Weekly Notebook

Electrical

First Semester 2024

INTRO TO ELECTRICAL ENG.

units :-

- Time → sec
- Electrical current → Amper (A)
- Voltage → volt
- mass → Kg
- Length → m
- power → watt

prefixes :-

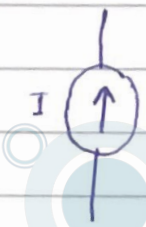
- milli → 10^{-3}
- micro → 10^{-6}
- nano → 10^{-9}
- pico → 10^{-12}
- Kilo → 10^{3}
- mega → 10^{6}
- giga → 10^{9}

power supply

- types
  - Voltage supply
  - current supply
- output signal
  - DC o/p signal
    - value of the supply is fixed with time
    - Ex: V = 3 volt, A = 2 A (constant)
  - AC o/p signal
    - It's value vary with time
    - Ex: $v(t) = 3\sin(4t)$, $i(t) = 2e^{-t}$ (vary)
- Dependency
  - Independent power supply
    - don't depend on any element in the circuit
    - symbol (Next page) →
  - dependent
    - It's value depend on or func of voltage or current

symbol of independent :-

independent voltage DC source



independent DC current source

independent AC voltage source

independent AC current source

Symbol of dependent :-

dep. DC voltage source

dep. DC current source

dep. AC voltage source

dep. AC current source

Loads :-

- Resistance — R
- inductive — L
- capacitive — C

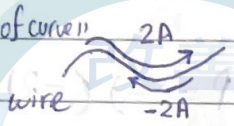
charge :- $\rightarrow Q$

$$q_e = 1.6 \times 10^{-19} \text{ C}$$

Electrical current :- $\rightarrow I$

charges in motion in a wire for a closed loop.

$$I = \frac{dQ}{dt} \quad \text{"slope of curve"}$$



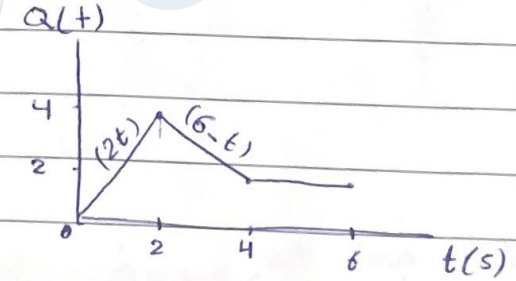
$$Q(t) = \int_{t_0}^{t_f} i(\tau)\, d\tau + \text{initial condition} \quad \text{("Area under the curve")}$$

Ex:- $Q(t) = 3e^{-t}$ C , find $i(t)$

Sol $i(t) = \frac{dQ}{dt} = 3(-1)e^{-t} \Rightarrow i(t) = -3e^{-t}$ A

Q Find $i(t)$?

$$i(t) = \frac{dQ}{dt}$$



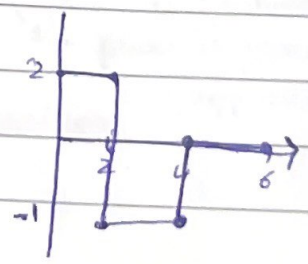
① for $0 \le t \le 2$

$Q(t) = 2t$

$i(t) = 2$ A

② for $2 \le t \le 4$ $\quad Q(t) = 6 - t$, $\quad i(t) = -1$ A

③ for $4 \le t \le 6$ $\quad Q(t) = 2$ c, $\quad i(t) = 0$ A

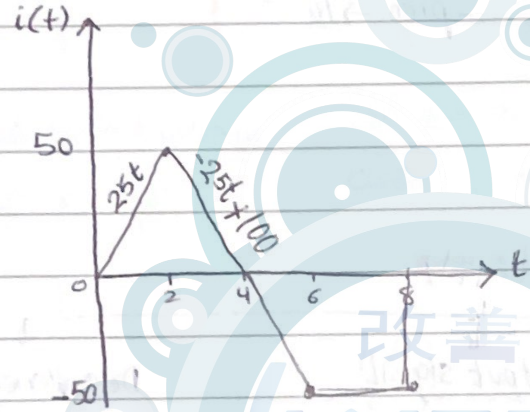


Find the :-

a) Total amount of charge $Q(t)$

b) Total amount of charge up to $t = 6$ sec

c) Total amount of charge between $t = 2$ sec to $t = 8$ sec



a) ⇓

$$Q(t) = \int_{t_0}^{t_f} i(\tau)\, d\tau + \text{initial}$$

① for $0 \le t \le 2$ $\quad i(t) = 25t$, initial $= 0$

$$Q = \int_0^t 25\tau \, d\tau$$

$$\frac{25\tau^2}{2}\Big|_0^t = \boxed{12.5\, t^2}$$

② for $2 \le t \le 6$

$i(t) = -25t + 100$, initial $= 2$

مشان اوجد ال initial cond. لازم اعوض قيمة ال 2 بالعلاقة بتعتي $12.5t^2$

$$Q(t) = \int_2^t (-25t + 100) + 50 \qquad = 12.5(2)^2 = 50 \text{ C}$$

$$= -\frac{25t^2}{2} + 100t \Big|_2^t + 50$$

$$(-12.5t^2 + 100t) - (-50 + 200) + 50$$

$$Q(t) = -12.5t^2 + 100t - 100$$

بعوض 6

③ for $6 \le t \le 8$ $\Rightarrow \int_6^t (-50)\, d\tau + Q(6) \Rightarrow Q(t) = -50(t-6) + 50$

b) amount of charge up to 6 sec

$Q = Q(6) - Q(0)$

$= 50 - 0 = 50\,c$

c) Total amount of charge between 2 sec to 8 sec

$Q(8) - Q(2)$

$-50 - 50 = -100\,c$

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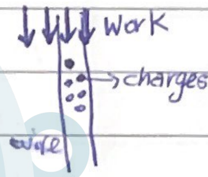
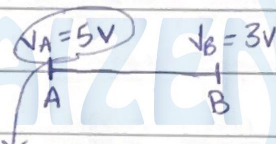
voltage :-

work needed to move unit charge in a wire

$V = \frac{work}{charge} = J/C = volt$



• voltage at ground = zero



Voltage at A related to the ground

$V_{AB} = V_A - V_B$ (refrence, related)

$= 5 - 3 = 2$ volt

$V_{BA} = V_B - V_A$

$= 3 - 5 = -2$ volt

} $V_{AB} = -V_{BA}$

power :-

work needed per unit time

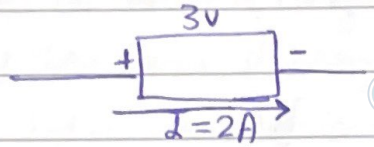
$P = \frac{work}{Time} = J/s = Watt$

$P = V * I$

if sign of power → + Power is dissipated (consumed) يعني تمّ استهلاكها
→ − power is generated (supplied) تمّ انتاجها

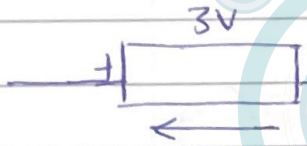
Power (+) + ← passive sign convention
Power (−) − ←

Examples-



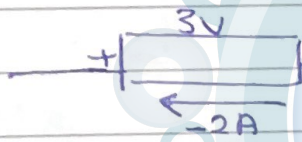
$P = (+)(2)(3V) = 6\,W$ consumed

لانه التيار خارج من السالب — القيمة مع اشارتها

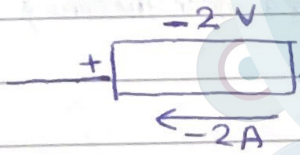


$P = (-)(2)(3V) = -6$ supplied

لانه التيار يمر من السالب الى الموجب

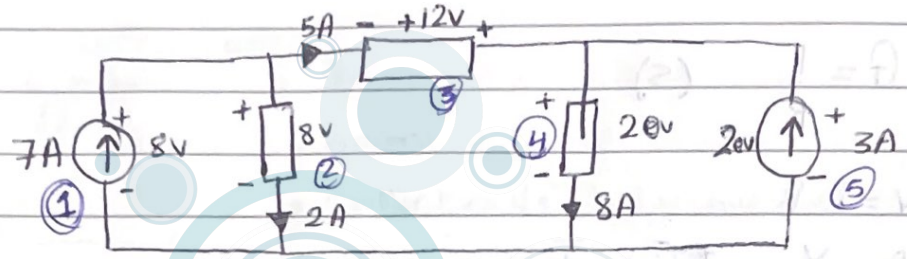


$P = (-)(-2)(3V) = +6$ consumed



$P = (-)(-2)(-2) = -4$ supplied

Example 8-



find power for each element

$P_1 = (-1)(7)(8) = -56$ watt (generated)

$P_2 = (+)(8)(2) = +16$ watt (consumed)

$P_3 = (-)(5)(12) = -60$ watt (generated)

$P_4 = (+)(20)(8) = +160$ watt (consumed)

$P_5 = (-)(2)(3) = -60$ watt (generated)

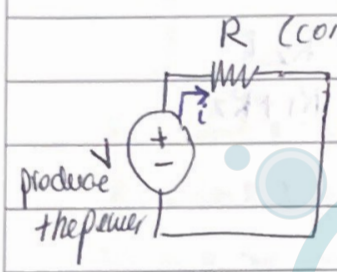
$\Sigma P_{consumed} = \Sigma P_{generated}$

$\Sigma P_{cons} = 16 + 160 = 176$ W

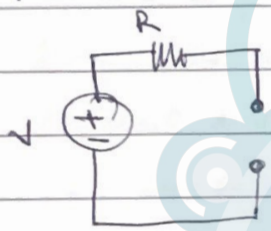
$\Sigma P_{generated} = 56 + 60 + 60 = 176$ W

Chapter 2 : Basic Laws

Basic concepts

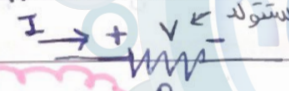


- current will pass
- current exits from the positive pole



$I = 0 \Rightarrow$ open circle

Ohm's Law

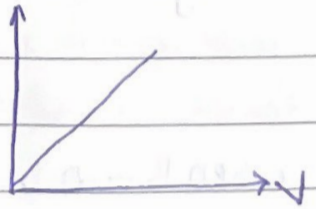


$V = IR$

for any conductor, If the current pass produce a voltage drop across this conductor

And so important to note that the current exits from (+)

$$P = I \cdot V$$
$$P = I \cdot I \cdot R = I^2 R$$
$$P = \frac{V}{R} \cdot V = V^2/R$$



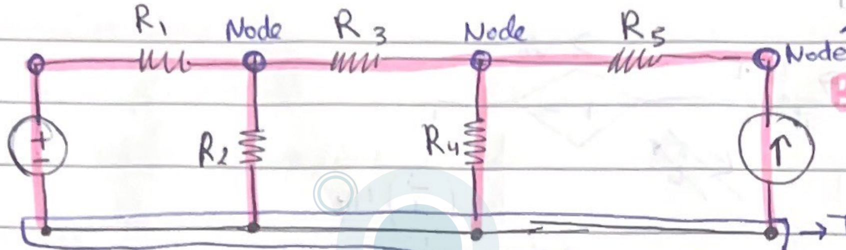
conductance (G)

$G = \frac{1}{R}$ (S)

$V = I \cdot R$

$R = \frac{V}{I}$ / $G = \frac{I}{V}$

نعتبرو node بحيث حسبنا 2 Branch بس اذا حسبناها One Branch It's Not node



Element: could be a current, voltage or resistance (7)

Branch: represent a single element or more.

Node: the point of connection between two or more Branches (3)

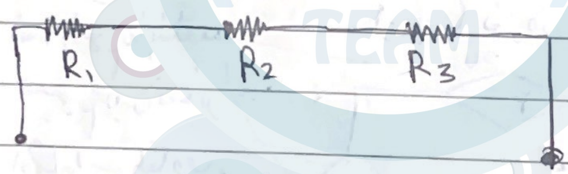
Loop: any closed Path (6)

Mesh: any closed Path but does not exist any closed path

- every mesh is a loop

NOTE • if the connection between nods has no element → considered 1 node

Series connection of Resistors:-

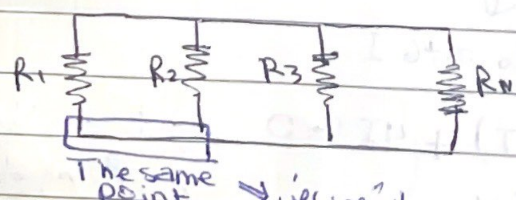


voltage across each resistor $V_1 = \frac{R_1 V}{R_1 + R_2}$

$R_{Total} = R_1 + R_2 + R_3$

- in series connection ⇒ same current pass through them
- in series connection ⇒ voltage is different for each element

parallel connection of Resistors:-



$$\frac{1}{R_{Total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$$

→ بيلمسوا بعض وبنقوو مع بعض

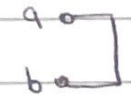
$$R_{Tot} = \frac{R_1 R_2}{R_1 + R_2}$$ special case for 2 Resistors

- in parallel connection ⇒ current divided between branches
- in parallel connection ⇒ Voltage is the same.

current across each resistor ⇒ $i_1 = \frac{R_2 i}{R_1 + R_2}$

Idealization of Resistance :-

① short circuit ⇒ R = zero



using ohm's Law $V = I * R$

$V = (I) * (zero) = zero$

when R = 0, V = 0 regardless to current passing through.

② open circuit ⇒ R = ∞

using ohm's Law $V = I * R$

$I = \frac{V}{R} = \frac{V}{\infty} = 0\ A$

when R = ∞, I = 0 regardless to voltage across it

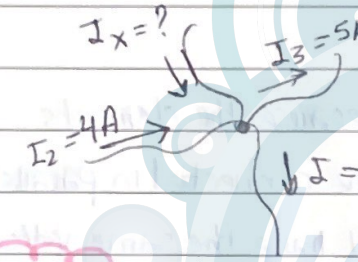
Kirchhoff's Law

- Kirchhoff's current Law KCL
- Kirchhoff's voltage law KVL

KCL : for any node $\Sigma I = zero$

$$\Sigma I_{in} = \Sigma I_{out}$$

example ⇓

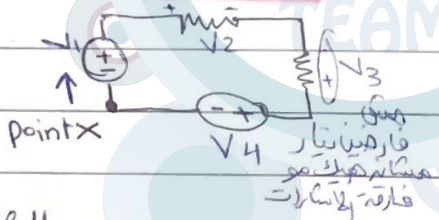


To find I_x :

$$I_x + I_2 = I_3 + I_1$$
$$I_x + 4 = 3 + 5$$
$$I_x = 4\,A$$

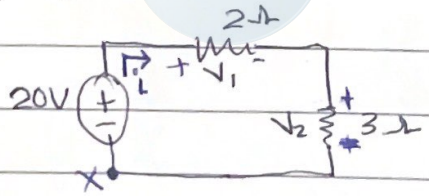
KVL : for any loop (closed path)

$$\Sigma V = 0$$



$$-V_1 + V_2 - V_3 + V_4 = 0$$

example ⇓



find I , V_1, V_2 = ?

