



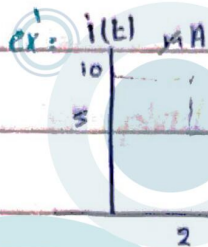
Electrical



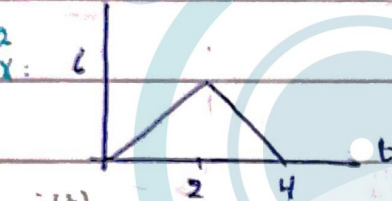
Lecture 1

• charge, current (I), $\bar{e} = 1.6 \times 10^{-19} \text{ C}$, $i = \frac{dq}{dt}$

$$Q = \int_{t_0}^{t_1} i \, dt$$

Find Q $t = 0 \rightarrow 2 \text{ s}$

$$Q = \int_0^2 i(t) \, dt = \text{area} = 20 \text{ C}$$

 $Q(t)$ ex: Q Find $i(t)$ & sketch itcurrent wave form $\rightarrow i(t)$

3 ex: $Q(t) = 10 - 10e^{-2t}$

Find $i(t)$ when $t = .5 \text{ sec}$

$$i(t) = \frac{dq}{dt} = -10e^{-2t} \times -2 = 20e^{-2t} \quad i(.5) = 90e^{-1} = 7.36 \text{ A}$$

ex4: $i(t) = 3t^2 - t \text{ A}$

Find Q $t=1$ Q $t=2$

$$q = \left. \frac{t^3}{3} - \frac{t^2}{2} \right|_1^2 = 5.5 \text{ C}$$

• voltages work per time, $V_A \rightarrow$ voltage of point A

$V_B \rightarrow$ " " " " B

charge 1 C $\rightarrow \infty$

* the work needed to bring a charge of 1 C to point a

from ∞

$$* J = Ws$$

$$i(t) = \frac{dq}{dt} \Leftrightarrow Q = \int i(t) dt$$

$$V = \frac{dW}{dq} \text{ (work)} \quad V = \frac{W}{Q} \text{ (energy)}$$

$$P = \frac{dW}{dt} \Rightarrow W = \int P dt \quad W = Pt$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$V = IR$$

$$V = \frac{dW}{dq} \quad (W \rightarrow \text{work}, q \rightarrow \text{charge})$$

ex $V_b = 3V$
 $V_a = 5V$ (5) Joule needed to bring 1C From ∞ to a)

$$V_{ab} = V_a - V_b \rightarrow V_{ab} = 2V$$

$$V_c = -4V \quad (4 \text{ Joule needed to bring 1C From } c \text{ to } \infty)$$

$$V_{ac} = 9V$$

watt
 • Power & energy
 (P)

$$P = \frac{dW}{dt} \quad \text{energy}$$

$$W = \int P dt$$

ex: Find to move charge From a to b the work needed

-30 J Find V if $q = 2C$. $a \rightarrow b$

$$V_{ab} = \frac{W}{Q} = \frac{-30}{2} = -15V$$

$$V_b > V_a$$

ex: Find the Power delivered to element at $t = 3ms$

if the current = $5 \cos 60\pi t$ & $v = 3i$.

$$P = IV \quad P = 5 \cos 60\pi t * 3(5 \cos 60\pi t) = 75 \cos^2 60\pi t$$

$$P|_{3 \times 10^{-3}} = 75 * \cos^2 60\pi (3 \times 10^{-3}) = 74.99 \text{ W (calculator in radian)}$$

ex: 100W bulb Per 2h, how much energy?

$$W = Pt \quad W = 100 * 2 * 3600 = 72 * 10^4 \text{ J}$$

Polarity $\leftarrow \pm$

Circuit elements

- power source [Produce - Power]
- load [Consume the Power]
- connection

ex: $3V$, $4V$... Ind. Voltage Source
independent Power Source $\oplus$

DC $\frac{+}{-}$

(continuous) $3V$ $\rightarrow$ t

direct current
portable → car charge
replacable mobile battery
remote control

ex: $2A$
 $5A$...

① ind. DC current source

∴ not a physical source

dependent Power Source (supply)

DC

dep. voltage source ex: $3i_x$
 $4V_y$
 $5V_z$

① dep. current source ex: $5V_y A$

Summary

Power supply

current

voltage

dep

indep

dep

indep



$3V_x$



$3A$

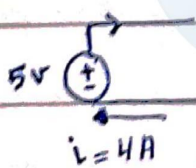


$3i_z$

$5V$



* For any electrical circuit (CKE) $\sum P = 0 \rightarrow \sum P_{in} = \sum P_{out}$



$$P = iV$$

∴ produce power

$$4 \times 5 = 20 \text{ watt}$$

الطاقة المنتجة = 20 واط



$$P = iV$$

Consume power

$$i = -3A$$

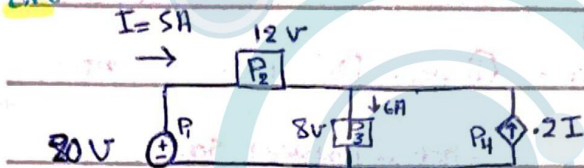
$$= -3 \times 5 = -15W$$

من البطارية

من داخل + البطارية - طاقة مستهلكة

lec 28

ex 8

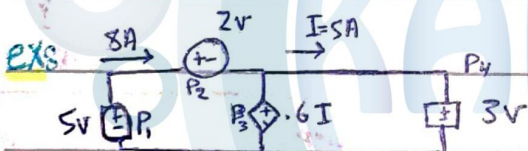


$$P_1 = -5 \times 20 = -100W \text{ (produce \& deliver)}$$

$$P_2 = 5 \times 12 = 60W \text{ (consumed)}$$

$$P_3 = 6 \times 8 = 48W \text{ (consumed)}$$

$$P_4 = -8W \quad \sum P = 0$$



$$P_1 = -5 \times 8 = -40W \text{ (delivered)}$$

$$P_2 = 8 \times 2 = 16W$$

$$P_4 = 5 \times 3 = 15W$$

$$P_3 = +9W$$

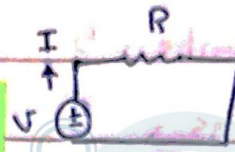
$$I = .6 \times \frac{1}{5} = 3A$$

* any long wire does not contain any element we consider it as point.

Basic concepts:

$$R = \frac{V}{I}, G = \frac{I}{V}$$

$$R = \frac{\rho L}{A}$$



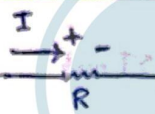
current will pass

(current from positive polar →)



$I = 0$ open circuit

ohm's law



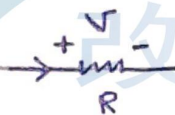
أي مقاومة يمر فيها تيار كهربائي يكون جود كهربائي

$$V = IR$$

$$P = IV$$

$$P = I(IR) = I^2 R$$

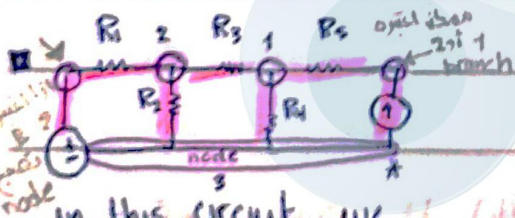
$$= \frac{V^2}{R}$$



where current enter, this polar will be positive

conductance $G = \frac{1}{R}$ ($S = \Omega^{-1}$)

$V = IR \Rightarrow R = \frac{V}{I}$, $G = \frac{I}{V}$ (↑ I ↑ V, R is a property) const.



in this circuit we

have 7 elements, 3 nodes

6 loops and 3 meshes

1) elements voltage or current source, R

2) Branches أي جزء في الدارة يتكون من عنصرين أو أكثر

3) Nodes أي نقطة اتصال بين العناصر

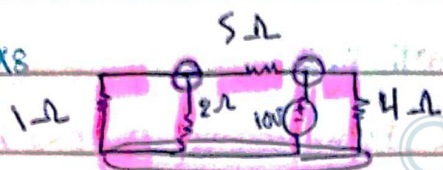
4) loops any closed path

5) Meshes any closed path but does not exist any other closed path.

* each mesh is loop

Sec 4

exs



Elements $\rightarrow 5$, Nodes $\rightarrow 3$

Branches $\rightarrow 5$, loops $\rightarrow 6$

Meshs $\rightarrow 3$

1. Kirchoff's current law KCL

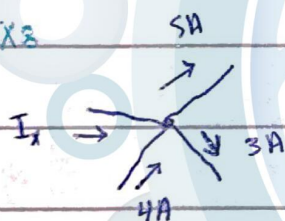
2. Kirchoff's voltage law KVL

① KCL : For any node $\sum I = 0 \rightarrow \sum I_{in} = \sum I_{out}$



$$I_x = I_1 - I_2 - I_3$$

exs



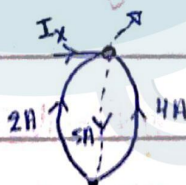
$$I_x + 5 = 3 + 4 \Rightarrow I_x = 2 \text{ A}$$

exs



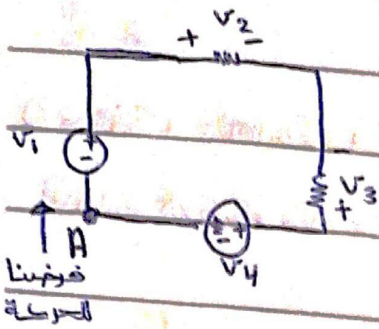
$$I_x + 2 + 4 = 5 \Rightarrow I_x = -1 \text{ A}$$

2 nodes



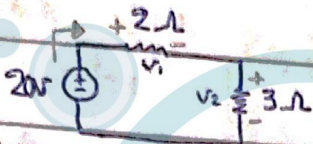
② KVL : For any loop (any closed path) the sum $\sum V = 0$

$$\sum V = 0$$



$$-V_1 + V_2 + V_4 - V_3 = 0$$

EX8



Find V_1, V_2, I

1) assign the current

2) assign the Polarity of each voltage [R]

3) apply KVL $\sum V = 0$

$$-20 + V_1 + V_2 = 0$$

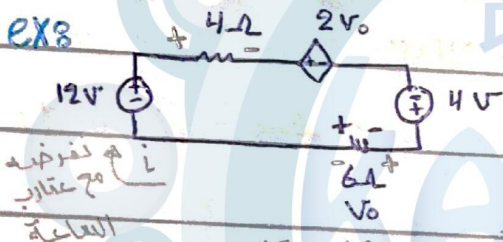
$$I = 4A$$

⊕ مع فرضنا

$$V_1 = 2 \times 4 = 8V, V_2 = 3 \times 4 = 12V$$

lec 4 1

EX8



Find V_0, i

• بفرض الاتجاه حسب اتجاه التيار الذي فرضناه

$$-12 + 4i + 2V_0 - 4 + 6i = 0 \quad i = -8A$$

$$V_0 = 6i$$

$$V_0 = 48V = -6 \times 8$$

بالسارعة

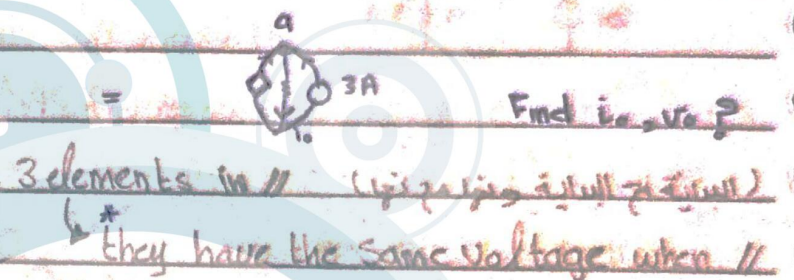
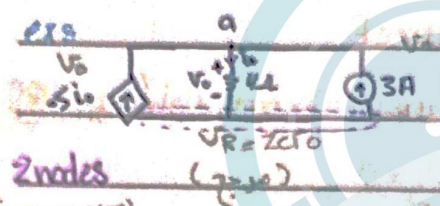
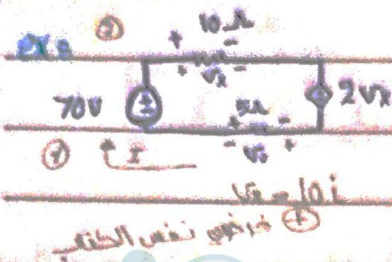
حل المسألة بالرجوع لفرضنا (عكس عقارب الساعة)

$$12 + 6i + 4 - 2V_0 + 4i = 0, V_0 = 6i$$

$$i = 8A, V_0 = 8 \times 6 = 48V$$

$$V_{4\Omega} = -8 \times 4 = -32V$$

Same V in //, Same I in series



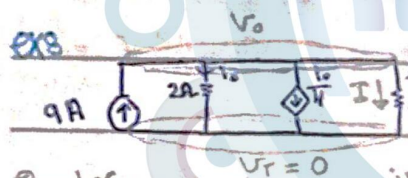
KCL (when in //)

$5i + 3 = i_o \rightarrow i_o = 6A$

Ohm's law

$V_o = IR = 6 \times 4 = 24V$

($Z_{do} = V_{ref}$ أن بنه أن)



KCL

$9 = i_o + i_o + I$

Ohm's law

$i_o = V_o \rightarrow i_o = V_o$

$9 = \frac{V_o}{2} + \frac{V_o}{8} + \frac{V_o}{8}$

$i_o = 6A, V_o = 12V$

$V_x = -V_o$

$I = 3A$

$V_x = -12V$

lec 5

• series elements have same current

$$R_{eq} = R_1 + R_2 + R_3 + \dots ; V_1 = \frac{V R_1}{R_1 + R_2}, V_2 = \frac{R_2}{R_1 + R_2} \cdot V$$

ex 9

Req? (أما في المثالين) b / a في المثالين

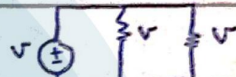
① series with ② : $1 + 5 = 6 \Omega$ ④

③ Parallel with ④ : $\frac{3 \times 6}{3 + 6} = 2 \Omega$ ⑤ ⑥ series with ③ : $2 + 2 = 4 \Omega$ ⑦

② Parallel with ③ : $\frac{4 \times 6}{4 + 6} = 2.4 \Omega$ ④ series with ⑥ & ⑦ : $4 + 2.4 + 8 = 14.4 \Omega = R_{eq}$

• Parallel elements have same voltage

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$



IF we have only 2 R we use $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$$i_1 = \frac{R_2}{R_1 + R_2} i, i_2 = \frac{R_1}{R_1 + R_2} i$$

ex 8

Req? ① & ② & ③ in series : $4 + 5 + 3 = 12 \Omega$ ④

② & ④ Parallel (||) : $\frac{12 \times 4}{12 + 4} = 3 \Omega$ ⑤

⑥ & ⑤ series : $3 + 3 = 6 \Omega$ ⑥

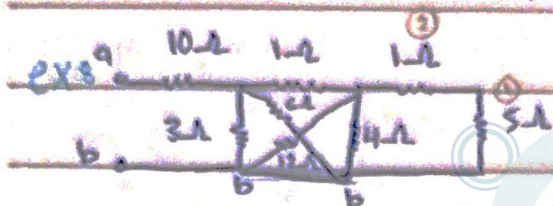
⑥ || ⑤ : $\frac{6 \times 6}{6 + 6} = 3 \Omega$ ⑦

④ & ⑦ series : $3 + 3 + 4 = 10 \Omega = R_{eq}$

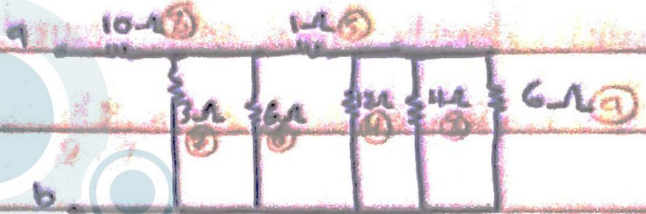
تكون المقاومة $\frac{1}{2}$ من المقاومة الأصلية

$$+G = \frac{1}{P}$$

Find Req?



① & ② in series $= 5 + 1 = 6\Omega$ ④



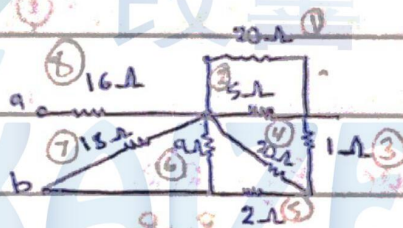
④ || ② $\frac{9 \times 4}{10} = 3.6\Omega$ ⑤

④ || ④ $\frac{12 \times 3.6}{14.4} = 3\Omega$ ⑥

④ series with ⑤ $= 1 + 2 = 3\Omega$ ⑦ • ⑥ || ⑥ $\frac{6 \times 3}{9} = 2\Omega$ ⑧

⑦ || ⑦ $\frac{2 \times 3}{5} = 1.2\Omega$ ⑨ • ⑧ series with ⑨ $= 1.2 + 10 = 11.2\Omega$ Req

Practice Problem 9



Find Req?

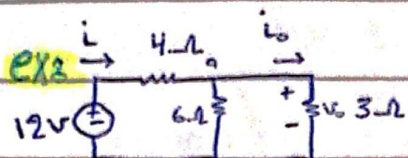
① || ② $\frac{20 \times 5}{25} = 4\Omega$ ③

③ series with ③ $4 + 1 = 5\Omega$ ④

④ || ④ $\frac{5 \times 20}{25} = 4\Omega$ ⑤ • ⑤ series with ⑤ $2 + 4 = 6\Omega$ ⑥

⑥ || ⑥ $\frac{6 \times 9}{15} = 3.6\Omega$ ⑦ • ⑦ || ⑦ $\frac{15 \times 3.6}{21.6} = 2.5\Omega$ ⑧

⑧ series with ⑧ $16 + 3 = Req = 19\Omega$



Find i_o , v_o ?

