Subject : Chapter 1

* linear system :-

$$x_1 + 4x_2 + x_3 = 5$$
$$x_1 + x_2 + 5x_3 = 12$$
$$3x_1 + x_2 + x_3 = 8$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 1 & 1 & 5 & 12 \\ 3 & 1 & 1 & 8 \end{array}\right] \rightarrow \text{Augmented matrix}$$

here we must have leader with value 1

must be zero

- To solve this system :- (using elementary row Operation)

① Make a leader with value 1, then make zeros under it :-

$-R_1 + R_2$:-
$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 0 & -3 & 4 & 7 \\ 3 & 1 & 1 & 8 \end{array}\right]$$

$-3R_1 + R_3$:-
$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 0 & -3 & 4 & 7 \\ 0 & -11 & -2 & -7 \end{array}\right] \rightarrow \frac{R_2}{-3} : \left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 0 & 1 & \frac{-4}{3} & \frac{-7}{3} \\ 0 & -11 & -2 & -7 \end{array}\right]$$

make it leader

must be zero under the leader

$11R_2 + R_3$:-
$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 0 & 1 & \frac{-4}{3} & \frac{-7}{3} \\ 0 & 0 & \frac{-50}{3} & \frac{-98}{3} \end{array}\right] \rightarrow \frac{-3}{50}R_3 : \left[\begin{array}{ccc|c} 1 & 4 & 1 & 5 \\ 0 & 1 & \frac{-4}{3} & \frac{-7}{3} \\ 0 & 0 & 1 & \frac{98}{50} \end{array}\right]$$

make leader

② Substitute :-

$$x_1 + 4x_2 + x_3 = 5$$
$$x_2 - \frac{4}{3}x_3 = -\frac{7}{3}$$
$$x_3 = \frac{98}{50}$$

③ then find the other variables.

Subject : .... / /

* Echelon forms :-

The first non-zero element must be a leader.

① Reduced row-Echelon form :-

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

- if we have a leader in a column, then all of ~~the~~ other element in same column must be zeros.

② Row - Echelon form :-

Same of Reduce... but without zeros in leaders columns, just up to leaders.

* Back substitution is to return the final matrix into equations.

* Gaussian elemination :- to make the matrix in row echelon form.

* Gaussian Jordan elimination :- to make the matrix in reduced row echelon form.

---

* Homo form :- the constant terms must be zero.

- in back substitution if there are free variables then assume a value for them like: t, s, w...

* $A_{i \times j}$, i : the row j : the column

* If we multiply a matrix by a constant, then all elements will be multiplied by this constant.

* $A_{m\times r}\ B_{r\times n} = (AB)_{m\times n}$

- # of columns in first matrix must be same of # of rows in second matrix to confirm multiplication.

- Ex.

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1*4+2*0+4*2 & \cdots \\ 2*4+6*0+0*2 & \end{bmatrix} \longrightarrow \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$

- Note: $A_{r\times n} * B_{m\times q} * \cdots D_{w\times y} = S_{r\times y}$ important !!

- $A_{r\times n}$, $A^T \longrightarrow A_{n\times r}$, T: transpose

- Trace of A: if the matrix is only **square** then tr(A) is the sum of entries on the main diagonal of A.

Ex.

$$A = \begin{bmatrix} 1 & 7 & 10 \\ 2 & -1 & 2 \\ 5 & 0 & 4 \end{bmatrix} \longrightarrow tr(A) = 4$$

- $AB \neq BA$

- Zero matrix: a matrix all of whose enteries are zero.

Subject : .... / /

Identity matrix :- (I_n)

a square matrix with 1 in the main diagonal with zero at remaining enteries.