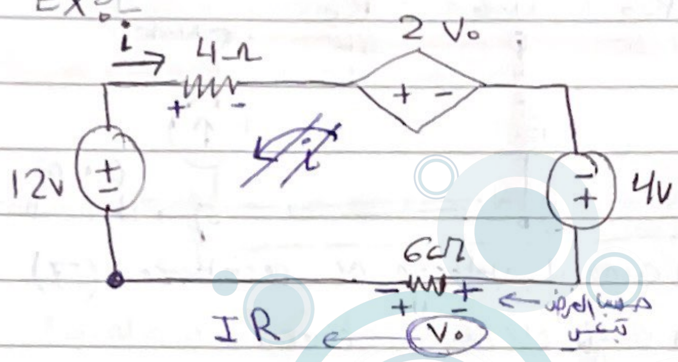
1- Assign the current يخرج من القطب الموجب

2- Assign the polarity of each voltage

3- Apply KVL $\Sigma V = 0$

$$-20 + V_1 + V_2 = 0$$
$$V_1 = IR = I2$$
$$V_2 = 3I$$
$$-20 + 2I + 3I = 0 \Rightarrow I = 4A,\ V_1 = 8V,\ V_2 = 12V$$

Ex:



لما نكون فارضين شي ومغيرين الإشارة مثل (قيمة) $V = IR$ وبالكتاب موجود العكس بنضيف (-)

IMPORTANT

$$-12 + 4I + 2V_o - 4 + 6I = 0$$

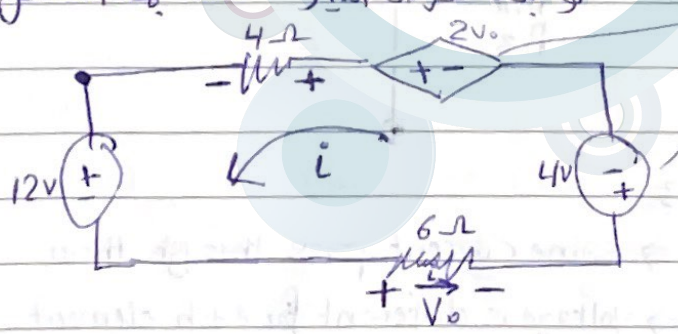
لانو فرضي عكس المفروض الموجود بدل ما نكون IR منحط (-)

$$-12 + 4I + 2(-6I) - 4 + 6I = 0$$

$$I = -8A$$ → يعني عكس الاتجاه المفروض

$$V_o = -6(-8) = 48v$$

لو بدنا نفرض التيار i بهلا الشكل



اشارات القيم ما بتتغير فبهم بس بتتغير اشارات المقاومة

$$+12 + 6I + 4 - 2V_o + 4I = 0$$

$$V_o = +6I$$

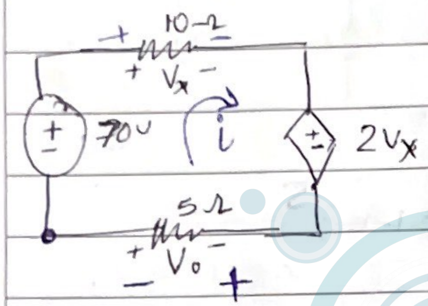
$$+12 + 6I + 4 - 2(6I) + 4I = 0 \Rightarrow I = +8A$$

اتجاه التيار صحيح

$$V_o = 6(8) = 48v$$

practice problem 2.6

Find V_x & V_o



$-70 + 10I + 2(10I) + 5I = 0$

$I = 2A$

$V_x = 10I = 20V$ فبتأكد انو

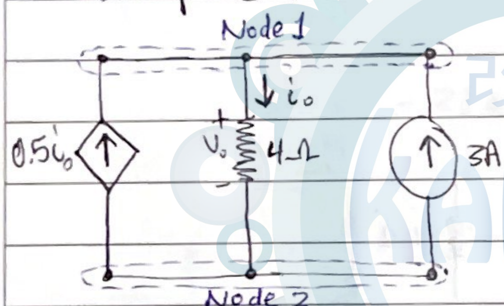
$V_o = 5I = 10V$ الاتجاه تبع الاشارة

اي احنا فرضناه نفس

الموجود بالكتاب

$V_o = -10V$ (عكس الفرض)

Example 2.7



• يجب ان تكون احدى ال Nodes ← refrence

Find the current & voltage ?

all element that are conecected in parallel have the same voltage

we assume the voltage in the node = zero

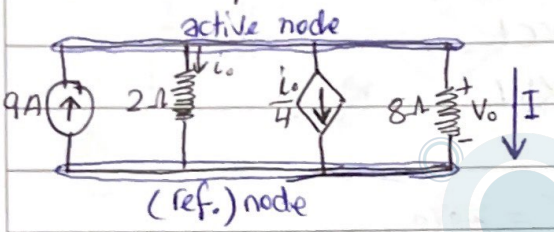
$3 + 0.5i_o = i_o$

$i_o = 6A$

$V_o = 4I_o = 4 \times 6 = 24V$

• If we have parallel elements we use KCL ($\Sigma I_{in} = \Sigma I_{out}$)

Practice Problem 2.7



We assume the current goes from the + (high voltage) to the − (low voltage) (From the active node to the ref.)

$$\Sigma I_{in} = \Sigma I_{out}$$

$$i_o + \frac{i_o}{4} + I = 9$$

$$\frac{5}{4} i_o + I = 9$$

$V = i_o R \rightarrow i_o = \frac{V_o}{2}$ ← because the elements are connected in parallel all have the same voltage

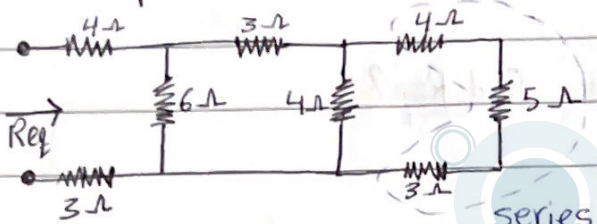
$V = I R \rightarrow I = \frac{V_o}{8}$

$$\frac{5}{4} \cdot \frac{V_o}{2} + \frac{V_o}{8} = 9$$

$$V_o = 12\,V$$

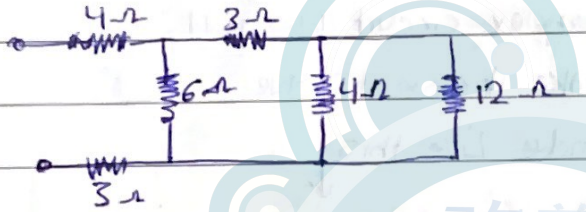
$$i_o = \frac{12}{2} = 6\,A$$

Example 2.9 (practice)

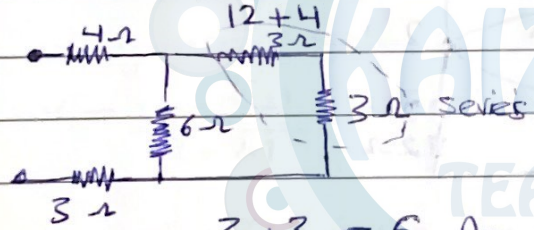


series

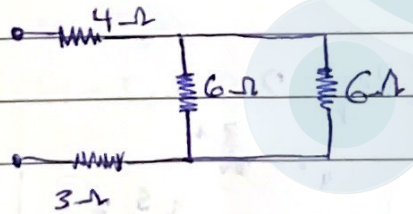
$R_x = 4+5+3 = 12\ \Omega$

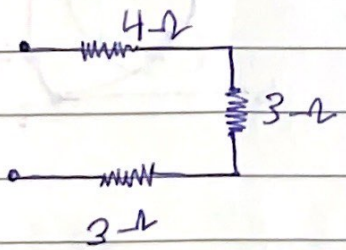


$$\frac{12 \times 4}{12+4} = 3\ \Omega$$



$3+3 = 6\ \Omega$





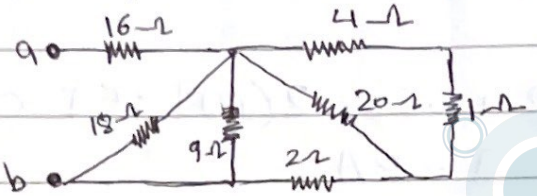
$R_{eq} = 4+3+3 = 10\ \Omega$

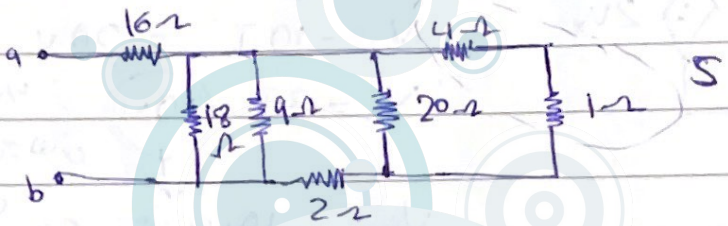
- بنلش حل من اول نقطة
- كل مرة لازم نرسم

NOTE

ما يكون عنا مقاومتين توازي
واليوم لقتين المقدار الناتج بتكوم
بكونه نفس وحدة فيهم

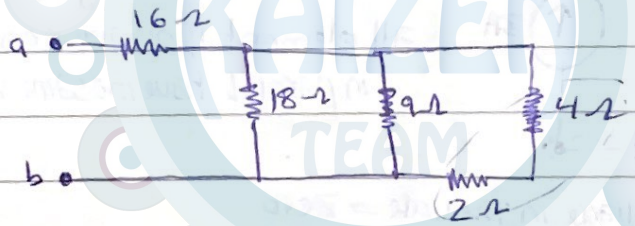
Example 2.10 (practice)





$4 + 1 = 5\ \Omega$

$$\frac{5 \times 20}{5 + 20} = 4\ \Omega$$



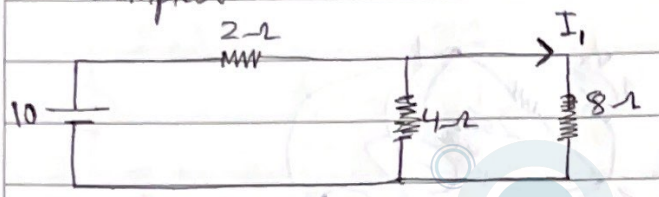
$4 + 2 = 6\ \Omega$

$$\frac{1}{6} + \frac{1}{9} + \frac{1}{18} = \frac{1}{R_{eq}}$$

$$R_{eq} = 3\ \Omega$$

$$16 + 3 = 19\ \Omega$$

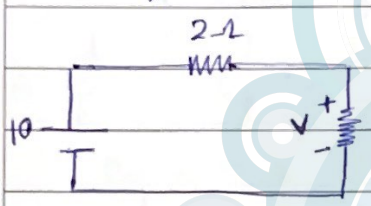
Examples:-



Find I_1?

- finding R_{eq}

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} \Rightarrow R_{eq} = 2.7\ \Omega$$



(V_{eq} is the same for the resistors that are linked together in parallel)

- using voltage divider

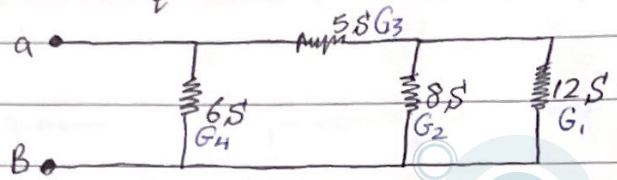
$$V = \frac{10\,(2.7)}{2+2.7} = 5.74 \text{ Volt}$$

- To finding I_1 (using ohm's law)

$$V = I_1 R$$

$$5.74 = I_1 (8) \Rightarrow I_1 = 0.71\ A$$

Find G_{eq} between a & B.



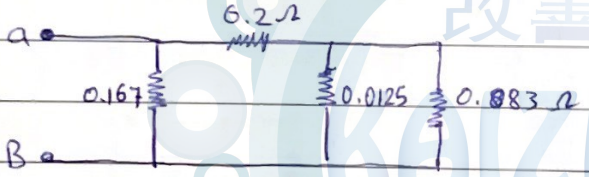
$$G = \frac{1}{R} \quad (\Omega, \Omega^{-1}, S)$$

$R_1 = \frac{1}{G_1} = 0.083\ \Omega$

$R_2 = \frac{1}{G_2} = \frac{1}{8} = 0.0125\ \Omega$

$R_3 = 0.2\ \Omega$

$R_4 = 0.167\ \Omega$

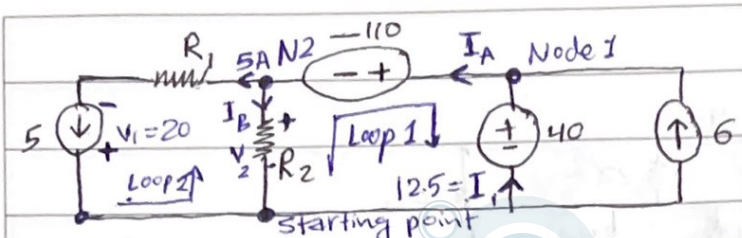


$$\frac{0.083 \times 0.0125}{0.083 + 0.0125} = 0.05\ \Omega$$

$0.05 + 0.2 = 0.25\ \Omega$

$$\frac{0.25 \times 0.167}{0.25 + 0.167} = 0.1\ \Omega$$

$$G_{eq} = \frac{1}{R_{eq}} = \frac{1}{0.1} = 10\ S$$



$P_{5A} = 100\ w$ (supplied) supplied means that the power(−)

$P_{40\,V} = 500$ (supplied)

find R_1 & R_2 ?

$P_{40\,V} = 500$

$40 \times I_1 = 500 \Rightarrow I_1 = 12.5\ A$

$P_{5A} = 100$

$5 \times V_1 = 100 \Rightarrow V_1 = 20$ volt

- by applying KCL at node 1

$6 + 12.5 = I_A \Rightarrow I_A = 18.5\ A$

- by applying KCL at node 2

$I_A = 5 + I_B$

$18.5 - 5 = I_B \Rightarrow I_B = 13.5\ A$

- by applying KVL at loop 1

$-13.5 R_2 - (-110) + 40 = 0$

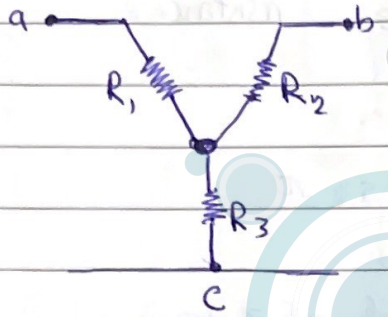
$R_2 = 11.11\ \Omega$

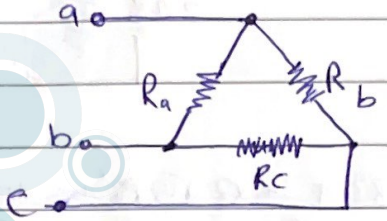
- by applying KVL at loop 2

$-11.11(13.5) + 5(R_2) - 20 = 0$

$R_2 = 33.99 \approx 34\ \Omega$

Delta-Wye connection for resistors:-

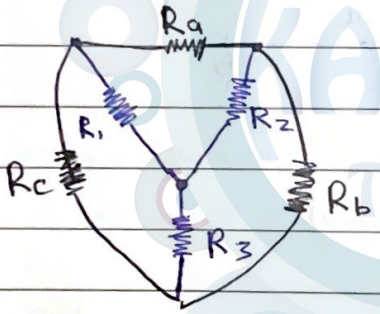




Δ connection

Wye (Y) connection

* conversion from Y – Δ :-

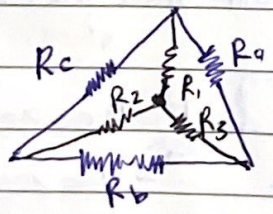


$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

* conversion from Δ to Y :-

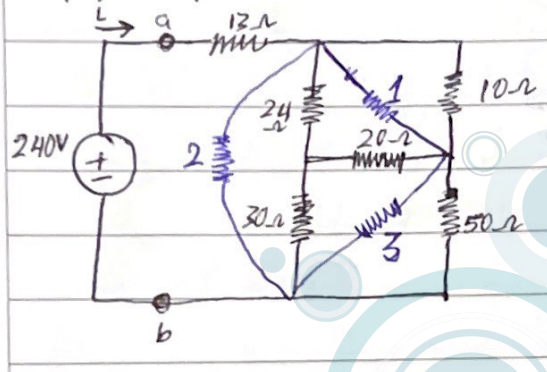


$$R_1 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_cR_b}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

practice problem 2.15

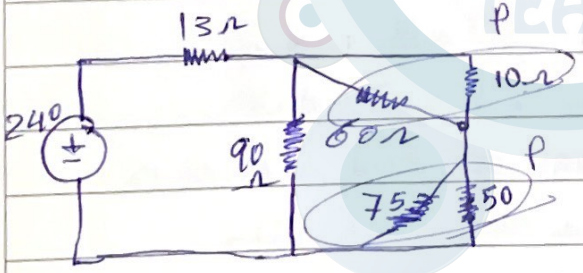


convert from Y to Δ

$1 = \frac{20\times24 + 20\times30 + 30\times24}{30} = \frac{1800}{30} = 60\,\Omega$

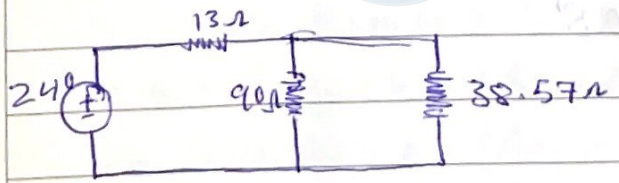
$2 = \frac{1800}{20} = 90\,\Omega$

$3 = \frac{1800}{24} = 75\,\Omega$



$\frac{10\times60}{10+60} = 8.57\,\Omega$

$\frac{50\times75}{50+75} = 30\,\Omega$



$8.57 + 30 = 38.57\,\Omega$

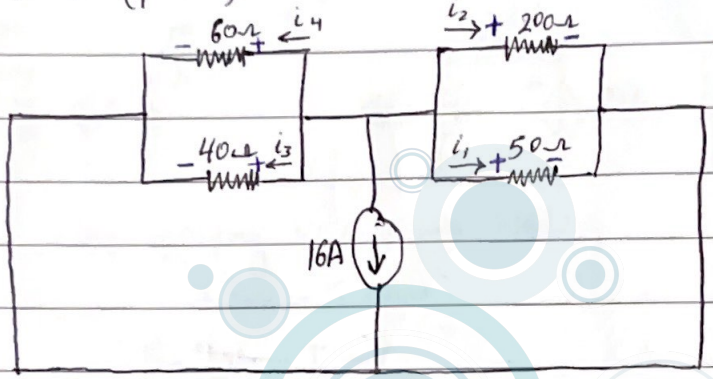
$\frac{38.57\times90}{38.57+90} = 26.99 \approx 27$

$R_{eq} = 13 + 27 = 40\,\Omega$

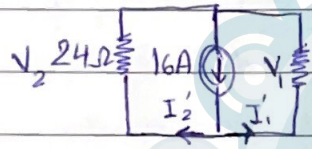
~~$V = \frac{240(27)}{13+27} = 162\text{V}$~~

$I = \frac{240}{40} = 6\text{A}$

2.32 (problem)



find i_1 through i_4 in the circuit.