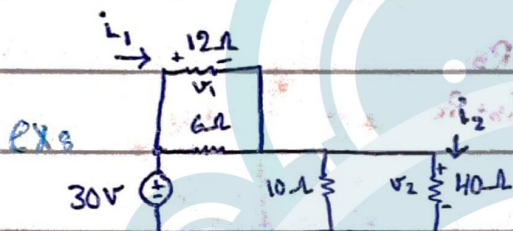
$$R_{eq} = 3 \parallel 6 = 2 \Omega + 4 \Omega = 6 \Omega$$

$$i = \frac{V}{R_{eq}} = \frac{12}{6} = 2 \text{ A} \quad , \quad v_o = \left(\frac{2}{2+4} \right) \times 12 = 4 \text{ V}$$

$$i_o = \frac{4}{3} \text{ A} = \frac{V}{R}$$

the Power in (3Ω) $\rightarrow P = \frac{4}{3} \times 3 = v_o i_o = 4 \text{ W}$

" " " $(12 \text{ V}) \rightarrow P = 12 \times 2 = 24 \text{ W}$



Find v_1 , v_2 , i_1 , i_2 ?

$$i = \frac{30}{4+8} = 2.5 \text{ A} \quad , \quad v_1 = 4 \times 2.5 = 10 \text{ V} \quad , \quad v_2 = 8 \times 2.5 = 20 \text{ V}$$

$$i_1 = \frac{10}{12} = .83 \text{ A} \quad , \quad i_2 = \frac{20}{40} = .5 \text{ A}$$

the Power dissipated in 12Ω & 40Ω ?

$$P_{12} = .83 \times 10 = 8.3 \text{ W} \quad , \quad P_{40} = 20 \times .5 = 10 \text{ W}$$

sec 6

• $\Delta \rightarrow Y$ (تحويل دلتا إلى Y)

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad \leftarrow \text{مقاومة بين } a \text{ و } b$$



$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

• $Y \rightarrow \Delta$ (تحويل Y إلى دلتا)

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad \leftarrow \text{مقاومة بين } a \text{ و } b$$

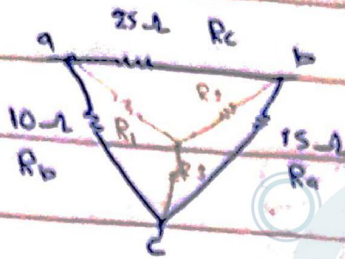
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

if $R_1 = R_2 = R_3 = R_Y$, $R_a = R_b = R_c = R_\Delta$

$$\Rightarrow R_Y = \frac{R_\Delta}{3} \quad \text{so} \quad R_\Delta = 3R_Y$$

EXS

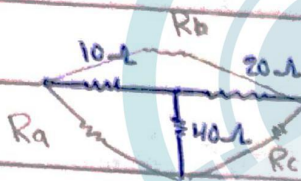


$$R_1 = \frac{25 \times 10}{25 + 15 + 10} = 5 \Omega$$

$$R_2 = \frac{15 \times 25}{50} = 7.5 \Omega$$

$$R_3 = \frac{15 \times 10}{50} = 3 \Omega$$

EXS

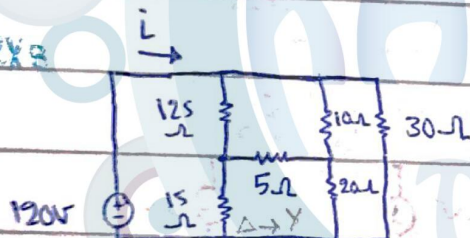


$$R_a = \frac{10 \times 20 + 10 \times 40 + 20 \times 40}{20} = 70 \Omega$$

$$R_b = \frac{1400}{40} = 35 \Omega$$

$$R_c = \frac{1400}{10} = 140 \Omega$$

EXS



Find i ?



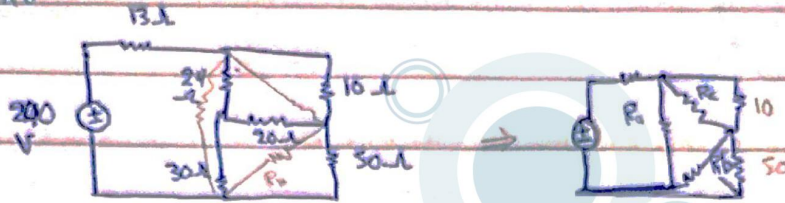
$$R_1 = \frac{15 \times 5}{40} = 1.875 \Omega, R_2 = \frac{20 \times 5}{40} = 2.5 \Omega, R_3 = \frac{20 \times 15}{40} = 7.5 \Omega$$

$$10 + 2.5 = 12.5 \Omega, 1.875 + 125 = 126.875 \Omega \Rightarrow \frac{1}{12.5} + \frac{1}{126.875} = \frac{1}{R} \Rightarrow R = 11.38 \Omega$$

$$11.38 + 7.5 = 18.88 \Omega \quad \text{①} \quad 30 \Omega \Rightarrow \frac{1}{30} + \frac{1}{18.88} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 11.58 \Omega$$

$$I = \frac{120}{11.58} = 10.356 \text{ A}$$

Ex 8



$$R_b = \frac{1800}{24} = 75\Omega, R_a = \frac{1800}{20} = 90\Omega, R_c = \frac{1800}{30} = 60\Omega$$

$$\frac{1}{10} + \frac{1}{60} \rightarrow R = 8.6\Omega / \frac{1}{50} + \frac{1}{75} \rightarrow R = 30\Omega \rightarrow 38.6\Omega$$

$$\frac{1}{38.6} + \frac{1}{90} \rightarrow R = 27\Omega, R_{eq} = 13 + 27 = 40\Omega$$

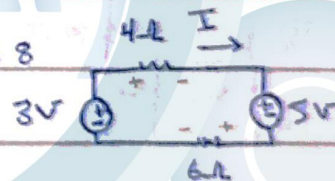
$$I = \frac{240}{40} = 6A$$

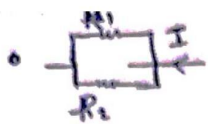
Solved Problems set 3

2.62 The current I in the circuit is 8

$$4I - 3 + 5 + 6I = 0 \therefore I = 0.2A$$

ans: -0.2A

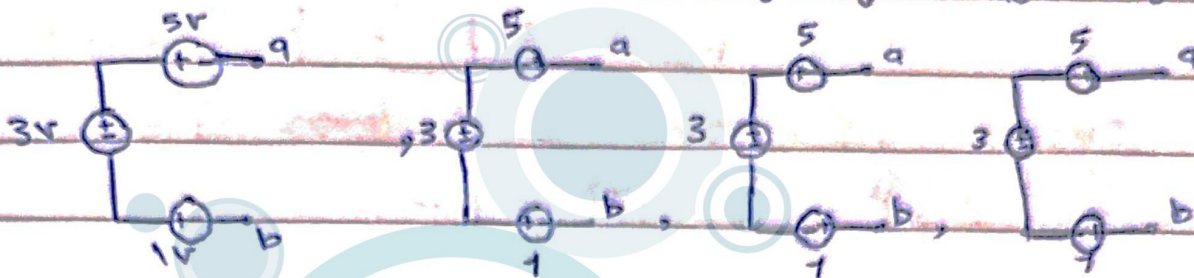




$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

2.98 which of the circuits will give you $V_{ab} = 7V$?



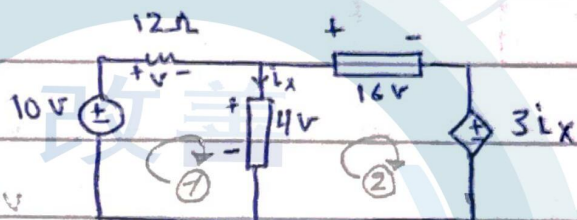
a → b

1) $-5 + 3 + 1 = -1V$ 2) $5 + 3 + 1 = 9V$

3) $-5 + 3 - 1 = -3V$ 4) $5 + 3 - 1 = 7V$ #

2.158 Find V , I_x

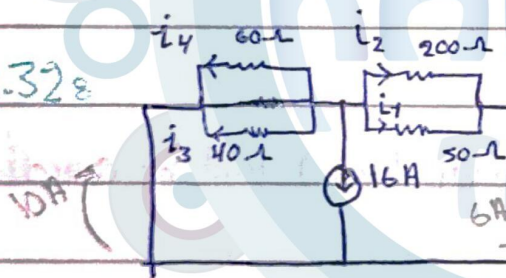
KVL



① $-10 + V + 4 = 0 \rightarrow V = 6V$

② $-4 + 16 + 3I_x = 0 \rightarrow I_x = -4A$

2.328



Find I

$\frac{1}{200} + \frac{1}{50} \rightarrow R_{eq} = 40\Omega$

$\frac{1}{60} + \frac{1}{40} \rightarrow R_{eq} = 24\Omega$

current division

$I_1 = 16 \times \frac{24}{24 + 40} = 6A$, $I_2 = 10A = 16 \times \frac{40}{64}$

للتيار I_1 في المقاومة 40Ω

③ $V = 40 \times 6 = 240V$, $V = 24 \times 10 = 240V$

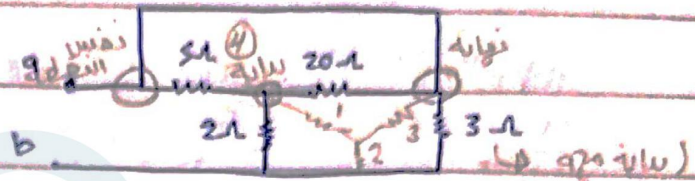
للتيار I_2 في المقاومة 24Ω

(+ من اليسار إلى اليمين)

$I_2 = \frac{240}{200} = 1.2A$, $I_1 = \frac{240}{50} = 4.8A$

$I_3 = -10 \times \frac{60}{100} = -6A$, $I_4 = -10 \times \frac{40}{100} = -4A$

2.448 obtain the R_{eq}



$$R_1 = \frac{20 \times 2}{25}$$

$$R_2 = \frac{3 \times 2}{25}$$

$$R_3 = \frac{20 \times 3}{25}$$

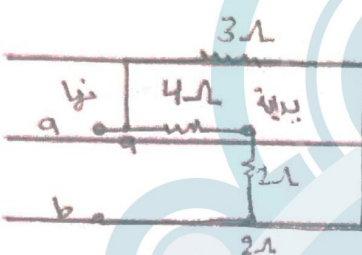
(20 و 2 في //)

not useful $\Delta \rightarrow Y$ 1, 2, 3 not in parallel

5Ω و 20Ω have the same start & end (so //)

$$4 + 9 = 13 \Omega$$

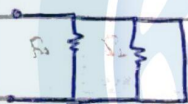
$$\frac{1}{10} + \frac{1}{5} \rightarrow R = \frac{4}{2}$$



$$\frac{1}{6} + \frac{1}{3} \Rightarrow R_{eq} = 2 \Omega$$

المقاومة الكلية بين الطرفين a و b

notes



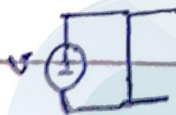
$$R_{eq} = \text{zero}$$



$$0 \times R_{eq} = 0$$

any resistance in // with short circuit (wire) will be cancelled (any element)

ex:



$$V = \text{zero}$$

Solved Problems sec 5

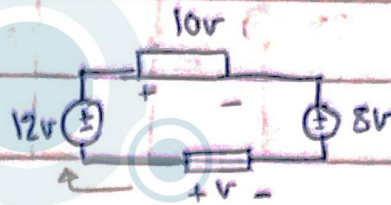
2.8s In the circuit V is

KVL

$$-12 + 10 + 8 - V = 0$$

(mesh)

$$V = +6V$$



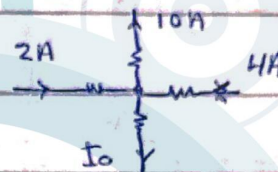
9.7s The current I_o

KCL

$$2 + 4 = 10 + I_o$$

(node)

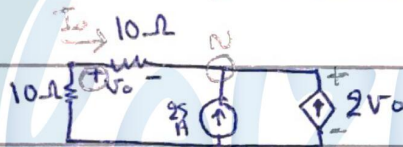
$$I_o = -4A$$



(in meq we choose the negative one)

2.97s V_o ?

Is the Power absorbed by the dep. source



• don't apply KVL

For any path has a current source

KVL (cannot be used here)

$$KCL \quad \text{at node } V_o \rightarrow I_o + 25 + 2V_o = 0 \rightarrow \frac{V_o}{10} + 2V_o = -25 \quad V_o = -11.9V$$

$$I_o = \frac{V_o}{10} = -1.19A$$

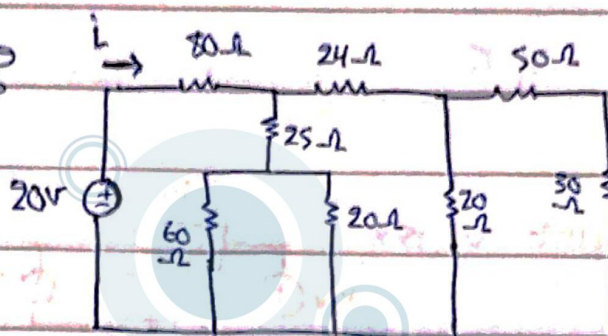
$$P = -I_o V_o = -(2 \times 11.9) \times 23.8 = \pm 566.44W$$

absorbed

• Power of the

$$20 \times 1.19 = 23.8V$$

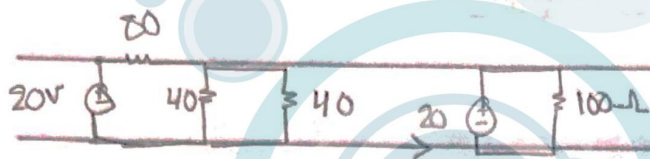
2.36a L & V_o ?



$$30 + 50 = 80 \Omega$$

$$\frac{1}{20} + \frac{1}{60} \rightarrow = \frac{1}{15} \Omega$$

$$15 + 25 = 40 \Omega$$



$$\frac{1}{80} + \frac{1}{20} \rightarrow 16 \Omega$$

$$16 + 24 = 40 \Omega$$

$$I = \frac{20}{100} = .2 A$$



$$I' = .2 \times \frac{40}{40 + 40} = .1 A$$

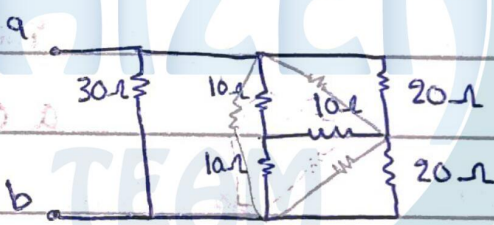
$$I'' = .1 \times \frac{20}{80 + 20} = .02 A$$

30 & 50 in series

So same current

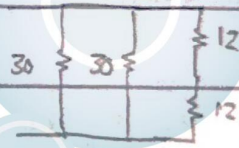
$$V_o = .02 \times 30 = .6 V$$

2.51a Req ?



$$\frac{1}{20} + \frac{1}{30} \rightarrow R = 12 \Omega$$

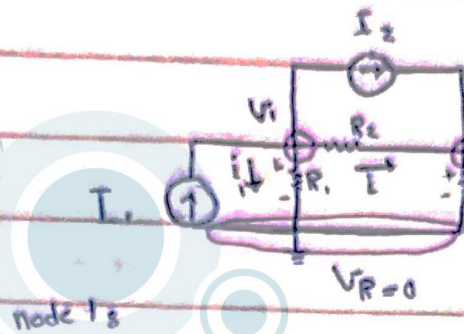
$$R_A = 3R_y = 30 \Omega$$



$$\frac{1}{24} + \frac{1}{30} \rightarrow R =$$

Nodal analysis 1

For R_2 it's your
choose the direction
of the current



$$I_1 = \frac{V_1 - V_R}{R_1} + \frac{V_1 - V_2}{R_2} + I_2$$

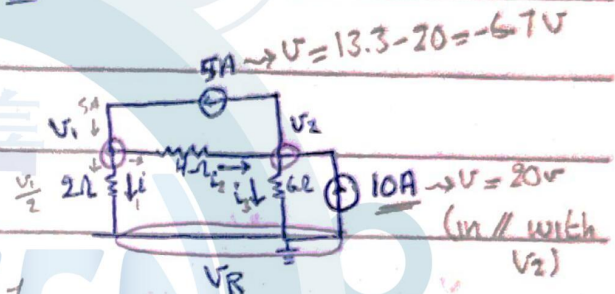
node 2s

$$I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2 - V_R}{R_3}$$

then solve V_1 & V_2 to find I .

ex: calculate the node voltages?

KCL



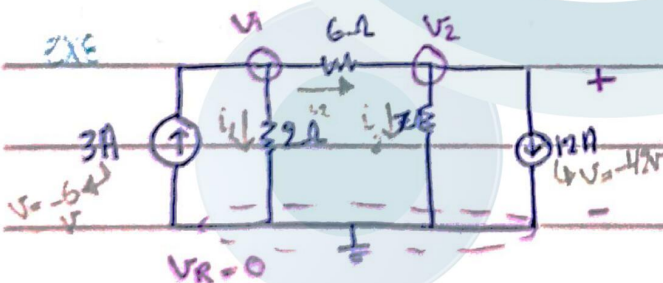
$$\text{node 1s } \frac{V_1}{2} + (V_1 - V_2) = 5 \quad \text{--- eq 1}$$

$$V_1 = 13.3 \text{ V} \quad V_2 = 20 \text{ V}$$

$$\text{node 2s } 10 + \frac{(V_1 - V_2)}{4} = \frac{V_2}{3} + 5 \quad \text{--- eq 2}$$

$$I_3 = \frac{20}{3} \text{ A} \quad I_1 = \frac{13.3}{2} \text{ A}$$

$$I_2 = \frac{-6.7}{4} (13.3 - 20) \text{ A}$$



$$\text{node 1s } 3 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$\text{node 2s } \frac{V_1 - V_2}{6} = 12 + \frac{V_2}{7}$$

$$V_1 = -6 \text{ V} \quad V_2 = -42 \text{ V}$$

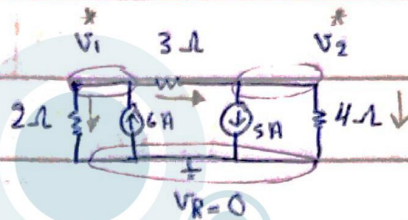
$$\text{Power in 12A } P = 12 \times -42 =$$

$$I_1 = \frac{-6}{2} \text{ A} \quad I_2 = \frac{-6 + 42}{6} \text{ A} \quad I_3 = \frac{-42}{7} \text{ A}$$

• Model 2

Ex 8

$$\text{N1: } \frac{V_1 + V_1 - V_2}{2} = 6$$



active nodes have labels

number of eq.

