

Subject: Chapter 2

* Current (I) = $\frac{dq}{dt}$, $Q = \int_{t_1}^{t_2} I dt$

* Voltage in point a :- the work needed to bring a charge of 1C to point (a) from ∞ .

$V = \frac{dw}{dq}$, w: work $V = \frac{W}{q}$

$V_{ab} = V_a - V_b$

- When the voltage is Negative: means its the work to move a charge from point to ∞ .

* power & Energy :-

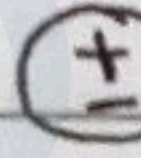
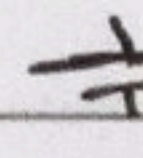
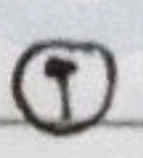
$P = \frac{dw}{dt}$, $w = \int P dt$, [P]: watt , [$\frac{W}{t}$]: Joule

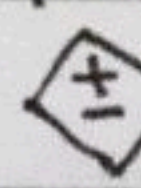
$P = IV$, $w = Pt$

* circuit elements :-

- power source [produce power]
- load [consumes the power]
- connection

① Power source:

a - independent voltage source   (Direct-current), (portable)  $\rightarrow 3A, 4A, 5A$ 

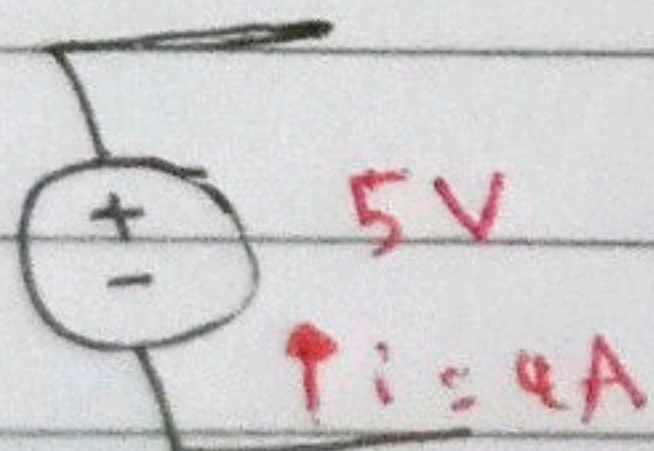
b - dependent power supply  (dependent voltage source)

Ex. $3ix$, $4Vy$, $5Vz$,  (the arrow means current)

(also Direct current DC)

* For any Electrical circuit:-

$\sum P = 0$, $\sum P_{in} = \sum P_{out}$



$P = iV = 4 * 5 = -20 \text{ watt}$ (produce power)

$\downarrow$ goes into negative side.

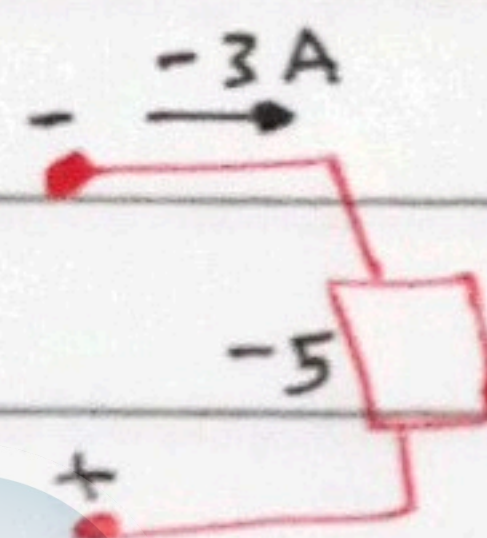
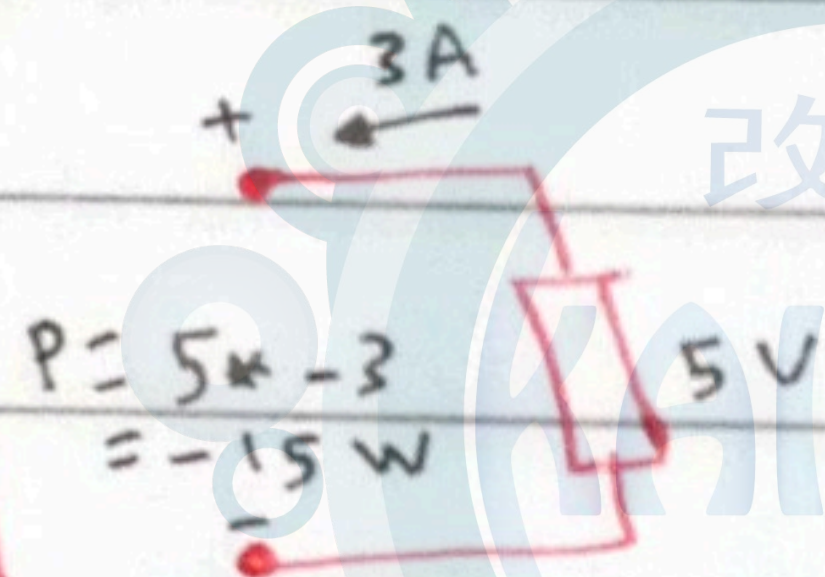
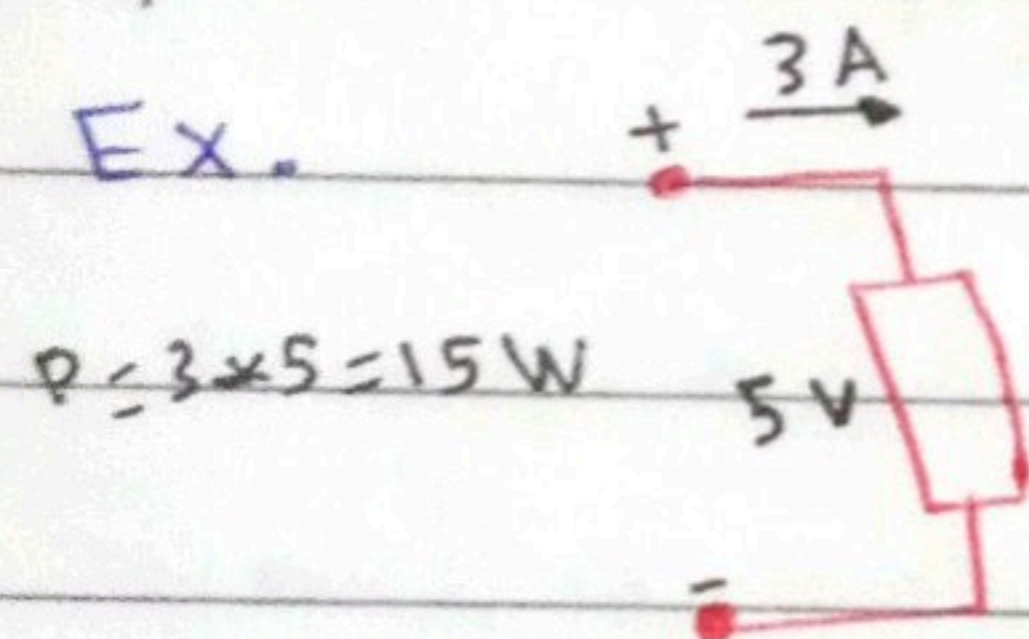
► Subject : _____

* if we got a positive power then its passive element and absorb this power and generate the negative of power.

if we got a negative power then its active element and absorb this power and generate the ~~negative~~ ^{positive} value.

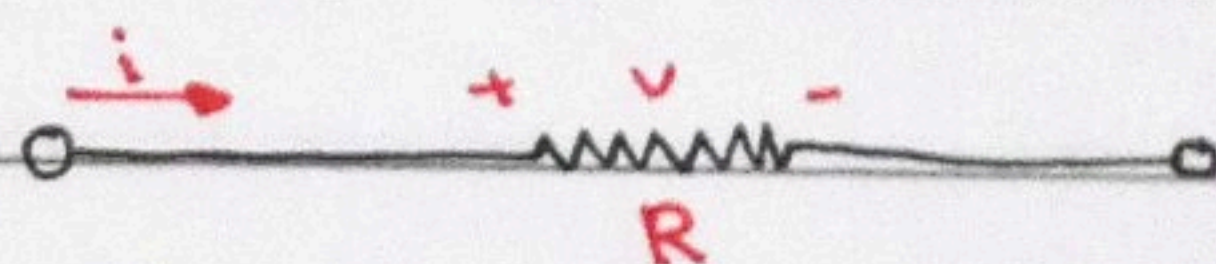
- $P = VI$

Ex.



① Ohm's law

- Resistance $(R) = \rho \frac{L}{A}$



- The resistor is a passive element (always absorb power)

- $V = Ri$

Ex. Find V : $V = Ri$ $V = (2)(1) = 2V$

- Short Circuit S.C: a circuit element with resistance approaching zero, $V = Ri = 0i = 0$, but the current could be anything (Wire).

- Open circuit O.C: a circuit element with resistance approaching to ∞ , $i = \lim_{R \rightarrow \infty} \frac{V}{R} = 0$, but the voltage could be anything.

- Conductance $(G) = \frac{1}{R} = \frac{i}{V}$, $[G] = \text{mhos or siemens (S)}$

- power:-

$$P = Vi = i^2 R = \frac{V^2}{R}$$

$$P = Vi = V^2 G = \frac{i^2}{G}$$

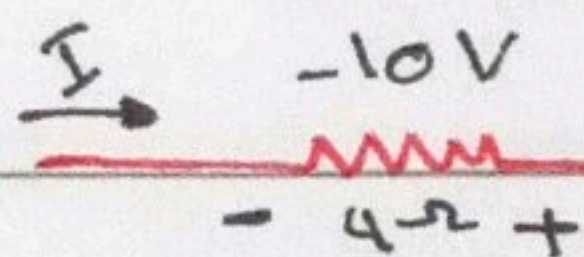
- Node: We can say that it is one node.

- Parallel: Series:

in parallel $\rightarrow$ Same voltage. $\frac{1}{\Sigma R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

in series $\rightarrow$ Same current. $\Sigma R = R_1 + R_2 + R_3 + \dots$

Ex. The current goes in negative in the circuit.



$$-10 = 4 * I \Rightarrow I = 2.5$$

* KCL: $\Sigma I_{in} = \Sigma I_{out}$

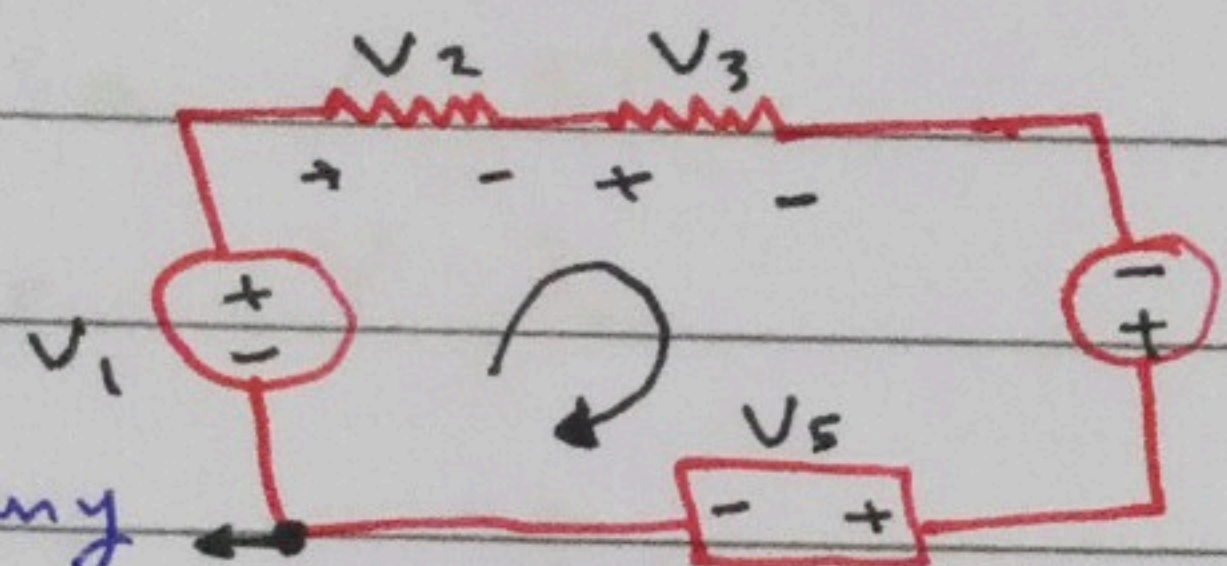
- if current sources are parallel then we can substitute them by an equivalent one.

- if voltage sources are in series then we can substitute them by an equivalent one.

► Subject :

⊗ KVL :- $\sum V$ around a closed path is zero.

Ex.

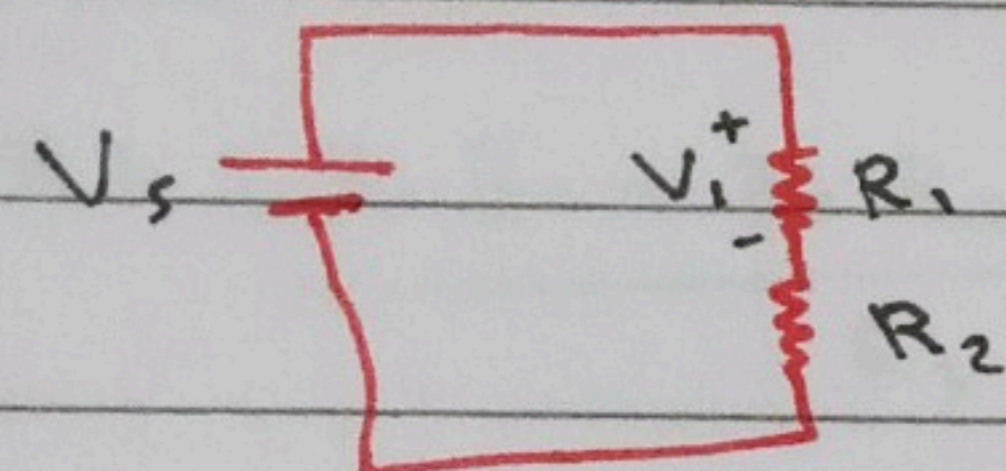


$$:- -V_1 + V_2 + V_3 - V_4 + V_5 = 0$$

choose any point

- Mesh: a loop doesn't contain a loop.

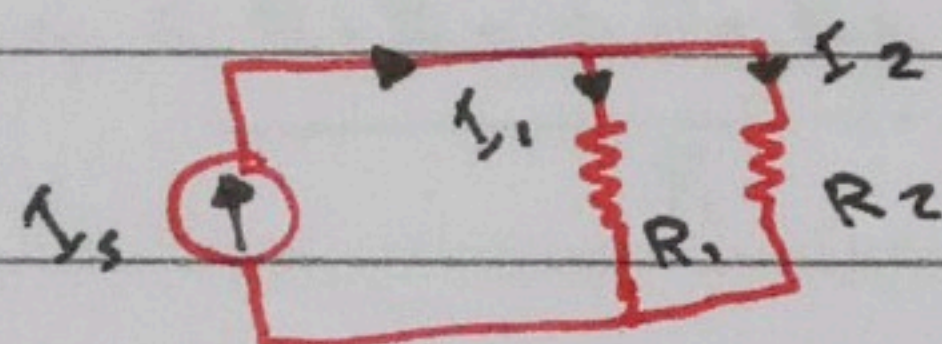
⊗ Voltage division :- (عند توزيع الجهد)



$$V_1 = V_s \times \frac{R_1}{R_1 + R_2}$$

$$V_n = V_s \times \frac{R_n}{\sum R}$$

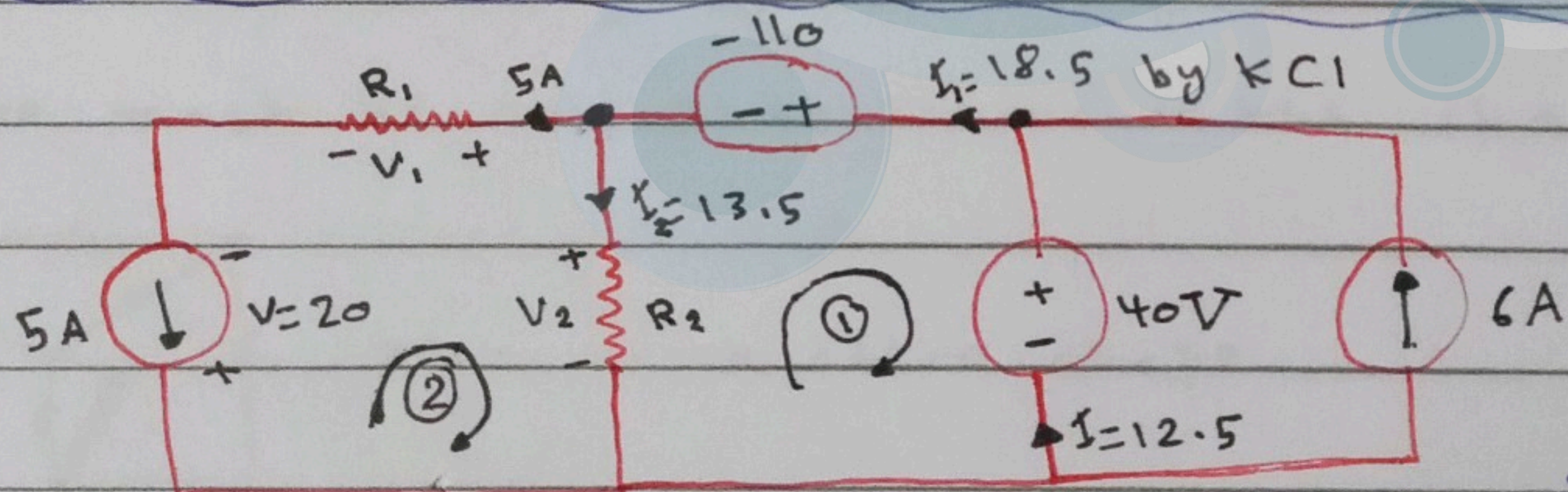
⊗ Current division :- (عند توزيع التيار)



$$I_1 = I_s \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$I_n = I_s \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots}$$

Ex.



Supplied $\rightarrow$ generated
means that the current goes into negative

given : $P_{5A} \text{ (supplied)} = 100 \text{ W} = IV = 5V$, $V = 20$

$P_{40V} \text{ (supplied)} = 500 \text{ W} = IV = 40I$, $I = 12.5$

- at loop ① :- $-13.5 R_2 - (-110) + 40 = 0$, $R_2 = \frac{150}{13.5}$, $V_2 = 150$

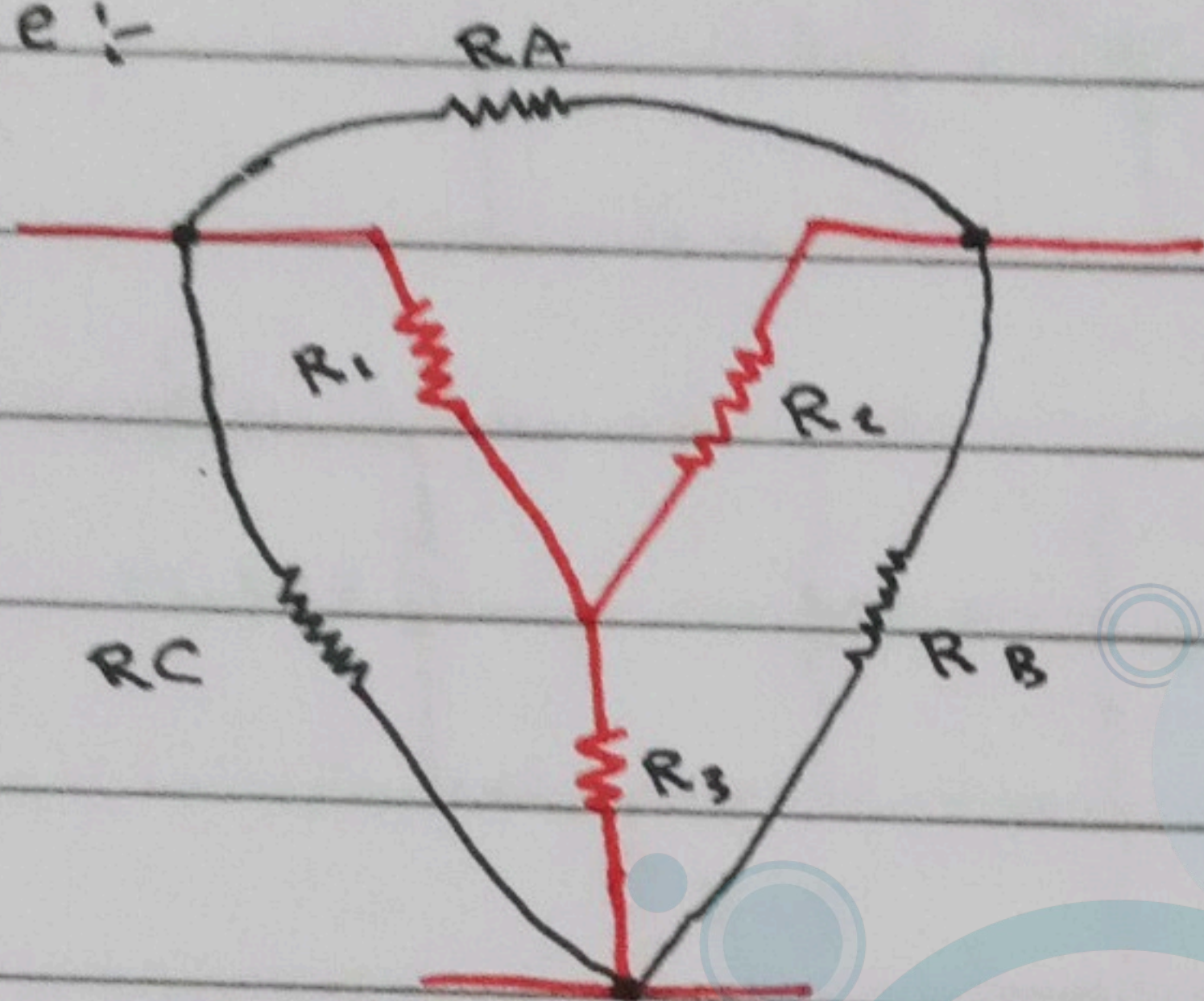
- at loop ② :- $20 - V_1 + 150 = 0$, $V_1 = 170$, $R_1 = \frac{170}{5} = 34$

⊗ Delta - wye - connection:-

- Delta $\triangle \nabla$

- Wye $Y \lambda$

① Wye:-

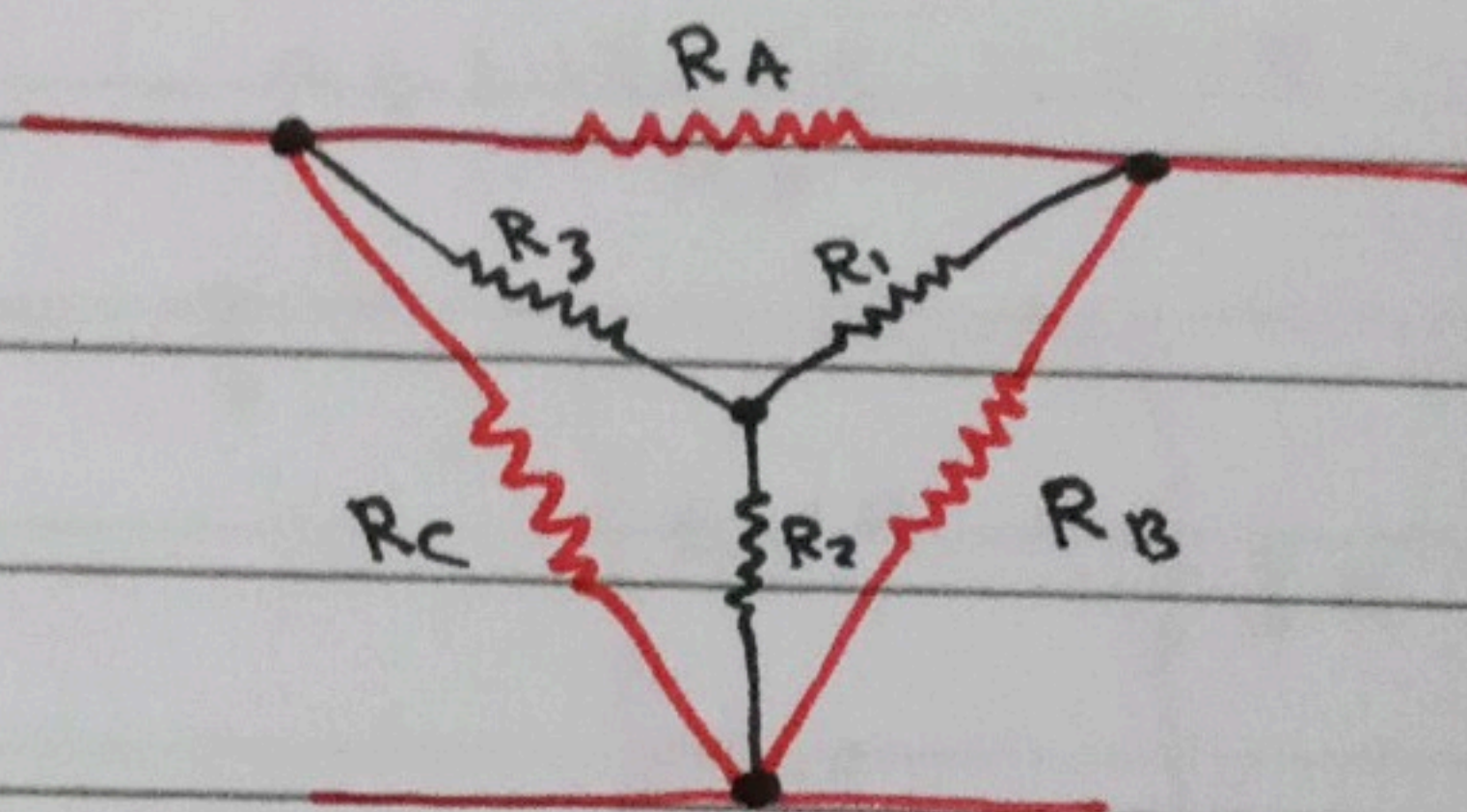


$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

② Delta:-



$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

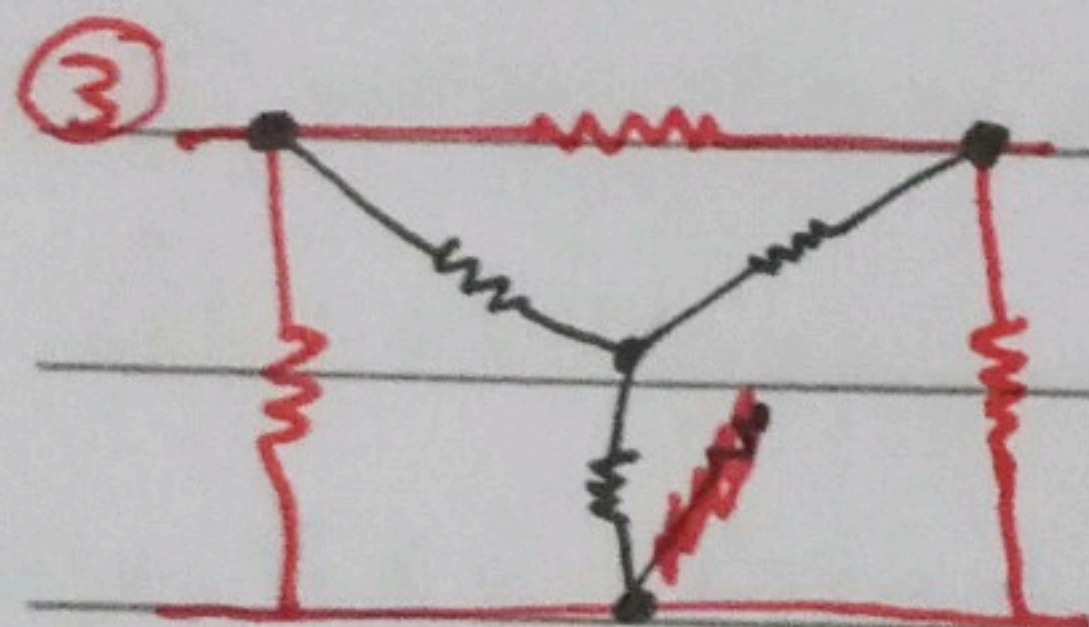
$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

- Notes:-

① Make delta $\rightarrow$ wye, Make wye $\rightarrow$ delta

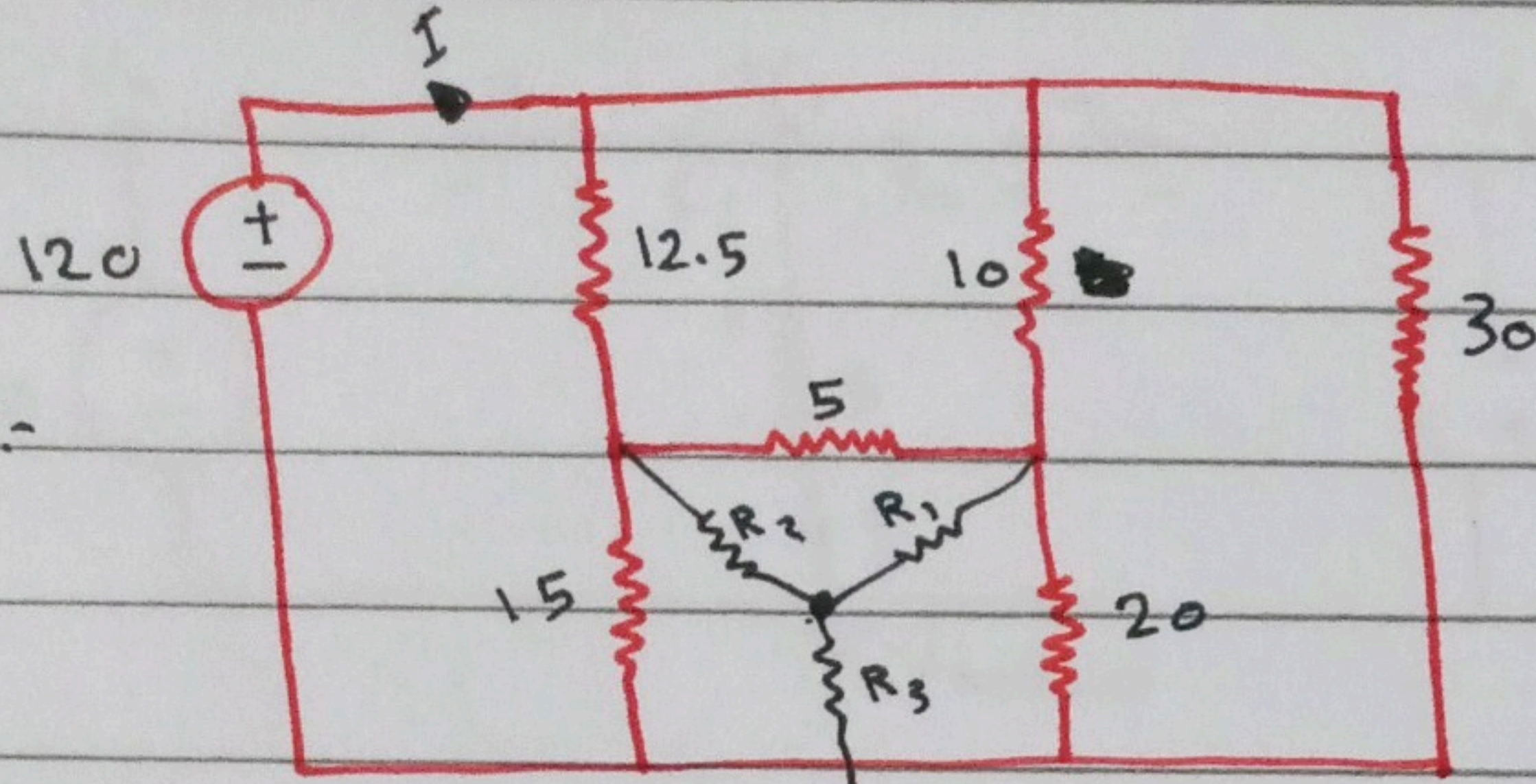
② There must be only 3 resistors in the shape.



This is a delta shape.

- Ex.

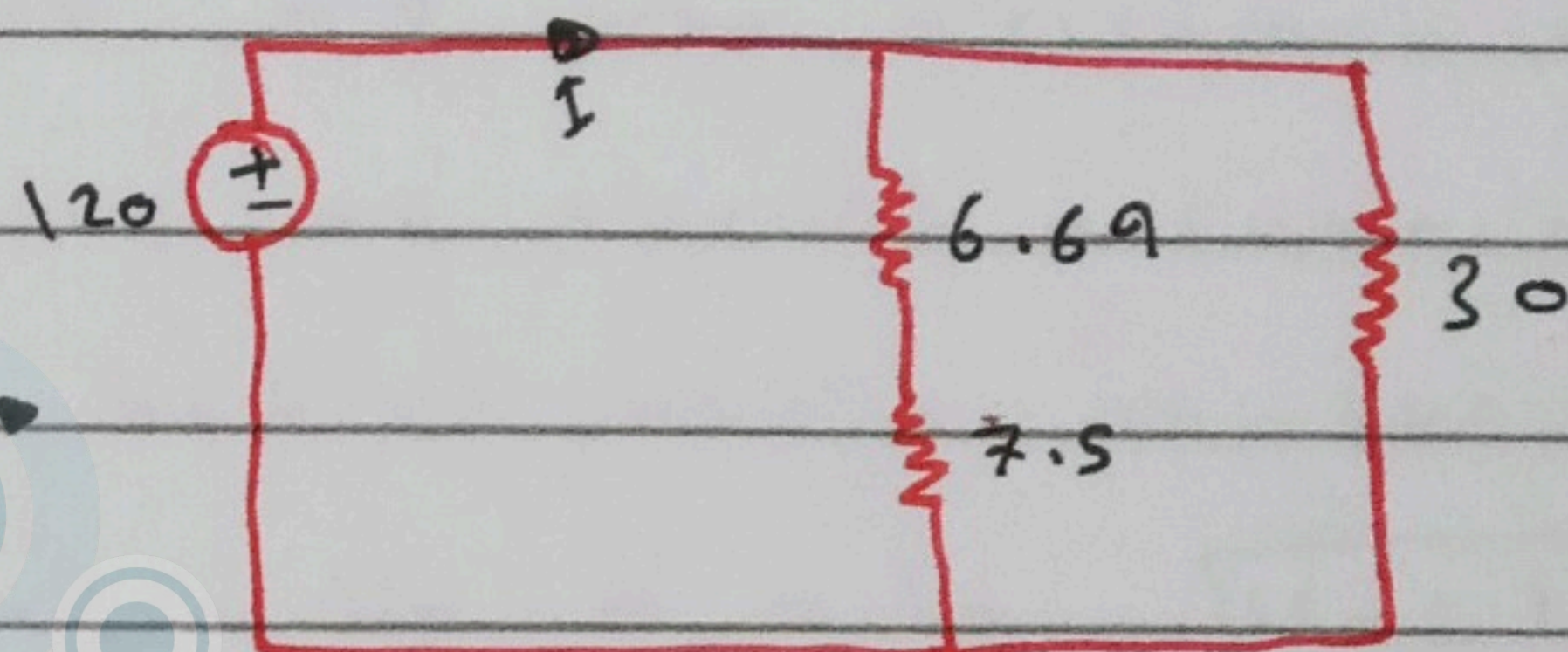
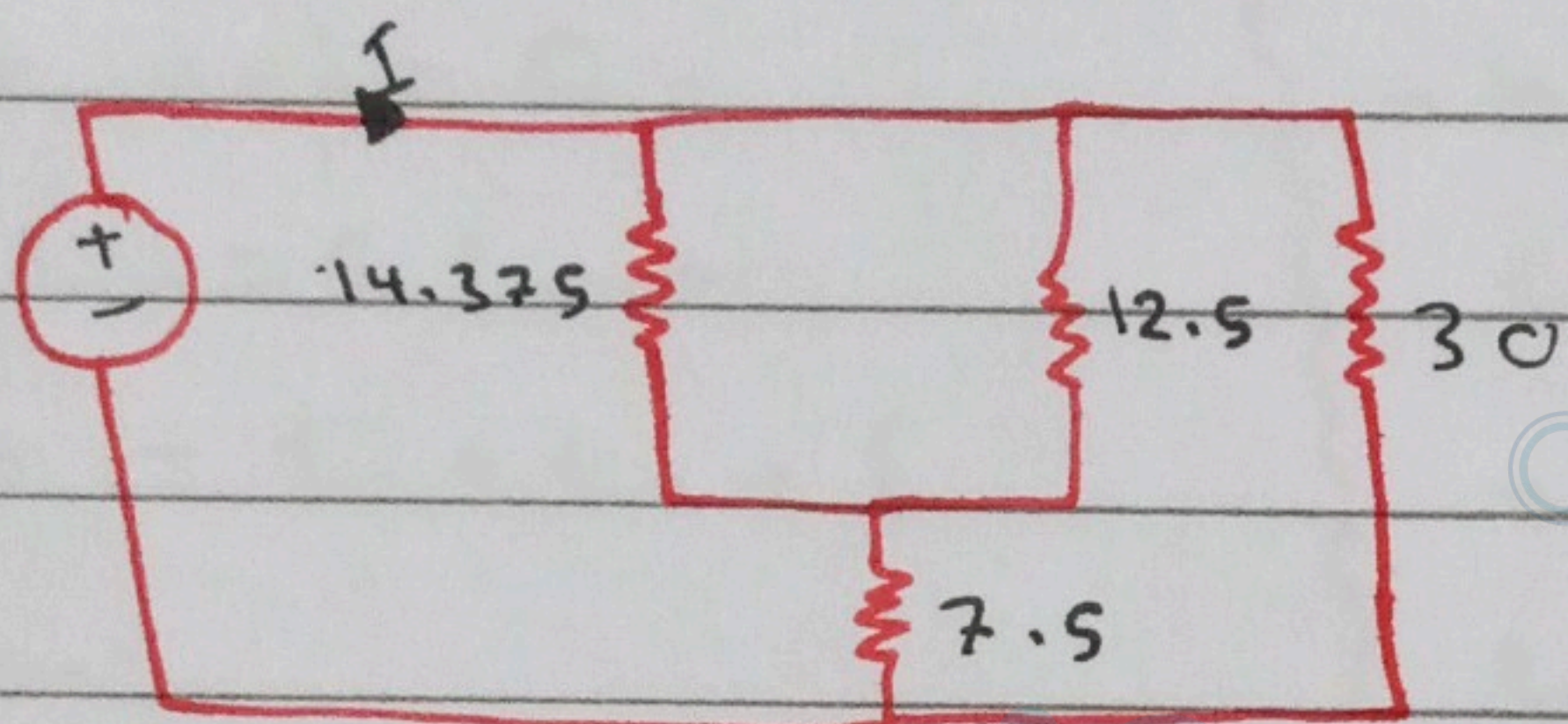
find I :



$$R_1 = \frac{5 \times 20}{40} = 2.5$$

$$R_2 = \frac{5 \times 15}{40} = 1.875$$

$$R_3 = \frac{15 \times 20}{40} = 7.5$$



* Fundamental techniques for ~~the~~ circuit analysis.

① Nodal analysis.

② Mesh analysis.

- First: Nodal analysis:-

① Determine the Nodes.

② assume one of the nodes as Reference ($V_{ref} = 0$).

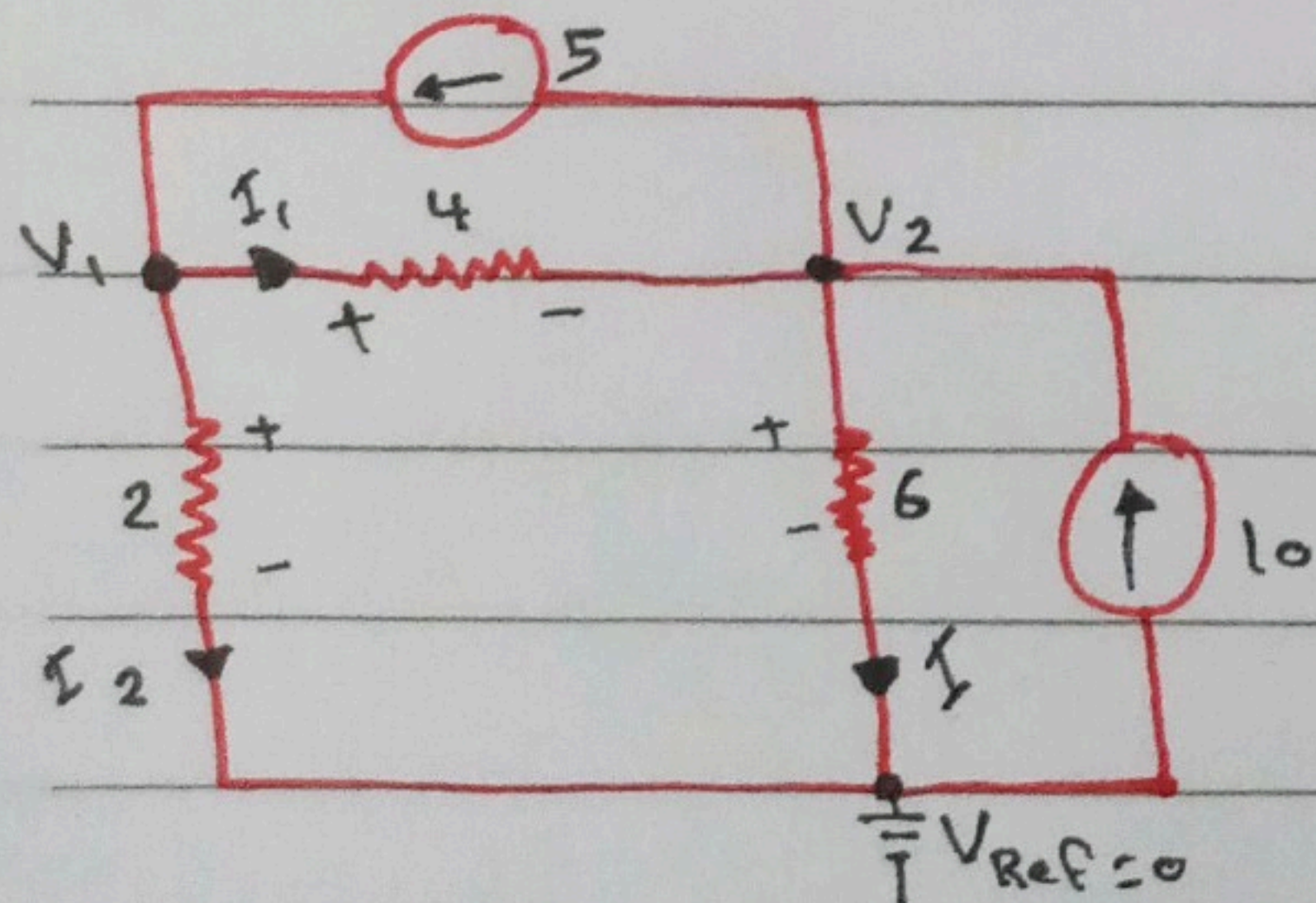
③ Name the nodes $\rightarrow V_1, V_2, \dots$

④ Apply KCL at each node.

⑤ use Ohm's law to convert $I \rightarrow V$. $I = \frac{V_{high} - V_{low}}{R}$

⑥ Solve.

- Ex. find I :



at node ①

$$\sum I_{in} = \sum I_{out}$$

$$5 = I_1 + I_2$$

$$5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$$

$$20 = V_1 - V_2 + 2V_1$$

$$20 = 3V_1 - V_2 \quad \text{--- (1)}$$

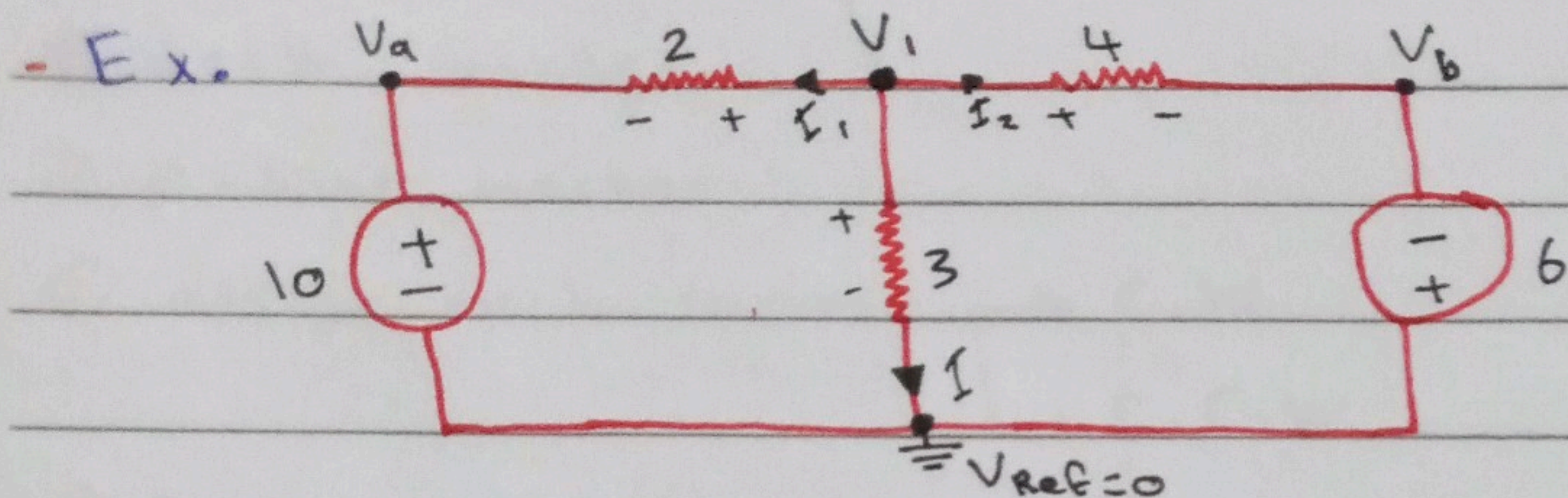
at node ②

$$\sum I_{in} = \sum I_{out}$$

$$10 + I_1 = I + 5$$

$$10 + \frac{V_1 - V_2}{4} = \frac{V_2 - 0}{6} + 5$$

Then solve Eqs.



Find I , $P(2\Omega)$ using Nodal.

at node ①:

$$\sum I_{in} = \sum I_{out}$$

$$0 = I_1 + I_2 + I$$

Then:

$$0 = \frac{V_1 - 10}{2} + \frac{V_1 + 6}{4} + \frac{V_1}{3}$$

- because of the existence of 10, 6 suppliers we should assume a virtual nodes before the suppliers.

for V_a : $V_a - 0 = 10$ $V_a = 10$

for V_b : $0 - V_b = 6$ $V_b = -6$

$$0 = 0.5V_1 - 5 + 0.25V_1 + 1.5 + \frac{V_1}{3}$$

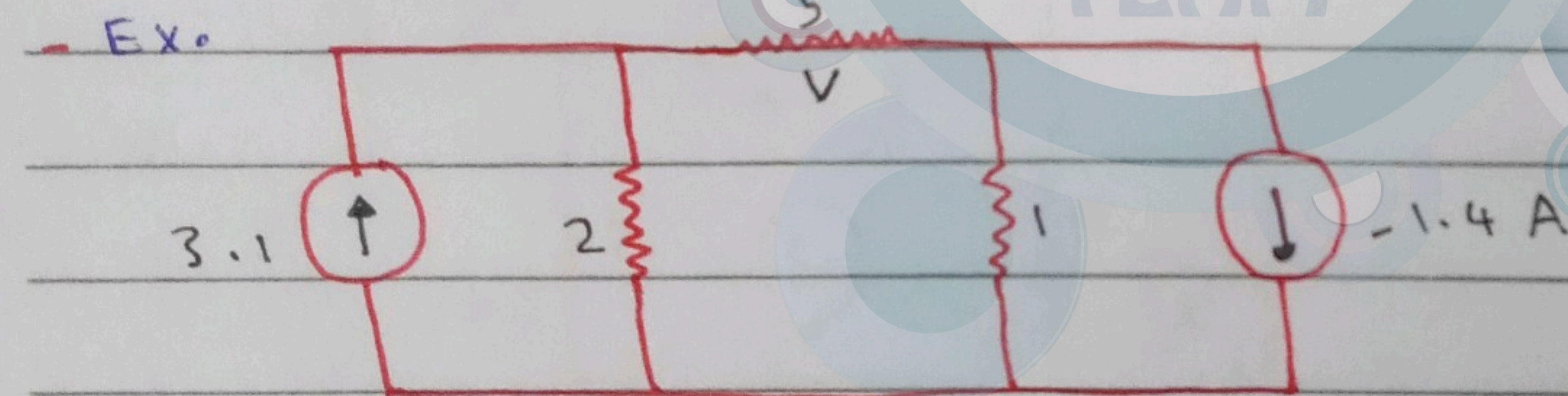
$$0 = 1.0833V_1 - 3.5$$

$$I_1 = -3.385$$

$V_1 = 3.23$

$$I = \frac{V_1}{3} = \text{1.077}$$

$$P(2\Omega) = I_1^2 R = \text{22.92 W}$$

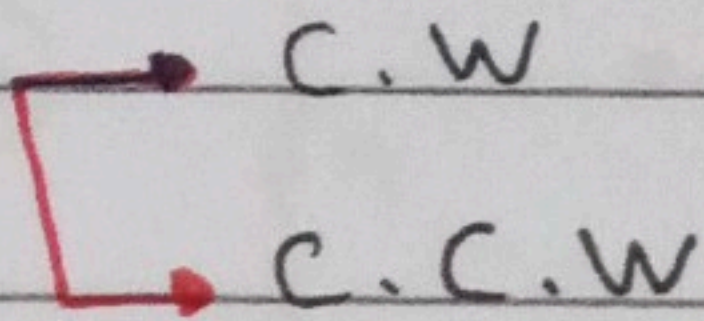


Find V , $P_{gen}(-1.4A)$ using Nodal.

③ Mesh analysis.

① Define meshes.

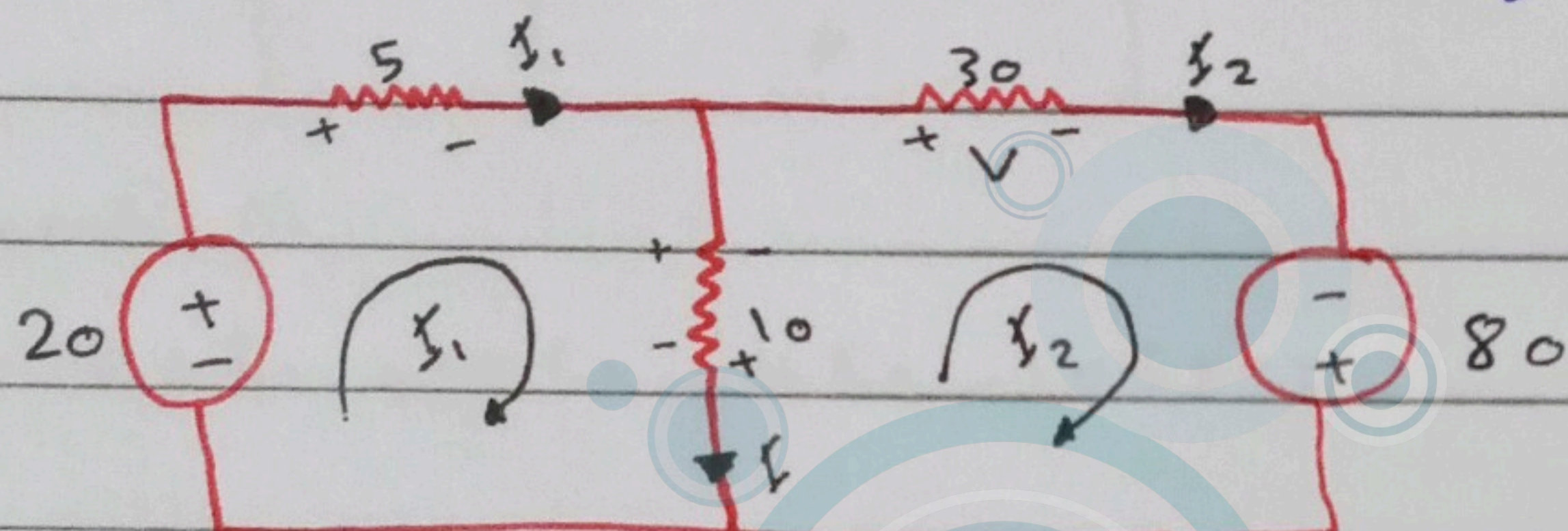
② assign mesh current



③ Apply KVL for each mesh.

④ use Ohm's law to express Voltages by currents.

Ex.



find V using mesh analysis.

at mesh ①

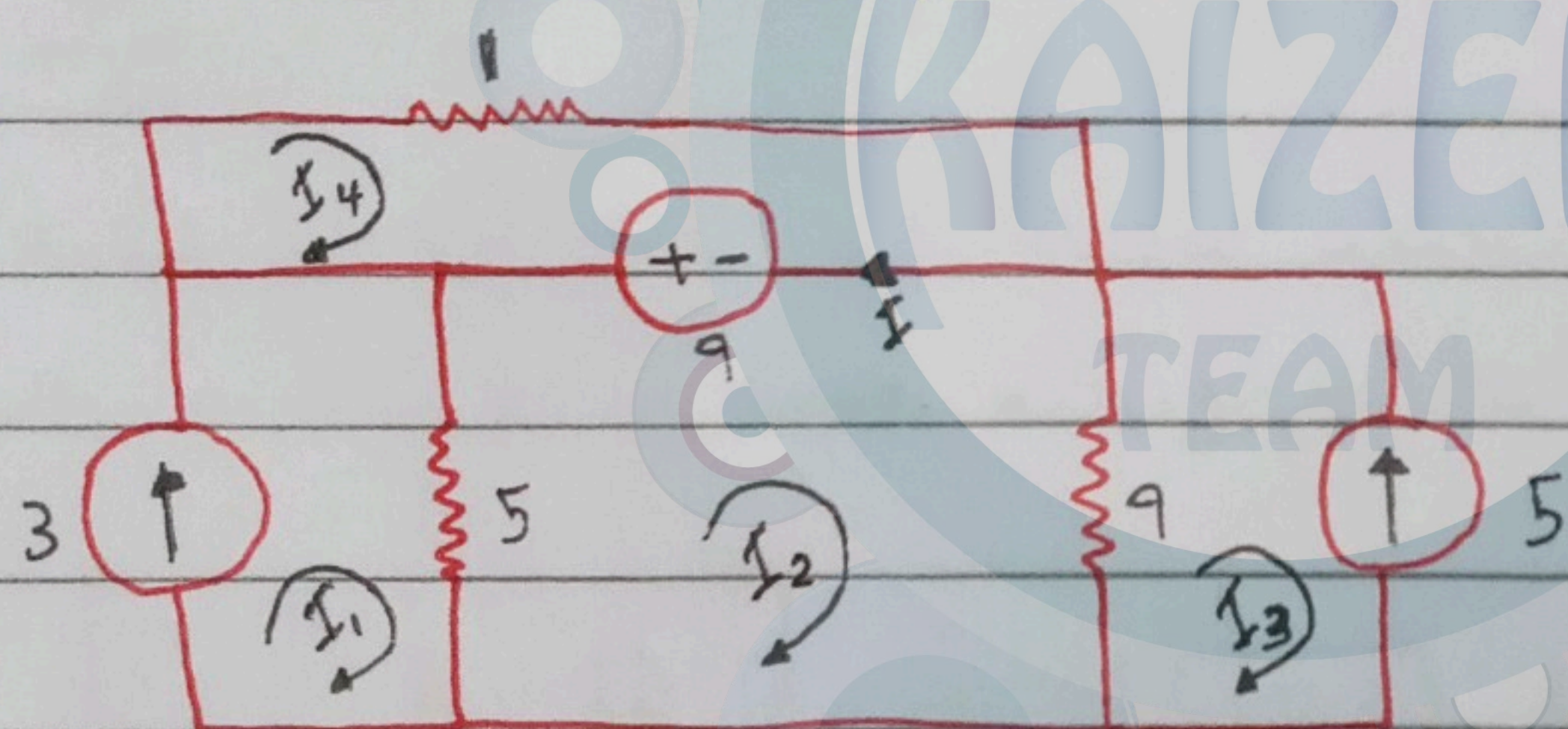
$$-20 + 5i_1 + 10i_1 - 10i_2 = 0$$

at mesh ②

$$10i_2 + 30i_2 - 80 - 10i_1 = 0$$

Then solve the eqs.

Ex.



find P_{gen} by (9V sup) using mesh analysis.

$$i_1 = 3$$

$$i_3 = -5$$

at mesh ②

$$5i_2 + 9 + 9i_2 - 5i_1 - 9i_3 = 0$$

$$14i_2 + 9 - 5(3) - 9(-5) = 0$$

$$i_2 = -2.7$$

at mesh ④

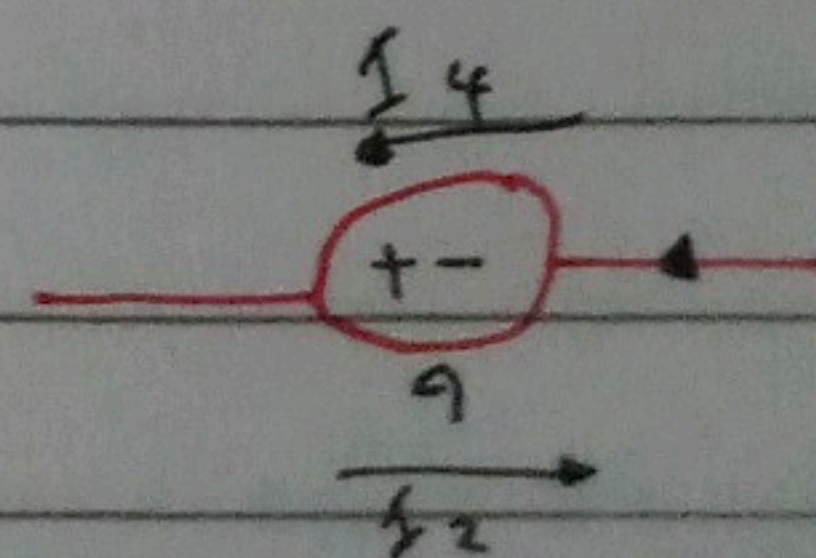
$$i_4 - 9 = 0$$

$$i_4 = 9$$

$$P_{(9V)} = iV, \quad i = i_4 + i_2 = 11.7$$

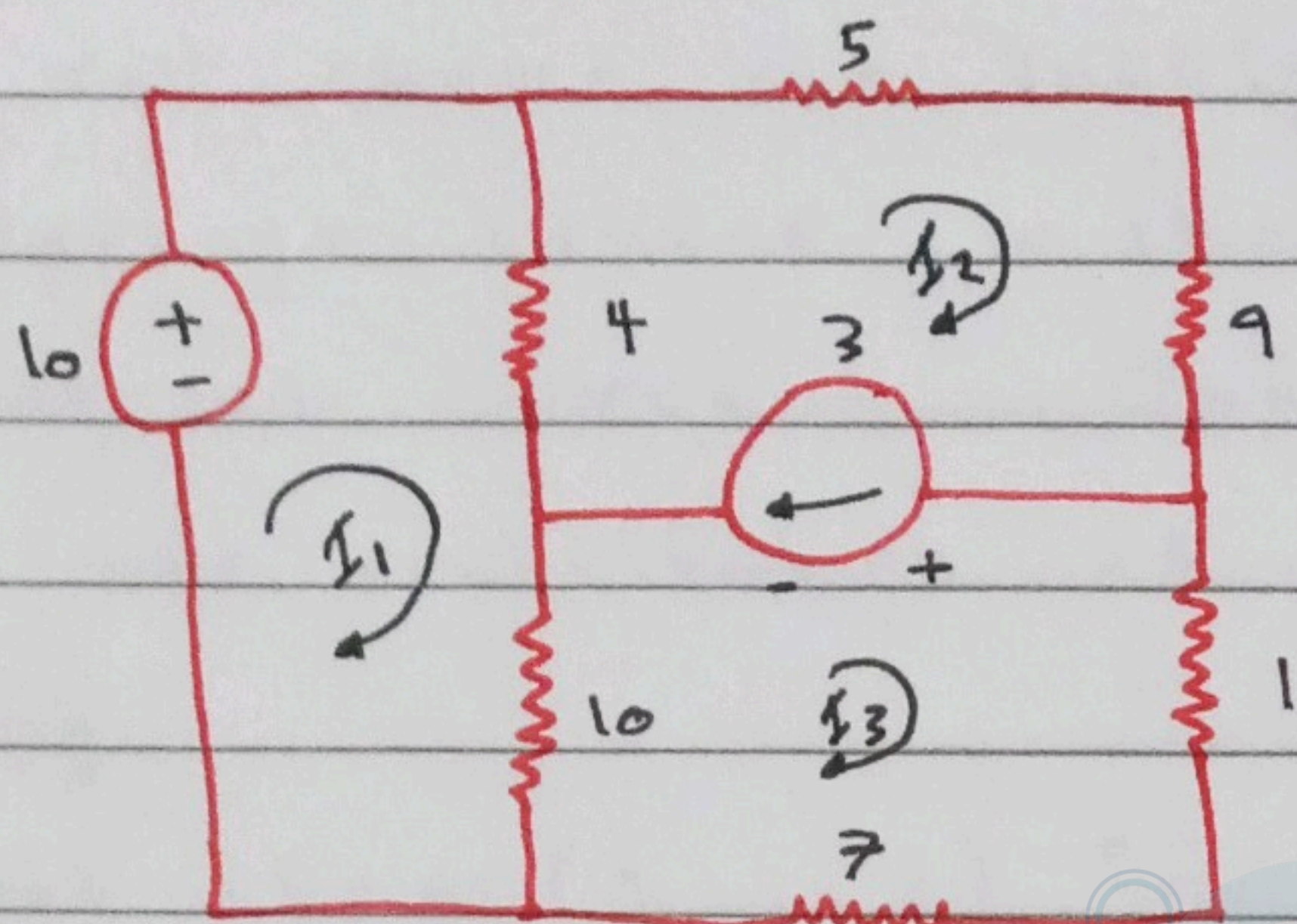
$$P = 11.7 \times 9$$

$$P_{gen} = 105.3 \text{ W}$$



- Super mesh:- special case of mesh.

Ex:-



• when there is a current source between two meshes then it is called a super mesh.

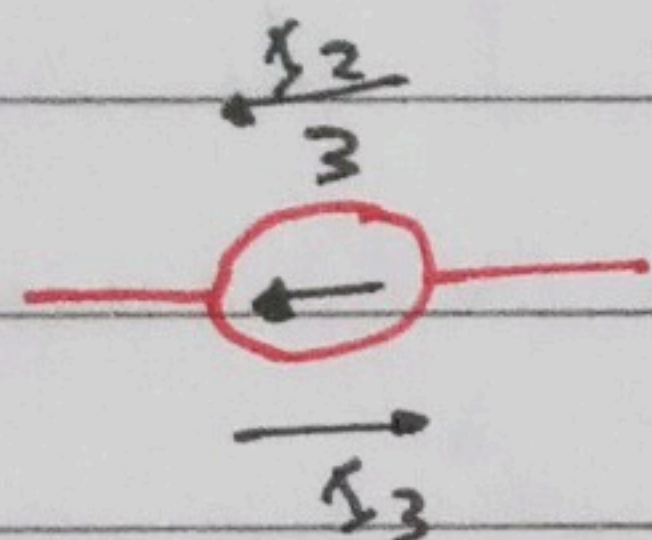
- find $P_{gen}(10V)$

• at mesh ①

$$-10 + 4i_1 + 10i_1 - 4i_2 - 10i_3 = 0$$

• at super mesh:-

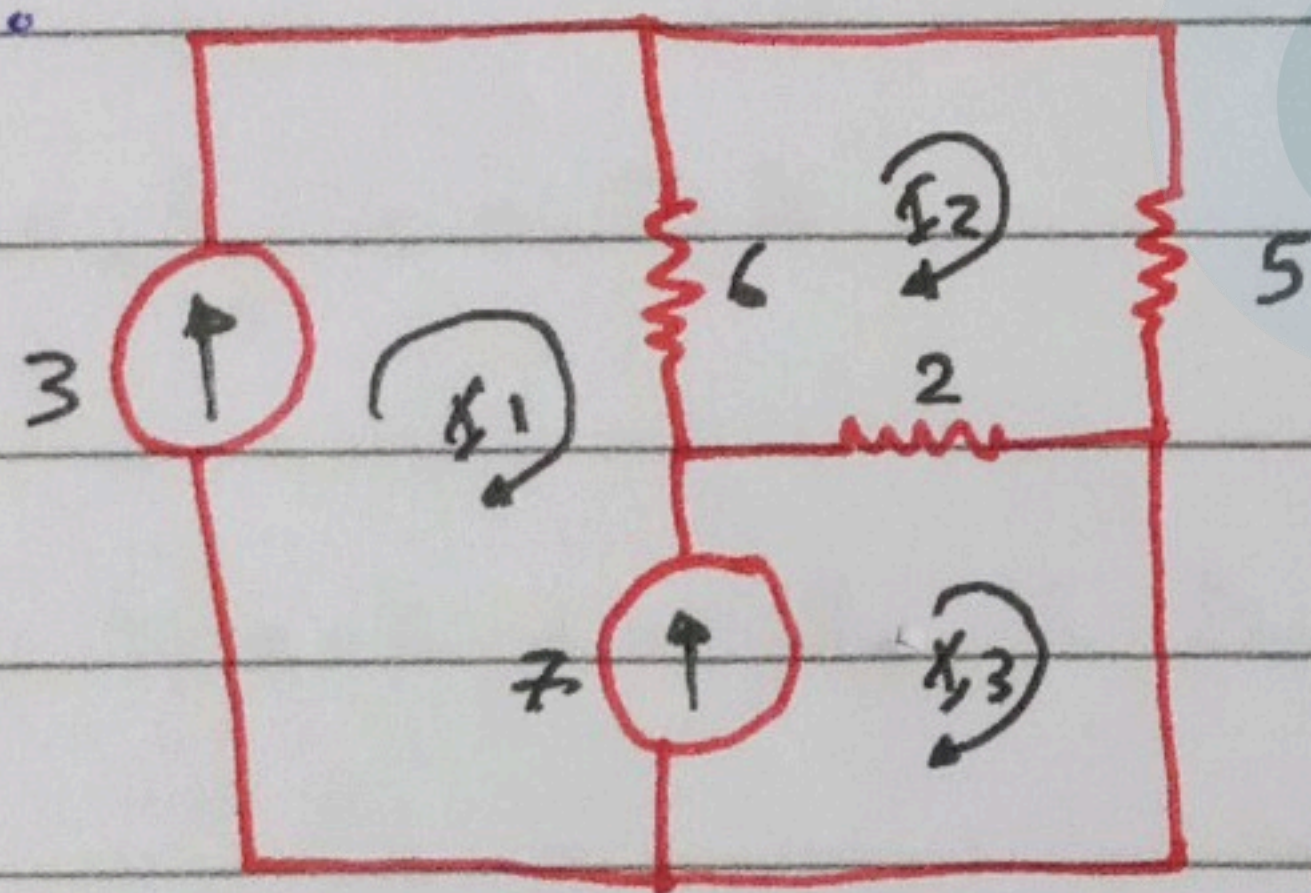
$$10i_3 + 4i_2 + 5i_2 + 9i_2 + i_3 + 7i_3 - 4i_1 - 10i_1 = 0$$



$$i_2 - i_3 = 3$$

- to find the Eq. of super mesh, consider the current source doesn't exist then a big mesh will occur.