→ we just convert the two parallel resistors into one resistor.
and then we assume a current that goes into these two resistors
we can find I_1' & I_2' using current divider

$V_1 = 40 \times 6 = 240$ volt

$V_2 = 24 \times 10 = 240$ volt

$I_1' = \frac{16 \times \frac{1}{40}}{\frac{1}{40}+\frac{1}{24}} = 6\ A$ $\quad I_2' = 16-6 = 10A$

$i_1 = \frac{240}{40} = 6\ A$

$i_2 = \frac{240}{200} = 1.2\ A$

neg. sign because the direction we assume it's the opposite.

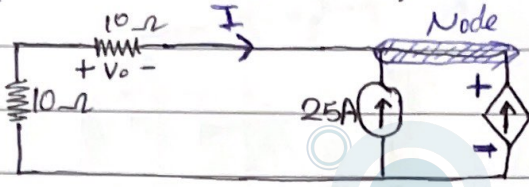
and then we will have two choice the first is to apply current divider twice & second is to use voltage divider.

$i_3 = \frac{240}{40} = 6\ A$

$i_4 = \frac{240}{60} = 4\ A$

IMPORTANT
dont apply KVL for any path that has a current source

problem 2.22 (IMP Q)



Find V_o?
& then the power, ? (by the dependent source)

- by applying KCL at the Node

$$\Sigma I_{in} = \Sigma I_{out}$$

$$25 + 2V_o + I = 0$$

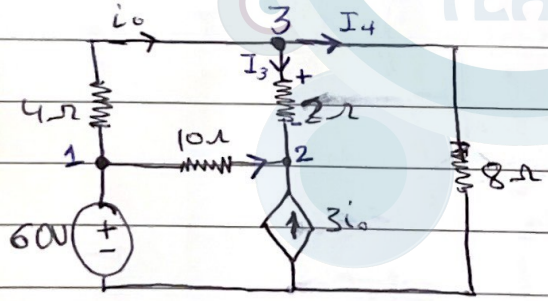
$$25 + 2V_o + \left(\frac{V_o}{10}\right) = 0 \Rightarrow V_o = -11.9 \text{ volt}$$

$$p = VI = \frac{2(-11.9)(-11.9)}{10} = (-)(2 \times -11.9)(23.8) = 566.44 \text{ WATT}$$

→ To find the voltage that applyied on the supply (KVL) ⇒ V = 23.8 volt

- the current enter the (+) pole in the resistor
- the current exit the (x) pole in the source (dependent or not)

problem 3.17 : find i_o?



at node 3:-

$$\Sigma I_{in} = \Sigma I_{out}$$

$$I_o = I_3 + I_4$$

$$\frac{V_3 - V_1}{4} = \frac{V_3 - V_2}{2} + \frac{V_3}{8}$$

$$8 \times \left(\frac{V_3}{4} - \frac{60}{4} - \frac{4V_3}{2} + \frac{V_2}{2} - \frac{V_3}{8}\right) = 0$$

$$4V_2 - 3V_3 = 120 \quad [1]$$

at node 2:-

$$\frac{V_3 - V_2}{2} + 3i_o = \frac{V_1 - V_2}{10}$$

$$\frac{V_3}{2} - \frac{V_2}{2} + 3\left(\frac{V_3 - V_1}{4}\right) = \frac{60}{10} - \frac{V_2}{10} \Rightarrow -15V_1 - 8V_2 + 25V_3 = 60 \quad [2]$$

Chapter 3 : METHODS OF ANALYSIS

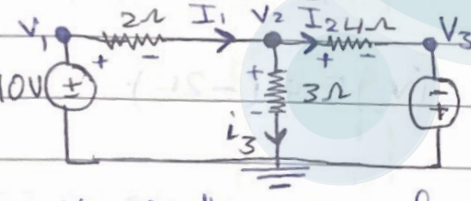
* Fundemental techniques for resistance circuit analysis
  - 1- Nodal analysis
  - 2- Mesh analysis
  - 3- super position
  - 4- source transformation
  - 5- Thevenin & Norton

NODAL ANALYSIS

Steps :-

1- Determine the nodes
2- assume one of the nodes as a refrence
3- name the nodes ⟹ using $V_1, V_2, \dots$
4- if we have voltage source between two node & refrence
   ↳ V_i at that node is Known.

Example : find i_3, P(2Ω) using nodal analysis



$V_1 = 10$ volt $= V_1 - ref = 10$ (as we talk at point 4 ↑↑)

$V_3 = -6$ volt

applying KCL at node (V_2)

$$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{4} + \underbrace{V_3/3}_{i_3} = 0$$

$V_2 = 3.24$ volt

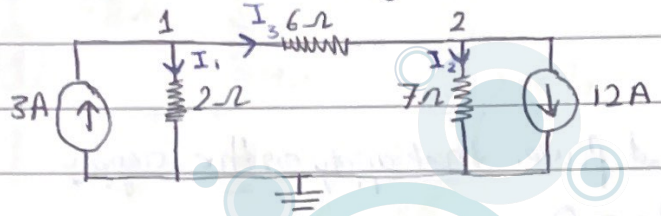
$$i_3 = \frac{3.24}{3} = 1.08A$$

$$P(2\Omega) = (+)\frac{(V_1 - V_2)^2}{2} = \frac{(10 - 3.24)^2}{2}$$

$= 22.8$ watt

practice problem 3.1

obtain the node voltages in the circuit



* the direction of I_1 & I_2 is from the active node towards the ref.

applying KCL at node 1 & node 2

$$3 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}, \quad \frac{V_1 - V_2}{7} = \frac{V_2}{6} + 12$$

$$6 \cdot \frac{V_1 - V_2 + 3V_1}{6} = 3 \cdot 6, \quad \frac{7(V_1 - V_2) - 6V_2}{42} = 12$$

$$4V_1 - V_2 = 24 \quad [1]$$

$$42 \cdot \frac{7V_1 - 7V_2 - 6V_2}{42} = 12 \cdot 42$$

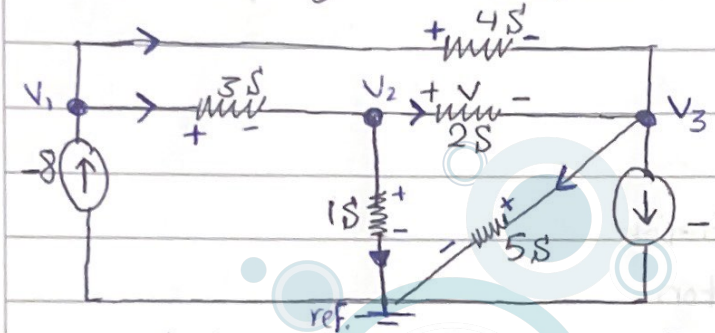
$$\begin{cases} 4V_1 - V_2 = 24 \\ 7V_1 - 13V_2 = 504 \end{cases} \qquad 7V_1 - 13V_2 = 504 \quad [2]$$

$$V_1 = \frac{\begin{vmatrix} 24 & -1 \\ 504 & -13 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ 7 & -13 \end{vmatrix}} = \frac{192}{-45} = -4.267\ V$$

$$V_2 = \frac{\begin{vmatrix} 4 & 24 \\ 7 & 504 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ 7 & -13 \end{vmatrix}} = \frac{1848}{-45} = -41.067\ V$$

↑ This Methed called Cramer's RULE

Find v using Nodal analysis :-



Conductance $\Rightarrow G = \frac{1}{R}$ $\quad V = IR \Rightarrow I = \frac{V}{R} \Rightarrow I = VG$

by applying KCL at node 1:-

$$-8 = \frac{(V_1 - V_2)}{3}(3) + (V_1 - V_3)(4)$$

$$3V_1 - 3V_2 + 4V_1 - 4V_3 = -8$$

$$7V_1 - 3V_2 = -8$$

KCL at node 2:-

$$(V_1 - V_2)(3) = (V_2 - V_3)(2) + (V_2)(1)$$

KCL at node 3:-

$$(V_1 - V_3)(4) + (V_2 - V_3)(2) = (V_3)5 + (-25)$$

1st eq $\Rightarrow 7V_1 - 7V_2 = -8$

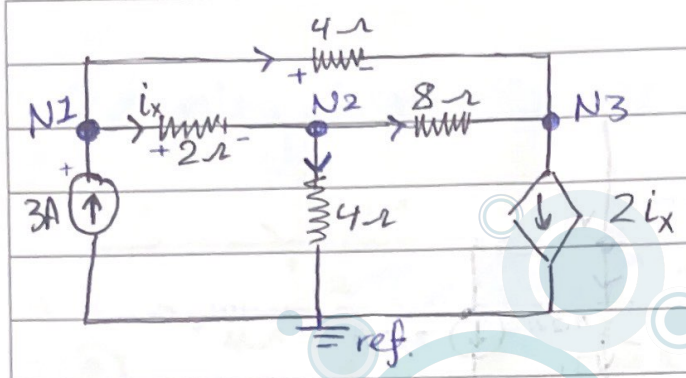
2nd eq $\Rightarrow 3V_1 - 6V_2 - 2V_3 = 0$

3rd eq $\Rightarrow 4V_1 + 2V_2 - 11V_3 = -25$

NUM OF EQ. Should equals TO NUM OF NODE

$V_1 = -2.89 \quad V_2 = -1.75 \quad V_3 = 0.91$

$$V = \frac{V_2 - V_3}{2} = \frac{-1.75 - 0.91}{2} = -1.33\ V$$



KCL at node 1

$$3 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} \Rightarrow 3V_1 - 2V_2 - V_3 = 12$$

KCL at node 2

$$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{8} + \frac{V_2}{4} \Rightarrow -4V_1 + 7V_2 - V_3 = 0$$

KCL at node 3

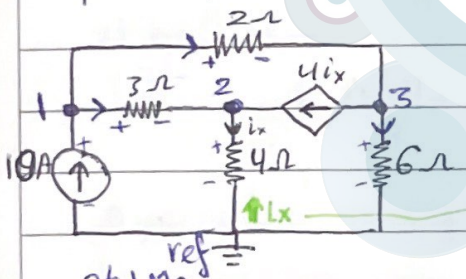
$$2i_x = \frac{V_2 - V_3}{8} + \frac{V_1 - V_3}{4} \Rightarrow 2V_1 + V_2 + 3V_3 = 16 i_x$$

i_x

$$2V_1 + V_2 + 3V_3 = 16\left(\frac{V_1 - V_2}{2}\right)$$

$$2V_1 - 3V_2 + V_3 = 0$$

practice problem 3.2



lets assume that i_x is in opposite Dir.
So i_x will be $-V_2/4$
لانو عكس الاتجاه إلي احنا فرضناه

at N1

$$4 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2}$$

at N2

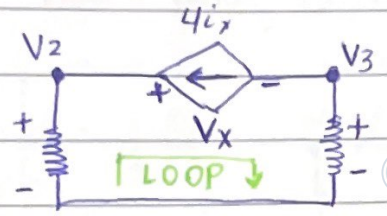
$$4i_x + \frac{V_1 - V_2}{3} = i_x \Rightarrow 4\left(\frac{V_2}{4}\right) + \frac{V_1 - V_2}{3} = \frac{V_2}{4}$$

at N3

$$4i_x + \frac{V_3}{6} = \frac{V_1 - V_3}{2}$$

$V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

cont.



we want to find power that apply on the supply

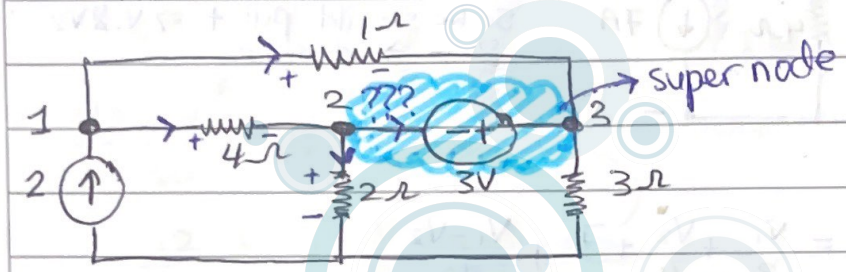
$-V_2 + V_x + V_3 = 0$

$-(-64) + V_x + 156 = 0$

$V_x = -220$

$P = (-)(-220)(4i_x) = +$ absorbed

Defining the super Node :



at node 2

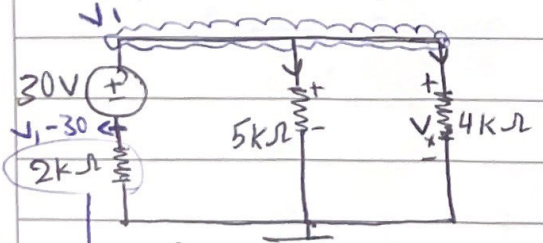
we can't know the current at branch (2,3) because we have voltage supply that gives us voltage that is independent for the current, so we assume that branch (2,3) is one big node.

