = \frac{1}{s} - \frac{1}{s + \frac{1}{\tau}}$

① First Order System

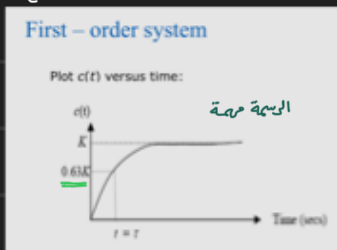
then with inverse Laplace transform, $C(t) = 1 - e^{-\frac{t}{\tau}}$

if $t = \tau \Rightarrow$ step response $C(t) = (1 - 0.37) = 0.63$.

in case of having only one real root (we could have 9 roots & one of them real).

Settling time when $t = 4\tau$.

$[\tau = RC] \rightarrow$ used here in time.



τ : could be used to measure the capacitor.

very slow sys.

Rise Time $\Rightarrow$ (from step response to go from 10% to 90%)

$T_r = 2.2 \tau$.

Settling time $\Rightarrow$ (2% from final value). $T_s = 4 \tau$.

x-axis. منسوب (tau) عن طريقه نأخذ (0.63 * final value) وبعده على

[ec(a) Tn.

② Second - Order System : way much better.

this system has a slight overshoot.

We will two complex root:

$G(s) = \frac{b}{s^2 + as + b} \rightarrow \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, ζ : determine if it's complex.

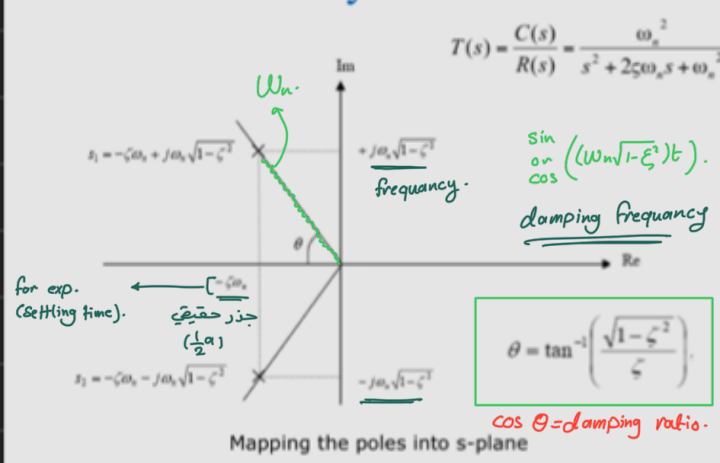
complex $\Rightarrow (\frac{1}{2}a)^2 < b$.

$\omega_n = \sqrt{b}$, $2\zeta\omega_n = a$, $\zeta = \frac{a}{2\omega_n}$

- $\zeta = 1 \Rightarrow$ repeated
- $\zeta > 1 \Rightarrow$ real.
- $\zeta < 1 \Rightarrow$ complex

Poles; $-\omega_n\zeta \pm \omega_n\sqrt{\zeta^2 - 1}$
 $-\omega_n\zeta \pm \omega_n\sqrt{1 - \zeta^2}$ Poles are complex if $\zeta < 1$!

Second - Order System



damping ratio $\rightarrow$ at x-axis = 1, y-axis = zero ($\cos \theta$)

$\zeta = \frac{a}{2\omega_n}$

- $\zeta = 1 \Rightarrow$ repeated Critically damped.
- $\zeta > 1 \Rightarrow$ real $\rightarrow$ Overdamped, 2 real roots.
- $\zeta < 1 \Rightarrow$ complex

$\zeta = 0 \rightarrow$ Undamped.

$(0 < \zeta < 1) \rightarrow$ Underdamped. (wanted!)

Given that the closed loop TF

$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

to find rise time $\Rightarrow T(0.9) - T(0.1)$.

" " settling time $\Rightarrow -\zeta\omega_n t = -4 \Rightarrow t = \frac{4}{\zeta\omega_n}$

(A) For transient response, we have 4 specifications:

- (a) Tr - rise time = $\frac{\pi - \cos^{-1}(\zeta)}{\omega_n\sqrt{1-\zeta^2}}$ $\cos^{-1}(\zeta)$ in Radian.
- (b) Tp - peak time = $\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
- (c) %MP - percentage maximum overshoot = $e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$
- (d) Ts - settling time (2% error) = $\frac{4}{\zeta\omega_n}$

لنظم معادل $s^2 + 4s + 36$

Example for Underdamped

$G(s) = \frac{36}{s^2 + 4.2s + 36}$: $\omega_n = 6$, $\zeta = 0.35$

$2\zeta\omega_n$: damping ratio = 4.2.

Example : Underdamped:

$G(s) = \frac{100}{s^2 + 15s + 100}$, $\omega_n = 10$, $\zeta = 0.75$

$T_s = \frac{4}{0.75 \times 10} = 0.533s$, $OS\% = e^{-\frac{\pi \times 0.75}{\sqrt{1-0.75^2}}} = 2.838\%$

$T_p = \frac{\pi}{10\sqrt{1-0.75^2}} = 0.475s$

Overdamped Response

$G(s) = \frac{b}{s^2 + as + b}$

$a = 9$

$C(s) = R * G(s)$

Overdamped system

$R(s) = \frac{1}{s}$

$C(s) = \frac{9}{s^2 + 9s + 9}$

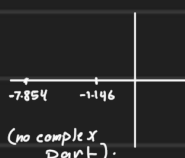
$\zeta > 1$

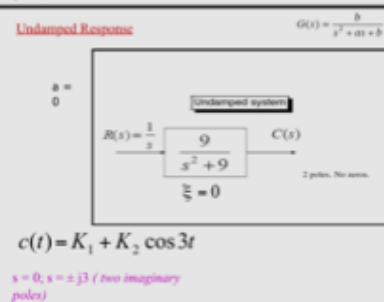
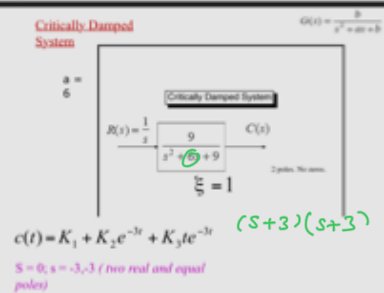
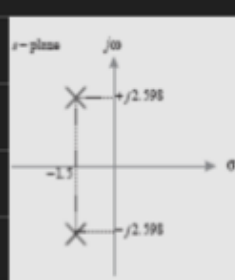
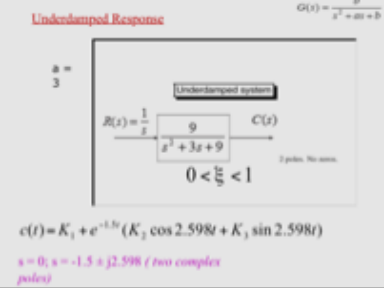
2 poles. No zeros.