- Ex.

$$I_3 : \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_2 : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Invertible:

$$AA^{-1} = A^{-1}A = I$$

- if there is no inverse for A then it's called singular matrix.
- not every squared matrix has an inverse.
- if we have a squared matrix then:-

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ to have inverse it must ~~be~~ ~~aei - ceg ≠ 0~~ ~~called determinant~~ be determinat $\neq 0$

- to find the inverse of a squared matrix:-

$$\begin{bmatrix} 3 & 1 & 5 \\ 2 & 4 & -1 \\ 7 & 1 & 2 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} 3 & 1 & 5 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ 7 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

then make it Reduced-Row-Echelon-Form

- the matrix has one inverse only.
- The matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \rightarrow ad - bc : \text{determinant}$$

- $\det(AB) = \det(A) \cdot \det(B)$ $\{$ - $I \cdot I = I$
- $(AB)^{-1} = B^{-1}A^{-1}$

- Making any row operation in an identity matrix will make it called an elementary matrix.

- Ex.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Then: } E_2(4) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3(-1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(4) ↓ can't be zero

$$E_{31} = E_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad 2R_3 + R_2 = E_{23}(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Powers of matrix :-

- $A^0 = I$ — $A^n = AAAA\ldots A$ (n times) — $A^{-n} = A^{-1}A^{-1}A^{-1}\ldots A^{-1}$ (n times)
- $A^r A^s = A^{r+s}$ — $(A^r)^s = A^{rs}$ — $(A^{-1})^{-1} = A$

- polynomial expressions :-

$$p(x) = a_n x^n + a_{n-1} x^{n-1} \ldots + a_0 x^0$$

$$p(A) = a_n A^n + a_{n-1} A^{n-1} \ldots + a_0 A^0$$

- Ex.

$$p(x) = 4x^2 + 5x + 7, \quad A = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} \quad \Big\{ \quad A^2 = A.A = \begin{bmatrix} 16 & 10 \\ 0 & 1 \end{bmatrix}$$

$$p(A) = 4 \begin{bmatrix} 16 & 10 \\ 0 & 1 \end{bmatrix} + 5 \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ continues...

- if we have an elementary matrix E and we need EA, then the result will be a matrix that the same row operation of E applying to A.

- Ex.

$$E_{23}(4), \quad EA = A_{23}(4) \longrightarrow 4R_3 + R_2$$

- all of elementary matrices are invertable, and the inverse will be also elementary.

- If A is invertable and $Ax=b$, $A^{-1}Ax=A^{-1}b$, $x=A^{-1}b$ is a matrix with only one solution. (NON-homo form)

- inverse operations:-

(a) $E_{ij}^{-1} = E_{ji}$

(b) $E_i(c)^{-1} = E_i(\frac{1}{c})$ with $c \neq 0$

(c) $E_{ij}(c)^{-1} = E_{ij}(-c)$ with $i \neq j$

- $E_k \dots E_2 E_1 A = I_n$ then $A^{-1} = E_k \dots E_2 E_1 I_n$

- trivial solution: $x_1 = 0$ $x_2 = 0$ $x_3 = 0$

- if we have $Ax = b_1$ $Ax = b_2$ then $[A | b_1 | b_2]$ then solve the system by Gaussian elemination to solve these two systems, then by back-substitution.

- when we obtain $0 = b$ then this system is inconsistant. $(b \neq 0)$ and then there is no answer.

- when we have an Augmented invertable matrix then it will have only one solution, also for a homo invertable system then it will have only one solution, and consistant for both.

- if AB is invertable, then A, B are both invertable (they must be squared with same size.

Subject :

- Q (important) $\quad 4x_1 + a x_2 = b$

$$c x_1 + d x_2 = e$$

① consistant 1 sol	② consistant ∞ sol	③ inconsistant
$4x_1 + 3x_2 = 0$	$4x_1 + a x_2 = b$	Same of ② but $e \neq b$
$0x_1 + x_2 = 0$	$4x_1 + a x_2 = b$	
a, b can be any values		

- if we can make a matrix as identity then its invertable.

⊛ Diagonal matrix : the main diagonal with any values, the other values must be zeros.