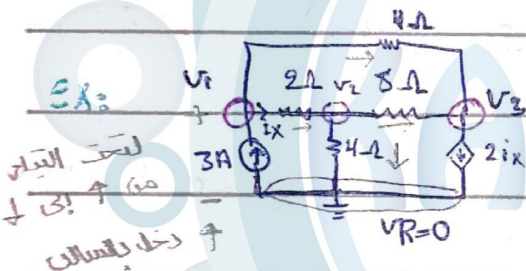
= # active nodes

$$5V_1 - 2V_2 = 36 \quad (1)$$

$$\text{N2: } \frac{V_1 - V_2}{3} = \frac{V_2 + 5}{4} \rightarrow 4V_1 - 7V_2 = 60 \quad (2) \quad V_1 = \quad V_2 =$$

• Kramers Rule $AX=B \rightarrow X=A^{-1}B$

$$\begin{bmatrix} 5 & -2 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 36 \\ 60 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 36 \\ 60 \end{bmatrix}$$



$$\text{N1: } 3 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} \rightarrow 3V_1 - 2V_2 - V_3 = 12$$

$$\text{N2: } \frac{V_1 - V_2}{2} = \frac{V_2 + V_2 - V_3}{4} \rightarrow -4V_1 + 7V_2 - V_3 = 0$$

$$V_1 = 4.8V, V_2 = -2.4V$$

$$V_3 = -2.4V$$

$$\text{N3: } \frac{V_1 - V_3}{4} + \frac{V_2 - V_3}{8} = 2i_x$$

$$\rightarrow 2V_1 - 3V_2 - V_3 = 0 \quad \text{if } \frac{V_1 - V_2}{2} = i_x$$

In 4 ohm (in middle) $V_2 = -2.4A$
assume it's order 4 4

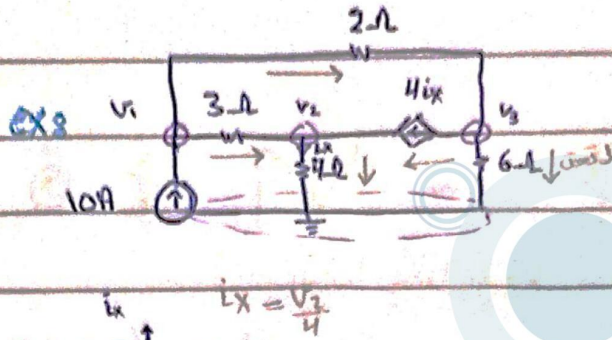
$$\text{in } 8 \text{ ohm } \frac{V_2 - V_3}{8} = \frac{2.4 + 2.4}{8}$$

voltage drop in $2i_x = V_3$

$$\text{in } \text{circle} = V_1$$

$$\text{Power in } 3A \quad P = -3 \times V$$

دخول الجهد



$$N1 \quad 10 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{2}$$

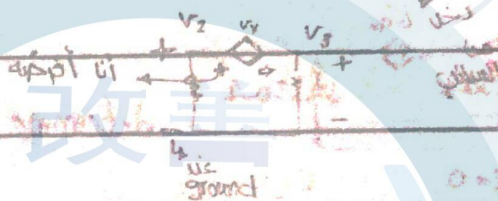
$$N2 \quad \frac{V_1 - V_2}{3} + 4i_x = \frac{V_2}{4}$$

$$N3 \quad \frac{V_1 - V_3}{2} = \frac{V_3}{6} + 4i_x \quad \text{where } i_x = \frac{V_2}{4}$$

$$V_1 = 80V, \quad V_2 = -64V, \quad V_3 = 156V$$

voltage drop on $\diamond$ $150 - -64$ ($V_3 - V_2 \uparrow - \downarrow$)

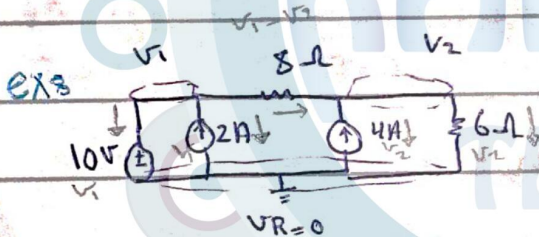
Power delivered or absorbed in $\diamond$ $P = IV = 4i_x(-220) = \oplus$ absorbed



$$V_1 + V_3 - V_2 = 0$$

$$V_x = V_2 - V_3 = -220V$$

applied voltage



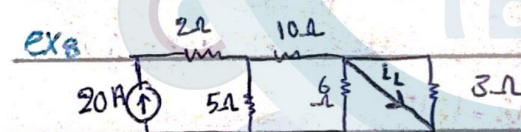
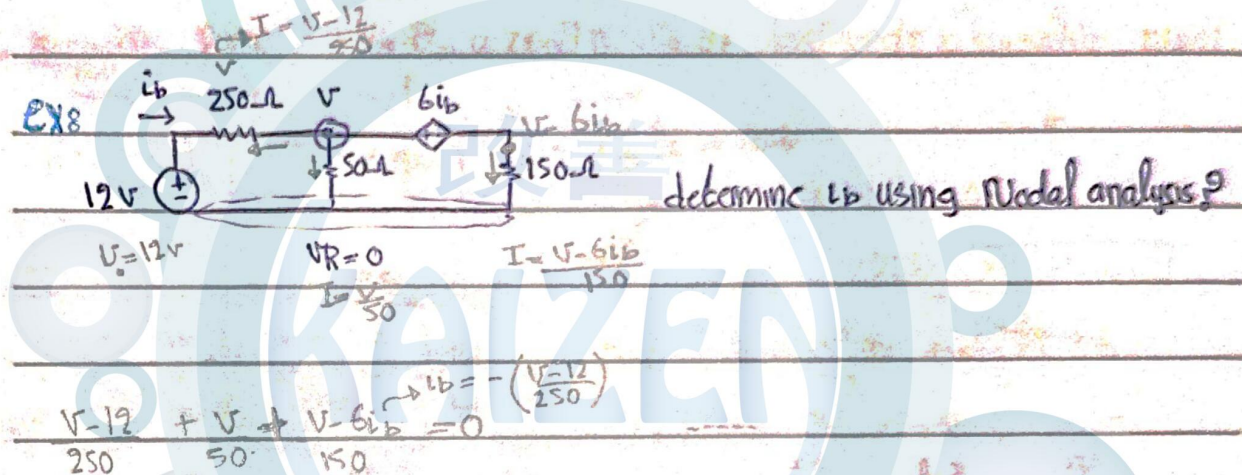
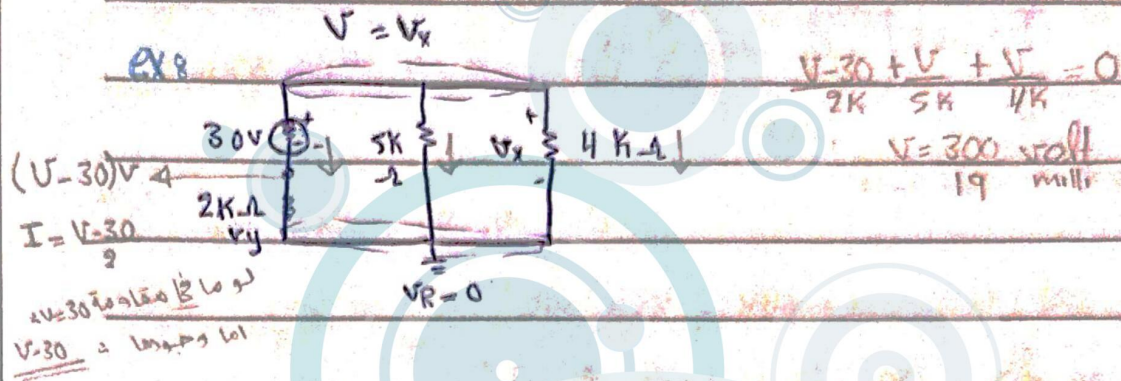
$$N1 \quad 2 = \frac{V_1 - V_2}{8} \quad (\text{down})$$

$$V_1 = 10V$$

$$N2 \quad \frac{10 + V_2}{8} + \frac{V_2}{6} + 4 = \frac{V_2}{6} \quad \Rightarrow V_2 = -10V$$

mins

• Nodal solved Prob



• any short circuit will cancel the elements in Parallel with it.

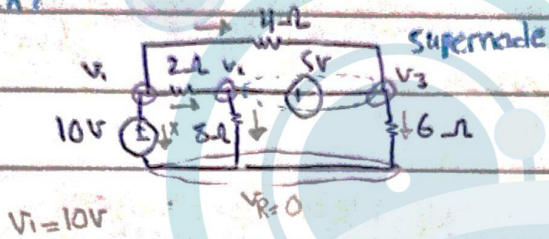
Since SC in // with 3Ω & 6Ω they will be cancelled





Supernode

ex:



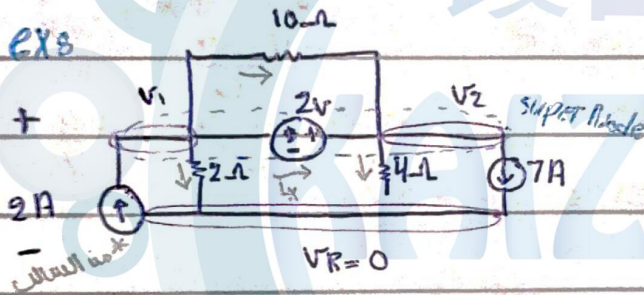
$$N1: V1 = 10V$$

$$\text{Supernode: } \frac{V1 - V2}{2} + \frac{V1 - V2}{4} = \frac{V2}{6} + Ix$$

(we need eq. for $V2$ & Ix)

$$V2 - V3 = 5 \quad V2 = 5 + V3$$

ex:



Supernode

$$2 + \frac{V1 - V2}{2} = \frac{V1}{10} + \frac{V2}{4} + \frac{V1 - V2}{10} + 7$$

$$V2 - V1 = 5 \quad \text{Substituting in the previous eq.}$$

Supernode // voltage source

$$V1 = -7.3V$$

$$V2 = -5.3V$$

$$P_{2A} = +IV = 2 \times -7.3 = -14.6W \quad \text{Produced, delivered}$$

$$P_{7A} = -IV = -7 \times -5.3 = 37.1W \quad \text{absorbed}$$

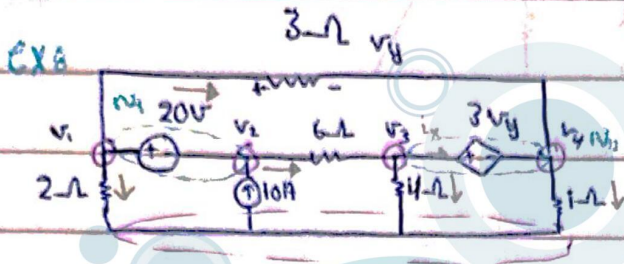
$$P_{7A} = +IV = 7 \times -5.3 = -37.1W$$

produced, delivered

$$Ix = ? \quad 2 = \frac{V1}{2} + \frac{V2 - V1}{4} + Ix$$

$$\frac{V_2 - V_3}{6} = \frac{V_3}{4} + I_x$$

$$P_{10W} = -I_v v = -10 * V_2 / P_{avg} = I_v v = 10 * 3V_y =$$



$$SP1 \quad \frac{V_1}{2} + \frac{V_1 - V_4}{3} + \frac{V_3 - V_3}{6} = 10$$

$$V_1 - V_2 = 20$$

$$V_R = 0$$

$$SP2 \quad \frac{V_2 - V_3}{6} + \frac{V_1 - V_4}{3} = \frac{V_3 + V_4}{4}$$

$$V_3 - V_4 = 3V_y$$

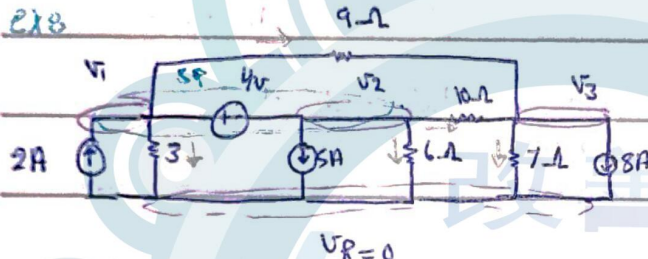
$$V_3 - V_4 = 3(V_1 - V_4) \quad (3)$$

$$V_1 - V_4 = V_y$$

$$V_1 = 26.7V / V_2 = 6.7V$$

$$V_3 = 173.3V / V_4 = -46.7V$$

Ex 8



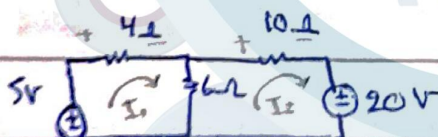
$$V_R = 0$$

$$V_1 - V_2 = 4$$

$$SP \quad 2 = \frac{V_1}{3} + \frac{V_1 - V_3}{9} + 5 + \frac{V_2}{6} + \frac{V_2 - V_3}{10}$$

$$SP \quad \frac{V_1 - V_3}{9} + \frac{V_2 - V_3}{10} = 8 + \frac{V_3}{7}$$

Mesh analysis



① assign # of Mesh

② assign a current for each mesh

③ apply KVL $\sum V = 0$

for R's

KCL

$$M1 \rightarrow -5 + 4I_1 + 6(I_1 - I_2) = 0$$

$$10I_1 - 6I_2 = 5$$

④ solve to find $I_1, I_2, \dots, I_n$

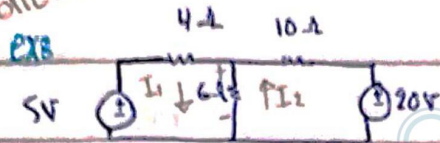
$$M2 \rightarrow 20 + 10I_2 + 6(I_2 - I_1) = 0$$

$$-6I_1 + 16I_2 = -20$$

$$I_1 = -0.37A, I_2 = 1.37A$$

$$V_{4A} = -0.37A \cdot 10 = -3.7V / P_{4A} = -3.7A \cdot 10 = -37W$$

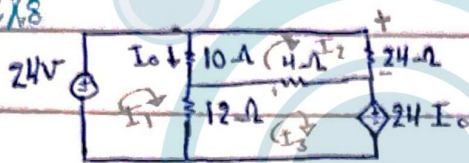
continued



$$V_{6\Omega} = (I_1 - I_2) \times 6 = -32 + 1.37 \times 6 = +1.63V$$

direction of I_1 is $I_1 - I_2$

ex8



• common R 's do not assume their polarity.

$$M_1 \quad -24 + 10(I_1 - I_2) + 12(I_1 - I_3) = 0$$

$$\rightarrow 22I_1 - 10I_2 - 12I_3 = 24$$

$$M_2 \quad 10(I_2 - I_1) + 24I_2 + 4(I_2 - I_3) = 0$$

$$\rightarrow 38I_2 - 10I_1 - 4I_3 = 0$$

$$M_3 \quad 24I_0 + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$I_1 = 2.25A, I_2 = .75A, I_3 = 1.5A$$

$$I_0 = I_1 - I_2$$

I_0 is $I_1 - I_2$

They are $\pm$ a right directions is assumed

$$I_0 = 1.5A \quad I_1 = .75A \quad (I_3 - I_2) = 1.5A \quad \text{or } -75A$$

$$P_{\diamond} = +I_0 V = 1.5 \times 24 = 36W \quad \text{absorbed}$$

ex8



$$M_1 \quad -12 + 10I_2 + 8(I_2 - 5) = 0$$

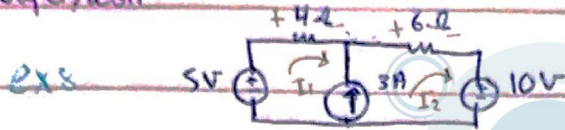
$$M_2 \quad I_1 = 5A \quad I_2 = \frac{52}{18}A$$

*any mesh contains a current source

we can't use KVL.

$$I_x = I_{8\Omega} = \frac{52}{18} - 5A \quad \uparrow I_x$$

• super mesh

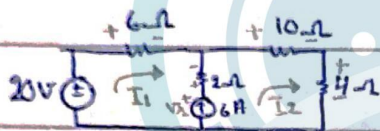


$$\text{loop } -5 + 4I_1 + 6I_2 + 10 = 0$$

$$\rightarrow 4I_1 + 6I_2 = -5$$

$$\rightarrow I_2 - I_1 = 3$$

exs



$$\text{loop } -20 + 6I_1 + 10I_2 + 4I_2 = 0$$

$$\rightarrow 6I_1 + 14I_2 = 20$$

$$I_1 = -3.2 \text{ A} \quad I_2 = 2.8 \text{ A}$$

$$\rightarrow I_2 - I_1 = 6$$

$$V_{6\Omega} = -3.2 \times 6 = -19.2 \text{ V}$$

$$V_{2\Omega} = 6 \times 2 = 12 \text{ V}$$

resultant

$$V_{10\Omega} = 2.8 \times 10 = 28 \text{ V}$$

to find V_x

$$-20 + V_{6\Omega} - V_{2\Omega} + V_x = 0$$

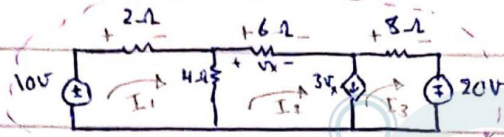
$$V_x = 51.3 \text{ V}$$

$$P_{6A} = -IV = -6 \times 51.2 = -307.2 \text{ W}$$

من البطارية

absorbed / produced

Ex 8



$$M1 \quad -10 + 2I_1 + 4(I_1 - I_2) = 0$$

$$\rightarrow 6I_1 - 4I_2 = 10$$

$$\text{By loop} \quad -10 + 2I_1 + 6I_2 + 8I_3 - 20 = 0$$

$$\rightarrow 2I_1 + 6I_2 + 8I_3 = 30$$

$$I_2 - I_3 = 3V_x$$

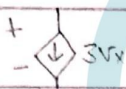
$$\hookrightarrow V_x = 6I_2$$

$$\rightarrow I_2 - I_3 = 18I_2$$

$$V_x = -6I_2$$

is 131

is 131



to find

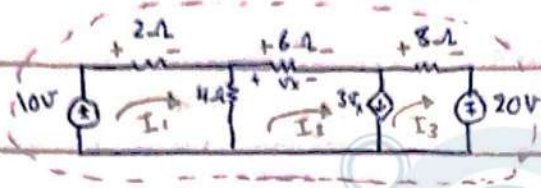
V_2

$$M3 \quad -V_2 + 8I_3 - 20 = 0$$

改善

KAIZEN
TEAM

Ex 8



$$M1 \quad -10 + 2I_1 + 4(I_1 - I_2) = 0$$

$$\rightarrow 6I_1 - 4I_2 = 10$$

$$\text{Big loop} \quad -10 + 2I_1 + 6I_2 + 8I_3 - 20 = 0$$

$$\rightarrow 2I_1 + 6I_2 + 8I_3 = 30$$

$$I_2 - I_3 = 3V_x$$

$$\hookrightarrow V_x = 6I_2$$

$$\rightarrow I_2 - I_3 = 18I_2$$

$$V_x = -6I_2$$

||| 1.1

is it with di?