$C(s) = \frac{9}{s(s^2 + 9s + 9)} = \frac{9}{s(s + 7.854)(s + 1.146)} = k_1 + k_2 e^{-7.854t} + k_3 e^{-1.146t}$

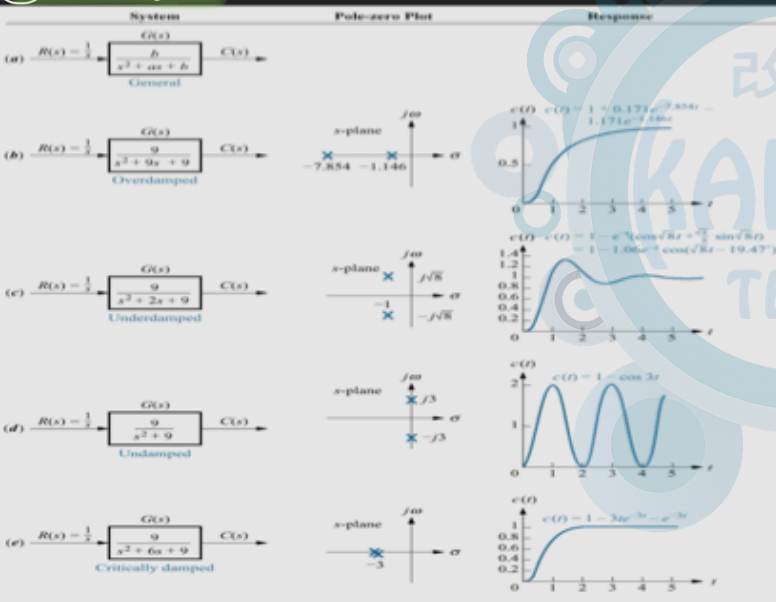
ζ : Damping ratio

$\rightarrow$ makes sys. slower.





Conclusion



real part $\Rightarrow$ Settling time
imaginary $\Rightarrow$ frequency.

First & Second order system:

First Order: $\frac{1}{\tau s + 1}$, $\tau = \frac{1}{a}$

$T_r = 2.2\tau$, with zero MP% } First order equation
 $T_s = 4\tau$

Example: $\frac{1}{3s + 2}$, $\tau = 3 \Rightarrow a = \frac{1}{3} = 0.33$

$T_r = 2.2 \times 3 = 6.6$, $T_s = 4 \times 3 = 12s$

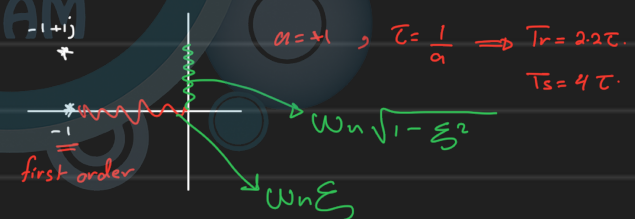
Second Order

Example: $\frac{C}{R} = \frac{s+3}{s^2 + s + 4.25}$, $\omega_n = \sqrt{4.25} = 2.06$
 $2\zeta\omega_n = 1 \Rightarrow \frac{1}{2 \times 2.06} = 0.2427$

$\cos \theta = \zeta$, $T_s = \frac{4}{\zeta \omega_n}$ } Second order equations
 $\omega_n = \sqrt{\text{real}^2 + \text{imag}^2}$,
 $T_r = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}}$, $T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$
 $MP = e^{\frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}} \times 100$

$\theta = \tan^{-1}(y/x)$ in Radian $\theta \times \frac{\pi}{180}$

in case first & Second Order in the same question.



Steady State Error Analysis :

Design in General around Critical damped.

The final value is an Indicator for the accuracy.

accuracy $\Rightarrow$ amount of error $\Rightarrow$ deviation from final value.

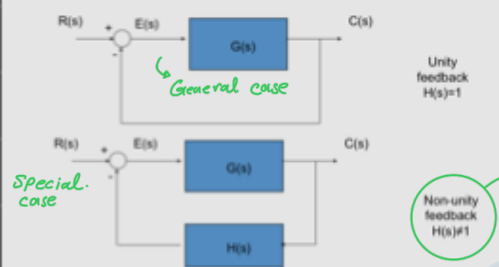
(*) المسألة متعلقة بجزء المقام التي قيمته حوض (أساس الحق) استقاة
 (* $\frac{1}{s}$) تكامل $(\frac{1}{\infty} = 0) \Leftarrow (\infty)$ عشان ادهل $(\frac{1}{s})$ تكامل

Test Waveform for evaluating steady-state error

Waveform	Name	Physical Interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

types of input $\Leftarrow$

Steady-state error analysis



case (2) if $R(s) = 250$

$f(s) = 2 \Rightarrow$ then

$E(s) = 0$ when $C(s) = 125$

Complicated.

Unity feedback $\Rightarrow E(s) = R(s) - C(s)$: sys. error.

Non-Unity feedback $\Rightarrow E(s) = R(s) - H(s) \cdot C(s)$: Actual error.

By Final Value Theorem:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) \equiv \lim_{s \rightarrow 0} s E(s) \quad \text{From final value theorem.}$$

$$\frac{1}{s} = \frac{1}{\infty} = \text{zero}$$

Unity feedback : $E(s) = R(s) - C(s)$, $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$

$$G(s) = \frac{K \prod_{i=1}^M (s + z_i)}{s^N \prod_{j=1}^N (s + p_j)}$$

high value for $G(s)$ to get small error.

or ∞

$G(s) = \frac{\text{constant}}{\text{zero}} = 0 \Rightarrow E(s) = \frac{1}{0} = \infty \Rightarrow$ most powerful control tool.

