

Weekly Notebook

OR1

Second
Semester
2024



Operation Research problems can be classified into:-

- 1- Deterministic model: All parameters are known with certainty.
- 2- Probabilistic model: The occurrence of specific event can't be perfectly predicted.

3.1 The prototype example

1- Decision variables

x_1 : no. of batches of product 1 / week

x_2 : no. of batches of product 2 / week

2- objectives

(maximizing total profit)

$$\hookrightarrow \text{obj. func } \max Z = 3000x_1 + 5000x_2$$

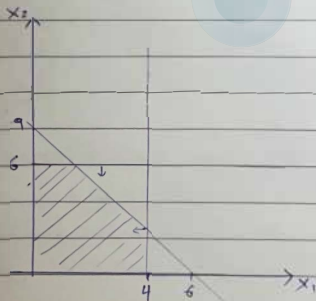
3- constraints

$$1x_1 + 0x_2 \leq 4$$

$$0x_1 + 2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$\underbrace{x_1 \geq 0, x_2 \geq 0}_{\text{non-negativity constraints}}$$



$$x_1 = 4, x_2 = 6$$

$$3x_1 + 2x_2 = 18$$

$$(x_1 = 6, x_2 = 9)$$

$$= (6, 0), (0, 9)$$

- the summation of all points in the feasible area $\rightarrow$ infinity no. of points
- the optimal solution is at one of the corner points.

We have in the previous example 5 corner points:

1- $(0, 0)$

2- $(4, 0)$

3- $(0, 6)$

4- ? } I don't have $\rightarrow$ so I should find them

5- ? } the coordinates $2x_2 = 12 \rightarrow x_2 = 6 \quad (6, 2)$

$3x_1 + 2x_2 = 18 \rightarrow x_1 = 2$

$x_1 = 4$

$3x_1 + 2x_2 = 18 \rightarrow x_2 = 3$

$(4, 3)$

substitute in the objective function:

- the first point $(0, 0) \rightarrow$ the profit will be zero

- the second point $(0, 6) \rightarrow 3000(0) + 5000(6) = 30,000$

I chose this point because of the higher contribution of x_2 in the profit (in the obj. function)

- $(2, 6) \rightarrow 3000(2) + 5000(6) = 36,000$ (OPTIMAL)

- $(4, 3) \rightarrow 3000(4) + 5000(3) = 27,000$

I can conclude that the optimal solution is 2 batches of product 1 and 6 batches of product 2

solving problem 3.1-7

1- decision variable

x_1 : number of wood-framed window to be produced per day

x_2 : number of aluminum-framed window to be produced per day

2- objective function

↳ maximize the profit

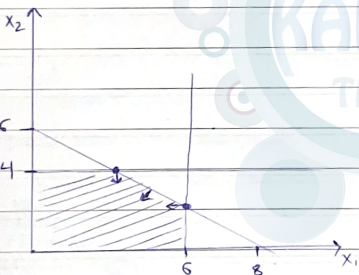
$$\max Z = 300x_1 + 150x_2$$

Subject to s-

$$x_1 \leq 6, \quad x_2 \leq 4$$

$$6x_1 + 8x_2 \leq 48$$

$$x_1 \geq 0, \quad x_2 \geq 0$$



to find the points:

$$x_1 = 6, \quad x_2 = 4$$

$$6x_1 + 8x_2 = 48$$

$$x_1 = 8, \quad x_2 = 6$$

the corner points that we have:

$$(0,0), (0,4), (6,0)$$

to find the other points by finding the intersection of the lines

$$x_2 = 4$$

$$6x_1 + 8x_2 = 48 \rightarrow x_1 = 2.67$$

$$x_1 = 6$$

$$6x_1 + 8x_2 = 48 \rightarrow x_2 = 1.5$$

to maximize the profit $\rightarrow (6, 1.5)$

$$\max Z = 300(6) + 150(1.5) = 2025$$

Design of Radiation Therapy

1- Decision variables

x_1 : dose in (kilorad) of beam 1

x_2 : dose in (kilorad) of beam 2

2- objective function

↳ minimize total dose absorbed by healthy anatomy

$$\text{Min } Z = 0.4x_1 + 0.5x_2$$

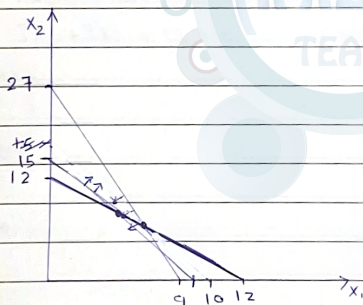
Subject to:

$$0.3x_1 + 0.1x_2 \leq 2.7$$

$$0.5x_1 + 0.5x_2 = 6$$

$$0.6x_1 + 0.4x_2 \geq 6$$

$$x_1 \geq 0, x_2 \geq 0$$



to find the points:-

$$0.3x_1 + 0.1x_2 = 2.7$$

$$x_1 = 9, x_2 = 27$$

$$(0, 27), (9, 0)$$

$$0.5x_1 + 0.5x_2 = 6$$

$$(12, 0), (0, 12)$$

$$0.6x_1 + 0.4x_2 = 6$$

$$(10, 0), (0, 15)$$

First intersection points

$$0.3x_1 + 0.1x_2 = 2.7$$

$$0.5x_1 + 0.5x_2 = 6$$

$$x_1 = 7.5, x_2 = 4.5 \rightarrow (7.5, 4.5)$$

$$\text{OP} \rightarrow Z = 0.4(7.5) + 0.5(4.5) = 5.25$$

$$Z = 0.4(6) + 0.5(6) = 5.4$$

sec. point :-

$$0.6x_1 + 0.4x_2 = 6$$

$$0.5x_1 + 0.5x_2 = 6$$

$$x_1 = 6, x_2 = 6$$

Solving 3-1-10

1- Decision Variable

x_1 : number of units of the hot dogs that need to be produced per week

x_2 : number of units of the hot dogs buns that need to be produced per week

2- Objective variable

$$\text{Max } Z = 0.88x_1 + 0.33x_2$$

subject to:

$$0.1x_2 \leq 200$$

$$0.25x_1 \leq 800$$

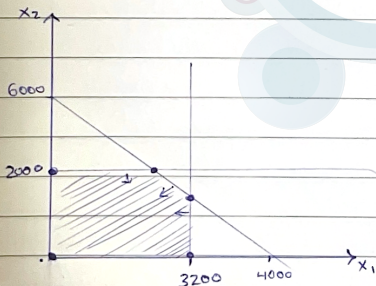
$$3x_1 + 2x_2 \leq 12000 \quad \begin{matrix} \downarrow \text{min} & \downarrow \text{min} & \downarrow 5 \text{ employee} & \downarrow 40 \text{ hr} \rightarrow \text{min} \\ & & & \rightarrow 40 \text{ hours per week} \end{matrix}$$

$$x_1 \geq 0, x_2 \geq 0$$

$$0.1x_2 = 200 \rightarrow x_2 = 2000$$

$$0.25x_1 = 800 \rightarrow x_1 = 3200$$

$$3x_1 + 2x_2 = 12000 \rightarrow (4000, 0), (0, 6000)$$



$$0.1x_2 = 200 \rightarrow x_2 = 2000$$

$$3x_1 + 2x_2 = 12000 \rightarrow 2666.67$$

$$(2666.67, 2000)$$

$$0.25x_1 = 800 \rightarrow x_1 = 3200$$

$$3x_1 + 2x_2 = 12000 \rightarrow x_2 = 1200$$

$$(3200, 1200)$$

$$\text{Max profit} = 0.88(3200) + 0.33(1200) = 3212$$

Reclaiming Solid Waste

Decision variables:

X_{A1} : amount in pound per week of waste material 1 that should be used to produce in grade A.

the same for X_{A2} , X_{A3} , X_{A4}

X_{B1} : amount in pound per week of waste material 1 that should be used to produce in grade B

the same for X_{B2} , X_{B3} , X_{B4}

X_{C1} : amount of waste material 1 in pound per week that should be used to produce in grade C.

the same for X_{C2} , X_{C3} , X_{C4}

OR

I can say that the Decision Variables is

X_{ij} : amount of material i to produced in grade j (pound/week)

$i: 1, 2, 3, 4$

$j: A, B, C$

Objective function:-

max. profit = (selling - amalgamation cost)

$$\text{Max } Z = (8.5 - 3) * (X_{1A} + X_{2A} + X_{3A} + X_{4A}) + (7 - 2.5) * (X_{1B} + X_{2B} + X_{3B} + X_{4B}) + (5.5 - 2) * (X_{1C} + X_{2C} + X_{3C} + X_{4C})$$