Ex.



$$i_1 = 3$$

• at mesh ②

$$6i_2 + 5i_2 + 2i_2 - 6i_1 - 2i_3 = 0$$

• at 7A current source

$$i_3 - i_1 = 7$$

$$i_3 = 10$$

⊕ Super position : used in linear circuits with multiple sources.

① We will choose one supply and Kill the others

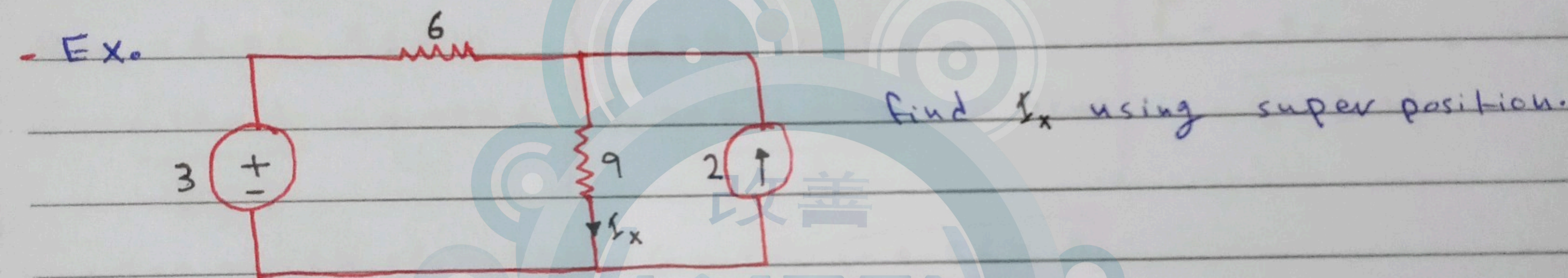
- if voltage source $\rightarrow$ short

- if current source $\rightarrow$ Open

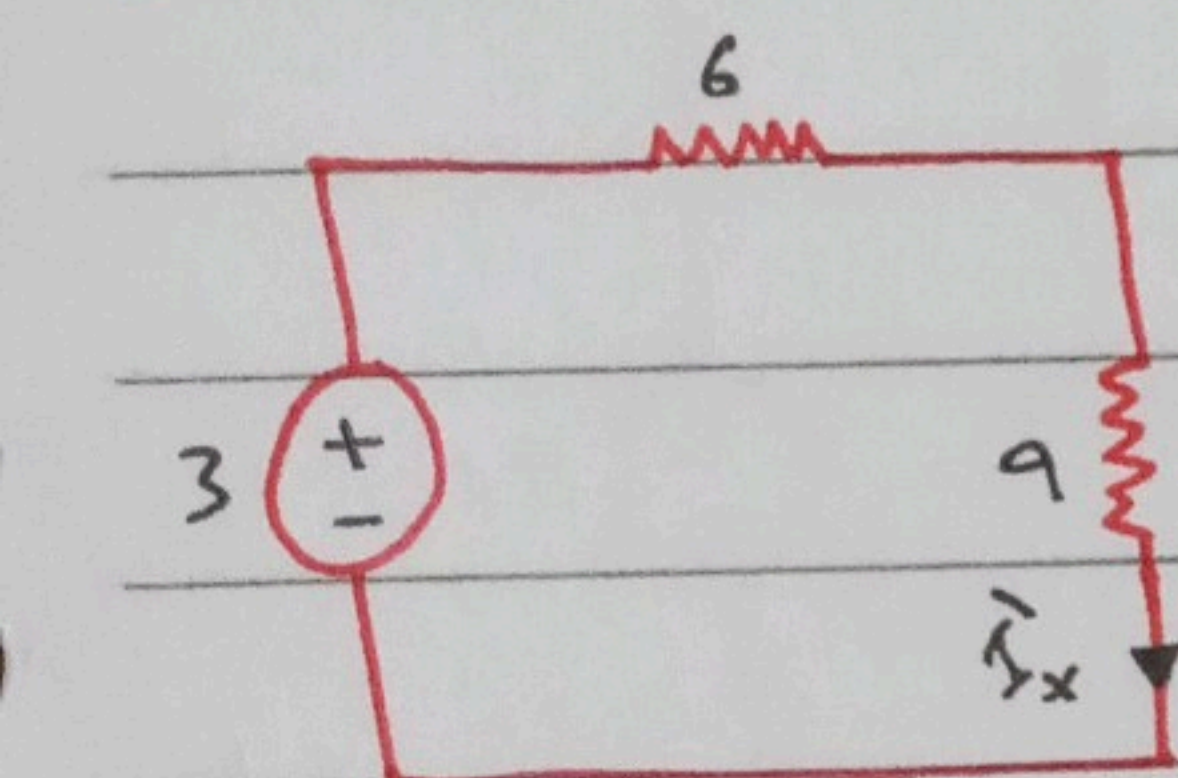
② Find the contribution of current and voltages due to that supply.

③ repeat steps (1 $\rightarrow$ 2) for each source.

④ The total current and voltages is the sum of all the contributes.

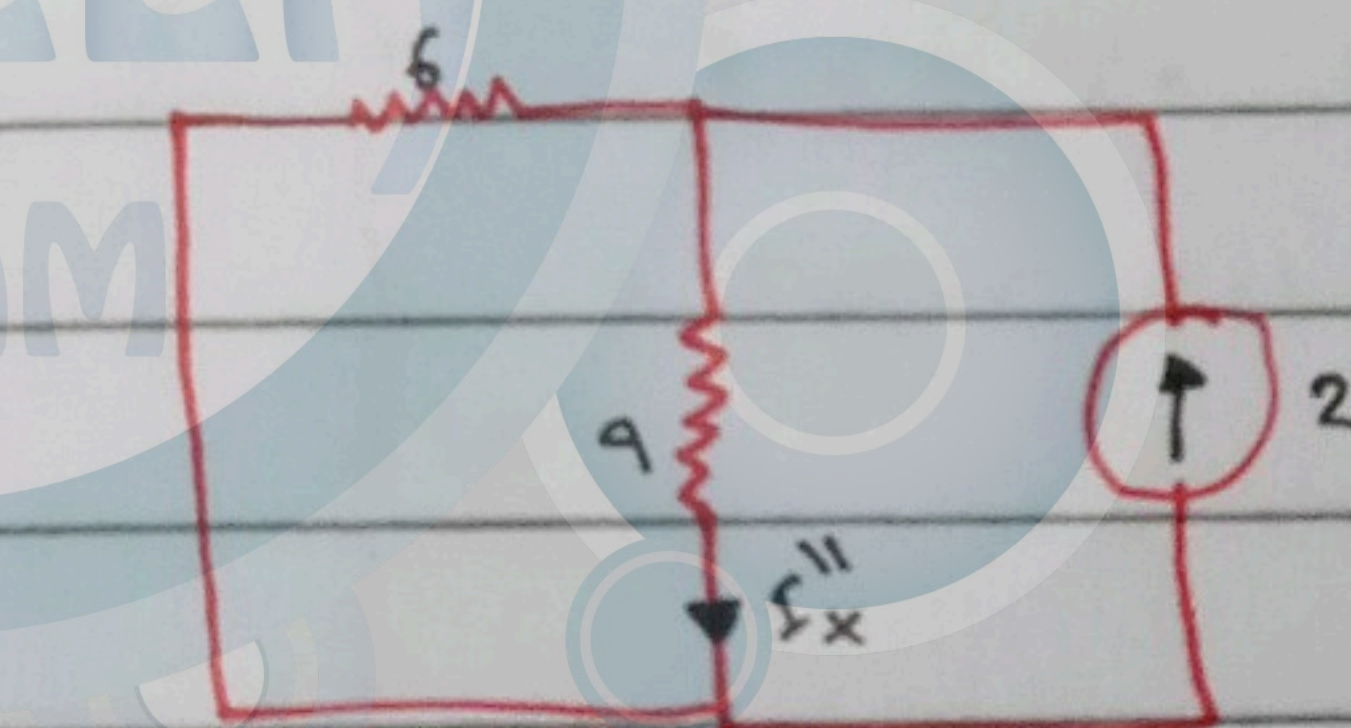


• leave (3V) and Kill (2A)



$$I'_x = \frac{3}{15} = 0.2 \text{ A}$$

• leave (2A) and Kill (3V)

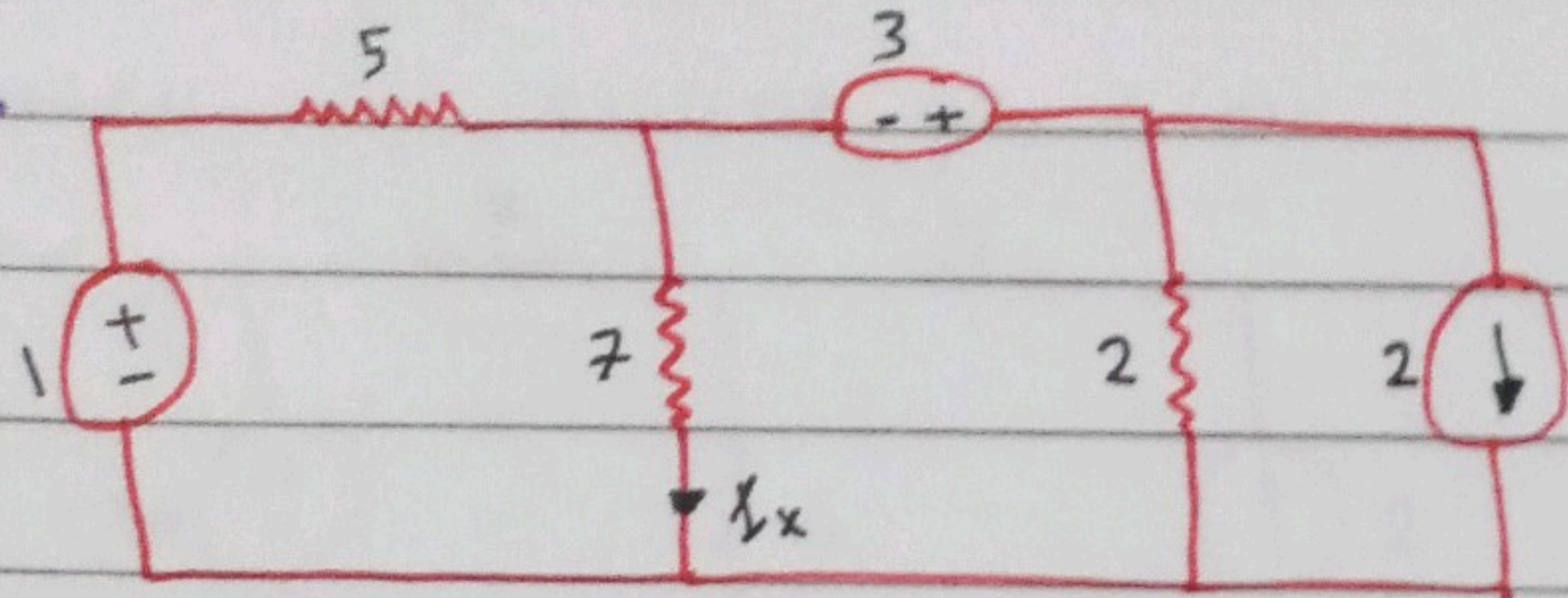


current
division

$$I''_x = 2 * \frac{\frac{1}{9}}{\frac{1}{6} + \frac{1}{9}} = 0.8 \text{ A}$$

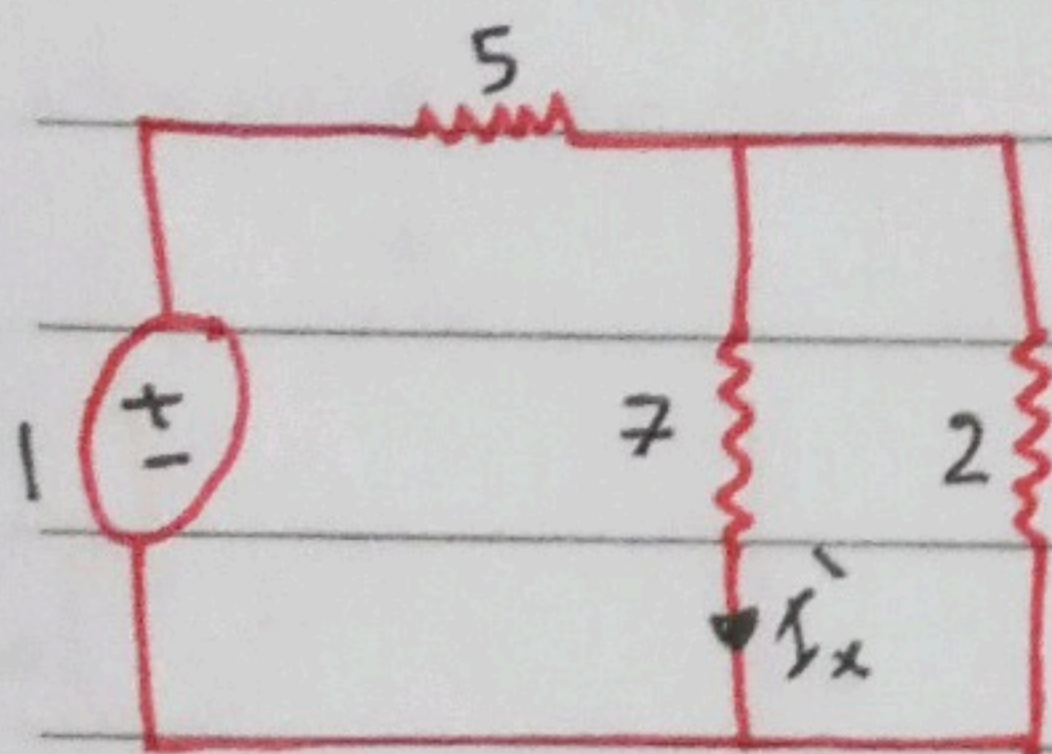
$$\therefore I_x = 0.2 + 0.8 = 1 \text{ A}$$

Ex.

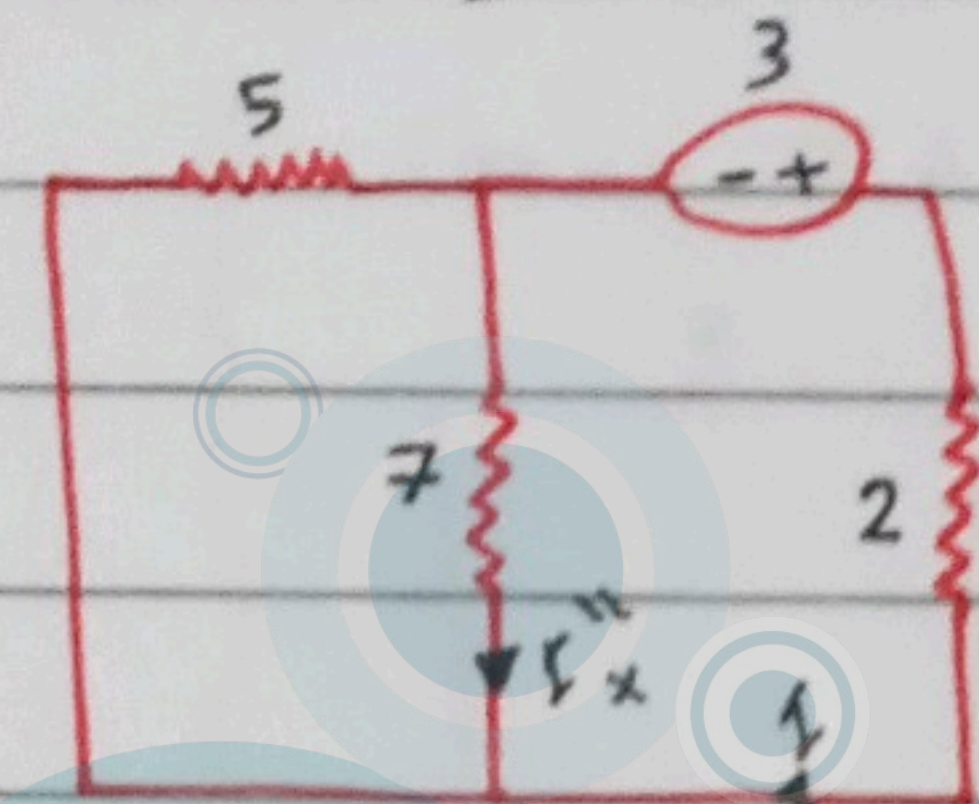


Find I_x using super position

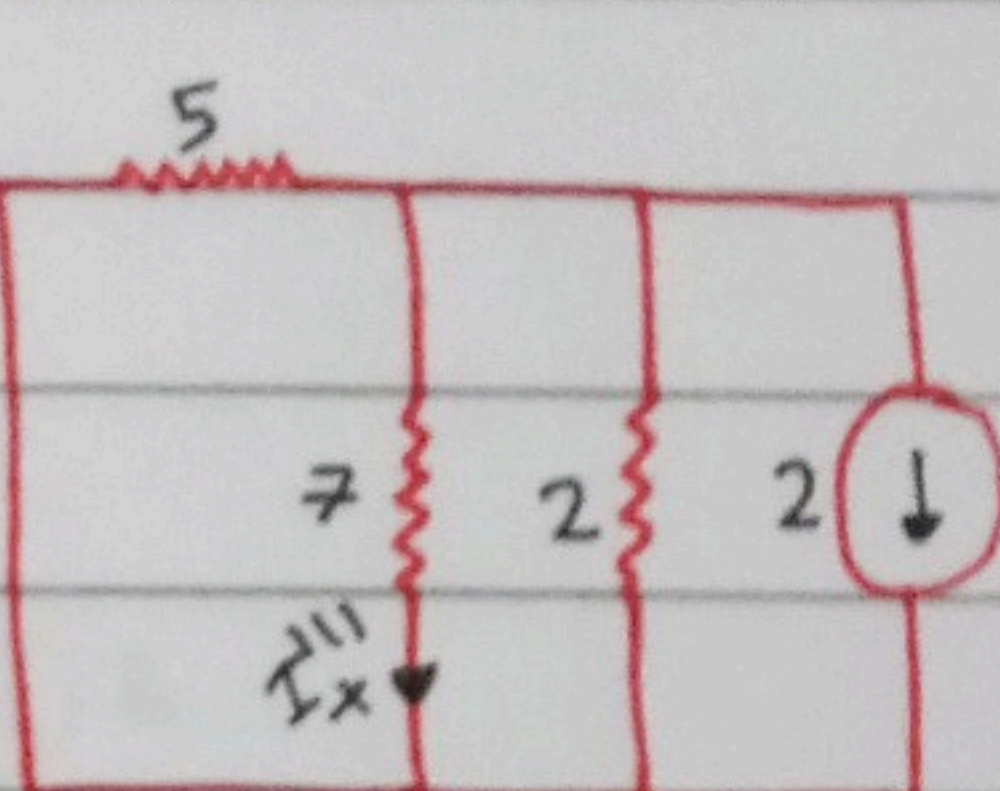
• leave 1V



• leave 3V



• leave 2A



$$R_{eq} = 6.55, I = 0.15$$

$$R_{eq} = 4.92, I = 0.61$$

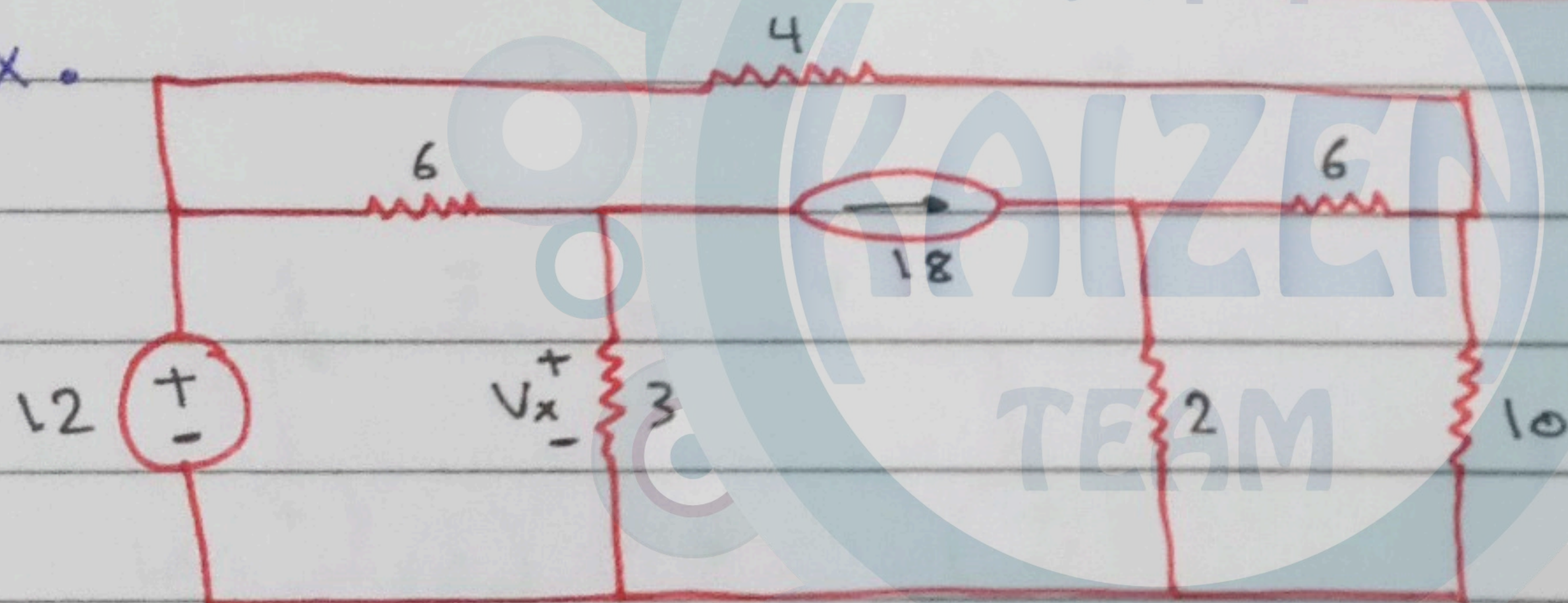
$$I_{x'} = 0.15 \times \frac{1}{\frac{1}{7} + \frac{1}{2}} = 0.034 \text{ A}$$

$$I_{x''} = -0.61 \times \frac{1}{\frac{1}{5} + \frac{1}{2}} = -0.254 \text{ A}$$

$$I_{x'''} = -2 \times \frac{1}{\frac{1}{2} + \frac{1}{7} + \frac{1}{5}} = -0.34 \text{ A}$$

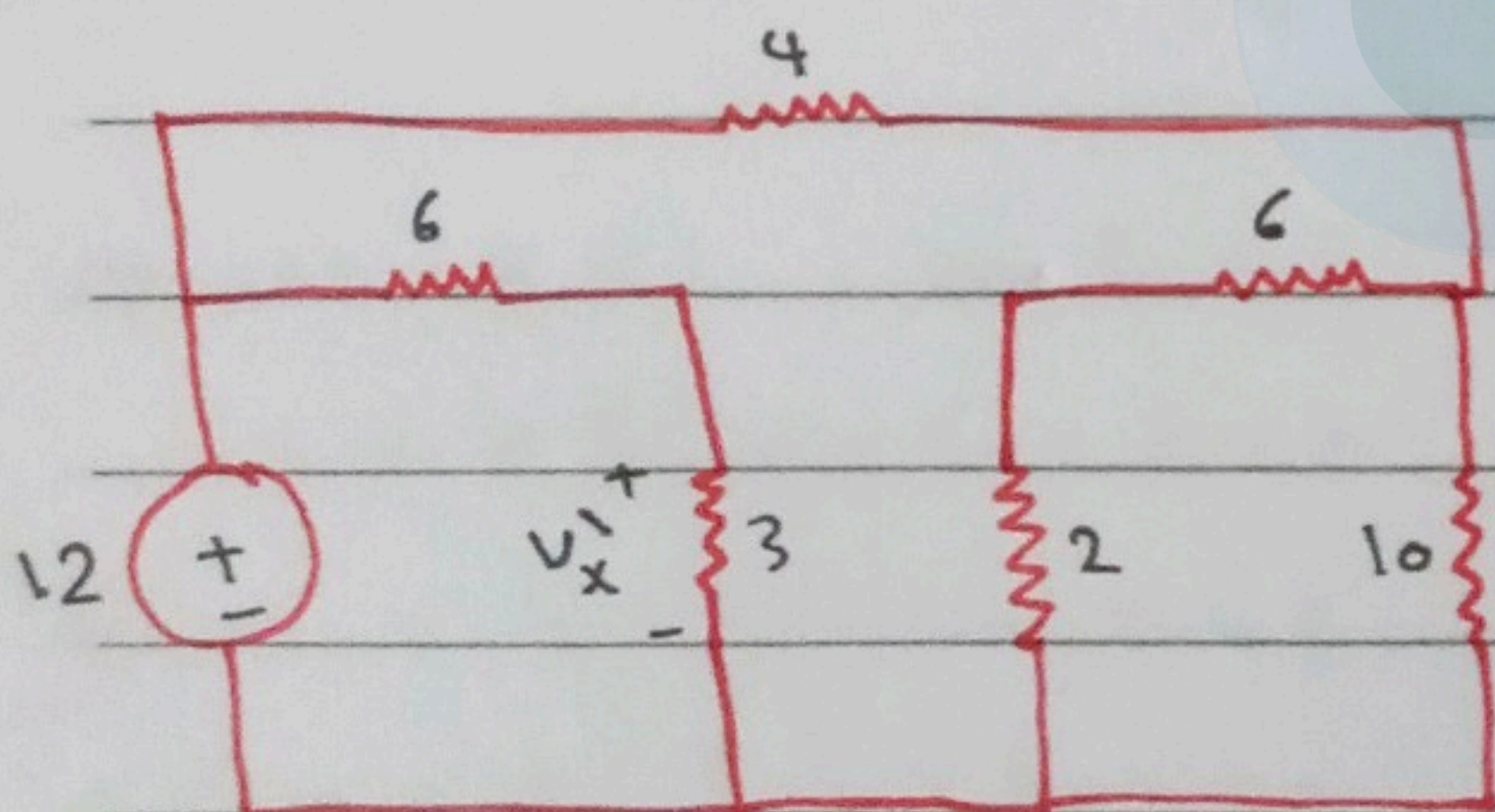
$$\therefore I_x = 0.034 + (-0.254) + (-0.34) = -0.564 \text{ A}$$

Ex.

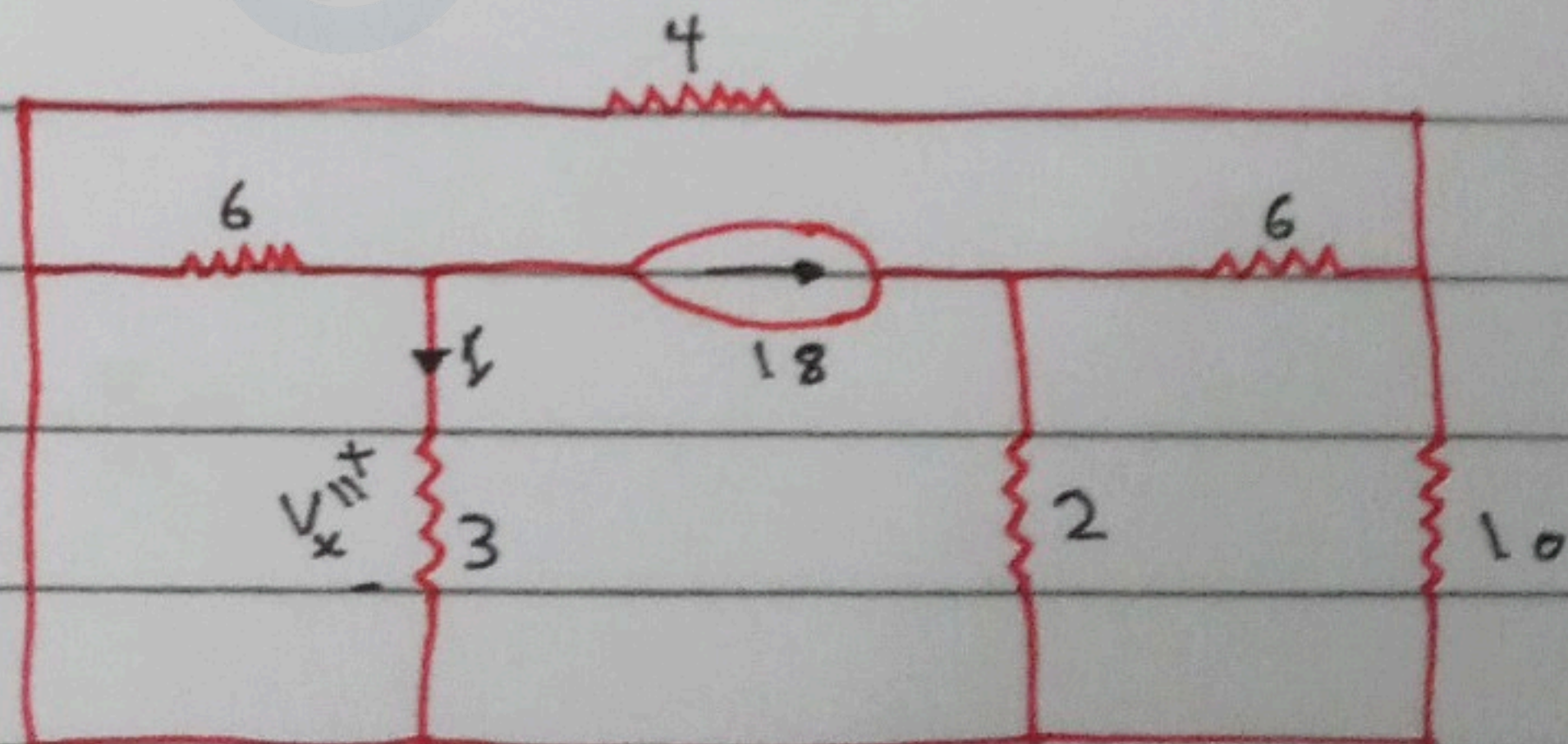


Find V_x using super position

• leave 12V



• leave 18A



$$V_{x'} = \frac{12 \times 3}{3 + 6} = 4 \text{ V}$$

$$\text{assume } I, I = -18 \times \frac{1}{\frac{1}{2} + \frac{1}{6}} = -12 \text{ A}$$

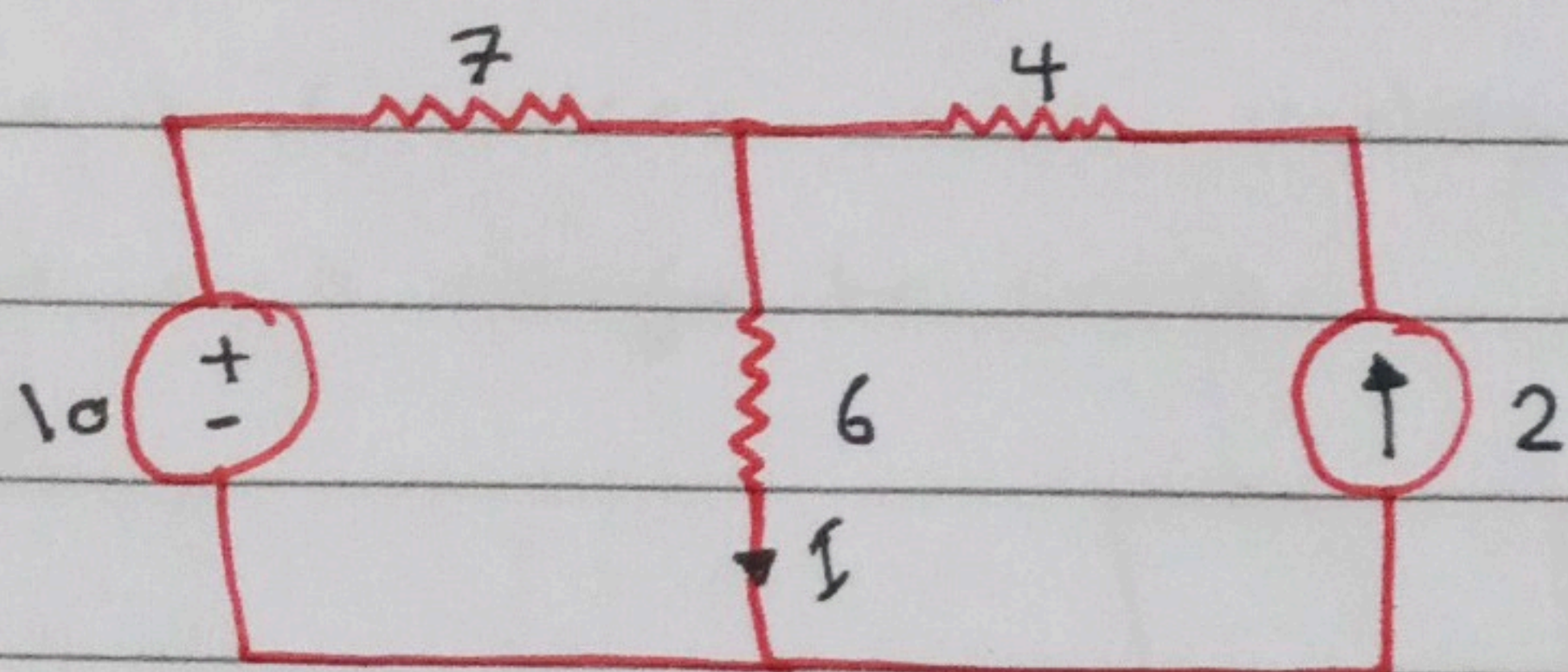
Or by KVL

$$V_{x''} = -12 \times 3 = -36 \text{ V}$$

$$\therefore V_x = 4 - 36 = -32 \text{ V}$$

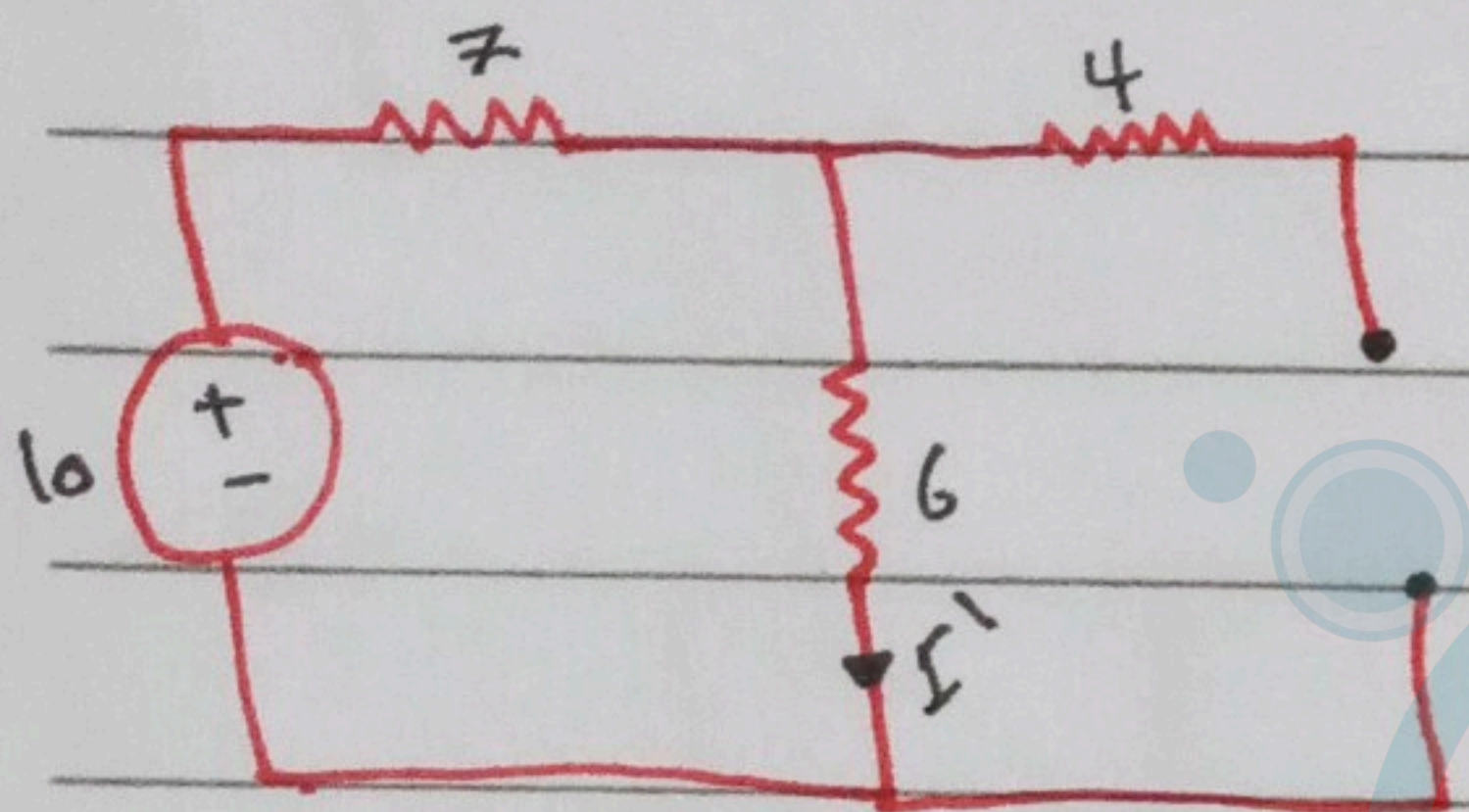
⊗ power in super position.

- Ex.

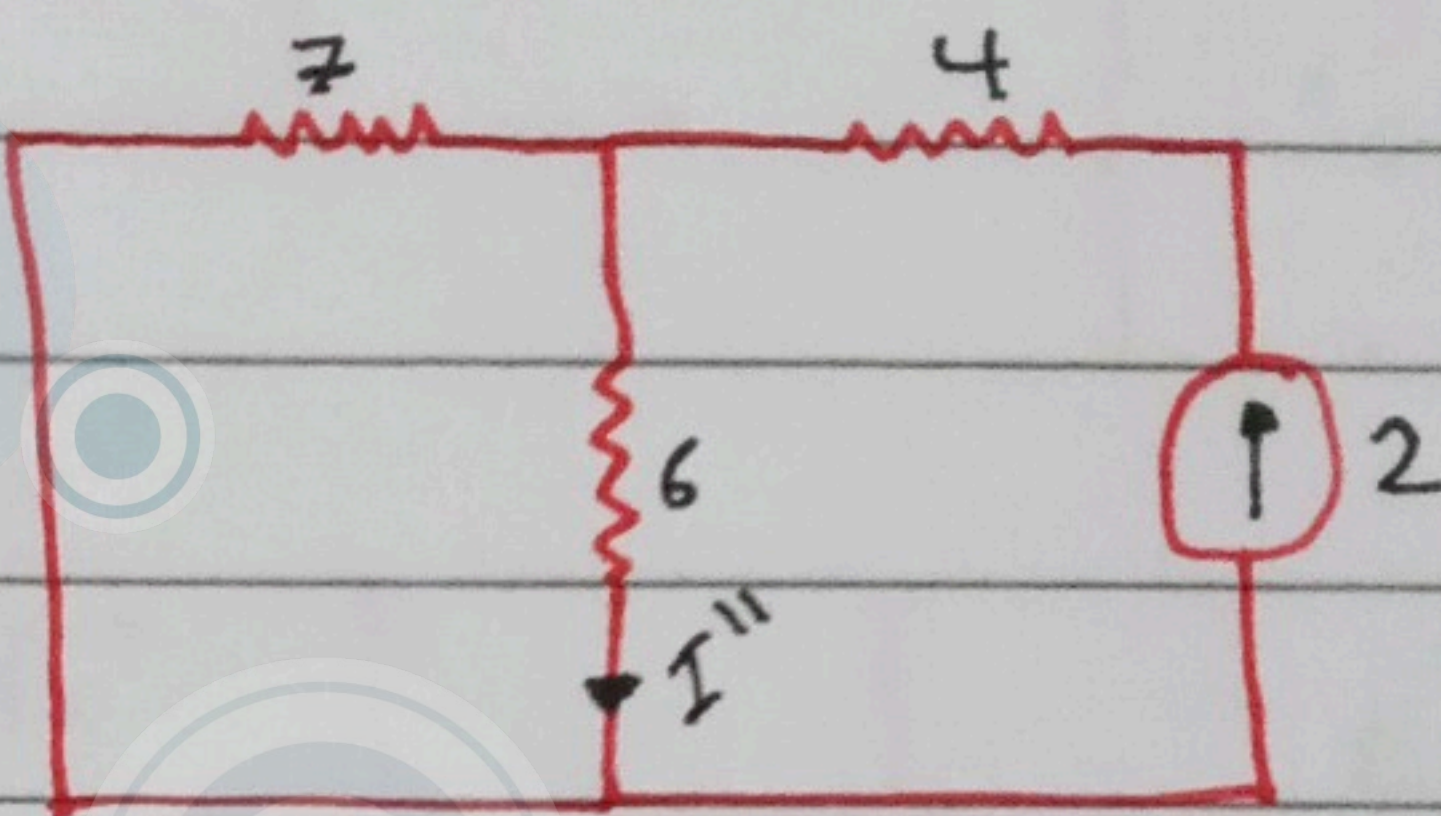


find power (6Ω) by super position.

• leave 10V



• leave 2A



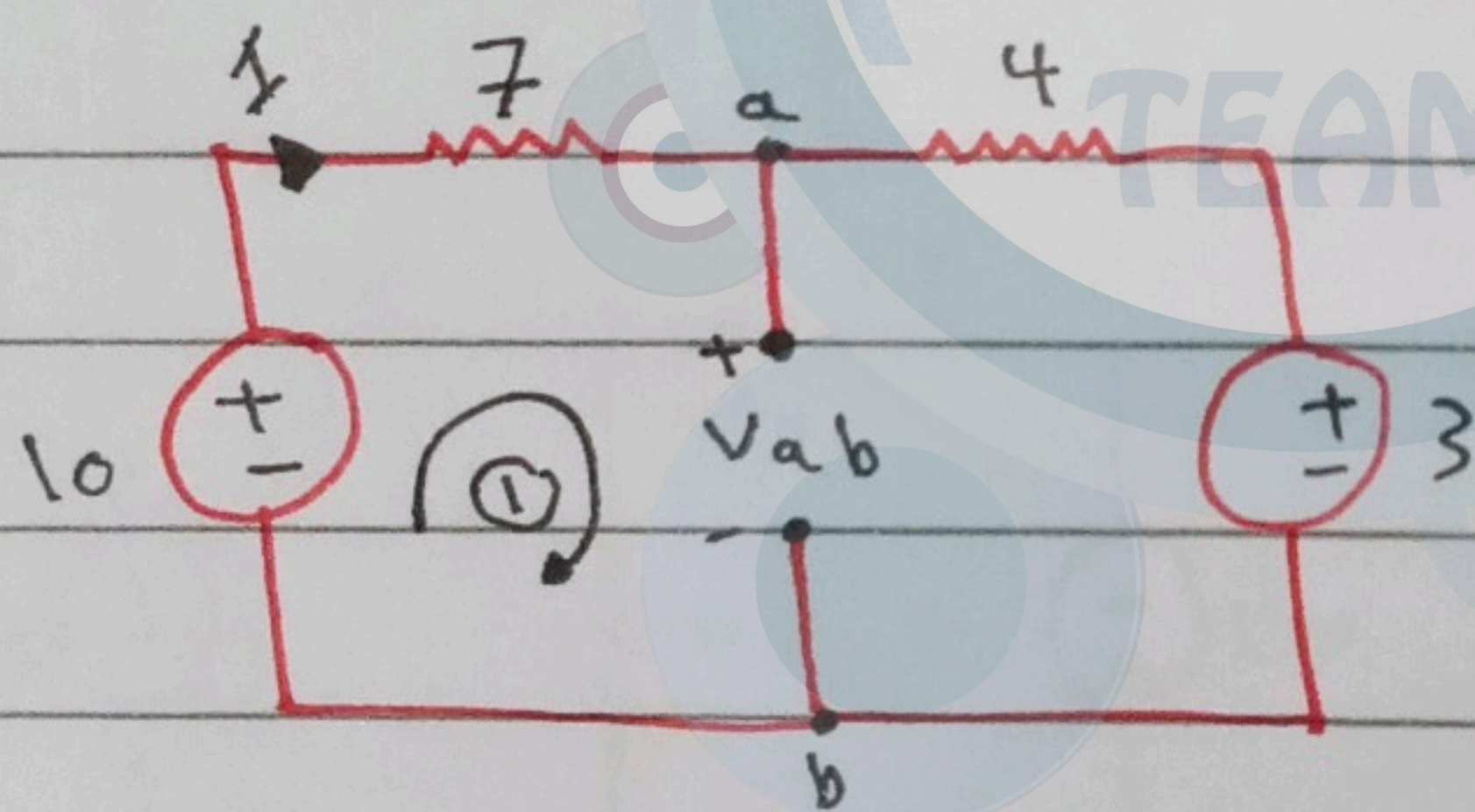
$$R_{eq} = 13, I' = 0.796 \text{ A}$$

$$I'' = 2 * \frac{1}{\frac{1}{2} + \frac{1}{6}} = 1.077 \text{ A}$$

→ $I = 0.796 + 1.077 = 1.87 \text{ A}$, So: $P = I^2 R = (1.87)^2 * 6 = 21.04$

- Note: You can't say: $P = P' + P''$

- Ex.



- to find V_{ab} :-

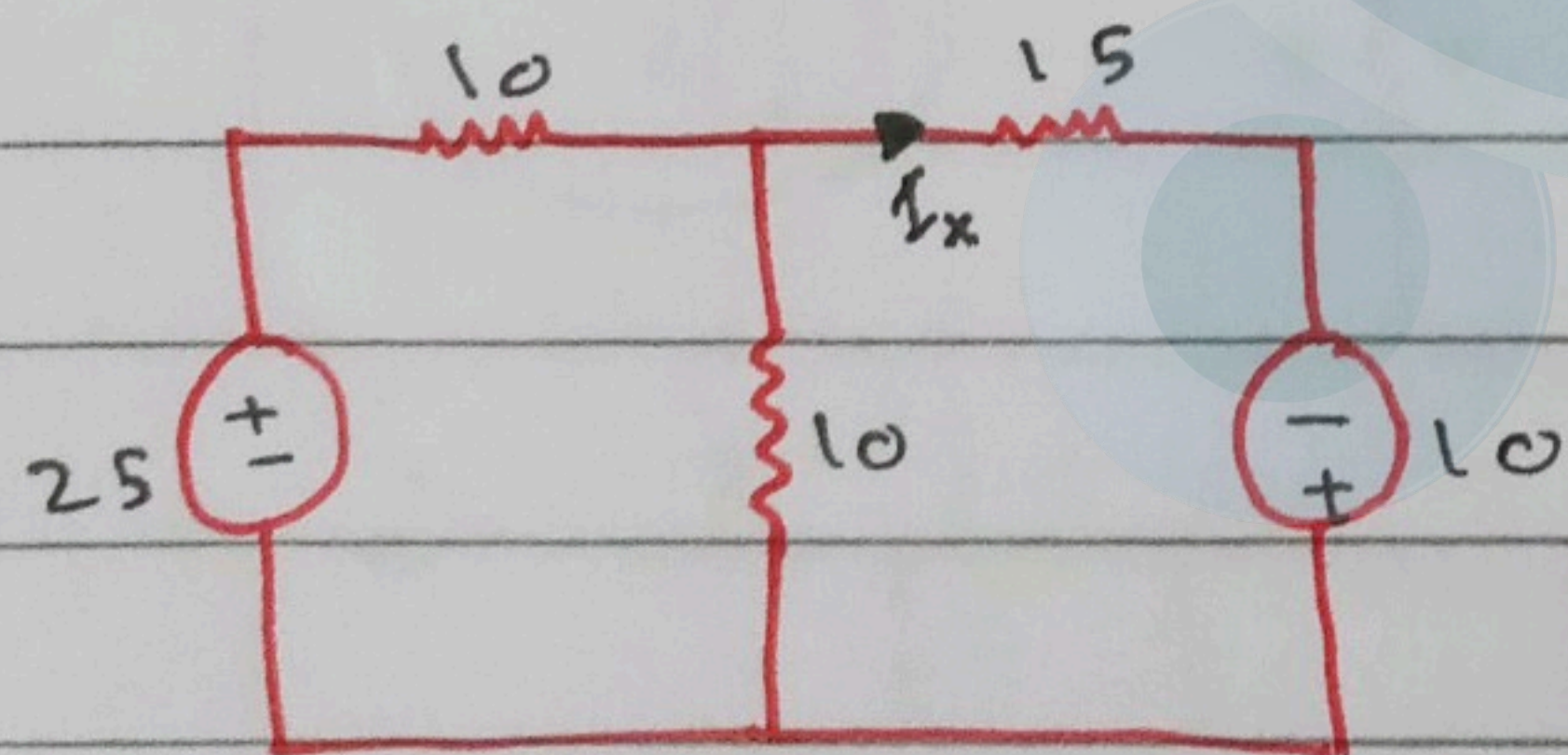
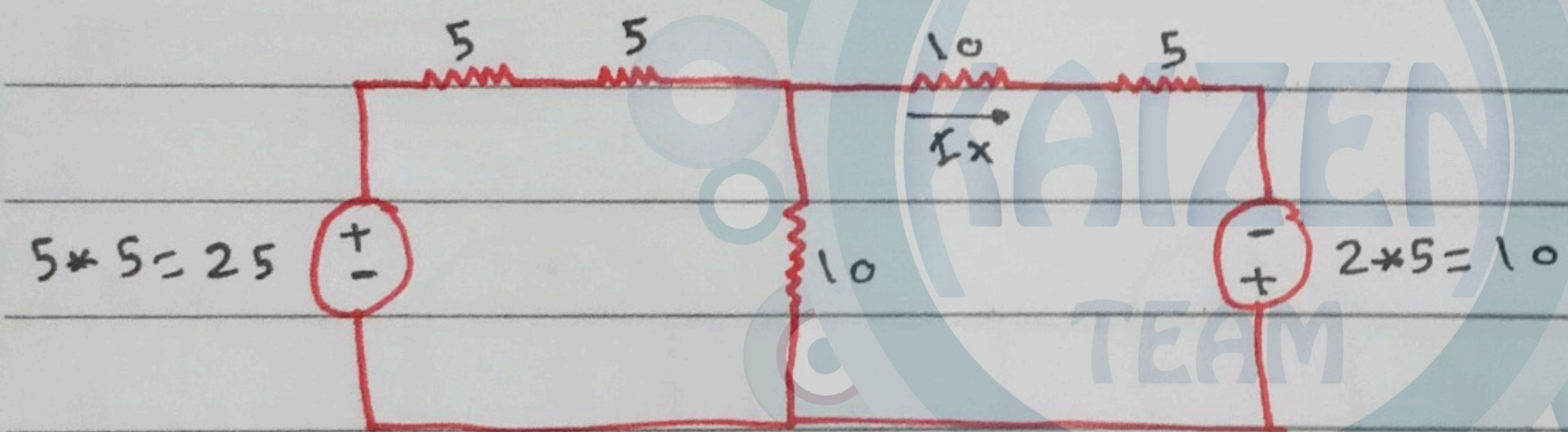
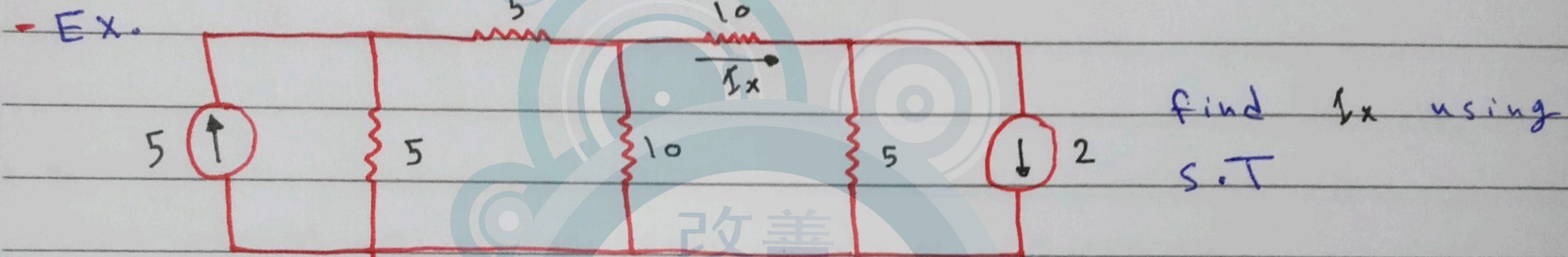
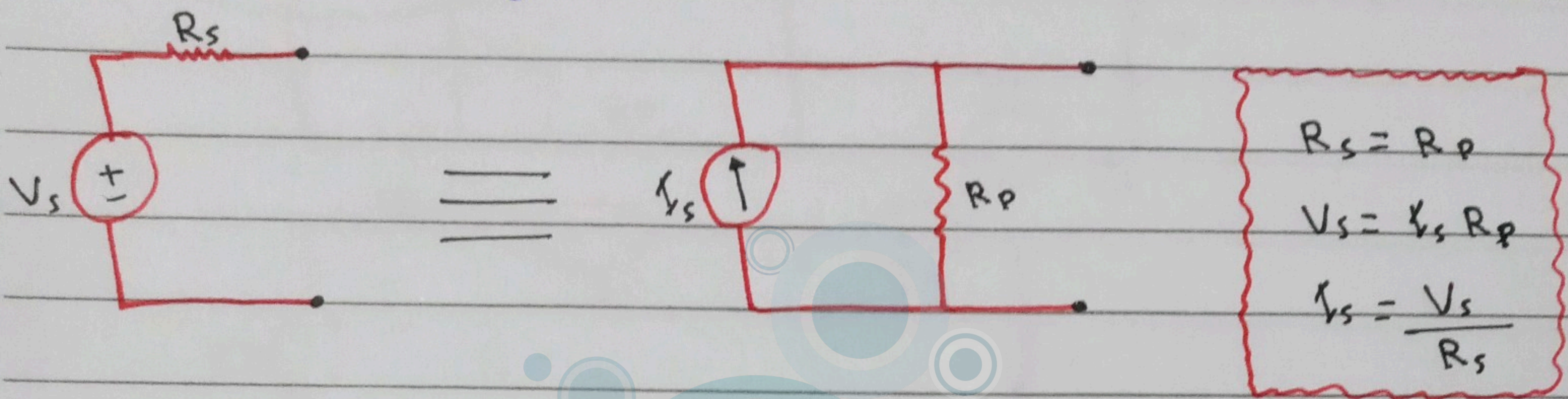
① use KVL to find I

$$-10 + 7I + 4I + 3 = 0, I = \frac{7}{11}$$

② use KVL to find V_{ab}

$$-10 + 7\left(\frac{7}{11}\right) + V_{ab} = 0$$

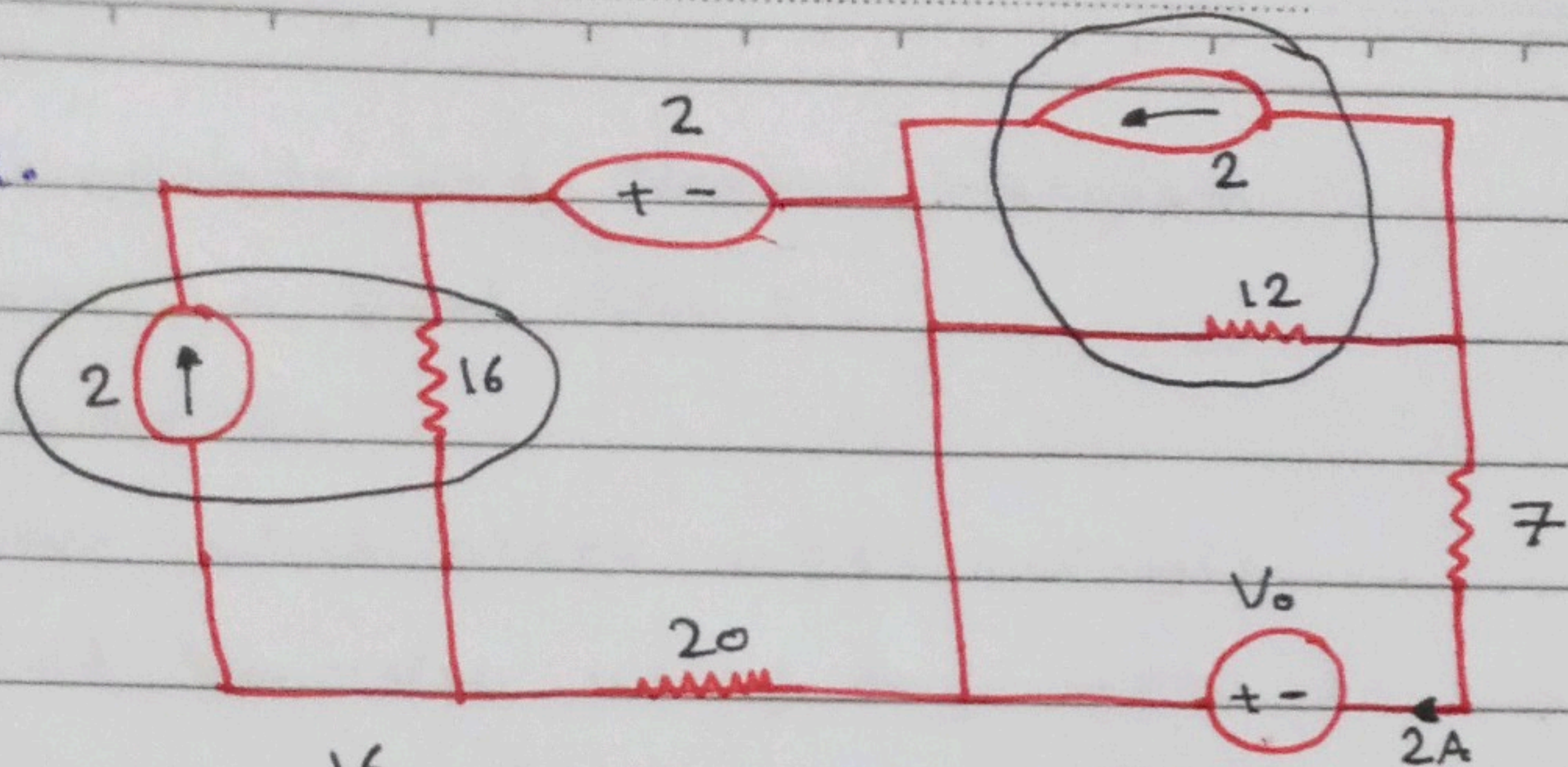
Source Transformation :- In some circuits, replacing current sources with voltage sources, make the circuit simple and easy to solve.



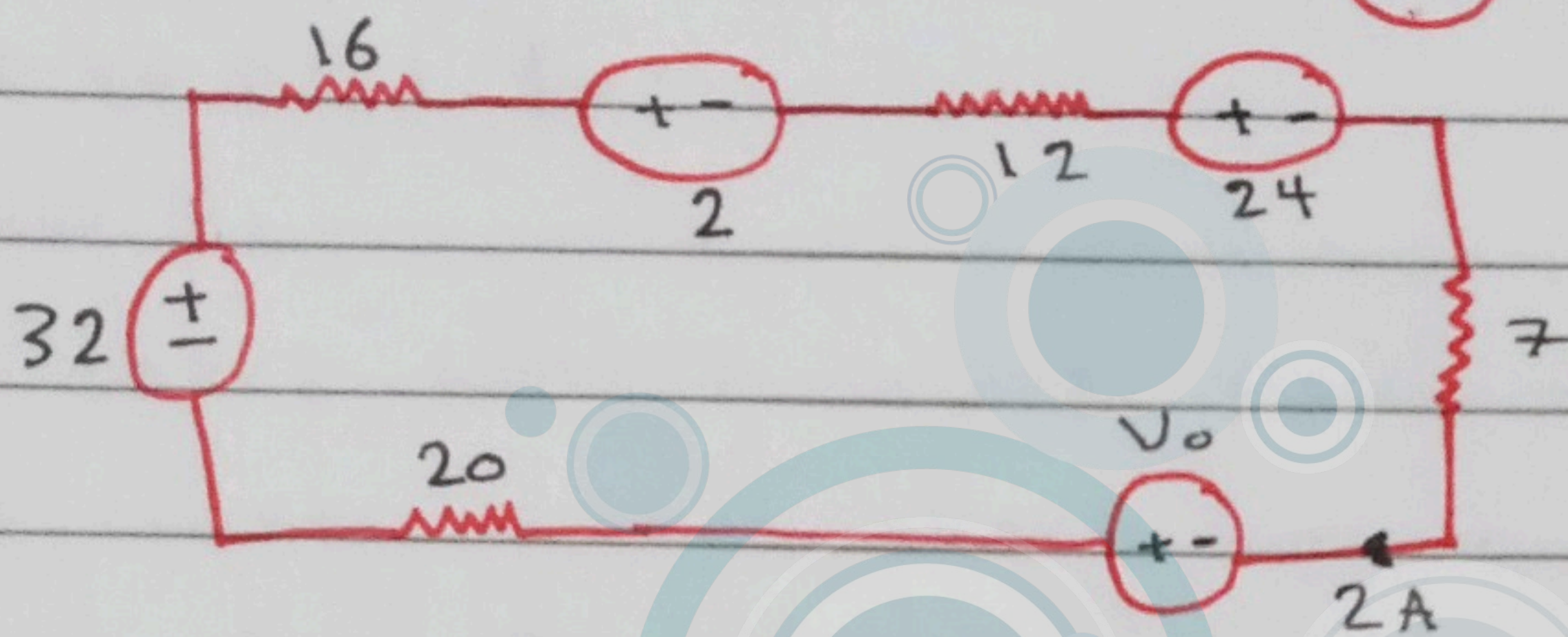
Then, in this Q use super position or any method. (The ~~easy~~ easiest)

Note : Take into consideration current direction.

Ex.



find V_o using S.T

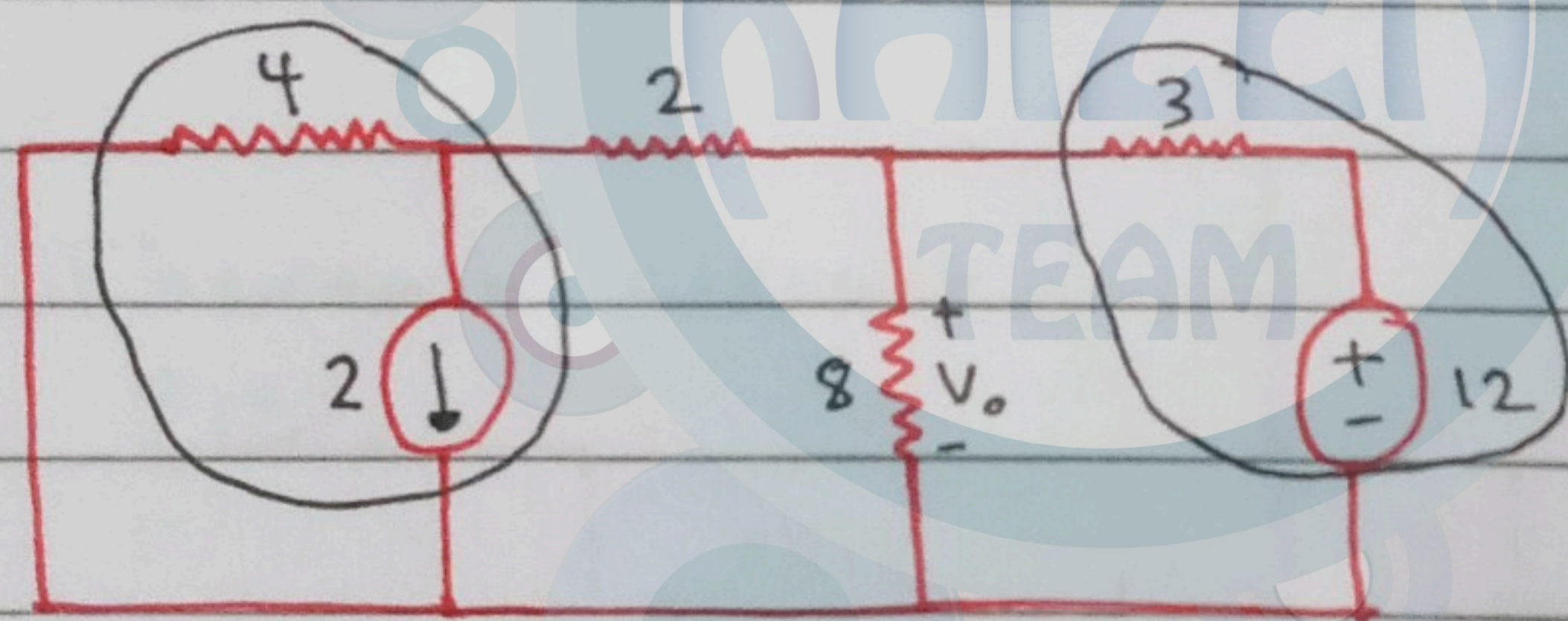


KVL:-

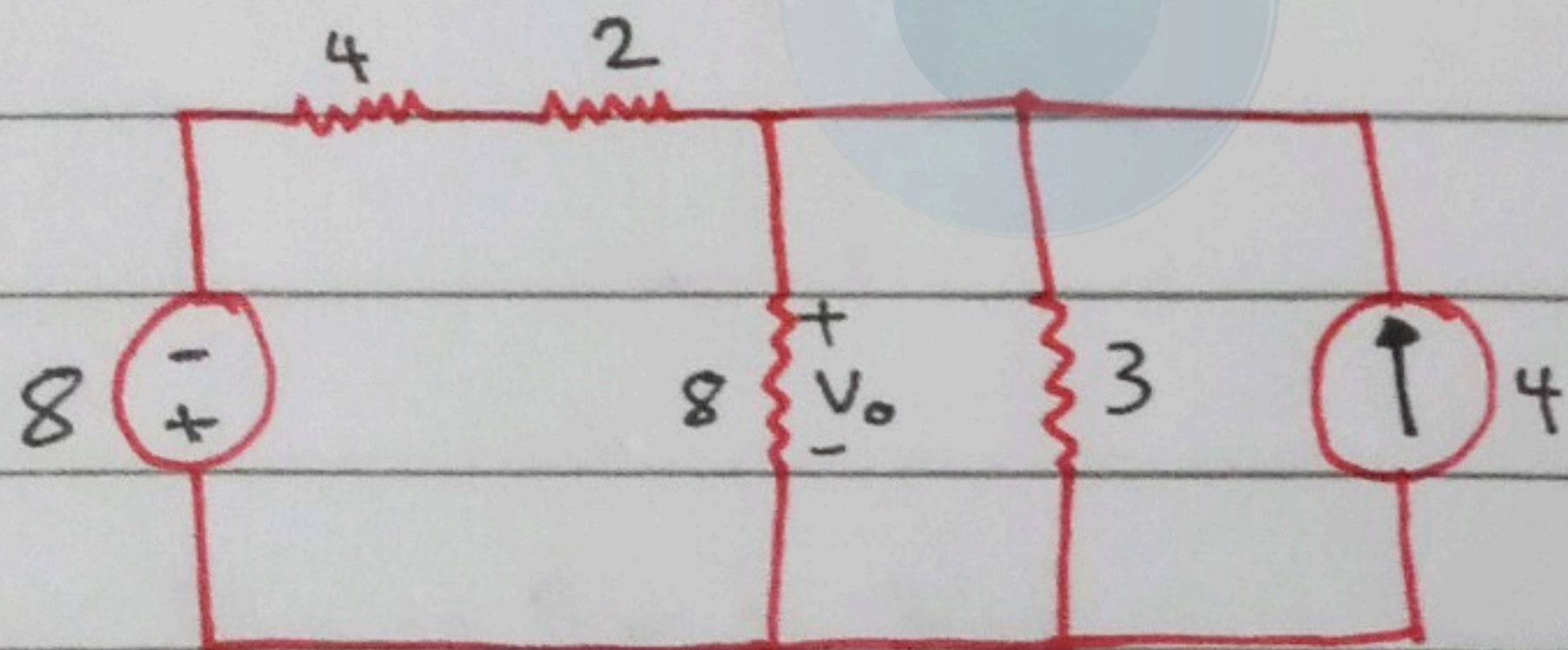
$$-32 + 16(2) + 2 + 12(2) + 24 + 7(2) - V_o + 20(2) = 0$$

$$V_o = 104 \text{ V}$$

Ex.



find V_o using S.T



Then by super position or any method easier.