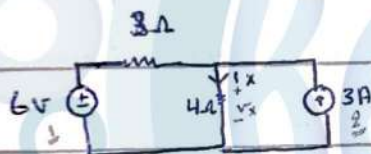


$$M3 \quad -V_2 + 8I_3 - 20 = 0$$

to find V_2

Super position

ex



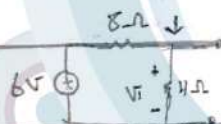
$I_x ?$
 $V_x ?$

$$I_x = I_{6V} + I_{3A}$$

1) Kill all the indep. sources except

one source $\left[\begin{array}{l} V \rightarrow \text{short circuit} \\ I \rightarrow \text{open circuit} \end{array} \right]$

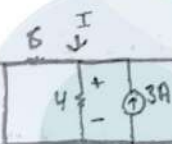
Source 1



$$V_1 = 6 \times \frac{4}{8} = 3V$$

$$I_{3A} = \frac{2}{4} = 0.5A$$

Source 2

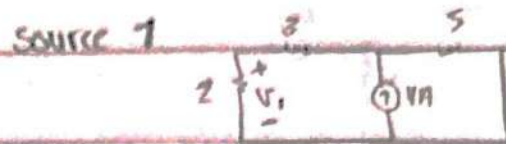
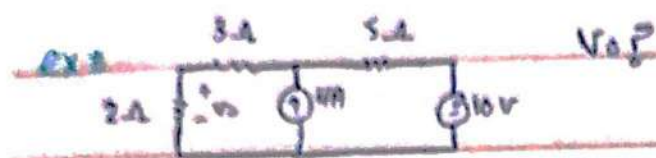


$$I_{3A} = 3 \times \frac{3}{12} = 0.75A$$

$$V_2 = 2 \times 4 = 8V$$

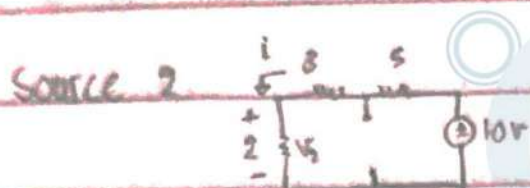
$\therefore V_x = 10V$ ($V_1 + V_2$ be same Polarity)

$$I_x = 2.5A$$



$$I_1 = 4 + 5 = 9A$$

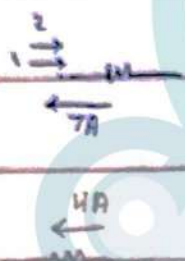
$$V_1 = 4V$$



$$I_2 = 10 - 1A \quad V_2 = 2V$$

$$V_o = 6V$$

Ex 2



3 sources

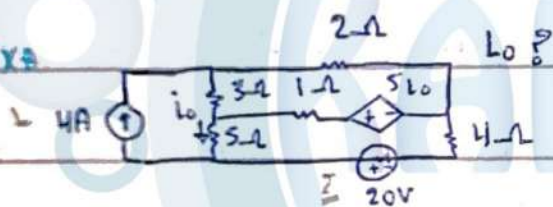
$$P_1 = 8W \quad P_2 = 2W \quad P_3 = 98W$$

$$P_{tot} = 108W$$

$$P_{tot} = 32W$$

We cannot apply superposition on the power (it is non linear)

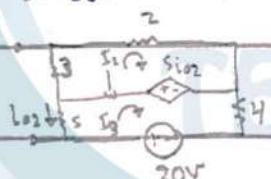
Ex 3



Source 1



Source 2



$$M1 \quad I_1 = 4A$$

$$M2 \quad 2I_2 - 5I_1 + 1(I_2 - I_3) + 3(I_2 - 4) = 0$$

$$M2 \quad 2I_2 - 5I_1 + 1(I_2 - I_3) + 3(I_2 - 4) = 0$$

$$M3 \quad 4I_3 + 5(I_3 - 4) + 1(I_3 - I_2) + 5I_1 = 0$$

$$M3 \quad 5I_2 + 4I_3 + 1(I_3 - I_2) + 5I_1 - 20 = 0$$

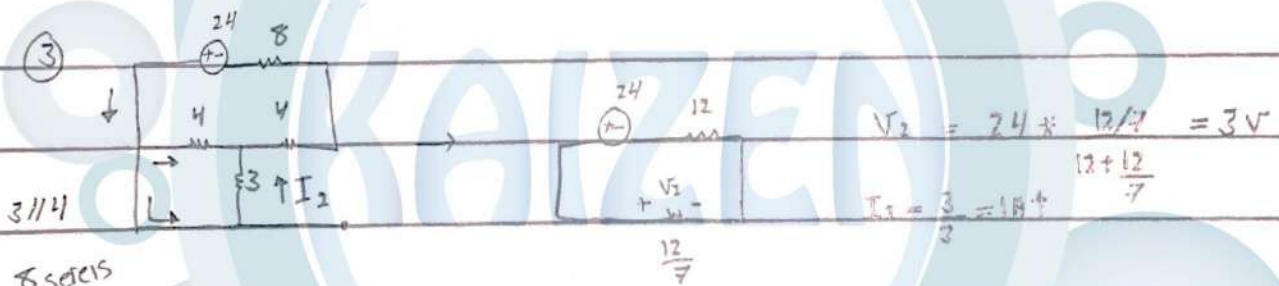
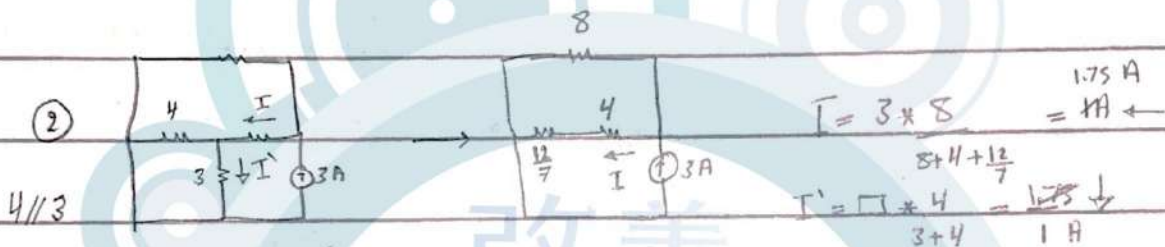
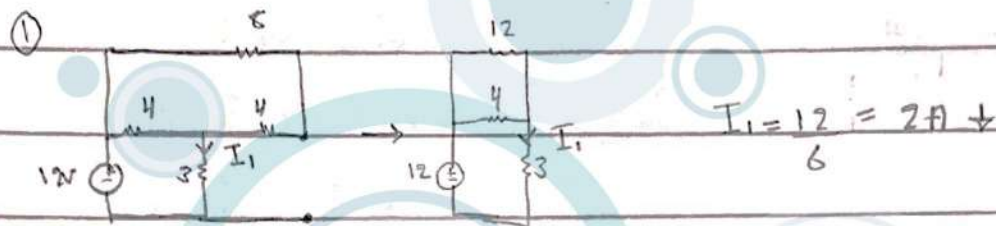
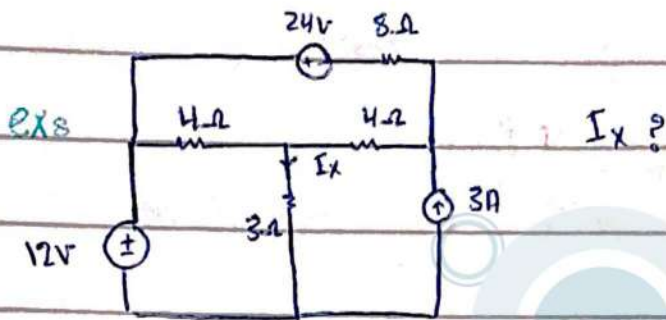
$$L_{21} = I_1 - I_2 = 4 - I_2$$

$$I_{02} = -I_2$$

$$L_{01} = \frac{52A}{17}$$

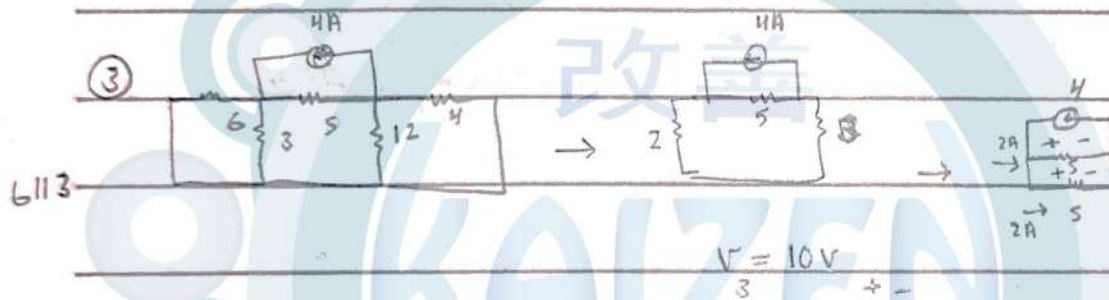
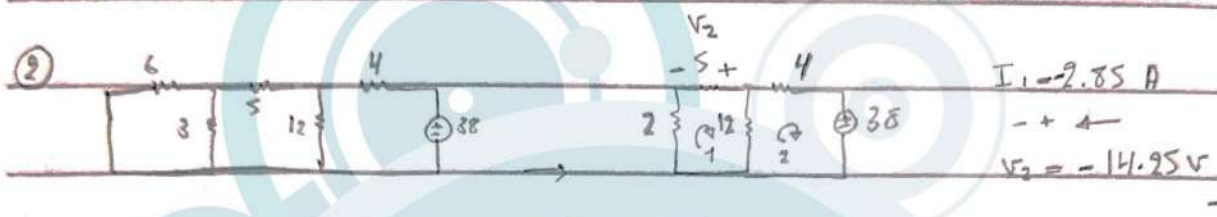
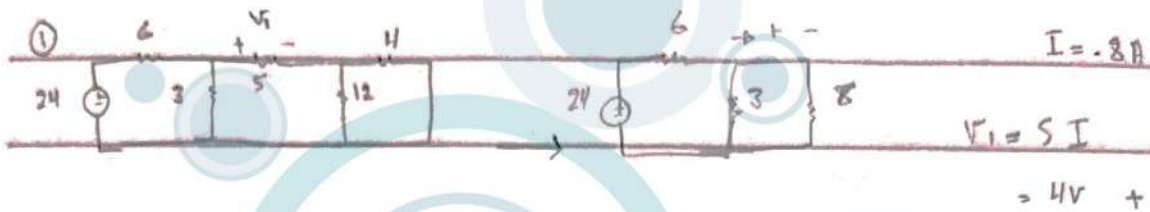
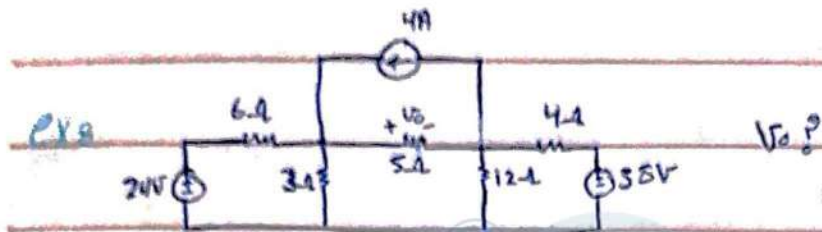
$$L_{02} = -\frac{60A}{17}$$

$$I_0 = -\frac{8A}{17}$$



$I_x = 2A + 1.75 - 1 = 2.75A \downarrow$

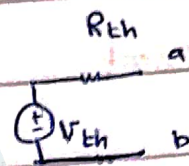
* R // voltage source when we make it short ckt the R will be cancelled.



$V_0 = 10 + 4 - 14.25 = -0.25V$

Ther & Noertons Max Power

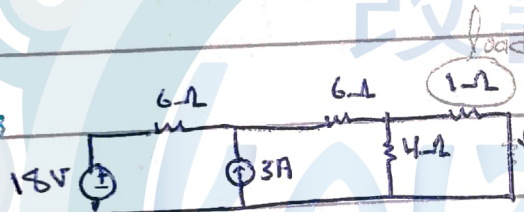
Thevenin theorem



Thevenin's equivalent

- ① cut the load
- ② Kill all the independent sources
- ③ calculate R_{th}
- ④ reconnect indep. sources
- ⑤ Find $V_{th} = V_o = V_{ab}$
- ⑥ Draw thevenin equivalent

Exs



Find I using thevenin's



(cut the load)



③ $R_{th} = 3\Omega$

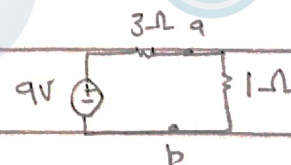
$12 \parallel 4 \rightarrow 3\Omega$



$$V_{ab} = V_{4\Omega} = 4 \times \frac{9}{4} = 9 \text{ volt} = V_{th}$$

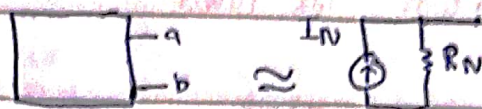
Big loop $-18 + 6I_1 + 6I_2 + 4I_2 = 0$

$$I_2 - I_1 = 3A \rightarrow I_2 = \frac{9}{4}A$$



$$I = \frac{9}{4}A$$

* Norton's equivalent



Source Trans Formation

$$R_N = R_{th}$$

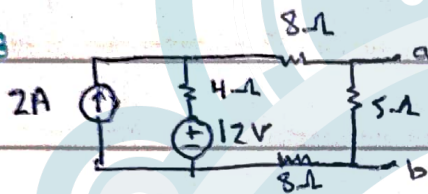
① cut the load

② kill the indep. sources

③ calculate R_N

④ short circuit between a & b (do not return the indep. sources)

ex:



Find I_N (or I_N ?)

1V done

②

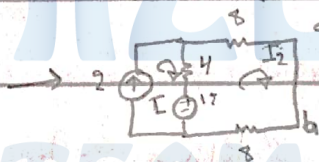


$$R_N = 4\Omega$$

$$2015 \rightarrow 4\Omega$$



5 cancelled (short CKt)

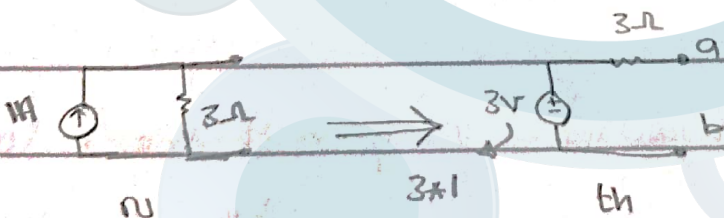


$$I_2 = I_N$$

$$= 1A$$

$$I_1 = 2A / -12 + 4(I_2 - I_1) + 16 I_2 = 0$$

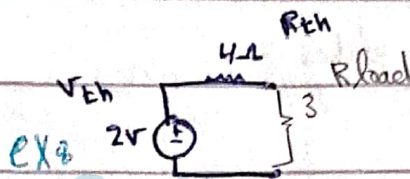
$$I_2 = 1A$$



Source Transformation

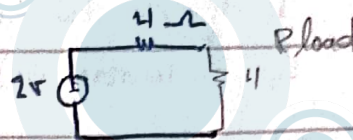
$$R_L = R_{th} \rightarrow P_{max} = \frac{V_{th}^2}{4R_L}$$

* to get maximum Power the R_L in the system should match.



$$I = \frac{2}{7} A$$

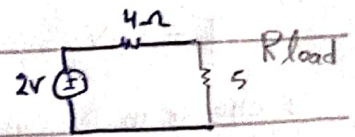
$$P_{load} = \frac{4}{49} \times 3 = \frac{12}{49} W = .244 W$$



$$I = \frac{2}{8} = .25 A$$

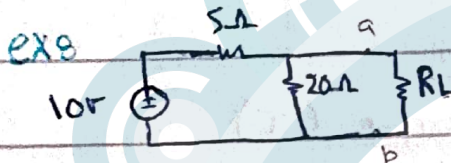
$$P_{load} = .25^2 \times 4 = .25 W$$

$$R_L = R_{th} \\ \therefore \text{Max Power}$$



$$I = \frac{2}{9} A$$

$$P_{load} = \frac{4}{81} \times 5 = .246 W$$



Find R_L to have maximum Power transfer?

$\therefore$ Find R_{th}



$$R_{th} = 4 \Omega$$

$$\therefore R_L = 4 \Omega$$

$$P_{max} = \frac{64}{4 \times 4} = 4 W$$

$$V_{ab} = V_{20\Omega} = \frac{10 \times 20}{25} = 8 V$$

$$-10 + 5I + 20I = 0$$

• Capacitor 1

energy storage elements

Capacitor 

$$C = \epsilon_0 \frac{A}{d}$$

A: area of the plate

d: distance btw two plates

ϵ_0 : permittivity (Air)

$$q = C \cdot v$$

q: charge in the capacitor

C: capacity (Farad)

v: voltage

$$* i = \frac{dq}{dt} \rightarrow q = \int i dt = C v \rightarrow v = \frac{\int i(t) dt}{C}$$

$$\text{or } i_c = \frac{d v(t)}{dt} \times C$$

$$* P = IV = C \frac{dv}{dt} \times v$$

energy

$$\leftarrow W = \frac{1}{2} C v^2$$

W: energy stored in the capacitor (J)

DC - circuit



voltage is const.

$$i_c = \text{zero} = v'$$

→ Capacitor acts as an open circuit in the DC circuits

30 PF, $v = 20V$

Ex 3



I, q?

$$q = 30 \times 20 \times 10^{-12} = 600 \text{ pC}$$

$$I = 0 = 20'$$

Ex 8

$C = 5 \mu F$, $v = 10 \cos 6000 t$, I, E?

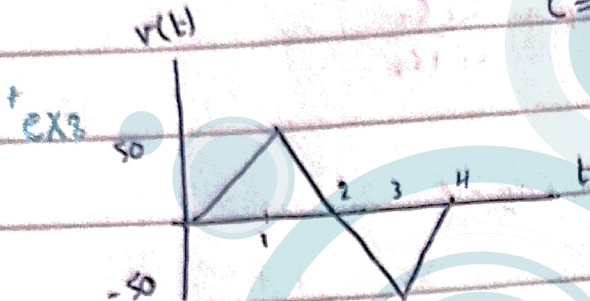
$$i = C \frac{dv}{dt} = 5 \times 10^{-6} (-10 \sin 6000 t (6000)) = -3 \sin 6000 t$$

$$W = E = \frac{1}{2} \times 5 \times 10^{-6} (10 \times 1)^2 = 2.5 \times 10^{-4} \text{ J}$$

ex8 $C = 2 \mu F$, $I(t) = 6e^{-3000t}$ mA $V?$

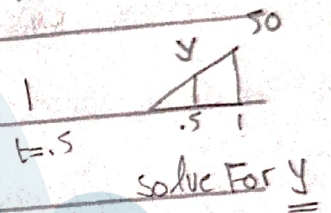
$$V = \frac{1}{C} \int I dt = \frac{1}{2 \mu F} \int 6e^{-3000t} dt = \frac{6}{2 \times 10^{-6}} \left(-\frac{1}{3000} e^{-3000t} \right) = -1000 e^{-3000t} V$$

$C = 200 \mu F$ sketch the current wave forms

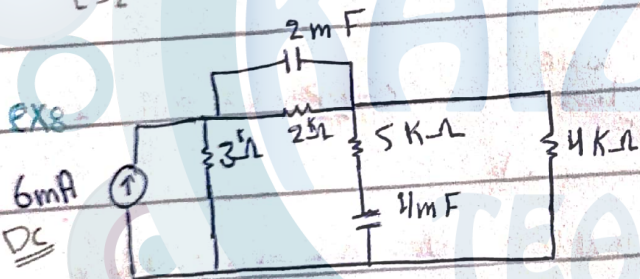


$$I(t) = 200 \mu \times 50 = 1 \times 10^{-2} A$$

$$E = W = \frac{1}{2} (V^2) = \frac{1}{2} \times 200 \mu \times 50^2$$



$E = \text{Zero } (V=0)$
 $t=2$



energy stored in $2mF$?