Case (1) $\Rightarrow R(s) = (1/s)$. $e_{ss} = \lim_{s \rightarrow 0} s E(s) = s \cdot \frac{1/s}{1 + G(s)} = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)}$

assuming $k_b = \lim_{s \rightarrow 0} G(s) \Rightarrow \lim_{s \rightarrow 0} \frac{1}{1 + G(s)} = \frac{1}{1 + k_b}$ k_b : static position Error constant.

how to get (∞)

① if $N=0 \Rightarrow k_p = \text{constant}$ $e_{ss} = \frac{1}{1 + k_p} = \text{finite} = \text{constant}$

② if $N \geq 1 \Rightarrow k_p = \text{infinite}$ $e_{ss} = \frac{1}{1 + k_p} = \frac{1}{\infty} = 0$

as the type of sys. increase ($N \geq 1$) the steady state error goes to zero.

Case (2) : $R(s) = (1/s^2)$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \left(\frac{1/s^2}{1 + G(s)} \right) = \lim_{s \rightarrow 0} \frac{1}{s + s \cdot G(s)} \quad \text{static velocity error constant.}$$

$$e_{ss} = \frac{1}{\lim_{s \rightarrow 0} s G(s)} = \frac{1}{k_v} \quad , \quad k_v = s \frac{\pi(s + z_i)}{\pi(s + p_j)}$$

① if $N=0$, $k_v=0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

② if $N=1$ و $k_v = \text{finite} \Rightarrow e_{ss} = \frac{1}{k_v} = \text{finite}$.

③ if $N \geq 2$, $k_v = \text{infinite} \Rightarrow e_{ss} = \frac{1}{\infty} = 0$.

تكون الحالت المطلوب
 $k = \infty$
 عشان بدنا نوصل
 للصفر.

Case (3) : $R(s) = (1/s^3)$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1/s^3}{1 + G(s)} = \lim_{s \rightarrow 0} \frac{1}{s^2 + s^2 G(s)} = \frac{1}{\lim_{s \rightarrow 0} s^2 G(s)} = \frac{1}{k_a}$$

k_a : static acceleration error constant $[k_a = \lim_{s \rightarrow 0} s^2 G(s)]$

① $N=0$, $k_a=0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

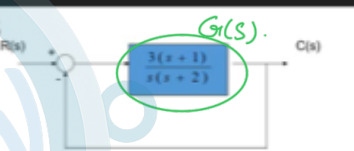
② $N=1$, $k_a=0 \Rightarrow e_{ss} = \frac{1}{0} = \infty$

③ $N=2$, $k_a = \text{constant} \Rightarrow e_{ss} = \frac{1}{\text{constant}} = \text{finite}$.

④ $N \geq 3$, $k_a = \text{infinite} \Rightarrow e_{ss} = \frac{1}{\infty} = 0$

increasing sys. type (N) will accommodate more different inputs.

Example 3



if $R(t) = (2 + 3t)u(t)$.

find steady-state error :

$$R(t) = (2 + 3t)u(t) \Rightarrow \frac{2}{s} + \frac{3}{s^2} = \left[\frac{2}{1 + k_p} + \frac{3}{k_v} \right]$$

$$1) k_p = \lim_{s \rightarrow 0} G(s) = \infty, \quad 2) k_v = \lim_{s \rightarrow 0} s G(s) = 3/2$$

$$\Rightarrow e_{ss} = \frac{2}{1 + \infty} + \frac{3}{3/2} = 2 \neq$$

في
 حال
 بالسرعة

Steady state error :

Types of errors :

① Step unit $e_{ss} = \frac{1}{1 + k_p}$, $k_b = \lim_{s \rightarrow 0} G(s)$

② ramp. $e_{ss} = \frac{1}{k_v}$ $k_v = \lim_{s \rightarrow 0} G(s) \cdot s$

③ parabolic $e_{ss} = \frac{1}{k_a}$ $k_a = \lim_{s \rightarrow 0} G(s) \cdot s^2$

Example : find the $e_{ss} = 3u(t) + 2t$

$$3u(t) \rightarrow \text{step unit} = \frac{1 \cdot (3)}{1 + k_p}$$

$$2t \rightarrow \text{ramp} = \frac{1 \cdot (2)}{k_v}$$

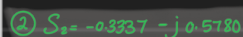
Controller

$$\text{ramp} : k_v = \lim_{s \rightarrow 0} G(s) \cdot s = \frac{k}{s}$$

$$\text{step} : k_p = \lim_{s \rightarrow 0} G(s) \cdot K$$

$$\text{parabolic} : k_a = \lim_{s \rightarrow 0} G(s) \cdot s^2 = \frac{k}{s^2}$$

Q2 a) (2.5 points) Find the steady state error to a step input the following



Lec (12) Root Locus (2)

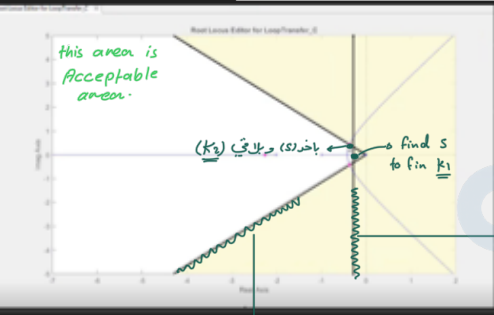
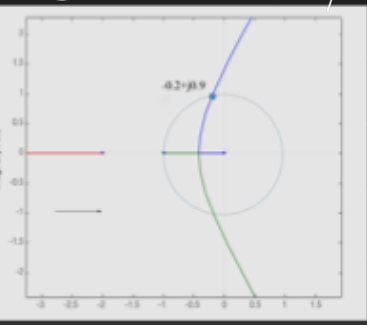
- 1) $S_1 = -0.3337 + j0.5780$
 - 2) $S_2 = -0.3337 - j0.5780$
- then we use it to find the value of (K). #