Extremely important Note:- if the question doesn't choose a method then you can solve like this, if it chooses S.T then solve by S.T without using any other method. (Keep simplifying the circuit to reach the simplest shape)

⊗ Thevenin and Norton Theorem.

- steps to find V_{th} :

- ① Remove R_L .
- ② Leave that place open circuit.
- ③ Find $V_{oc} = V_{th}$ using any method.

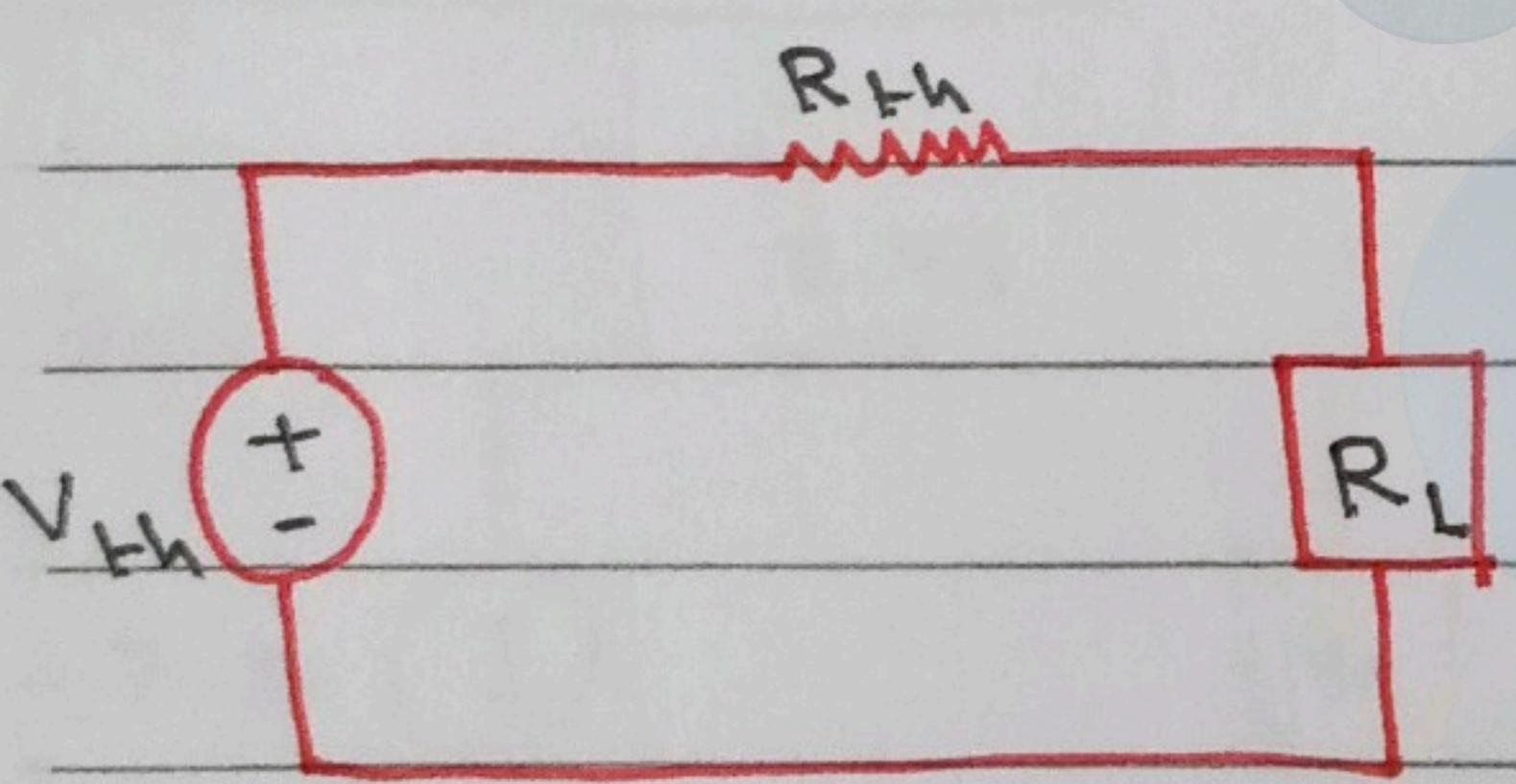
- Steps to find I_N :

- ① Remove R_L .
- ② put short circuit in the place of R_L .
- ③ Find current in that short circuit $I_N = I_{sc}$ using any method.

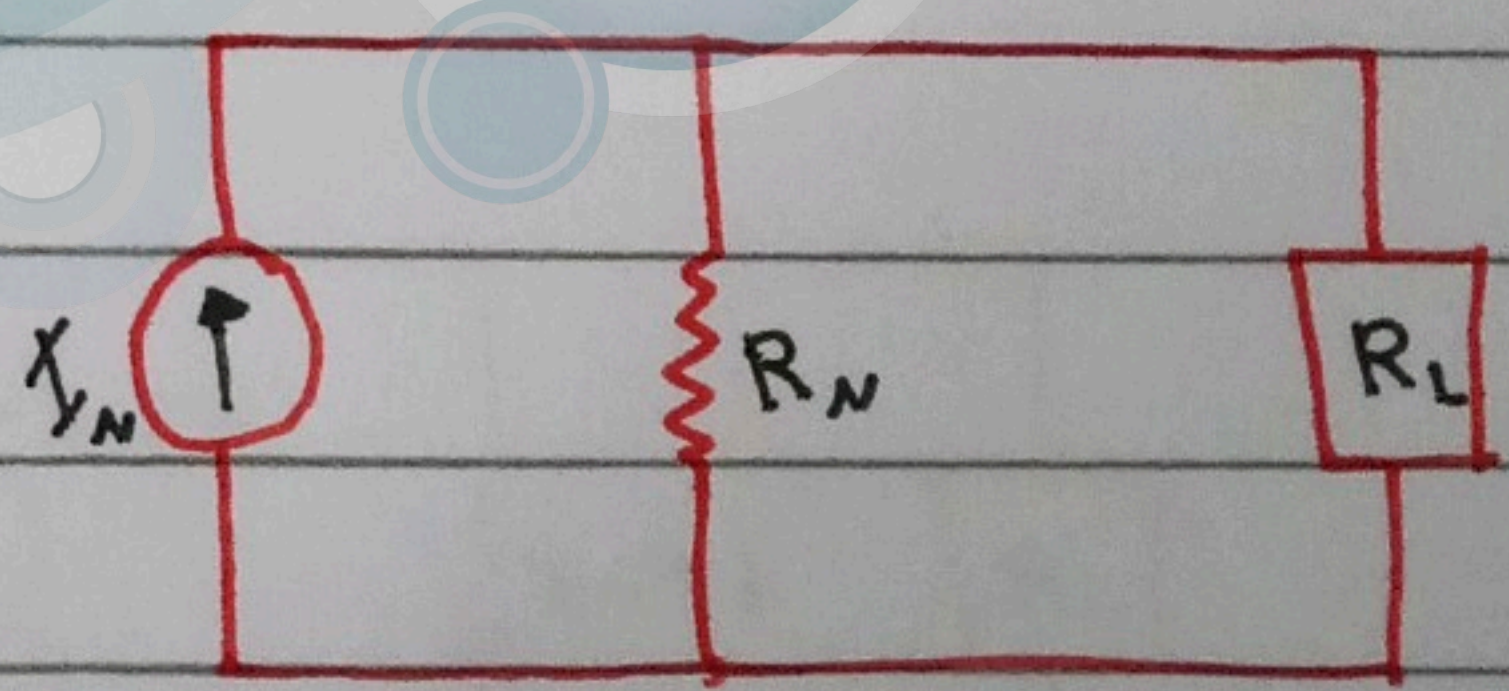
- Steps to find $R_{th} = R_N$:

- ① Remove R_L .
- ② Kill any sources
current → open
voltage → short
- ③ Find $R_{eq} = R_{th}$

Thevenin circuit



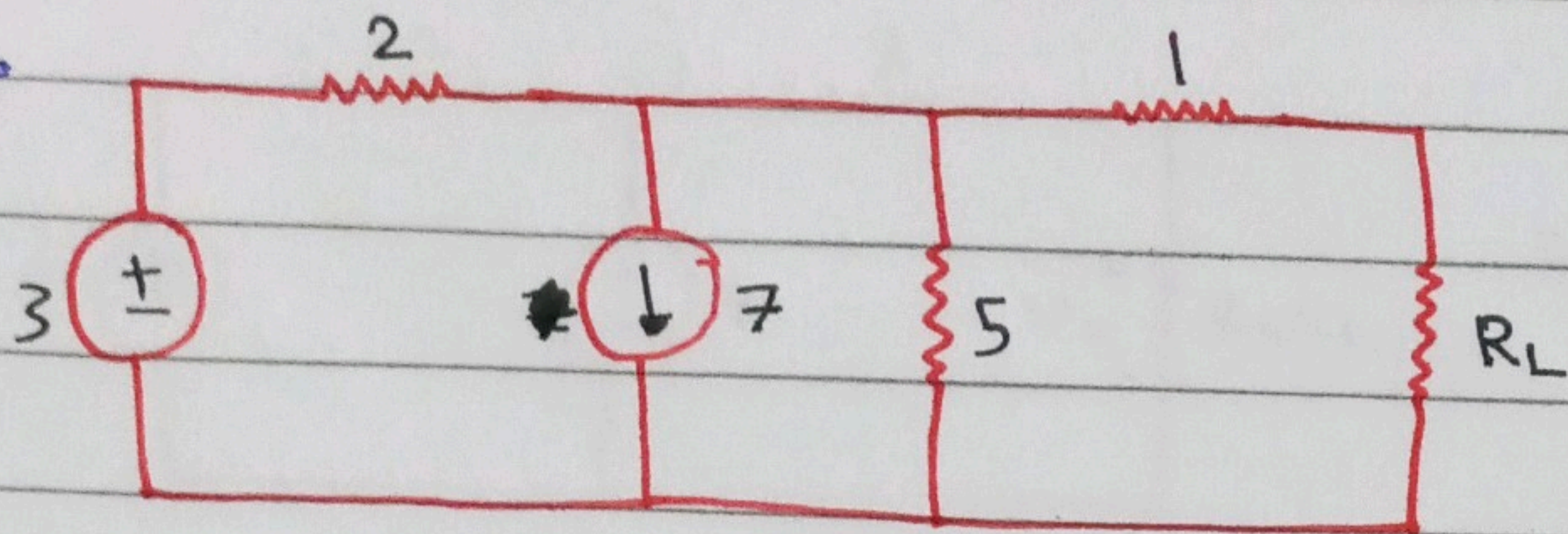
Norton circuit



- Note: $R_{th} = R_N = \frac{V_{th}}{I_N}$

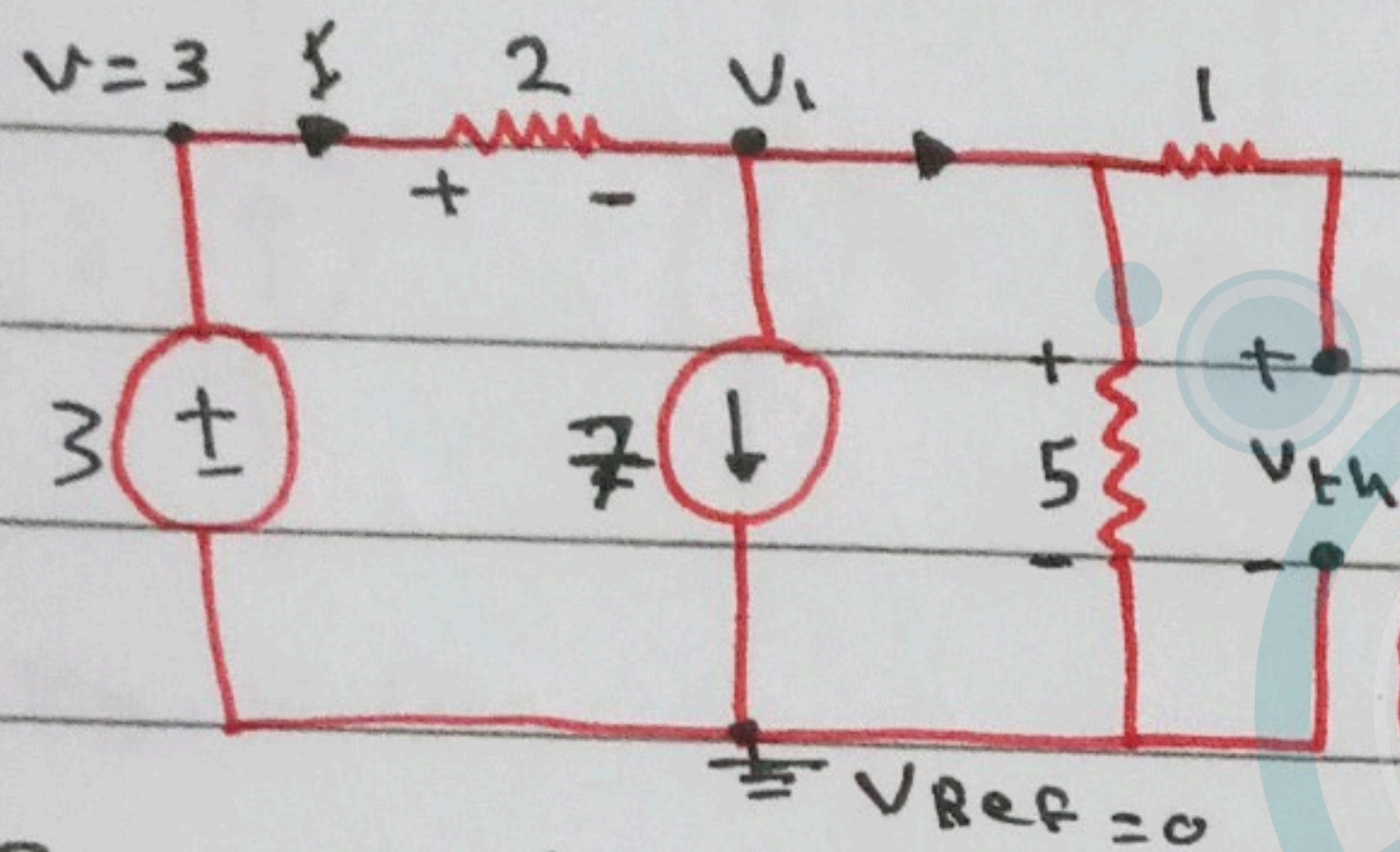
Subject: _____

- Ex.



Find Thevenin-norton
circuits seen from
 R_L .

• Find V_{th}



By Nodal

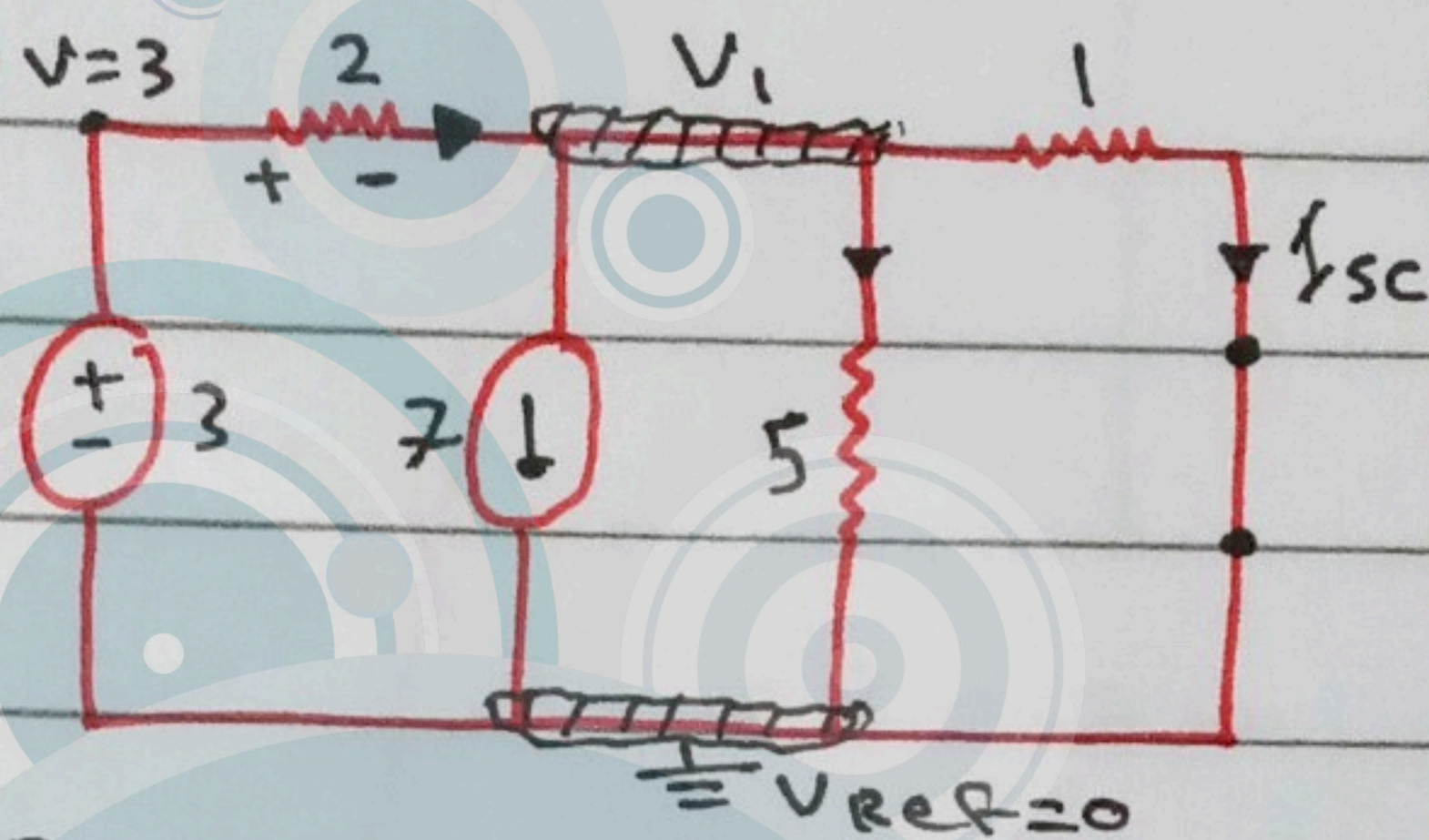
$$I_{in} = I_{out}$$

$$\frac{3 - V_1}{2} = \frac{7}{5} + \frac{V_1}{5}$$

$$V_1 = -7.86 \text{ V}$$

$$V_{oc} = V_{th} = -7.86$$

• Find I_N



By Nodal

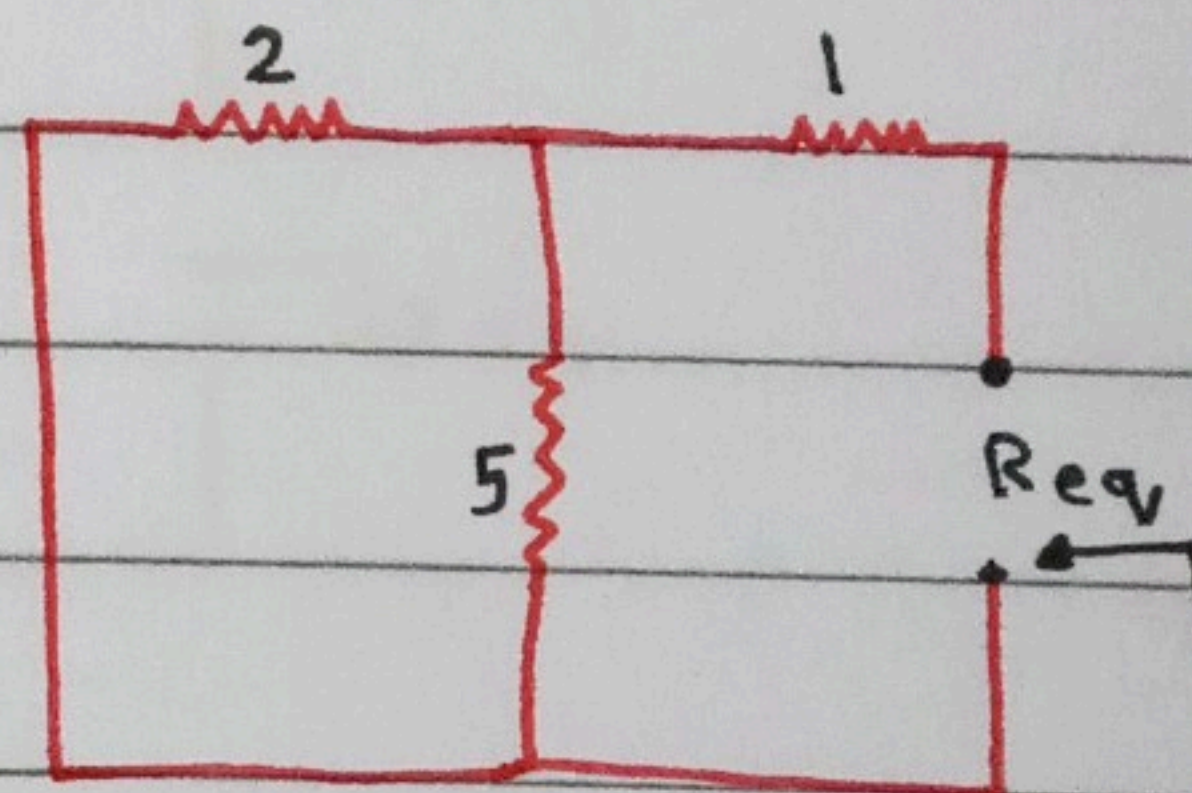
$$I_{in} = I_{out}$$

$$\frac{3 - V_1}{2} = 7 + \frac{V_1}{5} + V_1$$

$$V_1 = -3.24 \text{ V}$$

$$I_{sc} = \frac{V_1}{1} = -3.24$$

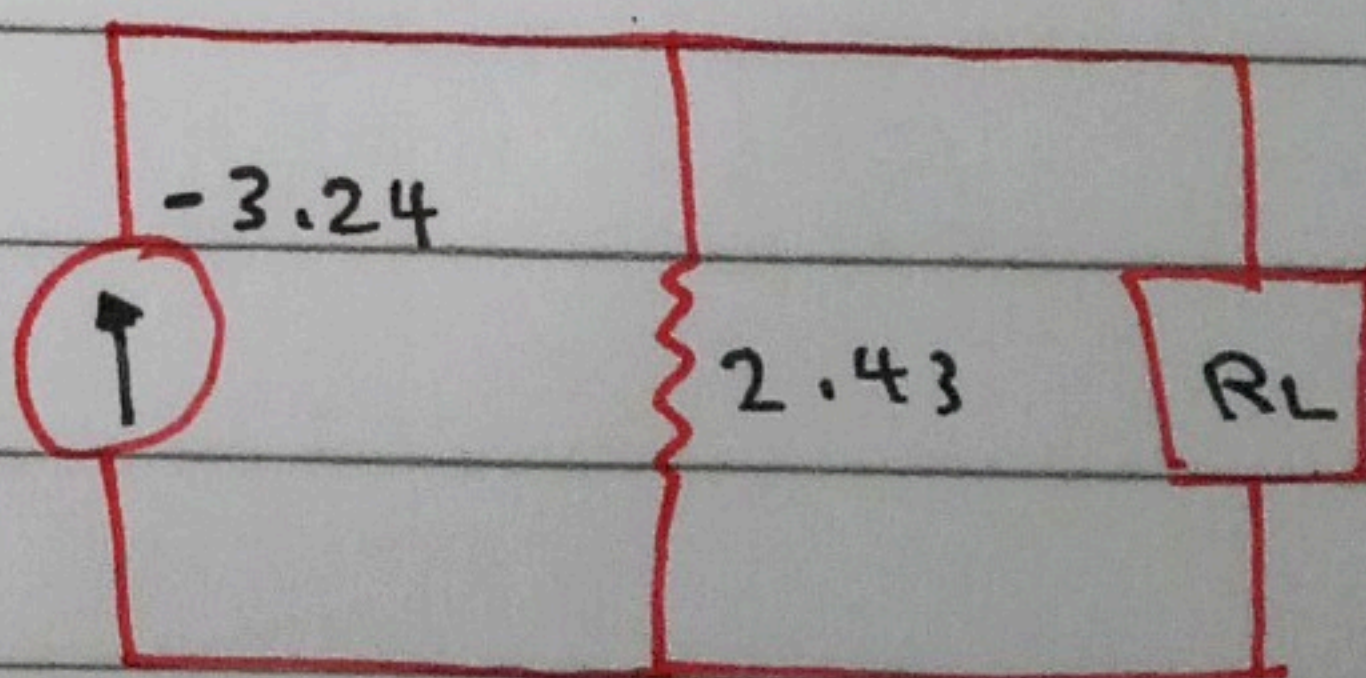
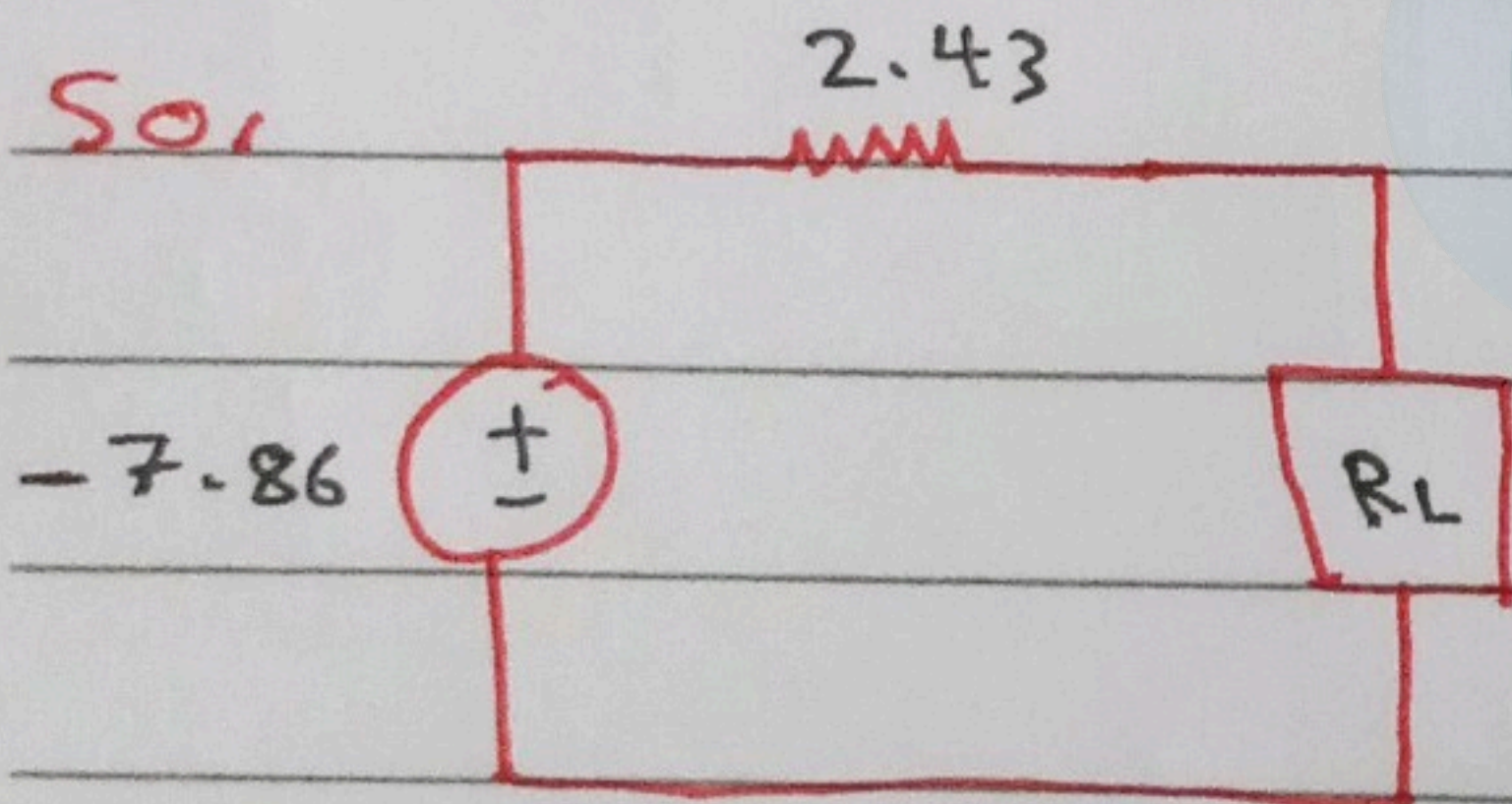
• Find $R_{th} = R_N$



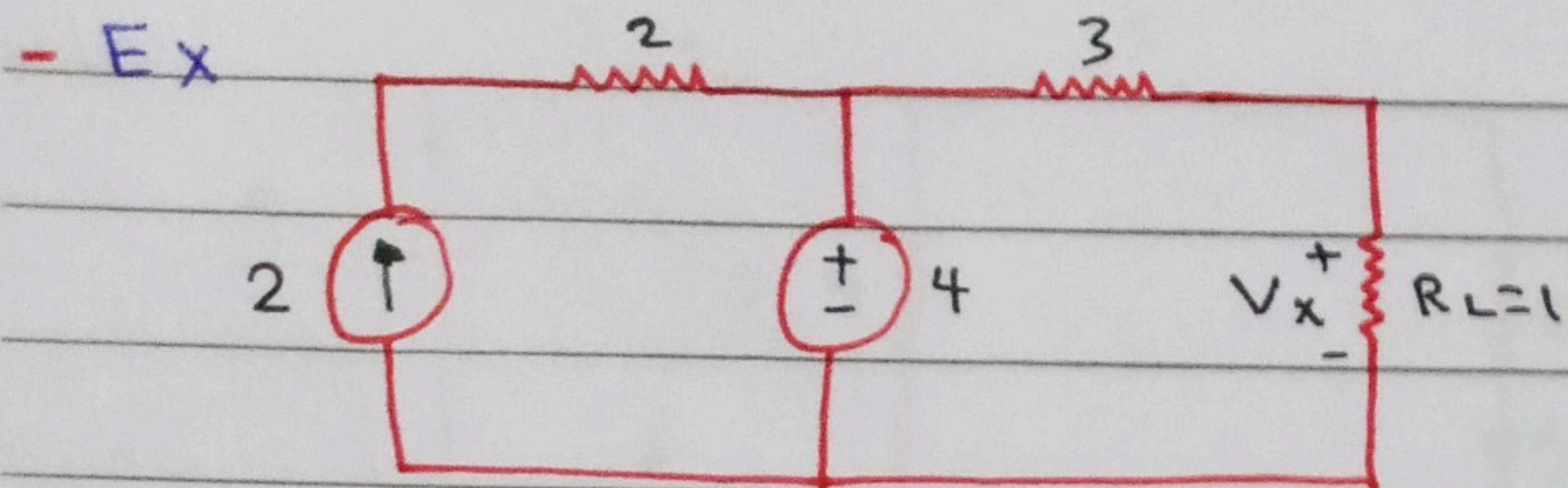
$$R_{eq} = \frac{10}{7} + 1$$

$$R_{th} = 2.43$$

Sol

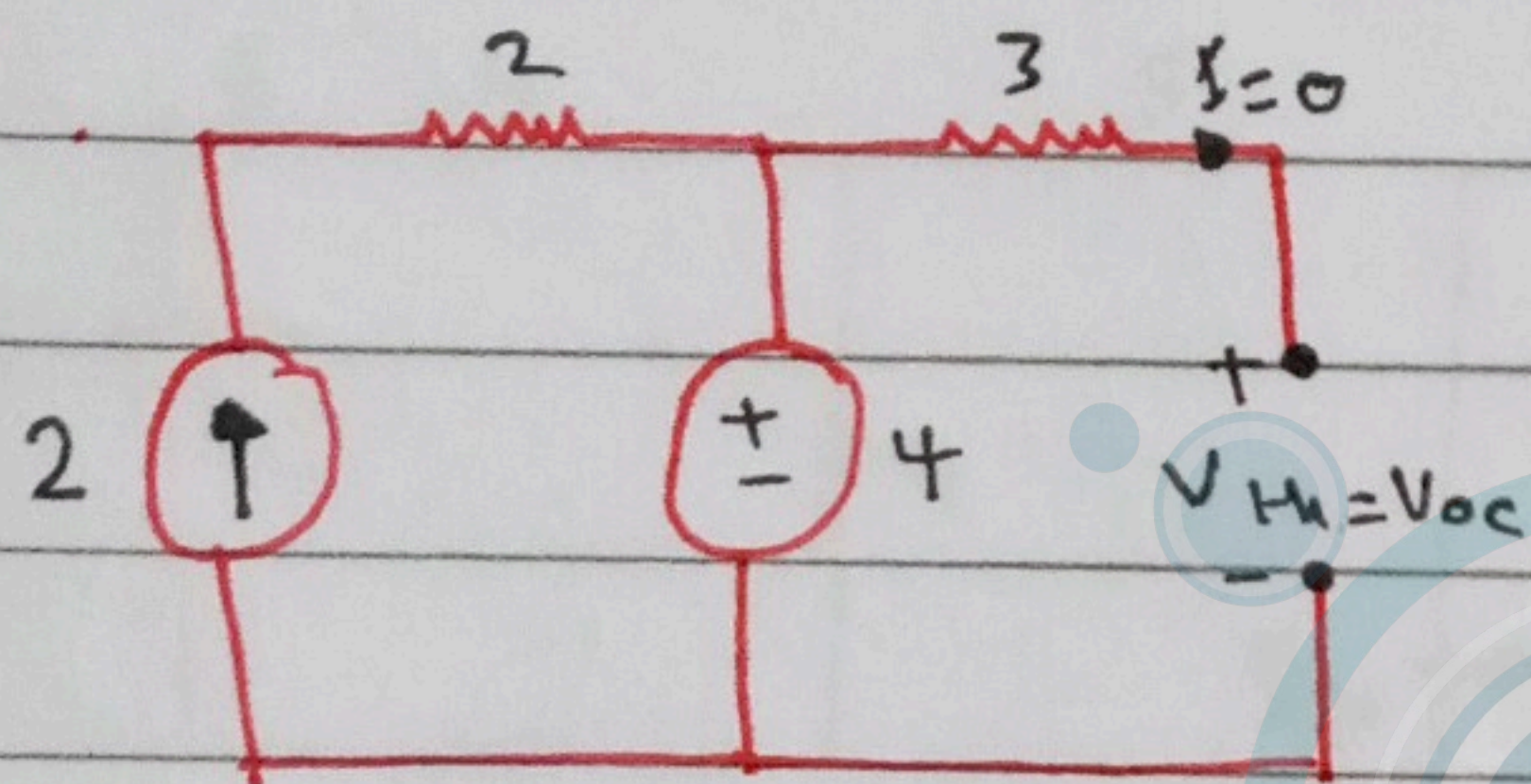


► Subject :

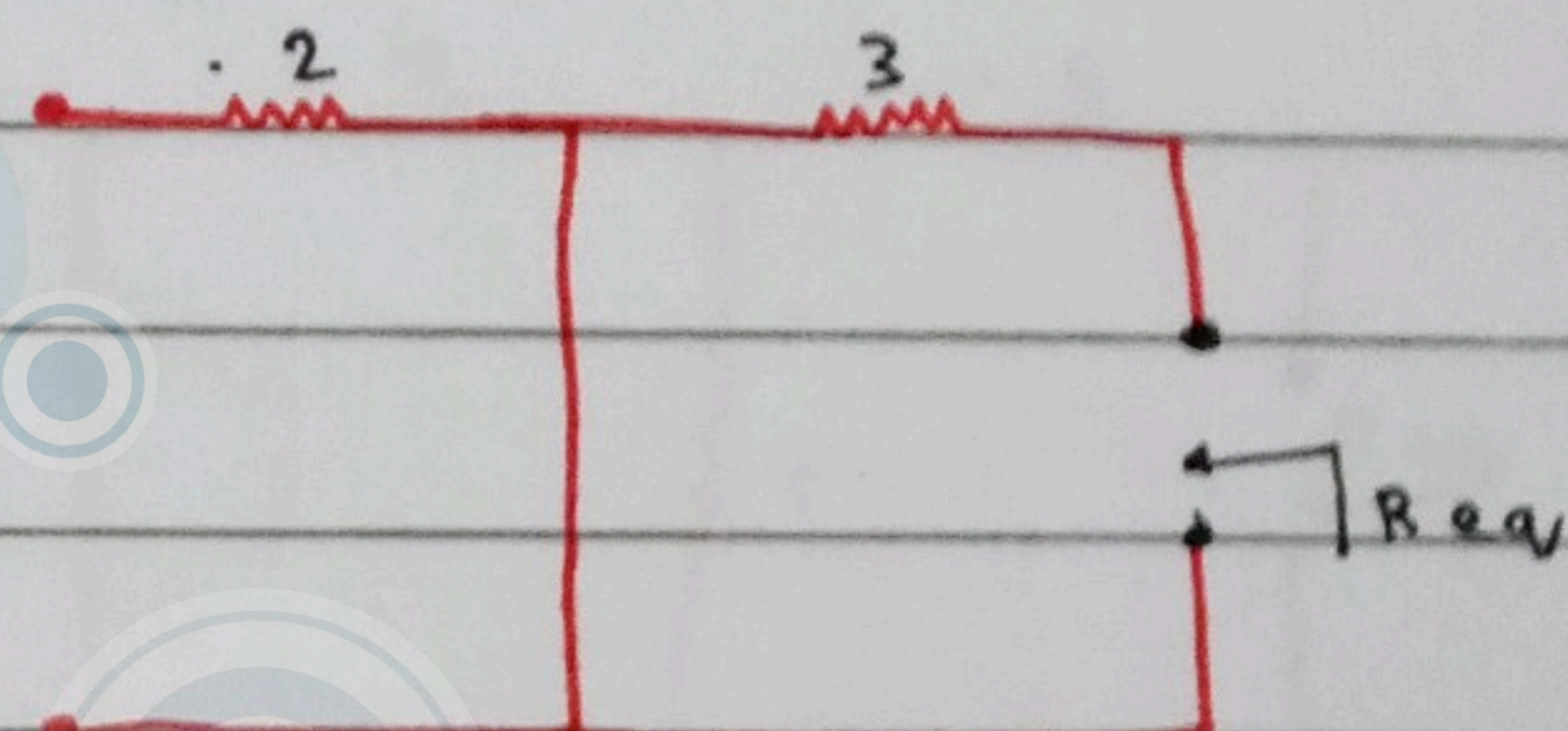


Find V_x using Thevenin

• Find V_{th}



• Find R_{th}

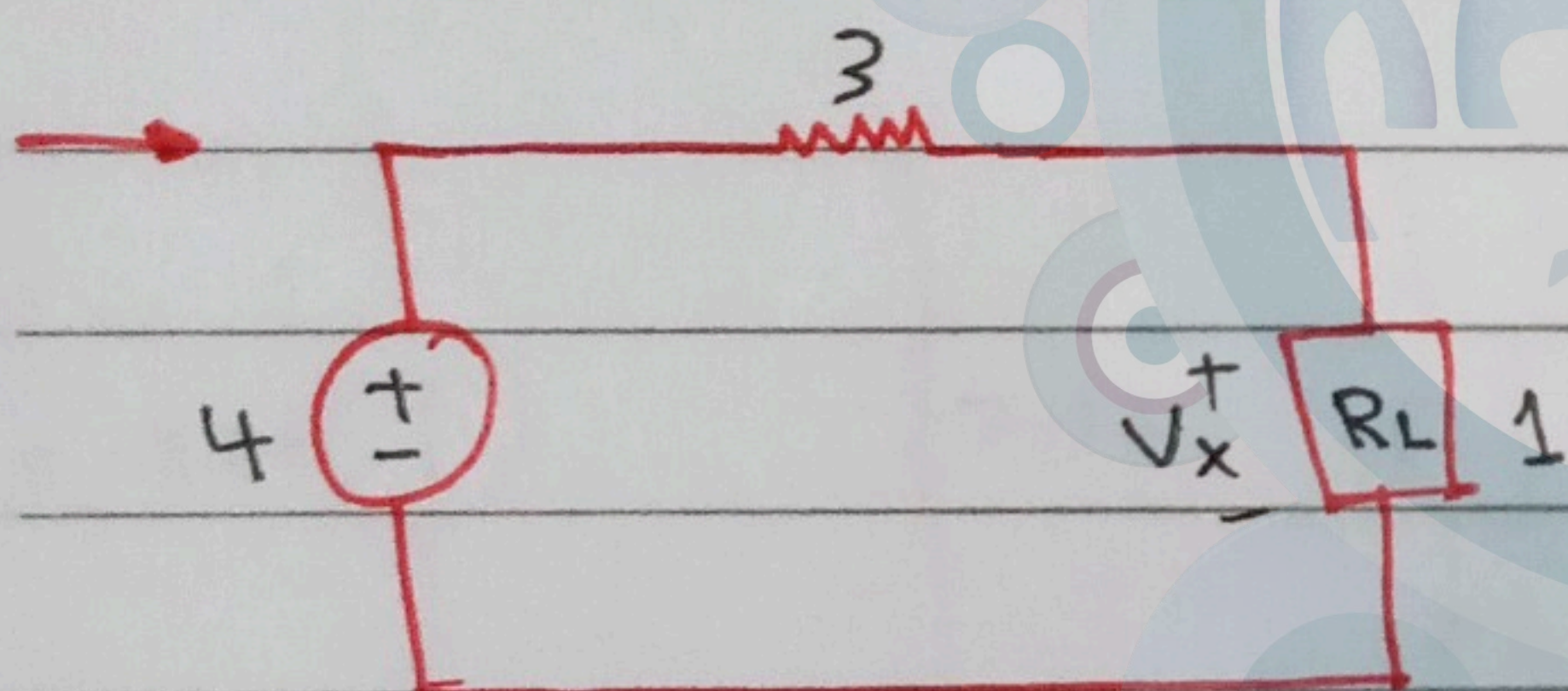


By KVL

$$-4 + 0 + V_{th} = 0$$

$$V_{th} = 4$$

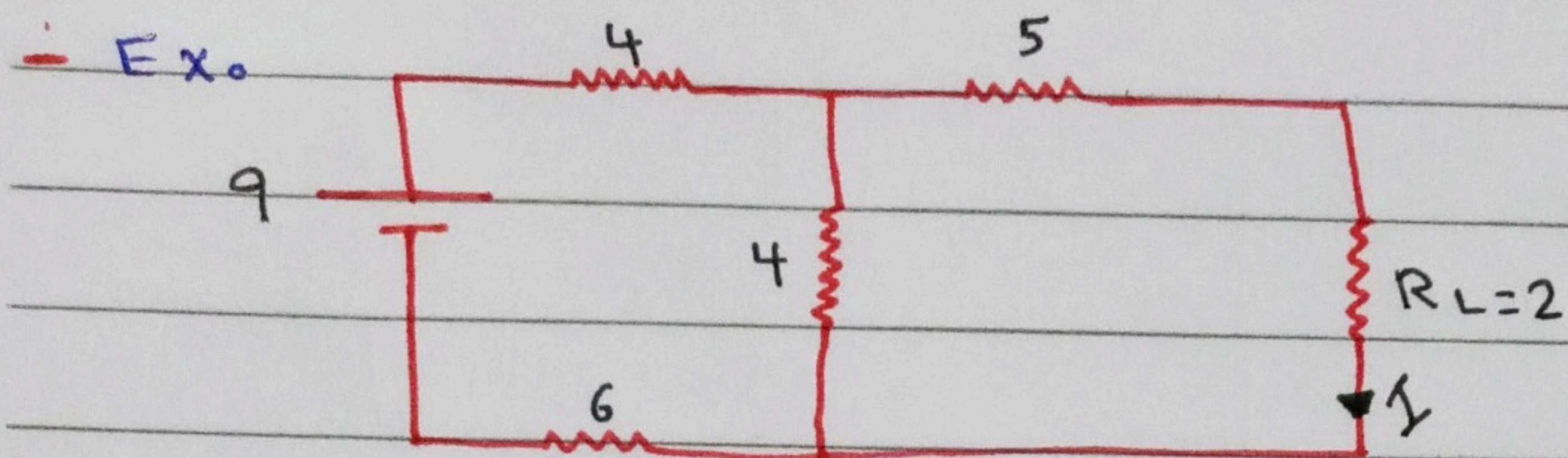
$$R_{eq} = 3 = R_{th}$$



Voltage division:

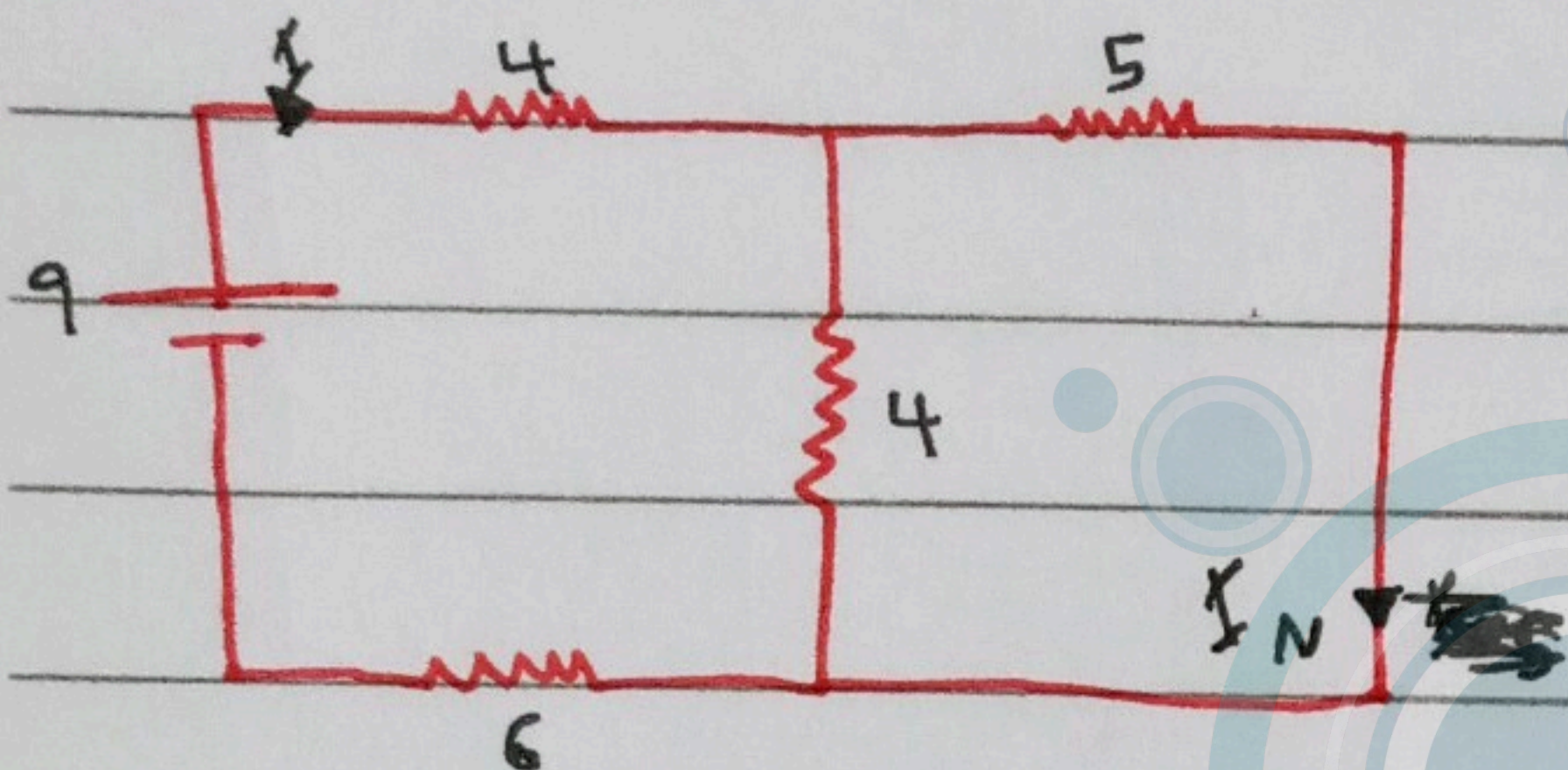
$$V_x = 4 \times \frac{1}{3 + 1} = 1 \text{ V}$$

► Subject : _____



Find I using Norton

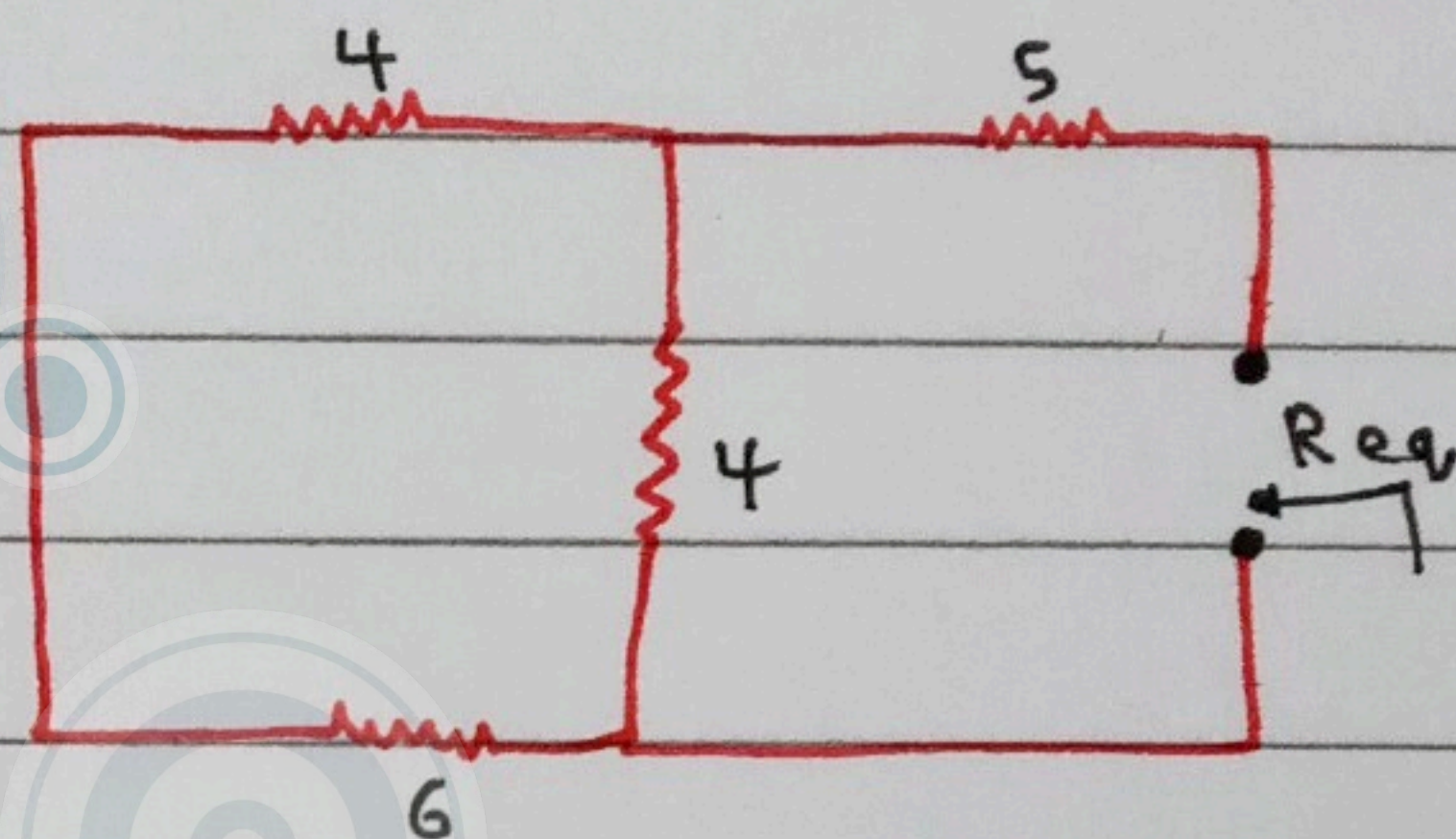
• Find I_N



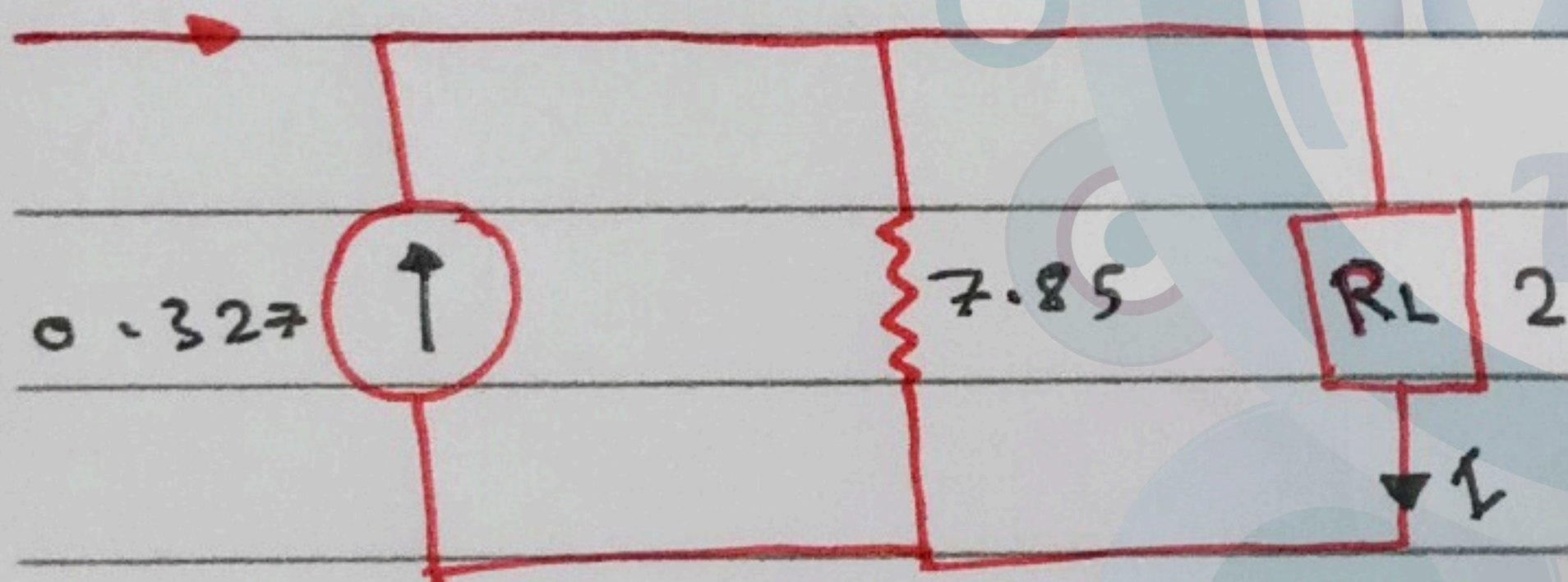
$$R_{eq} = 12.22, \quad I = 0.736$$

$$I_N = 0.736 \times \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{4}} = \boxed{0.327}$$

• Find R_N

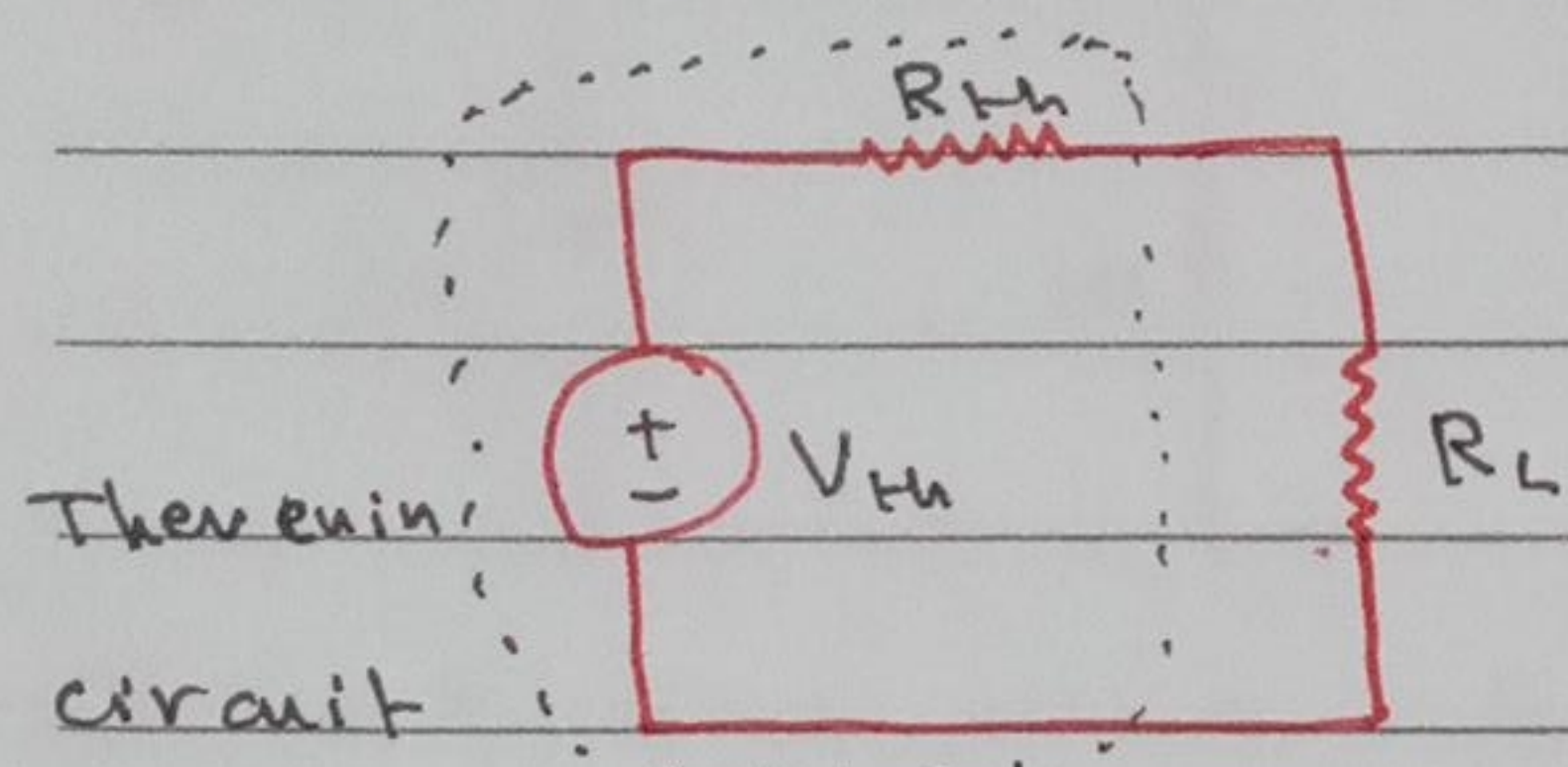


$$R_{eq} = 7.85$$



$$I = 0.327 \times \frac{\frac{1}{2}}{\frac{1}{7.85} + \frac{1}{2}} = \boxed{0.26 \text{ A}}$$

* Maximum Power Transfer



$$\text{KVL: } -V_{th} + I R_{th} + I R_L = 0$$

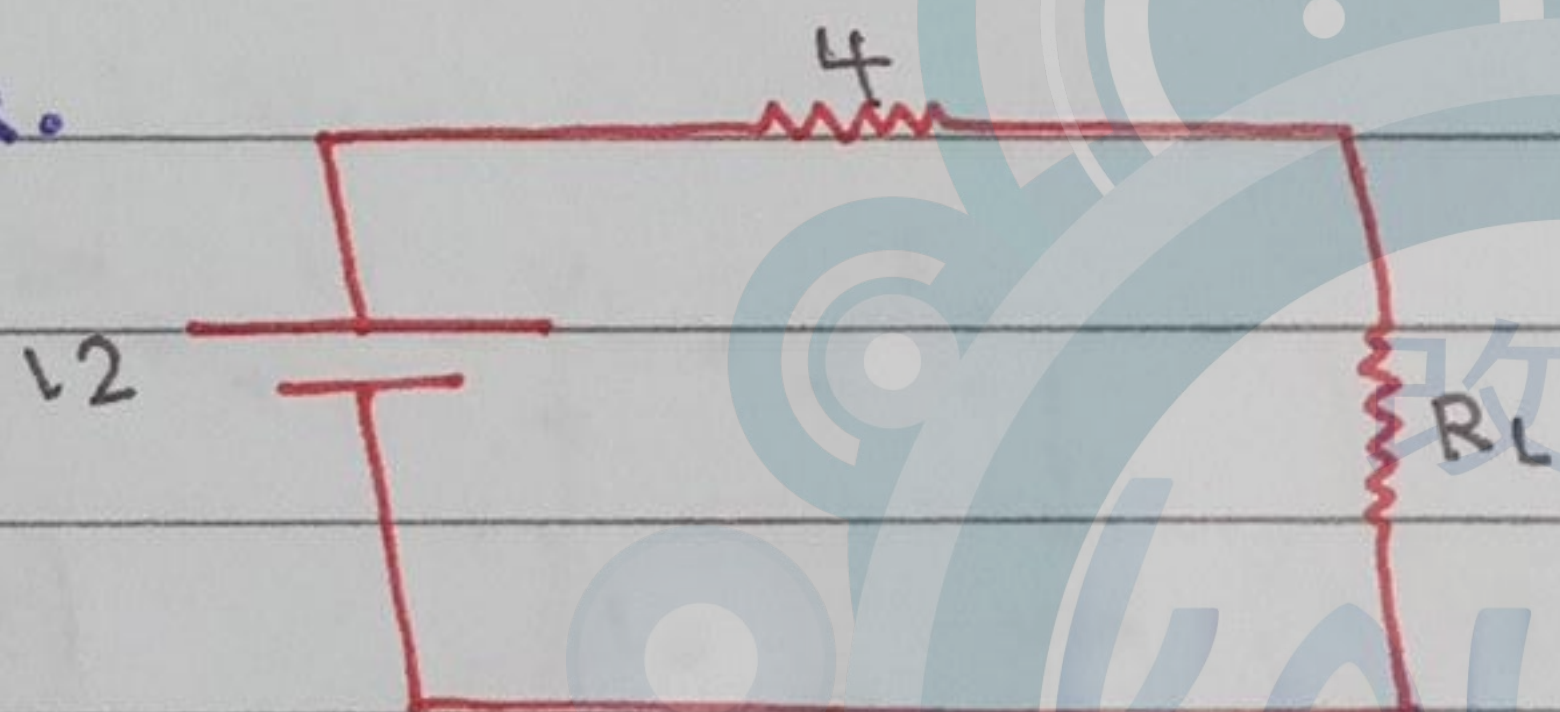
$$I = \frac{V_{th}}{R_{th} + R_L}$$

$$P = I^2 R \rightarrow \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 R_L$$

- Maximum power transfer occur when $R_{th} = R_L$

$$P_{max} = \frac{V_{th}^2}{4 R_{th}}$$

- Ex.



Find power at R_L when!