C will not appear in DC circuit (short circuit)

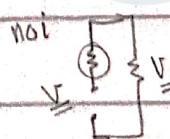


$$I_2 = 2mA \quad V_{2k} = 4V = V_{2mF}$$

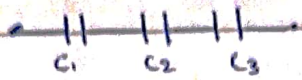
$$E = \frac{1}{2} \times 2m \times 4^2 = 16 mJ$$

$I = \text{Zero}$
 $2mF$

$$E = \frac{1}{2} \times 4m \times \frac{V(8)^2}{4k} =$$



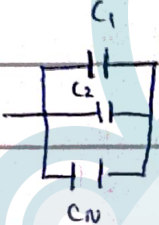
Capacitor 2

①  $\rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

 $\rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

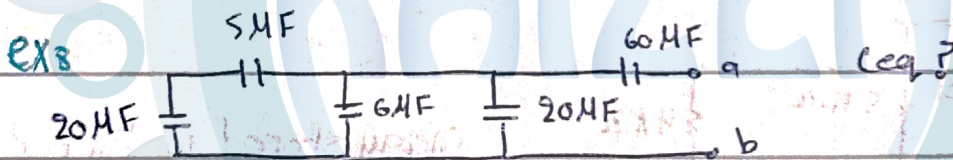
in series q is equal $q = C_{eq} \cdot V$

$(q_1 = q_2 = q_3 \dots)$

②  $C_{eq} = C_1 + C_2 + \dots + C_n$

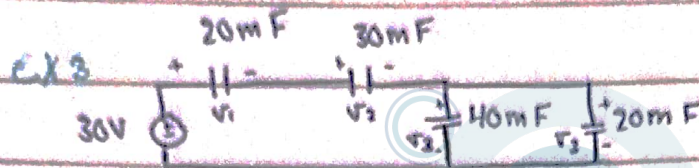
$V_1 = V_2 = V_3 \dots$

$q = q_1 + q_2 + \dots + q_n$



① $\frac{20 \times 5}{25} = 4 \mu F$ ② $4 + 6 = 10 \mu F$ ③ $10 + 20 = 30 \mu F$

④ $\frac{30 \times 60}{90} = \underline{\underline{20 \mu F}}$



Find V For each C ?

① $20 + 40 = 60 \text{ mF}$

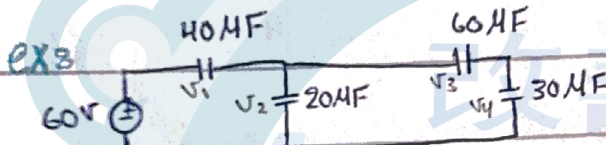
② $\frac{1}{60} + \frac{1}{30} + \frac{1}{20} = \frac{1}{C_{eq}} \quad C_{eq} = 10 \text{ mF}$

$\rightarrow q_{tot} = 10 \text{ m} \times 30 = .3 \text{ C} = 300 \text{ mC}$

$V_1 = \frac{300 \text{ m}}{20 \text{ m}} = 15 \text{ V}$

$V_2 = \frac{300 \text{ m}}{30 \text{ m}} = 10 \text{ V}$

$V_3 = 5 \text{ V} = \frac{300 \text{ m}}{(10+40) \text{ m}}$



Find the voltage across each capacitor

① $\frac{30 \times 60}{90} = 20 \text{ mF}$

② $90 + 20 = 40 \text{ mF}$

③ $\frac{40 \times 40}{80} = 20 \text{ mF}$

$q_{tot} = 60 \times 20 \text{ m} = 1200 \text{ mC}$

$\rightarrow V_1 = \frac{1200 \text{ m}}{40 \text{ m}} = 30 \text{ V}$

$V_2 = V_3 + V_4$
 $= 60 - 30$
 $= 30 \text{ V}$

$\rightarrow q_2 = 30 \times 20 \text{ m} = 600 \text{ mC}$

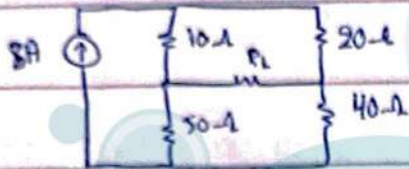
$\therefore V_3 = \frac{600 \text{ m}}{60 \text{ m}} = 10 \text{ V}$

$V_4 = 30 - 10 = 20 \text{ V}$ or $\frac{600}{30} = 20 \text{ V}$

Solved Problems

exs

Find R_L to transfer max Power :



$$R_L = R_{th}$$

①

$$20 + 10 = 30 \Omega, \quad 50 + 40 = 90 \Omega$$



$$30 \parallel 90 = \frac{30 \times 90}{120} = 22.5 \Omega = R_{th} = R_L$$



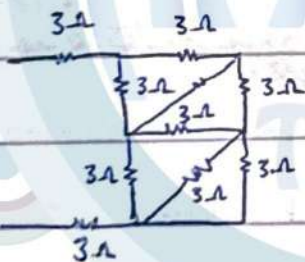
$$V_{ab} = V_{th} = V_{40} + V_{50}$$

$$= -4 \times 40 + 4 \times 50 = 40V$$

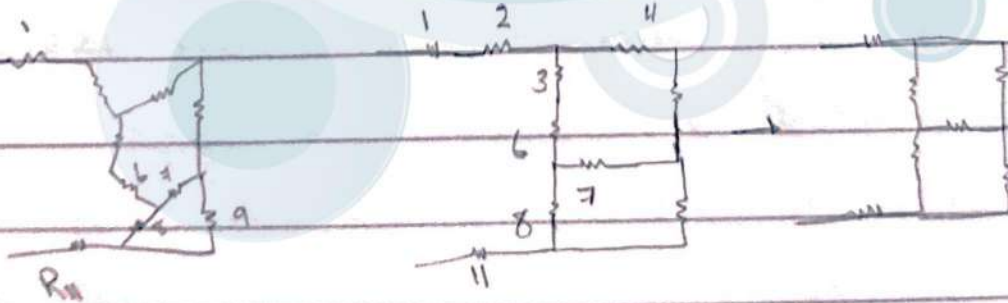


$$P = \frac{40^2}{4 \times 22.5} = \frac{1600}{90} W$$

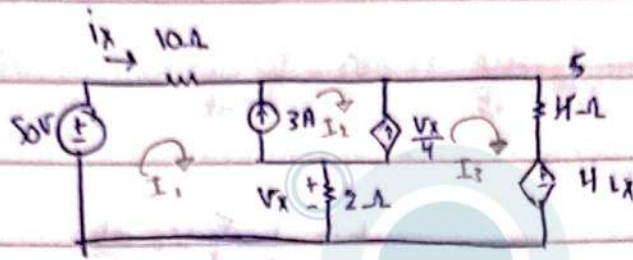
exs



$$3R_1 = R_{th}$$



ex 3



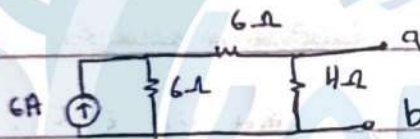
$$-50 + 10I_1 + 5I_3 + 4Ix = 0 \quad \text{--- (1)}$$

$$I_2 - I_1 = 3 \quad \text{--- (2)}$$

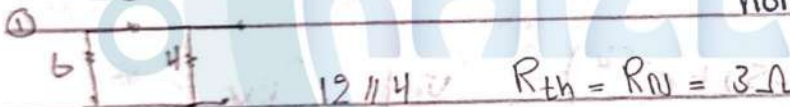
$$Ix = I_1$$

$$I_3 - I_2 = \frac{V_x}{4} \quad \text{--- (3)} \quad V_x = 2(I_1 - I_3) \quad \text{--- solve for } I\text{'s}$$

ex 8



Find the value of R_L
to transfer max Power,
norton's

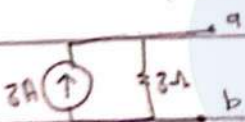


$$R_{th} = R_{N} = 3\Omega$$



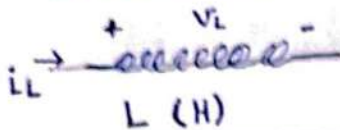
$$I_N = 6 \times \left(\frac{6}{12} \right) = 3A$$

$$V_{th} = 9V$$



$$P = \frac{9^2}{4 \times 3}$$

• inductor



$$i_L = \frac{1}{L} \int \frac{dV_L}{dt} dt$$

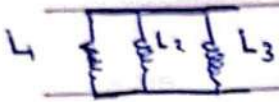
$$V_L = L \frac{di_L}{dt}$$

$$i_L = \frac{1}{L} \int V_L dt$$

$$I_L(t) = \frac{1}{L} \left(\int_{t_0}^t V_L dt \right) + I_L(t_0)$$



$$L_{tot} = L_1 + L_2 + L_3$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$W (\text{energy}) = \frac{1}{2} L I^2$$

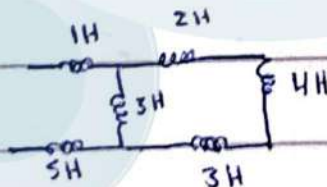
Ex: $i(t) = 10t e^{-5t}$, $L = .1 \text{ H}$, Find V_L & E :

$$V_L = .1 * (10 e^{-5t} + -50 t e^{-5t}) \quad V_L|_{t=0} = 1 \text{ V}$$

$$I|_{t=0} = \text{zero}$$

$$E|_{t=0} = \text{zero}$$

Ex: Find L_{eq}

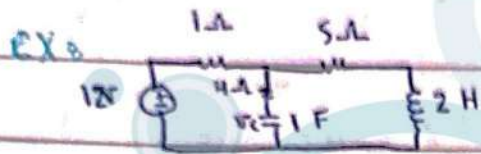


$$2 + 4 + 3 = 9 \text{ H}$$

$$\frac{1}{9} + \frac{1}{3} \rightarrow \frac{27}{12} \text{ H}$$

$$L_{eq} = 1 + 5 + \frac{27}{12} =$$

• Inductor acts as short circuit in the DC KtE while capacitor is open KtE.



Find V_c & E in inductor & capacitor



$$I = \frac{12}{6} = 2A$$

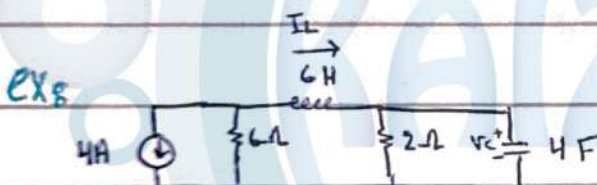
$$\textcircled{1} E_L = \frac{1}{2} \times 2 \times 2^2 = 4J$$

② There is no voltage drop on 4Ω

$$V_c = V_4 + V_5$$

$$= 2 \times 5 = 10V$$

$$E_c = \frac{1}{2} \times 1 \times 10^2 = 50J$$



Find V_c , I_L &

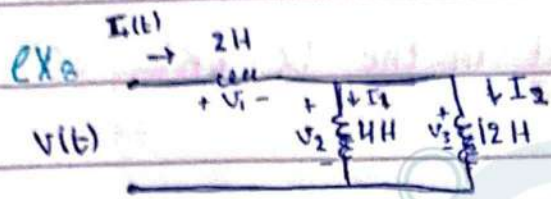


$$I_L = -4 \times \left(\frac{6}{8}\right) = -3A$$

$$E_L = \frac{1}{2} \times 6 \times 9 = 27J$$

$$V_c = -V_2 = -(2 \times 3) = -6V$$

$$E_c = \frac{1}{2} \times 4 \times 36 = 72J$$



$$I_1(t) = 4(2 - e^{-10t}) \text{ mA}$$

$$I_2(0) = -1 \text{ mA}$$

$$v(t), I_1(0) \text{ ?}$$

$$\textcircled{1} I_1(0) - I_2(0) = 4 - (-1) = 5 \text{ mA}$$

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12} \quad \textcircled{2} V(t) = L_{eq} \frac{dI}{dt} \rightarrow V(t) = 5 \times 4(10e^{-10t}) = 200e^{-10t}$$

$3+2=5H \leftarrow$

$$V_1 = 2 \times (40e^{-10t}) = 80e^{-10t} \quad , \quad V_2 = 4 \times \frac{dI_1}{dt} \quad \text{but I don't have } I_1$$

المقاومة الجارية

$$\text{So } V_2 = V(t) - V_1 = 120e^{-10t} \quad (-V(t) + V_1 + V_2 = 0)$$

$$I_1 = \frac{1}{4} \times \int 120e^{-10t} = -\frac{12}{4}e^{-10t} = -3e^{-10t}$$

• Sinusoidal Lead Lag

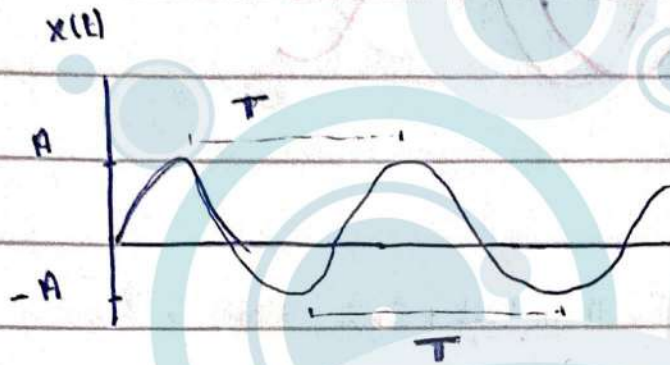
$$x(t) = A \cos[\omega t + \theta] \rightarrow \text{depends on time (current or voltage)}$$

A : Amplitude
 V, A

ω : radian

$$f = \frac{\omega}{2\pi}$$

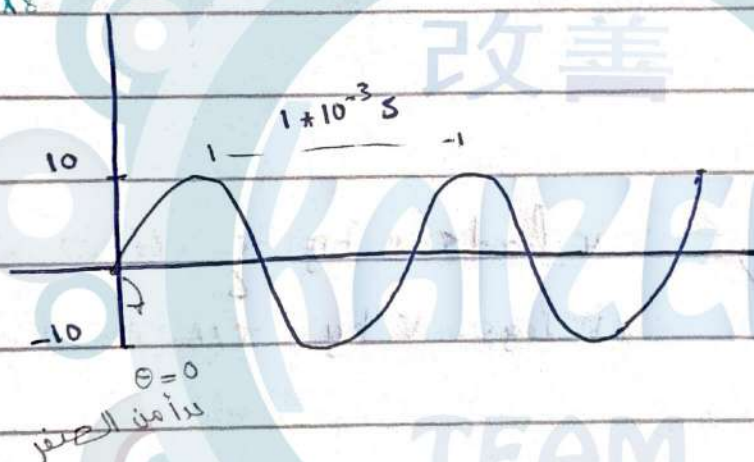
θ : Phase shift



$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$T = \frac{2\pi}{\omega} \text{ (periodic time)}$$

EX8



Find A, ω, f, T

$$A = 10 \text{ (Peak value)}$$

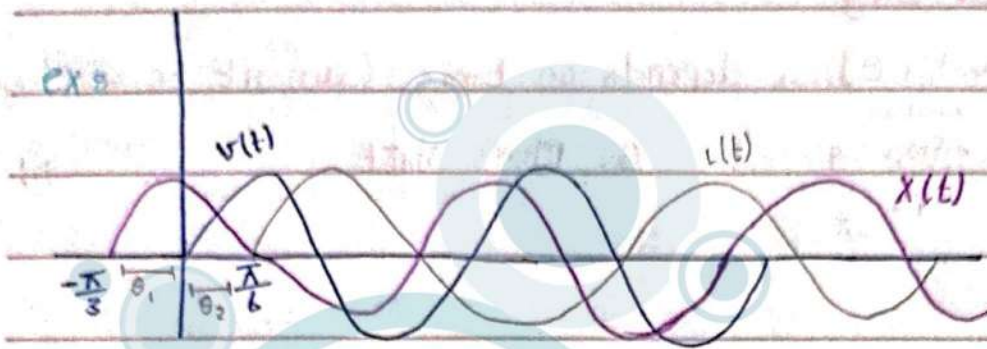
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1 \times 10^{-3}} \text{ rad/s}$$

$$T = 1 \times 10^{-3} \text{ s}, f = 10^3 \text{ Hz}$$

$$\theta = \text{zero}, v(t) = A \sin[\omega t + \theta]$$

$$= 10 \sin(2\pi \times 10^3 t)$$

* to compare two signals they have to have same F, Form (sin, cos)



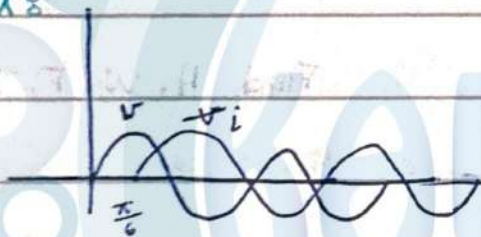
$x(t) \rightarrow$ leads $v(t)$ & $i(t)$

$v(t) \rightarrow$ lags $x(t)$

$$v(t) = A \sin(\omega t + 0), \quad x(t) = A \sin(\omega t + \theta_1)$$

$$i(t) = A \sin[\omega t - \theta_2]$$

ex 8



v leads i by $\frac{\pi}{6}$

i lags v by $\pi/6$

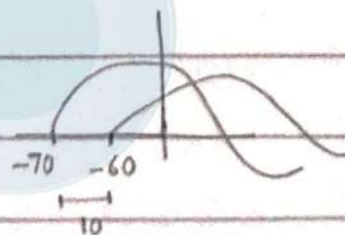
ex 8 $v(t) = 10 \cos(400t - 30^\circ)$, $i(t) = 20 \cos(600t + 40^\circ)$

• we cannot compare $\omega_1 = 400$ $\omega_2 = 600$

$$v(t) = 10 \cos[300t + 60^\circ] \quad i(t) = 5 \cos[300t + 70^\circ]$$

• we can

compare

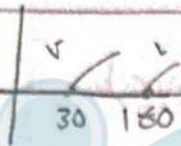


i leads by 10

$$\sin X = \cos(X - 90)$$

$$v(t) = 10 \cos[100t - 30] \quad i(t) = 5 \sin[100t - 70]$$

• we can compare $i(t) = 5 \cos[100t - 70 - 90] = 5 \cos[100t - 160]$



v leads i by 130°

$$-\sin X = \sin(X \pm 180), \quad -\cos X = \cos(X \pm 180)$$

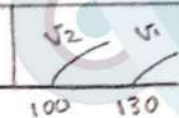
$$\pm \cos X = \sin(90 \pm X), \quad \pm \sin X = \cos(X \mp 90)$$