- D^2 = the same matrix with the $(\text{main diagonal})^2$, same for other powers.
- the diagonal matrix is invertable only if the values in the main diagonal are non zeros. (all of them)
- D^{-1} = the same matrix with the reciprocals to all values in the main diagonal.

⊛ Triangular matrix : Lower and Upper.

$$L: \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 \end{bmatrix} \qquad U: \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{bmatrix}$$

- unit triangular matrix : the determinant = 1 (the enteries at the main diagonal are 1)
- $L^T = U$ $\quad$ - $U^T = L$ $\quad$ / $U * U = U$ $\quad$ - $L * L = L$
- L, U are invertable if the enteries in the main diagonal $\neq$ zero.
- $L^{-1} = L$ $\quad$ - $U^{-1} = U$

Subject : .... / /

Symmetric matrix : a square matrix A for which $A^T = A$, so that $\langle A\rangle_{ij} = \langle A\rangle_{ji}$

- Ex. $\begin{bmatrix} 7 & -3 \\ -3 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$

- if A and B are symmetric with same size, then:

  A^T is symmetric

  $A + B$ and $A - B$ are symmetric

  kA is symmetric (k is constant)

- the product of two symmetric matrices is symmetric if only $AB = BA$
- if A is invertable symmetric, then A^{-1} is symmetric
- AA^T and A^TA are always symmetric.
- Let A be 2×3 matrix, then AA^T and A^TA are symmetric
- If A is invertable matrix, then AA^T and A^TA are invertable

Minor and Cofactor :

- The minor M_{ij} for any matrix means to delet the i row and j coloumn, then find the determinant

- Ex $\begin{bmatrix} 1 & 3 & 0 \\ 5 & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix}$ $M_{23} = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$ then $M_{23} = 4$

- The cofactor C_{ij} is $(-1)^{i+j} * M_{ij})$

in the previous example:-

$C_{23} = (-1)^{2+3} * M_{23} = -1 * 4 = -4$

- The determenant for 3x3 matrix:

Ex. $\begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & 2 \end{bmatrix}$, $\det = 3 \begin{bmatrix} -4 & 3 \\ 4 & 2 \end{bmatrix} - 1 \begin{bmatrix} -2 & 3 \\ 5 & 2 \end{bmatrix} + 0 \begin{bmatrix} -2 & -4 \\ 5 & 4 \end{bmatrix}$

$= 3(-4(2) - 4(3)) - 1($ ... continue as we know.

- The determinant for 4x4 matrix:

Ex. $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 3 & 1 & 2 & 2 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & 0 & 1 \end{bmatrix}$, $\det = 1 \begin{vmatrix} 1 & 2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 3 & 2 & 2 \\ 1 & -2 & 1 \\ 2 & 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{vmatrix}$

$- (-1) \begin{vmatrix} 3 & 1 & 2 \\ 1 & 0 & -2 \\ 2 & 0 & 0 \end{vmatrix}$ The signs are extremely important

So, The ~~det = C11 + C12 + C13 + C14~~

$\det = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} + a_{14} C_{14}$

- Adjoint of a matrix

if A in $n \times n$ matrix, then the matrix:

$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}$$

is called the matrix of cofators from A, and the transpose of this matrix is called the adjoint of A: $\text{adj}(A)$

- $A^{-1} = \dfrac{1}{\det(A)} \text{adj}(A)$

- If A is a square triangular matrix (upper or lower or diagonal) then $\det(A)$ is the product of the enteries in the main diagonal.

- If $Ax = b$ then $x_1 = \dfrac{\det A_1}{\det A}$, $x_2 = \dfrac{\det A_2}{\det A}$, $x_3 = \dfrac{\det A_3}{\det A}$

Ex.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 30 \\ 8 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix}$$

then find $x_1 = \dfrac{\det A_1}{\det A} \cdots$

- This method ~~called~~ is called (Cramer's Rule)

Subject : Chapter 3

Vectors

- linear combination:

if we have vectors $V_1, V_2, V_3, \dots$ then:

$W = k_1 V_1 + k_2 V_2 + k_3 V_3 + \dots + k_r V_r$

Ex.