① $R_L = 2$

$$P = \left(\frac{12}{4+2} \right)^2 \times 2 = 8 \text{ W}$$

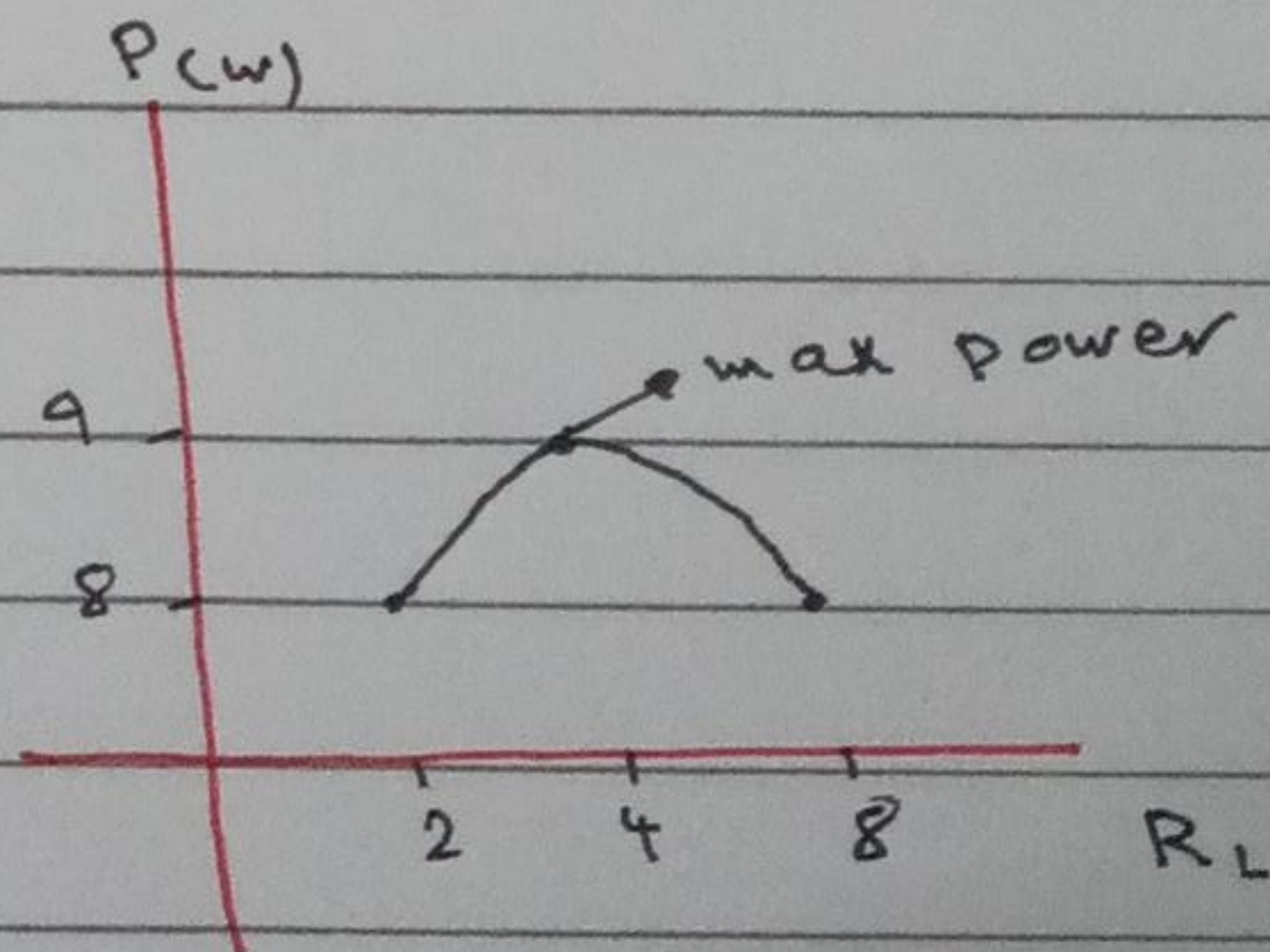
② $R_L = 4$

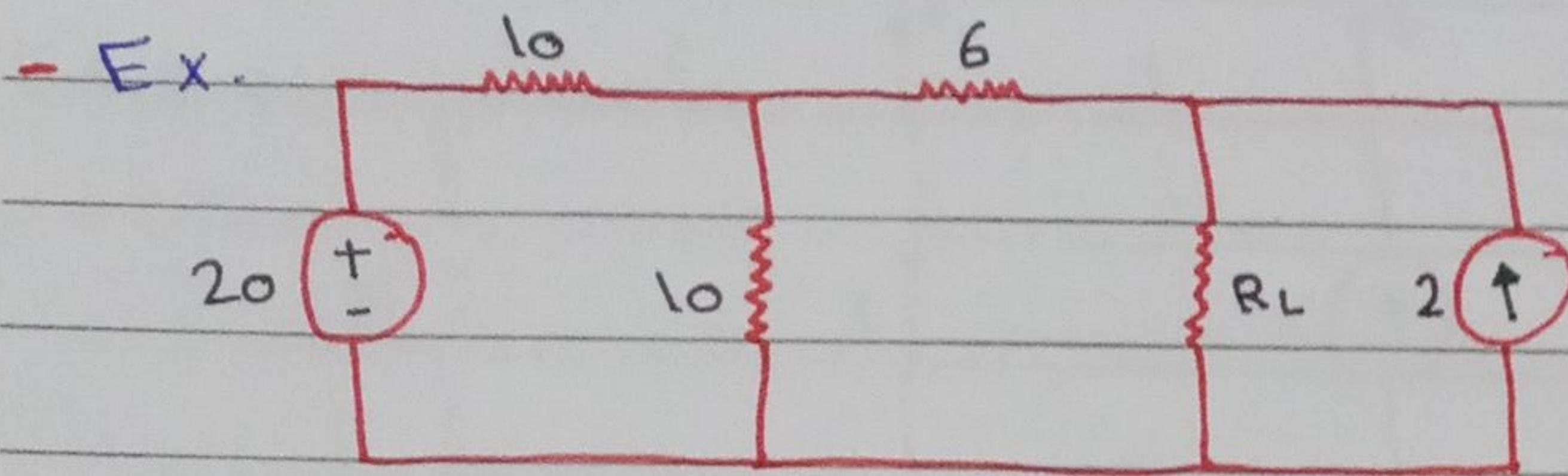
$$P = \left(\frac{12}{8} \right)^2 \times 4 = 9 \text{ W}$$

③ $R_L = 8$

$$P = \left(\frac{12}{12} \right)^2 \times 8 = 8 \text{ W}$$

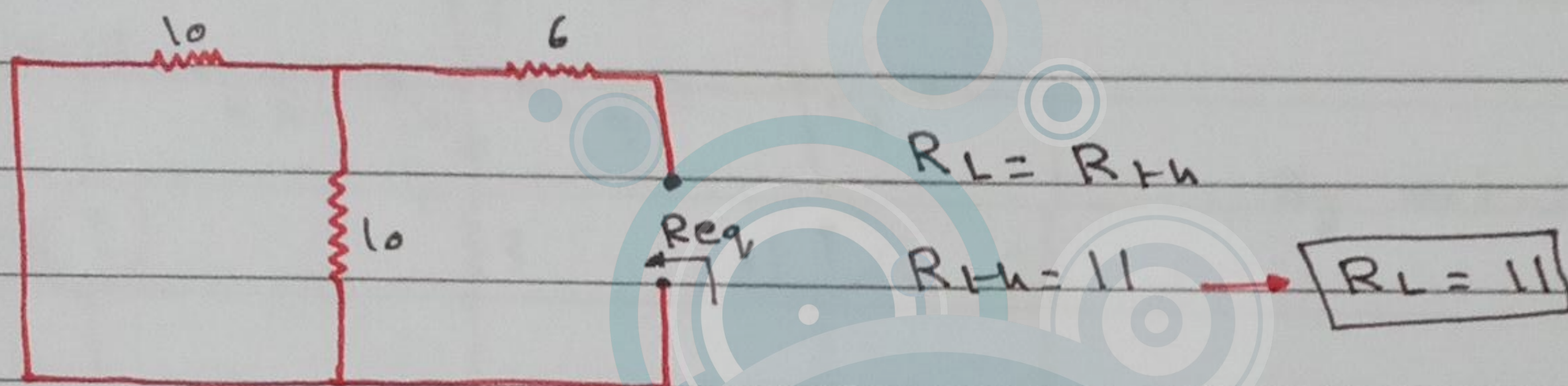
$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{144}{4 \times 4} = 9 \text{ W}$$





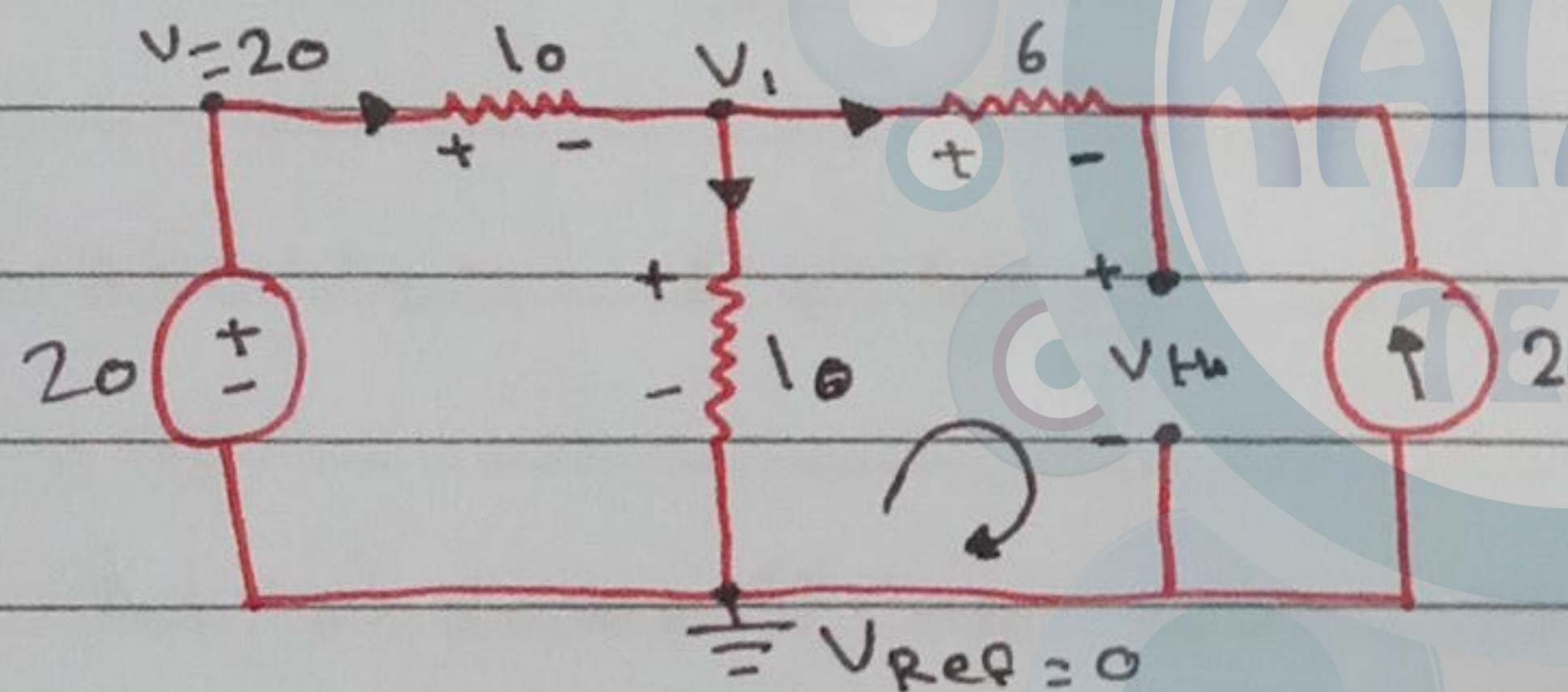
① Find the value of R_L that absorbs max power.

Find R_{Th} :



② Find max power:

Find V_{Th} :



By Nodal, at V_1 : $I_{in} = I_{out}$

$$\frac{20 - V_1}{10} = \frac{V_1}{10} + -2$$

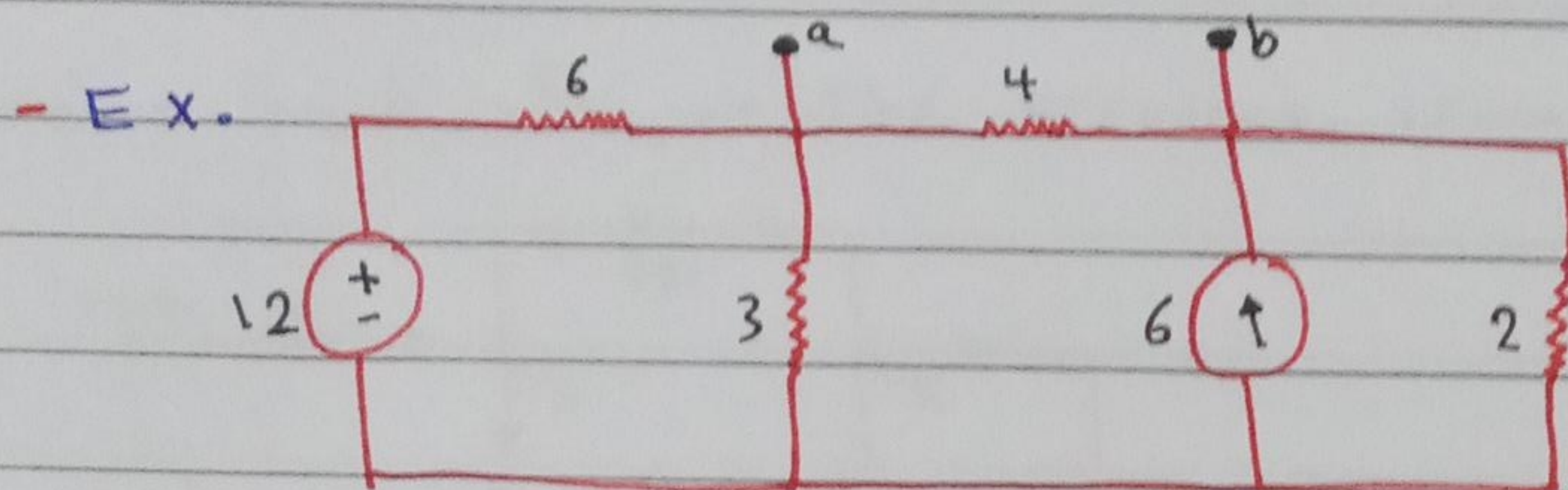
$$\boxed{V_1 = 20}$$

KVL:- $-V_1 + 6(-2) + V_{Th} = 0$

$$-20 - 12 = -V_{Th}$$

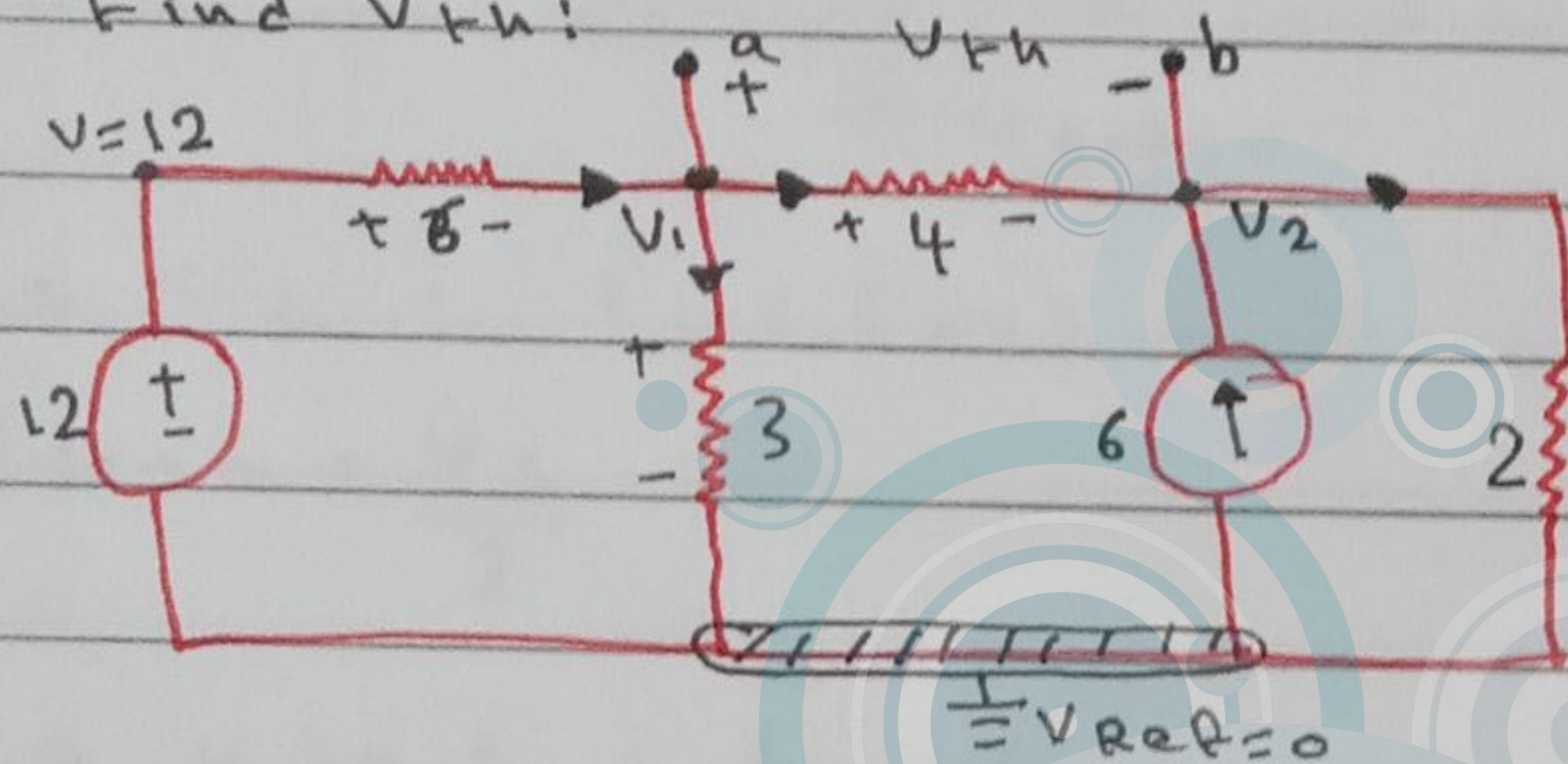
$$\boxed{V_{Th} = 32}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(32)^2}{4 \times 11} = \boxed{23.3 \text{ W}}$$



① Find Thevenin circuit seen between a, b:-

Find V_{th} :



By Nodal

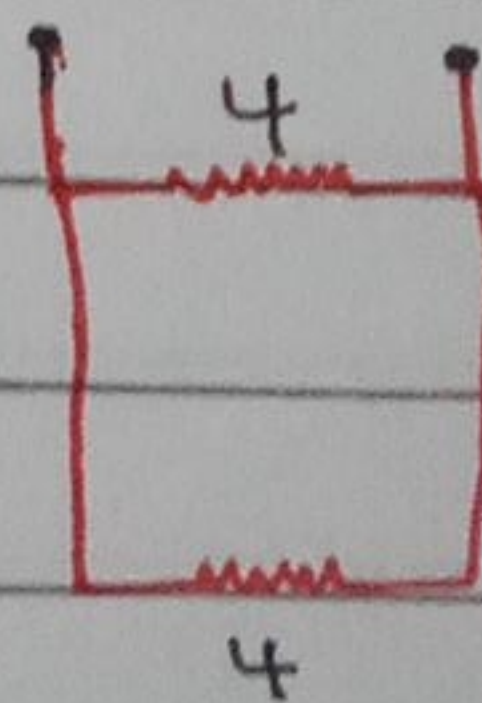
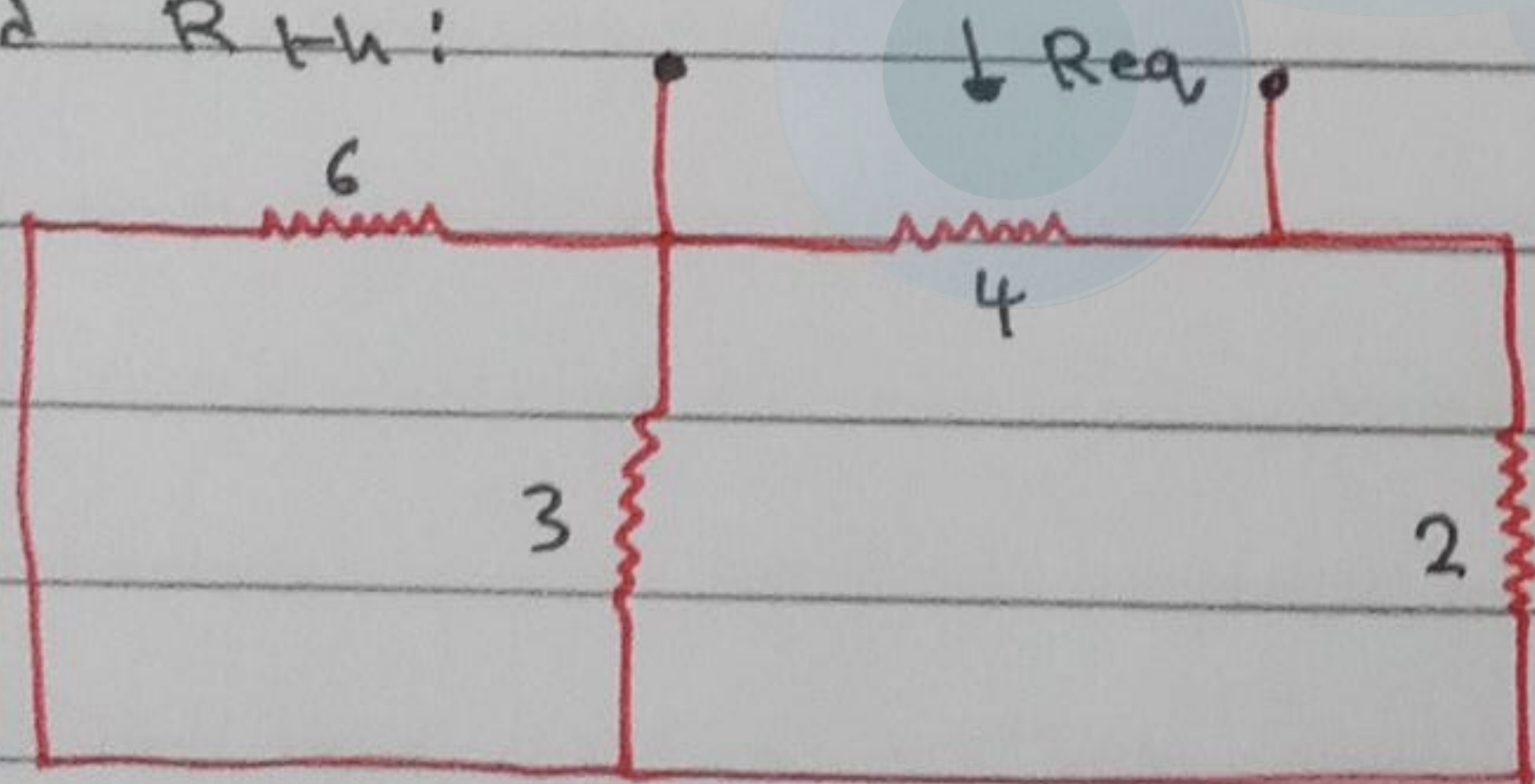
• at V_1 : $\frac{12 - V_1}{6} = \frac{V_1 - V_2}{4} + \frac{V_1}{3}$

• at V_2 : $6 + \frac{V_1 - V_2}{4} = \frac{V_2}{2}$ • $V_1 = 6$ $V_2 = 10$

$V_{th} = \text{Same } V \text{ (at } 4\Omega \text{ (parallel))}$

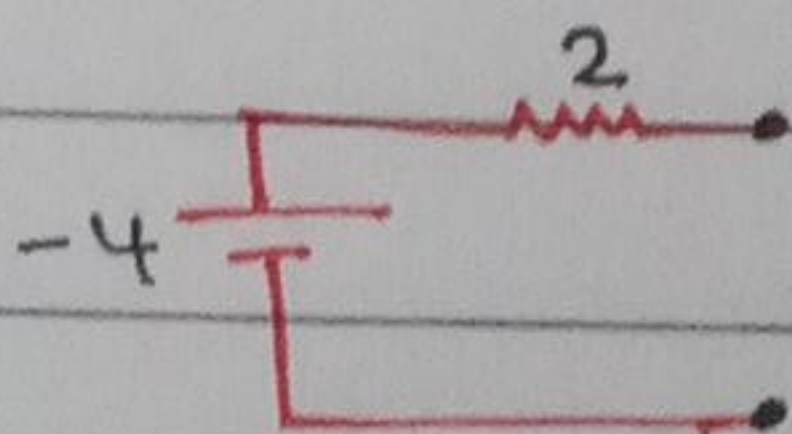
$= V_1 - V_2 = \boxed{-4V}$

Find R_{th} :



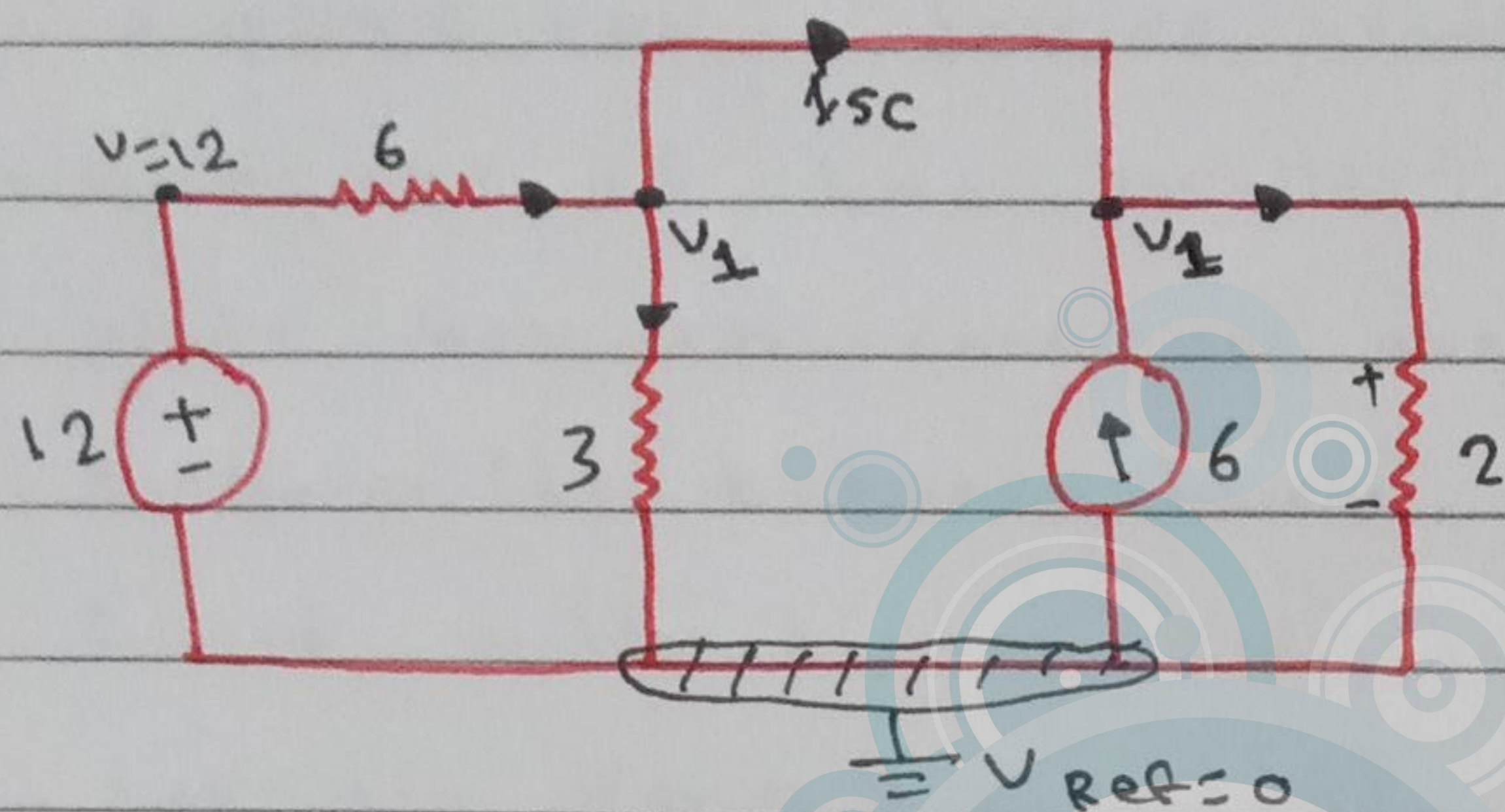
$R_{th} = 2$

Thevenin circuit:



② $P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(-4)^2}{4 \times 2} = 2W$

- To find I_N in the previous example:-



• at V_1 : $I_{in} = I_{out}$

$$I_{sc} + 6 = \frac{V_1}{2}$$

• at Node 1, $I_{in} = I_{out}$

$$\frac{12 - V_1}{6} + 6 = \frac{V_1}{3} + \frac{V_1}{2}$$

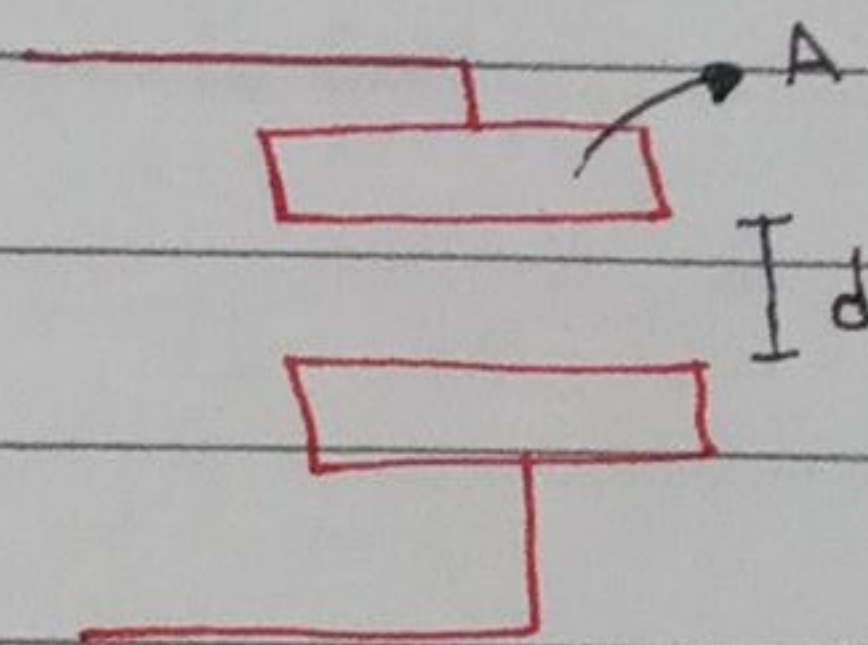
$$V_1 = 8$$

$$I_{sc} = I_N = -2 \text{ A}$$

* AC Network Analysis:-

① Capacitor: device store energy.

- Consists of two parallel plates, each plate has cross sectional area (A), have distance (d) between them, the separation filled with dielectrical material has certain permittivity. (ϵ)



- $C = \frac{\epsilon A}{d}$, $[C] \rightarrow F$, Farad

- symbol in electric circuit:

- charge on plates of capacitor: $Q = CV$, $q(t) = C V(t)$

- $i_c(t) = C \frac{d V_c(t)}{dt}$

- if $V(t) = \text{const} \equiv \text{DC Voltage} \rightarrow i_c(t) = 0$

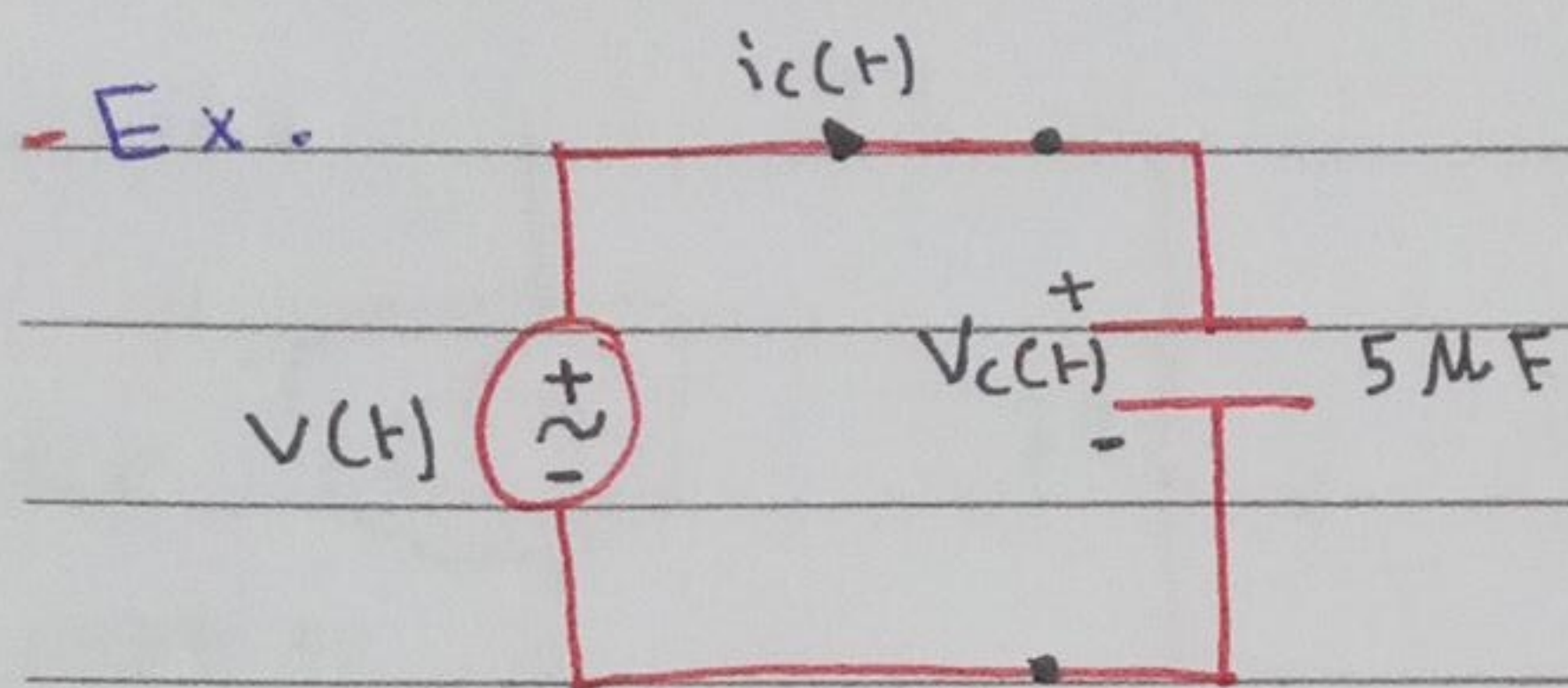
- Capacitor act like open circuit in DC circuit.

- $V_c(t) = \left[\frac{1}{C} \int_{t_0}^t i_c(\tau) d\tau \right] + V_c(t_0)$

- Energy stored in capacitor: $E = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}$, in Joule

- Capacitor on series = $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$

- Capacitors on parallel = $C_{eq} = C_1 + C_2 + C_3 + \dots$



$$v(t) = 12e^{-3t} \quad v_c(t) \text{ (in this Ex)}$$

Find: $i_c(t)$, $i_c(0)$, $i_c(\infty)$

$$\textcircled{1} i_c(t) = C \frac{dv_c(t)}{dt} = 5 \times 10^{-6} * (12 * -3 * e^{-3t}) = -180 \times 10^{-6} e^{-3t} \text{ A}$$

$$\textcircled{2} i_c(0) = -180 \times 10^{-6}$$

$$\textcircled{3} i_c(\infty) = 0 \text{ A}$$

Ex.

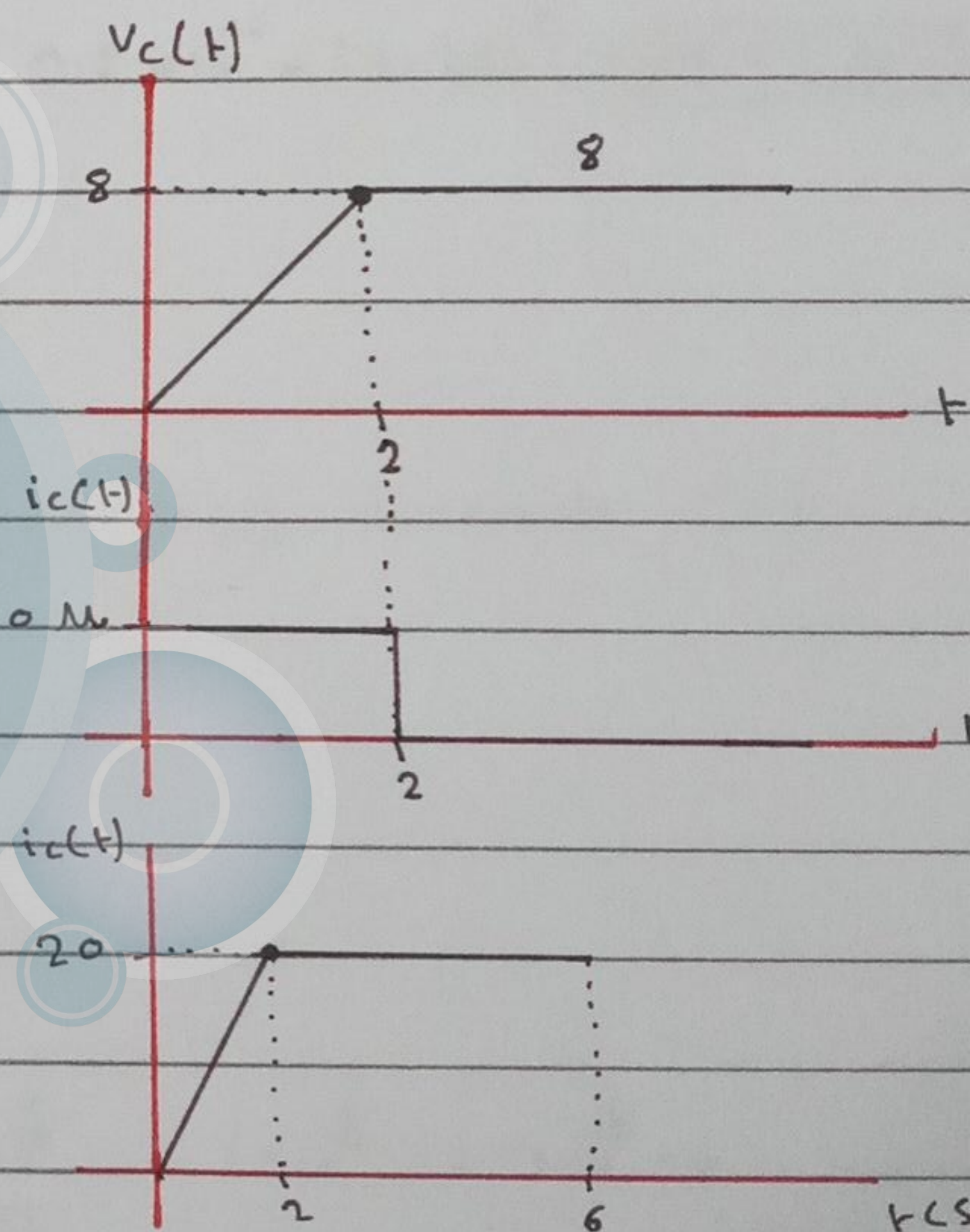
$C = 5 \text{ MF}$, sketch $i_c(t)$

$$\textcircled{1} \text{ For } 0 \leq t \leq 2 \quad v_c(t) = 4t$$

$$i_c(t) = C \frac{dv_c(t)}{dt} = 5 \times 10^{-6} * 4 = 20 \times 10^{-6}$$

$$\textcircled{2} \text{ For } 2 \leq t \leq \infty \quad v_c(t) = 8$$

$$i_c(t) = C \frac{dv_c(t)}{dt} = 0$$



Ex. $C = 1 \text{ MF}$ Find $v_c(t)$

$$v_c(t) = \left[\frac{1}{C} \int_{t_0}^t i_c(\tau) d\tau \right] + v_c(t_0)$$

$$\text{slope} = \frac{20}{2} = 10, \quad y - y_0 = m(x - x_0), \quad y = 10x \rightarrow i_c(t) = 10t$$

$$\textcircled{1} \text{ For } 0 \leq t \leq 2$$

$$v_c(t) = 10^6 \int_0^t 10\tau d\tau + v_c(0)$$

$$v_c(t) = 10^6 * 5\tau^2 \Big|_0^t$$

$$v_c(t) = 5 \times 10^6 t^2$$

$$\textcircled{2} \text{ For } 2 \leq t \leq 6$$

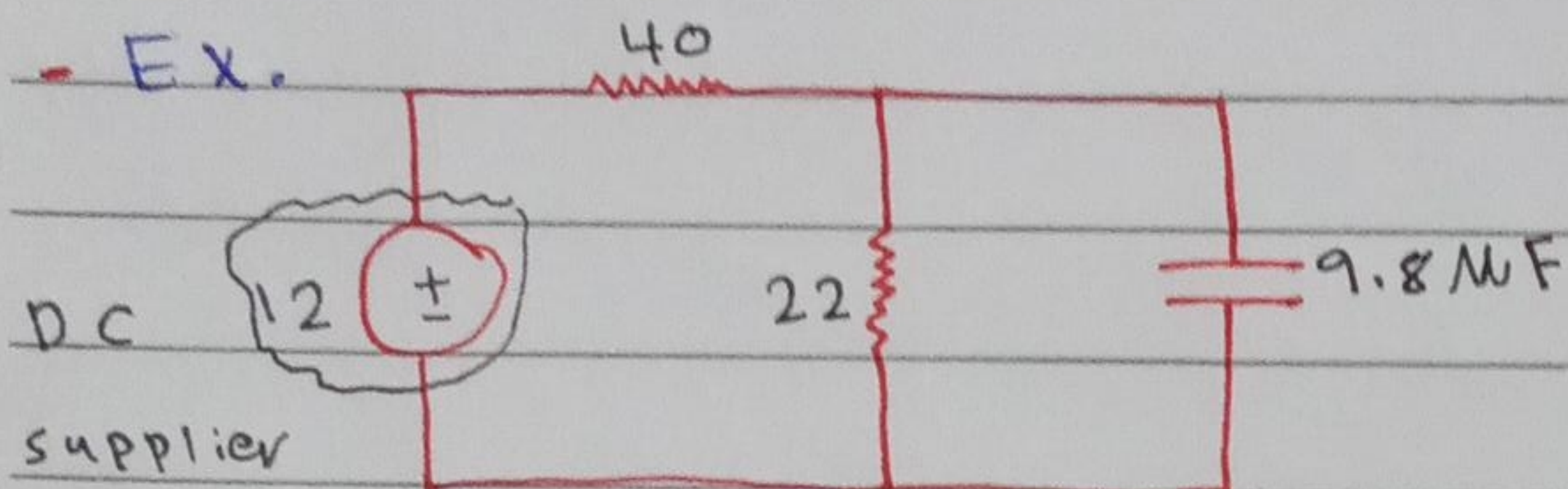
$$i_c(t) = 20 \quad v_c(t) = 10^6 \int_2^t 20 d\tau + v_c(2)$$

نحو يضاف في الفترة الاولى

~~$$v_c(t) = 20 \times 10^6 (t - 2) + 20$$~~

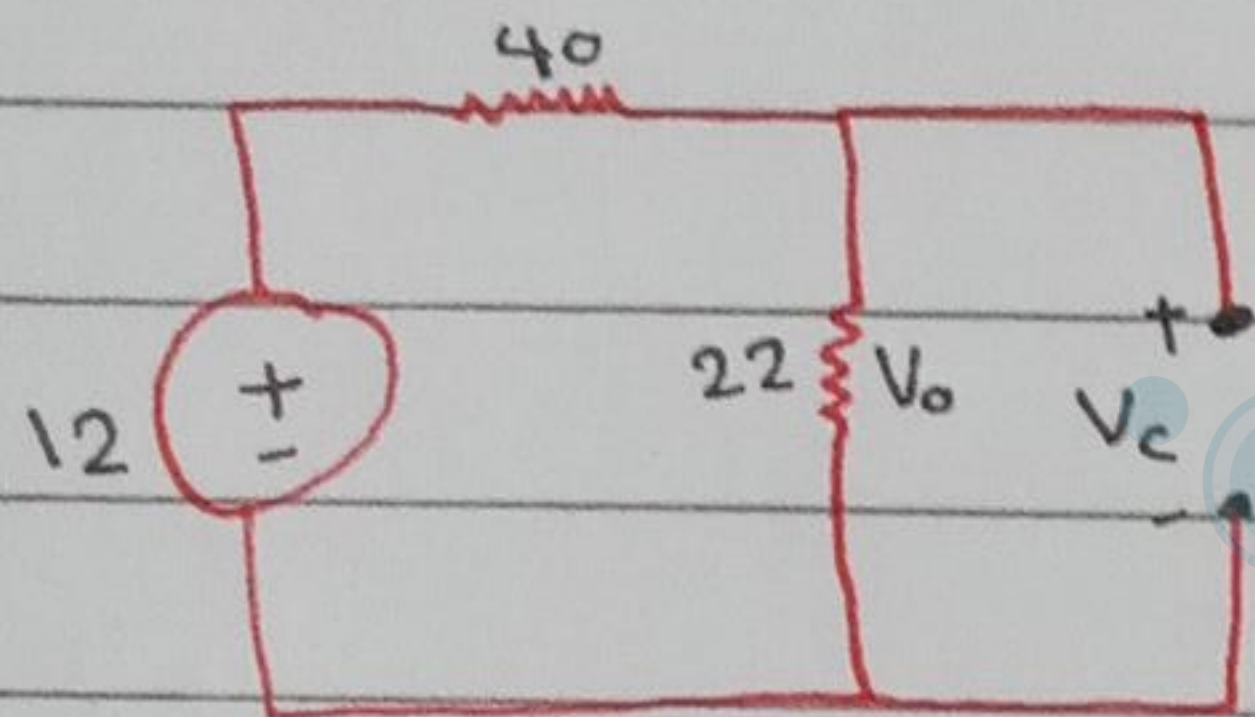
$$v_c(t) = 20 \times 10^6 (t - 2) + 20$$

- Ex.



① Energy in Cap:

The capacitor act like an open circuit in DC circuit:



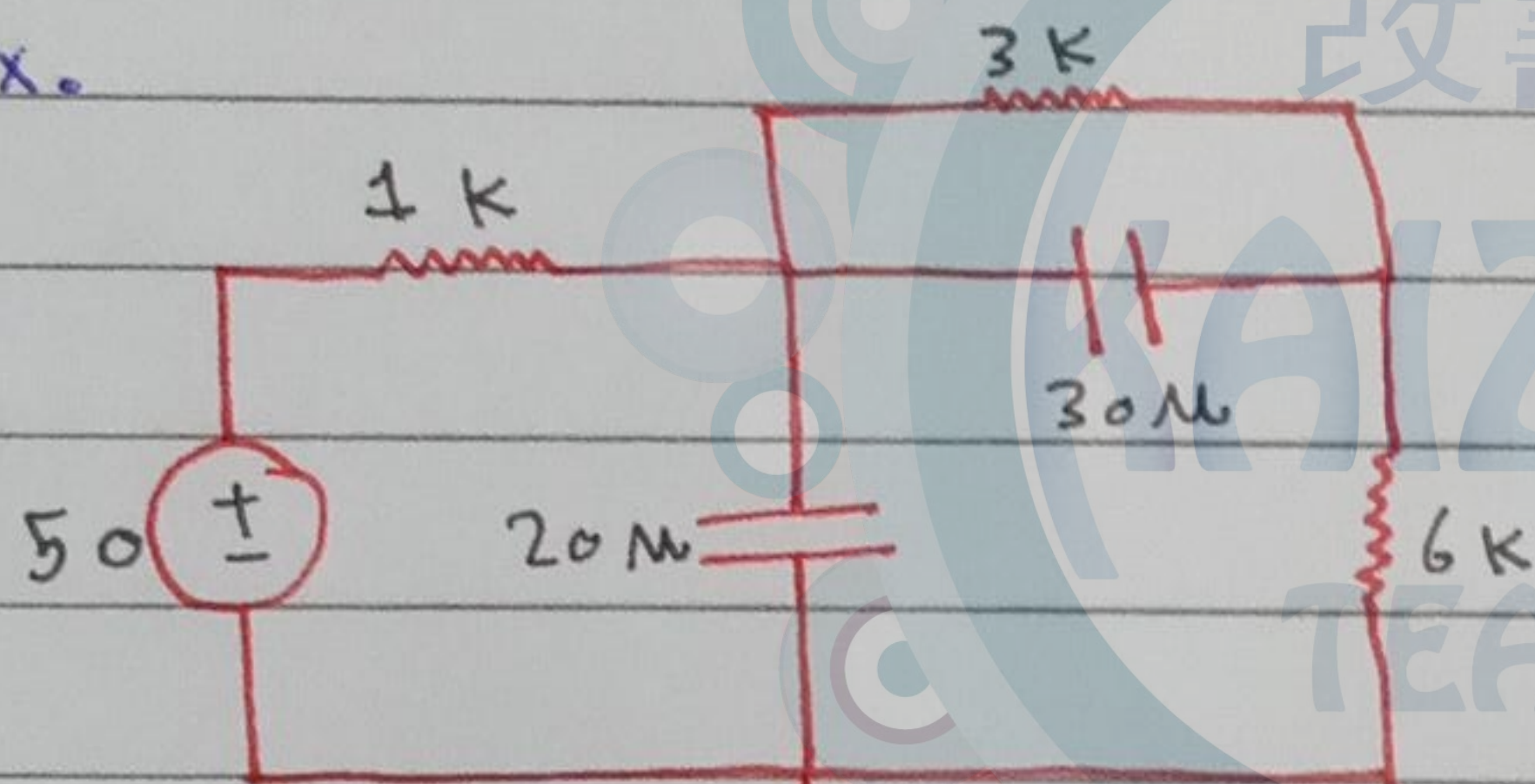
$$V_o = \frac{12 \times 22}{62} = 4.26 = V_c$$

$$E = \frac{1}{2} C V_c^2 = \frac{1}{2} \times 9.8 \times 10^{-6} \times (4.26)^2 = \boxed{88.92 \text{ MJ}}$$

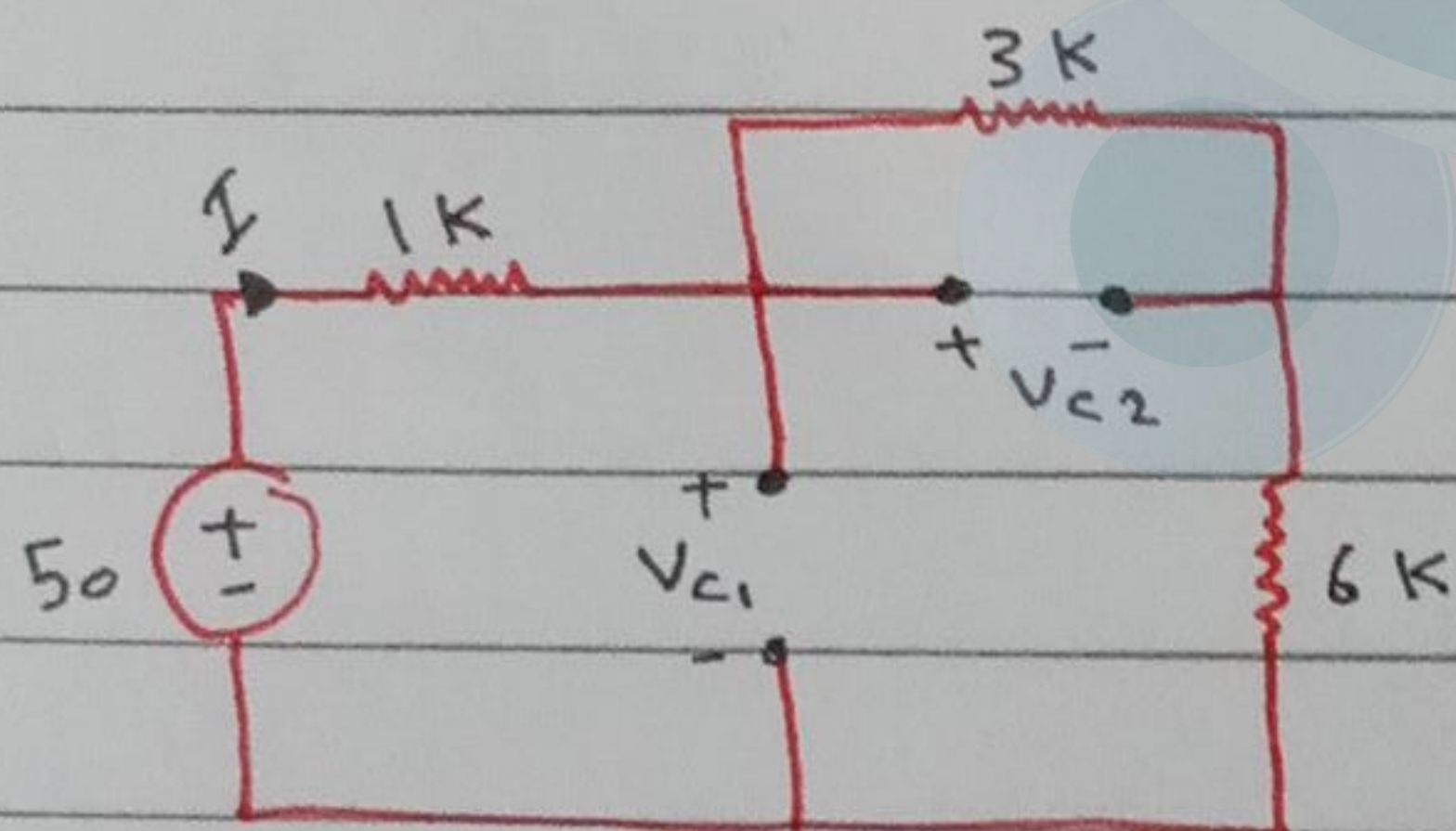
② Q in Cap:

$$Q = C V_c = 9.8 \times 10^{-6} \times 4.26 = \boxed{41.73 \text{ C}}$$

- Ex.



Find energy in each Cap



KVL

$$-50 + I \times 10^3 + 3 I \times 10^3 + 6 I \times 10^3 = 0$$

$$\boxed{I = 5 \times 10^{-3}}$$

$$V_{c2} = 5 \times 10^{-3} \times 3K = 15$$

$$E_{c2} = \frac{1}{2} C V_c^2 = \frac{1}{2} \times 30M \times (15)^2 = \boxed{3375 \text{ MJ}}$$

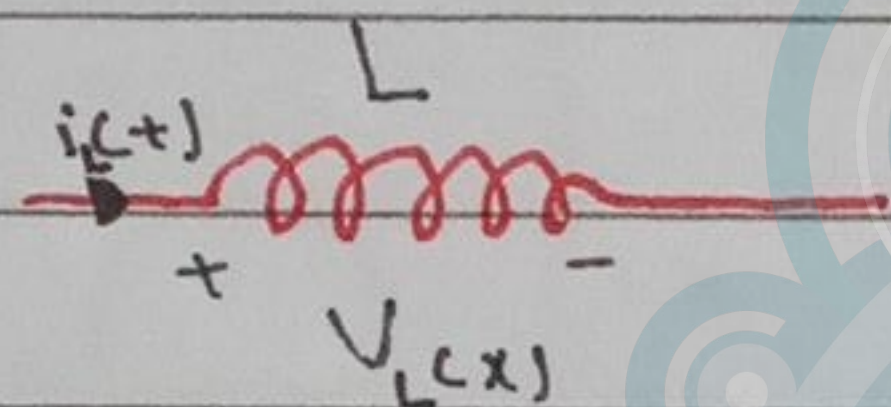
$$V_{c1} = 50 - V_{1K\Omega} = 50 - 5 = 45$$

$$E_{c1} = \frac{1}{2} \times 20M \times (45)^2 = \boxed{20250 \text{ MJ}}$$

② Inductor :-

- Store energy
- made of winding a coil of wire around a core.
The core could be ~~not~~ insulator or Ferro magnetic ~~material~~
material $\rightarrow \mu$: permeability.

- symbol :



- Unit : Henry

$$V_L(t) = L \frac{di_L(t)}{dt} \quad , \quad i_L(t) = \frac{1}{L} \int_{t_0}^t V_L(\tau) d\tau + i_L(t_0)$$

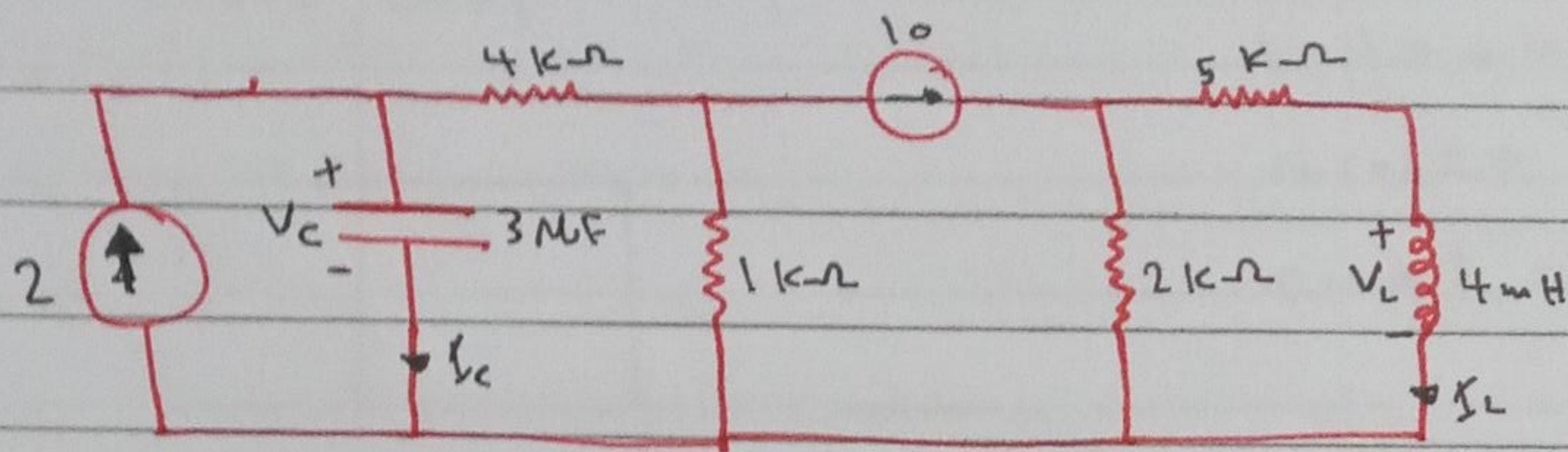
- Energy stored $\equiv E = \frac{1}{2} L i^2$ Joule.

- In DC circuit, the inductor act like short circuit.

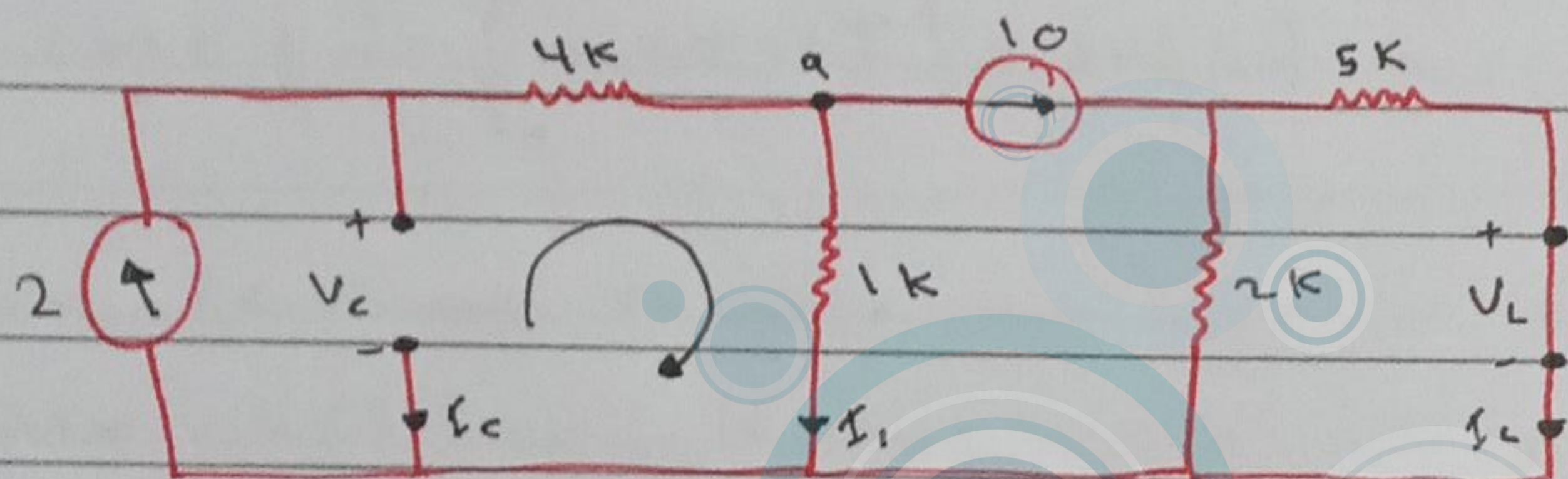
- Series connection: $L_{eq} = L_1 + L_2 + L_3 \dots$

- parallel connection: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots$

Ex.



Find: I_c , V_c , I_L , V_L :



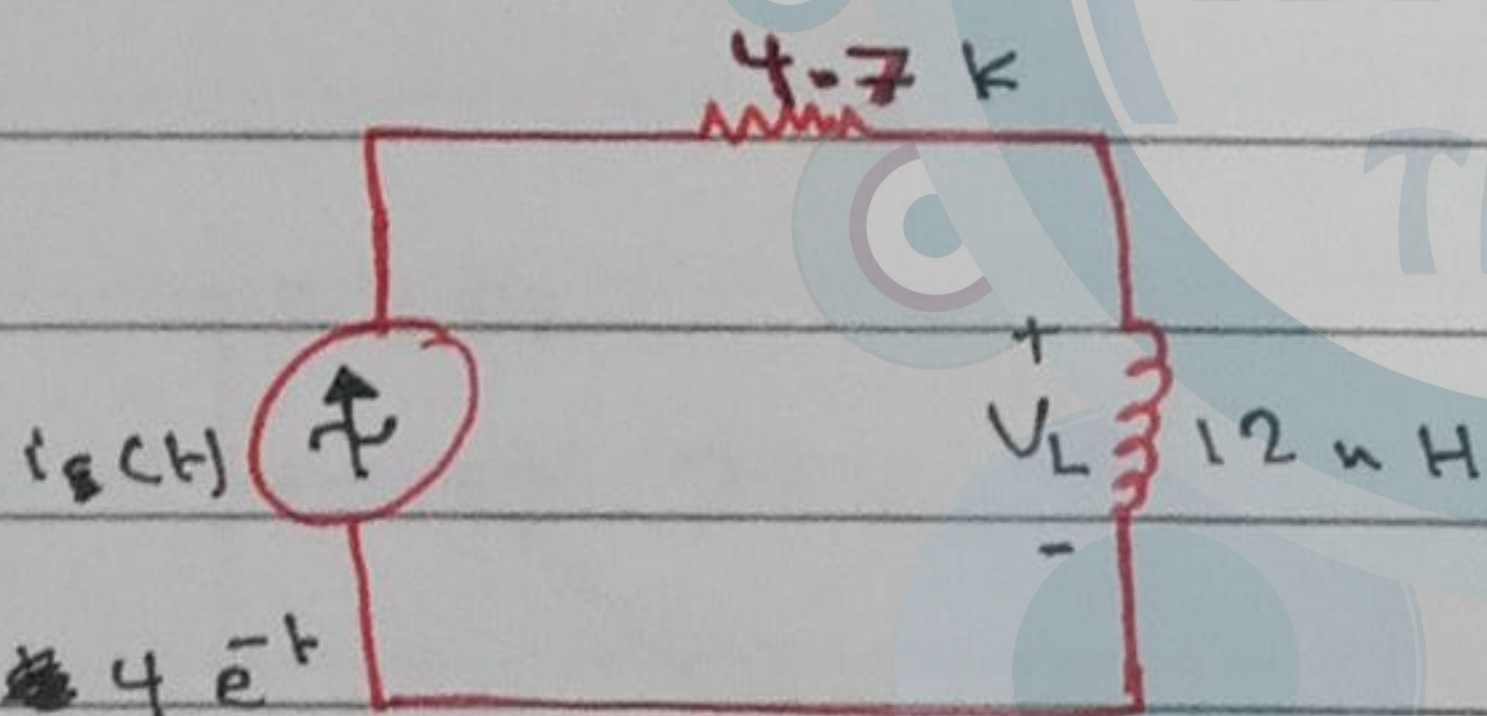
KCL at node a: $2 = 10 + I_1$, $I_1 = -8 \text{ A}$

KVL: $-V_c + 2(4\text{k}) + (-8)(1\text{k}) = 0 \rightarrow V_c = 0$, $I_c = 0$ (open)

改善

$I_L = 10 \times \frac{\frac{1}{5\text{k}}}{\frac{1}{2\text{k}} + \frac{1}{5\text{k}}} = 2.86 \text{ A}$, $V_L = 0$ (short)

Ex.



Find Energy stored in the inductor, $V_L(t)$

$E = \frac{1}{2} L I_c^2$ } $E = \frac{1}{2} (12 \times 10^{-9}) (4e^{-t})^2 = 96 \times 10^{-9} e^{-2t} \text{ Joule}$
 $I_{s(t)} = I_c(t)$

$V_L(t) = L \frac{di_L(t)}{dt} = 12 \times 10^{-9} \times -4e^{-t} = -48 \times 10^{-9} e^{-t}$

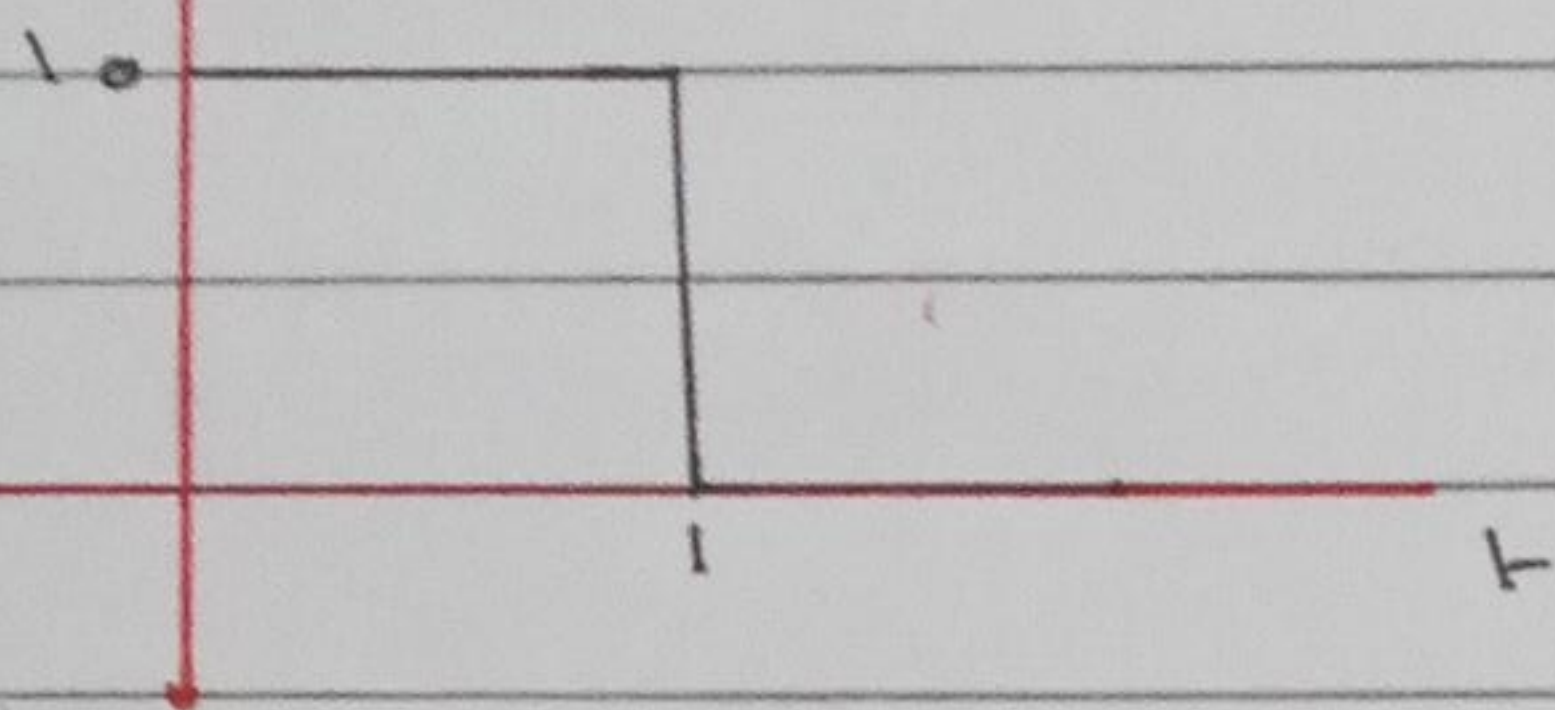
- Ex.

$V_L(t)$ (mV)

$L = 10 \text{ mH}$

$i_L(0) = 0$

Find $i_L(t)$



$$i_L(t) = \left[\frac{1}{L} \int_{t_0}^t V_L(\tau) d\tau \right] + i_L(t_0)$$

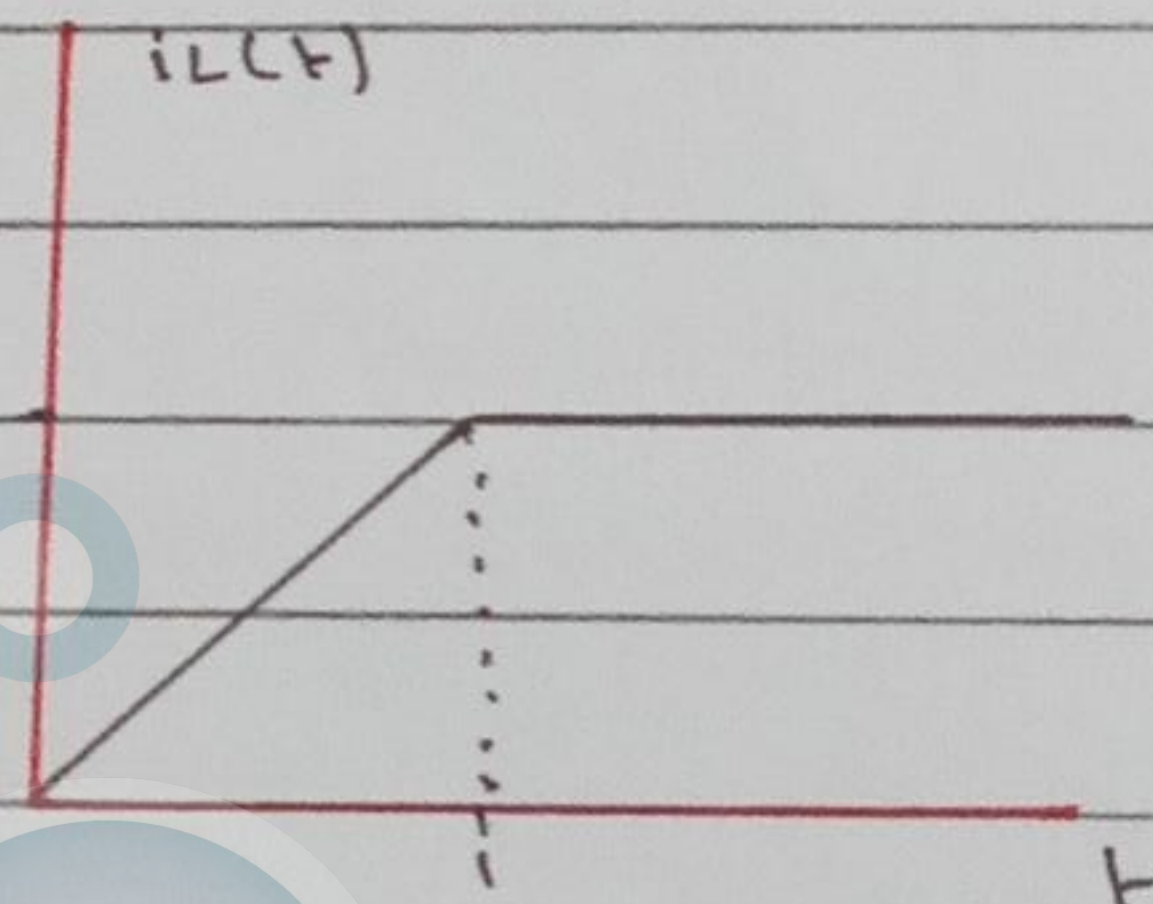
• For $t < 0 \rightarrow V_L(t) = 0 \rightarrow i_L(t) = 0$

• For $0 \leq t \leq 1 \rightarrow V_L(t) = 10 \text{ mV}$

$$i_L(t) = \left[\frac{1}{10 \times 10^{-3}} \int_0^t 10 \text{ mV} d\tau \right] + 0 = \frac{1}{10 \times 10^{-3}} \times 10 \times 10^{-3} t = \boxed{t = i_L(t)}$$

• For $t > 1 \rightarrow V_L(t) = 0 \rightarrow i_L(1) = 1$

$$i_L(t) = \frac{1}{10 \text{ m}} \int_1^t 0 d\tau + 1 = \boxed{1 \text{ A}}$$



- Ex.