EX 8 $v_1(t) = -10 \cos[\omega t + 50] \quad v_2(t) = 12 \sin[\omega t - 10]$

• we can compare $v_1 = 10 \cos[\omega t + 50 - 180] = 10 \cos[\omega t - 130]$
to keep it in the range of $(0, 180)$

$$v_2 = 12 \cos[\omega t - 100]$$

v_2 lead v_1 by 30°



$$i_1(t) = -4 \sin[377t + 25] \quad i_2(t) = 5 \cos[377t - 40]$$

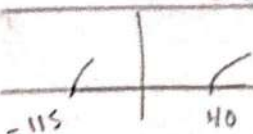
$$i_1 = 4 \sin[377t - 155]$$

$$i_2 = 5 \cos[377t - 945]$$

$$\text{or } i_1 = 4 \sin[377t + 205]$$

$$\text{or } i_2 = 5 \cos[377t + 115]$$

to keep it in $[0, 180]$



i_1 leads i_2 by 155°

• Impedance Concept

AC

$i(t), v(t)$

R, C, L

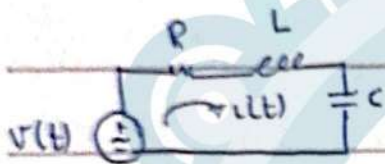
DC

I, V

load (Resistance)

Capacitor $\rightarrow$ open circuit

Inductor $\rightarrow$ short circuit



$$v(t) = v_R + v_L + v_C \quad (\text{KVL})$$

$$v(t) = i(t) \cdot R + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt$$

$$\frac{d}{dt} \rightarrow j\omega, \int \rightarrow \frac{1}{j\omega}$$

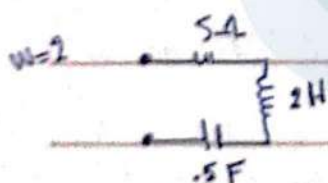
$$v(\omega) = I(\omega)R + L j\omega I + \frac{1}{j\omega C} I$$

$$V = IR, V = IZ, V_L = IZ_L, V_C = IZ_C, V_R = IR$$

Impedance

Impedance (R, L, C) $= Z(\Omega)$

$$R \rightarrow Z_R = R, L \rightarrow Z_L = j\omega L, C \rightarrow Z_C = \frac{1}{j\omega C} (\Omega)$$



$$Z_{tot} = 5 + j\omega L + \frac{1}{j\omega C}$$

$$= 5 + 3j \Omega$$

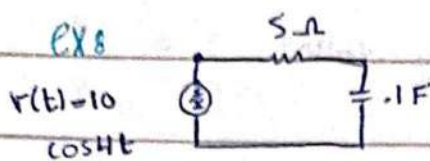
(is series ΣZ)

H. F. $\rightarrow$ time domain

$$= 5.8 \angle 31^\circ \Omega$$

$Z_R, Z_L, Z_C \rightarrow$ Frequency domain

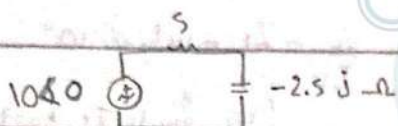
Ex 8



Find $i(t)$

$$\omega = 4$$

$$Z_R = 5 \Omega, Z_C = \frac{1}{j\omega C} = -j2.5 \Omega$$

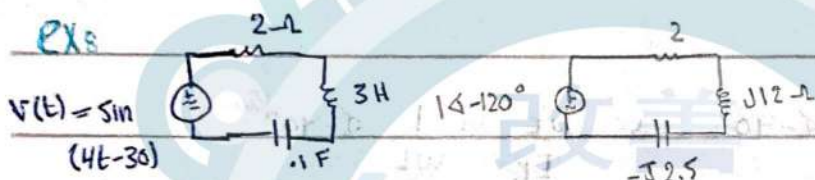


$$-10 \angle 0 + 5I - j2.5I = 0 \rightarrow (5 - j2.5)I = 10 \angle 0^\circ$$

$$I = 1.78 \angle 26.5^\circ \text{ A}$$

$$i(t) = 1.78 \cos(4t + 26.5^\circ)$$

Ex 8



$$v(t) \cos(4t - 120)$$

$$\omega = 4$$

$$-1 \angle -120 + 2I + j12I - j2.5I = 0$$

$$(2 + j9.5)I = 1 \angle -120^\circ$$

$$I = .1 \angle 161.9^\circ \text{ A}$$

$$i(t) = .1 \cos(4t + 161.9^\circ) \text{ A}$$

$$j \rightarrow 90^\circ$$

$$-j \rightarrow -90^\circ$$

YD AC

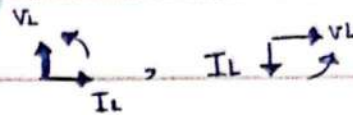
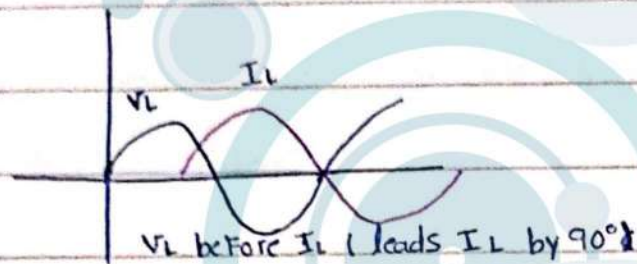
$$Z = j\omega L = \omega L \angle 90^\circ$$

$$Z = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

Inductor

$$* V = IZ, V_L = I_L Z_L = I_L \omega L \angle 90^\circ \rightarrow \frac{V_L}{I_L} = Z_L = \omega L \angle 90^\circ$$

(V_L leads I_L by 90°)



capacitor

$$* V_C = I_C Z_C = I_C \frac{1}{\omega C} \angle -90^\circ \rightarrow \frac{V_C}{I_C} = \frac{1}{\omega C} \angle -90^\circ$$

(I_C leads V_C by 90°)

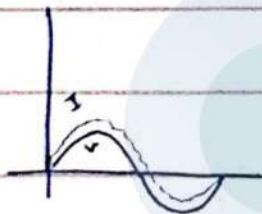
→ com. polar

$$4j \rightarrow 4 \angle 90^\circ$$

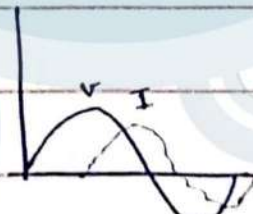
$$-3j \rightarrow 3 \angle -90^\circ$$



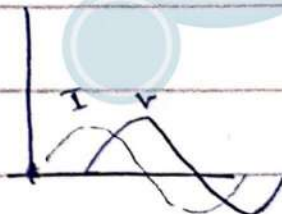
$R \angle 0^\circ$ (I & V are in the same Phase)



Resistor
(in Phase)

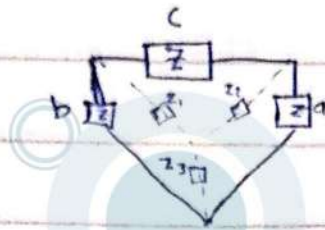


inductor
(V leads)



capacitor
(I leads)

$$\frac{1}{Z} = Y$$



$Y \rightarrow \Delta$

$$Z_a = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2}$$

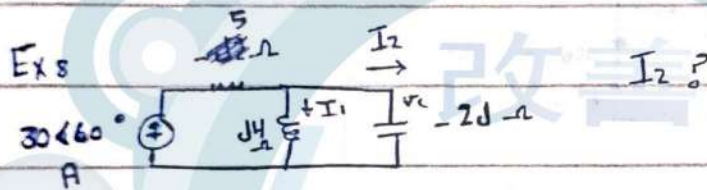
$$Z_c = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$

$\Delta \rightarrow Y$

$$Z_1 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$



$$I_2 = (30 \angle 60^\circ) + j4$$

$$\frac{j4 - j2}{j2} \rightarrow 2 \angle 90^\circ$$

$$I_1 = 30 \angle 60^\circ - 60 \angle 60^\circ = 30 \angle -120^\circ$$

$$= 60 \angle 60^\circ \text{ A}$$

(I leads V by 90°)

$$V_c = I_2 Z_c = (60 \angle 60^\circ) (-2j) = 120 \angle -30^\circ \text{ V}$$

$$V_L = (30 \angle -120^\circ) (j4) = 120 \angle -30^\circ$$

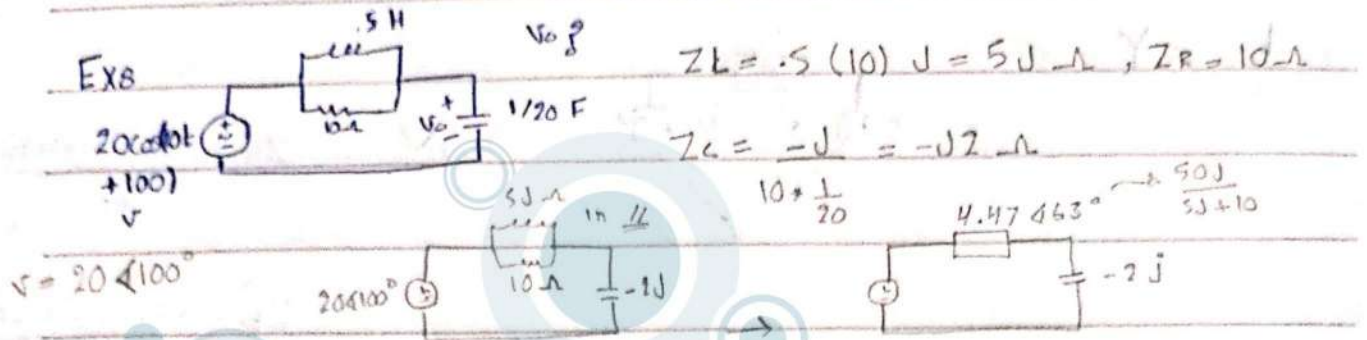
V leads I by 90°

$$I_R = 30 \angle 60^\circ$$

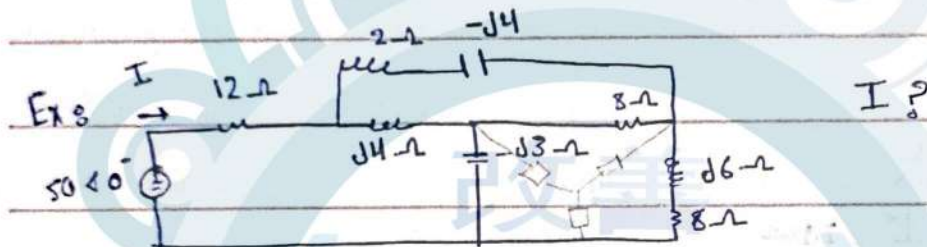
In Phase

$$V_R = (30 \angle 60^\circ) 5 = 150 \angle 60^\circ$$

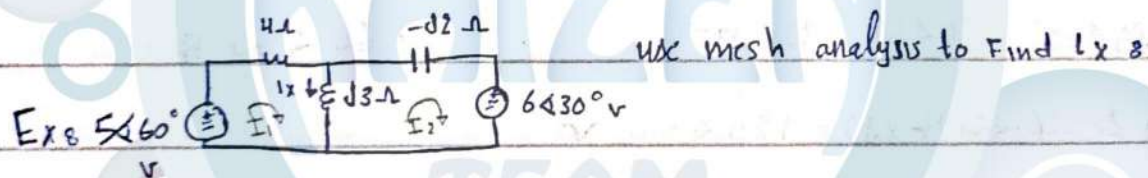
* convert From time domain to polar



$$V_o = 20 \angle 100^\circ \frac{-j2}{-j2 + 4.47 \angle 63^\circ} = 14.41 \angle -35^\circ \rightarrow 14.41 \cos(10t - 35^\circ)$$



• AC Mesh Nodal



use mesh analysis to find I_x

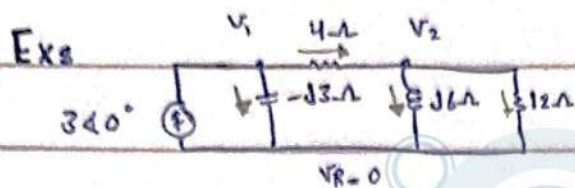
$$M1 \rightarrow -5 \angle 60^\circ + 4 I_1 + j3 (I_1 - I_2) = 0 \rightarrow (4 + j3) I_1 - j3 I_2 = 5 \angle 60^\circ$$

$$M2 \rightarrow -j2 I_2 + 6 \angle 30^\circ + j3 (I_2 - I_1) = 0 \rightarrow -j3 I_1 + j I_2 = 6 \angle -150^\circ$$

$$\begin{bmatrix} 4 + j3 & -j3 \\ -j3 & j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \angle 60^\circ \\ 6 \angle -150^\circ \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{13.9 \angle -70.36^\circ}{7.2 \angle 33.7^\circ}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{31.9 \angle -140.96^\circ}{7.2 \angle 33.7^\circ}$$



N1 $\rightarrow 340 = \frac{V_1}{-j3} + \frac{V_1 - V_2}{4}$

$340 = V_1 \left(\frac{1}{4} - \frac{1}{j3} \right) - \frac{1}{4} V_2$ ①

N2 $\frac{V_1 - V_2}{4} = \frac{V_2}{j6} + \frac{V_2}{12} \rightarrow \frac{V_1}{4} - V_2 \left[\frac{1}{4} + \frac{1}{j6} + \frac{1}{12} \right] = 0$

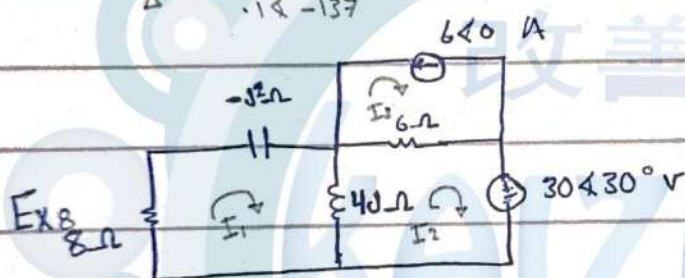
$\frac{1}{4} - \frac{1}{j3}$	$-\frac{1}{4}$	V_1	340
$\frac{1}{4}$	$-\frac{1}{4} + \frac{1}{j6} + \frac{1}{12}$	V_2	0

$\Delta = \begin{vmatrix} \frac{1}{4} - \frac{1}{j3} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} + \frac{1}{j6} + \frac{1}{12} \end{vmatrix}$

$\Delta = 1.14 \angle -137^\circ$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{1.11 \angle 153^\circ}{1.14 \angle -137^\circ}$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{.75 \angle 0^\circ}{1.14 \angle -137^\circ}$



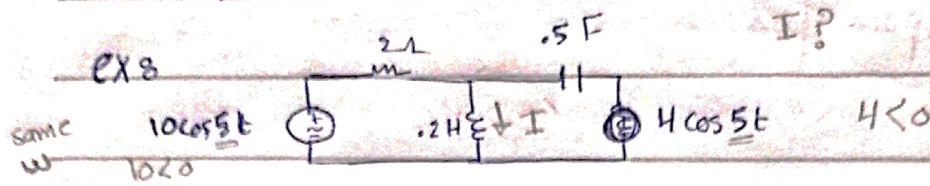
$I_3 = -6 \angle 0^\circ = 6 \angle 180^\circ \text{ A}$

N1 $\rightarrow 8I_1 - j2I_1 + j4(I_1 - I_2) = 0$

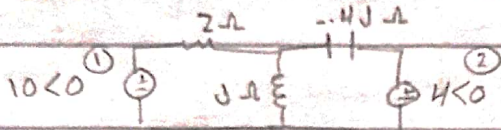
N2 $\rightarrow 30 \angle 30^\circ + 4j(I_2 - I_1) + 6(I_2 - 6 \angle 180^\circ) = 0$

then cramer's rule

AC superposition



$$Z_L = 2 \times 5 \times j = j10, Z_C = \frac{1}{.5 \times 5 \times j} = -j4$$



Kill ② & Keep ①

$$-j4 \parallel j10 = .67 \angle -90^\circ$$

$$V_1 = 10 \angle 0^\circ \left(\frac{.67 \angle -90^\circ}{2 + .67 \angle -90^\circ} \right)$$

$$I_1 = \frac{V_x}{j} \text{ A}$$

Kill ① & Keep ②

$$2 \parallel j10 = Z_{eq}$$

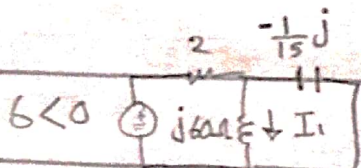
$$V_1 = 4 \angle 0^\circ \left(\frac{Z_{eq}}{-j4 + Z_{eq}} \right)$$

$$I_2 = \frac{V_x}{j} \text{ A}$$

$$I = I_1 + I_2$$



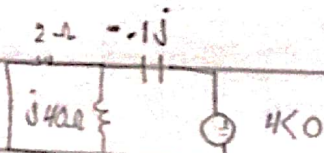
So, we have to use superposition



$$-j15 \parallel j60 = Z_{eq}$$

$$V_x = 6 \angle 0^\circ \left(\frac{Z_{eq}}{2 + Z_{eq}} \right) = V$$

$$I_1 = \frac{V}{60j}$$

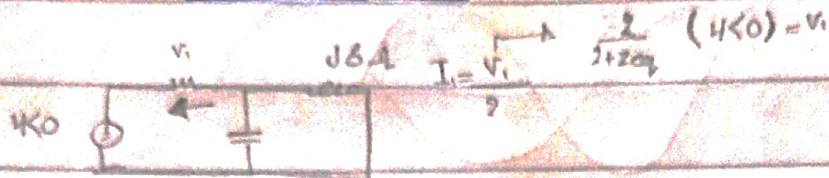
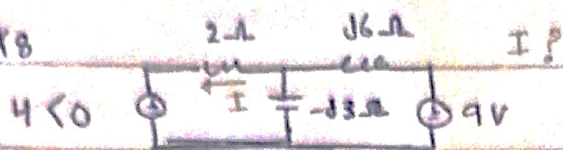


$$2 \parallel j40 = Z_{eq}$$

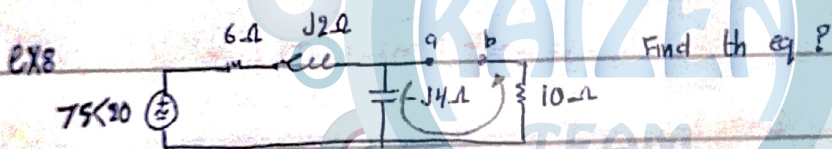
$$V_x = 4 \angle 0^\circ \left(\frac{Z_{eq}}{-j15 + Z_{eq}} \right) = V$$

$$I_2 = \frac{V}{60j}$$

EX8



AC Thevenin & Norton



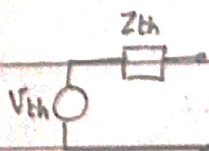
$$Z_{th} = 2j + 6 \parallel -j4 + 10 = 12.8 \angle 47^\circ \Omega = Z_{th}$$

$$= 12.4 - 3.2j \Omega$$

this is a capacitive load (the sign of j or angle is =

$$V_{ab} = V_{oc} + V_c = 47.4 \angle -51^\circ V$$

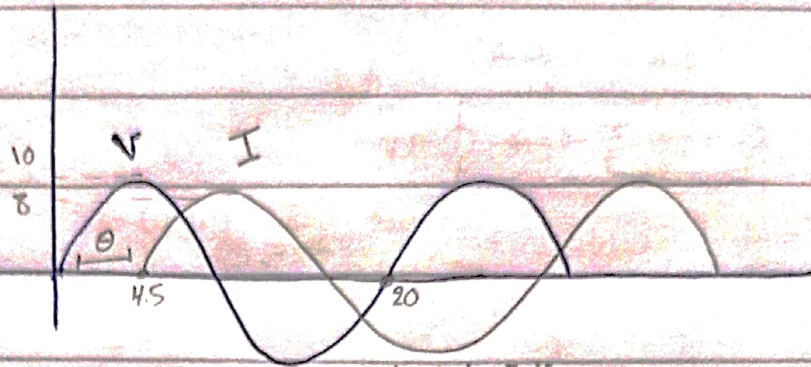
$$V_c = 75 \angle 30^\circ \left(\frac{-j4}{6 + j2 - j4} \right) = 47.4 \angle -51^\circ V$$



Q8 $Z = 4 + j7$ (inductive load) $\oplus$ $\frac{4 \Omega}{\parallel} \frac{j7 \Omega}{\text{series}}$

$Z = 3 - 2j$ (capacitive load) $\ominus$ $\frac{3}{\parallel} \frac{-2j}{\parallel}$

ex8



V leads I

phase shift?