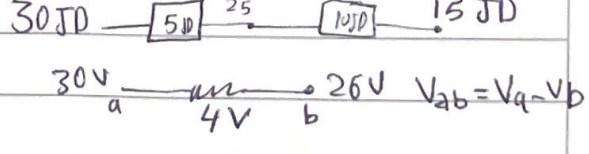
at node 1 & 2 = $\frac{V_1 - V_2}{4} + \frac{V_1 - V_3}{1}$ — 1

at node 2 —

$\frac{V_1 - V_2}{4} + \frac{V_1 - V_3}{1} = \frac{V_2}{2} + \frac{V_3}{3}$ — 2

supernode : $V_3 - V_2 = 3$ — 3

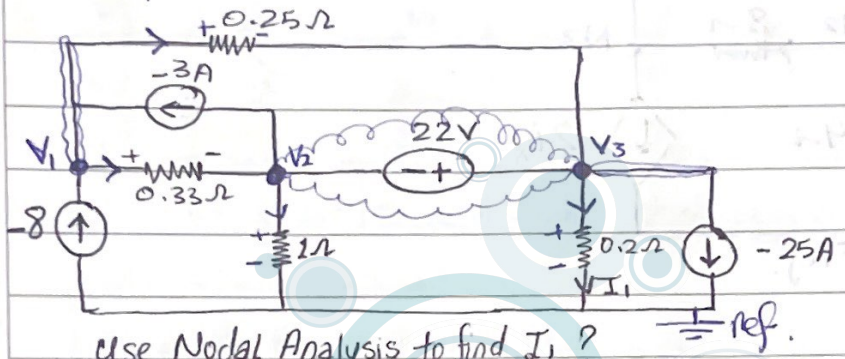




↳ If I want to discripe the current in this branch $\frac{V_1 - 30}{2}$

$\frac{V_1 - 30}{2} + \frac{V_1}{5} + \frac{V_1}{4} = 0$, $V_1 = V_x = 15.79$ mV

Example 8 -



Use Nodal Analysis to find I_1 ?

at node 1 :-

$$-8-3 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.25} \quad \text{---} \boxed{1}$$

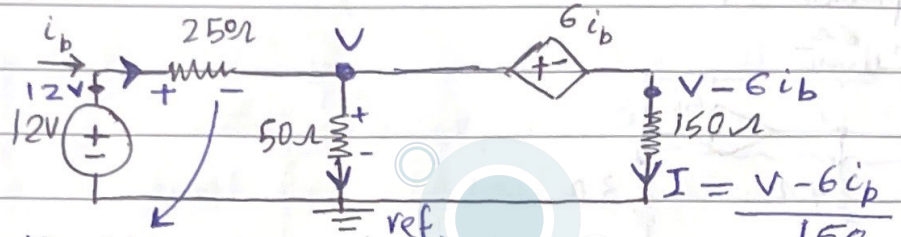
at node 2 :-

$$\frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.25} = -3 + \frac{V_2}{1} + \frac{V_3}{0.2} + (-25) \quad \text{---} \boxed{2}$$

using supernode

$$V_3 - V_2 = 22 \quad \text{---} \boxed{3}$$

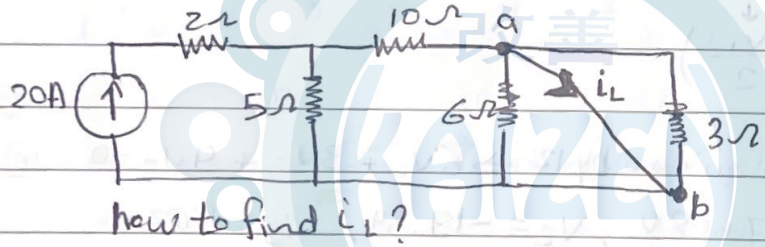
$$I_1 = \frac{V_3}{0.2} =$$



$\frac{12-V}{250}$

$$\frac{12-V}{250} = \frac{V}{50} + \frac{V-6i_b}{150}$$

Solve for V : 1.62 V

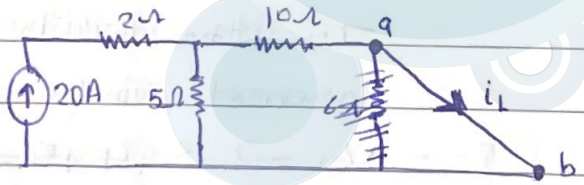


how to find i_L ?

Short circuit Any element in parallel with SC will be cancelled

⇒ Since SC in parallel with 3Ω & 6Ω they will be ~~os~~ cancelled

So the result will be



$$I_L = \frac{20 \times 5}{5+10} = \frac{100}{15} \text{ A}$$

MESH ANALYSIS

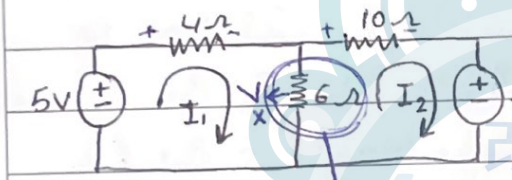
Steps:-

1- assign # of Mesh

2- assign a current for each mesh

3- Apply KVL for each mesh ($\Sigma V = zero$)

4- solve to find $i_1, i_2, \dots, i_n$



the common resistor we will keep it as it is without any polarity

MESH 1:-

$-5 + 4I_1 + 6(I_1 - I_2) = 0$

$10I_1 - 6I_2 = 5$ — [1]

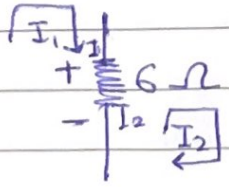
MESH 2:-

$10I_2 + 20 + 6(I_2 - I_1) = 0$

$-6I_1 + 16I_2 = -20$ — [2]

$I_1 = -0.32\ A, \quad I_2 = -1.37\ A$

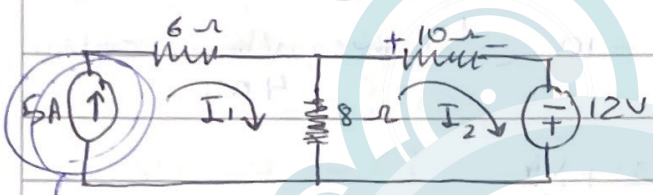
If we want to find V_x ?



$(I_1 - I_2)(6) = (-0.32 + 1.37)(6) = 6.3\ V$

I CAN'T FIND AN EXPRESSION FOR THE CURRENT PASSING THROUGH The VOLTAGE SOURCE
THE SAME FOR THE VOLTAGE THROUGH THE CURRENT SOURCE (→).

EXAMPLE 8-



→ In this mesh the current is already given So no need to find the (I) through applying KVL

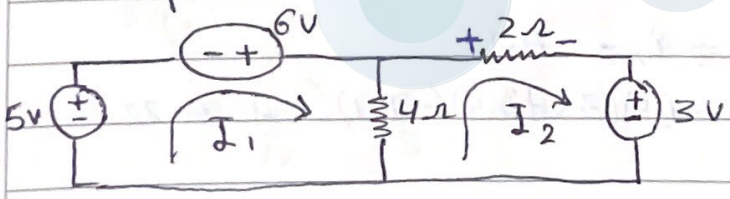
$I_1 = 5A$ — [1]

$10I_2 - 12 + 8(I_2 - 5) = 0$ — [2]

$18I_2 = 52$

$I_2 = \frac{52}{18}$ A — [2]

Example 8-

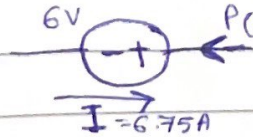


Find the absorbed power by 6V supply using Mesh analysis:-

at mesh 1: $-5 - 6 + 4(I_1 - I_2) = 0$

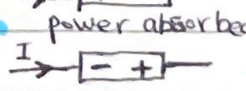
at mesh 2: $2I_2 + 3 + 4(I_2 - I_1) = 0$

$I_1 = 6.75A$, $I_2 = 4A$

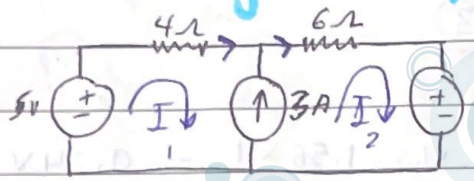


$\Rightarrow (-I)(6) = -40.5$ watt

NOTE



Defining the supermesh



Loop : $-5 + 4I_1 + 6I_2 + 10 = 0$

$4I_1 + 6I_2 = -5$ —[1]

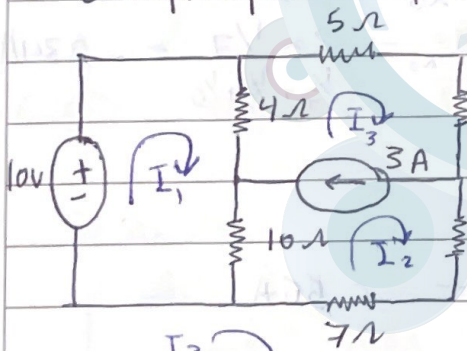
applying KCL at the node

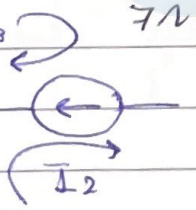
$3 = -I_1 + I_2$ —[2]

$I_1 = -2.3A$, $I_2 = 0.7A$

we make supermesh when there is a current source between two meshes ⇒ so we take larger loop that contain both meshes.

Example: Find the power generated by the 10V source.





$P = -(I_1)(10) = 12.86W$

For mesh 1:-

$-10 + 4(I_1 - I_3) + 10(I_1 - I_2) = 0$

$14I_1 - 14I_2 = 10$

supermesh:-

$5I_3 + 9I_3 + I_2 + 7I_2 + 10(I_2 - I_1) + 4(I_3 - I_1) = 0$

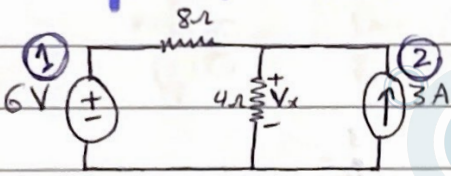
$-14I_1 + 18I_2 + 18I_3 = 0$

↳ The bigger current should be in the same direction of the current supply so we can subtract it from the smaller I.

$I_3 - I_2 = 3$

Chapter 4: Circuit THeorem

Super Position :-



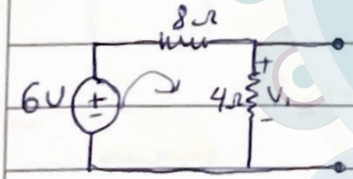
steps :-

1- Kill all the indp. sources except one source..

[V → short circuit , I → open circuit]

- Just for indp. sources not dependent sources because It's Not a source It's a resistor.

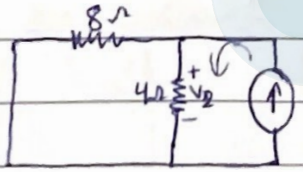
Source 1 :-



using voltage divider

$V_{x1} = \frac{6(4)}{8+4} = 2 \text{ volt} \pm$

Source 2 :-



using current division

$I_{(4\Omega)} = \frac{3(\frac{1}{4})}{1/8+1/4} = 2A \pm$

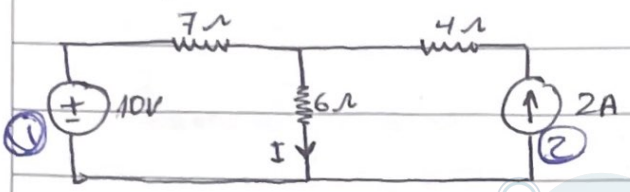
We must chick the polarity of them

$V_{x2} = 2 \times 4 = 8 \text{ volt.}$

$V_x = V_{x1} + V_{x2}$

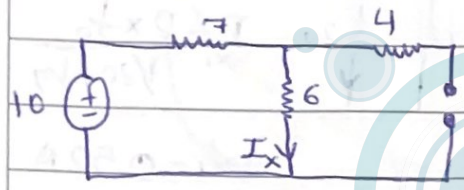
$= 2 + 8 = 10 \text{ volt}$

Example 8-



Find Power of (6Ω) by super-position 8-

Source 1 8-



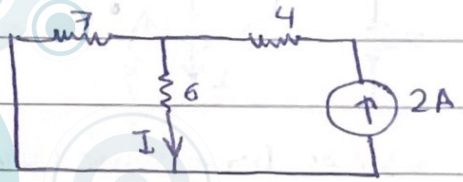
Using KVL 8-

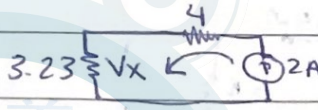
$-10 + 7I + 6I = 0$

$I = 0.77A$

$P_{(6\Omega)} = (0.77)^2(6) = 3.56W$

Source 2 8-





$V_x = 3.23 \times 2 = 6.46$ volt

$I = \frac{6.46}{6} = 1.08A$

$P_{(6\Omega)} = (1.08)^2(6) = 6.99W$

$I = 1.85A$

$P = (1.85)^2(6) = 20.5$ watt

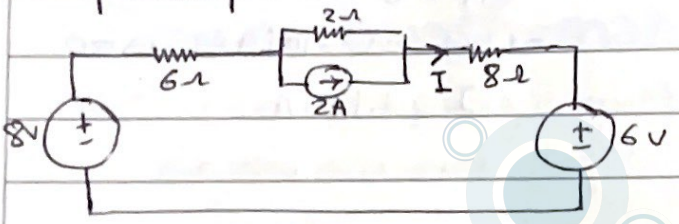
$P' + P'' = 3.56 + 6.99 = 10.55$

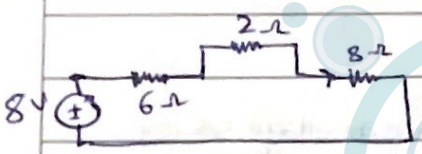
very imp note 8-

$I^2R \neq (I')^2R + (I'')^2R$

Super position not applied on Power

practice problem 4.5 :-





applying KVL :-

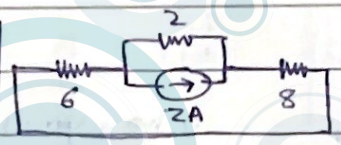
$-8 + 6I + 2I + 8I = 0$

$I = 0.5A$

OR

using ohms Law

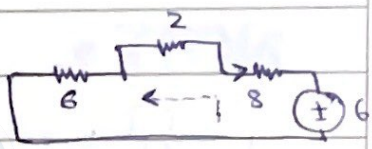
$I = \frac{V}{R} = \frac{8}{6+2+8} = 0.5A$



using current division

$\frac{2 \times 1/14}{1/14 + 1/2} = 0.25A$

$I_2 = 0.25A$



$-6 + 8I + 2I + 6I = 0$

$I = 0.375$

$I_3 = -I$

$I_3 = -0.375A$

$I_{Total} = 0.5 + 0.25 + (-0.375) = 0.375A = 375mA$

Source transformation :-

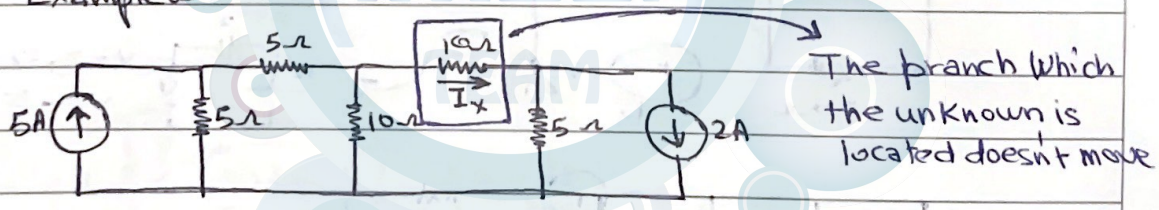
In some circuits replacing current sources with voltage sources or ~~vise versa~~ visa versa.

$$R_S = R_P$$

$$V = I R_P$$

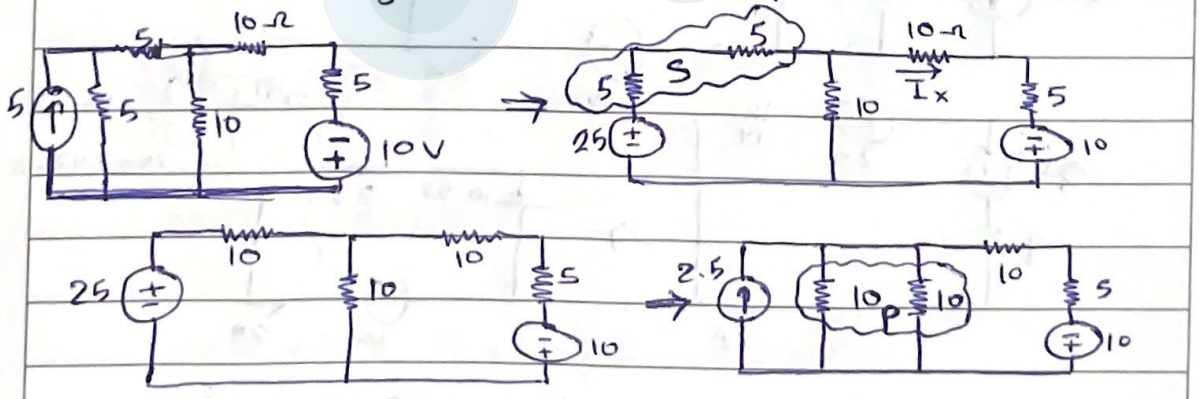
$$I = \frac{V}{R_S}$$

Example:-

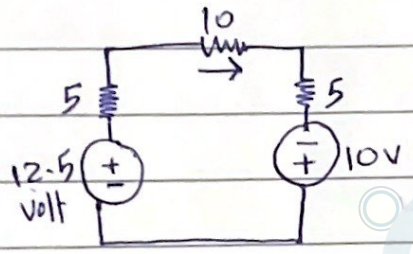


The branch which the unknown is located doesn't move

Find I_x using source transformation?