$i_L(t)$

15

(2, 15)

$L = 50 \text{ mH}$

Find and plot $V_L(t)$

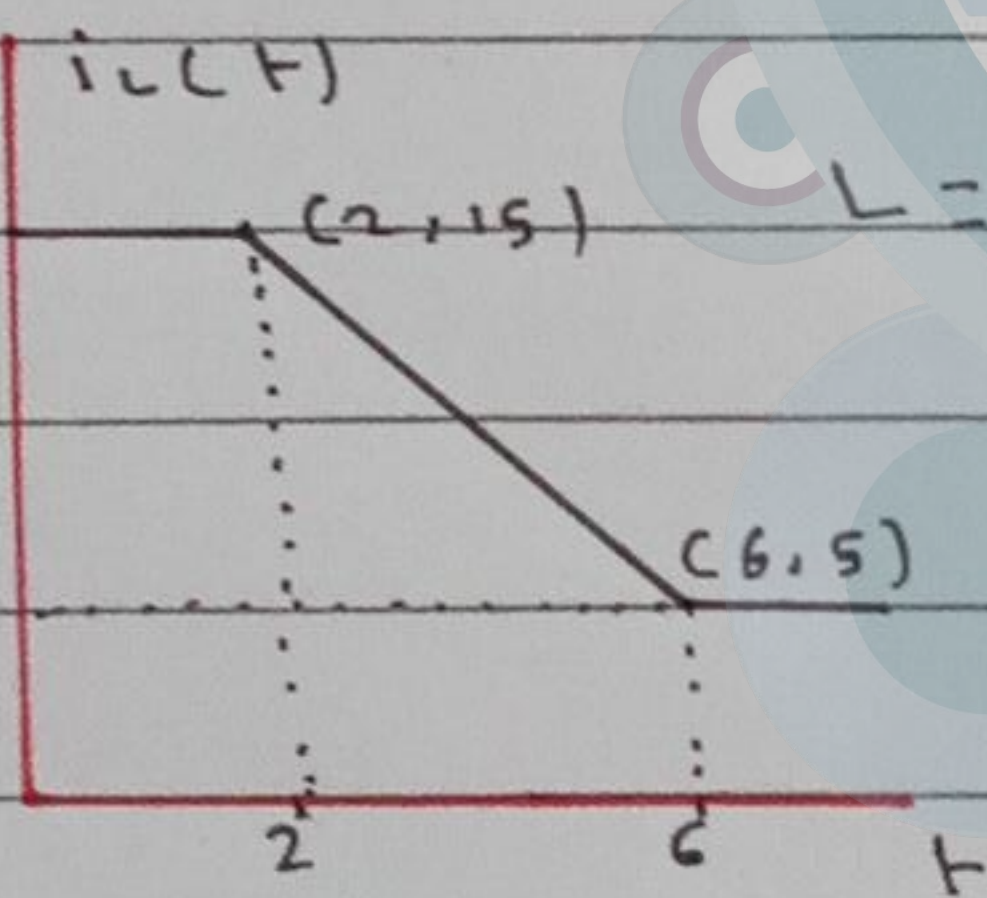
5

(6, 5)

slope = -2.5

$y - 5 = -2.5(x - 6)$

$y = -2.5x + 20$

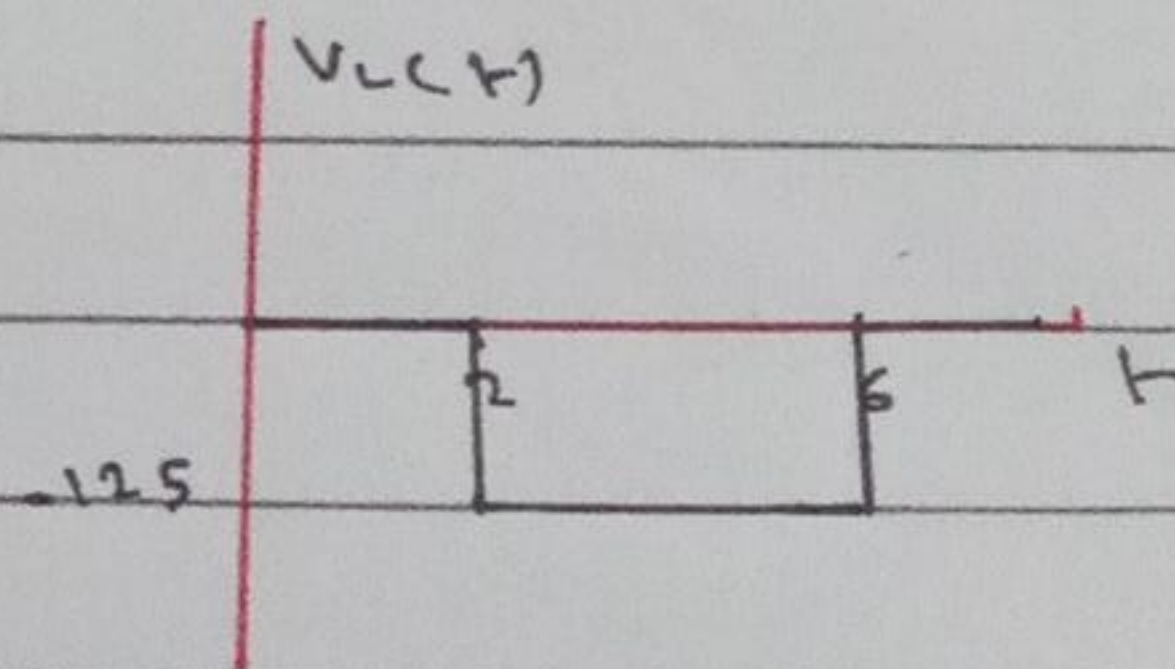


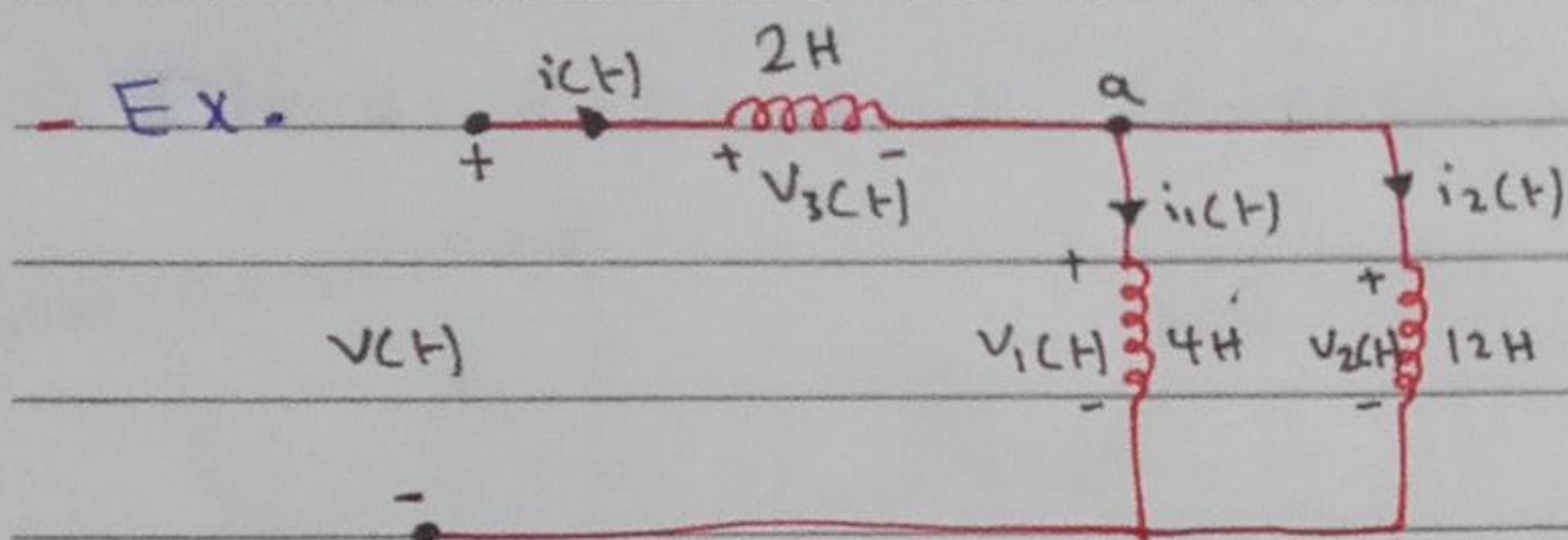
• For $0 \leq t \leq 2$, $i_L(t) = 15 \text{ A}$, $V_L(t) = 0$

• For $2 \leq t \leq 6$, $i_L(t) = -2.5t + 20$

$$V_L(t) = L \frac{di_L(t)}{dt} = 50 \text{ m} \times -2.5 = -125 \times 10^{-3} \text{ V}$$

• For $t > 6$, $i_L(t) = 5 \text{ A}$, $V_L(t) = 0$





$$i(t) = 4(2 - e^{-10t}) \text{ mA}, \quad i_2(0) = -1 \text{ mA}$$

Find:- $i_1(0)$, $V(t)$, $V_1(t)$

$V_2(t)$, $i_2(t)$, $i_1(t)$

$$i_1(t) = \left[\frac{1}{L} \int_{t_0}^t V_1(\tau) d\tau \right] + i_1(t_0)$$

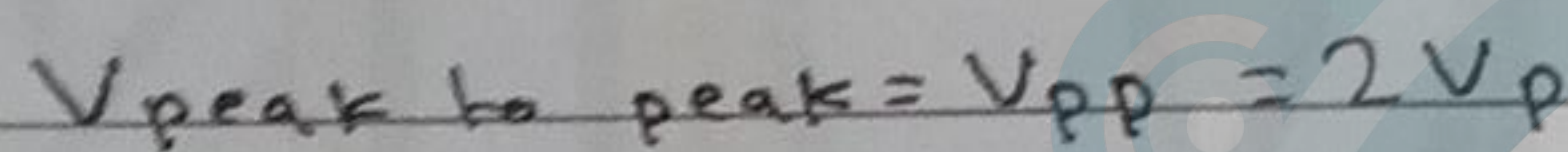
$i(0) = 4 \text{ mA} \rightarrow i_1(0) = 5 \text{ mA}$
by KCL at a

• To find $V_1(t)$:-

$$V(t) = L \frac{di(t)}{dt} = 5 \times 40 e^{-10t} = 200 e^{-10t}$$

$$V_3(t) = 80 e^{-10t} \rightarrow V_1(t) = V_2(t) = 120 e^{-10t}$$

Then find $i_1(t)$ and $i_2(t)$

$$y(t) = A \sin(\omega t + \theta)$$


تأخر lag سبق lead

① lead ② by a

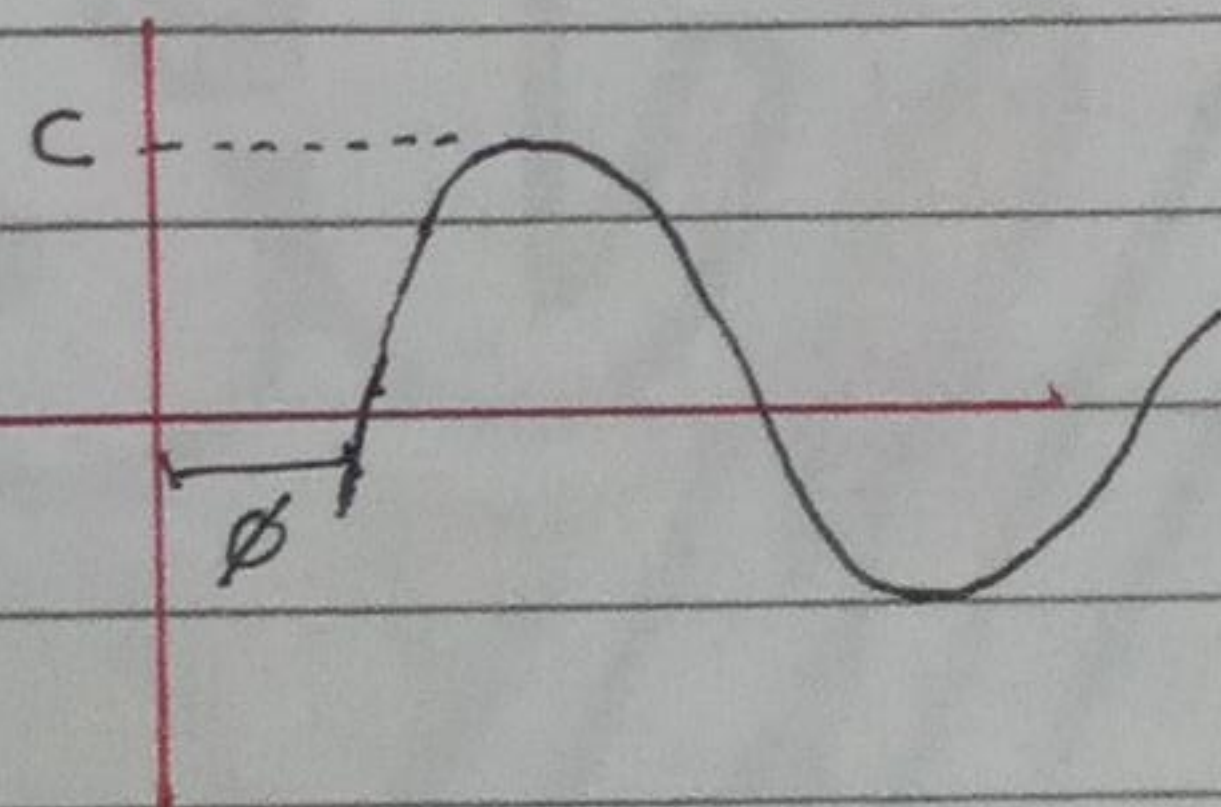
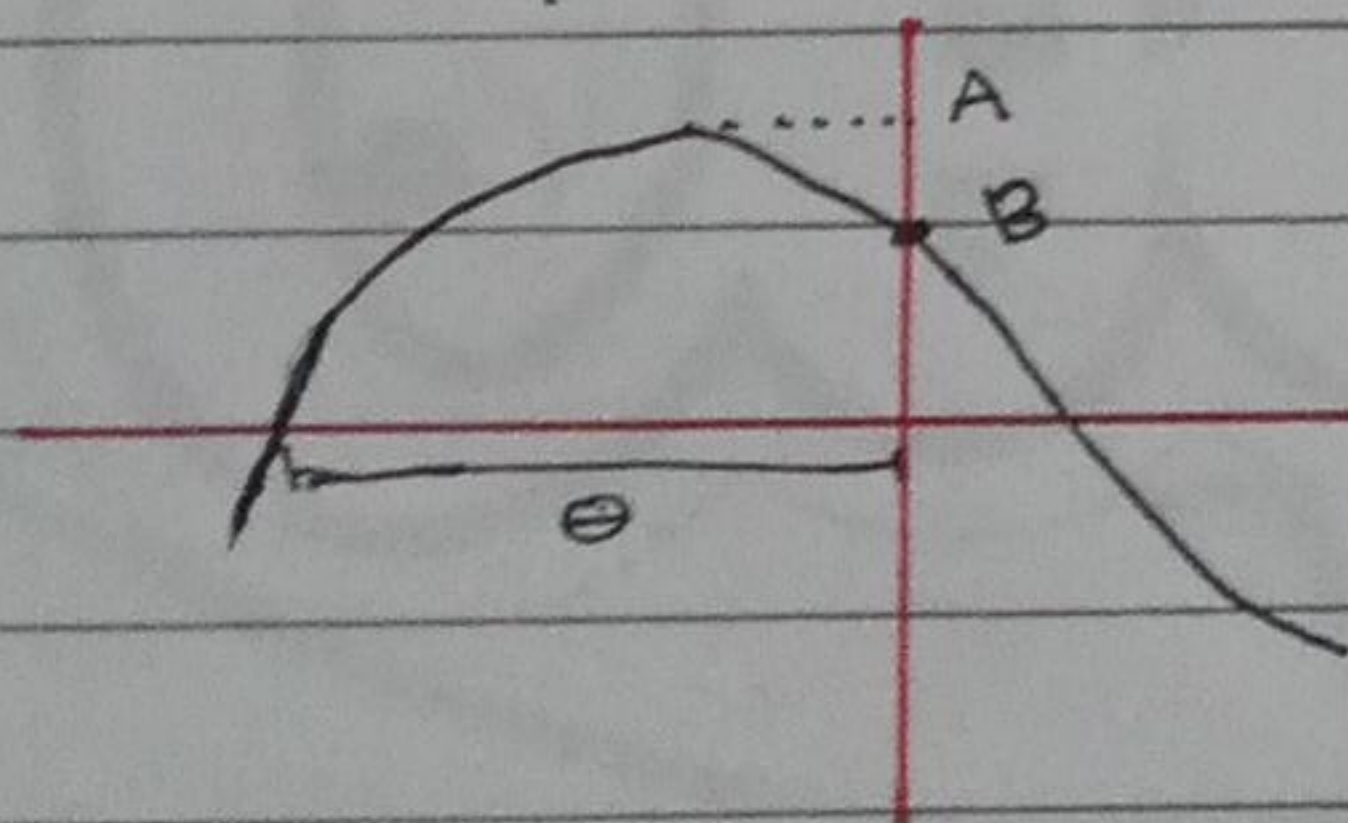
② lag ① by ③

V peak

$$\omega = 2\pi f \text{ rad/s}$$

phase shift

$$\bullet \theta = \frac{\Delta t * 360}{T}$$



$$y(t) = A \sin(\omega t + \theta)$$

$$y(0) = A \sin(0) = B$$

$$\theta = \sin^{-1} \left(\frac{B}{A} \right)$$

$$z(t) = C \sin(\omega t - \phi)$$

- Ex.

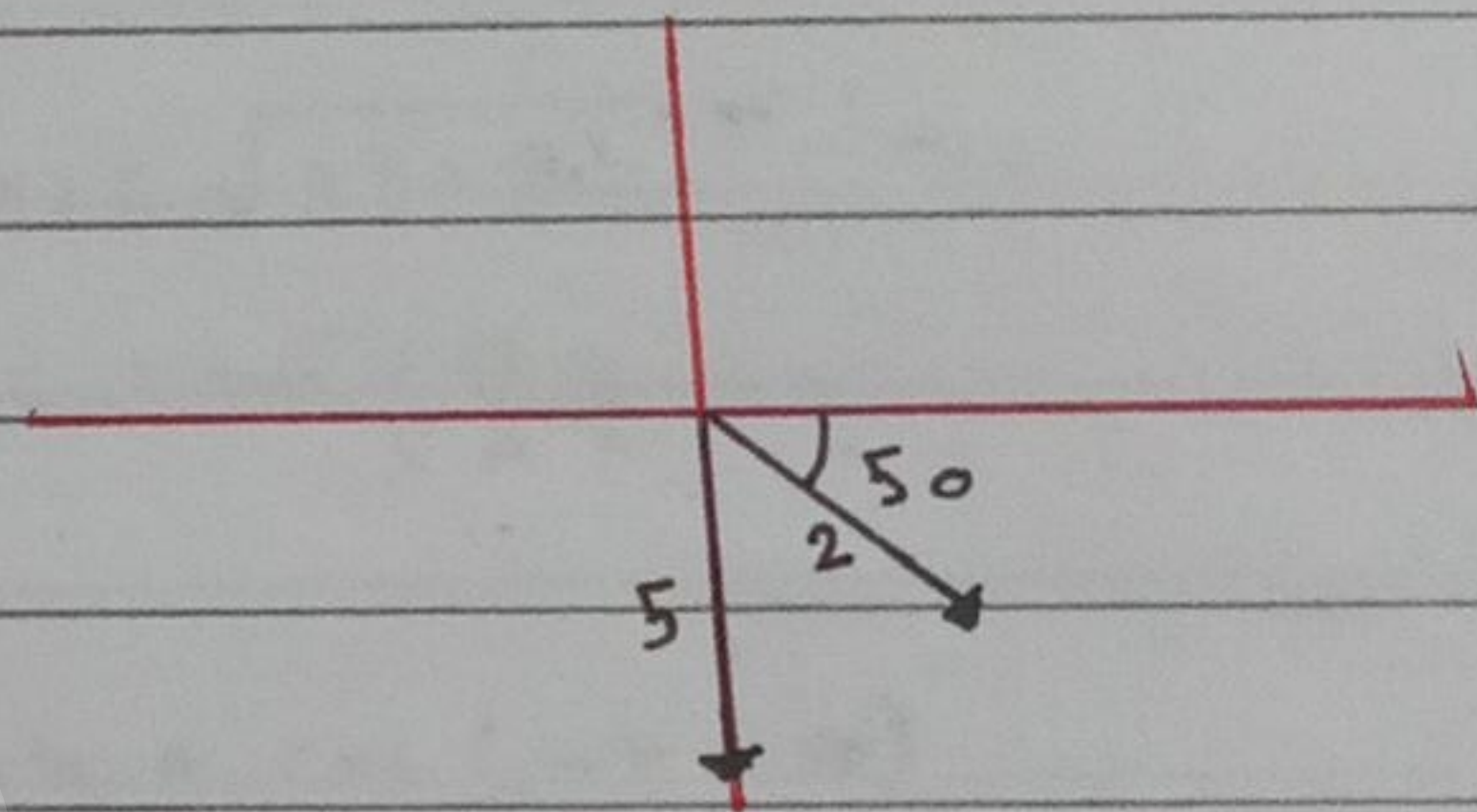
$$x(t) = 5 \sin(\omega t) + 2 \cos(\omega t - 50)$$

يجب ان تحول واحدة منهم

$$\cos(\omega t) = \sin(\omega t + 90)$$

$$\sin(\omega t) = \cos(\omega t - 90)$$

$$x(t) = 5 \cos(\omega t - 90) + 2 \cos(\omega t - 50)$$



- Ex.

$$i(t) = 10 \cos(377t + 30^\circ)$$

ω rad/s degree

$$I_{\max} = I_{\text{peak}} = 10$$

$$\omega = 377 \text{ rad/s} = 2\pi f \quad , \quad f = \frac{377}{2\pi} \text{ Hz}$$

$$\theta = 30^\circ$$

$$T = \frac{1}{f} = \frac{2\pi}{377} \text{ sec}$$

$$i(0.1) = 10 \cos(377(0.1) + 30^\circ) = 10 \cos(37.7 + \frac{\pi}{6})$$

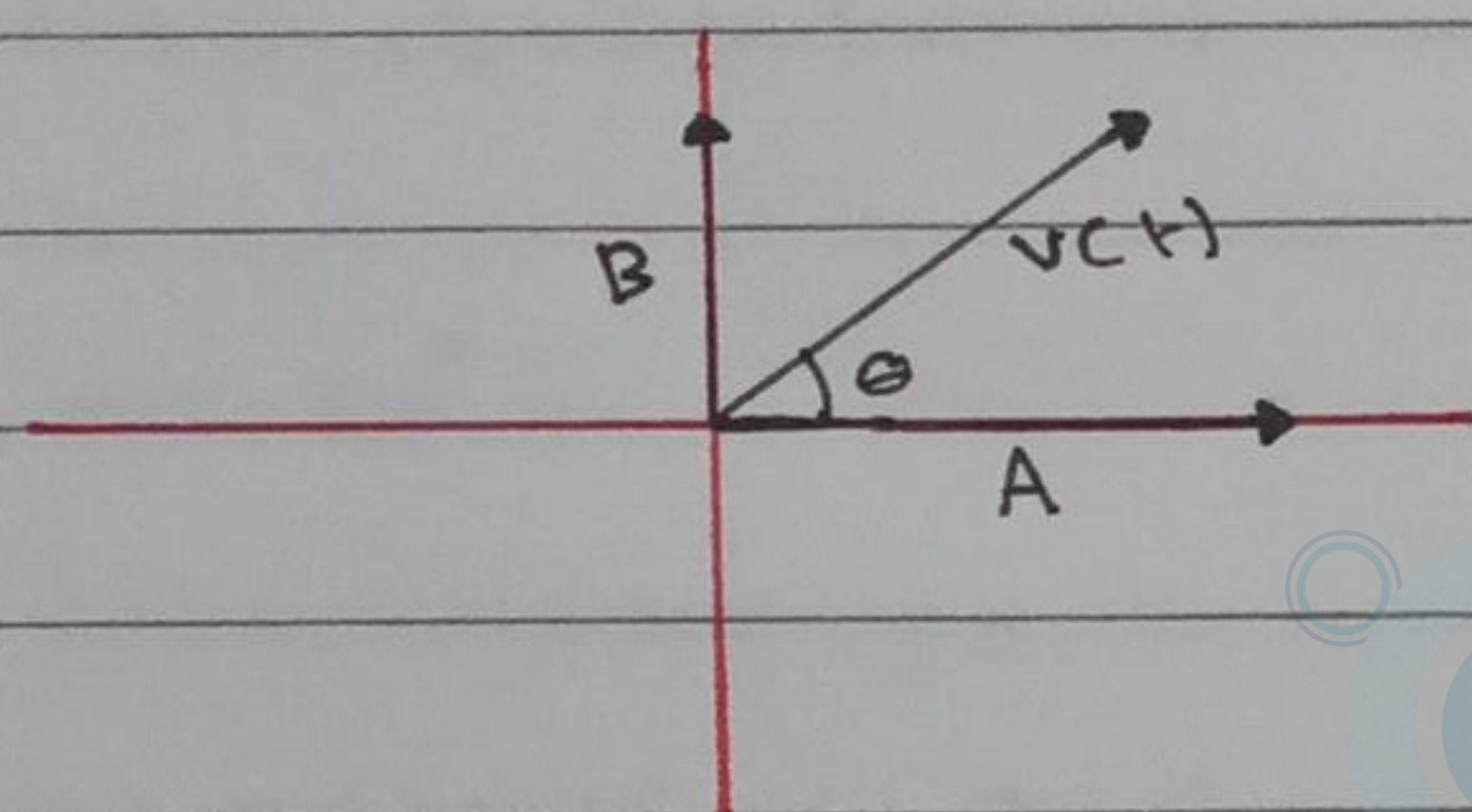
- Ex.

$$V_1 = A \cos(\omega t)$$

$$V_2 = B \sin(\omega t)$$

find $V(t) = V_1(t) + V_2(t)$

$$V_2(t) = B \sin(\omega t) = B \cos(\omega t - 90^\circ)$$



$$\text{length } V(t) = \sqrt{A^2 + B^2}$$

$$\text{angle } \theta = \tan^{-1}\left(\frac{B}{A}\right)$$

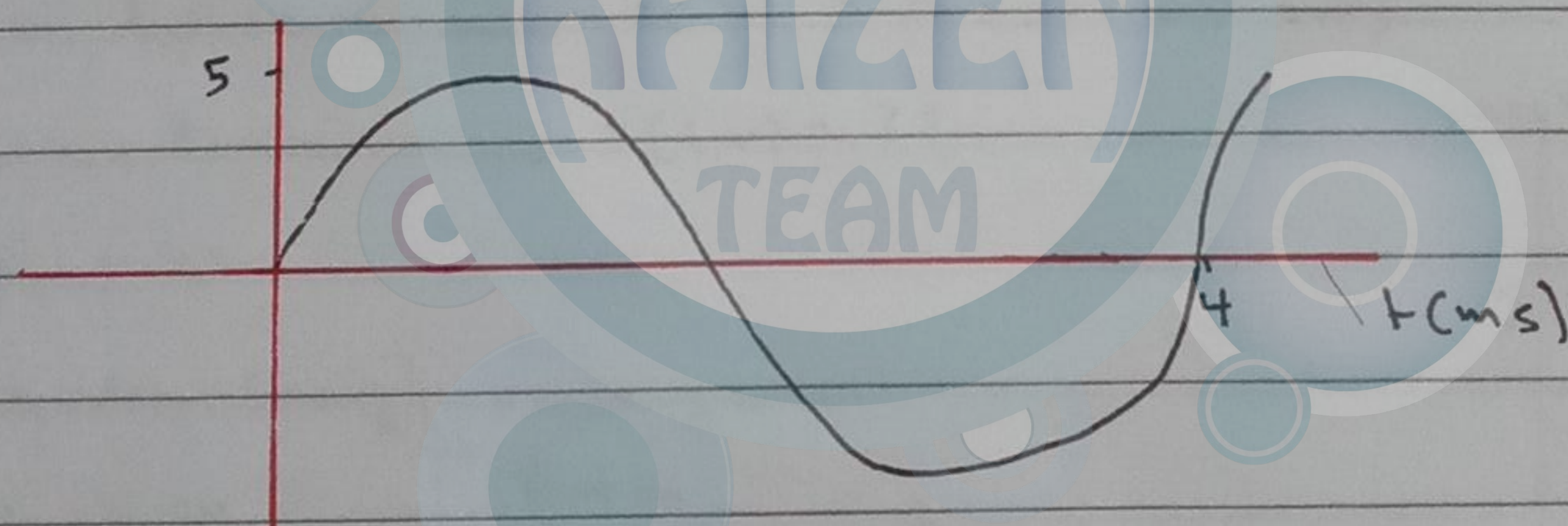
$$V(t) = \text{length} * \cos(\omega t + \theta)$$

- Note:

$$\sin(\omega t \pm 180^\circ) = -\sin(\omega t)$$

$$\cos(\omega t \pm 180^\circ) = -\cos(\omega t)$$

- Ex.



$$A = 5$$

$$\omega = 2\pi f \rightarrow f = \frac{1}{T} = \frac{1}{4\text{ms}} = 250 \text{ Hz}$$

$$\omega = 500\pi \text{ rad/s}$$

$$x(t) = 5 \cos(500\pi t + \theta)$$

$$x(t) = 5 \cos(500\pi t - 90^\circ)$$

because it is cos

that is shifted to the right by

90 degree.

To find θ

$$x(0) = 0$$

$$0 = 5 \cos(\theta)$$

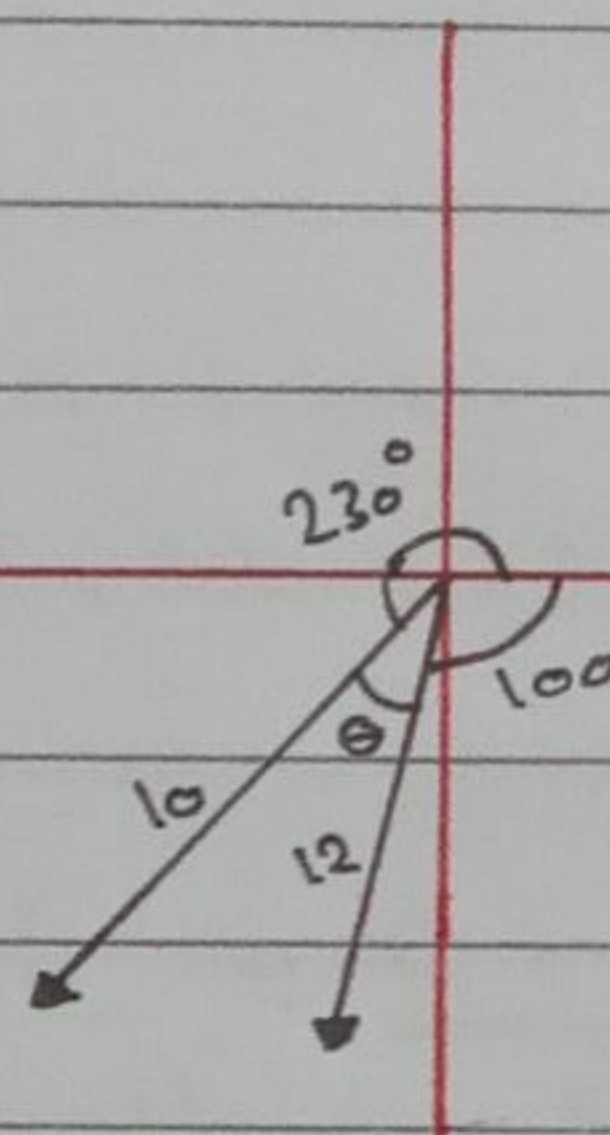
$$\theta = \frac{\pi}{2}$$

- Ex. $V_1(t) = -10 \cos(\omega t + 50)$ $V_2(t) = 12 \sin(\omega t - 10)$

① Find the angle between V_1, V_2 :-

$V_1(t) = -10 \cos(\omega t + 50) \rightarrow 10 \cos(\omega t + 230)$

$V_2(t) = 12 \sin(\omega t - 10) \rightarrow 12 \cos(\omega t - 100)$

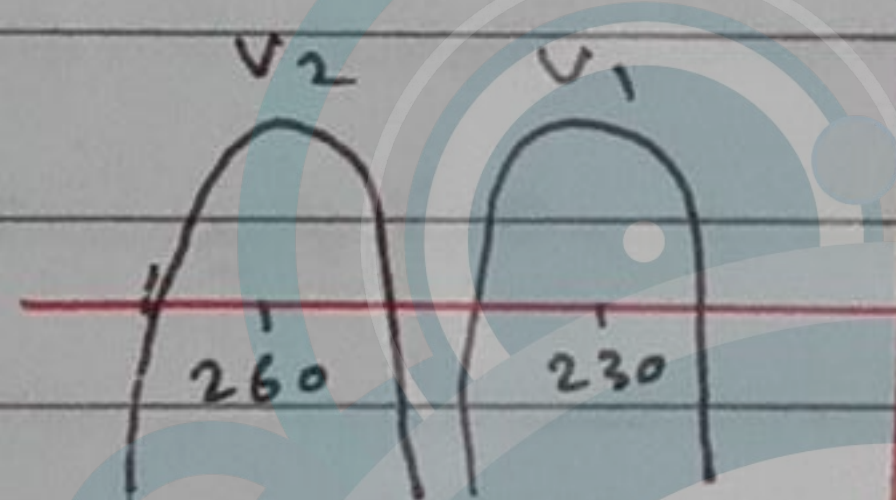


$\therefore \theta = 30^\circ$

② determine which lead or lag

$V_1(t) = 10 \cos(\omega t + 230)$

$V_2(t) = 12 \cos(\omega t + 260)$



V_2 lead V_1

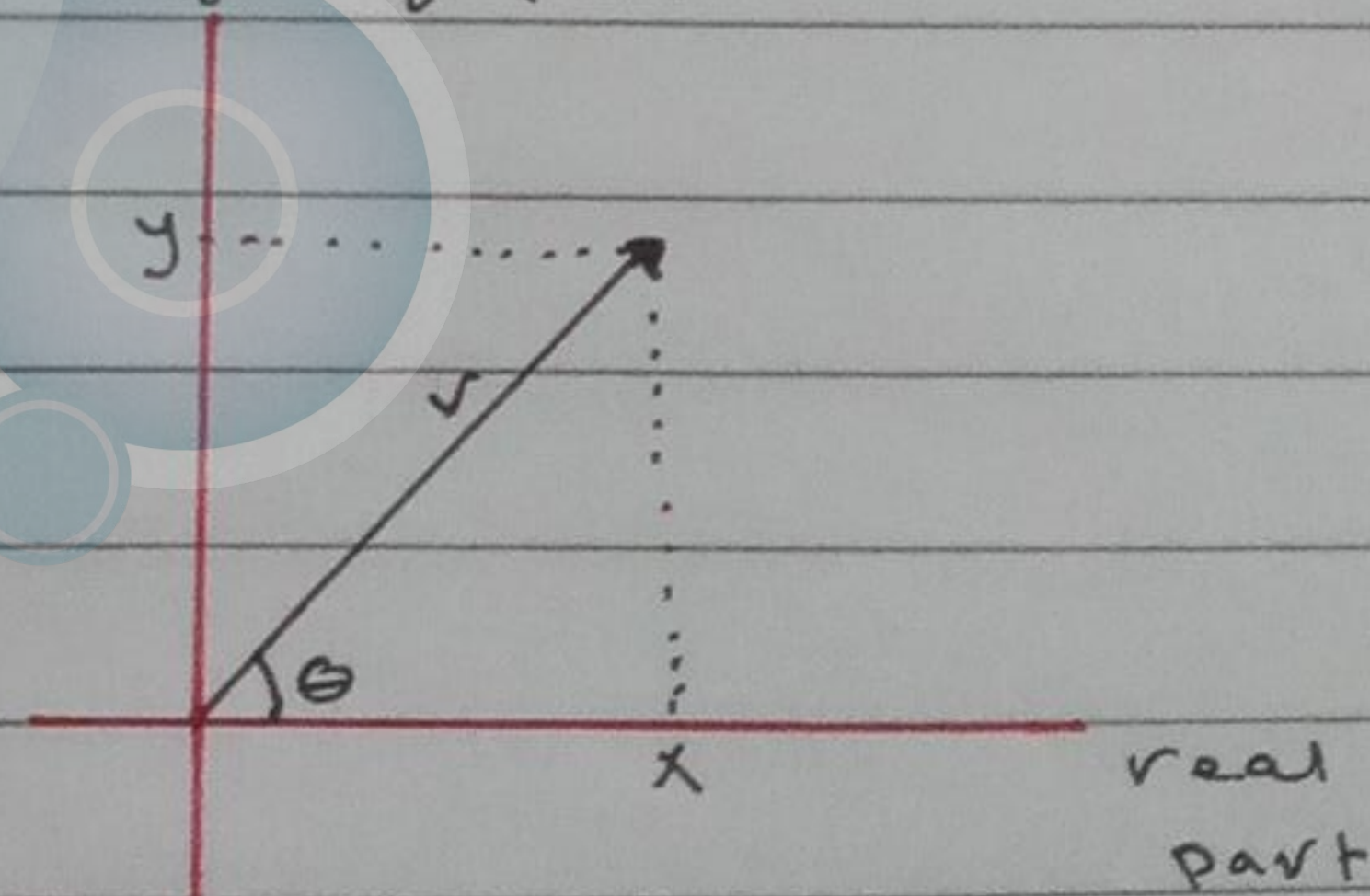
- time domain $x(t) = A \cos(\omega t + \theta)$

- phasor domain $x(i\omega) = ??$

- Complex :

$Z = \text{real} + j\text{imag}$
 $= x + jy$ } rectangular form

imaginary part



$Z = r \angle \theta$
 $\downarrow$
length of vector Z } Polar form

• To convert from rect to polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(\frac{y}{x})$

• To convert from polar to rect

$x = r \cos \theta$

$y = r \sin \theta$

operations at complex:-

Mathematical: $x_1 = a + jb$ $x_2 = c + jd$

① Addition $\rightarrow x_1 + x_2 = (a+c) + j(b+d)$

② Subtraction $\rightarrow x_1 - x_2 = (a-c) + j(b-d)$

③ Multiplication $\rightarrow x_1 x_2 = (a+jb)(c+jd) = ac + jad + jcb + -bd$
 $= (ac - bd) + j(ad + cb)$

④ Division $\rightarrow \frac{x_1}{x_2} = \frac{a+jb}{c+jd} \times \frac{c-jd}{c-jd} = \frac{(a+jb)(c-jd)}{c^2 + jdc - jdc + d^2} = \frac{(a+jb)(c-jd)}{c^2 + d^2} = \frac{(ac - bd) + j(ad + bc)}{c^2 + d^2}$

in polar: $x_1 = a \angle \theta_1$ $x_2 = b \angle \theta_2$

① addition and subtraction

$x_1 + x_2 = a \angle \theta_1 + b \angle \theta_2$ (convert to rectangular)

② Multiplication

$x_1 x_2 = (a \angle \theta_1)(b \angle \theta_2) = (ab) \angle \theta_1 + \theta_2$

③ Division

$\frac{x_1}{x_2} = \frac{a \angle \theta_1}{b \angle \theta_2} = \frac{a}{b} \angle \theta_1 - \theta_2$

- Note:- $j = \sqrt{-1}$ $\frac{1}{j} = -j$ $j = e^{j90}$

$e^{j\theta} = \cos \theta + j \sin \theta$

- Ex. $i_1(t) = 5 \cos(\omega t + 10^\circ)$, $i_2(t) = -2 \sin(\omega t - 10^\circ)$

$$i(t) = i_1(t) + i_2(t)$$

• $i_1(t) = 5 \cos(\omega t + 10^\circ)$

• $i_2(t) = -2 \sin(\omega t - 10^\circ)$
 $= 2 \cos(\omega t - 10^\circ + 180^\circ - 90^\circ)$
 $= 2 \cos(\omega t + 80^\circ)$

→ i_1 in phasor $= 5 \angle 10^\circ$

i_2 in phasor $= 2 \angle 80^\circ$

$$i(t) = 5 \angle 10^\circ + 2 \angle 80^\circ$$

convert to rectangular to combine then convert to time domain.

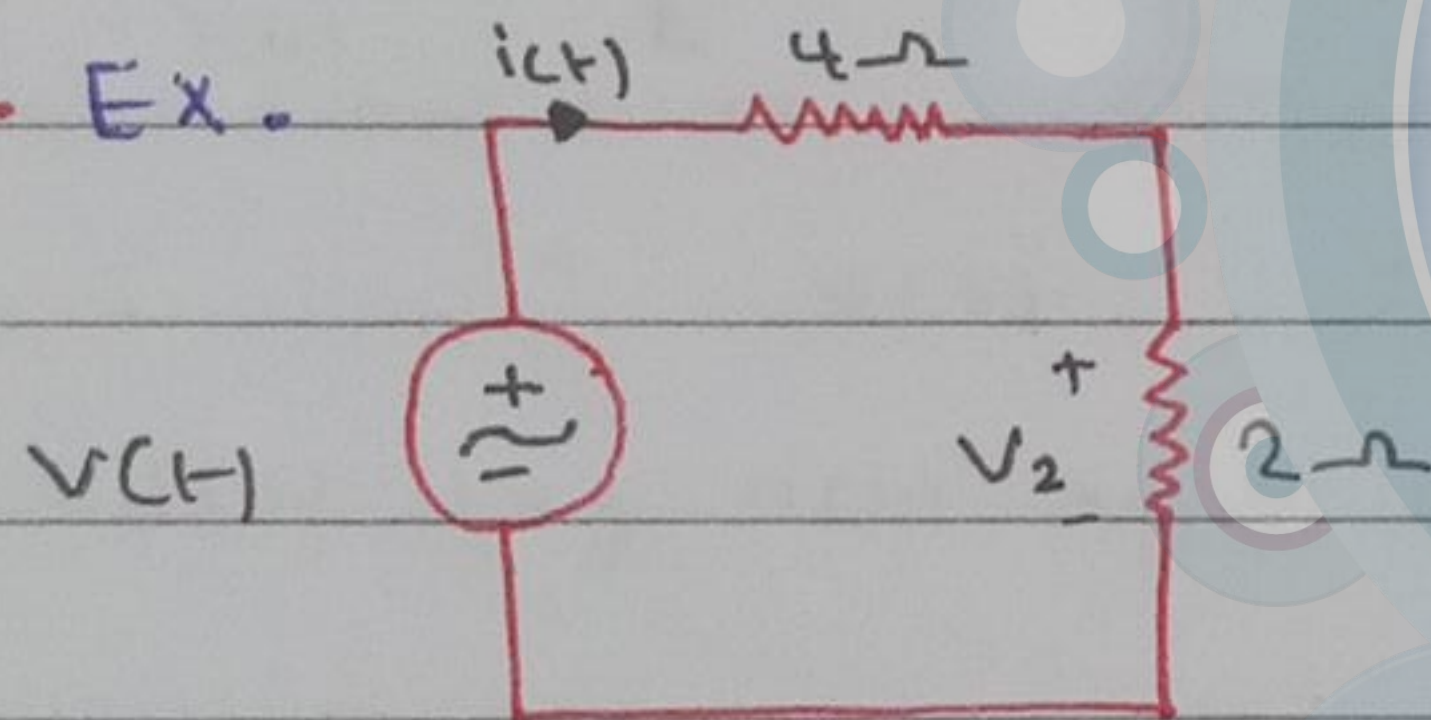
* important :-

to convert from time domain to phasor domain:-

① convert to cosine

② $A \cos(\omega t + \theta) \rightarrow A \angle \theta$

- Ex.



Find $V_2(t)$, $i(t)$

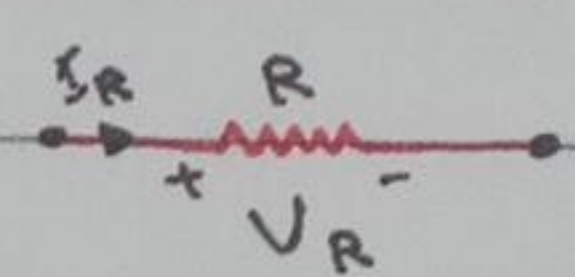
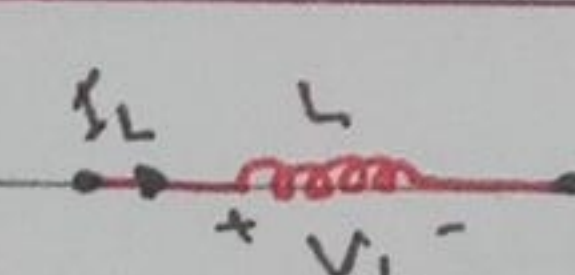
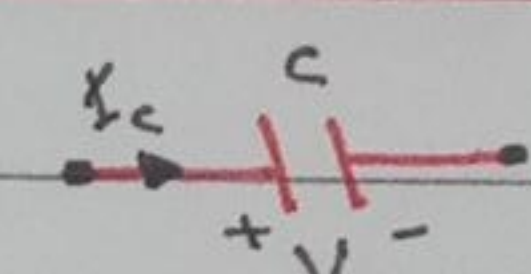
given: $V(t) = 12 \cos(377t + 40^\circ)$

• convert to phasor: $V = 12 \angle 40^\circ$

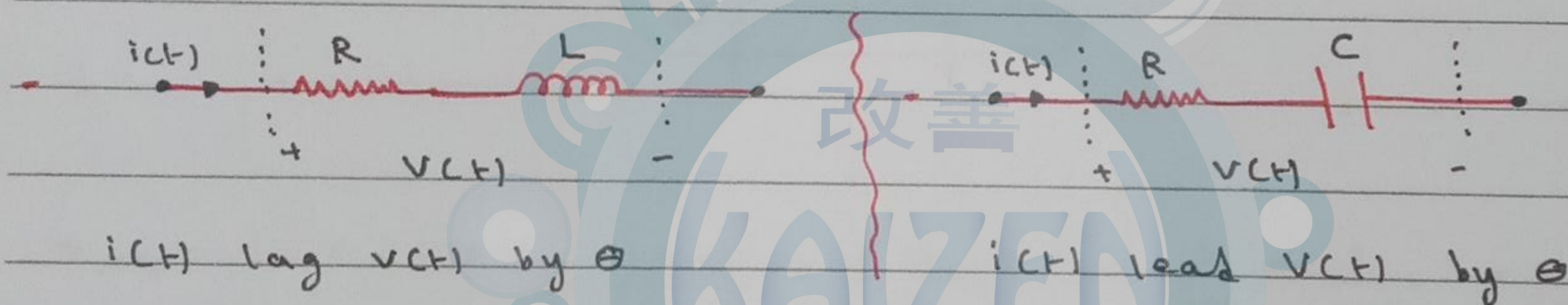
$$V_2 = \frac{12 \angle 40^\circ * 2}{6} = 4 \angle 40^\circ \rightarrow 4 \cos(377t + 40^\circ) = V_2(t)$$

$$I = \frac{V_2}{2} = \frac{4 \angle 40^\circ}{2} = 2 \angle 40^\circ \rightarrow 2 \cos(377t + 40^\circ) = i(t)$$

* phasor representation for circuit elements :-

	current	voltage	impedance
Resistor	$i_m < \theta$	$i_m R < \theta$	$Z_R = R$  i_R in phase V_R
inductor	$i_m < \theta$	$\omega L i_m < \theta + 90$	$Z_L = j\omega L$  i_L lag V_L by 90°
capacitor	$V_m \omega_c < \theta + 90$	$V_m < \theta$	$Z_C = \frac{-j}{\omega_c} = \frac{1}{j\omega_c}$  i_C lead V_C by 90°

- in phase mean neither lead nor lag.



- in DC — conductance $\equiv G = \frac{1}{R}$ (S)

AC — admittance $\equiv Y = \frac{1}{Z}$ (S)

- $j = < 90$, $-j = \frac{1}{j} = < -90$

- $Z = |Z| < \theta_2 = R + jX$

resistor reactance

$X_L = \omega L$ (inductor reactance)

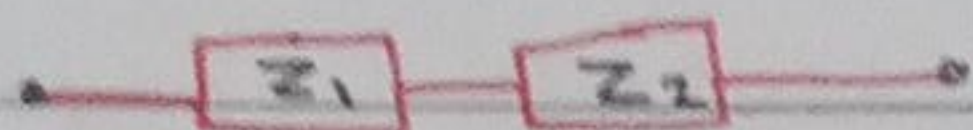
$X_C = \frac{1}{\omega_c}$ (capacitor reactance)

$R = Z \cos \theta_2$, $X = Z \sin \theta_2$

$Z = \sqrt{X^2 + R^2}$, $\theta_2 = \tan^{-1} \left(\frac{X}{R} \right)$

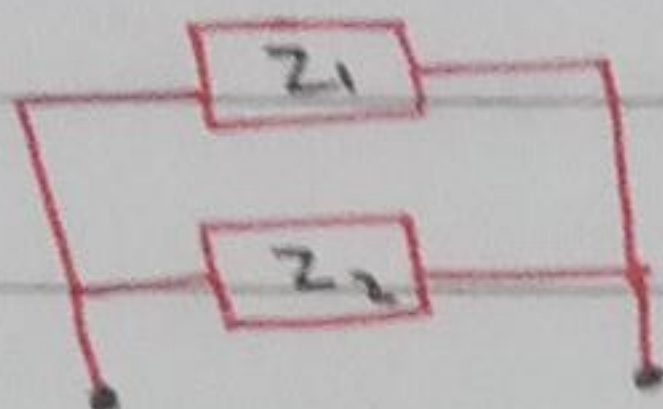
- $Z = \frac{V}{i} = \frac{V_m < \theta}{i_m < \phi} = Z < \theta_2$

⊕ Series



$$Z_{eq} = Z_1 + Z_2$$

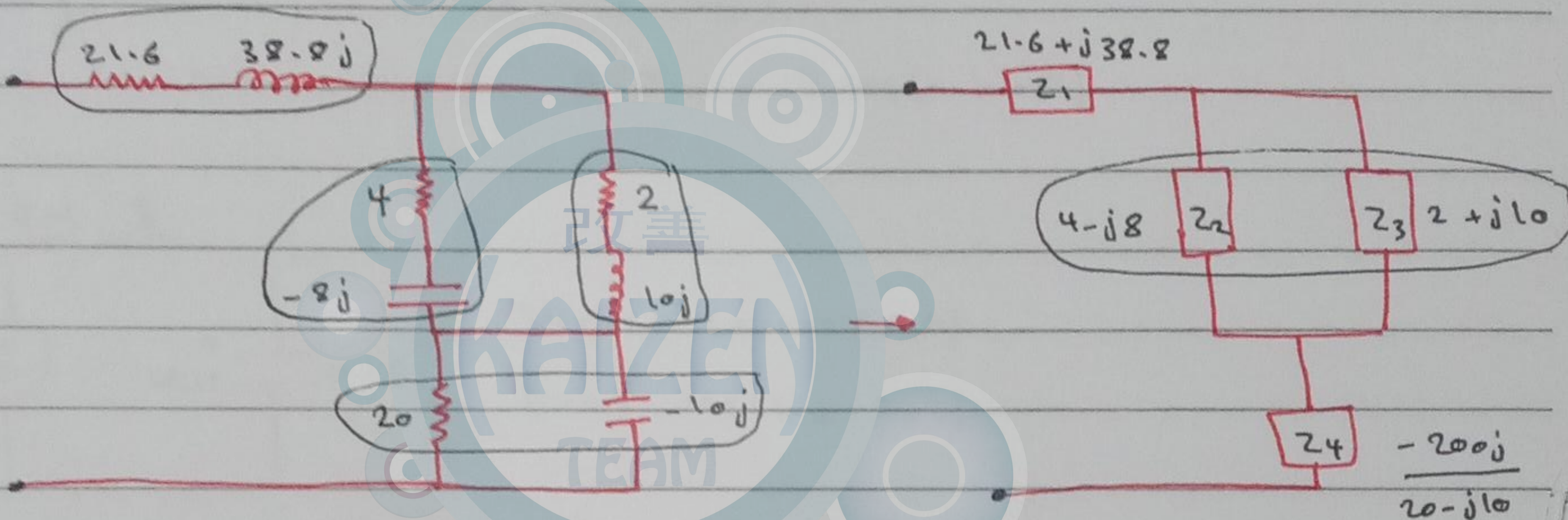
⊕ Parallel



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots}$$

⊕ Δ-y connection: same as (R) but with (Z)

Ex.



$$Z_{eq} = 21.6 + j38.8 - \frac{200j}{20 - j10} + (Z_{23})$$

$$\frac{1}{Z_{23}} = \frac{1}{2 + j10} + \frac{1}{4 - j8}$$

$$Z_{23} = 14.4 - 0.8j$$

Then solve in the calculator

$$Z_{eq} = 40 + 30j \rightarrow 50 \angle 36.87^\circ$$

⊕ AC-Analysis steps:-

① Find the value of ω (from the supply)

② convert all supplies to phasors

- convert to cosine $A \cos(\omega t + \theta)$
- convert to phasor $A \angle \theta$

③ convert all elements to impedance

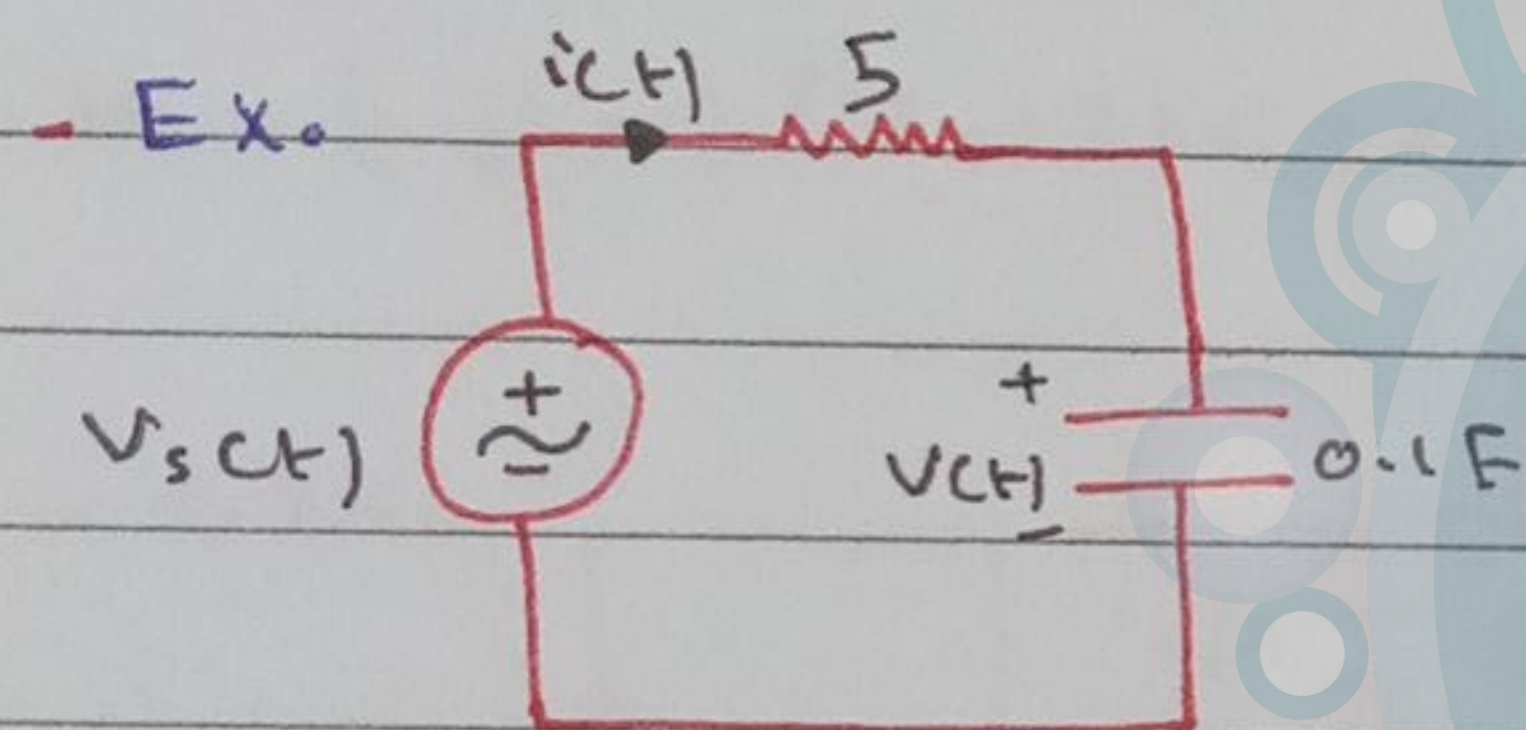
$$R \rightarrow R$$

$$L \rightarrow j\omega L$$

$$C \rightarrow \frac{-j}{\omega C}$$

④ use any method to solve the circuit

⑤ convert to time domain $B \angle \phi \rightarrow B \cos(\omega t + \phi)$



$$V_s(t) = 10 \cos(4t)$$

Find $v(t)$, $i(t)$

$$\omega = 4 \text{ rad/s}$$

$$V_s = 10 \angle 0^\circ \text{ V}$$

$$0.1 \text{ F} \rightarrow \frac{-j}{\omega C} = \frac{-j}{0.4} = -j 2.5 \Omega$$

KVL

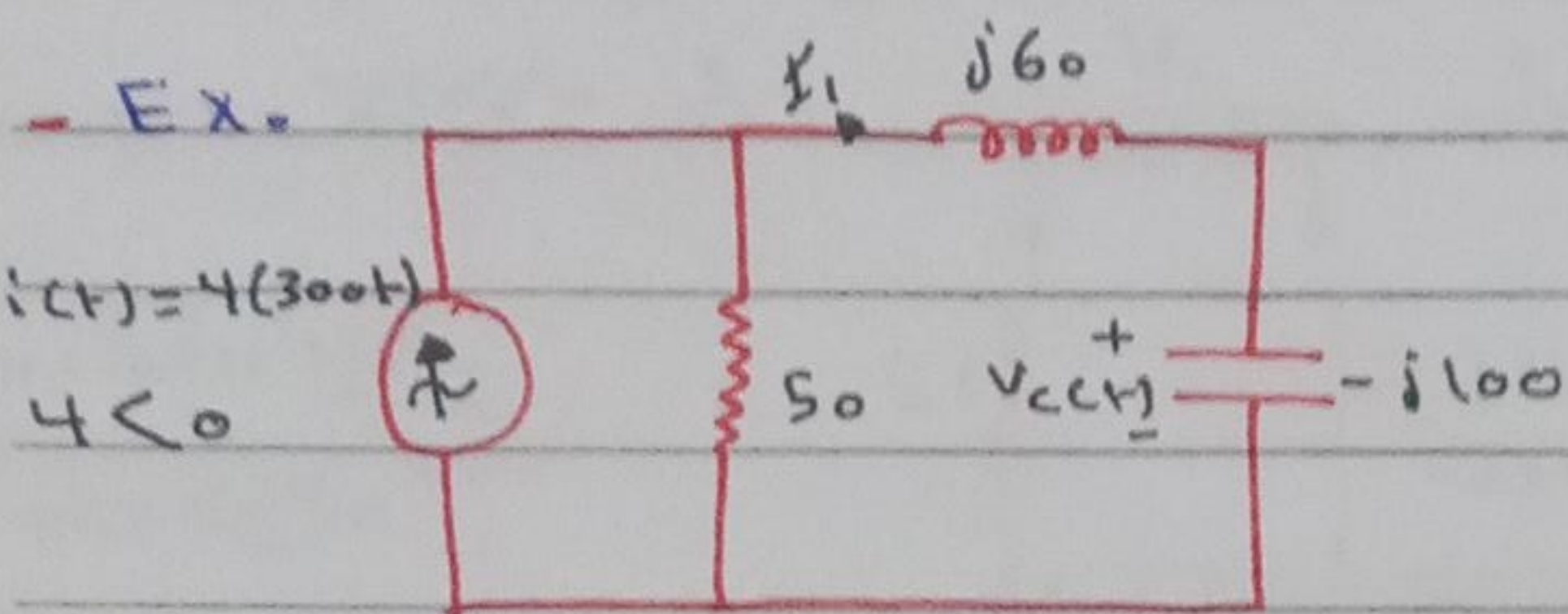
$$-10 \angle 0^\circ + 5I + I(-j 2.5) = 0$$

$$I = \frac{10 \angle 0^\circ}{5 - j 2.5} \quad (\text{on calculator})$$

$$I = 1.79 \angle 26.6^\circ \rightarrow 1.79 \cos(4t + 26.6^\circ)$$

$$V = I(-j 2.5) = 2 - j 4.47 \rightarrow 4.47 \angle -63.4^\circ$$

$$v(t) = 4.47 \cos(4t - 63.4^\circ)$$



• $\omega = 300$

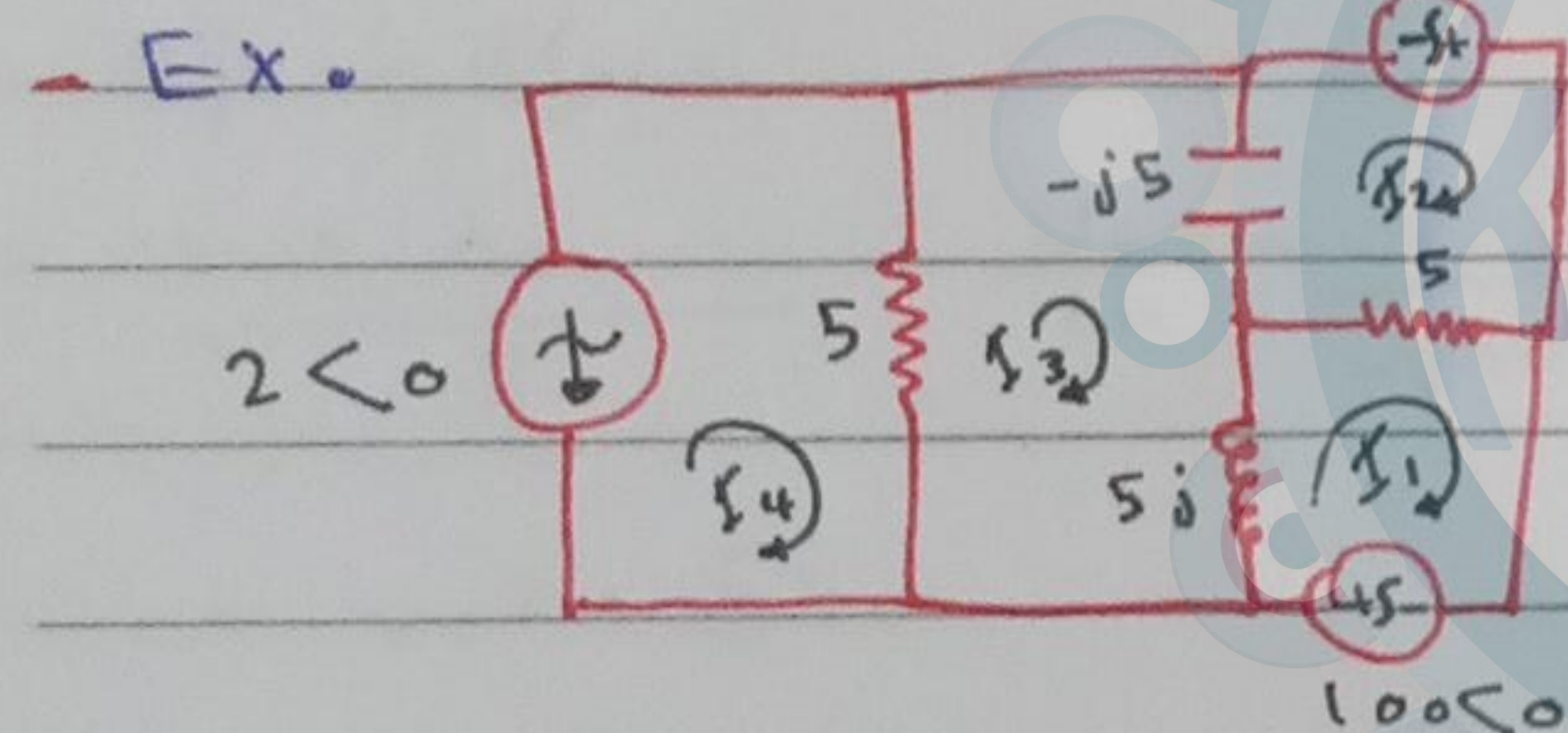
$i = 4 \angle 0^\circ$

Current division:

$$i_1 = 4 \angle 0^\circ \left(\frac{\frac{1}{-j40}}{\frac{1}{50} + \frac{1}{-j40}} \right) = 3.123 \angle -38.66^\circ$$

~~$v_c = (-j100) (3.123 \angle -38.66^\circ) = -312.35 \angle -51.34^\circ$~~

$v_c = (-j100) (3.123 \angle -38.66^\circ) = 312.35 \angle -51.34^\circ \rightarrow 312.35 \cos(300t - 51.34^\circ)$



write mesh equations:-

$i_4 = -2 \angle 0^\circ$

• Mesh ①

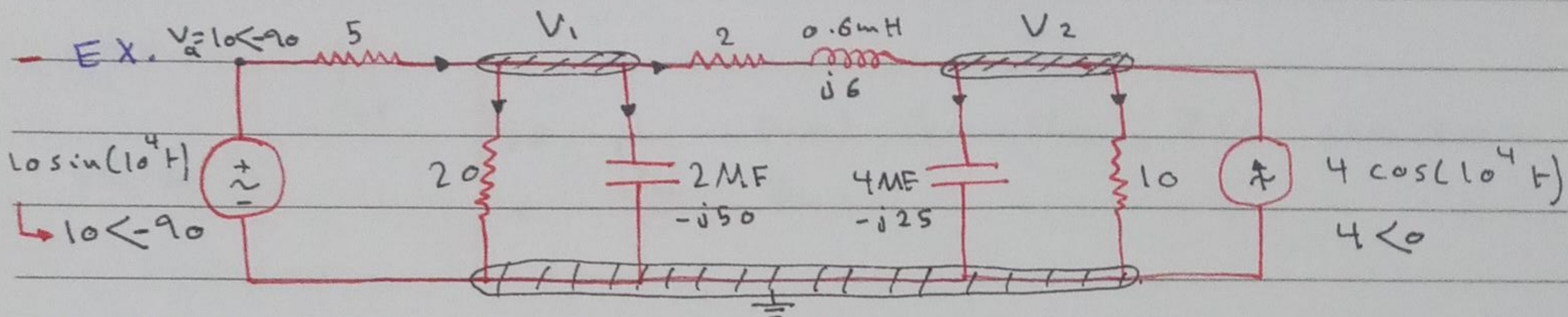
$$5j i_1 + 5 i_1 - 100 \angle 0^\circ - 5j i_3 - 5 i_2 = 0$$

• Mesh ②

$$-j5 i_2 - 50 \angle 0^\circ + 5 i_2 - 5 i_1 - (-5j) i_3 = 0$$

• Mesh ③

$$5 i_3 - 5j i_3 + 5j i_3 - 5 i_4 - 5j i_1 - (-5j) i_2 = 0$$



$\omega = 10^4$

$2 \text{ MF} \rightarrow \frac{-j}{10^4 \times 2 \times 10^{-6}} = -j50$ $4 \text{ MF} \rightarrow \frac{-j}{10^4 \times 4 \times 10^{-6}} = -j25$

$0.6 \text{ mH} \rightarrow j(10^4)(0.6 \times 10^{-3}) = j6$

at node ①

$\sum I_{in} = \sum I_{out}$

$$\frac{10 \angle -90 - V_1}{5} = \frac{V_1}{20} + \frac{V_1}{-j50} + \frac{V_1 - V_2}{2 + j6}$$

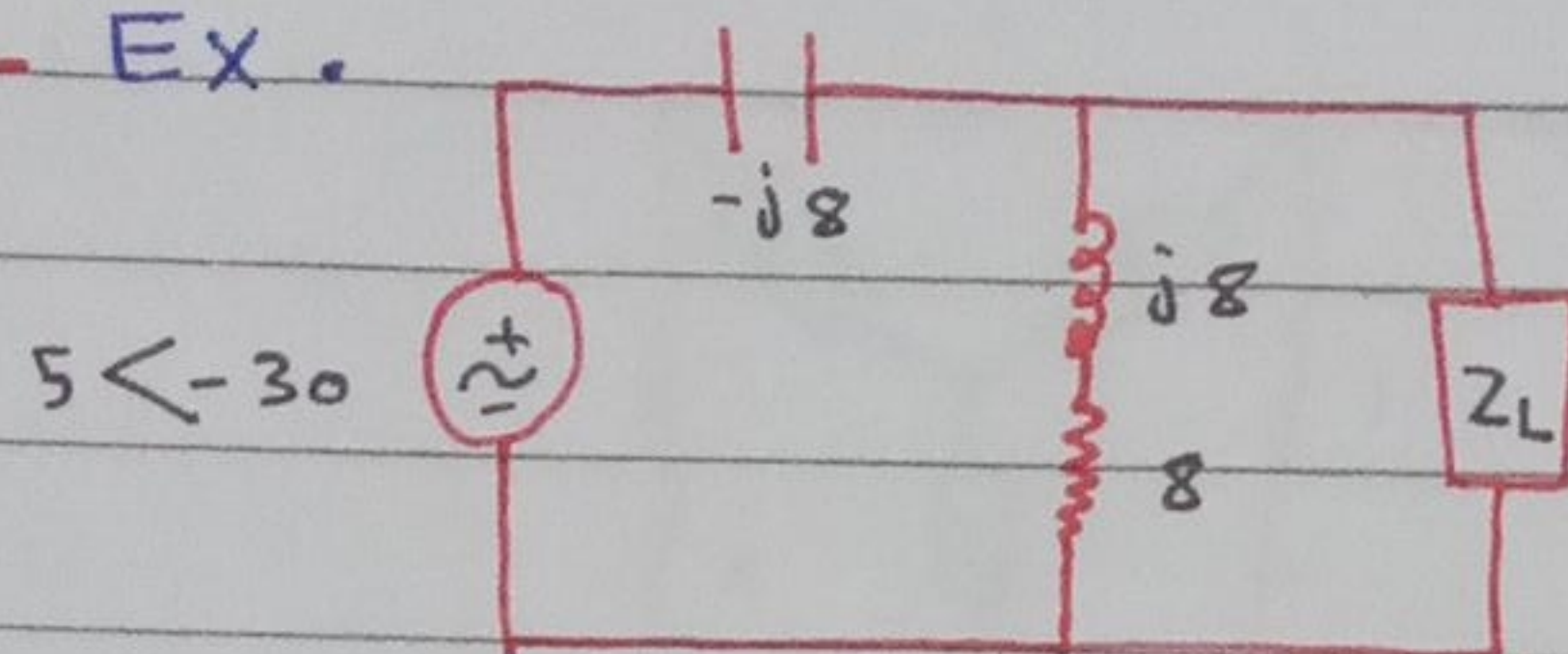
at node ②

$\sum I_{in} = \sum I_{out}$

$$\frac{V_1 - V_2}{2 + j6} + 4 \angle 0 = \frac{V_2}{10} + \frac{V_2}{-j25}$$

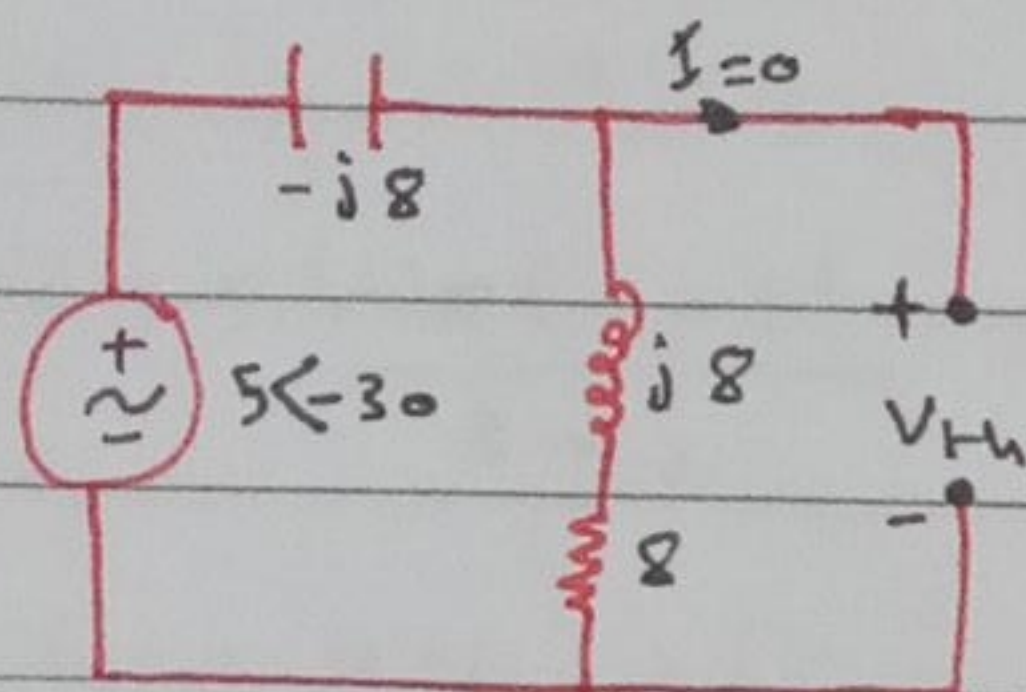
Continue ...