1 period of V

$$20 \rightarrow 360^\circ$$

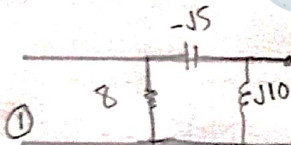
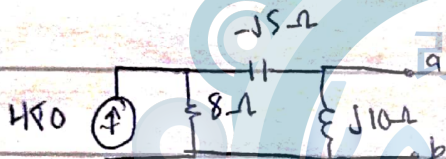
$$4.5 \rightarrow \theta$$

$$\theta = \frac{360 \times 4.5}{20} = 81^\circ$$

$$V = 10 \sin\left(\frac{2\pi}{20}t + 0\right)$$

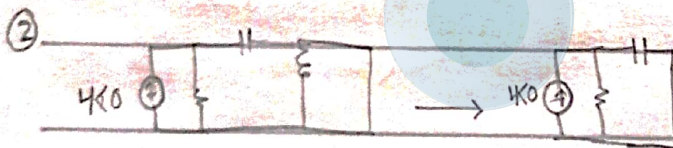
$$I = 8 \sin\left(\frac{2\pi}{20}t - 81^\circ\right)$$

ex8

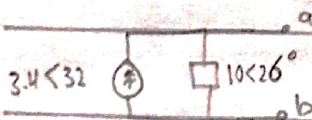


$$Z_N = 8 - j5 \parallel j10 = 10 \angle 26^\circ \text{ (inductive load } \oplus \theta)$$

$$= .98 + j1.38j$$



$$I_N = 4k \left(\frac{8}{8 - j5} \right) = 3.4 \angle 32^\circ \text{ A}$$



• AC Power 1

general Form

$$P = I^2 R = I \cdot V = \frac{V^2}{R} \quad (W)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

I inst. Power $P(t) = v(t) \cdot i(t) \quad (W)$

$$P(t) = V_m \cos(\omega t + \theta_v) \cdot I_m \cos(\omega t + \theta_i)$$

$$\rightarrow \cos A \cos B = \frac{1}{2} \cos(A-B) + \frac{1}{2} \cos(A+B)$$

$$P(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} \cos(2\omega t + \theta_v + \theta_i)$$

Indp. on t (DC)

Power

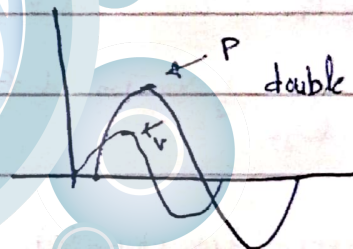
dep. on t (AC)

Power

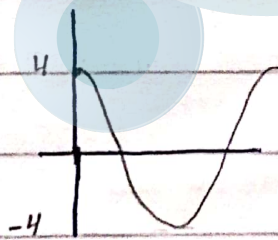
$$P_{DC} = \frac{1}{2} |V_m| |I_m| \cos(\theta_v - \theta_i) \quad \leftarrow \text{avg Power}$$

$$P_{AC} = \frac{1}{2} \cos(2\omega t + \theta_v + \theta_i)$$

2F



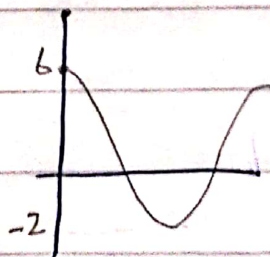
ex8 $4 \cos 10t$



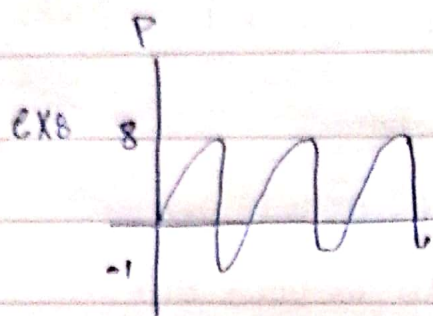
$$P_{DC} + 4 \cos 10t$$

DC Power

AC power



W For current = 5 rad/s



$$8 - 1 = 7$$

$$\therefore 3.5 = P_{DC}$$

$$* 3 \angle 20^\circ \cdot 4 \angle 50^\circ = 12 \angle 70^\circ$$

$$* \frac{3 \angle 20^\circ}{2 \angle 15^\circ} = 1.5 \angle 5^\circ$$

$$P_{DC} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$Z = \frac{V \angle \theta_v}{I \angle \theta_i} = \frac{|V|}{|I|} \angle \overbrace{\theta_v - \theta_i}^{\theta_z}$$

C $\text{---} \parallel P_{avg} = P_{DC} = \text{zero}$ (I leads v by 90° $\cos 90^\circ = 0$)

L $\text{---} \text{---} P_{avg} = P_{DC} = \text{zero}$ (v leads I by 90° $\cos 90^\circ = 0$)

R $\text{---} \text{---} P_{DC} = \frac{1}{2} V_m I_m$ ($\theta_v = \theta_i$ $\cos \theta = 1$)
 $P_{avg} =$

EX8 $v(t) = 4 \cos(10t - 30^\circ)$

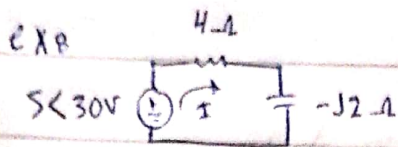
$i(t) = 2 \cos(10t + 40^\circ)$

$$P_{avg} \rightarrow P_{DC} = \frac{1}{2} \cancel{V_m}^4 \cancel{I_m}^2 \cos(-30 - 40) = 4 \cos(-70) = 1.37 \text{ W}$$

EX8 $I = 4 \angle 30^\circ$ $Z = 5 \angle 65^\circ$ $\rightarrow$ inductive load

$$P = \frac{1}{2} V_m I_m \cos(\overbrace{\theta_v - \theta_i}^{\theta_z}) = \frac{1}{2} \times 20 \times 4 \times \cos 53 = 24.07 \text{ W}$$

$$v = 4 \angle 30^\circ \cdot 5 \angle 53^\circ = 20 \angle 83^\circ$$



I ?

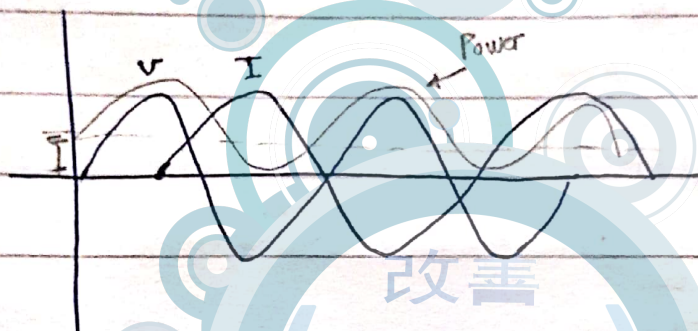
$$I = \frac{5\angle 30}{4-j2} = 1.12 \angle 56.6^\circ \text{ A}$$

$$P_{\text{source}} = P_{\text{DC}} = \frac{1}{2} \times 5 \times 1.12 \times \cos(30 - 56.6) = 2.5 \text{ W}$$

$$\text{Power consumed in } R = P_R = 2.5 \text{ W}$$

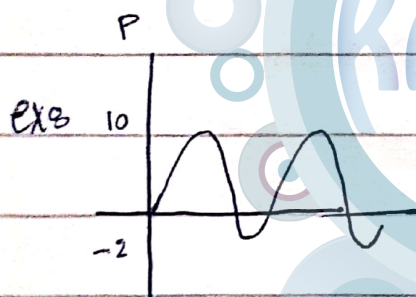
$$P_{\text{DC}} = \text{Zero}$$

$$= \frac{1}{2} \times I^2 \times R = 2 \times (1.12)^2 = 2.508 \text{ W} \#$$



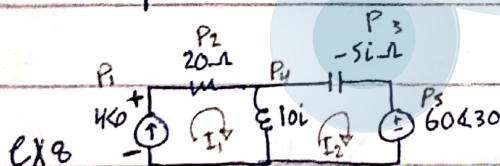
$$W_P = 2W_V = 2W_I$$

$$P_{\text{avg}} = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_I)$$



$$P_{\text{avg}} = 4 \text{ W}$$

$$\begin{matrix} 12 \\ -6 \end{matrix}$$



Find P For each elements

$$I_1 = 4 \angle 0, (10i - 5i) I_2 - 10i(4\angle 0) + 60\angle 30 = 0 \rightarrow I_2 = 10.58 \angle 79.1^\circ \text{ A}$$

$$P_1 = -\frac{1}{2} \times 4 \times 185 \times \cos(6.21 - 0) = -367.84 \text{ W}$$

$$P_3 = P_4 = \text{Zero}$$

المقدار

$$V_1 = V_{20\Omega} + V_{10i} = 20(I_1) + 10i(I_1 - I_2) = 185 \angle 6.21$$

$$P_2 = \frac{1}{2} \times 4^2 \times 20 = 160 \text{ W} \quad P_5 = \frac{1}{2} \times 60 \times 10.58 \cos(30 - 79.1) = 207.8 \text{ W}$$

$$\sum P_{\text{avg}} = \text{Zero} \#$$

Reactive Power is complex Power

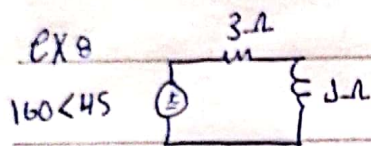
reactive $Q = \frac{1}{2} |V_m| |I_m| \sin(\theta_v - \theta_i)$ in VAR unit

for $R \rightarrow \sin \theta = \text{zero}$ so $Q = \text{zero}$ $L \rightarrow Q \checkmark \oplus$ $C \rightarrow Q \checkmark \ominus$

ex8 $v(t) = 4 \cos(10t + 60)$ $i(t) = 2 \cos(10t - 40)$ P_{DC} or P_{avg} or P_{max} ?

$P = \frac{1}{2} \times 4 \times 2 \times \cos(60 + 40)$ $P(t) = 4(\cos(10t + 60) \times 2 \cos(10t - 40))$

$Q = \frac{1}{2} \times 4 \times 2 \times \sin(60 + 40) = 4(\cos(100) + \cos(20t + 20))$



Find P

$I = \frac{160 \angle 45}{3 + j1} = 50.6 \angle 26.5^\circ$

$P_{\text{source}} = \frac{1}{2} \times 160 \times 50.6 \times \cos(45 - 26.5) = 3740 \text{ W}$

$P_L = \text{zero}$

$Q_R = \text{zero}$, $Q_{\text{source}} = \frac{1}{2} \times 160 \times 50.6 \times \sin(45 - 26.5) = 1284.4 \text{ VAR}$

$V_R = 50.6 \angle 26.5 \times 3 = 151.8 \angle 26.5 \Rightarrow P_R = \frac{1}{2} \times 151.8 \times 50.6 \times \cos 0 =$

$Q_L = \frac{1}{2} \times 50.6 \times 50.6 \times \sin 90 = 1280 \text{ VAR}$

$V_L = 50.6 \angle 26.5 \times j = 50.6 \angle 116.5^\circ \text{ V}$

complex $P \rightarrow \text{VA unit}$

$S = P + jQ$

$S_R = P$

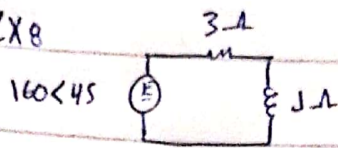
$S_L = jQ$, $S_C = -jQ$

$S = \frac{1}{2} \vec{V} \vec{I}^*$

$I = 3 + 4j \rightarrow I^* = 3 - 4j$

$I = 5 \angle 53^\circ \rightarrow I^* = 5 \angle -53^\circ$

EX8



$$I = 50.6 \angle 26.5^\circ \text{ A} \quad S ?$$

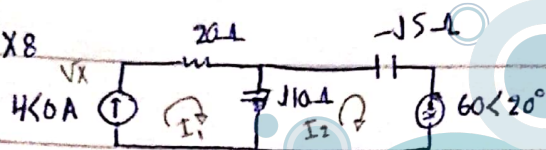
$$S_{\text{source}} = \frac{1}{2} * 160 \angle 45^\circ * 50.6 \angle -26.5^\circ = 4048 \angle 18.5^\circ \text{ VA}$$

$$P = 3838 \text{ W} \quad Q_S = 1284 \text{ VAR}$$

$$= \frac{3838 + j1284}{P}$$

$$S_R = P + jQ = \frac{1}{2} * 151.8 * 50.6 \cos 0 =$$

EX8



Find S For each element?

$$I_1 = 4 \angle 0^\circ \text{ A}, \quad -j5I_2 + 60 \angle 20^\circ + 10j(I_2 - I_1) = 0 \rightarrow I_2 = 10.58 \angle 79.1^\circ$$

$$S_{S1} = \frac{1}{2} (185 \angle 6.2^\circ) (4 \angle 0^\circ) = 370 \angle 6.2^\circ = \frac{P}{367.8} + j \frac{Q}{38.9}$$

$$V_X = V_{20} + V_{10j} = 20(4 \angle 0^\circ) + j10(I_1 - I_2) = 185 \angle 6.2^\circ \text{ V}$$

Power Factor $P.F. = \cos \theta = \cos \theta_s$

$$S_{S2} = \frac{1}{2} (60 \angle 20^\circ) (10.58 \angle -79.1^\circ) = 317.4 \angle -59.1^\circ = \frac{P}{163} - j \frac{Q}{272} \text{ VA}$$

• RMS & Power Factor

DC $R_L = R_{th}$ For max Power $P_{max} = \frac{V_{th}^2}{4 R_{th}}$

AC To have max Power $Z_L = Z_{th}^*$ ($Z_{th} = 3 \angle 60^\circ \rightarrow Z_L = 3 \angle -60^\circ$)

$P_{max} = \frac{V_{th}^2}{8 R_{th}}$ $Z_{th} = 5 \angle 53^\circ = 3 + j4$



$I = \frac{10 \angle 0}{12 - j6}$

$Z_{th} = 2.93 + j4.46$, $V_{th} = V_{8\Omega} + V_{-j6} = \frac{10 \angle 0 \cdot (8 - j6)}{8 + 4 - j6} = 7.45 \angle -10.3^\circ$

$P_{max} = \frac{7.45^2}{8(2.93)} = 2.36 \text{ W}$

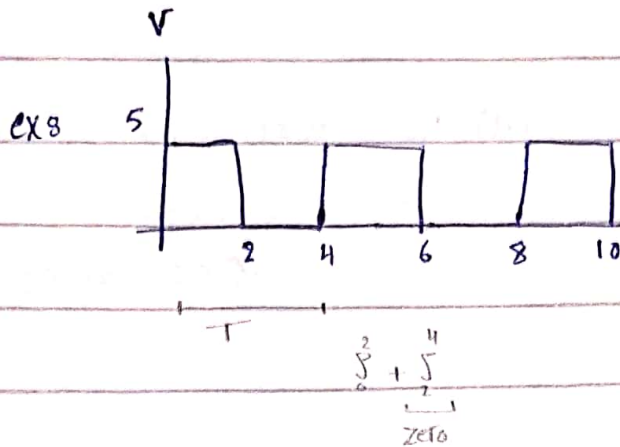
max value (Peak)
 $v(t) = A \cos(\omega t + \theta)$

Root Mean Square value

RMS

$X_{rms} = \sqrt{\frac{1}{T} \int_0^T X^2(t) dt}$ If sinusoidal $X_{rms} = \frac{A}{\sqrt{2}}$

ex9 $i(t) = 10 \sin(100t)$ $I_{max} = 10$, $I_{rms} = \frac{10}{\sqrt{2}}$



$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

$$= \sqrt{\frac{1}{4} \int_0^2 5^2 dt}$$

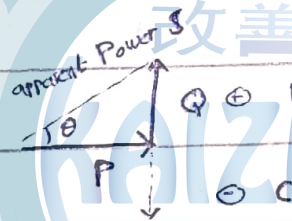
$$= \sqrt{\frac{1}{4} \times 25 \times 2} = \sqrt{12.5}$$

$$\rightarrow P_{avg} = \frac{1}{2} |V_m| |I_m| \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \sqrt{2} V_{RMS} \sqrt{2} I_{RMS} \cos(\theta_v - \theta_i) = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

$$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)$$

Power triangle



$$\theta = \tan^{-1} \left(\frac{Q}{P} \right)$$

$$\cos \theta = \frac{P}{S} \quad (\text{Power Factor})$$

$$0 < PF < 1$$

$$S = P \pm jQ$$

① inductor $\rightarrow$

$$\theta > 0$$

leading Power Factor

$$\theta > 0 \quad \theta < 0$$

② capacitor $\rightarrow$

$$\theta < 0$$

lagging P F

$$\theta < 0 \quad \theta > 0$$

$$PF = \cos \theta_z = \cos(\theta_v - \theta_i), \quad \theta = \tan^{-1} \left(\frac{Q}{P} \right), \quad \theta = \cos^{-1} \left(\frac{P}{S} \right)$$

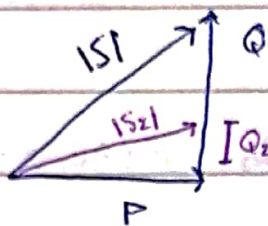
ex8 $i(t) = 4 \cos(100\pi t + 10^\circ) \text{ A}$, $v(t) = 120 \cos(100\pi t - 20^\circ) \text{ V}$

$|S| = \frac{1}{2} \times 120 \times 4 = 240 \text{ VA}$

$PF = \cos(10 - 20 - 10) = .866 \text{ (leading)}$

$\phi < 0 \text{ leading}$

Power Factor correction



we can correct the PF by adding a Capacitor in Parallel with voltage source $(-Q)$

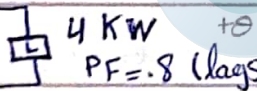
$Q = P \tan \theta_1$

$Q_2 = P \tan \theta_2 \rightarrow Q_c = Q_1 - Q_2 = P \tan \theta_1 - P \tan \theta_2$

$I^2 Z = Q_c = \frac{V_{rms}^2}{V_c} = \frac{V_{rms}^2}{\frac{1}{\omega C}} \rightarrow C = \frac{Q_c}{\omega V_{rms}^2} \rightarrow C = \frac{P \tan \theta_1 - P \tan \theta_2}{\omega V_{rms}^2}$

ex8

120
V_{rms}



4 KW
PF = .8 (lags)

P = 4 KW

$\theta_1 = \cos^{-1}(PF_1) = 36.8^\circ$

f = 60 Hz

Find C to PF = .95?

