chapter 9 + 10

$\Rightarrow \alpha = P(\text{type I error}) = P(\text{reject } H_0, H_0 \text{ True})$

$\Rightarrow \beta = P(\text{type II error}) = P(\text{Fail to reject } H_0, H_0 \text{ False})$

$\Rightarrow$ Power of test $= 1 - \beta$

$\Rightarrow$ Mean :-

$\Rightarrow$ Hypothesis test :-

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$ or $\mu < \mu_0$ or $\mu > \mu_0$

$\Rightarrow$ Test statistic :-

Three cases :-

- σ^2 known

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- σ^2 unknown, $n \geq 40$

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

- σ^2 unknown, $n < 40$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$\Rightarrow$ Confidence intervals :-

Three cases :-

- σ^2 known

$$P\left(\bar{x} - Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

- σ^2 unknown $n \geq 40$

$$P\left(\bar{x} - Z_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}\right) = 1 - \alpha$$

- σ^2 unknown $n < 40$

$$P\left(\bar{x} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}\right) = 1 - \alpha$$

$$\Rightarrow s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}$$

$\Rightarrow$ Variance :-

$\Rightarrow$ Hypothesis test :-

$H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 \neq \sigma_0^2$ or $\sigma^2 > \sigma_0^2$ or $\sigma^2 < \sigma_0^2$

$\Rightarrow$ Test statistic :-

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$\Rightarrow$ Confidence interval :-

$$P\left(\frac{(n-1)s^2}{\chi^2} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2}\right) = 1 - \alpha$$

$\Rightarrow$ proportion :-

$\Rightarrow$ Hypothesis test :-

$H_0: p = p_0$

$H_1: p \neq p_0$ or $p > p_0$ or $p < p_0$

$\Rightarrow$ Test statistic :-

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$\Rightarrow$ Confidence interval :-

$$P\left(\hat{p} - Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 1 - \alpha$$

$\Rightarrow$ If σ_1^2, σ_2^2 unknown, $\sigma_1^2 \neq \sigma_2^2$

$$\Rightarrow T_0^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\Rightarrow$ degrees of freedom :-

$$V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

always round down

$\Rightarrow$ If σ_1^2, σ_2^2 unknown, $\sigma_1^2 = \sigma_2^2$

$$\Rightarrow S_p^2 = \frac{(n_1 - 1)S_1^2 - (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$\Rightarrow T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p^2 \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\Rightarrow P\left((\bar{x}_1 - \bar{x}_2) - t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot S_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot S_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right) = 1 - \alpha$$

$\Rightarrow$ Degrees of freedom: $n_1 + n_2 - 2$

$\Rightarrow$ Paired T test :-

$\Rightarrow H_0: \mu_D = \Delta_0$

$H_1: \mu_D \neq \Delta_0$

$$\Rightarrow T_0 = \frac{\bar{d} - \Delta_0}{S_D / \sqrt{n}}$$

$$\Rightarrow P\left(\bar{d} - t_{\frac{\alpha}{2}, n-1} \cdot S_D/\sqrt{n} \leq \mu_D \leq \bar{d} + t_{\frac{\alpha}{2}, n-1} \cdot S_D/\sqrt{n}\right) = 1 - \alpha$$

$\Rightarrow$ F-Distribution :-

$\Rightarrow H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$, $\sigma_1^2 > \sigma_2^2$, $\sigma_1^2 < \sigma_2^2$

$\Rightarrow$ Test statistic :-

$$F_0 = \frac{S_1^2}{S_2^2}$$

Tables give upper tail $\alpha \leq 0.25$

$$F_{1-\alpha, n_1-1, n_2-1} = \frac{1}{F_{\alpha, n_2-1, n_1-1}}$$

$\Rightarrow$ confidence interval :-

$$P\left(\frac{S_1^2}{S_2^2} \cdot f_{1-\frac{\alpha}{2}, n_2-1, n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} \cdot f_{\frac{\alpha}{2}, n_2-1, n_1-1}\right)$$

Ratio: $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$

Goodness of Fit :-

⟹ Test statistic :-

$\chi_0^2 = \frac{(O_i - E_i)^2}{E_i}$

⟹ Binomial Distribution :-

- $P(X=x) = C^n_x \cdot p^x \cdot (1-p)^{n-x}$
- $E(X=x) = (\text{number of observations})(P)$

⟹ Poisson Distribution :-

- $P(X=x) = \frac{(\lambda t)^x}{x!} \cdot e^{-\lambda t}$
- $E(X=x) = (\text{number of observations})(P)$

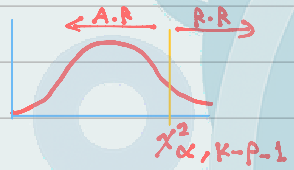
⟹ If $E(X=x) < 3$ ⟹ sum with previous $E(X=x)$

⟹ Degrees of freedom :-

$= k - p - 1$

- $p = 0$ ⟹ if all parameters given in question
- $p = 1$ ⟹ if any parameter calculated in question

⟹ only one side



Contengency Table Test :-

⟹ Test independence.