Ex.



Find I_N , V_{Th} , Z_{Th} seen from Z_L .

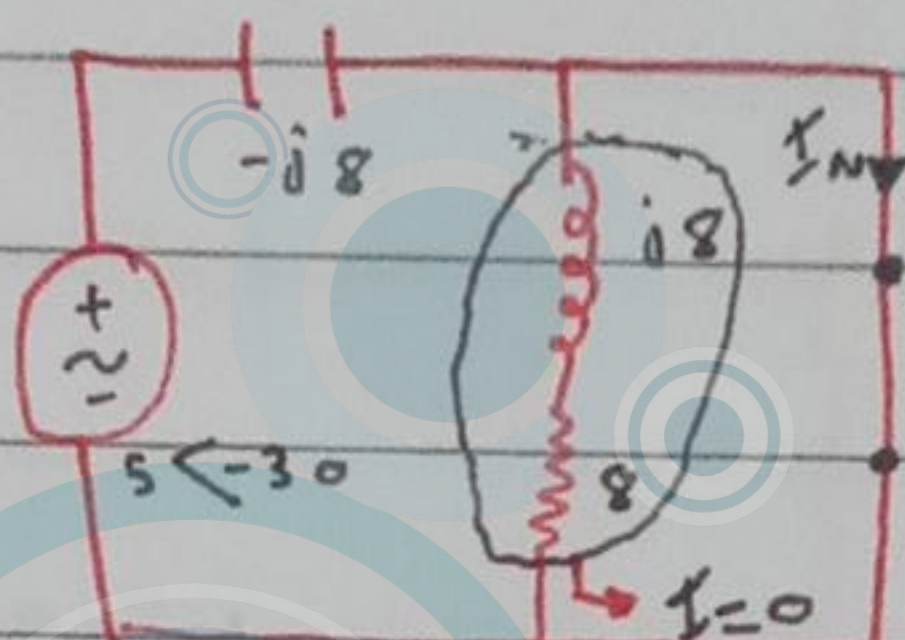
• find V_{Th}



$$V_{Th} = \frac{5 \angle -30 (8 + j8)}{-j8 + 8 + j8}$$

$$= 7.07 \angle 15 \text{ V}$$

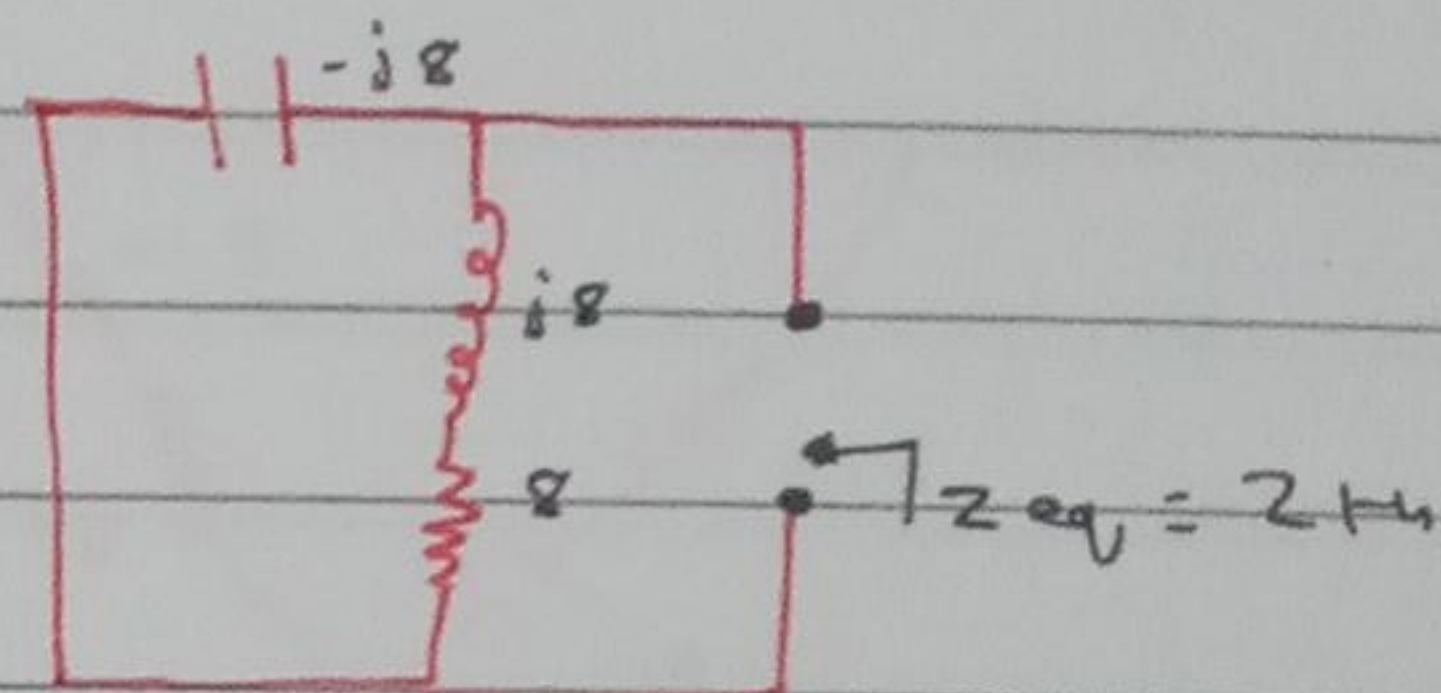
• find I_N



$$I_N = \frac{5 \angle -30}{-j8}$$

$$= 0.625 \angle 60$$

• find Z_{Th}



$$\frac{1}{Z_{Th}} = \frac{1}{8 + j8} + \frac{1}{-j8}$$

$$Z_{Th} = 11.31 \angle -45$$

- check :- $Z_{Th} = \frac{V_{Th}}{I_N} = \frac{7.06 \angle 15}{0.625 \angle 60} = 11.31 \angle -45$ ✓

- Note: In AC circuits, max power delivered to load when:

$$Z_L = Z_{Th}^* \text{ conjugate}$$

$$P_{max} = \frac{|V_{Th}|^2}{8 R_{Th}}$$

- The conjugate :-

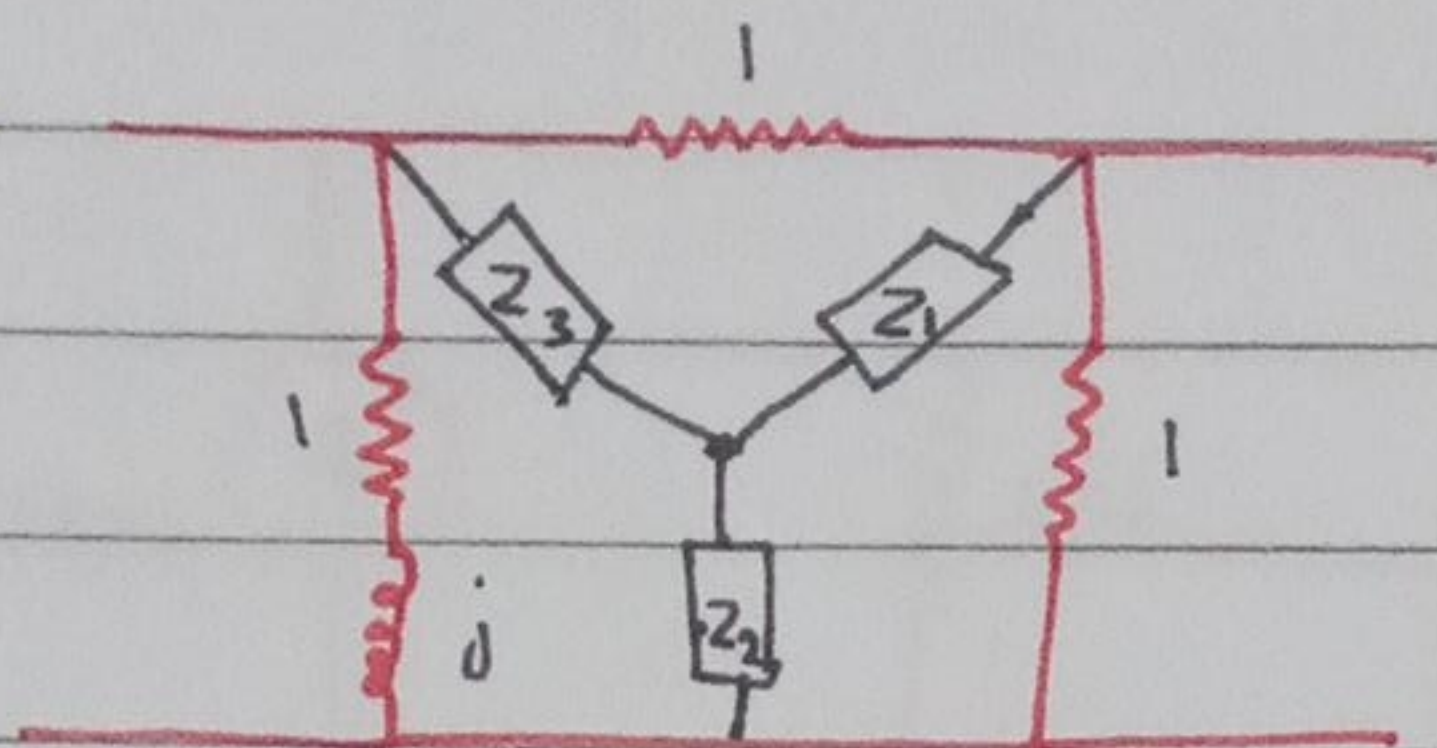
$$Z_1 = 5 + j2 \rightarrow Z_1^* = 5 - j2$$

$$Z_2 = 3 - j2 \rightarrow Z_2^* = 3 + j2$$

$$Z_3 = 5 \angle 40 \rightarrow Z_3^* = 5 \angle -40$$

$$Z_4 = 5 \angle -20 \rightarrow Z_4^* = 5 \angle 20$$

Ex.



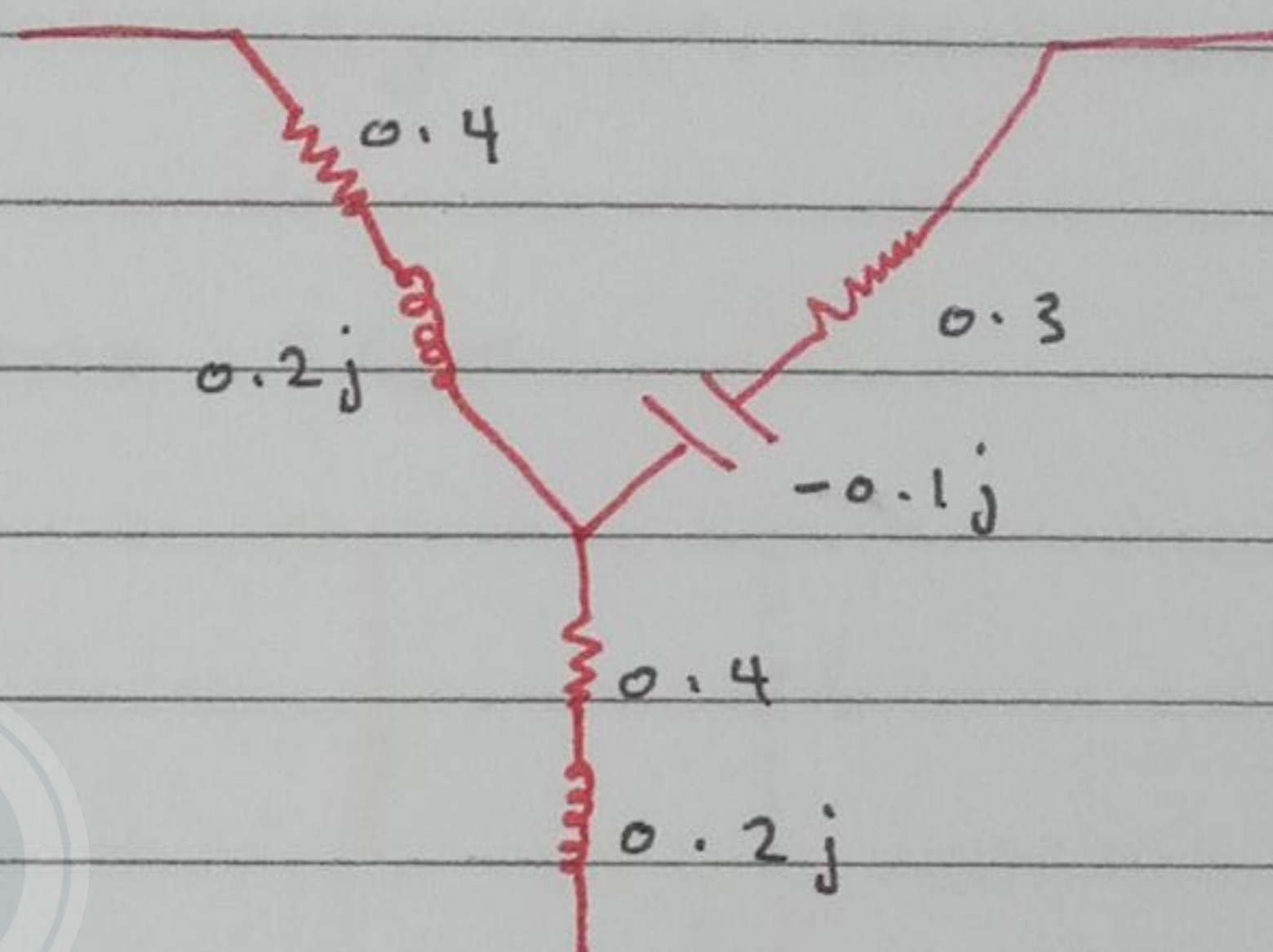
convert from Δ to Y

$$Z_1 = \frac{(1)(1)}{1+1+1+j} = \frac{1}{3+j} = 0.3 - 0.1j$$

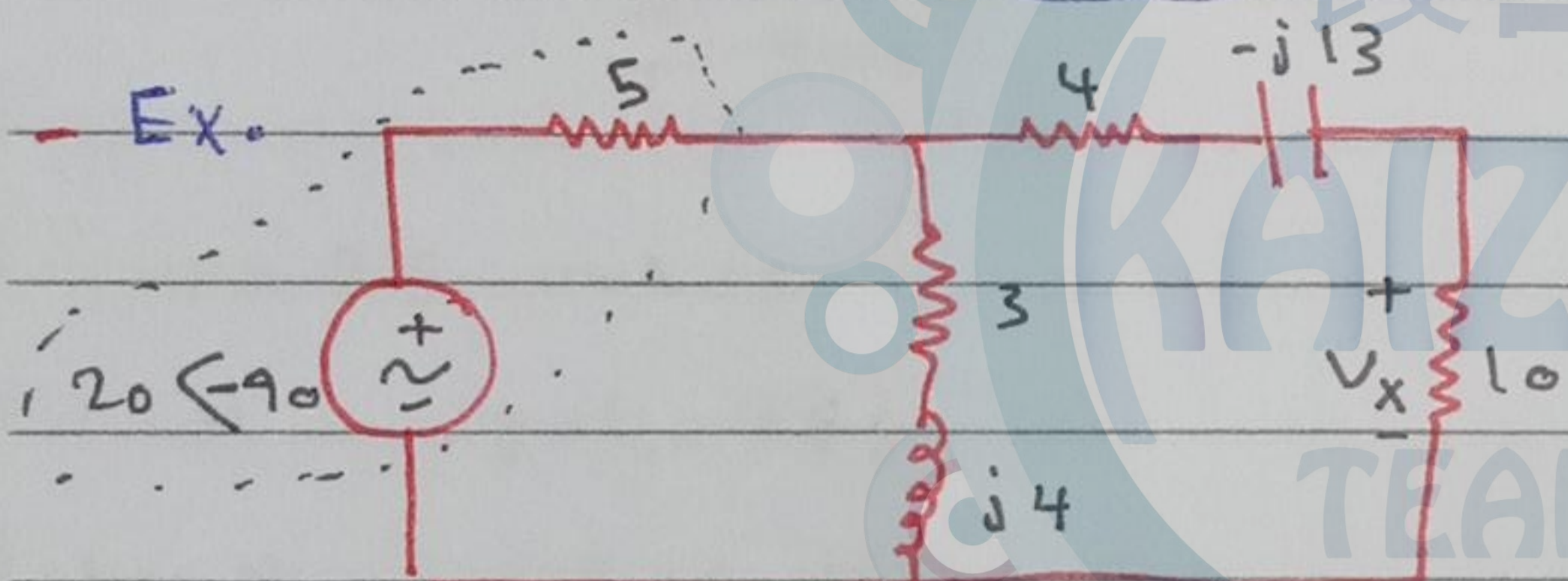
The Result

$$Z_2 = \frac{(1)(1+j)}{3+j} = 0.4 + 0.2j$$

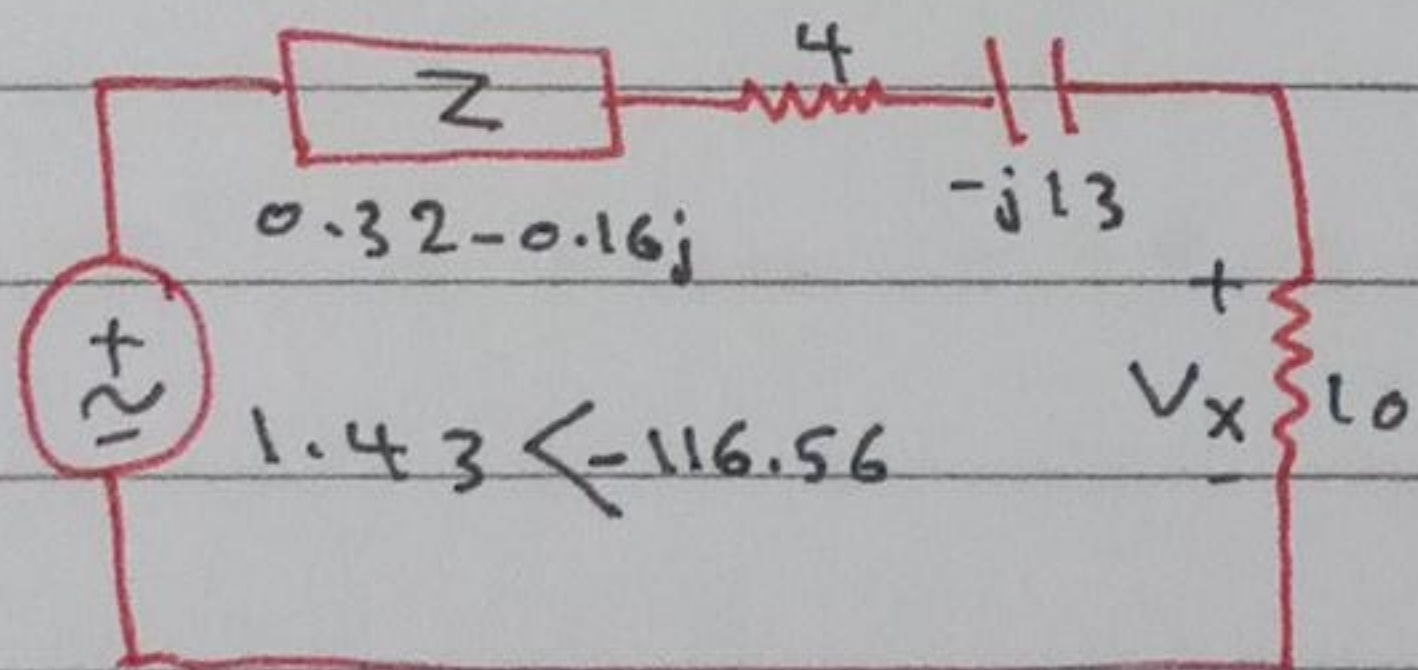
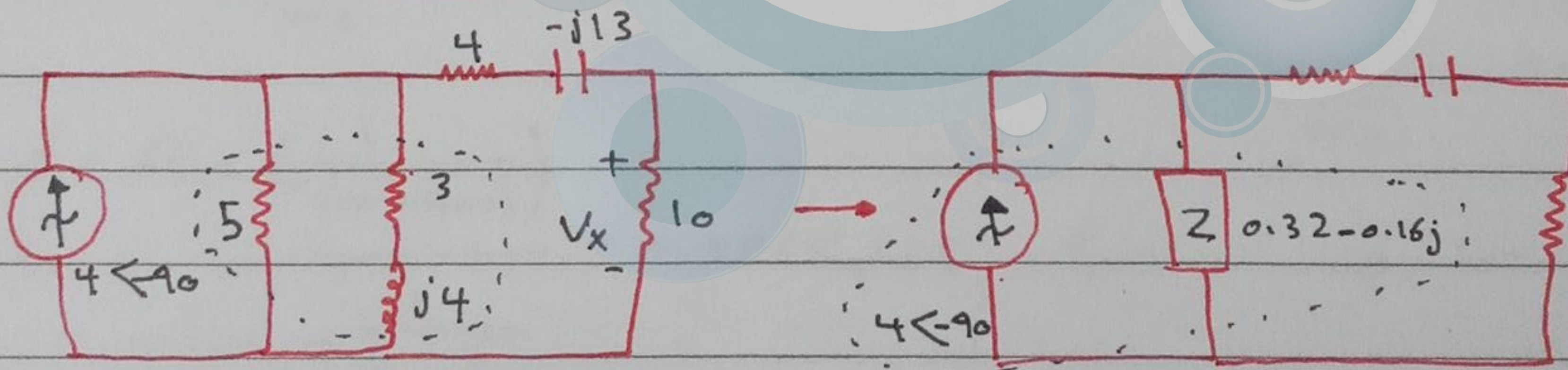
$$Z_3 = \frac{(1)(1+j)}{3+j} = 0.4 + 0.2j$$



Ex.



use source transformation
to find V_x

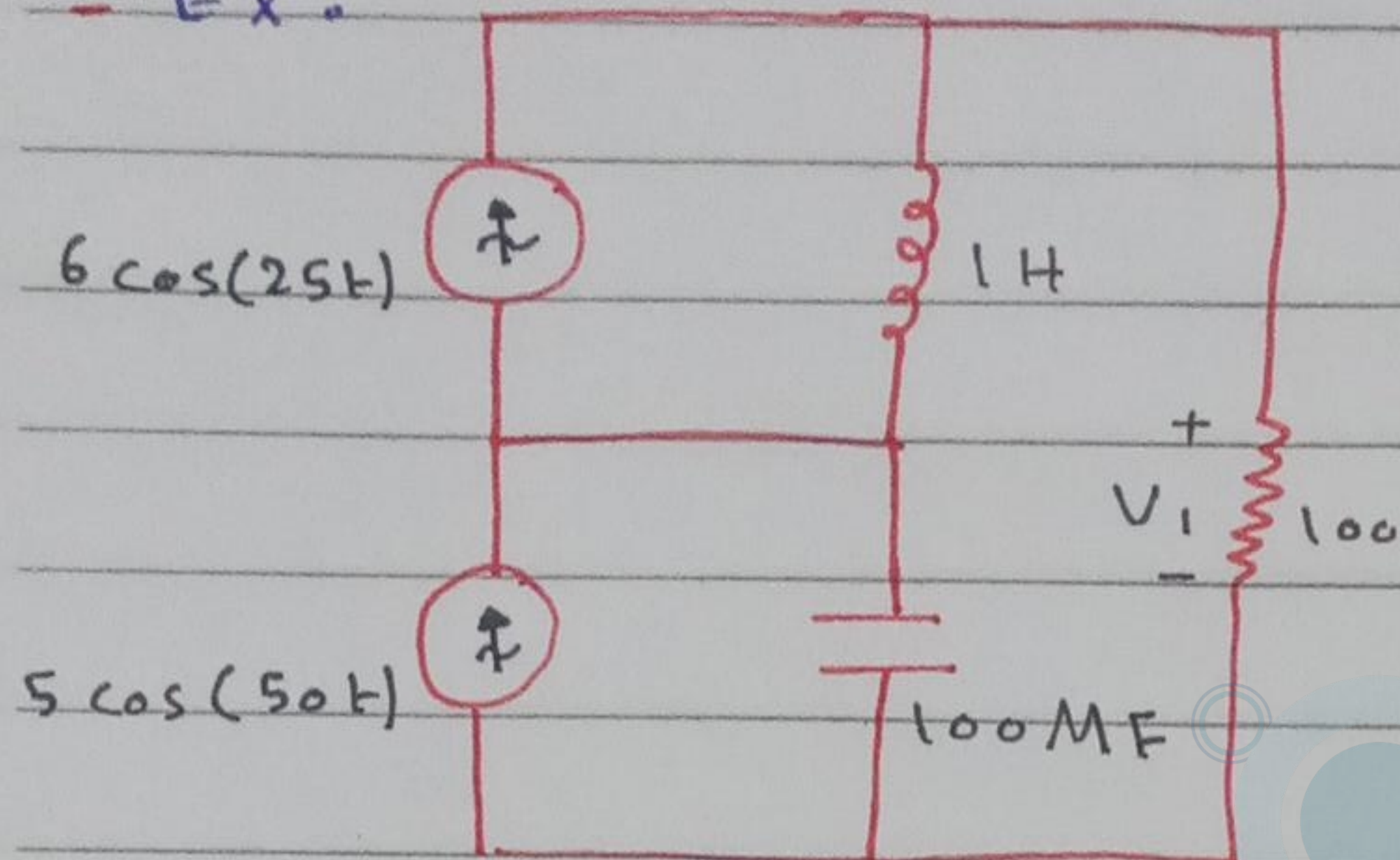


$$V_x = \frac{(1.43 \angle -116.56)(10)}{10 + 4 + (-j13) + (0.32 - 0.16j)}$$

$$V_x = 0.2 - 0.7j$$

$$= 0.735 \angle -73.98$$

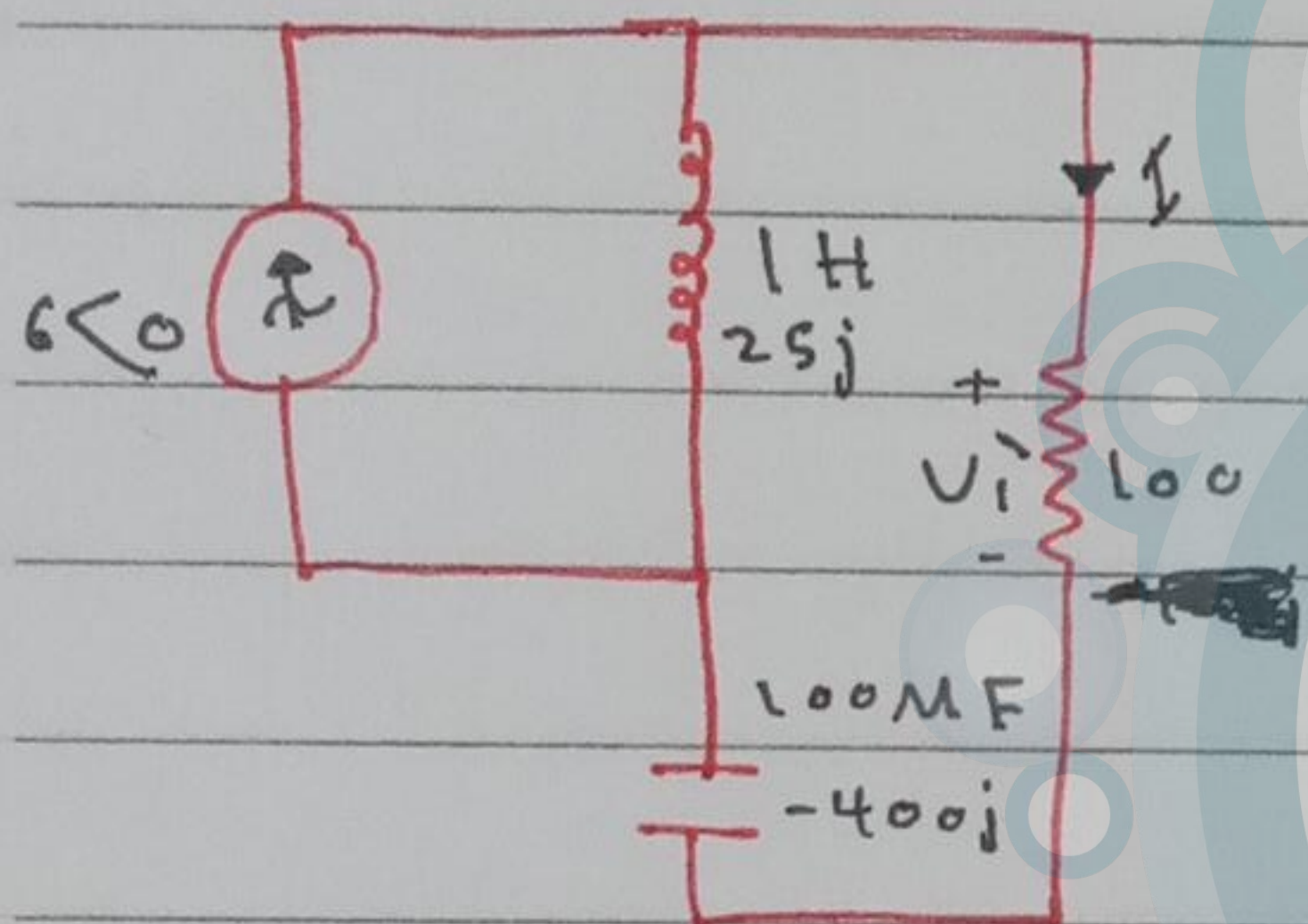
EX.



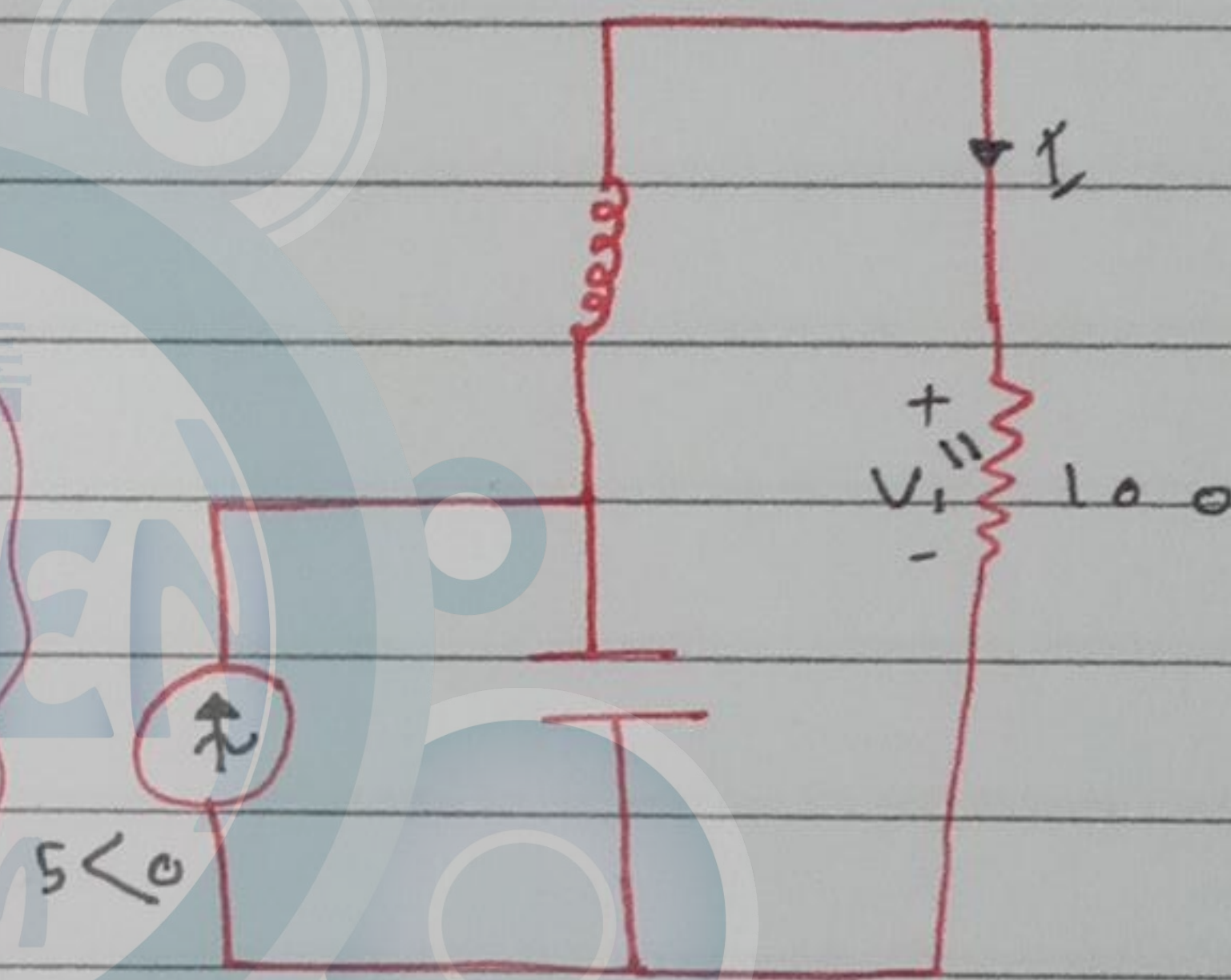
Find V_1

Note: if ω for the supplies is different, only solve by super position.

• Leave $6 < 0$



• Leave $5 < 0$



$$\omega = 25 \text{ rad/s}$$

$$1 \text{ H} \rightarrow j\omega L = 25j$$

$$100 \text{ mF} \rightarrow \frac{-j}{\omega C} = -400j$$

$$I = 6 < 0 \left(\frac{1}{100 - 400j} \right)$$

$$\frac{1}{25j} + \frac{1}{100 - 400j} = 0.386 < 165$$

$$\frac{1}{25j} + \frac{1}{100 - 400j}$$

$$V_1' = I * 100 = 38.6 < 165$$

$$5 < 0$$

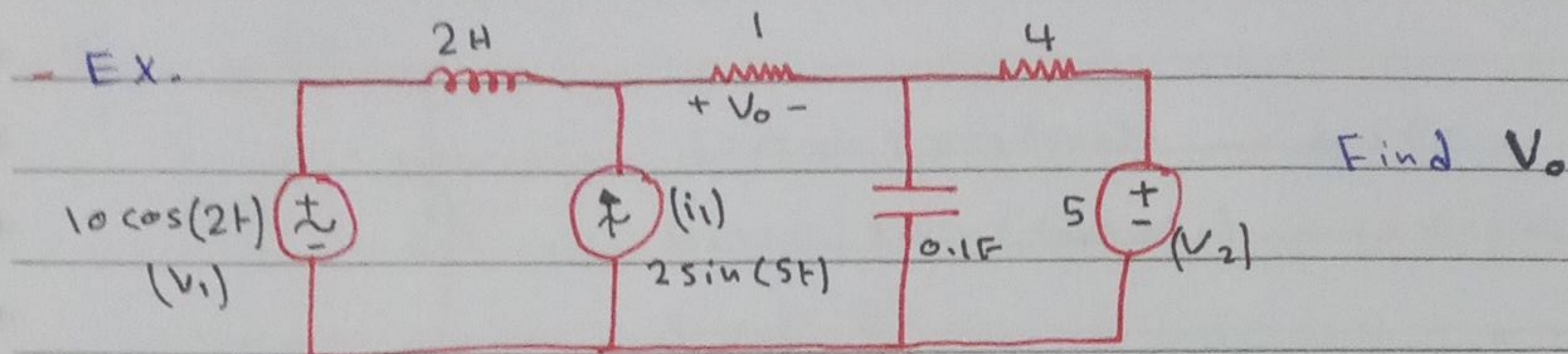
$$\omega = 50$$

$$1 \text{ H} \rightarrow j\omega L = 50j$$

$$100 \text{ mF} \rightarrow \frac{-j}{\omega C} = -200j$$

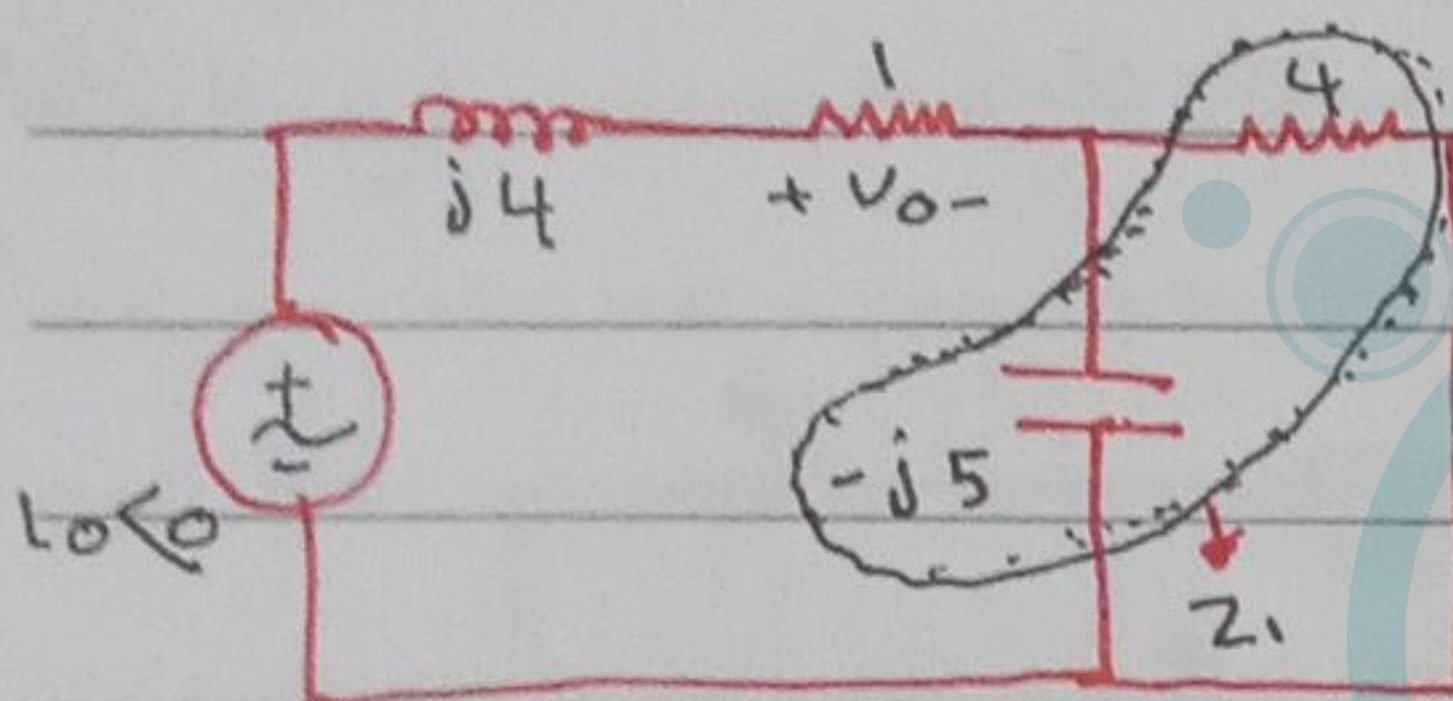
$$I =$$

Can find ...

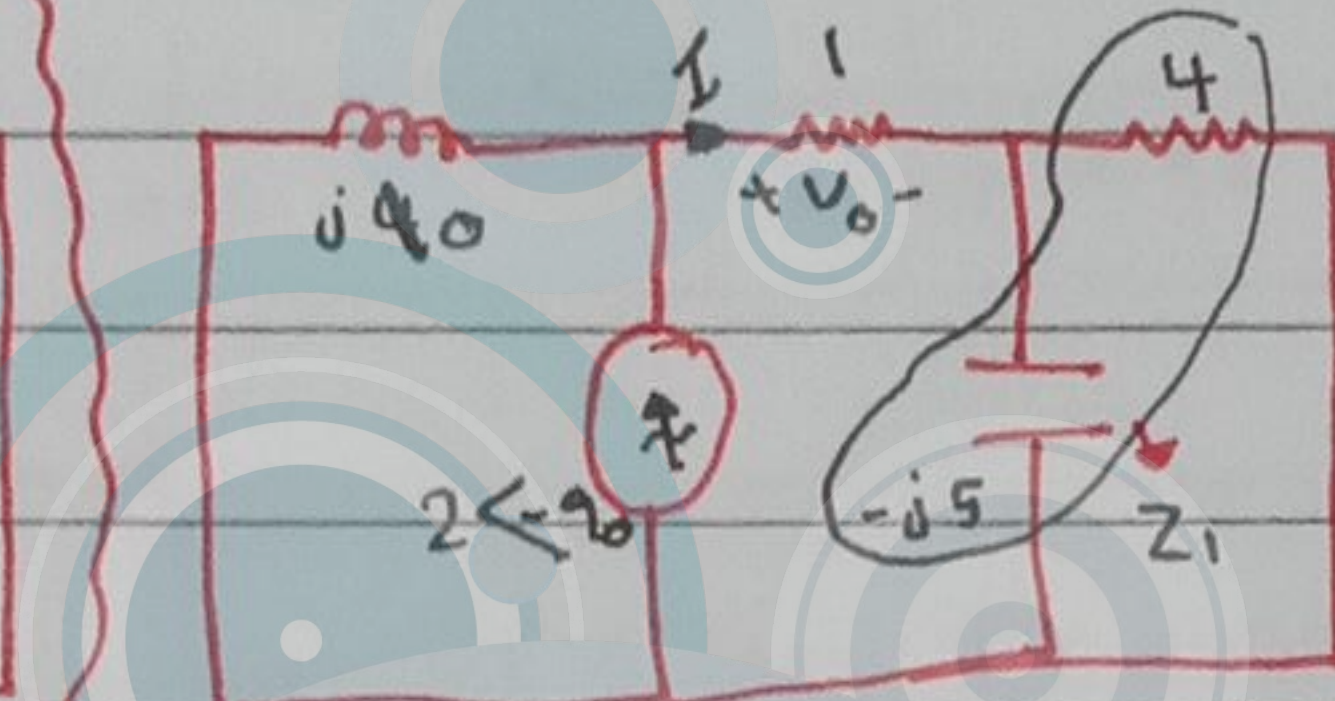


• Since different ω only superposition

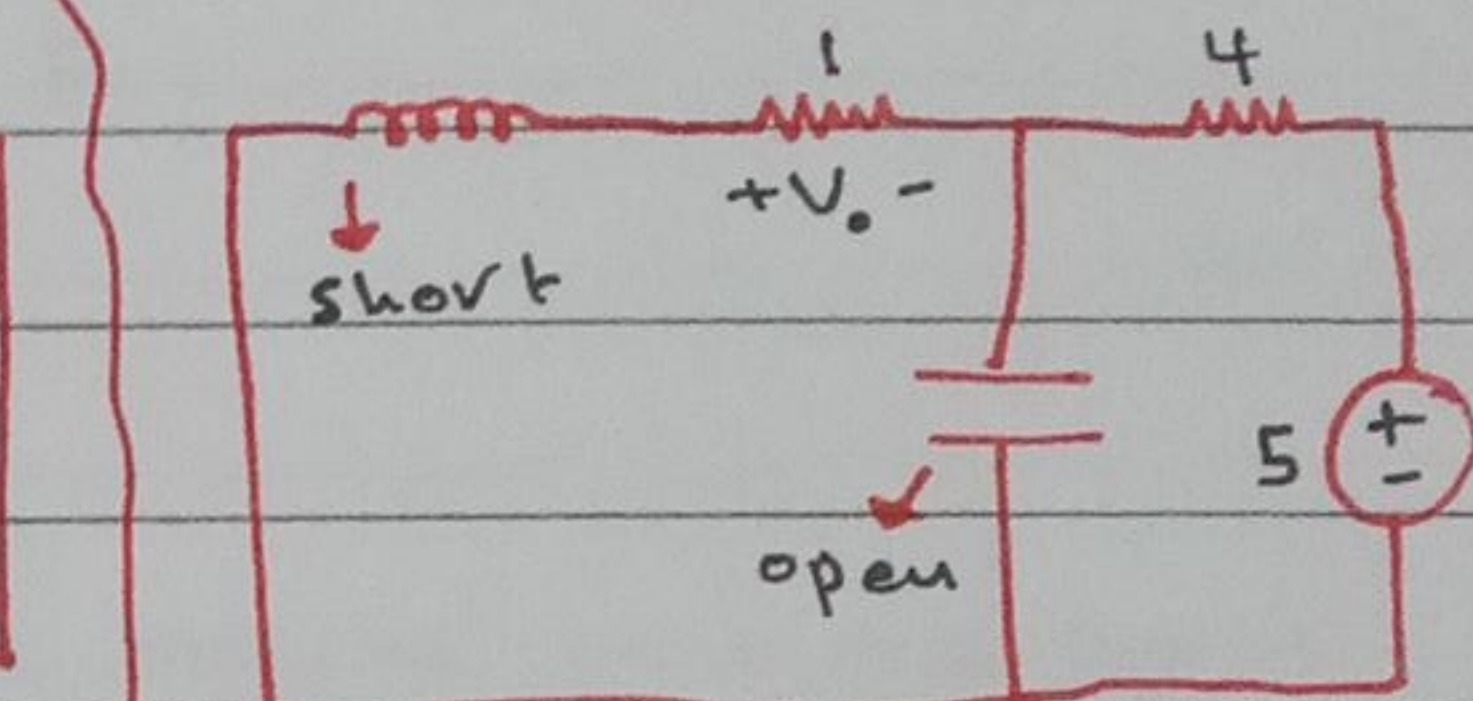
• leave V_1



• leave i_1



• leave V_2



$$V_0' = \frac{10 \angle 0^\circ \times 1}{j4 + 1 + Z_1}$$

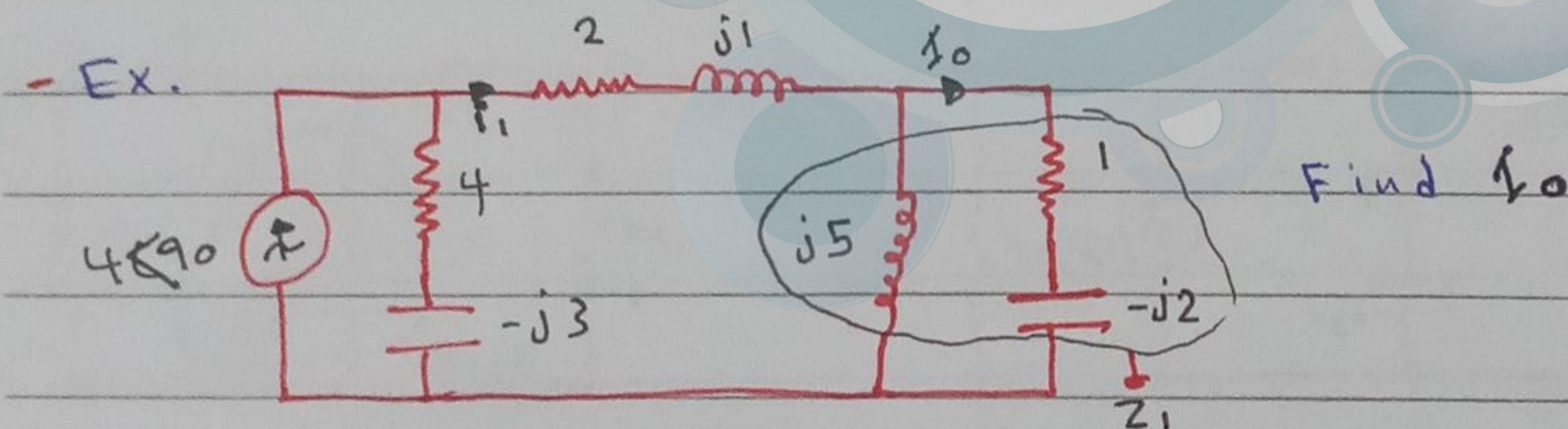
$$I = 2 \angle -90^\circ \times \frac{1}{1 + Z_1}$$

$$\frac{1}{1 + Z_1} + \frac{1}{j10}$$

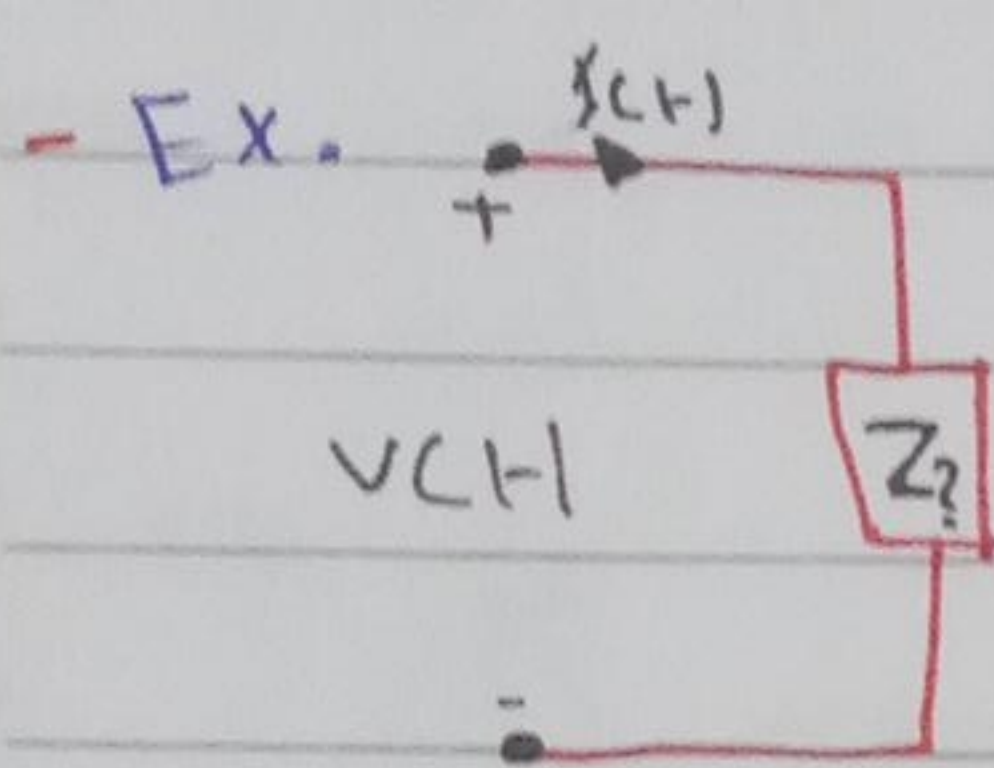
$$V_0''' = -5 \angle 1^\circ$$

$$V_0'' = I \times 1$$

⊗ $V_0 \neq V_0' + V_0'' + V_0'''$ • $V_0(t) = V_0'(t) + V_0''(t) + V_0'''(t)$



Find Z_1 then Find i_1 by current division, then Find I_0 by current division.



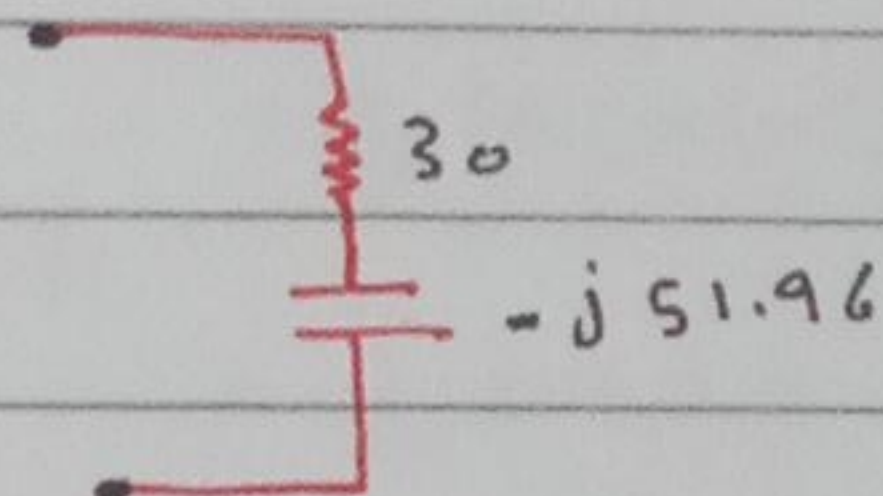
$$i(t) = 0.2 \cos(500t) \rightarrow 0.2 \angle 0$$

$$v(t) = 12 \sin(500t + 30) \rightarrow 12 \angle -60$$

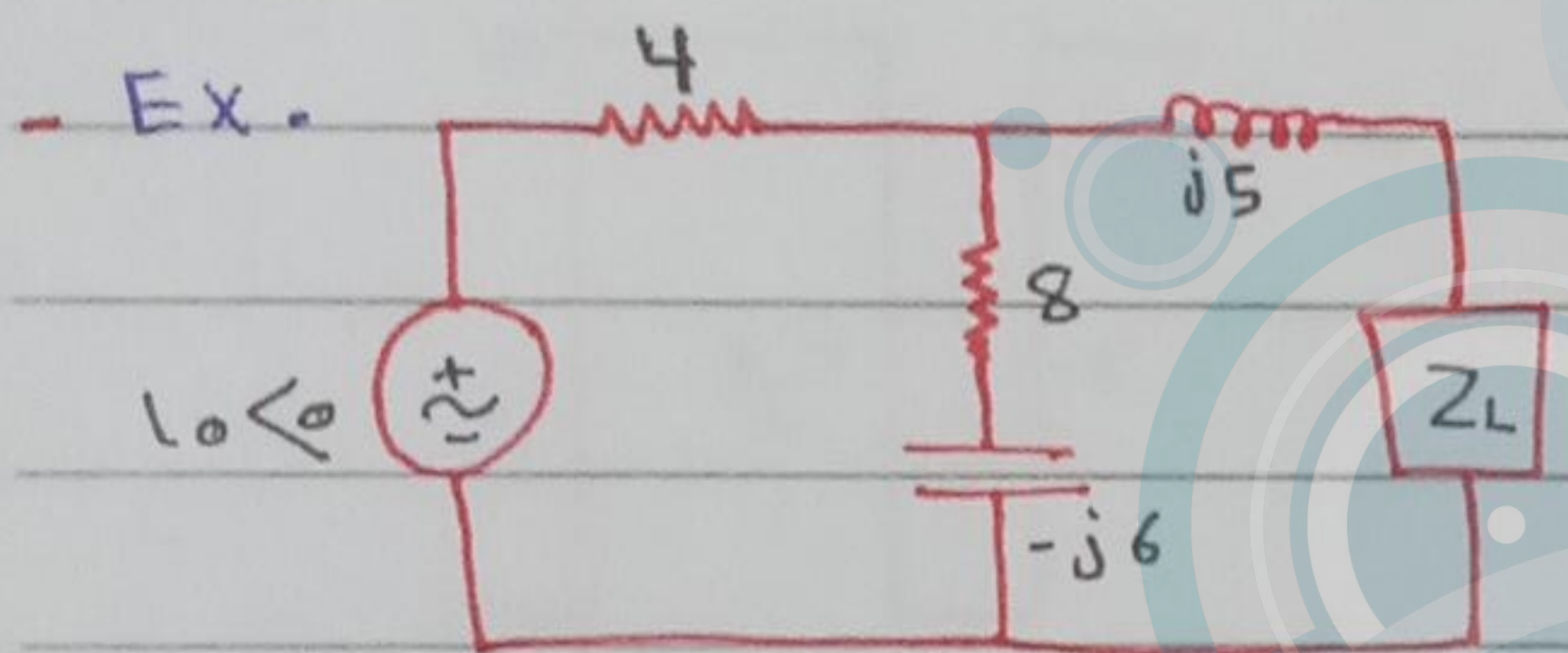
Find Z

$$Z = \frac{V}{I} = \frac{12 \angle -60}{0.2 \angle 0} = 60 \angle -60 \rightarrow 30 - j51.96$$

$R \quad Z_C$



$$Z_C = \frac{-j}{\omega C} \rightarrow -j51.96 = \frac{-j}{500C} \quad C = \frac{1}{500 \times 51.96} \text{ F}$$



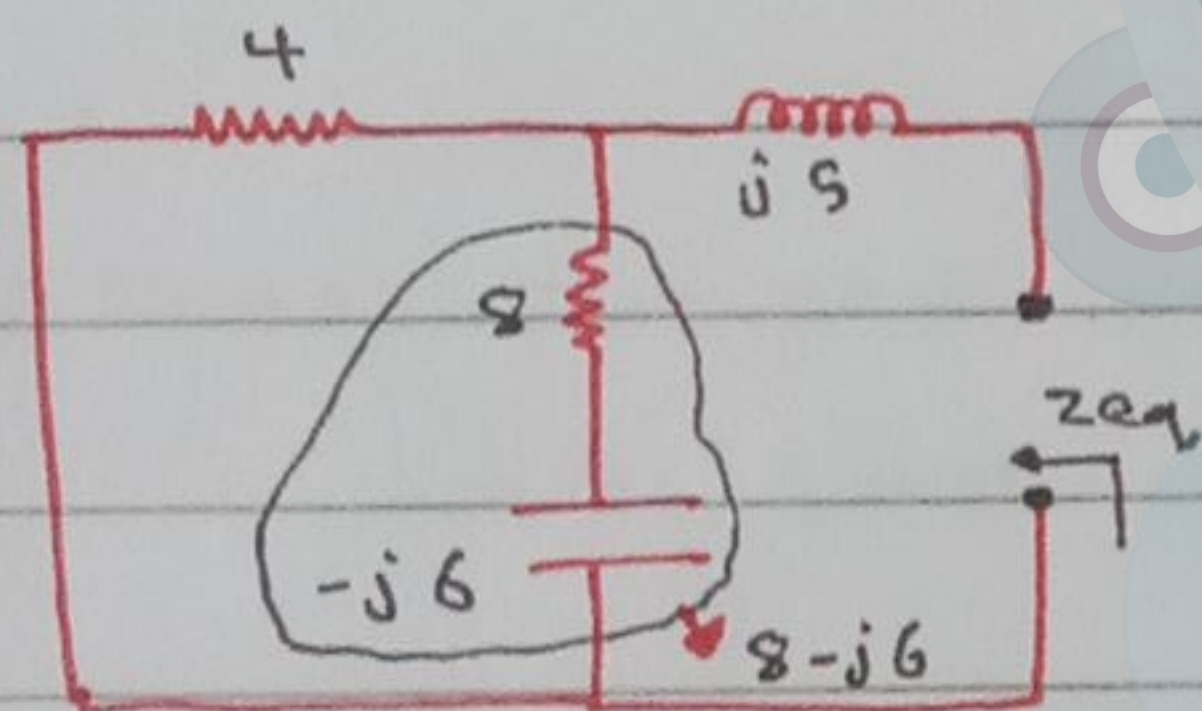
① Find Z_L that absorbs max avg power

② Find max avg power

• For max power transfer $\rightarrow Z_L = Z_{Th}^*$

$$\text{max power} = P_{max} = \frac{|V_{Th}|^2}{8 R_{Th}}$$

• Find Z_{Th}

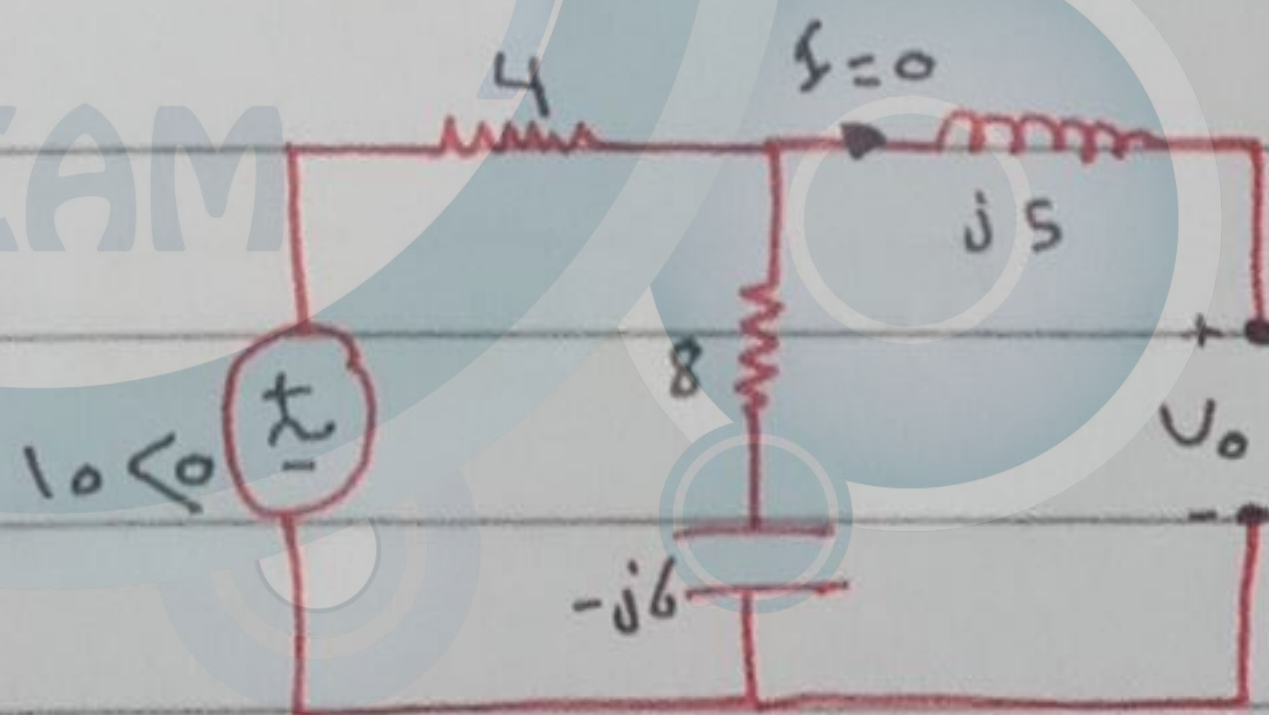


$$Z_{eq} = Z_{Th} = \left(\frac{4(8 - j6)}{4 + (8 - j6)} + j5 \right)$$

$$= 2.94 - j5.5$$

R_{Th}

• Find V_{Th}



V_o = Voltage at $8 - j6$

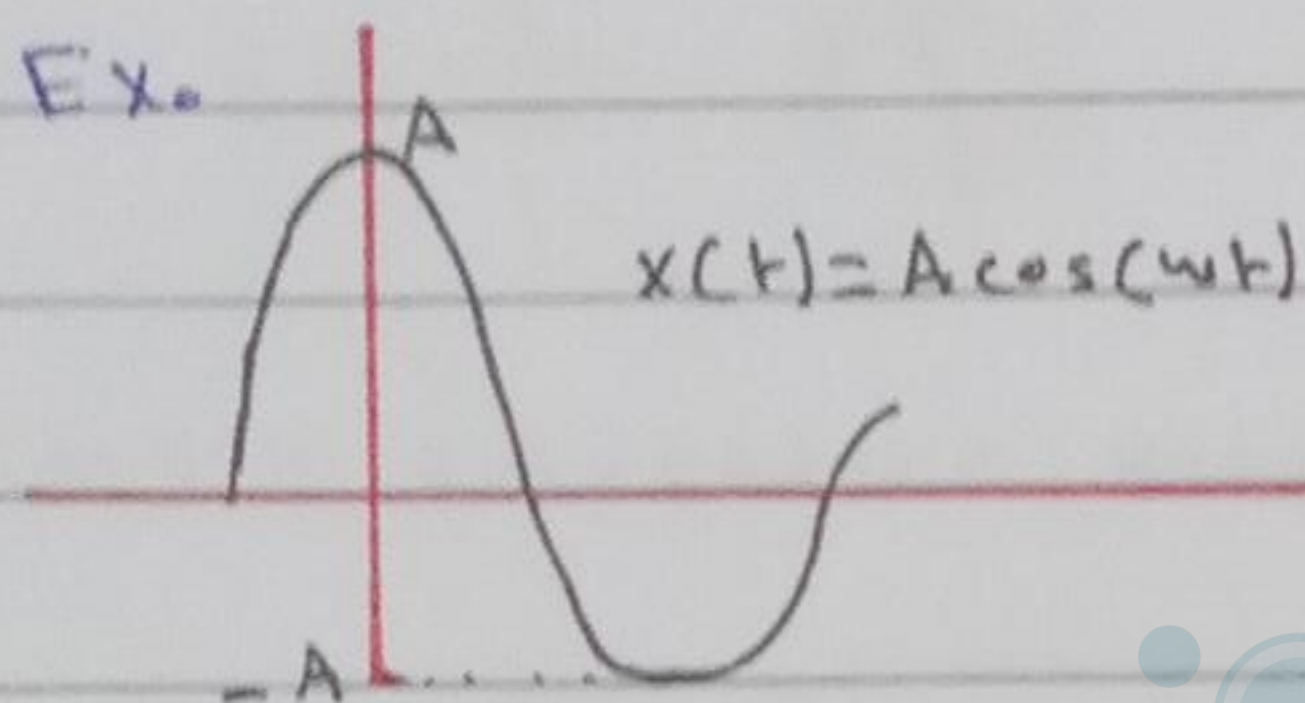
$$V_o = \frac{10 \angle 0 \times (8 - j6)}{4 + 8 - j6} = 7.5 \angle -10.3$$

$$P_{max} = \frac{(7.5)^2}{8 \times 2.94} = 2.34 \text{ W}$$

⊗ Average Value $\langle x(t) \rangle = \frac{1}{T} \int_0^T x(\tau) \cdot d\tau$

↑
period

- Ex.