$\theta_2 = \cos^{-1}(.95) = 18.2^\circ$

$C = \frac{4 \times 10^3 (\tan 36.8 - \tan 18.2)}{2\pi(60) \times 120^2} = 310.5 \mu\text{F}$

$\cos \theta_1 \cdot S_{old} = P \rightarrow S_{old} = \frac{4000}{\cos .8} = 5000 \text{ VA}$, $S_{new} = \frac{4000}{\cos .95} = 4210 \text{ VA}$

$$V_{rms} = 110 \text{ V} \quad F = 60 \text{ Hz}$$

EX 8 140 KVAR, $PF_1 = .85$ (lag) $\rightarrow PF_2 = 1$ Find C

$$\theta_1 = \cos^{-1}(.85) = 31.8^\circ, \cos^{-1}(1) = 0 = \theta_2, Q = P \tan(\theta_1)$$

$$\tan(\theta_1) = .62$$

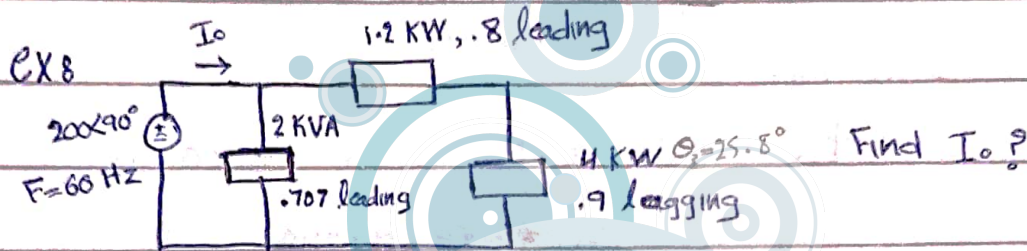
$$\tan(0) = 0$$

$$P = \frac{140 \times 10^3}{.62} = 225806.5$$

$$\frac{225806.5 (.62 - 0)}{2\pi 60 \times 110^2} = .03 \text{ F}$$

$$2\pi 60 \times 110^2$$

$$\theta_2 = -36.9$$



$$S_T = S_{11} + S_{12} + S_{13}$$

$$= 2 \angle \cos^{-1}(.707) + (P_2 + jQ_2) + (P_3 + jQ_3)$$

$$= 2 \angle 45^\circ + (1.2 - j(1.2 \tan \theta_2)) + (4 + j(4 \tan \theta_3))$$

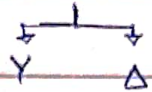
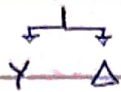
$$S_T = 6.61 - j.04 = 6.62 \angle -3.5^\circ = V_{rms} I_{rms}^*$$

$$I^* = \frac{6.62 \angle -3.5^\circ}{200 \angle 90^\circ}$$

• 3 Phase Y Y

3 Sources with 3 loads

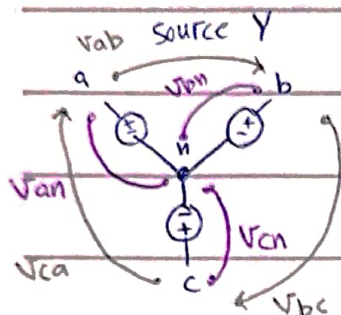
Y-Y * will be required



Y-Δ*

Δ-Y

Δ-Δ*



+ VC Seq.

$$V_{an} = V_P \angle \theta, V_{bn} = V_P \angle \theta - 120^\circ$$

$$V_{cn} = V_P \angle \theta - 240^\circ$$

- VC Seq.

$$V_{an} = V_P \angle \theta$$

$$V_{bn} = V_P \angle \theta + 120^\circ$$

$$V_{cn} = V_P \angle \theta + 240^\circ$$

ex8 $V_{an} = 10 \angle 30^\circ$ + VC Seq. $\rightarrow V_{bn} = 10 \angle -90^\circ, V_{cn} = 10 \angle -210^\circ$

$V_{an} = 50 \angle -10^\circ$ + VC Seq. $\rightarrow V_{bn} = 50 \angle -130^\circ, V_{cn} = 50 \angle -250^\circ$

$V_{an} = 10 \angle 60^\circ$ - VC Seq. $\rightarrow V_{bn} = 10 \angle 180^\circ, V_{cn} = 10 \angle 300^\circ$

V_{an}

V_{bn}

V_{cn}

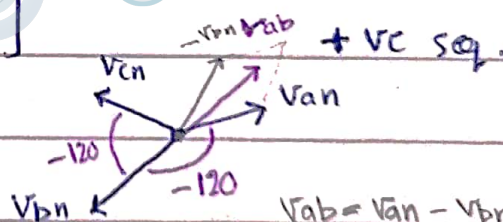
Phase voltage

V_{ab}

V_{bc}

V_{ca}

line voltage



* IF I have 1 Phase voltage

I can Find the other

Phase voltages at the line voltages

Phase

line

ex 8

$$V_{an} = 10 \angle 50^\circ$$

$$\rightarrow V_{ab} = \sqrt{3} \cdot 10 \angle 50 + 30$$

$$V_{bn} = 10 \angle -70^\circ$$

$$V_{bc} = \sqrt{3} \cdot 10 \angle 80 - 120$$

$$V_{cn} = 10 \angle -190^\circ$$

$$V_{ca} = \sqrt{3} \cdot 10 \angle -40 - 120$$

$$\Rightarrow * \sqrt{3} \angle +30^\circ$$

$$\frac{1}{\sqrt{3}} \angle -30^\circ \leftarrow$$

ex 1 F $V_{ac}^{ac} = 100 \angle 0$ Find all the voltages

$$V_{ac} = -V_{ca} \rightarrow V_{ca} = 100 \angle 180^\circ$$

120

$$V_{bc} = 100 \angle 300^\circ$$

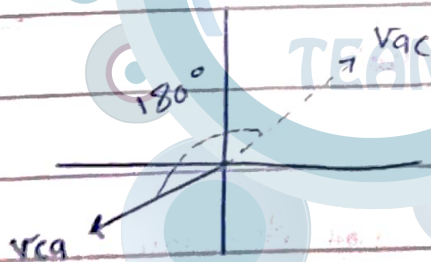
$$V_{ab} = 100 \angle 60^\circ$$

$$\rightarrow 300 + 120 = 420 - 360 = 60$$

$$V_{an} = \frac{100}{\sqrt{3}} \angle 60 - 30$$

$$V_{bn} = \frac{100}{\sqrt{3}} \angle -90^\circ$$

$$V_{cn} = \frac{100}{\sqrt{3}} \angle -210^\circ$$



ex 8 $V_{an} = 200 \cos(\omega t + 10) = 200 \angle 10 \rightarrow V_{ab} = 200 * \sqrt{3} \angle 40$

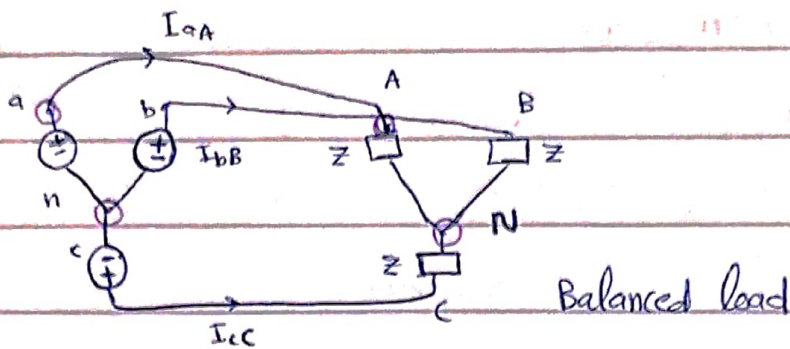
$$V_{bn} = 200 \angle -110^\circ$$

$$V_{bc} = 200 \sqrt{3} \angle -80^\circ$$

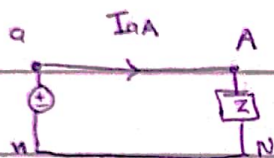
$$V_{cn} = 200 \angle -230^\circ$$

$$V_{ca} = 200 \sqrt{3} \angle -200^\circ$$

YY 2



Because I have Balanced load



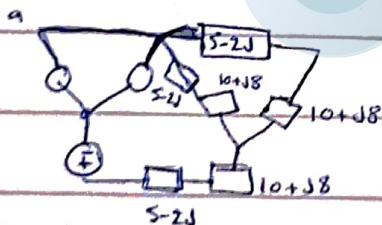
$$I_{aA} = \frac{V_{an}}{Z_y} = I \angle \theta$$

$$I_{bB} = |I| \angle \theta - 120^\circ$$

$$I_{cC} = |I| \angle \theta - 240^\circ$$

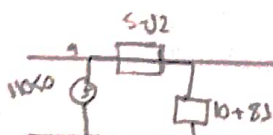
$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = \text{zero} \quad (\because \text{balanced load})$$

Ex 8



$$V_{an} = 110 \angle 0^\circ, \quad Z_1 = 5 - j2, \quad Z_2 = 10 + j8$$

Find I_p



$$I_{aA} = \frac{110 \angle 0^\circ}{15 + j6} = 6.81 \angle -21.8^\circ \text{ A}$$

$$I_{bB} = 6.81 \angle -141.8^\circ \text{ A}$$

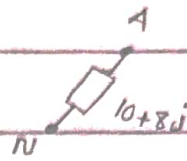
$$I_{cC} = 6.81 \angle -261.8^\circ \text{ A}$$

complex Power consumed by the load

$$V_{AN} = 6.81 \angle -21.8^\circ \times (10 + j8)$$

$$= I_{AN} \times Z_L$$

$$= 87.2 \angle 16.85^\circ \text{ V (V}_m\text{)}$$



$$S_L = \frac{1}{2} V_{AN} I_{AN}^* = \frac{1}{2} (87.2 \angle 16.85^\circ) (6.81 \angle 21.8^\circ) = 296.5 \angle 38.6^\circ$$

$$PF = \cos 36.6$$

$$S_{L_{Total}} = 3 \times S_L$$

complex P in the source (mean all sources)

$$S_s = 3 \times \left(\frac{1}{2} \times 11040 \times 6.8 \angle 21.8^\circ \right)$$

$$= 1192 \angle 21.8^\circ$$

$$P.F. = \cos (21.8)$$

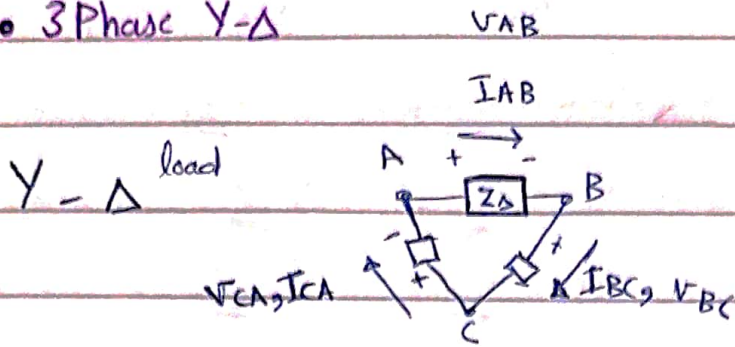
Source lag $\rightarrow$ $\theta \pm$

改善

KAIZEN

TEAM

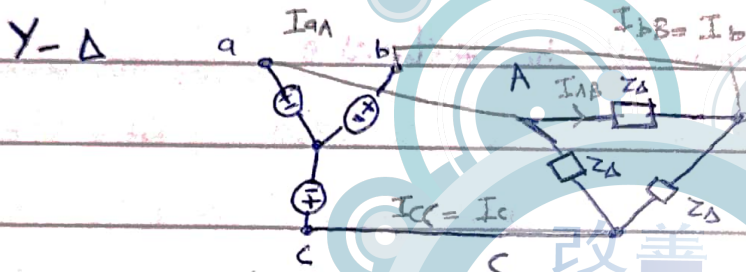
• 3Phase Y-Δ



Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}, \quad I_{BC} = \frac{V_{BC}}{Z_{\Delta}}, \quad I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$$

-120° -120°



• convert the load to

$$Y \rightarrow Z_Y = \frac{Z_{\Delta}}{3}$$

Line current

$$I_{AA} = \sqrt{3} I_{AB} \angle -30^\circ$$

-120° -120°

EX8 Y-Δ system $V_{an} = 100 \angle 0^\circ$ Find phase current & line current s

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{\sqrt{3} 100 \angle 30^\circ}{8 + j4} = 19.36 \angle 33.43^\circ \text{ A}$$

$$Z_{\Delta} = 8 + j4 \Omega$$

or convert load to Y

$$I_{BC} = 19.36 \angle -120 + 33.43^\circ \text{ A}, \quad I_{CA} = 19.36 \angle -240 + 33.43^\circ \text{ A}$$

$$Z_Y = 2.98 \angle 26.5^\circ$$

$$I_{AA} = \sqrt{3} 19.36 \angle 33.43 - 30^\circ$$

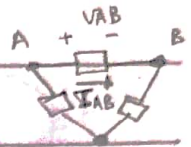
$$I_{AA} = \frac{V_{an}}{Z_Y} = 33.5 \angle -16.5^\circ$$

$$I_{BB} = \text{ // } \angle 33.43 - 150^\circ$$

ex8 $Z_{\Delta} = 20 - j15 \Omega$ $V_{ab} = 330 \angle 0^\circ$ Find Phase & line I & P?

$$I_{AB} = \frac{330 \angle 0^\circ}{20 - j15} = 13.2 \angle 36.8^\circ \text{ A}, I_{BC} = -120^\circ, I_{CA} = -120^\circ$$

$$I_{aA} = \frac{1}{\sqrt{3}} (13.2) \angle -6.8^\circ \text{ A}, I_{bB} = -120^\circ, I_{cC} = -120^\circ$$



Complex P of load & P.F. $\rightarrow S = \frac{1}{2} \times 3 \times (330 \angle 0^\circ) \times (13.2 \angle -36.8^\circ) = 78.74 \angle -36.8^\circ$

In Source $\rightarrow \frac{1}{2} \times 3 \times V_{an} \times I_{aA}^*$

$$= 575 - j431$$

P(W) Q(VA)

$$PF = \cos(-36.8^\circ) = 0.8 \text{ leading}$$

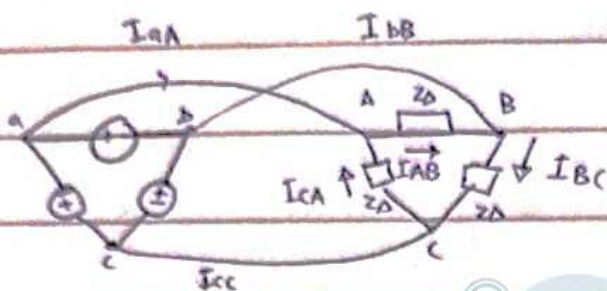
Summary :

Y-Y $S_L = \frac{1}{2} \times 3 \times V_{an} \times I_{aA}^*$ $S_S = \frac{1}{2} \times 3 \times V_{an} \times I_{aA}^*$

Y- Δ $S_L = \frac{3}{2} \times V_{AB} \times I_{AB}^*$ $S_S = \frac{3}{2} \times V_{an} \times I_{aA}^*$

Δ - Δ $S_L = S_S = \frac{1}{2} \times 3 \times V_{AB} \times I_{AB}^*$

3 Phase Δ - Δ



$$V_{ab} = V_{AB}, \quad I_{AB} = \frac{V_{AB}}{Z_{\Delta}} \angle -120^\circ$$

$$I_{BC} = \angle -120^\circ$$

$$I_{CA} = \angle -120^\circ$$

ex8 $Z_{\Delta} = 20 - j5 \Omega$, $V_{ab} = 330 \angle 0^\circ \text{ V}$, Find Phase current & line c

$$I_{AB} = \frac{330 \angle 0^\circ}{20 - j5} = 13.2 \angle 36.86^\circ \text{ A}, \quad I_{BC} = 13.2 \angle -83.14^\circ, \quad I_{CA} = 13.2 \angle -203.14^\circ$$

$$I_{aA} = \sqrt{3} \times 13.2 \angle -6.86^\circ \text{ A}, \quad I_{bB} = 22.86 \angle -170^\circ \dots$$

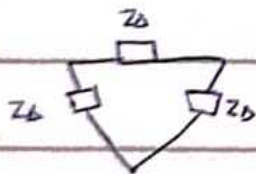
complex Power consumed by the load

$$S = \underbrace{\frac{1}{2}}_{\text{not rms}} \times 3 \times (330 \angle 0^\circ)(13.2 \angle -36.86^\circ) = 718.74 \angle -36.86^\circ = \underbrace{575}_{P} - \underbrace{j431.14}_{Q}$$

PF = $\cos(-36.86^\circ) =$ leading

capacitive load

ex8 $Z_{\Delta} = 18 + j12 \Omega$, $I_{aA} = 9.6 \angle 35^\circ$ Find I_{AB} & V_{AB} s



$$I_{AB} = \frac{9.6 \angle 35^\circ + 30^\circ}{\sqrt{3}}, \quad V_{AB} = I_{AB} \times Z_{\Delta} = 5.54 \angle 65^\circ (18 + j12)$$

$$= 119.84 \angle 98.7^\circ \text{ V}$$

complex Power in load $S = 3 \times (119.84 \angle 98.7^\circ)(5.54 \angle 65^\circ)$