$K = |s(s+1)(s+2)|_{s=-0.3337+j0.5780} = |-1.03829 - 1.28 \times 10^{-3}j|$

$K = 1.0383$

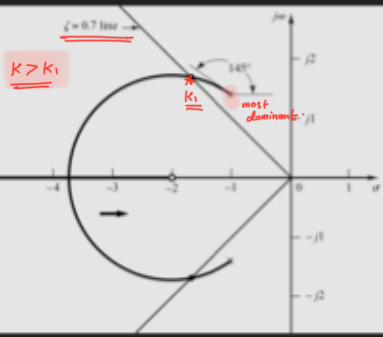
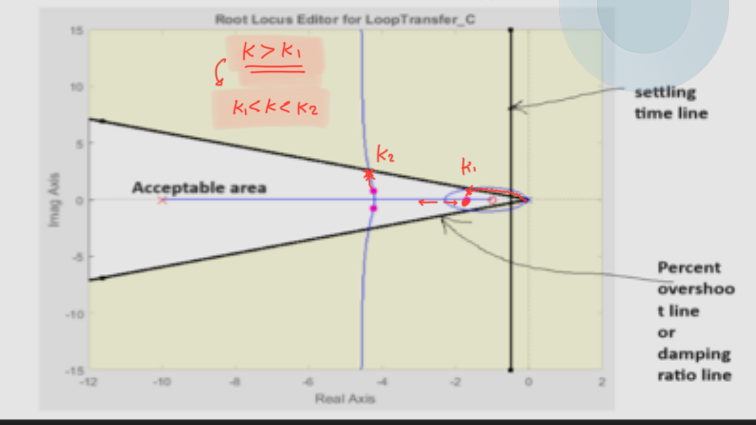
the type of questions is to determine the value of (K) such that - - - -

3) Natural frequency : البعد المركزي
القائم الي على نفس الدائرة عندهم
نفس البعد



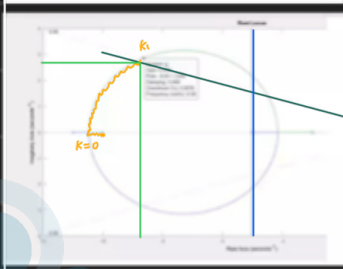
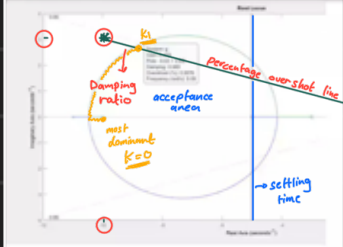
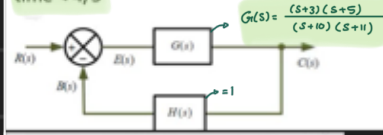
Lec (13). Root Locus (3).

We care about damping ratio or settling time.



Lec (14) Root Locus (4)

Q5) For the following system design a controller K to achieve a damping ratio of > 0.95 and settling time < 4/5



in Matlab :
 $\tan(a \cos(0.95)) = 0.3287$
 $\tan(a \cos(0.95)) \times (180/\pi) = 18.1949$
اذا مشتت على (X) بقدر (10)
امشي على (10) مقدار
 $10 \times 0.3287 = 3.287$

settling time : $\frac{4}{5} = -4 \Rightarrow Ts = -5$
the solution $0 < K < K_1$

$S_1 = -8.8 + j2.75$ in matlab

$S = -8.8 + 2.7j$

$g = (s+3)(s+5) / ((s+10)(s+11))$

$K_1 = \text{abs}(1/g) = 0.3451$

final answer $0 < K < 0.3451$

Root locus : find K $\begin{cases} \text{damping ratio} \\ \text{settling time} \end{cases}$

1) if settling time < 3 $\Rightarrow -3\sigma = -4 \Rightarrow (\sigma = 4/3)$ --- 1

2) Damping ratio or %MP $\Rightarrow \zeta = \frac{\ln(\frac{MP\%}{100})}{\sqrt{\pi^2 + \ln^2(\frac{MP\%}{100})}}$ we care about (ζ).
 ζ given.

$\zeta = \cos \theta \Rightarrow \theta = \cos^{-1}(0.7)$ --- 2

then $\tan(\theta) = \dots$

$\tan(\cos^{-1}(\zeta)) = \underline{\underline{c}} \Rightarrow$ نحدد قيمة (X) ونضربها بـ (c)
عشان عند قيمة (y) --- 3

find the (S)'s value for K_1 & K_2 --- 4

$\zeta > X \Rightarrow$ always in the acceptable region.

$T_s < X \Rightarrow$ --- 5

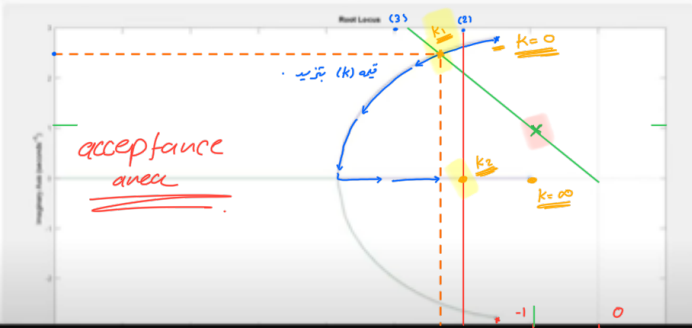
منزوح مع اتجاه بـ ي اتجاه قيمتها بتكبر كل ما غشيت القيمة بتزيد

$K = \frac{1}{|G(s)H(s)|}$ --- 6

Root locus :

For the following system determine K that achieves a damping ratio of > 0.7 and settling time < 3 s

$$GH(s) = \frac{(s+1)}{(s^2+3s+10)}$$



$$\zeta > 0.7 \Rightarrow \theta = \cos^{-1}(0.7) = \underline{45.57} \rightarrow \text{Scale} = \tan(\theta) = \underline{1.02}$$