⟹ H_0: .... independent

H_1: .... not independent

⟹ Test statistic :-

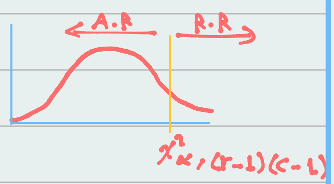
$\chi_0^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

⟹ $E_{ij} = \frac{(\text{row total})(\text{column total})}{\text{Grand total}}$

⟹ Degrees of freedom :-

$= (r-1)(c-1)$ r: row , c: column

⟹ only one side



⟹ $E = \frac{\sigma}{\sqrt{n}}$

⟹ $E = \frac{Z_{\alpha/2} \cdot \sigma}{\sqrt{n}}$

⟹ $E = Z_{\alpha/2} \cdot \sqrt{\frac{p(1-p)}{n}}$

Sample size all round up

Chapter 11

$\Rightarrow y = B_0 + B_1 x + \epsilon$

$\Rightarrow \hat{y} = \hat{B}_0 + \hat{B}_1 x$

$\Rightarrow S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$

$\Rightarrow S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$

$\Rightarrow S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$

$\Rightarrow \hat{B}_0 = \bar{y} - \hat{B}_1 \bar{x}$

$\Rightarrow \hat{B}_1 = \frac{S_{xy}}{S_{xx}}$

$\Rightarrow$ Coefficient of Determination $(R)^2$

$R^2 = \frac{SSreg}{SST} = 1 - \frac{SSE}{SST} \qquad 0 \le R^2 \le 1$

$\Rightarrow$ pearson corelation (r):-

$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} \qquad -1 \le r \le 1$

$\Rightarrow$ ANOVA Table:-

SOV	DF	SS	MS	F
reg	1	SSreg	$\frac{SSreg}{1}$	$\frac{MSreg}{MSE}$
E	n-2	SSE	$\frac{SSE}{n-2}$	
T	n-1	SST		

$\Rightarrow SSreg = \hat{B}_1 \cdot S_{xy}$

$\Rightarrow SS_T = S_{yy}$

$\Rightarrow SSE = SST - SSreg$

$\Rightarrow 1-R^2$: coefficient of randomness

$\Rightarrow \hat{\sigma}^2 = MSE = \frac{SSE}{n-2}$

$\Rightarrow H_0: B_1 = B_{1,0} \qquad T = \frac{\hat{B}_1 - B_{1,0}}{S.E(\hat{B}_1)}$

$H_1: B_1 \neq B_{1,0}$

$\Rightarrow H_0: B_1 = 0 \qquad T = \frac{\hat{B}_1 - 0}{S.E(\hat{B}_1)}$ significance of regression

$H_1: B_1 \neq 0$

$\Rightarrow H_0: B_0 = B_{0,0} \qquad T = \frac{\hat{B}_0 - B_{0,0}}{S.E(\hat{B}_0)}$

$H_1: B_0 \neq B_{0,0}$

$\Rightarrow H_0: B_0 = 0 \qquad T = \frac{\hat{B}_0 - 0}{S.E(\hat{B}_0)}$

$H_1: B_0 \neq 0$

$\Rightarrow S.E(\hat{B}_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$

$\Rightarrow S.E(\hat{B}_0) = \sqrt{\hat{\sigma}^2\left(\frac{1}{n} + \frac{(\bar{x})^2}{S_{xx}}\right)}$

$\Rightarrow P(\hat{B}_1 - t_{\frac{\alpha}{2}, n-2} \cdot S.E(\hat{B}_1) \le B_1 \le \hat{B}_1 + t_{\frac{\alpha}{2}, n-2} \cdot S.E(\hat{B}_1)) = 1 - \alpha$

$\Rightarrow P(\hat{B}_0 - t_{\frac{\alpha}{2}, n-2} \cdot S.E(\hat{B}_0) \le B_0 \le \hat{B}_0 + t_{\frac{\alpha}{2}, n-2} \cdot S.E(\hat{B}_0)) = 1 - \alpha$