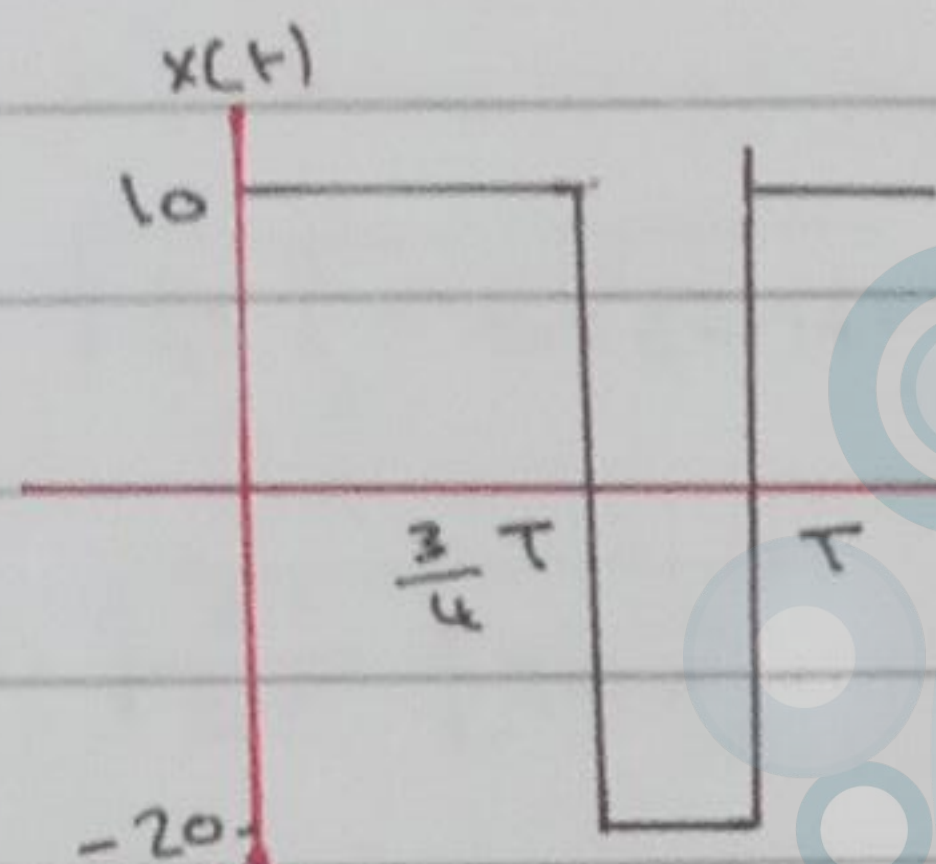


$$\langle x(t) \rangle = \frac{1}{T} \int_0^T A \cos(\omega \tau) d\tau$$

$$= \frac{1}{T} (A \sin(\omega \tau) \omega) \Big|_0^T$$

$$= \frac{1}{T} (0) = 0$$

- Ex.



$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(\tau) d\tau$$

$$= \frac{1}{T} \left[\int_0^{3T/4} 10 d\tau + \int_{3T/4}^T -20 d\tau \right]$$

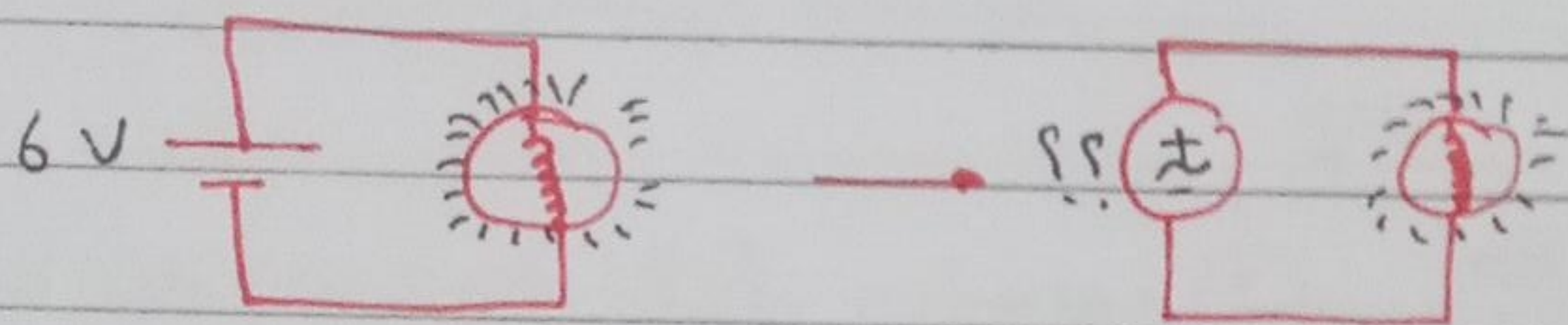
$$= \frac{1}{T} \left[\frac{30T}{4} - 20 \left(T - \frac{3T}{4} \right) \right]$$

$$= \frac{1}{T} \left[\frac{30T}{4} - \frac{20T}{4} \right] = \frac{1}{T} \left(\frac{10T}{4} \right)$$

$$= \boxed{2.5}$$

⊗ Note: The average of the sinusoidal signal is zero, independent of Amplitude or frequency

⑤ Root mean square (RMS) :-



RMS = effective value of varying voltage or current

$$X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} \equiv X_{\text{effective}}$$

- Ex. $x(t) = A \cos(\omega t)$

$$\begin{aligned} X_{rms} &= \sqrt{\frac{1}{T} \int_0^T A^2 \cos^2(\omega t) dt} = \sqrt{\frac{A^2}{2T} \int_0^T 1 + \cos(2\omega t) dt} \\ &= \sqrt{\frac{A^2}{2T} [T + 0]} = \frac{A}{\sqrt{2}} = X_{rms} \end{aligned}$$

- Note: For sinusoidal signal $X_{rms} = \frac{\text{Peak value}}{\sqrt{2}}$

For square wave signal $X_{rms} = A$ (max value)

For triangular signal $X_{rms} = \frac{A}{\sqrt{3}}$

For D-C signal $X_{rms} = \text{DC value}$

- Ex. $V_{rms} = 220 \text{ volt}$ (sin wave)

$$V_{peak} = V_{rms} \times \sqrt{2} = 220 \sqrt{2}$$

- Note :-

$$V(t) = V_1 \cos(\omega t) + V_2 \sin(\omega t) + \dots + V_0 \rightarrow \text{DC}$$

$$V_{rms} = \sqrt{(V_{oc})^2 + \sum_{i=1}^n \frac{V_i^2}{2}} \quad \text{OR} \quad R_{MS_{\text{total}}} = \sqrt{(RMS_1)^2 + (RMS_2)^2 + (RMS_3)^2 + \dots}$$

- Ex. $V(t) = 4 \cos(100t) + 12 \sin(100t) - 6 \cos(300t) + 5$

$$V_{rms} = \sqrt{25 + \frac{16 + 144 + 36}{2}} = \sqrt{123}$$

Power in AC circuits :-

① The instantaneous power

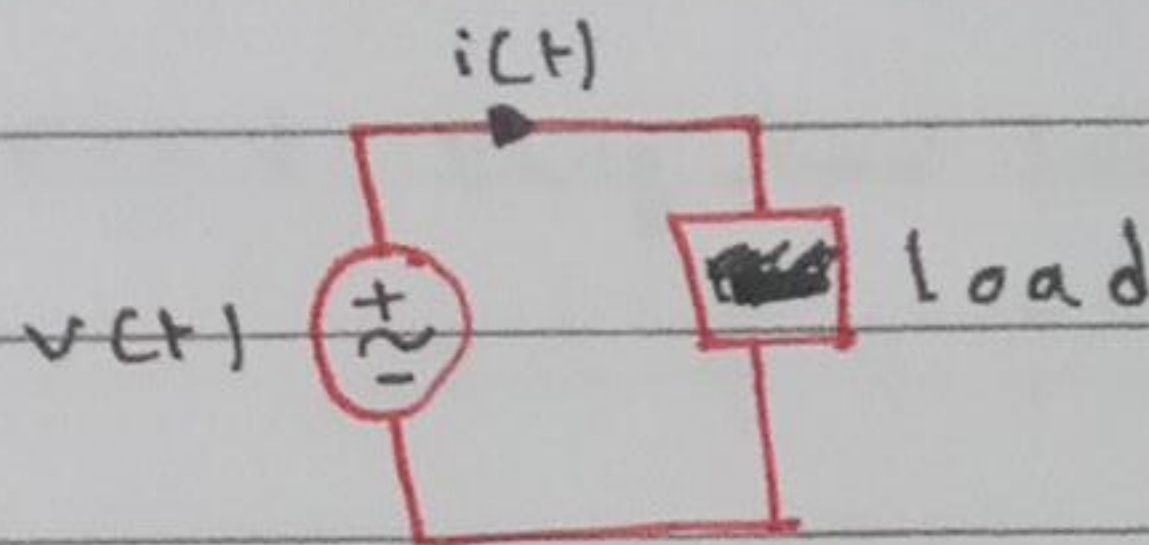
i f

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

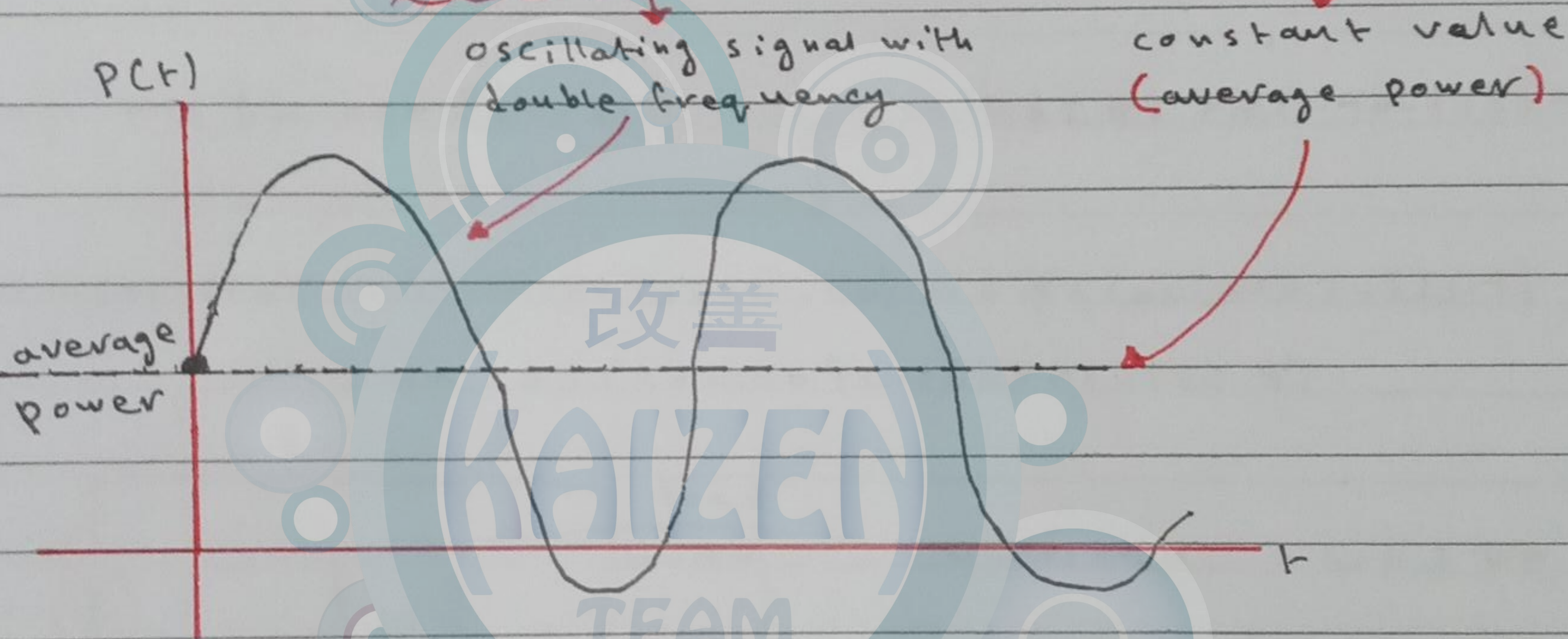
peak

phase

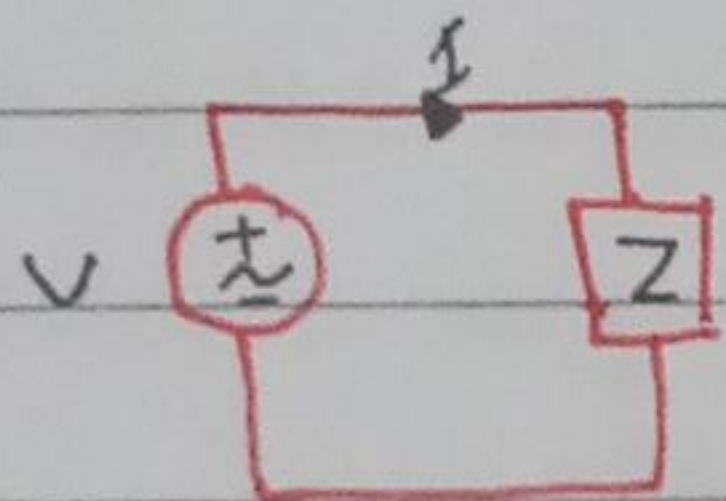


$$p(t) = v(t) i(t) = V_m \cos(\omega t + \theta_v) \times I_m \cos(\omega t + \theta_i)$$

$$= \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$



$$I = I_m \angle \theta_i$$

$$V = V_m \angle \theta_v$$

$$Z = \frac{V}{I} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$\rightarrow |Z| = \frac{V_m}{I_m} \quad \theta_z = \theta_v - \theta_i$$

$$P_{avg} = \frac{1}{2} I_m^2 |Z| \cos(\theta_v - \theta_i) \quad \text{and} \quad P_{avg} = \frac{1}{2} \frac{V_m^2}{|Z|} \cos(\theta_v - \theta_i)$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

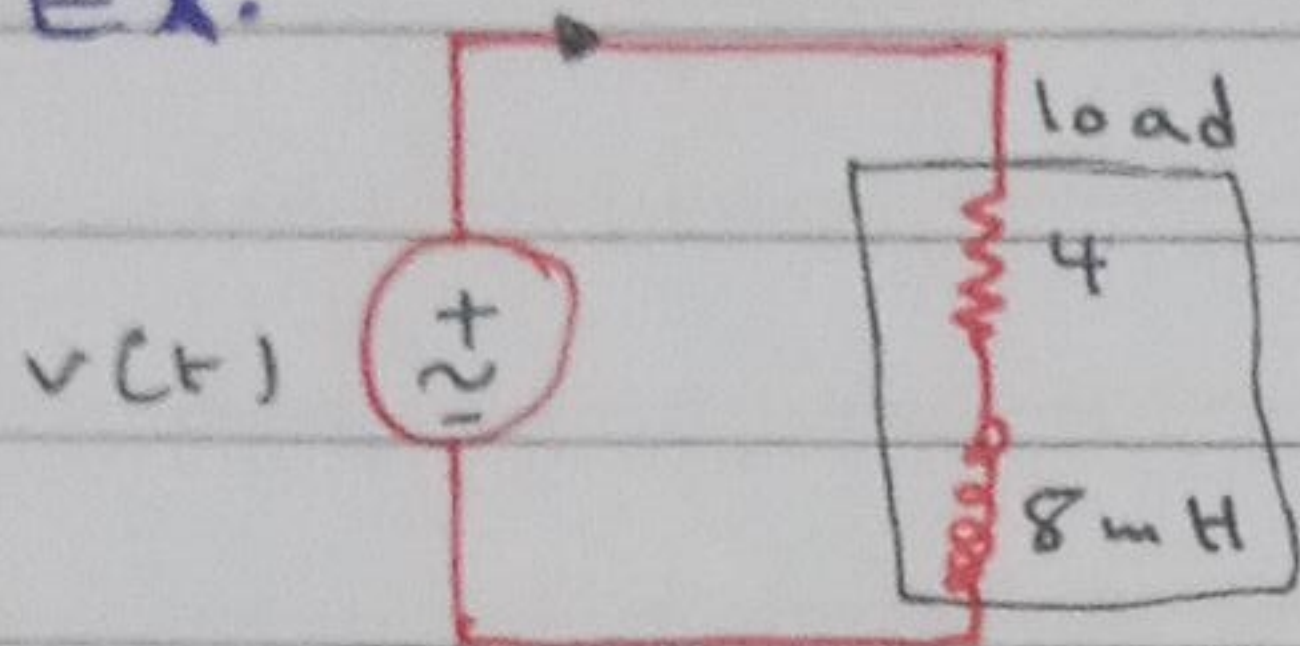
$$P_{avg} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$P_{avg} = I_{rms}^2 |Z| \cos(\theta_v - \theta_i)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = \frac{V_{rms}^2}{|Z|} \cos(\theta_v - \theta_i)$$

Ex.



$$v(t) = 14.14 \cos(377t - 90) \rightarrow 14.14 \angle -90$$

Find: P_{avg} for load and $p(t)$

$$\omega = 377$$

$$8 \text{ mH} \rightarrow j3$$

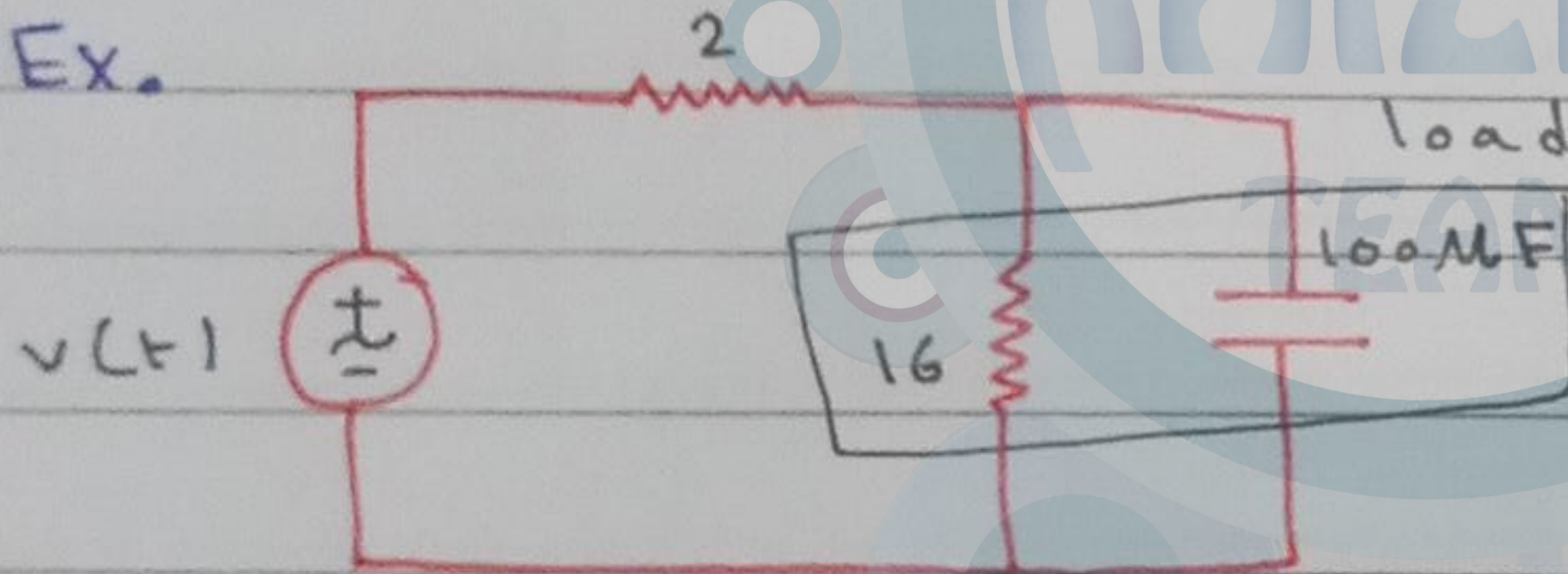
$$Z_{load} = 4 + j3 \rightarrow 5 \angle 36.9$$

$$i = \frac{V}{Z} = \frac{14.14 \angle -90}{5 \angle 36.9} = 2.83 \angle -126.9$$

$$P_{avg} = \frac{1}{2} V_m i_m \cos(\theta_v - \theta_i) = \frac{1}{2} \times 14.14 \times 2.83 \cos(-90 - (-126.9)) = 16 \text{ W}$$

$$p(t) = v(t) i(t) = 14.14 \cos(377t - 90) \times 2.83 \cos(377t - 126.9) = 40 \cos(377t - 90) \cos(377t - 126.9)$$

Ex.



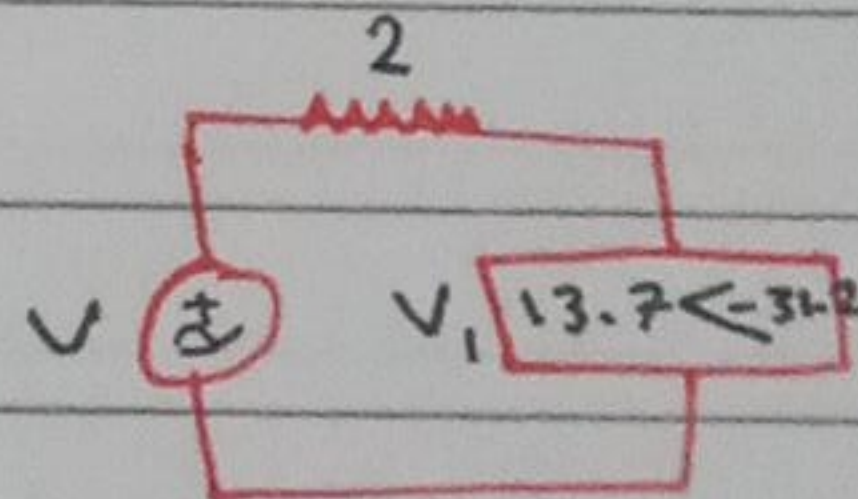
$$V = 110 \angle 0$$

$$\omega = 377$$

Find P_{avg} for the load

$$100 \text{ mF} \rightarrow -j26.5$$

$$Z_{load} = \frac{16(-j26.5)}{16 - j26.5} = 13.7 \angle -31.2$$



$$V_1 = \frac{110 \angle 0 (13.7 \angle -31.2)}{2 + 13.7 \angle -31.2} = 97.5 \angle -3.85$$

$$P_{avg} = \frac{1}{2} \frac{V_m^2}{|Z|} \cos(\theta_v - \theta_i) = \frac{1}{2} \times \frac{97.5^2}{13.7} \cos(-3.85) = 296.8 \text{ W}$$

⊗ complex power

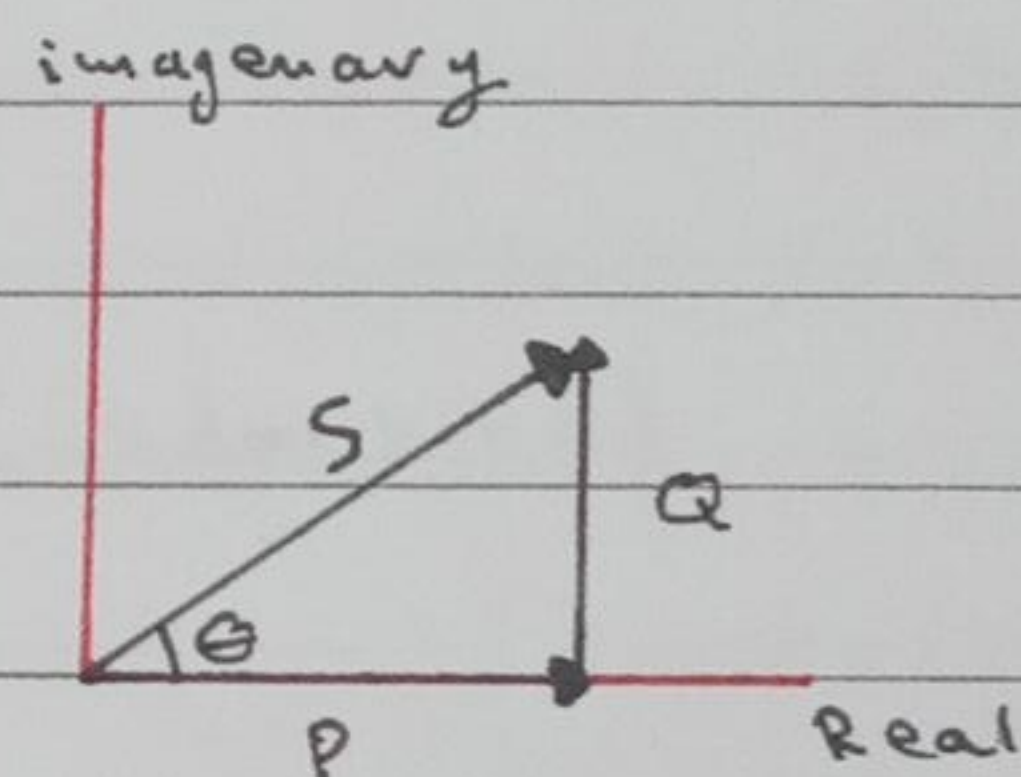
$$S = V_{rms} I_{rms}^* \rightarrow (V_{rms} \angle \theta_v) (I_{rms} \angle -\theta_i) \rightarrow \boxed{V_{rms} I_{rms} \angle \theta_v - \theta_i}$$

- When we make S in rectangular form:-

$$S = P + jQ \rightarrow \begin{matrix} \text{Complex power / apparent power} \\ \text{(VA)} \end{matrix} \quad \begin{matrix} \text{Real power} \\ \text{Average power} \\ \text{(Watt)} \end{matrix} \quad \text{Reactive power (VAR)}$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \text{ W}$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \text{ VAR}$$

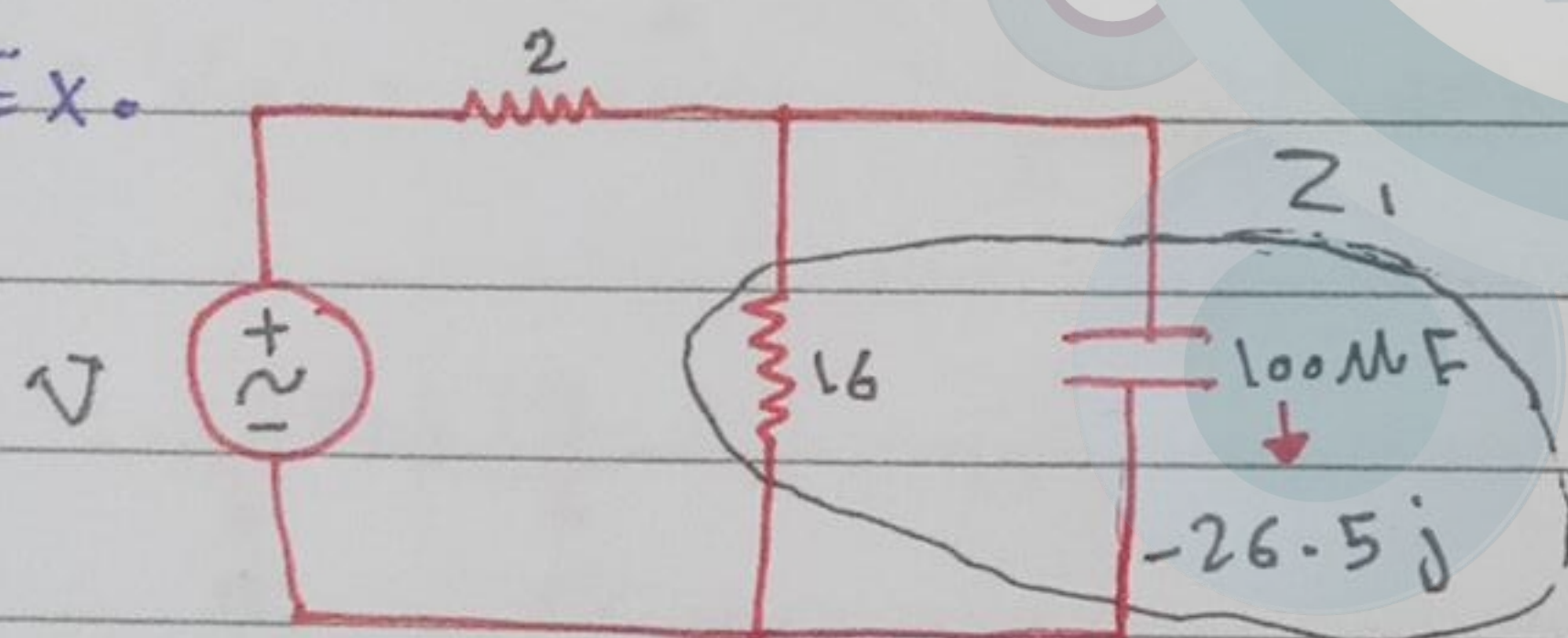


$$Z = R + jX \quad z = \frac{V_{rms}}{I_{rms}} \rightarrow S = \underbrace{I_{rms}^2 R}_P + j \underbrace{I_{rms}^2 X}_Q$$

- Q is the measure of Energy that exchanges between the source and the reactive impedance (inductors and capacitors)

- P is power dissipated in resistors

- Ex.



$$V = 110 \angle 0^\circ, \omega = 377$$

Find S, P, Q, PF

$$Z_1 = 13.7 \angle -31.2^\circ$$

$$I = \frac{V}{2 + Z_1} = 7.116 \angle -27.35^\circ$$

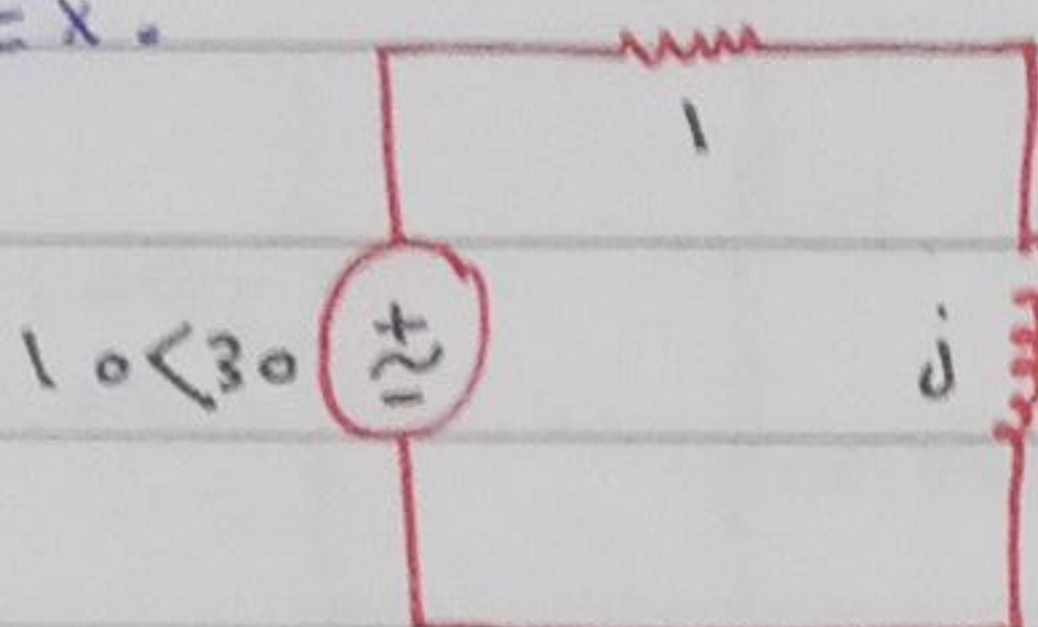
$$S = V_{rms} I_{rms}^* \left(\frac{110 \angle 0^\circ}{\sqrt{2}} \right) \left(\frac{7.116 \angle -27.35^\circ}{\sqrt{2}} \right) = \underbrace{347.63}_P + j \underbrace{179.81}_Q$$

PF

$$\text{PF} = \frac{P}{|S|} = \cos(\theta_v - \theta_i) = 0.89 \text{ leading (capacitance)}$$

$$Q > 0$$

- Ex.



assume rms values (means $10\angle30$ is rms)

Find S, P, Q, PF

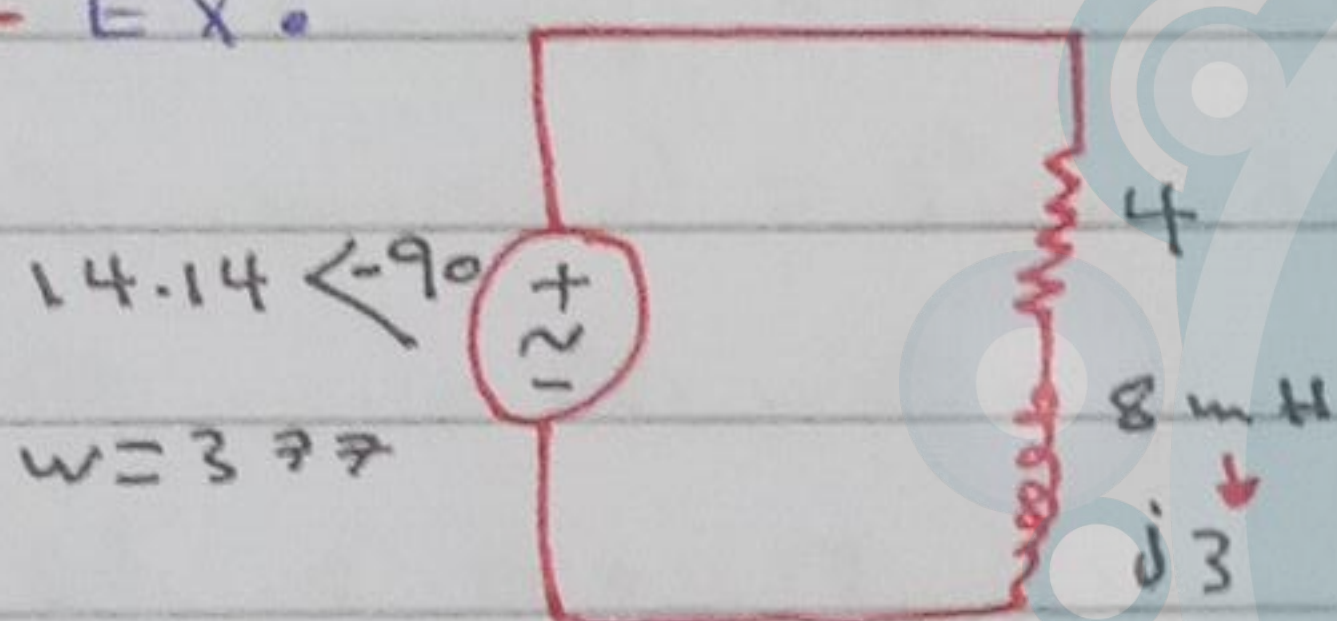
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{10\angle30}{1+j} = 7.07\angle-15$$

$$S = V_{rms} I_{rms}^* = (10\angle30)(7.07\angle15) = 70.7\angle45$$

$$= \underbrace{50}_P + j\underbrace{50}_Q$$

$$PF = \cos(30 - -15) = \cos(45) = 0.707 \text{ lagging (inductive)}$$

- Ex.



Find S, P, Q, PF at source

$$I = \frac{V}{Z} = \frac{14.14\angle-90}{4+j3} = 2.828\angle-126.8$$

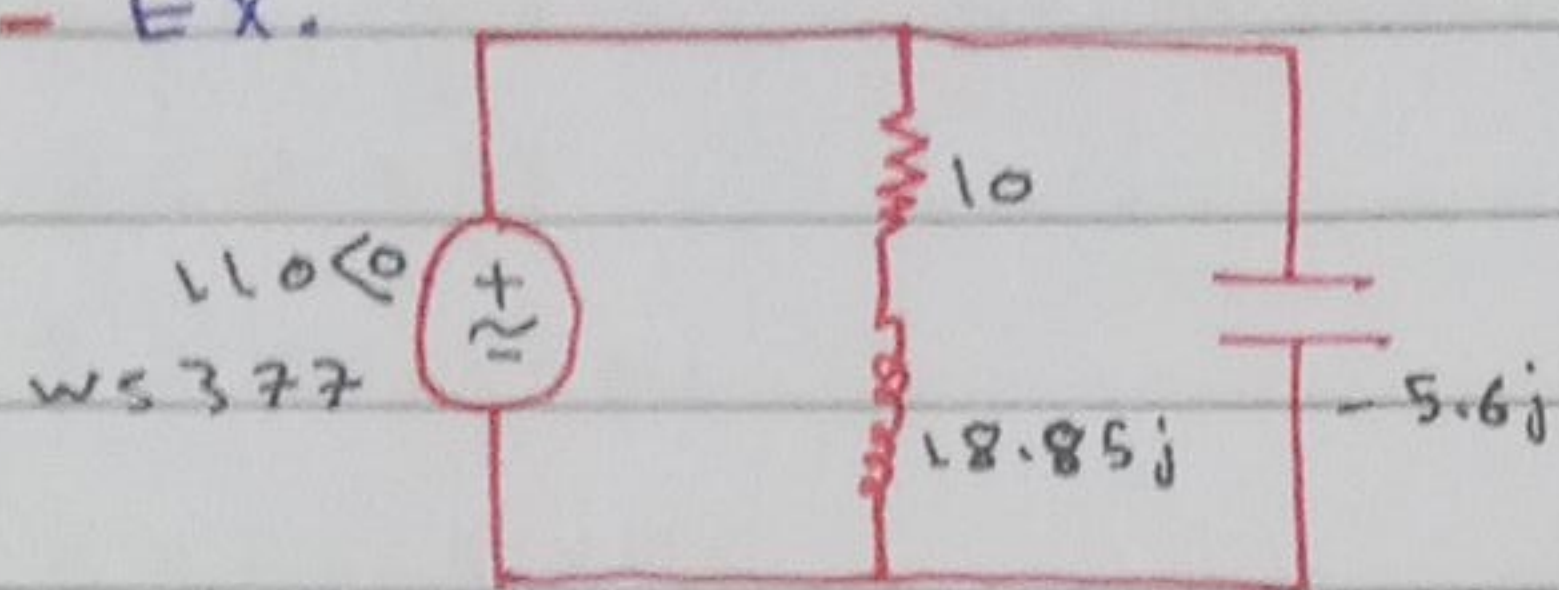
$$S = V_{rms} I_{rms}^* = \left(\frac{14.14}{\sqrt{2}}\angle-90\right) \left(\frac{2.828}{\sqrt{2}}\angle126.8\right)$$

$$= \underbrace{16}_P + j\underbrace{12}_Q$$

$$PF = \cos(-90 - -126.8) = \cos(36.8) = 0.8 \text{ lagging (inductive)}$$

- Note: PF (efficiency measure) $0 < PF < 1$

- Ex.



assume rms values

Find: ① Dissipated power ② reactive power

③ complex power at supply ④ power factor at supply

$$Z_{eq} = 7.2 \angle -80.9$$

$$I = \frac{V}{Z} = 15.3 \angle -80$$

$$S = V_{rms} I_{rms}^* = (110 \angle 0) (15.3 \angle -80) = \underline{265.7} - j \underline{165.9}$$

dissipated power reactive power

$$PF = \cos(0 - 80) = 0.17 \text{ leading}$$

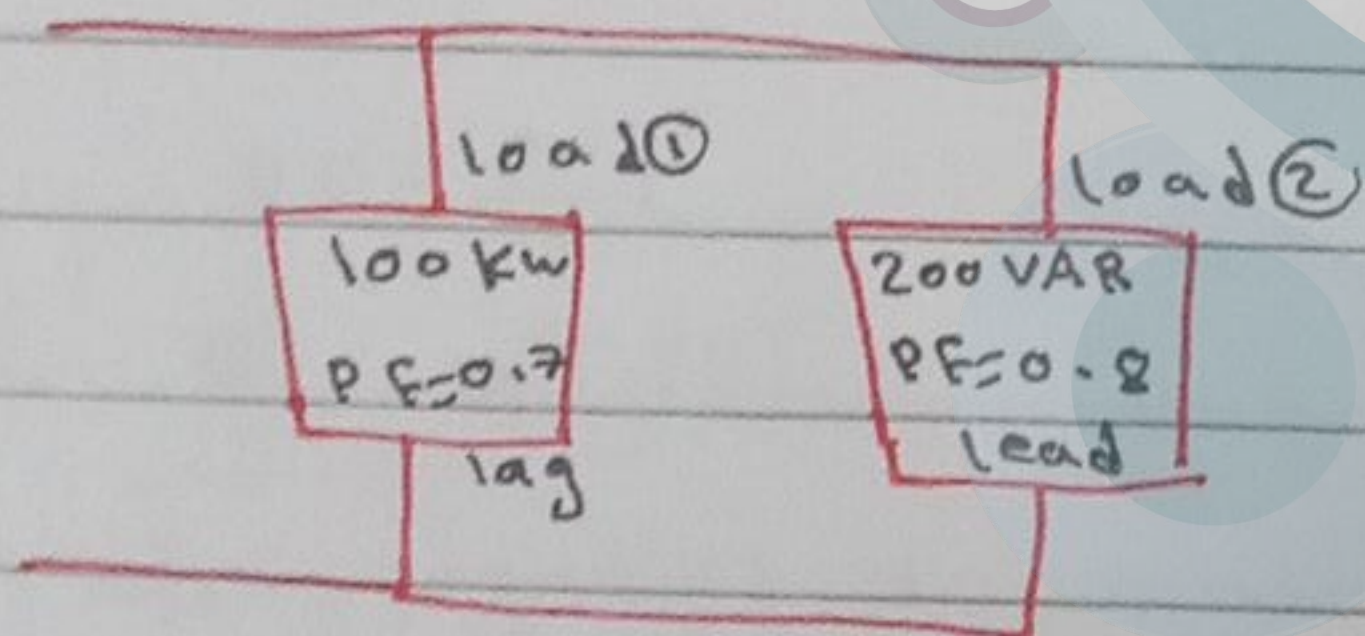
$$Q(\text{inductor}) = I_{rms}^2 X_L$$

$$Q(\text{capacitor}) = I_{rms}^2 X_C$$

$$Q_{total} = Q_L - Q_C \text{ or } |Q_L| + |Q_C|$$

$$S|_{sup} = P + jQ_{total}$$

- Ex.



$$S_{supply} = S_{load1} + S_{load2}$$

• at load ①

$$PF = 0.7 = \frac{P}{|S|} = \frac{100 \text{ kW}}{|S|}$$

$$|S| = 142.857 \text{ VA}$$

$$PF = \cos(\theta_s) = 0.7$$

$$\theta_s = 45.6^\circ \text{ (+ve)}$$

• at load ②

$$0.8 = \cos(\theta_s)$$

$$\theta_s = 36.87^\circ \text{ (-ve)}$$

$$Q = |S| \sin \theta_s$$

$$200 = |S| \times 0.6$$

$$|S| = 333.33$$

* Impedance triangle

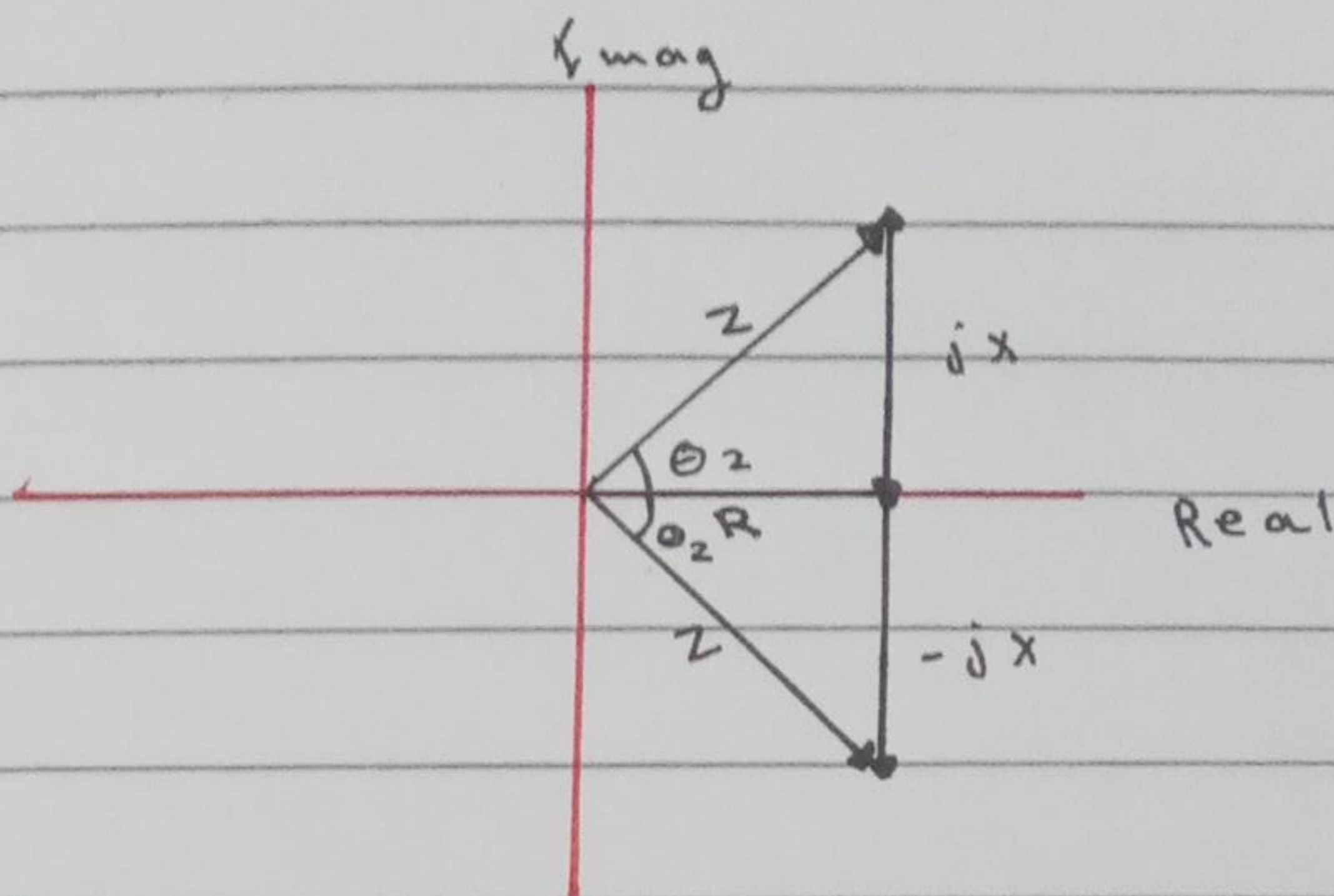
$$Z = R + jX$$

$$R = |Z| \cos \theta_z$$

$$X = |Z| \sin \theta_z$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_z = \tan^{-1} \left(\frac{X}{R} \right)$$



* Power triangular

$$|S| = \sqrt{P^2 + Q^2}$$

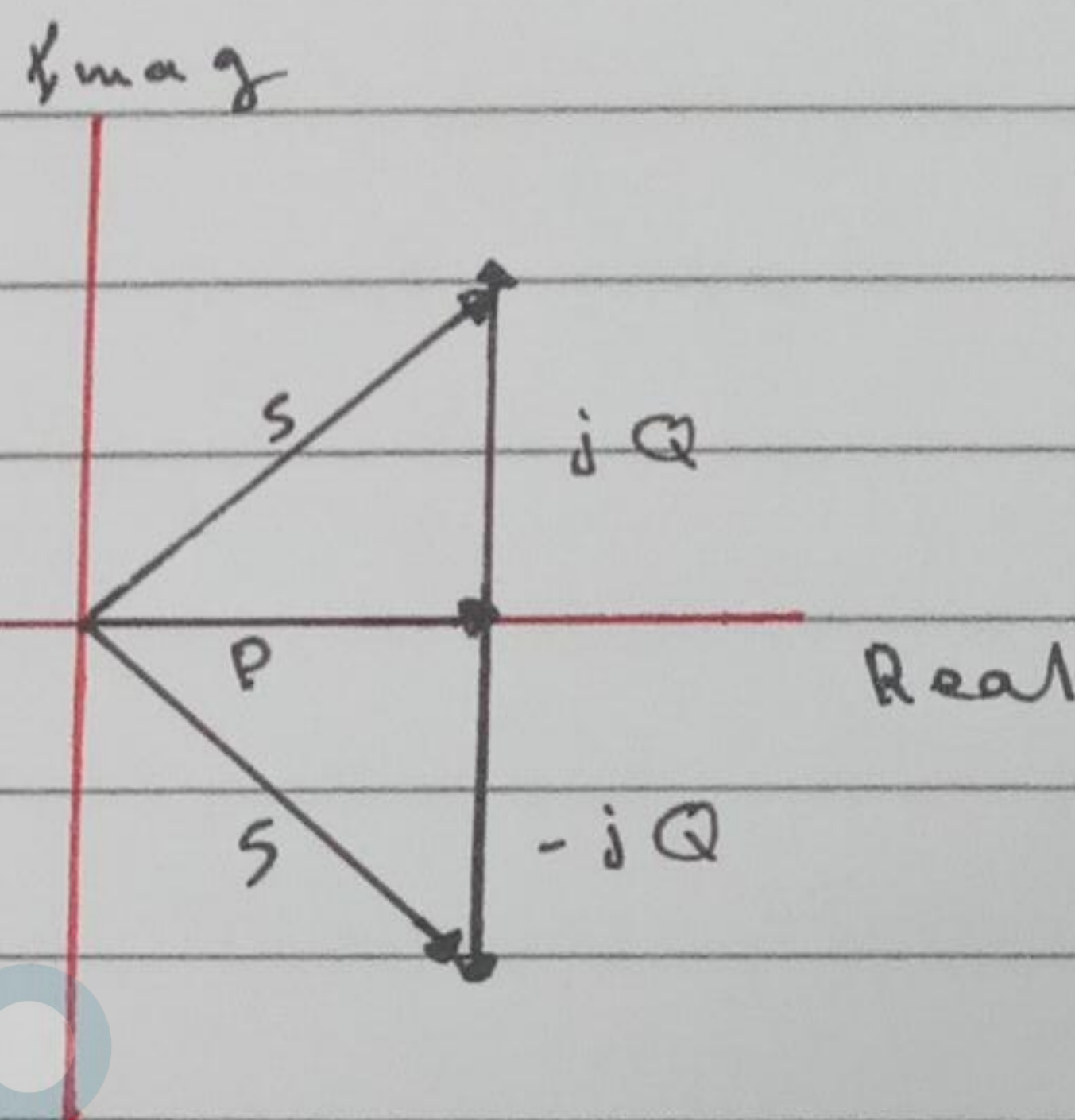
$$\theta_s = \tan^{-1} \left(\frac{Q}{P} \right)$$

$$P = |S| \cos \theta_s$$

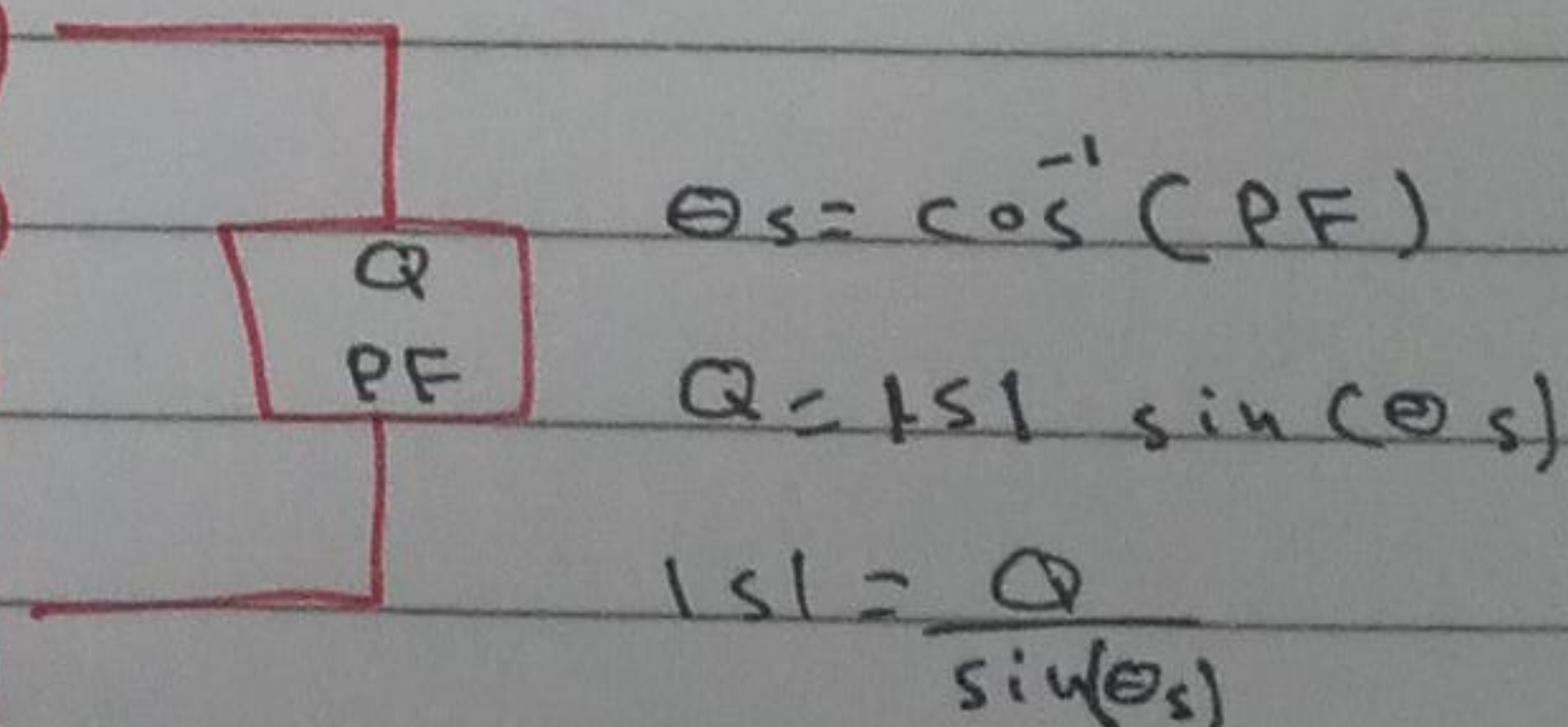
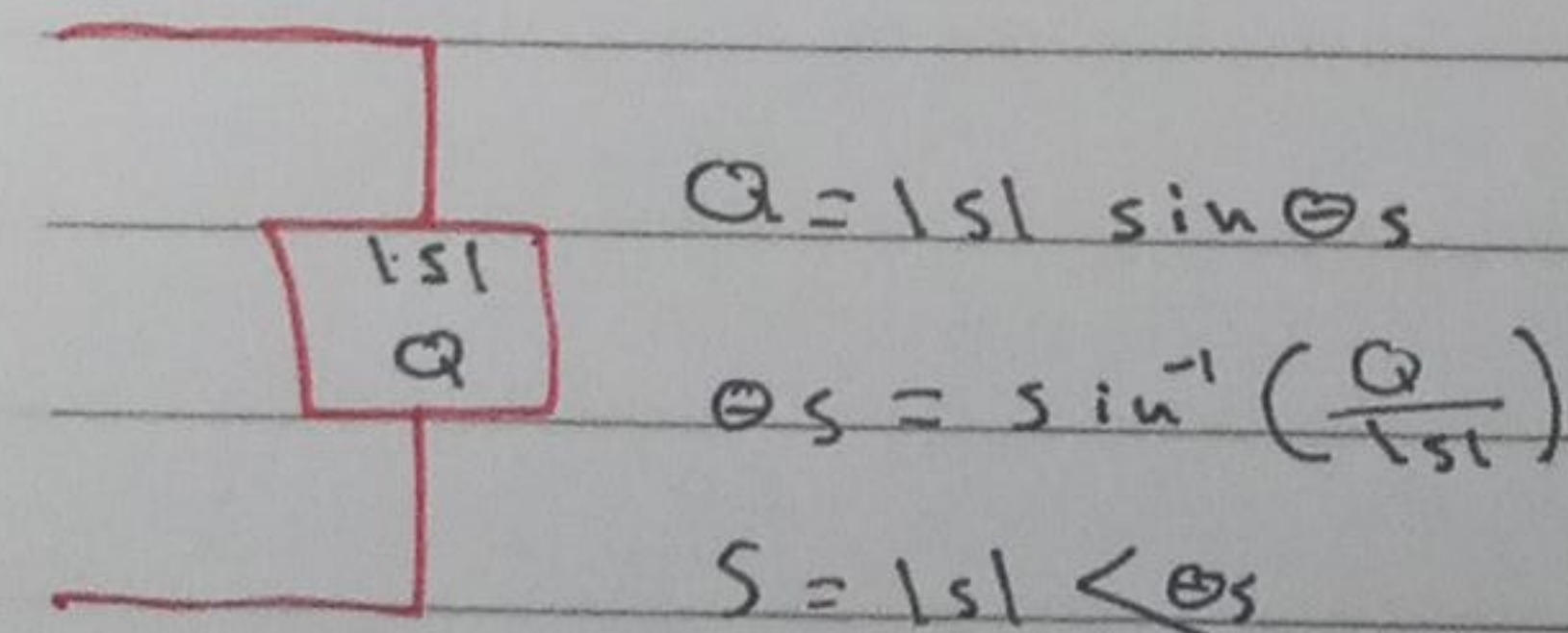
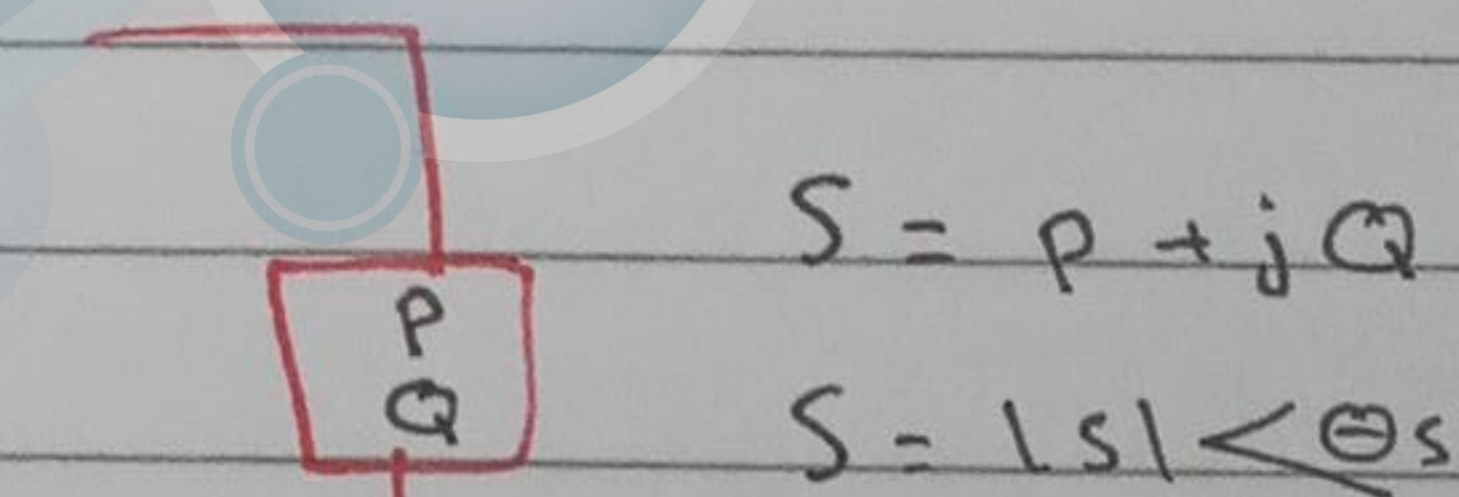
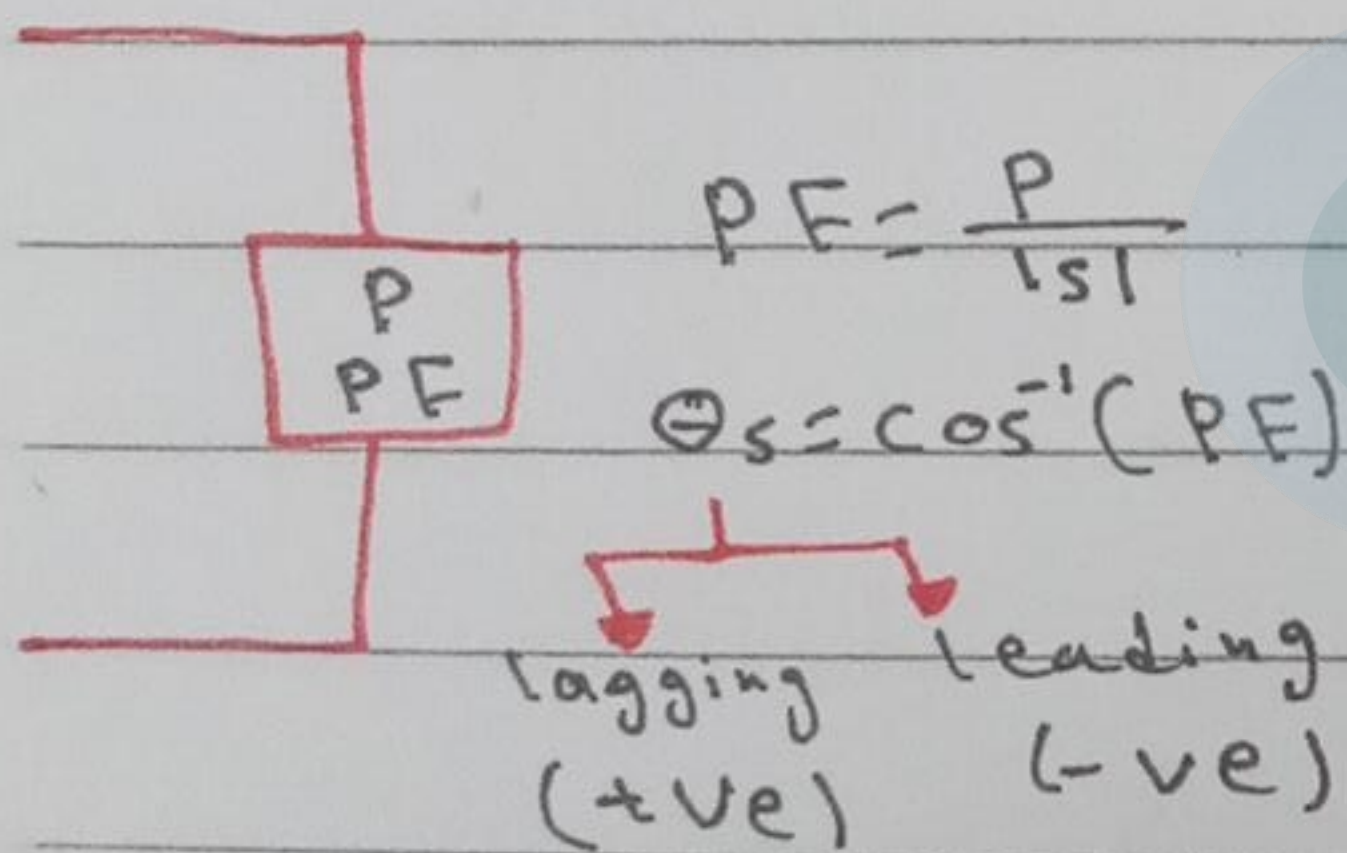
$$Q = |S| \sin \theta_s$$

$$S = P + jQ$$

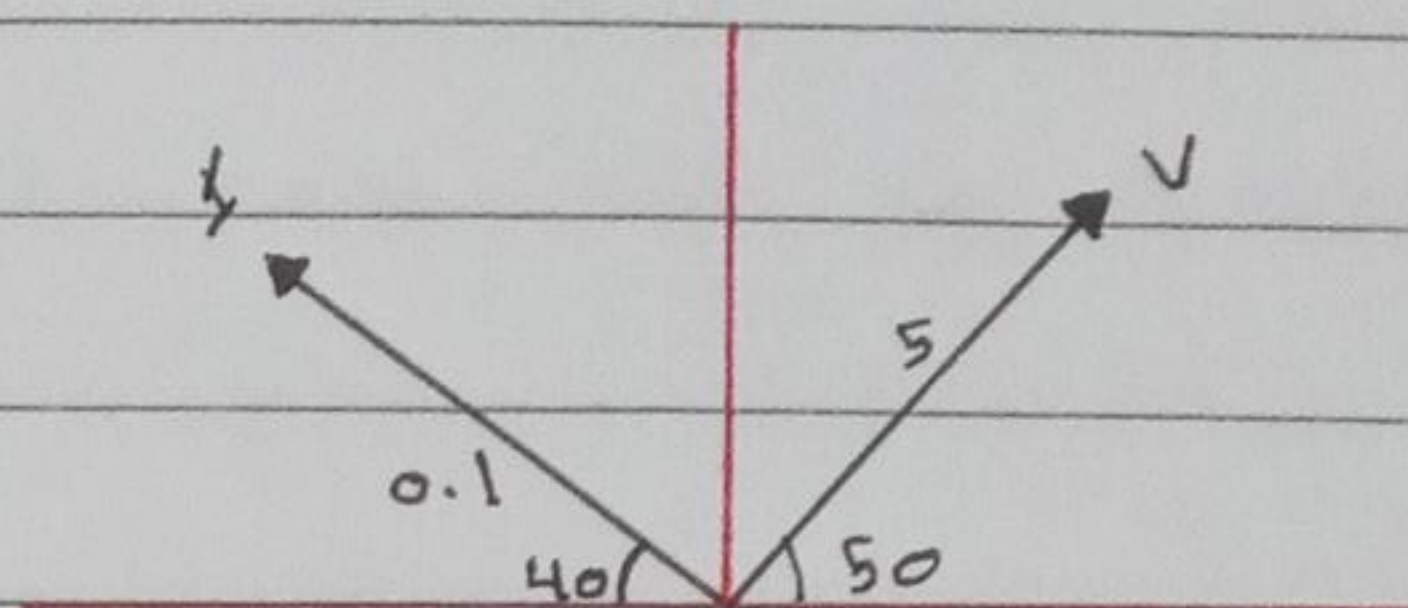
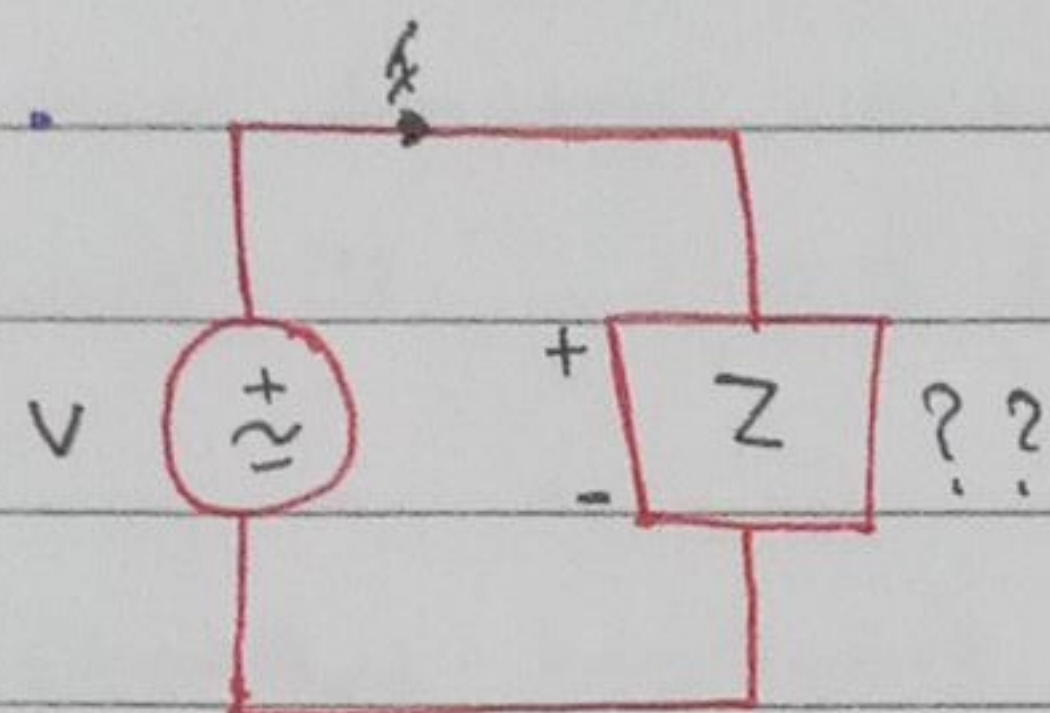
$$PF = \cos(\theta_v - \theta_i) = \frac{P}{|S|} = \cos \theta_s$$



* cases



Ex.