Cont. →

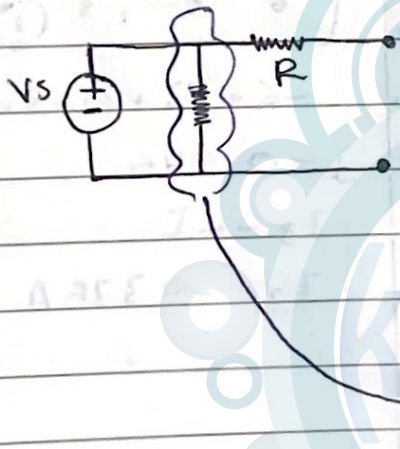


applying KVL :-

$-12.5 + 5I + 10I + 5I - 10 = 0$

$I = 1.125\,A$

NOTE :-

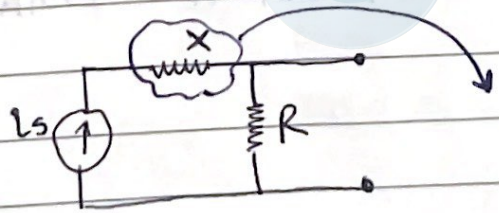


When you have a voltage source you care about the resistor that is in series with that source.

And you don't care about anything else in the middle

Because

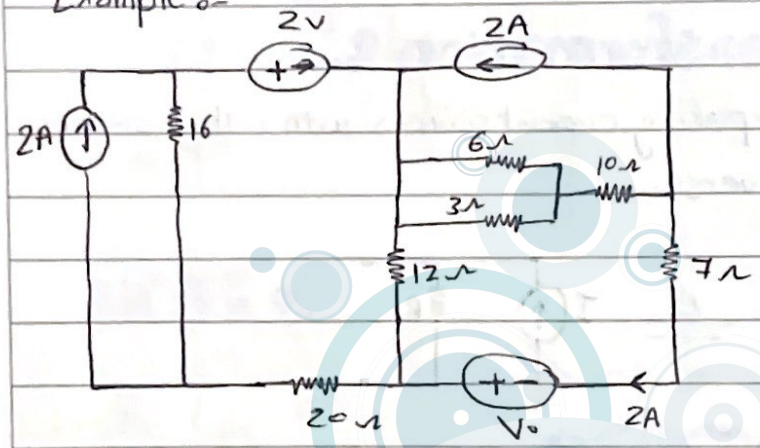
When you think about it this resistance it's sitting in paralllel with the voltage source so the voltage across the ~~resit~~ resistor is the same that goes into the source.

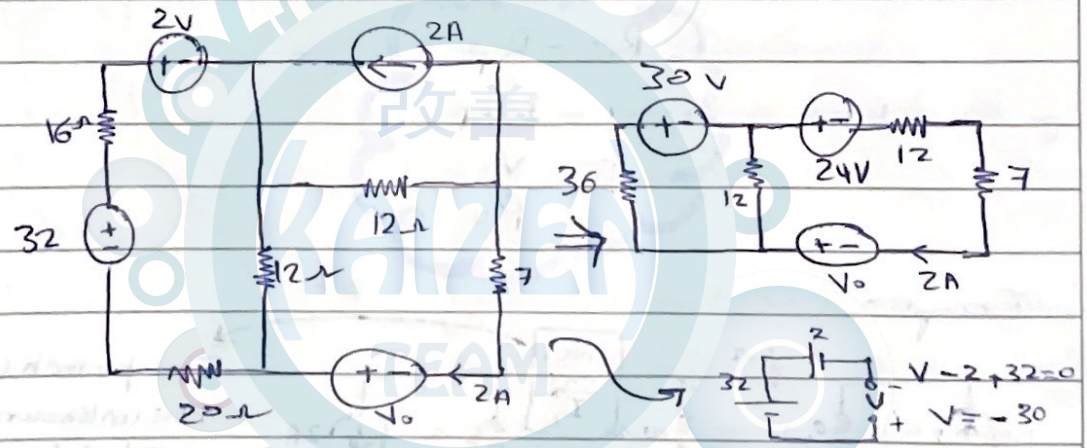


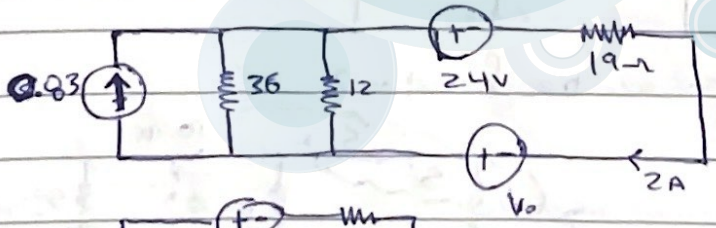
the same amount of current entering in the (series resistor)

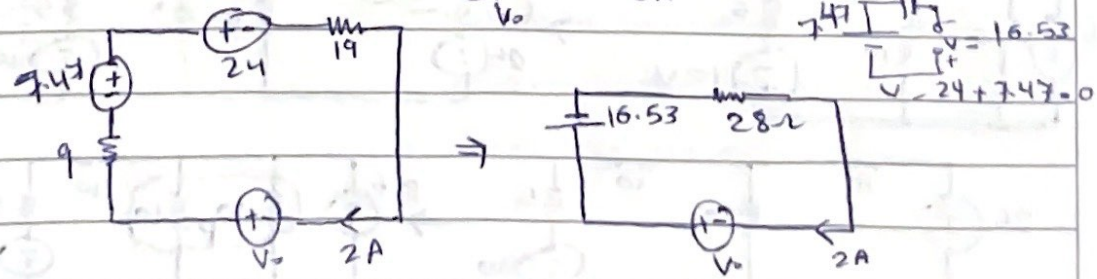
So we can just ignore it

Example :-



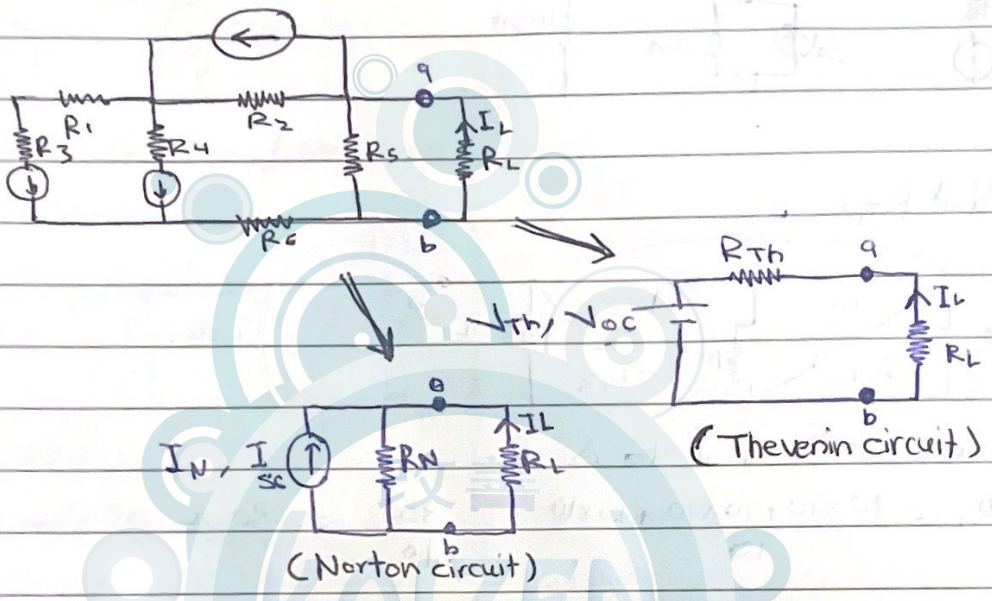






$16.53 + 28(2) + (-V_o) = 0$ $\Rightarrow V_o = 72.53$ volt

Thevenin & Norton :-



Steps to find $V_{Th} = V_{oc}$:-

1- remove R_L

2- leave that place open circuit

3- find V_{op} at that terminal

Steps to find $I_N = I_{sc}$:-

1- remove R_L

2- put short circuit in the place of R_L

3- find current in that short circuit

Steps to find ($R_N = R_{Th}$) :-

$R_{Th} = \frac{V_{op}}{I_{sc}}$ or

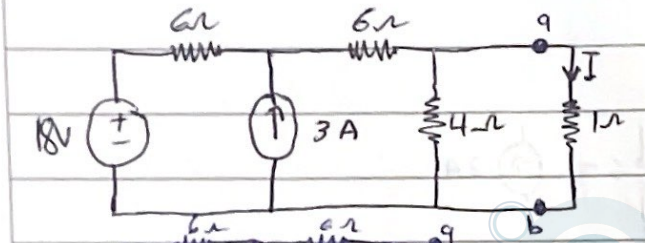
1- Remove R_L

2- Kill all sources

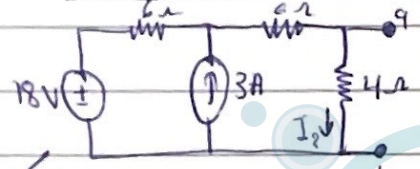
3- find R_{eq} seen at that terminals

$V_{Th} = I_N \times R_{Th}$

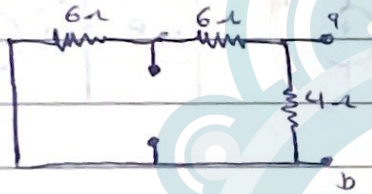
$I_N = \frac{V_{Th}}{R_{Th}}$



Find I using Thevenin:-



1- Killing the sources



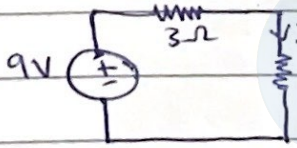
$R_{Th} = 6+6 \,||\, 4 = 3\Omega$

To find V_{Th}

$-18 + 6I_1 + 6I_2 + 4I_2 = 0$

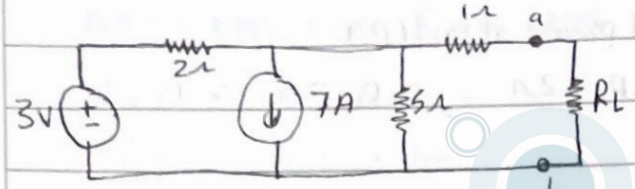
$I_2 - I_1 = 3A \longrightarrow I_2 = 9/4 A$

$V_{Th} = \frac{9}{4} \times 4 = 9 volt$



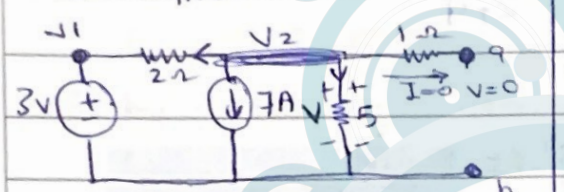
$= \frac{9}{4} A$

Ex:-



find Thevenin & Norton circuit?

Find V_{Th}:-



$V_1 = 3v$

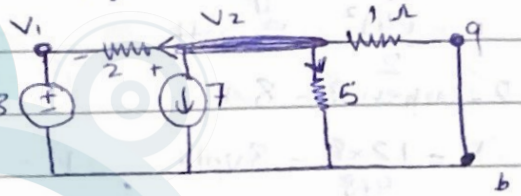
at node 2:-

$$\frac{V_2 - 3}{2} + 7 + \frac{V_2}{5} = 0$$

$V_2 = -7.86$ volt

$V_2 = V_{Th} = -7.86$ volt

Find I_{sc}:-



$V_1 = 3$ volt

at node 2:-

$$\frac{V_2 - 3}{2} + 7 + \frac{V_2}{5} + \frac{V_2}{1} = 0$$

$V_2 = -3.24$ volt

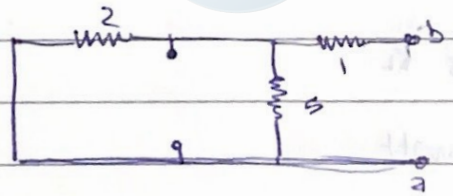
$I_{sc} = \frac{V_2}{1} = -3.24$ A

To find R_{Th}:-

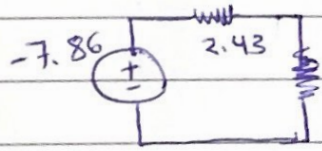
1- $R_{Th} = \frac{V_{op}}{I_{sc}} = \frac{-7.86}{-3.2} = 2.43\ \Omega$

or

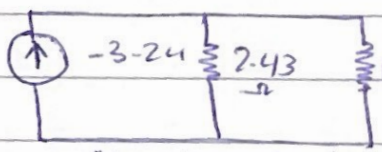
2-



$R_{eq} = 2 // 5 + 1 \Rightarrow 2.43$

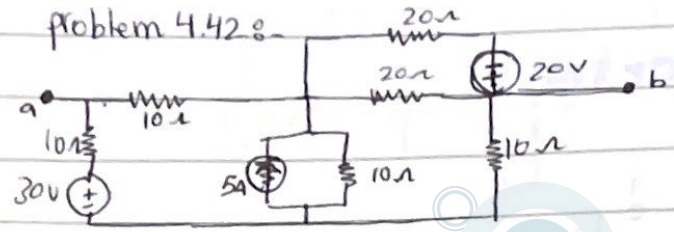


(Thevenin circuit)

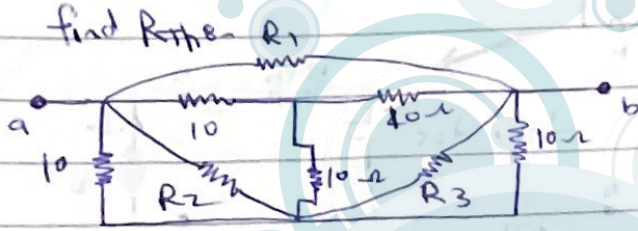


(Norton circuit)

Problem 4.42 :-



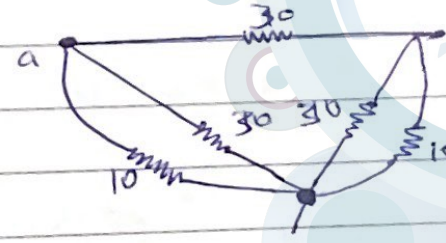
find R_{Th} :-



Convert from Y to Δ :-

$$R_1 = \frac{10\times10 + 10\times10 + 10\times10}{10} = \frac{300}{10} = 30\,\Omega$$

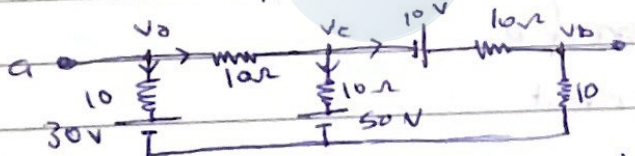
$$R_2 = R_3 = R_1 = 30\,\Omega$$



$$Req = 7.5\times2 = 15\,\Omega$$

$$= \frac{15\times30}{15+30} = 10\,\Omega$$

To find V_{Th} :-



$V_{ab} = V_{Th}$

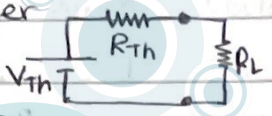
node A :-

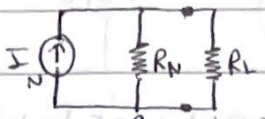
$$\frac{V_A - 30}{10} + \frac{V_A - V_C}{10} = 0$$

Node C :-

$$\frac{V_A - V_C}{10} = \frac{V_C - 50}{10} + \frac{V_C + 10 - V_B}{10}$$

Maximum Power transfer :-

- We concern about power consumption by Load after simplification of the circuit either 

or 

1- what is the value of R_L to get (absorbed) max power from the source

2- $P_{Load} = I^2 R_L$ [1] , $I = \frac{V_{Th}}{R_{Th} + R_L}$ [2]

3- by putting eq 2 into 1

$$P_{Load} = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \quad [3]$$

4- for max power Load $\Rightarrow \frac{dP_L}{dR_L} = 0$

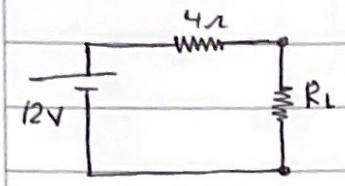
Result $R_L = R_{Th}$ [4]

So for any circuit to get the maximum power at the load

5- by putting eq 4 in 3

$$P_{L(max)} = \frac{V_{Th}^2}{4R_{Th}} = \frac{I_N^2 R_L}{4}$$

Ex8-



Find power at load (R_L)

1- $R_L = 2\Omega$, 2- $R_L = 8\Omega$, 3- $R_L = 4\Omega$

1- when $R_L = 2\Omega$

using voltage division $V = \frac{12 \times 2}{2+4} = 4$ volt

$P = \frac{(4)^2}{2} = 8$ watt

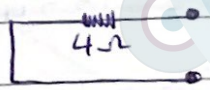
2- when $R_L = 8\Omega$

$V = \frac{12 \times 8}{4+8} = 8$ volt , $P = \frac{8^2}{8} = 2$ watt

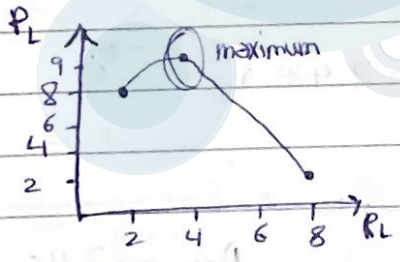
3- when $R_L = 4\Omega$

$V = \frac{12 \times 4}{4+4} = 6$ volt , $P = \frac{(6)^2}{4} = 9$ watt ← (*)

Refer to circuit & find R_{Th}:-



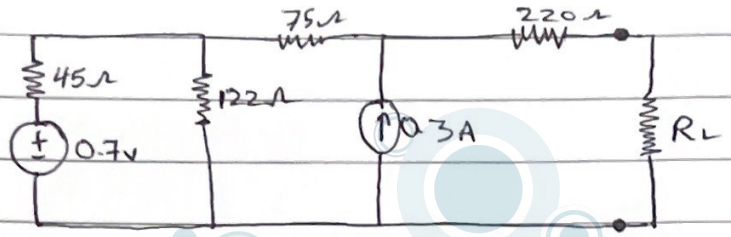
$R_{Th} = R_{eq} = 4\Omega$



P max when $R_L = R_{Th}$

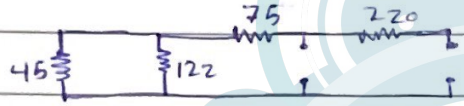
$P_{L(max)} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(12)^2}{4 \times 4} = 9$ watt

Example:



Find R_L to get max. power, $P_{L(max)}$ = ?