↓
selling
price

↓
amalgamation
cost

↓
total quantity of grade C

Subject to :-

$$X_{1A} \leq 0.3 * (X_{1A} + X_{2A} + X_{3A} + X_{4A})$$

$$X_{2A} \geq 0.4 * (X_{1A} + X_{2A} + X_{3A} + X_{4A})$$

$$X_{3A} \leq 0.5 * (X_{1A} + X_{2A} + X_{3A} + X_{4A})$$

$$X_{4A} = 0.2 * (X_{1A} + X_{2A} + X_{3A} + X_{4A})$$

$$X_{1B} \leq 0.5 (X_{1B} + X_{2B} + X_{3B} + X_{4B})$$

$$X_{2B} \geq 0.1 (X_{1B} + X_{2B} + X_{3B} + X_{4B})$$

$$X_{4B} = 0.1 (X_{1B} + X_{2B} + X_{3B} + X_{4B})$$

$$X_{1C} \leq 0.7 (X_{1C} + X_{2C} + X_{3C} + X_{4C})$$

$$1500 \leq X_{1A} + X_{1B} + X_{1C} \leq 3000$$

$$1000 \leq X_{2A} + X_{2B} + X_{2C} \leq 2000$$

$$2000 \leq X_{3A} + X_{3B} + X_{3C} \leq 4000$$

$$500 \leq X_{4A} + X_{4B} + X_{4C} \leq 1000$$

$$3(X_{1A} + X_{1B} + X_{1C}) + 6(X_{2A} + X_{2B} + X_{2C}) + 4(X_{3A} + X_{3B} + X_{3C}) \\ + 5(X_{4A} + X_{4B} + X_{4C}) = 30000$$

$$X_{ij} \geq 0$$

Solving 3.4-10

Decision Variables

X_{fm} = # of full time consultant must be hired to work during morning shift

X_{Fa} = # of full time = , , , , afternoon.

$X_{Fe} = \#$ of _____ evening

X_{pi} = # of part time consultant to be hired to work in shift i .

$X_{pr} = \dots = \dots = \dots = \dots = \text{shift}2$

$X_{p3} = \dots \dots \dots \text{shift 3}$

Xpu = ; ; ; ; ; ; shift 4

Objective Function :-

$$Z = 40 \times 8 \times (X_{Fm} + X_{Fa} + X_{Fe}) + 30 \times 4 \times (X_{P1} + X_{P2} + X_{P3} + X_{P4})$$

Subject to:

$$X_{fm} + X_{p1} \geq 4$$

$$X_{fm} + X_{fa} + X_{p2} \geq 8$$

$$X_{fa} + X_{fe} + X_{p3} \geq 10$$

$$X_{Fe} + X_{P4} \geq 6$$

$$x_{fm} \geq 2x_{pi}$$

$$X_{fm} + X_{fa} > 2X_{P2}$$

$$X_{Fe} + X_{Fe} \gg 2 X_{P_3}$$

$$x_{Fe} \geq 2x_{py}$$

$$x_{ij} \geq 0, \text{ for all } x_{ij}$$

Solving 3-4.14

Decision Variables:-

x_{ij} : amount of cargo i to be accepted and stored in the j compartment

$i : 1, 2, 3, 4$

$j : \text{Front, center, back.}$

Objective Function:-

$$Z = 320 (x_{1F} + x_{1C} + x_{1B}) + 400 (x_{2F} + x_{2C} + x_{2B}) + 360 (x_{3F} + x_{3C} + x_{3B}) + 290 (x_{4F} + x_{4C} + x_{4B})$$

Subject to :-

$$x_{1F} + x_{1C} + x_{1B} \leq 20$$

$$x_{2F} + x_{2C} + x_{2B} \leq 16$$

$$x_{3F} + x_{3C} + x_{3B} \leq 25$$

$$x_{4F} + x_{4C} + x_{4B} \leq 13$$

$$(x_{1F} + x_{2F} + x_{3F} + x_{4F}) \leq 12$$

$$(x_{1C} + x_{2C} + x_{3C} + x_{4C}) \leq 18$$

$$(x_{1B} + x_{2B} + x_{3B} + x_{4B}) \leq 10$$

$$500x_{1F} + 700x_{2F} + 600x_{3F} + 400x_{4F} \leq 7000$$

$$500x_{1C} + 700x_{2C} + 600x_{3C} + 400x_{4C} \leq 9000$$

$$500x_{1B} + 700x_{2B} + 600x_{3B} + 400x_{4B} \leq 5000$$

$$\frac{x_{1F} + x_{2F} + x_{3F} + x_{4F}}{12} = \frac{x_{1C} + x_{2C} + x_{3C} + x_{4C}}{18} =$$

$$\frac{x_{1F} + x_{2F} + x_{3F} + x_{4F}}{12} = \frac{x_{1B} + x_{2B} + x_{3B} + x_{4B}}{10}$$

$$x_{ij} \geq 0 \quad \forall i, j$$

Solving 3-4.15

Decision variables:-

X_{ij} = number of hrs that i operator will work on j day.