$V_1 = \langle 3, 4, 5 \rangle$ $\quad V_2 = \langle 1, -1, 0 \rangle$ $\quad V_3 = \langle 2, -1, 1 \rangle$

$W = 3V_1 + 5V_2 + 0V_3$

$\rightarrow W = \langle 3*3 + 5*1 + 0*2,\ 3*4 + 5*-1 + 0*-1,\ \dots \rangle$

- Norm (magnitude)

$V = \langle a, b, c \rangle \rightarrow |V| = \sqrt{a^2 + b^2 + c^2}$

unit vector $u = \dfrac{V}{|V|}$

- $\langle 2, -3, 4 \rangle \rightarrow 2i - 3j + 4k$

$\langle 1, -2, 5, 3 \rangle \rightarrow e_1 - 2e_2 + 5e_3 + 3e_4$

- if we have: $W = \langle 4, 5, 7 \rangle$, $\quad V_1 = \langle 3, 4, 5 \rangle$

$V_2 = \langle 1, -1, 0 \rangle$

$V_3 = \langle 2, 1, 4 \rangle$

then:

$$\boxed{\begin{aligned} 3c_1 + c_2 + 2c_3 &= 4 \\ 4c_1 - c_2 + c_3 &= 5 \\ 5c_1 + 0c_2 + 4c_3 &= 7 \end{aligned}}$$

- Note: we can write W as linear eqs if the matrix is in vertable and $W = \langle a, b, a+b \rangle$

- If we have ~~vectors~~ Points v, u then the distance

$$d(v,u) = \sqrt{(v_1-u_1)^2 + (v_2-u_2)^2 + \dots + (v_n-u_n)^2}$$

- Dot product

$u = \langle u_1, u_2, u_3 \rangle$ $\quad$ $v = \langle v_1, v_2, v_3 \rangle$

- $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $u \cdot v = |u|\,|v| \cos\theta \longrightarrow \cos\theta = \dfrac{u \cdot v}{|u|\,|v|}$

- Standard unit vector:

① 3D:

$i \to \langle 1,0,0 \rangle$, $j \to \langle 0,1,0 \rangle$, $k \to \langle 0,0,1 \rangle$

② 4D:

$u_1 \to \langle 1,0,0,0 \rangle$, $u_2 \to \langle 0,1,0,0 \rangle$

$u_3 \to \langle 0,0,1,0 \rangle$, $u_4 \to \langle 0,0,0,1 \rangle$

- in cube:

$D = \langle d, d, d \rangle$, $\quad E = \langle d, 0, 0 \rangle$

ⓐ $D \cdot E = d^2$ $\quad$ ⓑ $|D| = \sqrt{3}\, d$ $\quad$ ⓒ $|E| = d$

- $u \cdot u = |u|^2$

$|v+u|^2 = |v|^2 + 2\, v \cdot u + |u|^2$

- Orthognality: vectors u and v are said to be orthognal if $u \cdot v = 0$.

- If we have ~~vectors~~ Points V, u then the distance

$$d(V,u) = \sqrt{(V_1-u_1)^2 + (V_2-u_2)^2 + \dots + (V_n-u_n)^2}$$

- Dot product

$u = \langle u_1, u_2, u_3 \rangle$ $V = \langle V_1, V_2, V_3 \rangle$

- $u \cdot V = u_1V_1 + u_2V_2 + u_3V_3$
- $u \cdot V = |u|\,|V| \cos\theta \longrightarrow \cos\theta = \dfrac{u \cdot V}{|u|\,|V|}$

- standard unit vector:

① 3D:

$i \to \langle 1,0,0 \rangle$, $j \to \langle 0,1,0 \rangle$, $k \to \langle 0,0,1 \rangle$

② 4D:

$u_1 \longrightarrow \langle 1,0,0,0 \rangle$, $u_2 \to \langle 0,1,0,0 \rangle$

$u_3 \longrightarrow \langle 0,0,1,0 \rangle$, $u_4 \to \langle 0,0,0,1 \rangle$

- in cube:

$D = \langle d,d,d \rangle$, $E = \langle d,0,0 \rangle$

ⓐ $D \cdot E = d^2$ ⓑ $|D| = \sqrt{3}\,d$ ⓒ $|E| = d$

- $u \cdot u = |u|^2$

$|V+u|^2 = |V|^2 + 2\,V \cdot u + |u|^2$

- Orthognality: vectors u and v are said to be orthogonal if $u \cdot v = 0$

* projection: if we have vectors u, a then:

$\text{Proj}_a u$ is the vector component of u along a.

$$\text{Proj}_a u = \frac{u \cdot a}{|a|} a$$

- The vector component of u orto to $a = u - \text{proj}_a u$
- If u, v are orthogonal then: $|u+v|^2 = |u|^2 + |v|^2$

* Point-line and point-plane distance
- In R^2 the distance between point $P_0(x_0, y_0)$ and the line $ax + by + c = 0$:

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

- I R^3 the distance between point $P_0(x_0, y_0, z_0)$ and the plane $ax + by + cz + d = 0$:

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

- Eq of a line through point x_0 and parallel to the non-zero vector V is: $x = x_0 + tV$
- Eq of a plane through point x_0 and parallel to non-colinear vectors V_1 and V_2 is: $x = x_0 + t_1 V_1 + t_2 V_2$

if $x_0 = 0$ then the plane passes through origin

Subject :

Cross product :

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$ continue...

- $u \times v$ is Orthogonal to u and v
- $|u \times v|^2 = |u|^2 |v|^2 - (u \cdot v)^2$
- $u \times (v \times w) = (u \cdot w) v - (u \cdot v) w$

  $(u \times v) \times w = (u \cdot w) v - (v \cdot w) u$
- $|u \times v| = |u| \, |v| \sin \theta$

Subject: Chapter 4

Vector space:

- The axioms in slide 4 are very important
- The triangular matrix isn't in vector space if we take upper + lower: $u + L \neq$ triangular.
- Identity matrices aren't are not in vector space: $I + I \neq I$
- Diagonal matrix is in Vector space.
- we can consider any object is in vector space if all axioms met.
- The last 5 axioms are scalar * object.
- 2D vectors and 3D are in vector space.
- functions are in vector space
- polynomials are in vector space
- integers are not because axiom 5 doesn't apply.
- R^n: vectors in n space.
- If we test lines and planes check $(u + v)$

  Ex. 2 lines with intercept 5 are not V.S because if we combine them the intercept will be 10.
- Every plane through origin in a vector space.

Subject : .... / /

- Subspaces : A subset W of a vector space V is called a subspace of V (if W is vector space)

- Ex. a squared matrix is vector space → upper triangular matrix is subspace of squared matrix.

- We can consider an object is a subspace if only it is a vector space.

- Homo system → 4 eqs , 5 var → Sol : 5D $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

- Solutions of homo system are subspaces with respect to their space R^n

- This two points aren't working in non homo sys $AX = b$.

Subject : .... / /

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- Solutions of homo system are subspaces with respect to their space R^n

- This two points aren't working in non homo sys $AX = b$.

- Zero object (number, matrix, vector) is vector space.

- integers are not vector space because we can multiply them by a number that make them not integers ($\frac{1}{2}$, ...)

- homo matrix with ind eqs = # variables is vector space because it will have only trivial Solution and it is a zero object (the trivial Solution)

Subject : ....

- $R = \langle 4, 0, 2 \rangle$, $V_1 = \langle 4, 5, 7 \rangle$ $V_2 = \langle 1, 2, 3 \rangle$ $V_3 = \langle 1, 0, 2 \rangle$

write them as linear system and solve it, if it is consistant then we can write R as linear system, so if the coefficient matrix is invertable then it is consistant.
if its not invertable then solve it by Gaussian elemenation or Gaussian Jordan elemenation to test the consistancy.