- for max power $R_L = R_{Th}$

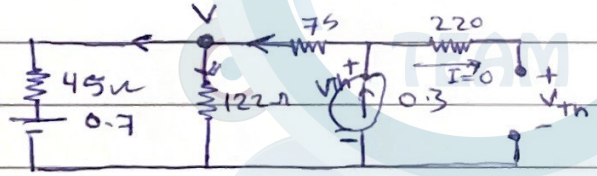


$R_{Th} = 327.9\Omega$

So for max. power transfer $R_L = 327.9\Omega$

- $P_{L(max)} = \frac{V_{Th}^2}{4R_{Th}}$

to find V_{Th}



by using nodal analysis

$$\frac{V-0.7}{45} + \frac{V}{122} = 0.3 \Rightarrow V_1 = 10.45 \text{ volt}$$

by KVL:-

$$-10.45 - 95(0.3) + V_{Th} = 0 \Rightarrow V_{Th} = 32.95 \text{ volt}$$

$$P = \frac{32.95^2}{4 \times 327.9} = 0.827 \text{ W}$$

Chapter6 : Capacitor & Inductors

We will talk about energy storage elements

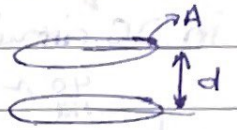
1- Capacitor

1- device (element) that store energy

2- consists of two parallel plates, each plate has a cross sectional (A) have distance (d) between them.

& it filled with dielectric material.

has certain permitivity (ϵ)



$$C = \frac{\epsilon A}{d}$$

3- unit of capacitance $\Rightarrow$ (F)

4- charge on plates of capacitor $Q = CV$ (Fixed voltage)

$q(t) = Cv(t)$ (variable voltage)

5- $i = \frac{dQ}{dt} \Rightarrow i_c(t) = C\frac{dv_c(t)}{dt}$

6- $v_c = \frac{1}{C}\int_{t_0}^{t} i_c(\tau)\, d\tau + v_c(t_0)$

if $v_c(t)$ = DC value = fixed

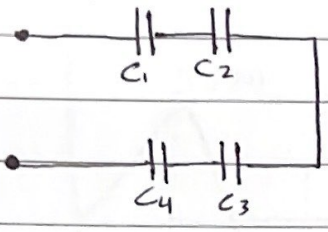
$\Rightarrow i_c(t)$ = zero

means that capacitor act like open circuit in DC-Circuit

7- energy stored in the capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$

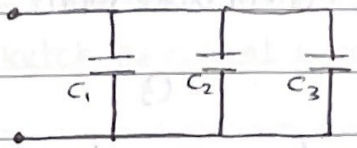
the calc should be RAD

capacitor in series



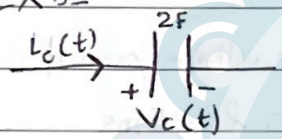
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

Capacitor in parallel



$$C_{eq} = C_1 + C_2 + C_3$$

Ex:-



$V_c(t) = 5\sin(t)$

1- energy stored

2- Q 3- I

$$E = \frac{1}{2} C \times V^2 = \frac{1}{2} \times 2 \times (5\sin t)^2 = 25\sin^2(t) \text{ Joul}$$

$$Q = CV = 2 \times 5\sin(t) = 10\sin(t) \text{ colonmb}$$

$$I = C\frac{dv_c(t)}{dt} = 2 \times (+5\cos(t)) = 10\cos(t) \text{ A}$$

Note that capacitors are used to block DC, pass AC

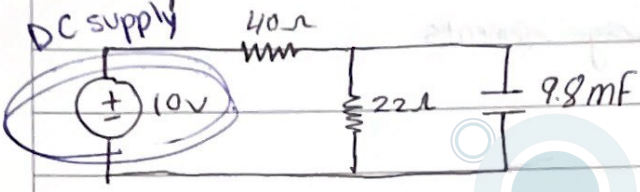
Shift phase, store energy, start motors & supperssnoise dv = finite

The capacitor doesn't allow the sudden change in voltage ($dt \approx 0$)

$$i = C\frac{dv}{dt} = C\frac{\text{finited}}{\text{zero}}$$

$i \approx \infty$ not possible

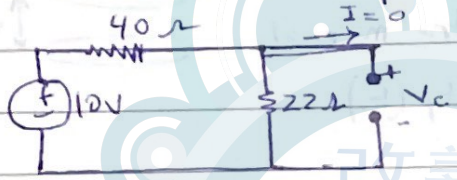
Examples-



Find 1- energy stored in the capacitor

2- q on the capacitor

- in DC circuit the capacitor act like open circuit

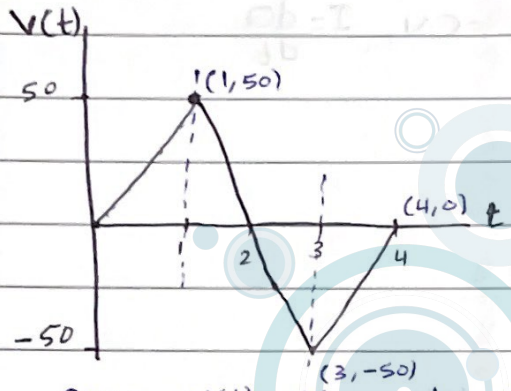


1- $E = \frac{1}{2}CV^2 = \frac{1}{2}(9.8\times10^{-3})(4.26)^2 = 88.9$ mJoul

by voltage division $V = \frac{10\times22}{22+40} = 4.26$ volt

2- $Q = CV = 9.8\times10^{-3} \times 4.26 = 41.75$ mC

IMP & exam question



$C = 200\,mF$

sketch the current waveform

assum $v(t) = Y$, $t = X$

$$i(t) = \begin{cases} 50 & (0<t<1) \\ -50 & (1<t<3) \\ 50 & (3<t<4) \end{cases} \times 200\times10^{-3} = \begin{cases} 10mA \\ -10mA \\ 10mA \end{cases} \Rightarrow$$ cont.

from $0<t<1$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \Rightarrow y - 0 = \frac{50}{1}(x - 0)$$

$$y = 50x \Rightarrow \underline{i = 50t}$$

from $1<t<3$

$$y - 50 = \frac{-50-50}{3-1}(x-1)$$

$$y - 50 = -50x + 50$$

$$y = -50x + 100 \Rightarrow \underline{i = -50t + 100}$$

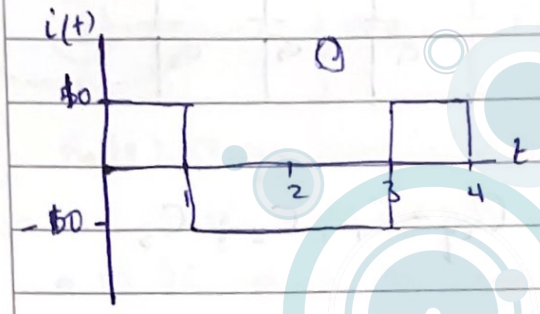
from $3<t<4$

$$y + 50 = \frac{50}{1}(x-3)$$

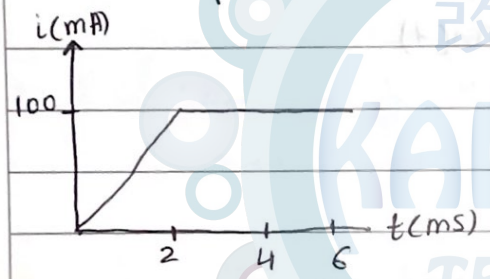
$$y = 50x - 150 - 50 \Rightarrow \underline{i = 50t - 200}$$

$$i(t) = C\frac{dv}{dt}$$

the current waveform is -



Practice problem 6.4



calculate the voltage across it at $t = 2$ ms & $t = 5$ ms

$0 < t < 2 \quad y - 0 = \frac{100}{2}(x - 0) \Rightarrow y = 50x \quad i = 50t$

$0 < t < 6 \quad i = 100$ ms

$$\begin{cases} 50t & 0 < t < 2 \\ 100 & t > 2 \end{cases}$$

1- $\frac{1}{1 \times 10^{-3}} \left[\frac{50}{2} t^2 \right]_0^2 = 10$ mV

2- $\frac{1}{1 \times 10^{-3}} \left[100 \times 10^{-3} t \right]_0^t + 100 \times 10^{-3} = 0.4$ mV

Practice problem 6.5

To find V_2 :-

$3+6 = \underline{9\,k\Omega}$ This resistor is parallel to the capacitor V_2

So $V_2 = 9k\Omega \times I = 45$ volt

KVL $\Rightarrow I = 5$ mA

Or

we can use voltage divider

$$V_2 = \frac{50 \times 9}{9+1} = 45 \text{ volt}$$

To find V_1 &

also we can use voltage divider

$$V_1 = \frac{50 \times 3}{3+6+1} = 15 \text{ volt}$$

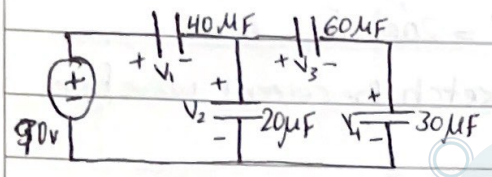
the question want us to find the energy stored in the capacitors

$$E = W = \frac{1}{2} QV = \frac{1}{2} CV^2$$

1 $\frac{1}{2} \times 30\times10^{-6} \times (15)^2 = 3375\times10^{-6}$ J

2 $\frac{1}{2} \times 20\times10^{-6} \times (45)^2 = 20250\times10^{-6}$ J

practice problem 6.7



$Q = CV, \quad I = \frac{dQ}{dt}$

1- Total charge

60 & 30 series $\frac{60 \times 30}{60+30} = 20\ \mu F$

20 // 20 parallel $20+20 = 40\ \mu F$

40 & 40 series $\frac{40 \times 40}{40+40} = 20\ \mu F$

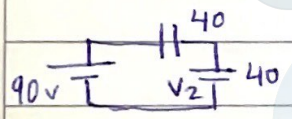
$C_{eq} = 20\ \mu F$

$Q = CV = 20\ \mu F \times 90 = 1800\ \mu C$

To find V_1 &-

$V_1 = \frac{Q}{C} = \frac{1800\ \mu C}{40 \times 10^{-6}} = 45\ V$

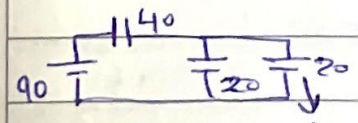
To find V_2 &-



$V_2 = \frac{Q}{C} = \frac{1800\ \mu C}{40 \times 10^{-6}} = 45\ V$

To find V_3 & V_4 &-

$V_2 = V_3 + V_4$



for V_3 & V_4

$Q = 20 \times 10^{-6} \times 45 = 900\ \mu C$

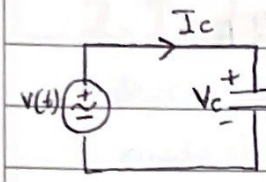
at 60 μF capacitor

$V_3 = \frac{Q}{C} = \frac{900\ \mu C}{60\ \mu F} = 15\ volt$

at 30 μF capacitor

$V_4 = \frac{900\ \mu C}{30\ \mu F} = 30\ volt$

Ex:-



$v(t) = 12e^{-3t}$

Find: 1- $i_c(t)$ 2- $i_c(0)$ 3- $i_c(\infty)$

$i_c(t) = C\frac{dv_c(t)}{dt}$, $v_c(t) = v(t) = 12e^{-3t}$

because they are in parallel

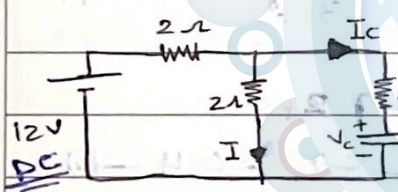
1. $i_c(t) = 5\times10^{-6} \times (12)(-3)e^{-3t}$

$= -180e^{-3t}\ \mu A$

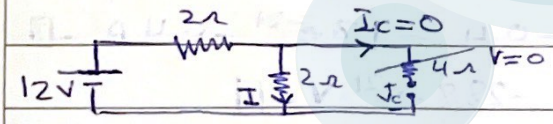
2. $i_c(0) = -180e^{-3(0)}\ \mu A = -180\ \mu A$

3. $i_c(\infty) = -180e^{-3(\infty)}\ \mu A = 0\ A$

Ex:-



Note that the capacitor in DC circuit act like open circuit so $I_c = 0A$



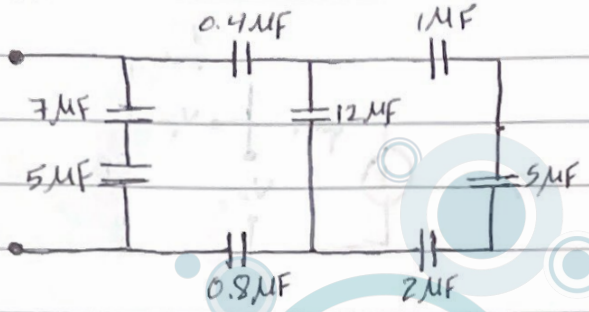
by KVL : $-12 + 2I_2 + 2I_2 = 0$

$I_2 = 3A$

$V_{(2\Omega)} = 2\times3 = 6$ volt

$V_{2\Omega} = V_c = 6$ volt

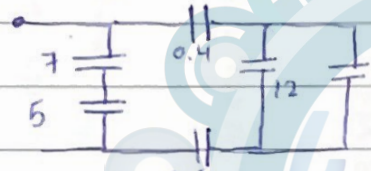
Ex8-



Find Ceq:-

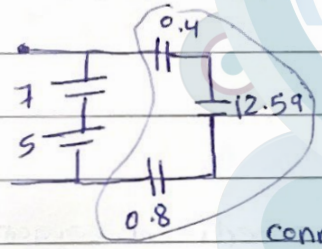
1MF & 5MF & 2MF are connected in series

$$\frac{1}{1\times10^{-6}} + \frac{1}{5\times10^{-6}} + \frac{1}{2\times10^{-6}} = \frac{1}{Ceq} \qquad Ceq = 0.59\,\mu F$$



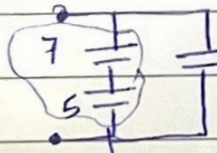
0.59 μF || 12 μF

0.59 + 12 = 12.59 μF



connected in series

$$\frac{1}{0.4\,\mu F} + \frac{1}{12.59\,\mu F} + \frac{1}{0.8\,\mu F} = \frac{1}{Ceq} \qquad Ceq = 0.261\,\mu F$$



$$\text{Series} = \frac{7\times5}{7+5} = 2.92\,\mu F$$

$$2.92 + 0.261 = 3.18\,\mu F$$

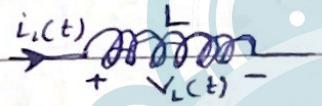
2. Inductor

1- device (element) that store energy

2- made of winding a ~~coil~~ coil of wire around a core.