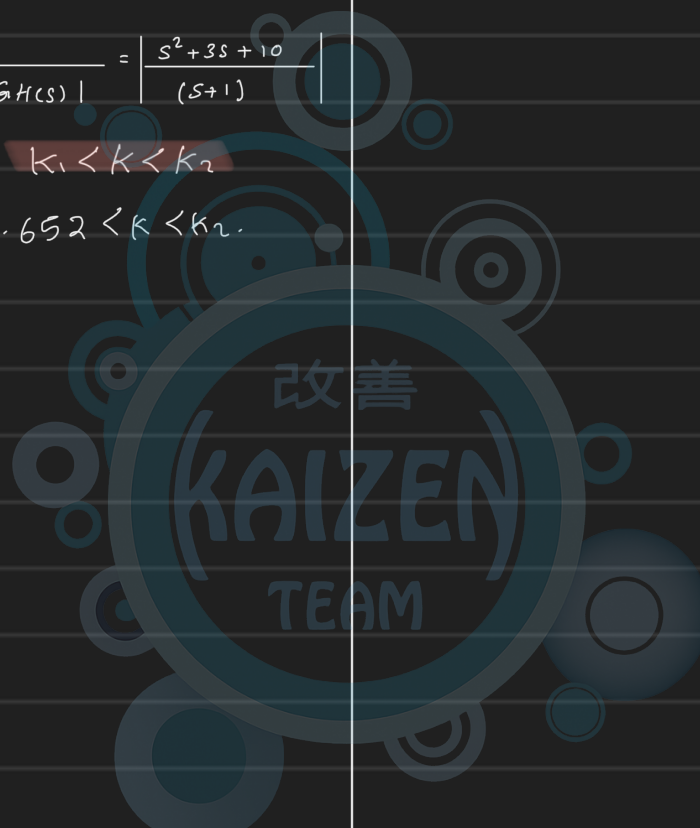
$$\text{if } x=1 \Rightarrow y=1.02$$

$$\text{settling time} < 2 \Rightarrow \frac{4}{\omega \zeta} = 2 \Rightarrow \omega \zeta = 2 \quad (*)$$

$$\left. \begin{array}{l} s_1 = -2.3 + j2.45 \\ s_2 = -2 \end{array} \right\} \Rightarrow k = \frac{1}{|GH(s)|} = \left| \frac{s^2 + 3s + 10}{(s+1)} \right|$$

$$k_1 = 1.652, \quad k_2 = 8 \Rightarrow k_1 < k < k_2$$

$$1.652 < k < 8$$



11/8

Matlab

Control system Toolbox :

Transfer function assuming zero initial condition

Example : $y'' + 6y' + 5y = 4u' + 3u$. ① Transfer function.

$$s^2 Y(s) + 6s Y(s) + 5Y(s) = 4s U(s) + 3U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5}$$

>> num = [4 3];

>> den = [1 6 5]; G = tf([4 3], [1 6 5])

>> G = tf(num, den)

② Zero-pole-gain (ZPK)

$$\frac{Y(s)}{U(s)} = \frac{4s+3}{s^2+6s+5} = \frac{4(s+0.75)}{(s+1)(s+5)}, \quad G = \text{ZPK}([-0.75], [-1, -5], 4)$$

Normal : $S = \text{tf}('s')$

$$G = (4*s + 3) / ((s^2) + 6*s + 5)$$

③ State-Space Model (SS)

$$\frac{Y}{R} = \frac{4s+3}{s^2+6s+5}, \quad \frac{X}{R} = \left(\frac{1}{s^2+6s+5} \right), \quad \frac{Y}{X} = (4s+3)$$

$$R = x'' + 6x' + 5x, \quad Y = 4x' + 3x$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} R$$

$$G = \text{SS}([0 \ 1; -5 \ -6], [0; 1], [3 \ 4], 0)$$

Symbolic :

⇒ Solve the diff. equ. $\frac{dy}{dx} = e^{-\frac{1}{x}} \frac{1}{x^2}$:

Syms Y(x)

$$\text{eqn} = \text{diff}(Y) == \exp(-1/x) / x^2$$

$$\text{cond} = [Y(0) == 1];$$

$$\text{sol}(x) = \text{dsolve}(\text{eqn}, \text{cond})$$

⇒ system of differential equ.

$$\frac{dy}{dt} = z, \quad \frac{dz}{dt} = y$$

Syms Y(t) Z(t)

$$\text{eqns} = [\text{diff}(Y, t) == z, \quad \text{diff}(Z, t) == -Y]$$

$$\text{sol} = \text{dsolve}(\text{eqn})$$

sol.Z

sol.Y

$$(1+t^2) \ddot{w} + 2t \dot{w} + 3w = 2$$

2 order

$$t_0 = 0, t_f = 5, w(t_0) = 0, \dot{w}(t_0) = 1$$

$$\text{function } dx = \text{diff}(t, x)$$

$$[m, n] = \text{size}(X)$$

$$dx = \text{zeros}(m, n)$$

$$dx(1) = x(2)$$

$$dx(2) = (2 - 2*t*x(2) - 3*x(1)) / (1+t^2)$$

$$tspan = [0 \ 5]$$

$$x_0 = [0; 1]$$

$$[t, x] = \text{ode23}(@\text{diff}, tspan, x_0)$$

$$\text{plot}(t, x(:, 1))$$

$$\text{plot}(t, x(:, 1), t, x(:, 2))$$

write MATLAB code using ODE45 to solve the following differential equations

$$\ddot{x} + x = \log(t), \quad \dot{x}(0) = 0, \quad x(0) = 0$$

Plot(y) and plot(x) on same figure for a time interval 0 to 20 seconds.

ODE45 : Numerical answer :

Convention $\Rightarrow$ there is a second order

$$\begin{aligned} x_1, x_2 & \quad \ddot{x} + x = \log(t) \\ \dot{x}_1 = x_2 & \quad \text{--- ①} \\ \dot{x}_2 = \log(t) - x_1 & \quad \text{--- ②} \end{aligned}$$

function dx = sos(t,x)

[m,n] = size x ;

dx = zeros(m,n) ;

dx(1) = x(2) ;

dx(2) = log10(t) - x(1) ;

tspan = [0, 20]

x0 = [0; 0] ;

[tx] = ode45(@sos, tspan, x0)

plot(t, x)

plot(t, x(:,1))

Normal first order equation :

$$\dot{x} + ax = b \Rightarrow \dot{x} + 2x = b \quad \dot{x} = b - ax$$

function dx = sos(t,x)

a = 2 ;

b = 4 ;

dx = b - a*x

tspan = [0 20] ;

x0 = [0]

[tx] = ode45(@sos, tspan, x0)

plot(t, x)

Changing parameters :

$$\dot{x} + ax = b \quad \dot{x} = b - ax$$

function dx = sos(t,x,param)

a = param(1)

b = param(2)

dx = b - a*x

tspan = [0 20] ;

x0 = [0] ;

param[,] ;

[tx] = ode45(@sos, tspan, x0, [], param)

plot(t, x)