From the graph $\rightarrow V = 5 \angle 50^\circ$, $I = 0.1 \angle 140^\circ$

$$Z = \frac{V}{I} = \frac{5 \angle 50^\circ}{0.1 \angle 140^\circ} = 50 \angle -90^\circ = -j50 \quad (\text{capacitance})$$

$$-j50 = \frac{-j}{\omega C} \quad C = \frac{1}{50\omega}$$

Ex.

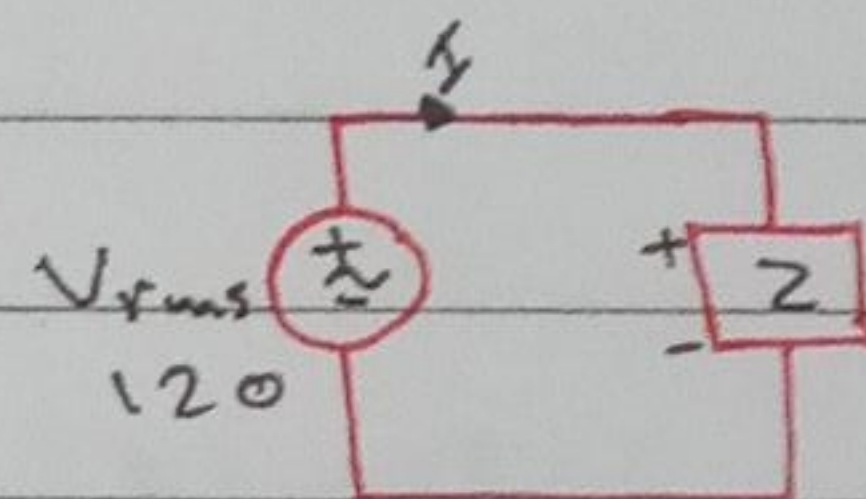
load (Z) draws 12 KVA at PF 0.856 lagging from 120 Vrms source, Find:-

① P_{avg} at ~~load~~

② Q at load

③ peak value

④ Z



$$|S| = 12 \text{ KVA}, \quad |V_{rms}| = 120 \rightarrow |S| = |V_{rms}| |I_{rms}|$$

$$|I_{rms}| = \frac{12 \text{ KVA}}{120} = 100 \text{ A} \quad \text{So, } I_{peak} = 100\sqrt{2}$$

$$\theta_s = \cos^{-1}(0.856) = 31.13^\circ \rightarrow S = 12 \times 10^3 \angle 31.13^\circ \rightarrow \underline{10.2 \text{ KW}} + j \underline{6.2 \text{ KVAR}}$$

$P_{avg} \quad \text{lag} \quad Q$

$$Z = \frac{V_{rms} \angle \theta_v}{I_{rms} \angle \theta_i} = \frac{V_{rms}}{I_{rms}} \angle (\theta_v - \theta_i) \rightarrow \theta_s$$

$$Z = 1.2 \angle 31.13^\circ \quad \text{--- R --- L ---}$$

⊕ power factor correction :-

reduce θ $\rightarrow$ PF $\uparrow$ add capacitor in parallel

$$C = \frac{P(\tan \theta_{old} - \tan \theta_{new})}{\omega |V_{rms}|^2}$$

P $\rightarrow$ real power at supply

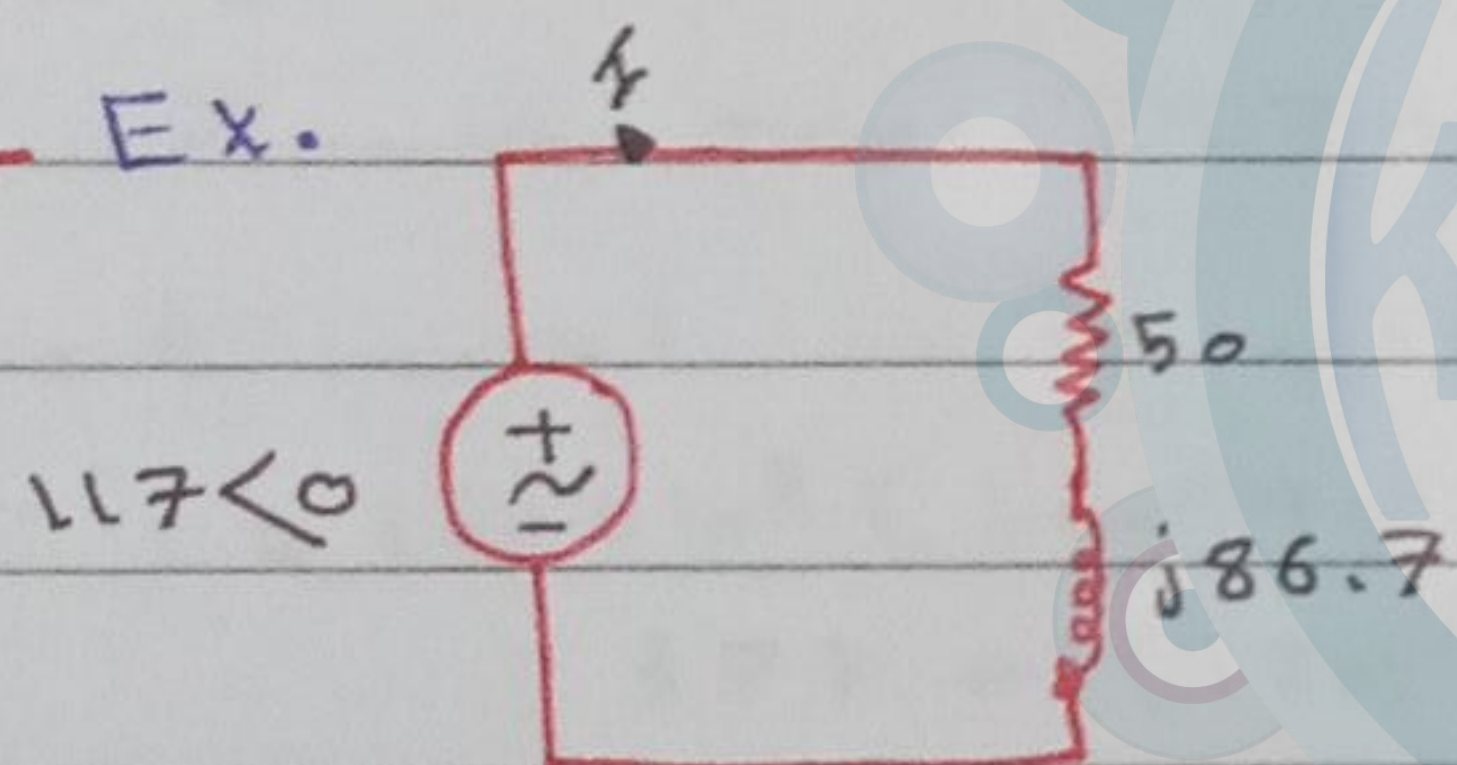
θ_{old} $\rightarrow$ for old PF, $\theta_{old} = \cos^{-1}(PF_{old})$

θ_{new} $\rightarrow$ for new PF, $\theta_{new} = \cos^{-1}(PF_{new})$

ω $\rightarrow$ radian frequency

$|V_{rms}|$ $\rightarrow$ magnitude of the rms voltage at supply

Ex.



assume rms values, $\omega = 377 \text{ rad/s}$

Find:- ① S at source

② C to get $PF = 1$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{117\angle 0}{50 + j86.7} = 1.17\angle -60^\circ \text{ A}$$

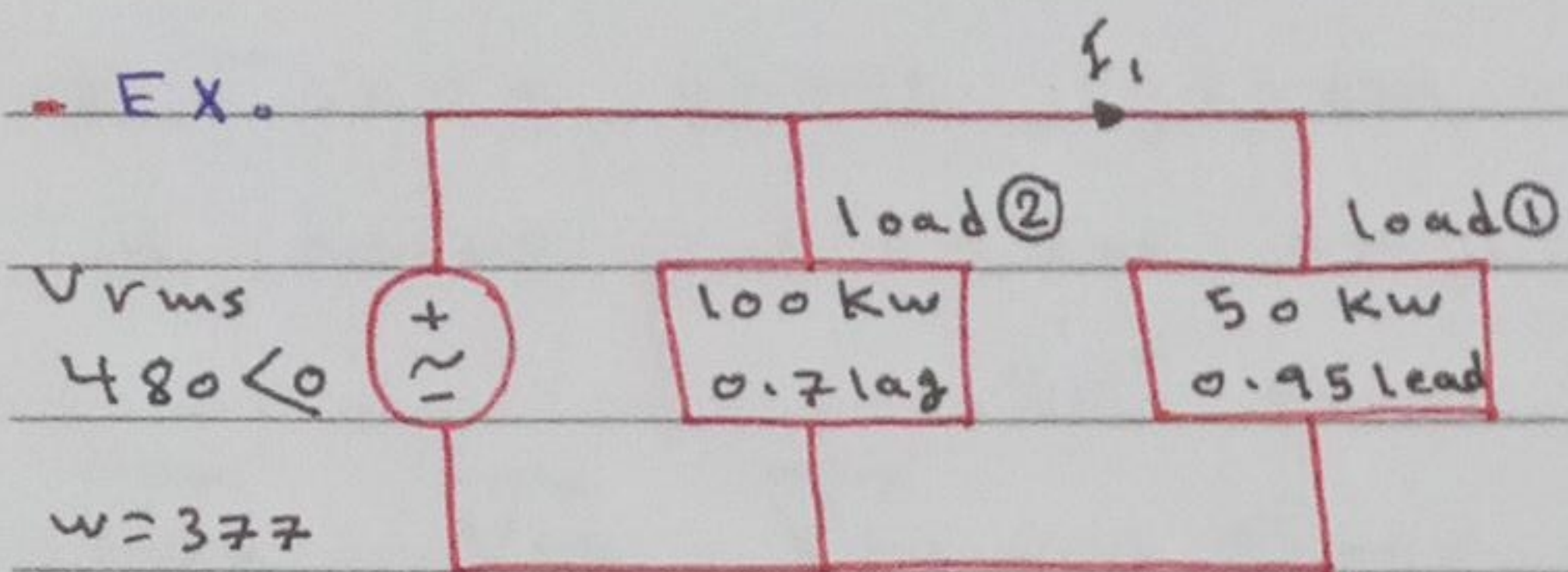
$$S = (117\angle 0)(1.17\angle 60) = 137\angle 60 = \cancel{68.5} \underline{68.5} + j118.5$$

$\downarrow$
 P

$$\theta_{old} = \cos^{-1}(PF_{old}) = \theta_s = 60$$

$$\theta_{new} = \cos^{-1}(1) = 0$$

$$C = \frac{68.5(1.73 - 0)}{377(117)^2} = 2.3 \times 10^{-5} \text{ F}$$



Find:- ① S at source ② PF at source
 ③ C to get PF=1 ④ Z at load ①

at load ①

$$\theta_s = \cos^{-1}(0.95) = 18.19^\circ \text{ (-ve)}$$

$$|S| = \frac{P}{PF} = \frac{50 \text{ kW}}{0.95} = 52.63 \text{ KVA}$$

at load ②

$$\theta_s = \cos^{-1}(0.7) = 45.6^\circ \text{ (+ve)}$$

$$|S| = \frac{P}{PF} = \frac{100 \text{ kW}}{0.7} = 142.86 \text{ KVA}$$

$$S_{\text{source}} = S_{\text{load ①}} + S_{\text{load ②}} = 52.63 \angle -18.19^\circ + 142.86 \angle 45.6^\circ$$

$$= 172.8 \angle 29.74^\circ \text{ KVA} = 150 \text{ kW} + j 85.7 \text{ KVAR}$$

~~PF source = cos(29.74) = 0.87 lag~~

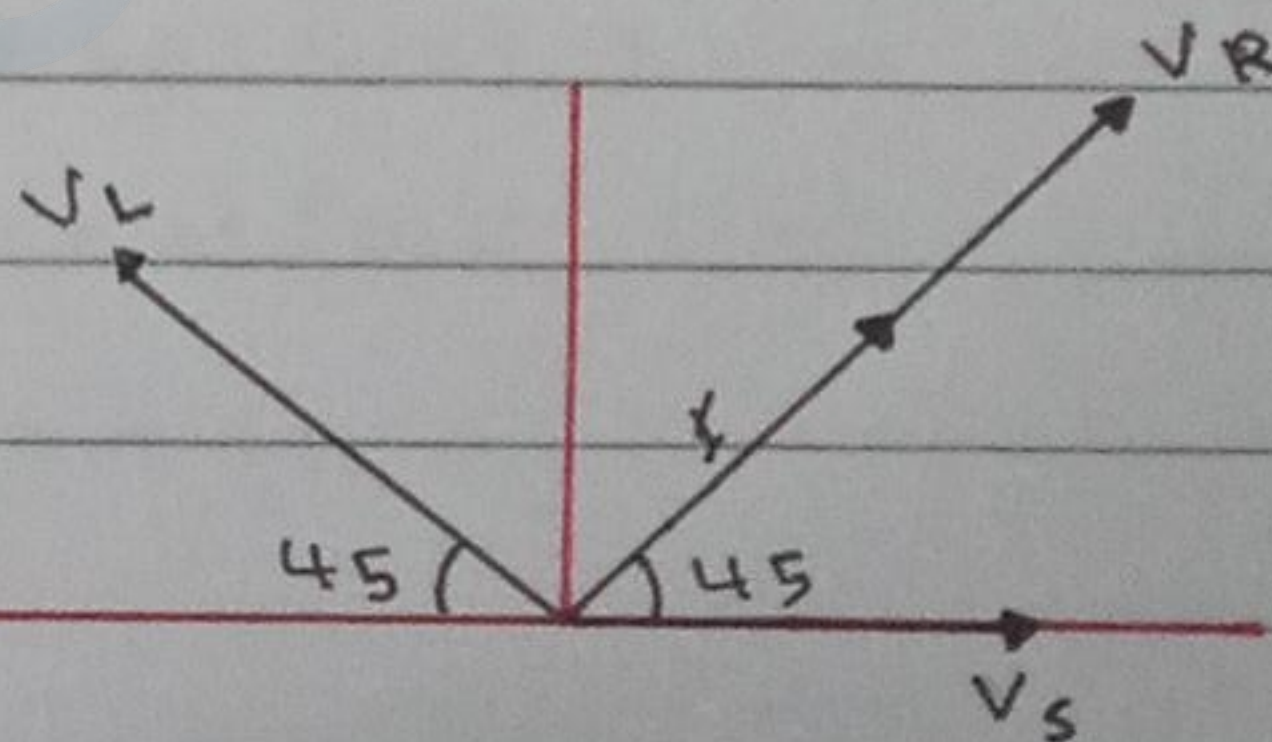
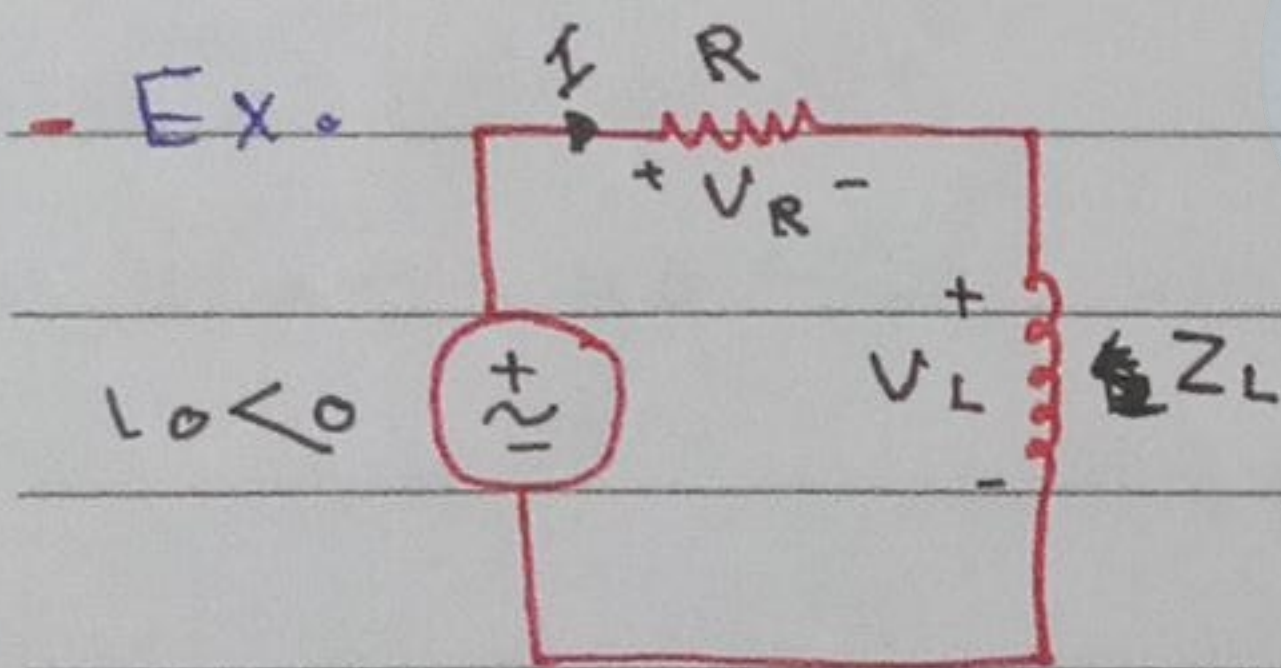
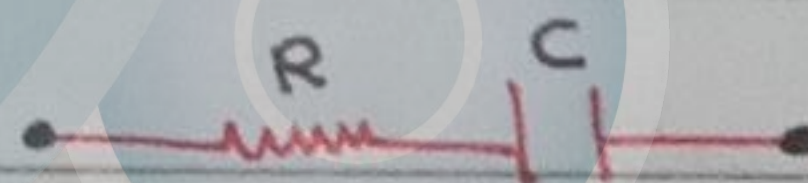
$$PF_{\text{source}} = \cos(29.74) = 0.87 \text{ lag (because } \theta_s \text{ is positive)}$$

$$PF_{\text{new}} = 1 \quad \theta_{\text{new}} = 0$$

$$C = \frac{150 \times 10^3 (\tan(29.74) - \tan(0))}{377 \times (480)^2} = 985 \text{ } \mu\text{F}$$

$$|I_{rms ①}| = \frac{|S|_{①}}{|V_{rms}|} = \frac{52.63 \text{ KVA}}{480} = 109.6 \text{ A}$$

$$Z_{①} = \frac{|V_{rms}| \angle \theta_v - \theta_i}{|I_{rms}|} = 4.37 \angle -18.19^\circ$$



given :-

$$I = 2 \angle 45^\circ \text{ mA}$$

$$V_R = 7.07 \angle 45^\circ \text{ V}$$

$$V_L = 7.07 \angle 135^\circ \text{ V}$$

draw phasor diagram:-

V_L lead V_R by 90°

V_R lead V_s by 45°

V_s lag I by 45°

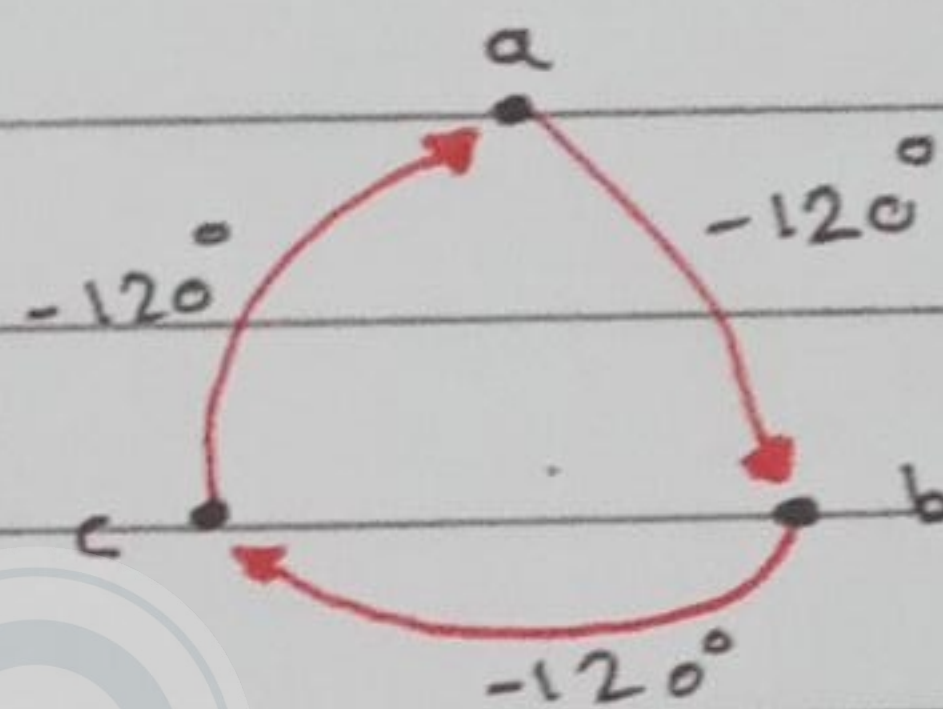
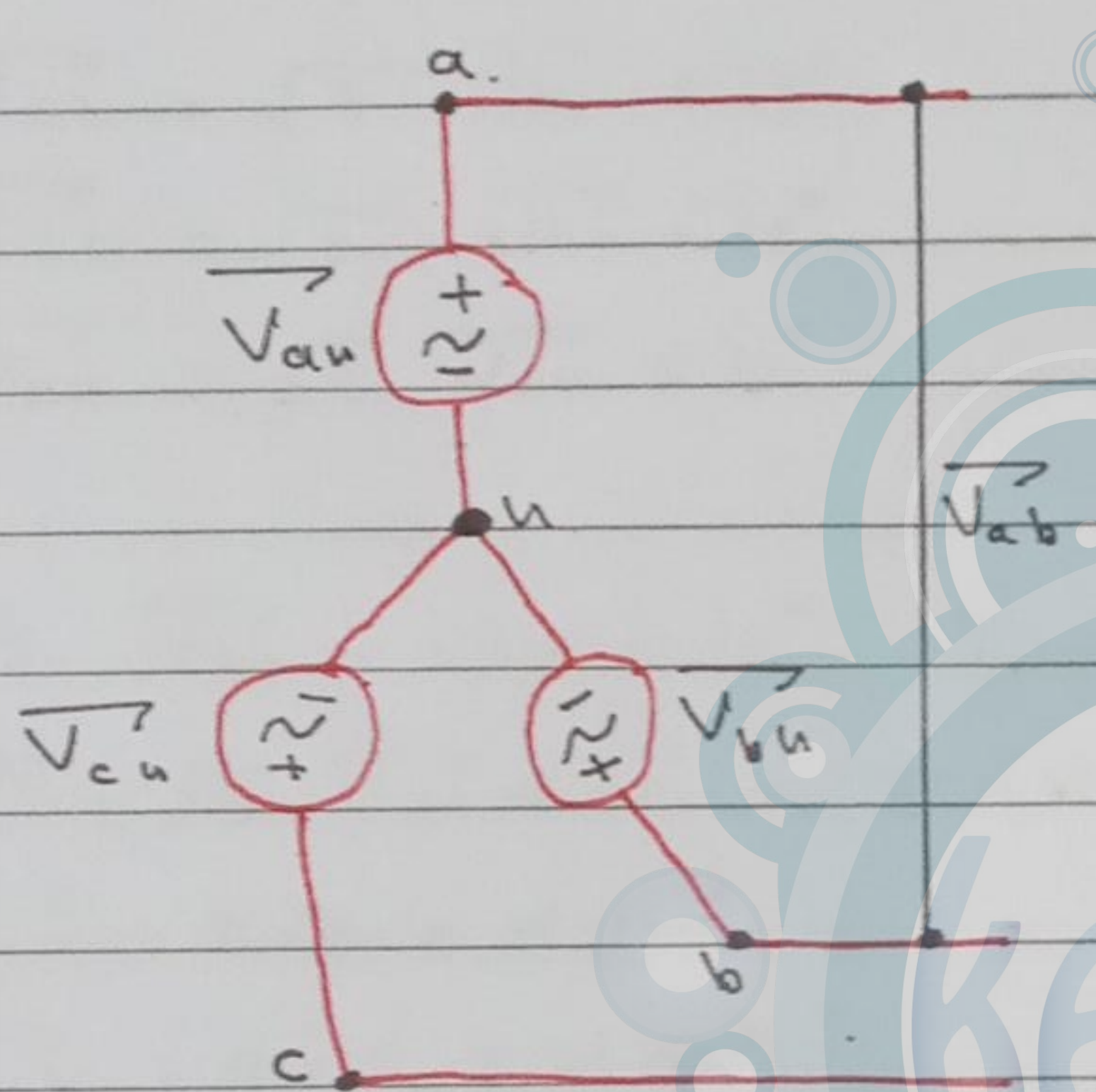
I in phase with V_R

* Three phase system

3 sources equal Amplitude

-120° phase shift

$\vec{V}_{an}$, $\vec{V}_{bn}$, $\vec{V}_{cn}$ → phase voltages



$$\vec{V}_{an} = V_o \angle \theta$$

$$\vec{V}_{bn} = V_o \angle \theta - 120^\circ$$

$$\vec{V}_{cn} = V_o \angle \theta - 240^\circ$$

Ex. if $\vec{V}_{an} = 5 \angle 10^\circ$ find $\vec{V}_{bn}$, $\vec{V}_{cn}$:-

$$\vec{V}_{bn} = 5 \angle -110^\circ$$

$$\vec{V}_{cn} = 5 \angle -230^\circ$$

Note:- $\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$

Ex. if $V_{bn}(t) = 240 \cos(\omega t + 10^\circ)$ find $V_{an}(t)$, $V_{cn}(t)$:-

$$\vec{V}_{bn} = 240 \angle 10^\circ$$

$$\vec{V}_{an} = 240 \angle 130^\circ \rightarrow V_{an}(t) = 240 \cos(\omega t + 130^\circ)$$

$$\vec{V}_{cn} = 240 \angle -110^\circ \rightarrow V_{cn}(t) = 240 \cos(\omega t - 110^\circ)$$

$\vec{V}_{an}, \vec{V}_{bn}, \vec{V}_{cn}$ — phase voltages

$\vec{V}_{ab}, \vec{V}_{bc}, \vec{V}_{ca}$ — line voltages

line voltage = $\sqrt{3}$ * phase voltage $\angle 30^\circ$

$$\vec{V}_{ab} = \sqrt{3} \vec{V}_{an} \angle 30^\circ$$

$$\vec{V}_{bc} = \sqrt{3} \vec{V}_{bn} \angle 30^\circ$$

$$\vec{V}_{ca} = \sqrt{3} \vec{V}_{cn} \angle 30^\circ$$

if $\vec{V}_{bn} = 120 \angle -60^\circ$ find $\vec{V}_{ab}, \vec{V}_{bc}, \vec{V}_{ca}, \vec{V}_{ac} :-$

$$\vec{V}_{bc} = \sqrt{3} \vec{V}_{bn} \angle 30^\circ = \sqrt{3} (120 \angle -60^\circ) \angle 30^\circ = 207.8 \angle -30^\circ$$

$$\vec{V}_{ca} = 207.8 \angle -150^\circ$$

$$\vec{V}_{ab} = 207.8 \angle 90^\circ$$

$$\vec{V}_{ac} = -\vec{V}_{ca} = \vec{V}_{ca} \angle 180^\circ = 207.8 \angle -150^\circ + 180^\circ = 207.8 \angle 30^\circ$$

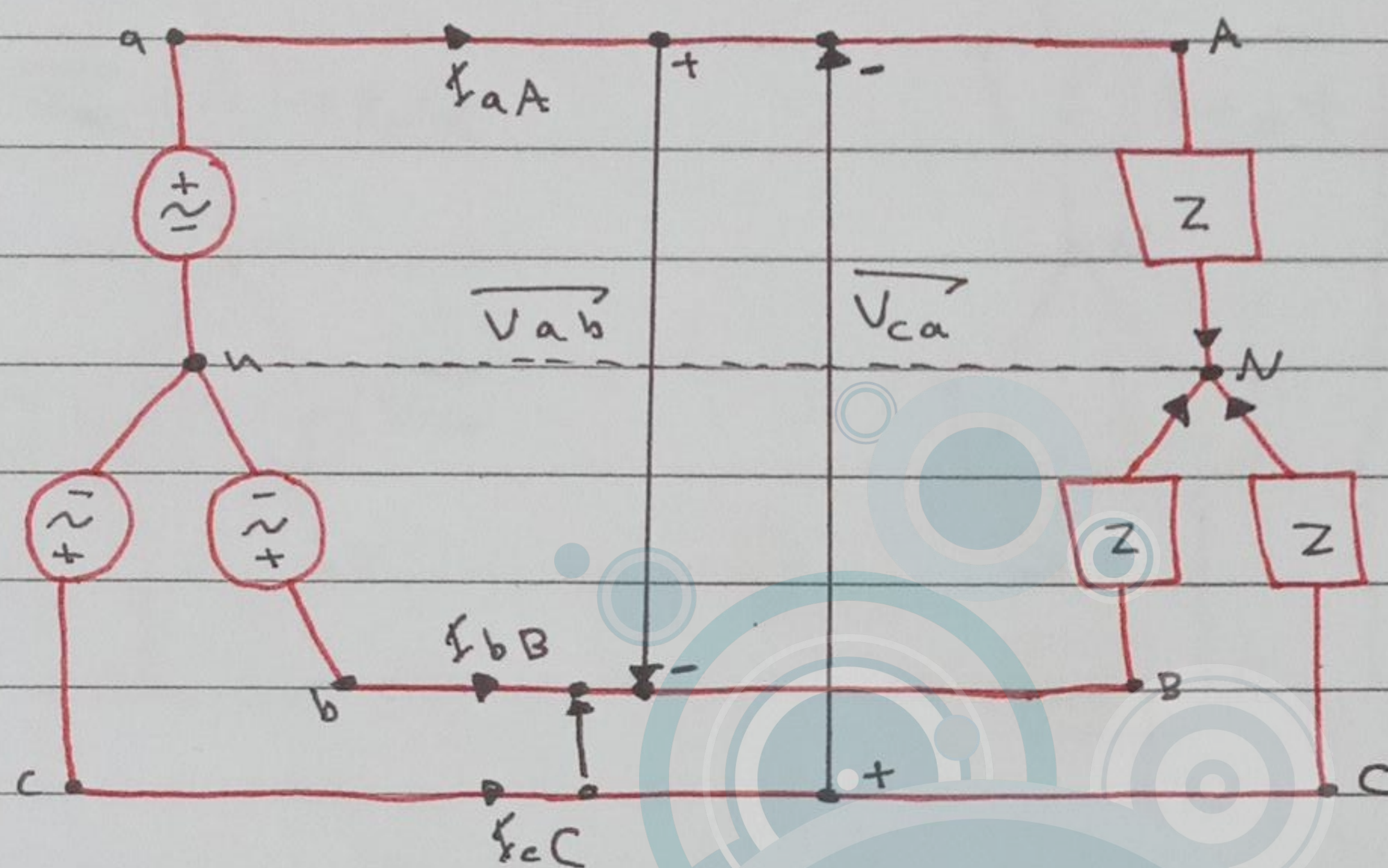
EX. if $\vec{V}_{bc} = 381.1 \angle -90^\circ$ find $\vec{V}_{ab}, \vec{V}_{ca}, \vec{V}_{an}, \vec{V}_{bn}, \vec{V}_{cn} :-$

$$\vec{V}_{ab} = 381.1 \angle 30^\circ \quad \vec{V}_{ca} = 381.1 \angle -210^\circ$$

$$\vec{V}_{bc} = \sqrt{3} \vec{V}_{bn} \angle 30^\circ \rightarrow \vec{V}_{bn} = \frac{\vec{V}_{bc}}{\sqrt{3} \angle 30^\circ} = \frac{381.1 \angle -90^\circ}{\sqrt{3} \angle 30^\circ} = 220.2 \angle -120^\circ$$

$$\vec{V}_{an} = 220.2 \angle 0^\circ \quad \vec{V}_{cn} = 220.2 \angle -240^\circ$$

- Y-Y connection :-



- in Y-Y connect
phase current $\equiv$ line current

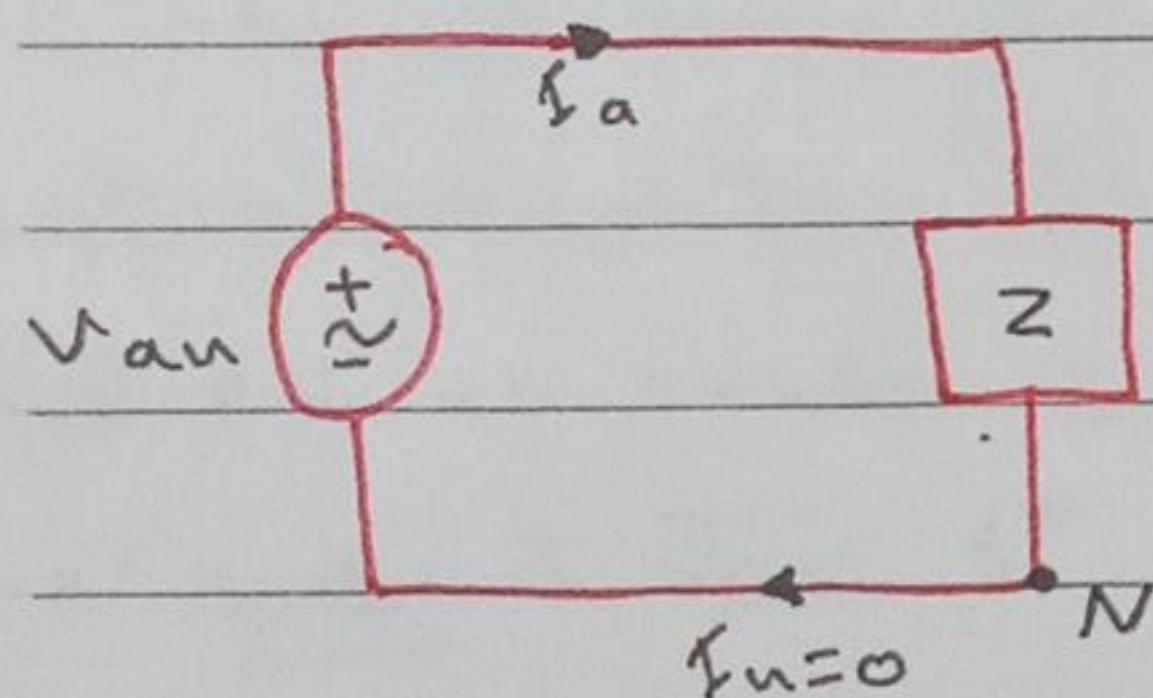
$$\begin{aligned} I_n &= I_a + I_b + I_c \\ &= \frac{\vec{V}_{an}}{Z} + \frac{\vec{V}_{bn}}{Z} + \frac{\vec{V}_{cn}}{Z} \\ &= \frac{1}{Z} (\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn}) \\ &= \frac{1}{Z} \times 0 = 0 \end{aligned}$$

and this is called

Balanced load &
Balanced source
(Balanced system)

- $V_{nN} = 0$, $I_n = 0$

- to solve, we can work per phase:-

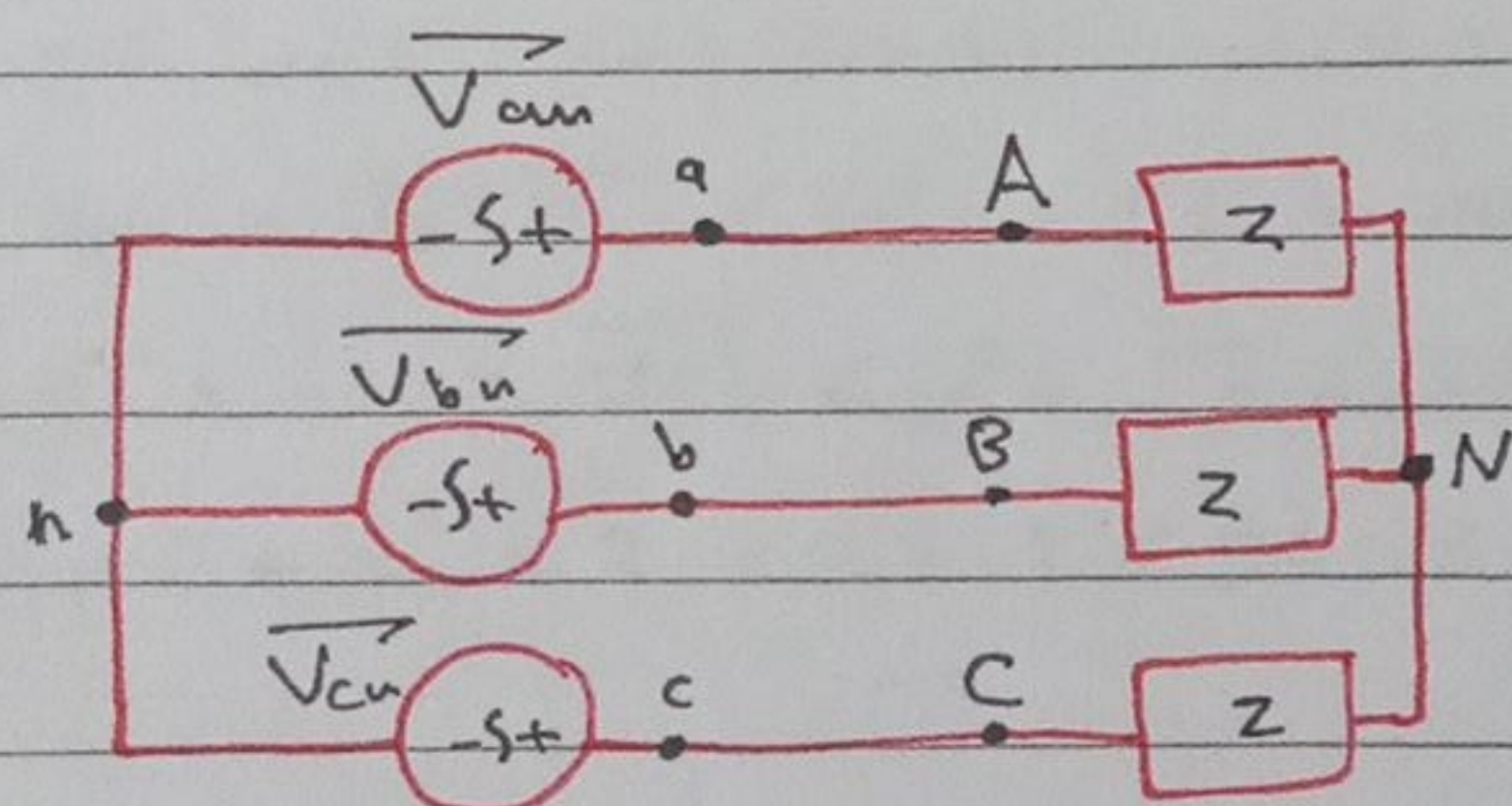


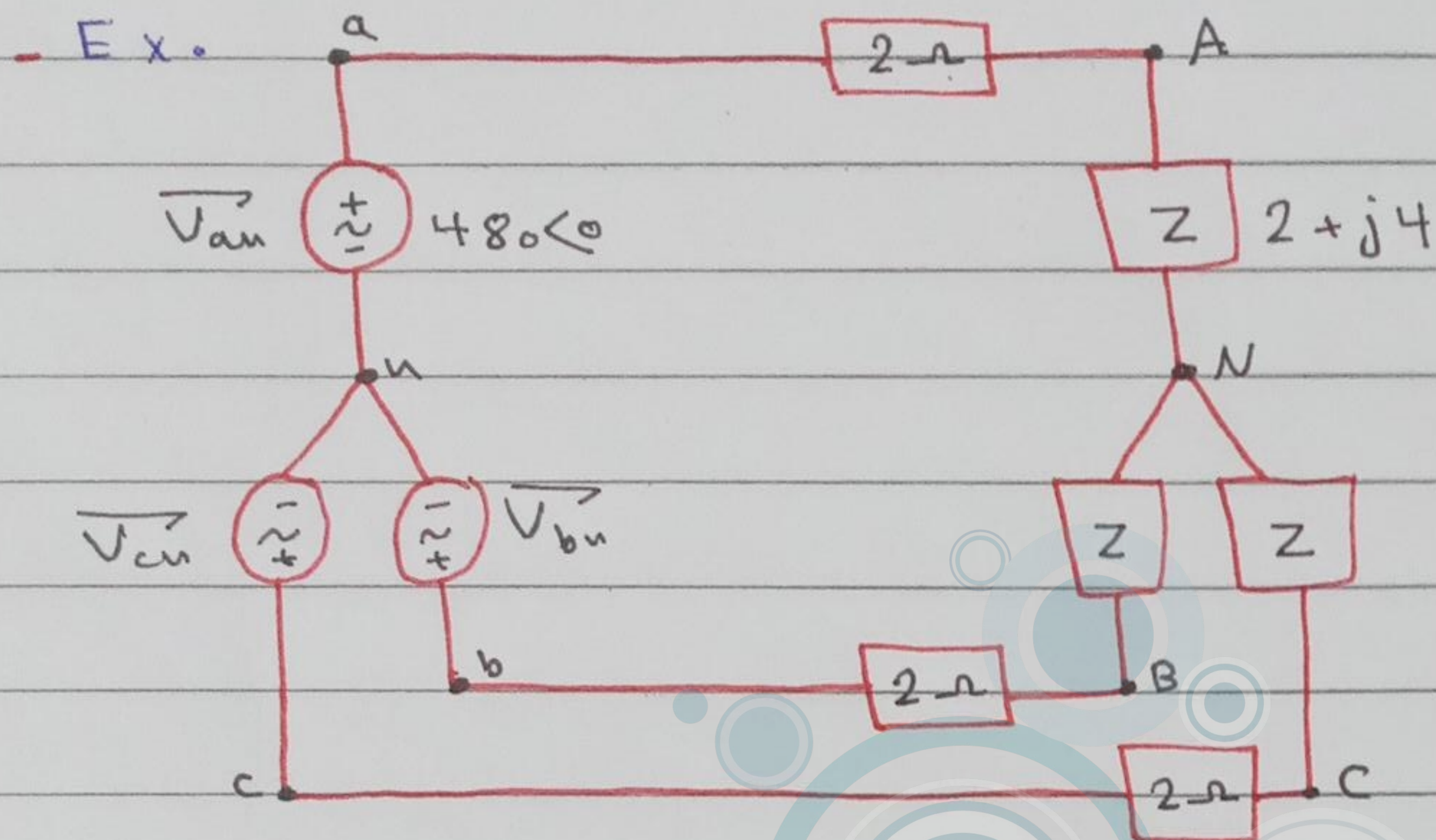
$$I_a = \frac{\vec{V}_{an}}{Z}$$

$$I_b = |I_a| \angle \theta_a - 120^\circ$$

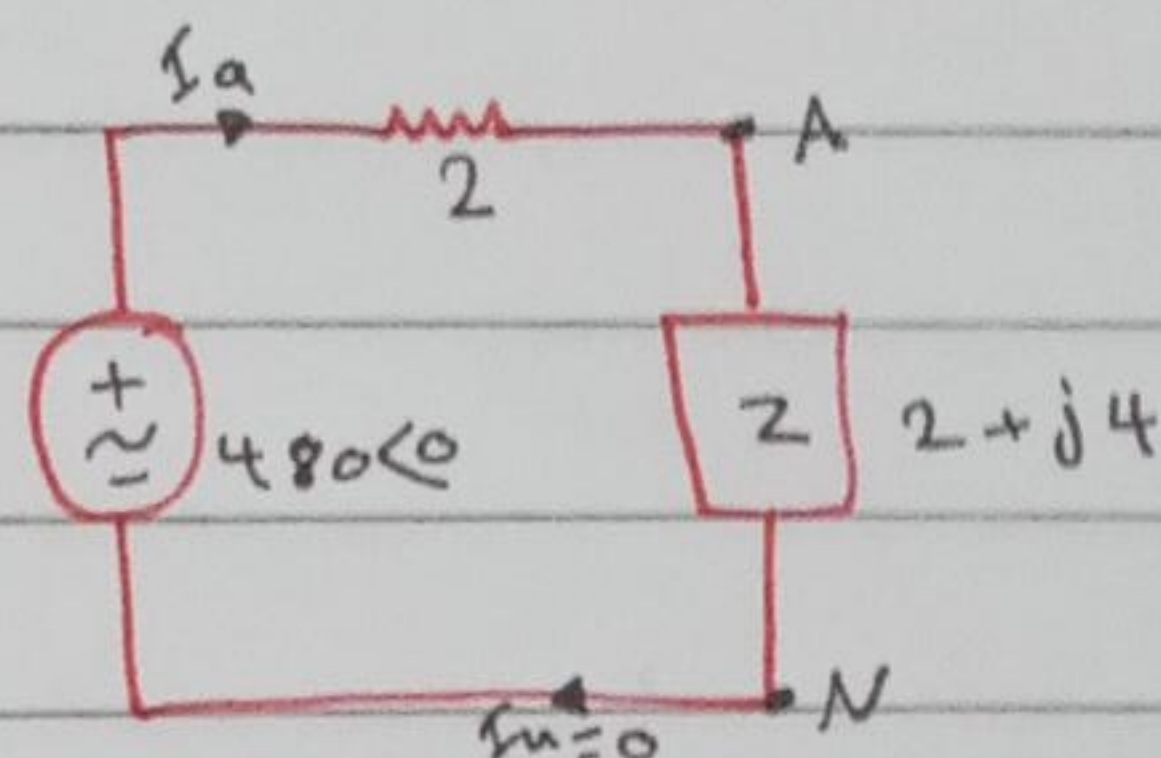
$$I_c = |I_a| \angle \theta_a - 240^\circ$$

- another shape for Y-Y connection:-





Find:- I_a, I_b, I_c



$$I_a = \frac{480 \angle 0^\circ}{4 + j4} = 84.85 \angle -45^\circ$$

$$I_b = 84.85 \angle -165^\circ$$

$$I_c = 84.85 \angle -285^\circ$$

Power in Three phase :-

$$S_{\text{phase}} = V_{\text{rms phase}} * I_{\text{rms phase}}^* \quad S_{3\text{ph}} = 3 V_{\text{rms ph}} * I_{\text{rms ph}}^*$$

Ex. assume rms value in the previous example and find:-

- ① S at loads ② V_{ca}, V_{ea} ③ power losses in wires
- ④ PF at loads ⑤ PF at source ⑥ S at source

• $S_{\text{source}} = 3 V_{an} I_a^* = 3 * 480 \angle 0^\circ * 84.85 \angle 45^\circ = 122.2 \angle 45^\circ \text{ KVA}$

• PF source = $\cos(45) = 0.707$

• $S_{\text{load}} = 3 V_{AN} I_a^* = 3 * 379.46 \angle 18.4^\circ * 84.85 \angle 45^\circ = 965.9 \angle 63.4^\circ \text{ KVA}$

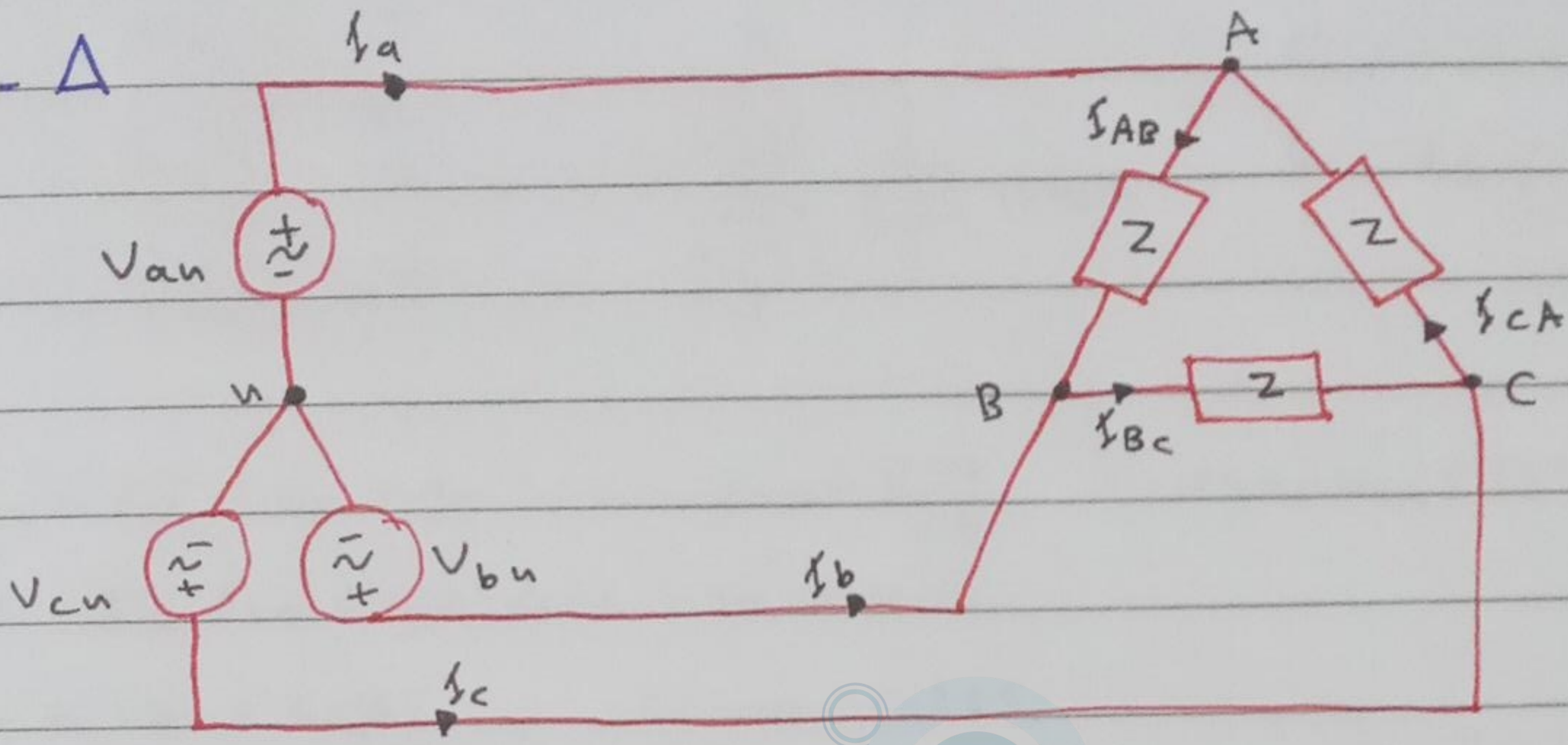
• PF load = $\cos(63.4) = 0.447$

• $V_{ca} = \sqrt{3} V_{cn} \angle 30^\circ = \sqrt{3} * 480 \angle -240^\circ * \angle 30^\circ = 831.38 \angle -210^\circ$

$V_{CA} = \sqrt{3} V_{CN} \angle 30^\circ = \sqrt{3} * 379.46 \angle 18.4 - 240^\circ * \angle 30^\circ = 657.24 \angle -191.6^\circ$

• $P = 3 * I_a^2 * 2 = 43.2 \angle -90^\circ \text{ Kw}$

- Y-Δ



Notice the direction of I !!

• if no TL impedance:-

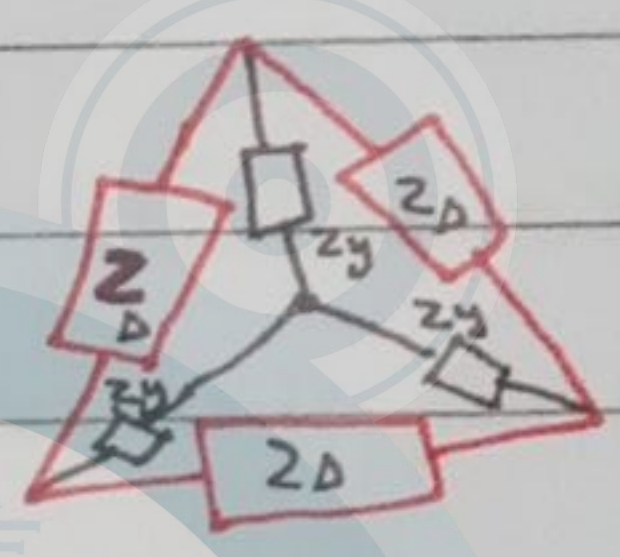
$$V_{AB} = V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$

$$S_{3ph} = \sqrt{3} V_{line} I_{line}^*$$

• if there is TL impedance:-
convert $\Delta \rightarrow Y$



$$Z_Y = \frac{Z_\Delta}{3}$$

Then solve per phase

- Ex. in previous figure, assume $V_{an} = 100 \angle 10^\circ$, $Z_\Delta = 8 + j4 \Omega$:-

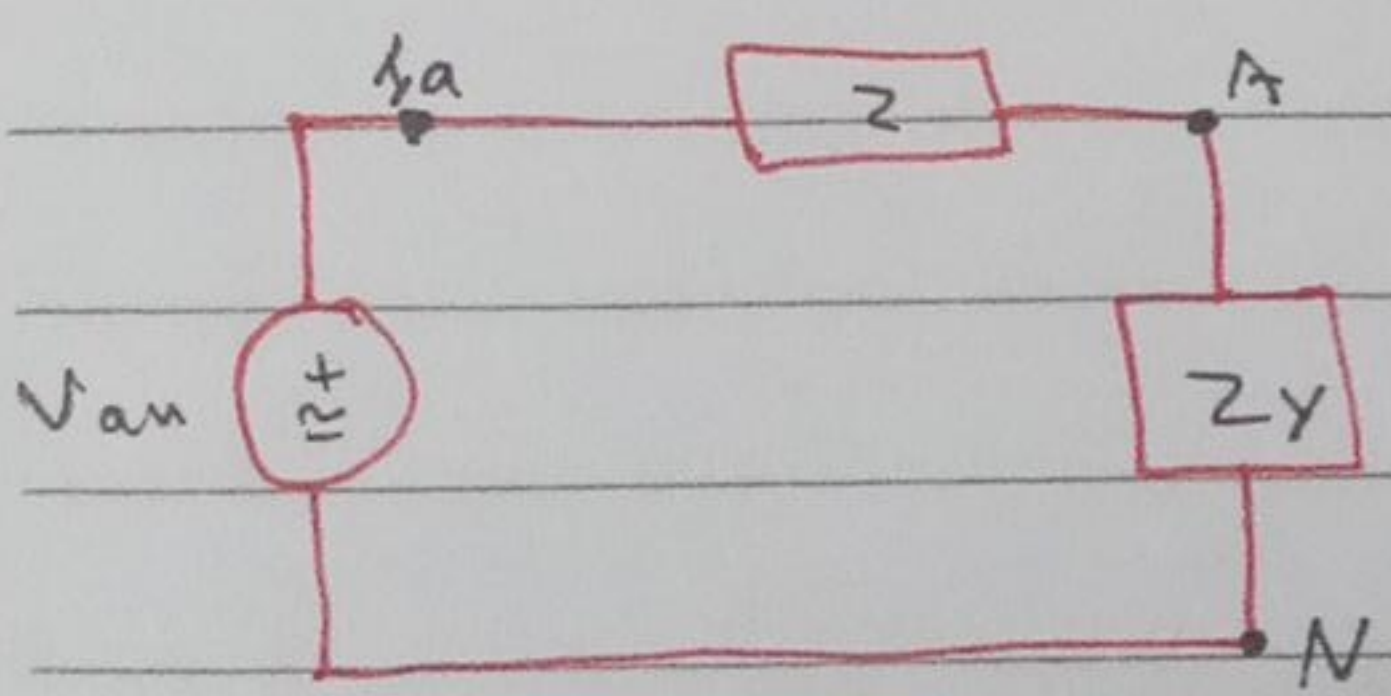
$$V_{AB} = V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = \sqrt{3} (100 \angle 10^\circ) \angle 30^\circ = 173.2 \angle 40^\circ$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = 19.5 \angle 13.4^\circ \rightarrow V_{BC} = 19.5 \angle -106.6^\circ, V_{CA} = 19.5 \angle -226.6^\circ$$

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ = 33.8 \angle -16.6^\circ \rightarrow I_b = 33.8 \angle -136.6^\circ, I_c = 33.8 \angle -256.6^\circ$$

- Note: you can convert Δ to Y in both cases about TL impedance.

- when we convert to Y-Y connection:-



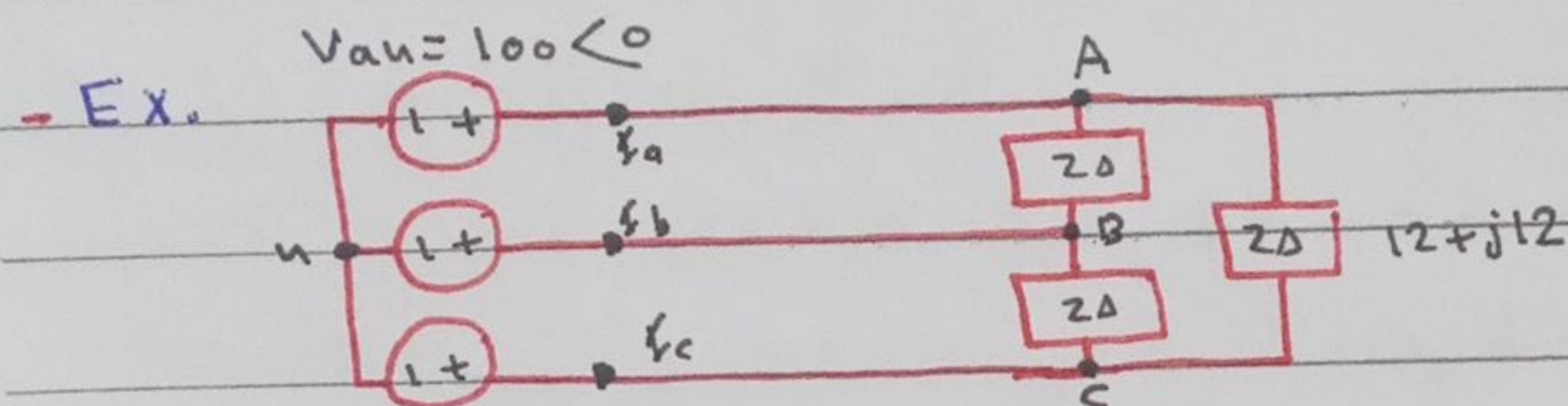
$$S_{load} = V_{AN} I_a^*$$

$$S_{total load} = 3 V_{AN} I_a^* = 3 |I_a|^2 Z_Y$$

$$S_{source} = V_{an} I_a^*$$

$$S_{source total} = 3 V_{an} I_a^*$$

$$S_{total} = 3 V_{phase rms} I_{phase rms}^*$$



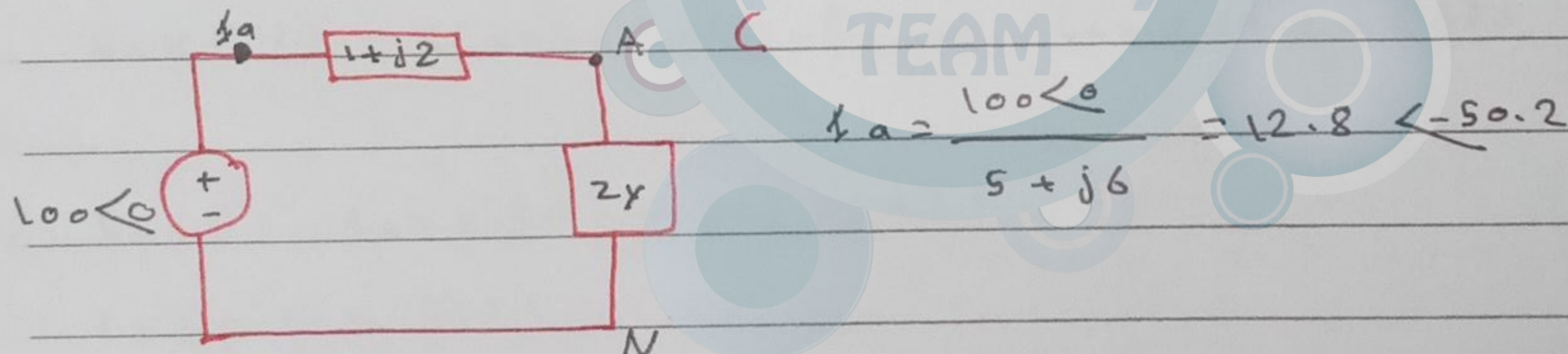
Find line currents:-

I_a, I_b, I_c

- $I_a = \sqrt{3} I_{AB} \angle -30^\circ \rightarrow I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$
 $V_{AB} = V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = 173.2 \angle 30^\circ$
 $= \sqrt{3} (10 \angle -15^\circ) \angle -30^\circ = 17.7 \angle -45^\circ$
- $I_b = 17.7 \angle -165^\circ, I_c = 17.7 \angle -285^\circ$

- Note :- $S_{load} = 3 (V_{AB} \times I_{AB}^*) = P_{3ph} + j Q_{3ph}$

convert the previous example to Y-Y $\rightarrow Z_Y = \frac{Z_{\Delta}}{3} = 4 + j4$
 now work per phase:- (assuming there is $1 + j2$ TL impedance)



- $S_{load} = 3 V_{AN} \times I_a^* \rightarrow V_{AN} = 72.4 \angle -5.2^\circ$
 $= 3 (72.4 \angle -5.2^\circ) (12.8 \angle 50.2^\circ) = 2780.2 \angle 45^\circ \text{ VA}$

- $PF = \cos(\theta_s) = 0.707$

- Ex. 3 ph system Y- Δ

load draws 30 KW at PF = 0.6 lagging, the line voltage = 240 KV rms at 60 HZ freq, ($V_{AB} = V_{ab}$ since no T.L impedance)

Find:- ① S_{load} , P_{load} , Q_{load} , PF load

② line currents

③ Z_{load}

$$0.6 = \frac{P}{|S|} = \frac{30 \text{ K}}{|S|} \rightarrow |S| = 50 \text{ KVA} \quad \theta_s = \cos^{-1}(0.6) = 53.1$$

$$S = 50 \angle 53.1 \text{ KVA} = \underbrace{30 \text{ K}}_{P_{load} \text{ 3 ph}} + j \underbrace{40 \text{ K}}_{Q_{load} \text{ 3 ph}}$$

$$S_{3ph} = 3 V_{phase} \cdot I_{phase}^* = 3 V_{AB} I_{AB}^*$$

$$50 \text{ K} \angle 53.1 = 3(240 \text{ K} \angle 0) I_{AB}^* \rightarrow I_{AB}^* = 69.4 \text{ mA} \angle -53.1 \quad I_{AB} = 69.4 \text{ mA} \angle -53.1$$

$$I_a = \sqrt{3} I_{AB} \angle -30 = 0.12 \angle -83.1$$

$$I_b = 0.12 \angle -123.1$$

$$I_c = 0.12 \angle -173.1$$

$$Z = \frac{V_{AB}}{I_{AB}} = 3458.213 \text{ K} \angle 53.1$$