⇒ core could be insulator or fercomagnetic material that have μ (permability)

3- Symbol



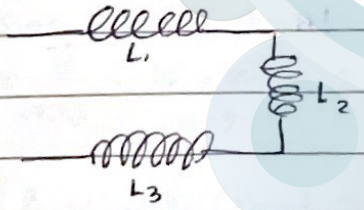
(Henry)

4- $V_L(t) = L \frac{di_L(t)}{dt}$ ، $i_L(t) = \frac{1}{L}\int_{t_0}^{t} V_L(\tau)d\tau + i_L(t_0)$

5- Energy stored : $E = \frac{1}{2} L I^2$ (Joul)

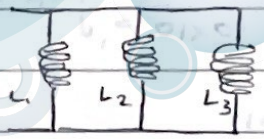
6- In DC-circuit, the inductor act like Short circuit $V_L = 0$

Inductors in series



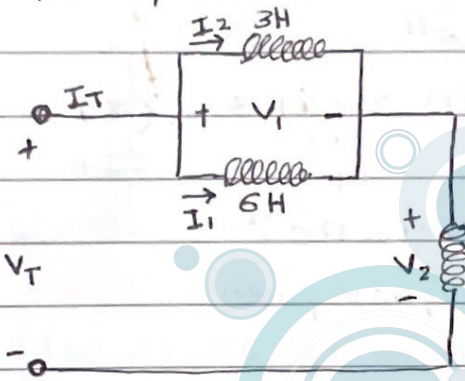
$L_{eq} = L_1 + L_2 + L_3$

Inductors in parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Practice problem 6.12



$I_1(t) = 0.6e^{-2t}$ A

If $i(0) = 1.4$A

Find:

1- $I_2(0)$ 3- $I(t)$ 5- $V_2(t)$

2- $I_2(t)$ 4- $V_1(t)$ 6- $v(t)$

$I_T = I_1 + I_2$

$1.4 = 0.6e^{-2(0)} + I_2 \Rightarrow I_2 = 0.8$ A — [1]

To find $I_2(t)$ we should have the value of v

$V_1 = 6 \times 0.6 \times -2e^{-2t} = -7.2e^{-2t}$ volt — [4]

$I_2(t) = \frac{1}{H}\int_0^t v(\tau)\,d\tau + I_2(0) = \frac{1}{3}\int_0^t -7.2e^{-2t}\,dt + 0.8$

$\frac{-7.2}{-6}\left[\frac{e^{-2t}}{-2} + 1\right] + 0.8 = 1.2e^{-2t} + 1.2 + 0.8$

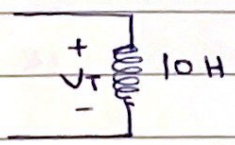
$= 1.2e^{-2t} + 3.2 - 0.4$ — [2]

$I(t) = I_1 + I_2$

$= 0.6e^{-2t} + 1.2e^{-2t} - 0.4 = 1.8e^{-2t} - 0.4$ A — [3]

$V_2 = 8 \times 1.8 \times -2 \times e^{-2t} = -28.8e^{-2t}$ V — [5]

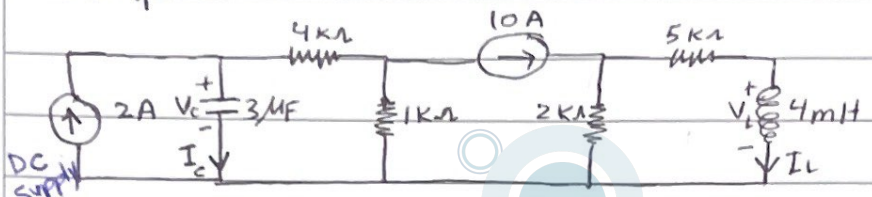
for V_T



$V = 10 \times 1.8 \times -2 \times e^{-2t}$

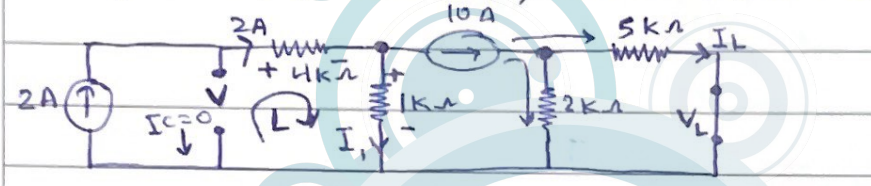
$= -36e^{-2t}$ V — [6]

Example :-



Find I_c, V_c, I_L, V_L :-

capacitor → open circuit , inductor → short circuit



by using KCL :-

$2 = 10 + I_1 \Rightarrow I_1 = -8A$

by KVL

$-V_c + 4(2) + (-8) = 0 \Rightarrow V_c = 0$ volt

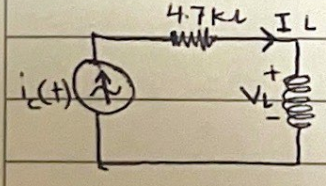
& $I_c = 0$ (open circuit)

using current division (current & two parallel resistors)

$$I_{(5\Omega)} = \frac{10 \times 1/5}{1/5 + 1/2} = 2.86\ A = I_L$$

& $V_L = 0$ (short circuit)

Example :-

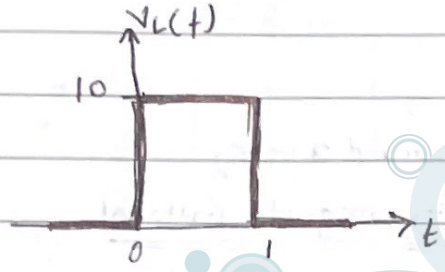


$i_s(t) = 4e^{-t}$, Find Energy Stored in the inductor

$$E = \frac{1}{2} L I^2 = \frac{1}{2} \times 12 \times 10^{-9} \times (4e^{-t})^2$$

$$= 96 \times 10^{-9} e^{-2t}\ \text{Joul}$$

Examples.



$L = 10\,mH$

$i_L(0) = 0\,A$

Find $i_L(t) = ?$

$$i_L(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i_L(t_0)$$

for $t < 0 \longrightarrow i_L(t) = 0$

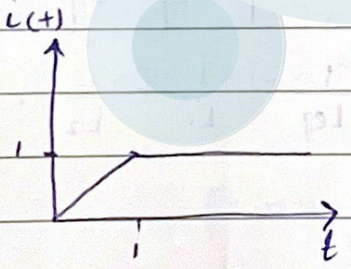
for $0 < t < 1 \Longrightarrow V_L(t) = 10\,mV$

$$i_L(t) = \frac{1}{10\times10^{-3}}\,10t\times10^{-3}\Big]_0^t = t$$

for $t > 1 \longrightarrow V_L(t) = 0$

$$i_L(1) = 1A$$

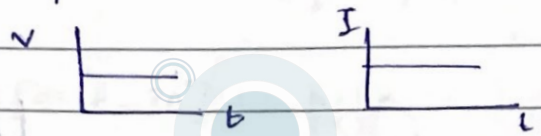
$$i_L(t) = \frac{1}{10\times10^{-3}}\int_1^t 0\,d\tau + 1 = 1A$$



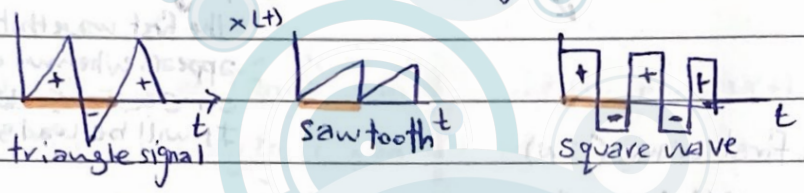
Chapter 9 : Sinusoids & Phasors

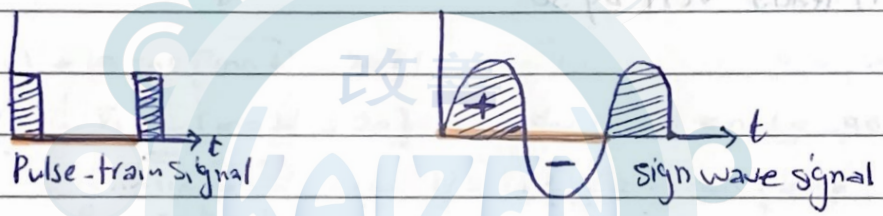
Time Dependent signal sources:-

DC. - sources → fixed value & direction



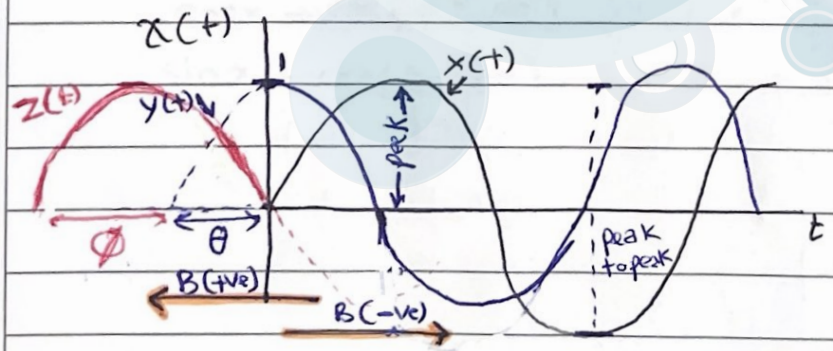
AC. - sources → generate voltage or current varying with time





periodic signal : It's a signal Repeat it self after one periode (T)

Sinusoidal signal :-



$X(t) = A \cos(\omega t - \theta)$

$Y(t) = B \cos(\omega t)$

$Z(t) = C \cos(\omega t + \phi)$

A: amplitude of signal (peak value)

peak to peak = 2 x peak

= 2 x A

cont. →

ω: radian freq → unit rad/s

$\omega = 2\pi f$ $\quad$ f: natural freq (Hz)

$T = \frac{1}{F}$ (sec)

θ : phase shift

$\theta = \frac{\Delta T \text{ (distance between two signal in sec)} \times 360^\circ}{T \text{ (period)}}$

$\sin(\omega t) = \cos(\omega t - 90)$

Example:

$i(t) = 10\cos(377t + 30^\circ)$ → moved to left

$I_{peak} = 10\ A$ $\quad$ $I_{peak\ to\ peak} = 2 \times 10 = 20\ A$

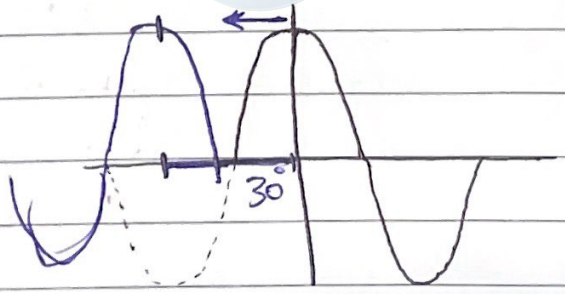
$\theta = 30^\circ$

$\omega = 377$ rad/s

$F = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60$ Hz

$T = \frac{1}{F} = \frac{1}{60} = 0.016$ sec

$i(0.1\text{ sec}) = 10\cos(377(0.1) + 30^\circ) = 7.8\ A$



To compare the signals (leading or lagging)
They should have the same frequency & same formulas

examples :-

$v(t) = 10 \cos[400t - 30^\circ]$
$i(t) = 3 \cos[600t - 60^\circ]$ NO

$v(t) = 10 \cos[300t + 60^\circ]$ $i(t)$ leads $v(t)$ by 10°
$i(t) = 5 \cos[300t + 70^\circ]$ ✓

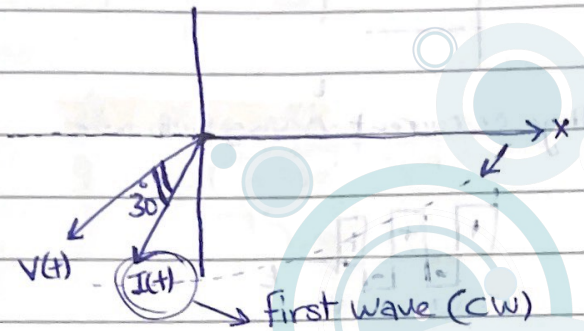
$v(t) = 10 \cos[100t - 30]$
$i(t) = 5 \sin[100t - 70]$
I can transfer from sin to cos

$\sin(x) = \cos(x - 90)$
$i(t) = 5 \cos(100t - 160)$
$v(t)$ leads $i(t)$ by 130°
$i(t)$ lags $v(t)$ by 130°

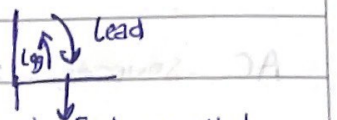
$-\sin x = \sin(x \mp 180^\circ)$
$-\cos x = \cos(x \mp 180^\circ)$
$\sin x = \cos(x - 90^\circ)$

Compare between $V(t) = -10\cos(\omega t + 50^\circ)$, $I(t) = 12\sin(\omega t - 10^\circ)$

$V(t) = 10\cos(\omega t - 130^\circ)$ $I(t) = 12\cos(\omega t - 100^\circ)$



I(t) leads V(t) by 30°



the first wave that appears whenwe go on clockwise direction It will be leads.

Complex Numbers:

j or $i = \sqrt{-1}$

$(j)^2 = -1$

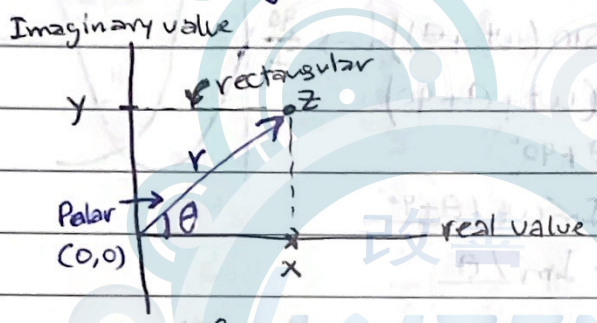
Complex number has two forms

1- Rectangular form

$Z = x + jy$

2- polar form

$Z = r \angle \theta$



convertion from Rect. to polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ "Just for 1st & 4th Quarter"

convertion from polar to Rect.