Symbolic :

write MATLAB symbolic code to solve the following differential equations

$$\ddot{x}y + y = \log(t), \quad \ddot{y}x + \dot{y} = t, \quad \dot{x}(0) = 0, \quad \dot{y}(0) = 0, \quad x(0) = 0, \quad y(0) = 0$$

x(t), y(t)

Syms x(t) y(t)

eqn = [diff(x,t,2)*y + y == log10(t)

, diff(y,t,2)*x + diff(y,t) == t] ;

dx = diff(x,t) ;

dy = diff(y,t) ;

cond = [dx(0)==0, dy(0)==0, x(0)==0, y(0)==0] ;

sol = dsolve(eqn, cond)

في حال لغت المعادلة التفاضلية

Syms x(t) y(t)

eqn = [diff(x,t,2)*y + y == log10(t)

, diff(y,t,2)*x + diff(y,t) == t] ;

dx = diff(x,t) ;

dy = diff(y,t) ;

cond = [dx(0)==0, dy(0)==0, x(0)==0, y(0)==0] ;

sol = dsolve(eqn, cond)

بس متلغني هاي

Routh Hurwitz :

عشان اتق الجذور الموجبة للـ polynomials. حقه لو في زوج.

steps:

1 Block-diagram reduction : $\left[\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \right]$

$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$

parameter = (highest power + 1)

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	-	-	-
s	-	-	-
s^0	-	-	-

Example : $\frac{1000}{s^3 + 10s^2 + 31s + 1030}$

s^3	1	31	0
s^2	10	1030	0
s	-	-	-
s^0	-	-	-

row(2) $\Rightarrow \frac{10}{10}$

s^3	1	31	0
s^2	1	103	0
s^1	-	-	-
s^0	-	-	-

$(1 \times 31) - (1 \times 103) = -72$
 $(1 \times 0) - (1 \times 0) = 0$

s^3	1	31
s^2	1	103
s^1	$31 \times 1 - 1 \times 103 = -72$	$0 \times 1 - 1 \times 0 = 0$
s^0	$-72 \times 103 - 1 \times 0 = 103$	$-72 \times 0 - 1 \times 0 = 0$

عدد الجذور الموجبة
يساري عدد تغيرات الإشارة
في العناصر الموجبة.

* Special Cases :

Zero in the first column ... (1)

Entire row is zero ... (2).

Example : the characteristic equation $[s^4 + 6s^3 + 11s^2 + 6s + k = 0]$

s^4	1	11	k	0
s^3	6	6	0	0
s^2	10	k		
s^1	$\frac{60-6k}{10}$			
s^0	k			

for stability : $\frac{60-6k}{10} > 0$
 $k > 0$

$0 < k < 10$

row 3

$10s^2 + 10 = \text{zero}$

$s^2 = -1j$

Example : Construct a Routh table & find the # of roots with

positive real parts : $[2s^3 + 4s^2 + 4s + 12 = 0]$

s^3	2	4	0
s^2	4	12	0
s^1	-2	0	
s^0	12		

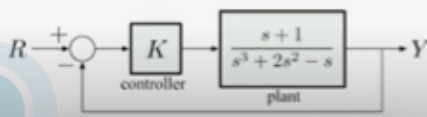
2 changes of sign in the first column

so 2 real roots positive.

* Stability :

determin (k) for stability :

Q1) Determine K for stability



feed back :

$\frac{G(s)}{1 \pm G(s) \cdot H(s)}$

$\left(\frac{K(s+1)}{s^3+2s-s} \right) \Rightarrow \frac{K(s+1)}{(s^3+2s^2-s) + K(s+1)} = \frac{K(s+1)}{s^3+2s^2-s+Ks+K}$

$s^3 + 2s^2 - s + Ks + K = 0 \Rightarrow s^3 + 2s^2 + (K-1)s + K = 0$

s^3	1	(K-1)	0
s^2	2	K	0
s^1	$\frac{K-2}{2}$	0	
s^0	K		

$\frac{K-2}{2} \geq 0 \Rightarrow K \geq 2$

$K \geq 0$

$\left. \begin{matrix} K \geq 2 \\ K \geq 0 \end{matrix} \right\}$

in stability the answer should be positive.

تنبيه
القسم

و دايم
الضرب
مع اقل
عامود

PID $\Rightarrow$ [proportional Integral Derivative].

$$C_{pid} = (K_p e(t)) + (K_i \int e(t) dt) + (K_d \frac{de(t)}{dt})$$

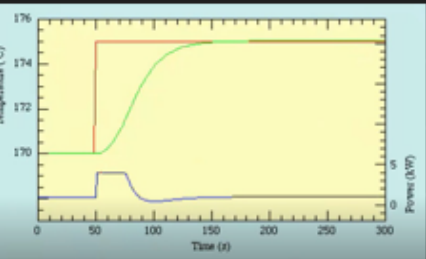
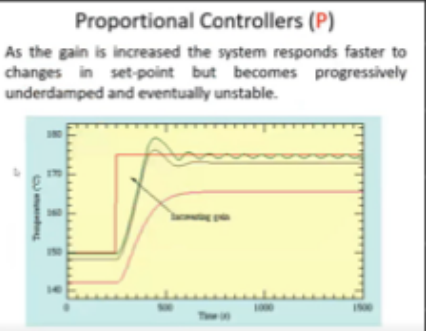
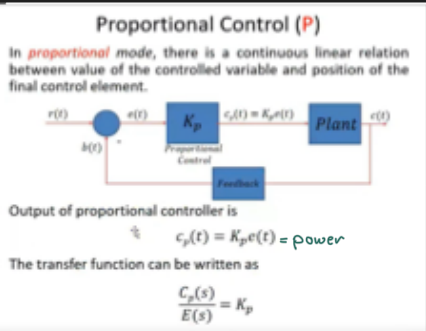
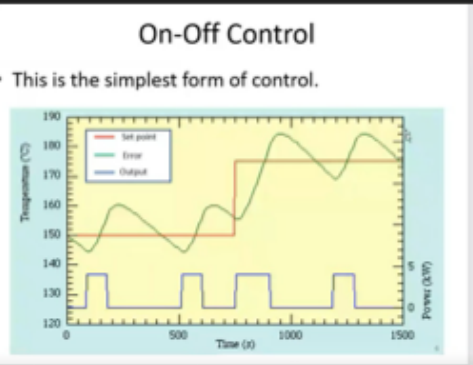
↳ amount of the electrical power

ما بيستعمل اى (K) مكونات دوا بقى المقياس

K_p : to get less rise time

$K_d \frac{d}{dt}$: less overshoot

$K_i \int$: more accuracy.