$\Rightarrow P\left(\hat{\mu}_{y|x_0} - t_{\frac{\alpha}{2}, n-2} \cdot \sqrt{\hat{\sigma}^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)} \le \mu_{y|x_0} \le \hat{\mu}_{y|x_0} + t_{\frac{\alpha}{2}, n-2} \cdot \sqrt{\hat{\sigma}^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}\right) = 1 - \alpha$

Chapter 13

$\Rightarrow$ ANOVA Table:-

SOV	DF	SS	MS	F
Tr	a-1	SSTr	$\frac{SSTr}{a-1}$	$\frac{MSTr}{MSE}$
E	a(n-1)	SSE	$\frac{SSE}{a(n-1)}$	
T	an-1	SST		

$\Rightarrow SST = \sum_{i=1}^{a}\sum_{j=1}^{n} (y_{ij})^2 - \frac{(y_{..})^2}{an}$

$\Rightarrow SSTr = \frac{1}{n}\sum_{i=1}^{a} (y_{i.})^2 - \frac{(y_{..})^2}{an}$

$\Rightarrow SSE = SST - SSTr$

$\Rightarrow$ residual:-

$e = y - \bar{y}$

$\Rightarrow P\left(\bar{y} - t_{\frac{\alpha}{2}, a(n-1)} \cdot \sqrt{\frac{MSE}{n}} \le y \le \bar{y} + t_{\frac{\alpha}{2}, a(n-1)} \cdot \sqrt{\frac{MSE}{n}}\right) = 1 - \alpha$

$\Rightarrow P\left((\bar{y}_i - \bar{y}_j) - t_{\frac{\alpha}{2}, a(n-1)} \cdot \sqrt{\frac{2MSE}{n}} \le y_i - y_j \le (\bar{y}_i - \bar{y}_j) + t_{\frac{\alpha}{2}, a(n-1)} \cdot \sqrt{\frac{2MSE}{n}}\right) = 1 - \alpha$

$\Rightarrow$ ANOVA Table with blocks:-

SOV	DF	SS	MS	F
A	a-1	SSA	$\frac{SSA}{a-1}$	$\frac{MSA}{MSE}$
B	b-1	SSB	$\frac{SSB}{b-1}$	$\frac{MSB}{MSE}$
E	(a-1)(b-1)	SSE	$\frac{SSE}{(a-1)(b-1)}$	
T	ab-1	SST		

$B \rightarrow$ Block

$\Rightarrow SST = \sum_{i=1}^{a}\sum_{j=1}^{b} (y_{ij})^2 - \frac{(y_{..})^2}{ab}$

$\Rightarrow SSTr = \frac{1}{b}\sum_{i=1}^{a} (y_{i.})^2 - \frac{(y_{..})^2}{ab}$

$\Rightarrow SSB = \frac{1}{a}\sum_{j=1}^{b} (y_{.j})^2 - \frac{(y_{..})^2}{ab}$

$\Rightarrow SSE = SST - SSA - SSB$

$\Rightarrow$ LSD: least significance Difference

$\Rightarrow LSD = t_{\frac{\alpha}{2}, a(n-1)} \cdot \sqrt{\frac{2MSE}{n}}$

$\Rightarrow \bar{y}_{i.} - \bar{y}_{j.} < LSD \rightarrow$ difference

$\bar{y}_{i.} - \bar{y}_{j.} < LSD \rightarrow$ no difference

chapter 14

$\Rightarrow$ ANOVA Table of two factors:-

SOV	DF	SS	MS	F
A	a-1	SSA	$\frac{SSA}{a-1}$	$\frac{MSA}{MSE}$
B	b-1	SSB	$\frac{SSB}{b-1}$	$\frac{MSB}{MSE}$
AB	(a-1)(b-1)	SSAB	$\frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$
E	ab(n-1)	SSE	$\frac{SSE}{ab(n-1)}$	
T	abn-1	SST		

$\Rightarrow SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} (y_{ijk})^2 - \frac{(y_{...})^2}{abn}$

$\Rightarrow SSA = \frac{1}{bn}\sum_{i=1}^{a} (y_{i..})^2 - \frac{(y_{...})^2}{abn}$

$\Rightarrow SSB = \frac{1}{an}\sum_{j=1}^{b} (y_{.j.})^2 - \frac{(y_{...})^2}{abn}$

$\Rightarrow SSAB = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} (y_{ij.})^2 - \frac{(y_{...})^2}{abn} - SSA - SSB$

$\Rightarrow SSE = SST - SSA - SSB - SSAB$