- if # of eqs > # of variables (over determin system) then its inconsistant.

- two non colinear vectors pass in (0,0) will give us plane pass in (0,0)
$S = k_1 V_1 + k_2 V_2$

- if $W = k_1 V_1 + k_2 V_2 + k_3 V_3 + \dots$ then $W = \text{span}\{V_1, V_2, V_3, \dots\}$

▸ Subject : .... / /

① Linear Independence

- To determine wheather the vectors are dependent or not:

① find the determinant, if det = 0 so not invertable and dependent, otherwise its independent.

② Solve the system by Gaussian elemenation, So if we obtain 0 = 0, that means the system is dependent. (solve in homo form)

- To test the dependancy between objects → multiply by a scalar like:

if we have x , sin x are they dependent or not?

• lets multiply sin x by a scalar as example, can we obtain x by multipling sin x by a scalar?

→ of course no, so they are independent.

- The standard vectors are independent because they will make an identity matrix which is independent.

- In set of two or more vectors, they are linearly dependent if and only if at least one of the vectors is expressible as a linear combination of the other vectors, in the other hand if there no vectors expressible as a linear combination of the other vectors then they are linearly independent.

(check Slide 43)

Subject :

- A finite set of vectors that contains the zero vector is linearly dependent.

- A set with exactly two vectors is linearly independent if only neither vector is a scalar multiple of the other.

- In R^2 and R^3, a set of two vectors is linearly independent if only the vectors dont lie on the same line when they are placed with their initial points at the origin.

- In R^3, a set of 3 vectors is linearly independent if only the vectors dont lie in the same plane when they are placed with their initial points at the origin.

- if we have $S = \{V_1, V_2, \dots, V_r\}$ a set of vectors in R^n and $r > n$ (# of vectors > dimention) then S is linearly dependent.

Subject : .... / /

- Non rectangular coordination system $\rightarrow \angle\theta$

θ = any value except 90°

Basis

- if we have set of vectors $S = \{v_1, v_2, v_3 \dots v_n\}$ in vectorspace V, the we can consider S as ~~the~~ basis of V if:-

1. S is linearly independent.
2. S spans V.

- if we have n objects in R^n, to check if they are basis find the determinant to check the independency, and because # of objects = the dimension of vector space R^n so the two conditions will occur together.

- if # of objects > dimension of the vector space $\rightarrow$ dependent
  if # of objects < dimension of the vector space $\rightarrow$ we cant span them

- Standard basis for polynomials :- $(a_0, a_1x^1, a_2x^2, a_3x^3, \dots, a_nx^n)$

- Dimension :-

$\dim(R^n) = n$, $\dim(P_n) = n+1$, $\dim(M_{mn}) = mn$

- if we get:

$x_1 = -s-t$, $x_2 = s$, $x_3 = -t$, $x_4 = 0$, $x_5 = t$

$$\rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \underbrace{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{\text{The basis for the homo solution}}$$

- dim = 2 because two objects.

(they span the solution, not respect to R^5)

- Plus theorem

Ex. R_3 , $\{V_1, V_2\} + \{V_3\}$

if V_1, V_2 are independent and we add V_3 to the system and still independent, so we can add V_3.

- Minus theorem

- Ex. R_3 , $S\{V_1, V_2, V_3, V_4\} - \{V_4\}$

if V_4 express alinear combination of the other vectors so S and $S-\{v\}$ span the same space $\rightarrow$ $span(S) = span(S-\{v\})$

$V_1(2,4,7)$, $V_2(3,0,1)$, $V_3(4,0,10)$, $V_4(5,7,6)$

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 0 & 0 \\ 7 & 1 & 10 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix}$$

Solve, if it is consistant then you can write V_4 as span of the other vectors (find the determinant in this example)