The Characteristics of P, I, and D controllers

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
$K_p \uparrow$	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small Change	Decrease	Decrease	Small Change

PID part (3) :

PID :

type (1)

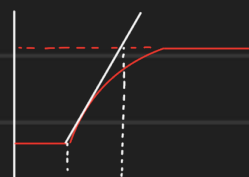


Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

K_{cr} ? given

P_{cr} ?

type (2)



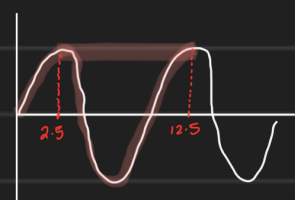
	K_p	T_r	T_d
P	$\frac{V_0}{V_o}$		
PI	$\frac{K_p T_o}{0.9 V_o}$	$3T_o$	
PID	$\frac{K_p T_o}{1.2 V_o}$	$2T_o$	$0.5T_o$

V_0 ?

K_0 ?

T_0 ?

Example 8 find the PID : ($K_{cr} = 3.5$).



$$\Rightarrow P_{cr} = 12.5 - 2.5 = 10$$

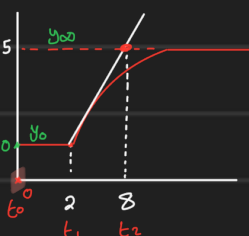
وسنكمل القيم من الجدول .

$$PID = 0.6 K_{cr} = 0.6 * 3.5 = 2.1$$

$$0.5 P_{cr} = 0.5 * 10 = 5$$

$$0.125 P_{cr} = 0.125 * 10 = 1.25$$

Example : find the final PID ? $u_{\infty} = 1.25$
 $u_0 = 0.25$



$$① V_0 = t_2 - t_1 = 6$$

$$② T_0 = t_1 - t_0 = 2$$

$$③ K_0 = \frac{y_{\infty} - y_0}{u_{\infty} - u_0} = \frac{5}{1} = 5$$

(,)
given

وبعدنا على الجدول .

$$PID = \frac{1.2 V_0}{K_0 T_0} = \frac{1.2 * 6}{5 * 2}$$

$$2 T_0 = 4$$

$$0.5 T_0 = 1$$

Arduino : Temperature Sensors



مصمم بطريقة تحيداً عنشان يقيس voltage

يستقبل من البطارية (Zero 5 V)



تتغير المقاومة فيه (تتزل مع الحرارة)

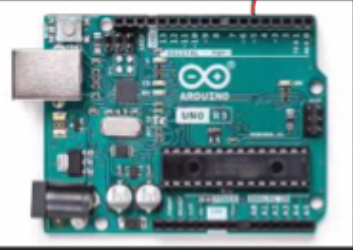
```
// We'll use analog input 0 to read Temperature Data const int temperaturePin = 0;
void setup()
{ Serial.begin(9600); }
void loop()
{ float voltage, degreesC, degreesF;
  voltage = getVoltage(temperaturePin);
  // Now we'll convert the voltage to degrees Celsius.
  // This formula comes from the temperature sensor datasheet:
  degreesC = (voltage - 0.5) * 100.0;
  // Send data from the Arduino to the serial monitor window
  Serial.print("voltage: ");
  Serial.print(voltage);
  Serial.print(" deg C: ");
  Serial.println(degreesC);
  delay(1000);
  // repeat once per second (change as you wish!)
}
float getVoltage(int pin)
{ return (analogRead(pin) * 0.004882814); }
// This equation converts the 0 to 1023 value that analogRead()
// returns, into a 0.0 to 5.0 value that is the true voltage
// being read at that pin.
```

```
// Read Temperature Values from NTC Thermistor
const int temperaturePin = 0;
void setup()
{ Serial.begin(9600); }
void loop()
{ int temperature = getTemp();
  Serial.print("Temperature Value: ");
  Serial.print(temperature);
  Serial.println("°C");
  delay(1000);
}
double getTemp()
{
  // Inputs ADC Value from Thermistor and outputs Temperature in Celsius int RawADC =
  analogRead(temperaturePin);
  long Resistance;
  double Temp;
  // Assuming a 10k Thermistor. Calculation is actually: Resistance = (1024/ADC)
  Resistance = ((10240000/RawADC) - 10000);
  // Utilizes the Steinhart-Hart Thermistor Equation:
  // Temperature in Kelvin = 1 / {A + B[ln(R)] + C[ln(R)]^3}
  // where A = 0.001129148, B = 0.000234125 and C = 8.76741E-08 Temp = log(Resistance);
  Temp = 1 / (0.001129148 + (0.000234125 * Temp) + (0.0000000876741 * Temp * Temp *
  Temp)); Temp = Temp - 273.15;
  // Convert Kelvin to Celsius return Temp;
  // Return the Temperature
}
```

Lecture 1

Arduino :

digital [10]
on/off $\Rightarrow$ on = 5V , off = 0V



input : Button 10 Output LED 10 :

const int Button = 10; ^{منشيطا لما نغير}
LED = 9; resistance (LED) دايما في

void setup() كيف بخله بالحدالة ؟

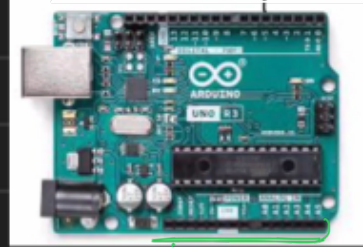
{ pinMode(Button, INPUT); في حال كان على (9)
pinMode(LED, OUTPUT); Button على (10)

void loop()

{ int x;

x = digitalRead(Button);

if (x == 1)
digitalWrite(LED, HIGH);
else
digitalWrite(LED, LOW);
}



Analog pin [1024]

using analogRead

analogWrite

delay(1000);

$$R_{total} - 1000 + R_T = \frac{V}{I} = \frac{1024}{\frac{V}{1800}}$$

float getTemp(int Tpin)