$x = r\cos\theta$

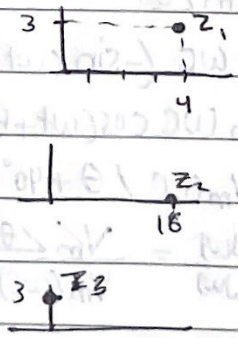
$y = r\sin\theta$

example:-

$Z_1 = 4 + 3j$ (4: real, 3: imaginary)

$Z_2 = 16$ (real)

$Z_3 = 3j$



MATHMATICAL:-

* Addition

For example $X_1 = a + jb \quad X_2 = c + jd$

$$X_1 + X_2 = (a+jb) + (c+jd)$$
$$= (a+c) + j(b+d)$$

* Subtraction

$$X_1 - X_2 = (a+jb) - (c+jd)$$
$$= (a-c) + j(b-d)$$

* Multiplication

$$X_1 \times X_2 = (a+jb) * (c+jd)$$
$$= ac + jda + jbc - bd$$
$$= (ac - bd) + j(da + bc)$$

* division

$$\frac{X_1}{X_2} = \frac{a+jb}{c+jd} \times \left(\frac{c-jd}{c-jd}\right)$$

$$\frac{ac+bd}{c^2+d^2} + j\,\frac{bc-ad}{c^2-d^2}$$

For polar

$$\left.\begin{aligned} X_1 &= r_1 \angle\theta_1 \\ X_2 &= r_2 \angle\theta_2 \end{aligned}\right\} \Rightarrow$$

$$X_1 * X_2 = (r_1\angle\theta_1)(r_2\angle\theta_2)$$
$$= r_1 r_2 \angle\theta_1 + \theta_2$$

$$\frac{X_1}{X_2} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle\theta_1 - \theta_2$$

$$X_1 \pm X_2 = \underbrace{r_1\angle\theta_1 \pm r_2\angle\theta_2}_{\text{convert to rect.}}$$

Sinusoidal signals in Freq. domain

time domain $\rightarrow x(t) = A\cos(\omega t + \theta)$

Freq. domain $\rightarrow \bar{X}(j\omega) = A \angle\theta$ A: peak value θ: phase shift ref. to cosine signal

Examples

$V_1(t) = 15\cos(377t + \frac{\pi}{4})$,

$V_2(t) = 15\cos(377t + \frac{\pi}{12})$

Find $V_s(t) = V_1(t) + V_2(t)$

$V_1(j\omega) = 15\angle\pi/4$ $V_2(j\omega) = 15\angle\pi/12$

$V_s = V_1(j\omega) + V_2(j\omega)$

$= 25.1 + j14.5 \xrightarrow{P} 28.98\angle\pi/6$

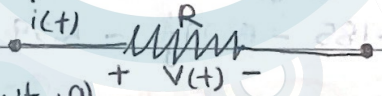
$V_s(t) = 28.98\cos(377t + \frac{\pi}{6})$

Impedance:

→ we will describe the circuit element (R, L, C) in AC circuit

« they will have impedance »

1- Resistor



if $v(t) = V_m\cos(\omega t + \theta)$

$\Rightarrow \bar{V}(j\omega) = V_m\angle\theta$

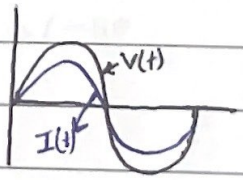
to find $I(j\omega) = \frac{V(j\omega)}{R} = \frac{V_m\angle\theta}{R} = \frac{V_m}{R}\angle\theta$

$I(t) = \frac{V_m}{R}\cos(\omega t + \theta)$

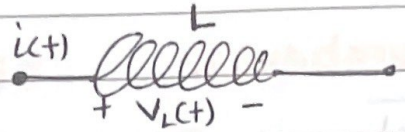
same phase shift between voltage & current

$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{V_m\angle\theta}{\frac{V_m}{R}\angle\theta} = R\angle 0 \rightsquigarrow$ there is no phase shift

$Z_R(j\omega) = R$



2- Inductor



voltage : $V(t) = L \frac{di_L(t)}{dt}$

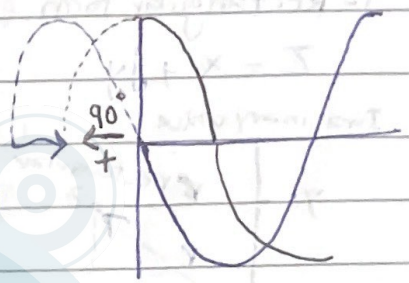
if $i_L(t) = I_m \cos(\omega t + \theta)$

$i_L(j\omega) = I_m \angle\theta$

so $V_L(t) = I_m(\omega L)\,(-\sin(\omega t + \theta))$

$= I_m \omega L \cos(\omega t + \theta + 90)$

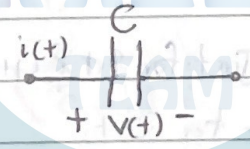
$V(j\omega) = I_m \omega L \angle\theta + 90^\circ$



$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{I_m \omega L \angle\theta + 90^\circ}{I_m \angle\theta}$

$Z(j\omega) = \omega L \angle\theta + 90 - \theta = \omega L \angle 90^\circ = j\omega L$

3- capacitor



voltage : $\frac{1}{C}\int_0^t i(\tau)\,d\tau$

if $V_c(t) = V_m \cos(\omega t + \theta)$

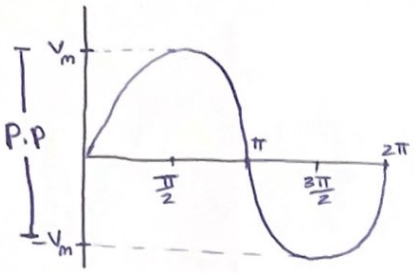
$V(j\omega) = V_m \angle\theta$

so $I_c(t) = V_m \omega C\,(-\sin(\omega t + \theta))$

$= V_m \omega C \cos(\omega t + \theta + 90^\circ)$

$I_c(j\omega) = V_m \omega C \angle\theta + 90^\circ$

$Z_c(j\omega) = \frac{V_c(j\omega)}{I_c(j\omega)} = \frac{V_m \angle\theta}{V_m(\omega C)\angle\theta + 90} = \frac{1}{C\omega} \angle\theta - \theta - 90 = \frac{1}{C\omega} \angle -90$



$V(t) = V_m \sin(\omega t + \phi)$

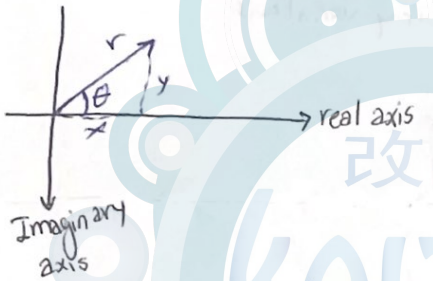
the begining of the wave

If we want to compare two waves
- same ω
- same sin, cos
- positive (+)

$\sin \rightarrow \cos \quad \phi - 90°$
$\cos \rightarrow \sin \quad \phi + 90°$
$-\sin \rightarrow \sin \quad \phi \pm 180°$
$-\cos \rightarrow \cos \quad \phi \pm 180°$

phasor (complex number)

$A + jB \rightarrow$ Rectangular , $r\angle\theta \rightarrow$ Polar



$r = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$
$x = r\cos\phi$
$y = r\sin\phi$

$Z_1 \mp Z_2$ (must be rectangular form) $= (x_1 + jy_1) \mp (x_2 + jy_2)$
$(x_1 + x_2) \mp j(y_1 + y_2)$

$Z_1 * Z_2$ (must be in polar form) $= r_1\angle\theta_1 * r\angle\theta_2$
$r_1 * r_2 \quad \angle\theta_1 + \theta_2$

$\frac{Z_1}{Z_2}$ (must be in polar form) $= \frac{r_1}{r_2} \angle\theta_1 - \theta_2$

$\frac{1}{Z} = \frac{1}{r\angle\theta} = \frac{\angle -\theta}{r}$

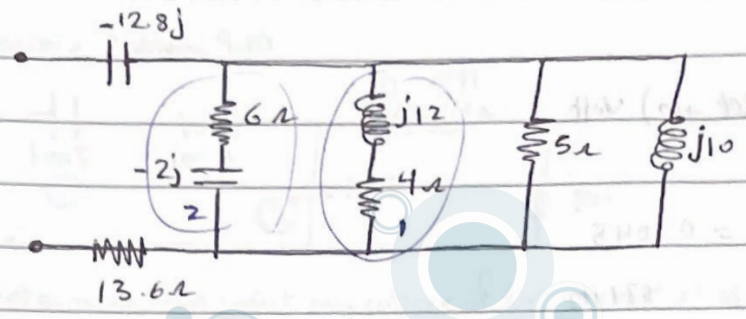
$\sqrt{Z} = \sqrt{r\angle\theta}$ must be in polar $= \sqrt{r}\ \angle\theta/2$

$Z^* = X \ominus jy$ (convert the sign from + to − & vice versa)

$= r\angle\theta \longrightarrow r\angle\ominus\theta$

$j = \frac{1}{-j}$; $-j = \frac{1}{j}$; $\frac{1}{j} = -j$; $\frac{1}{-j} = j$

$\int v\, d(t) = \frac{V}{j\omega}$; $\frac{dv}{dt} = j\omega v$



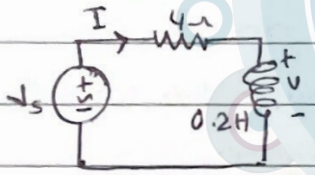
1: $4+12j$

2: $6-2j$

$$\frac{1}{Zeq} = \frac{1}{j10} + \frac{1}{5} + \frac{1}{4+12j} + \frac{1}{6-2j} \Rightarrow Z_{eq} = 2.4+0.8j$$

$$(-12.8j)+(2.4+0.8j)+(13.6) = 16-12j\ \Omega$$

practice problem 9.9



determine $v(t)$ & $I(t)$

$v_s = 20\sin(10t+30°)V$

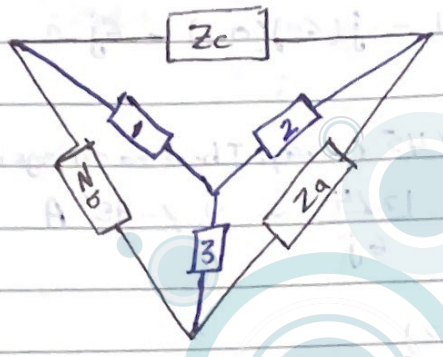
$X_L = j\omega L = j(10)(0.2) = j2\Omega$

$$I = \frac{V_s}{4+2j} = \frac{20\angle 30°}{4+2j} = 4.47\angle 3.43°$$

$$V = \frac{20\angle 30}{4+2j} \times 2j = 8.944\angle 93.43° \quad \text{OR} \quad 4.47\angle 3.43° \times 2j$$

$I(t) = 4.47\sin(10t+3.43°)\ A$

$v(t) = 8.944\sin(10t+93.43°)\ V$



balanced $\Rightarrow Z_1 = Z_2 = Z_3$

equal in magnitude

$\Delta \rightarrow Y \quad Z_Y = \frac{1}{3} Z_\Delta$

$\Delta \rightarrow Y$

$$Z_1 = \frac{Z_c Z_b}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

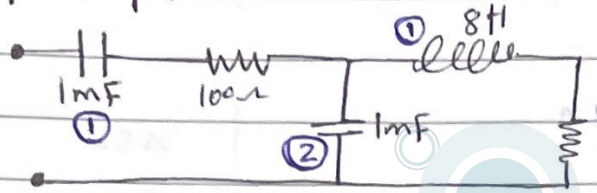
$Y \rightarrow \Delta$

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$

Practice problem 9.10



Determine the input impedance of the circuit at $\omega = 10$ rad/s

$$X_{C1} = \frac{1}{j\omega C} = \frac{1}{j(10)(1\times10^{-3})} = -100j$$

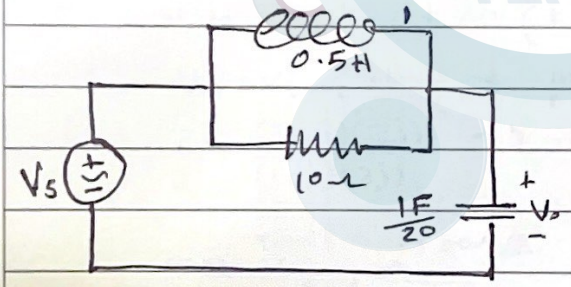
$$X_{C2} = \frac{1}{j(10)(1\times10^{-3})} = -100j$$

$$X_L = jL\omega = j(8)(10) = 80j$$

$$200 + 80j \;||\; -100j = 49.5 - 95.05j$$

$$-100j + 100 + (49.5 - 95.05j) = 149.5 - 195j$$

Practice problem 9.11



$V_s = 50\cos(10t + 30°)$

Calculate V_o ?

$$X_L = j\omega L = j(10)(0.5) = 5j$$

$$Z = 2 + 4j$$

$$X_C = \frac{1}{C\omega j} = \frac{1}{1/20\,(10)\,j} = -2j$$

$$V_o = \frac{V_s \times X_C}{X_C + Z}$$

$$= \frac{50\angle 30° \times (-2j)}{(-2j) + 2 + 4j}$$

$$= 35.36\angle -105$$

$$= 35.36\cos(10t - 105)$$

More problemss.

$V(t) = 50\cos(30t + 10)$ Volt

$V_m = 50$ volt

$T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = 0.204\,S$

$F = 2\pi\omega = 2\pi \times 30 = 4.77$ Hz

$V(t)$ at 10 ms $= 50\cos(\underbrace{30 \times 10 \times 10^{-3}}_{\text{rad/s}} + \underbrace{10}_{\text{deg}})$ volt

I can't sum them up because they are different

$$\text{rad} = \frac{\pi}{180^\circ} \times \text{deg.}$$

$$\text{deg} = \frac{180}{\pi} \times \text{rad.}$$

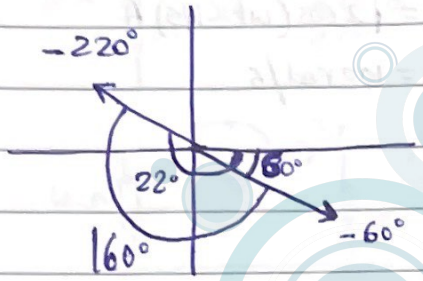
$$\delta_0 = \frac{180}{\pi} \times 30 \times 10 \times 10^{-3} = 17.19$$

$$50\cos(\underbrace{17.9 + 10}_{\text{deg}}) = 44.19 \text{ volt}$$

$v(t) = 10\cos(4t - 60°)$, $I(t) = -4\sin(4t + 50°)$

$= 4\sin(4t - 130°)$

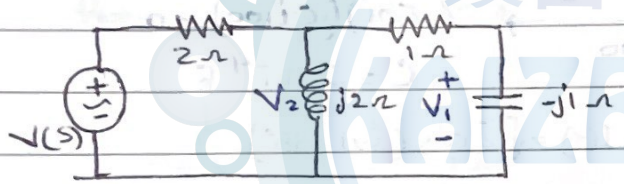
$= 4\cos(4t - 220°)$



v(t) leads I(t) by 160°

problem 9.44

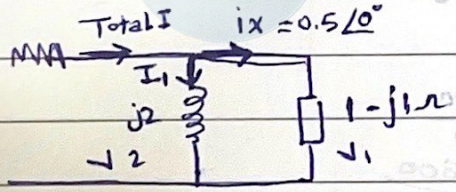
Find v(s) in the circuit if the current $i_x = 0.5\sin(200t)$ A through the 1 Ω resistor.

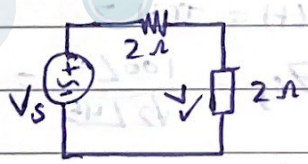


$V_1 = IZ = 0.5\angle 0\,(1 - j) = 0.707\angle -45$ volt

$V_1 = V_2$ (they are parallel)

$Z = \frac{(1-j)(2j)}{(1-j)+(2j)} = 2\,\Omega$



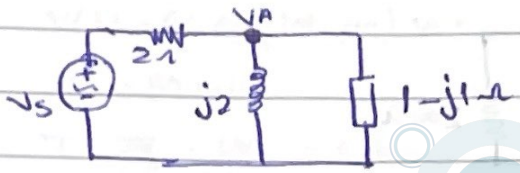


to find $I_1 = \frac{0.707\angle -45}{2j} = 0.3535\angle -135°$

$I_{Total} = I_1 + I_x = 0.35\angle -135 + 0.5\angle 0 = 0.35\angle -44.99$

$-V_s + 2(0.35\angle -45) + 2(0.707\angle -45) = 1.41\angle -45°$

another answer for problem 9.49



If we want to take KCL at node V_A

$$\sum I_{in} = \sum I_{out}$$

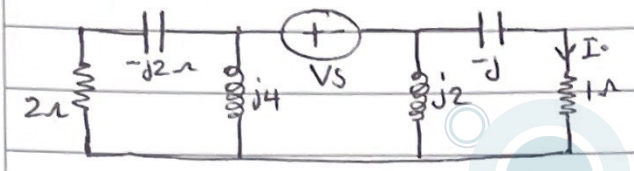
$$\frac{V_S - V_A}{2} = \frac{V_A}{j2} + \frac{V_A}{1-j1}$$

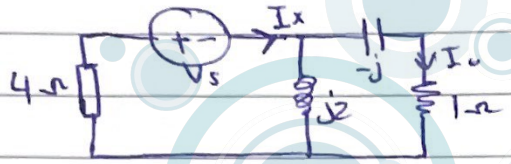
noting that $V_A = IZ = 0.5\angle 0^\circ (1-j1) = 0.5 - j0.5$ volt

$$\frac{V_S}{2} - \frac{0.5-j0.5}{2} = \frac{0.5-j0.5}{j2} + 0.5\angle 0^\circ$$

$$V_S = 1 - j \text{ volt} = 1.414\angle -45^\circ$$

problem 9.54





To find Ix we have to use current divider

$$I_x = I_x \left(\frac{j2}{j2 + (1-j)} \right) = 1-j$$

then $(1-j) \,||\, (2j) = 2\Omega$

using KVL:-

$$4I_x + V_s + 2I_x = 0$$

$$-V_s = I_x(4+2)$$

$$V_s = -6I_x$$

$$V_s = 8.49 \angle 135^\circ$$