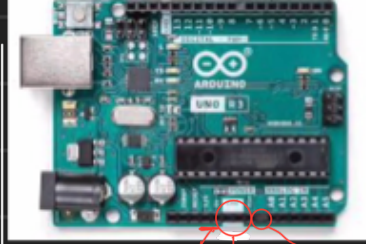
{ int V = analogRead(Tpin);

float R = (1024000 / V) - 10000

float logR = log(R);

return (1 / (0.00129148 + 0.000234125 * log(R)
+ 8.76741E8 * log R * log R * log R))

- 273.15 $\rightarrow$ if it's in (C)



THP 36 : Ground. الطرف اليمين مع الـ



البار مع (5V)

الوسط مع A0

Const int Tpin = 0;

Void setup()

{ Serial.begin(9600);

}

Void loop()

{ float voltage, temperature;

0 1024

0V 5V

voltage = getVoltage(Tpin);

$\Rightarrow 512 \rightarrow 2.5V$

temperature = (voltage - 0.5) * 100;

$= \frac{5}{1024}$

Serial.print("T is ");

Serial.println(temperature);

delay(1000);

}

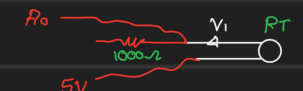
float getVoltage(int Tpin)

{ return (analogRead(Tpin) * 5 / 1024)

تايه



NTC Thermistor



voltage div

5V $\rightarrow$ 1024

Const int Tpin = 0;

Void setup()

{ Serial.begin(9600);

Void loop()

{ float Temp;

Temp = getTemp(Tpin);

Serial.print("T is ");

Serial.print(Temp);

Serial.println(Temp);

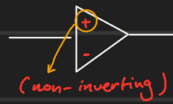
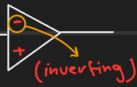
delay(1000);

}

Electronic sys., Active circuit - (AC)

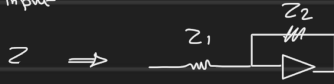
$$T.f = \frac{\text{output}}{\text{input}}$$

Cases : 1] Inverting 2] Non-inverting.

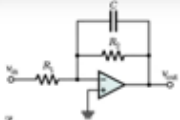


$$T.f = \frac{-Z_2}{Z_1} = \frac{\text{output}}{\text{input}}$$

$$T.f = 1 + \frac{Z_2}{Z_1}$$



Find out the transfer function of the following circuit.



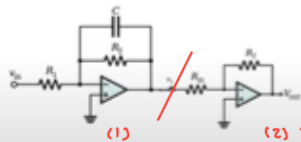
inverting

$$Z_2 = \frac{R_2 + \frac{1}{C \cdot s}}{R_2 + \frac{1}{C \cdot s}}$$

$$T.f = \frac{-Z_2}{Z_1}$$

$$Z_1 = -R$$

Find out the transfer function of the following circuit.



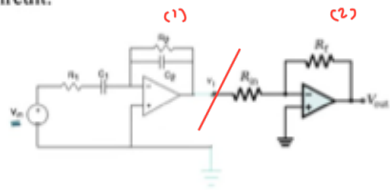
$$T.f_{(tot)} = T.f_{(1)} \cdot T.f_{(2)}$$

$$(1) : \frac{-R_2}{1 + R_2 C \cdot s}$$

$$(2) : \frac{-R_2}{R_{in}}$$

view (2) is wrong

Find out the transfer function of the following circuit.



$$(2) : \frac{-R_2}{R_{in}}$$

$$(1) : \frac{R_2}{1 + R_2 C \cdot s}$$

$$(R_1 + \frac{1}{C \cdot s})$$

Gear ratio :

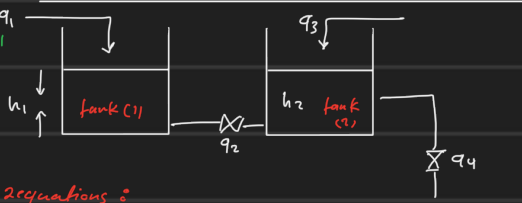
$$\frac{\# \text{ of teeth of input gear}}{\# \text{ of teeth of output gear}} = \frac{\text{Input Torque}}{\text{Output Torque}} = \frac{\text{out. p. speed}}{\text{inp. speed}}$$

مع (q) المقاومة خاصة فيزياء

Tanks :

$$q = \frac{h_{in} - h_{out}}{R_1} \quad \dots (1)$$

$$C \frac{dh}{dt} = q_{in} - q_{out} \quad \dots (2)$$



2 equations :

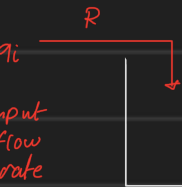
$$\text{Tank (1)} : C_1 \frac{dh_1}{dt} = q_1 - q_2 \quad \text{Step (1)}$$

$$\text{Tank (2)} : C_2 \frac{dh_2}{dt} = q_2 + q_3 - q_4$$

for (q) $\Rightarrow$ (R) :

$$R_1 = \frac{h_{in} - h_{out}}{q_1} = \frac{0 - h_1}{q_1}$$

$$R_2 = \frac{h_1 - h_2}{q_2}, \quad R_3 = \frac{-h_2}{q_3}, \quad R_4 = \frac{h_2}{q_4}$$



R : Resistance of valve

anti proportional relationship

$$q_0 \uparrow \quad R \downarrow$$

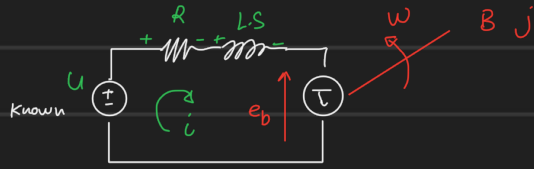
$$q_0 \propto \frac{1}{R}$$

$$q_0 \uparrow \quad H \uparrow$$

$$q_0 \propto H$$



*] DC motor :



$$-U + RI + LI + e_b = 0$$

$$T = K_t \cdot I, \quad I = \frac{T}{K_t}, \quad e_b = K_b \omega$$

$$T = J \dot{\omega} + B \omega$$

$$U = \frac{J s^2 + B s}{K_t} (R + L s) + (K_b \omega)$$

$$U = \omega \left(\frac{J L s^2 + (J R + B L) s + B R + K_b K_t}{K_t} \right)$$

$$\left(\frac{\omega}{U} = \frac{K_t}{J L s^2 + (J R + B L) s + B R + K_b K_t} \right)$$