$i = 1, 2, 3, 4, 5, 6$

$j = \text{mon, tue, thurs, } \overbrace{\text{Wed, Fri}}$

Objective $\rightarrow$ Min cost

$$\begin{aligned} \text{Min } Z = & (X_{11} + X_{12} + X_{13} + X_{14} + X_{15}) 25 + 26(X_{21} + X_{22} + X_{23} + X_{24} + X_{25}) \\ & + 24(X_{31} + X_{32} + X_{33} + X_{34} + X_{35}) + 23(X_{41} + X_{42} + X_{43} + X_{44} + X_{45}) + 28(X_{51} + X_{52} + X_{53} + X_{54} + X_{55}) \\ & + 30(X_{61} + X_{62} + X_{63} + X_{64} + X_{65}) \end{aligned}$$

$$X_{11} \leq 6, X_{12} \leq 0, X_{13} \leq 6, X_{14} \leq 0, X_{15} \leq 6$$

$$X_{21} \leq 0, X_{22} \leq 6, X_{23} \leq 0, X_{24} \leq 6, X_{25} \leq 0$$

$$X_{31} \leq 4, X_{32} \leq 8, X_{33} \leq 4, X_{34} \leq 0, X_{35} \leq 4$$

$$X_{41} \leq 5, X_{42} \leq 5, X_{43} \leq 5, X_{44} \leq 0, X_{45} \leq 5$$

$$X_{51} \leq 3, X_{52} \leq 0, X_{53} \leq 3, X_{54} \leq 8, X_{55} \leq 0$$

$$X_{61} \leq 0, X_{62} \leq 0, X_{63} \leq 0, X_{64} \leq 6, X_{65} \leq 2$$

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 12$$

$$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} = 12$$

and so on for all days.

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \geq 8$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \geq 8$$

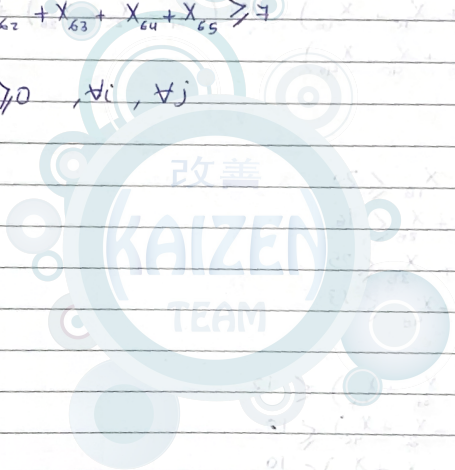
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \geq 8$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} \geq 8$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} \geq 7$$

$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} \geq 7$$

$$x_{ij} \geq 0, \forall i, \forall j$$



$$\begin{aligned} \text{max } z &= 100x_{11} + 120x_{12} + 150x_{13} + 180x_{14} + 200x_{15} \\ \text{max } z &= 120x_{21} + 150x_{22} + 180x_{23} + 200x_{24} + 220x_{25} \\ \text{max } z &= 150x_{31} + 180x_{32} + 200x_{33} + 220x_{34} + 250x_{35} \\ \text{max } z &= 180x_{41} + 200x_{42} + 220x_{43} + 250x_{44} + 280x_{45} \\ \text{max } z &= 200x_{51} + 220x_{52} + 250x_{53} + 280x_{54} + 300x_{55} \\ \text{max } z &= 220x_{61} + 250x_{62} + 280x_{63} + 300x_{64} + 320x_{65} \end{aligned}$$

$$s.t. \quad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 8$$

$$s.t. \quad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 8$$

" Chapter four "

Simplex Method:-

$$\text{Max } Z = 3x_1 + 5x_2$$

subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

1- I have to check that the mathematical model written in the standard form

- ↳ the objective function $\rightarrow$ maximization
- ↳ all constraints $\leq$ (+ve)
- ↳ all the decision variables should be ≥ 0 .

2- identifying an initial solution

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

$$(x_3, x_4, x_5) \geq 0$$

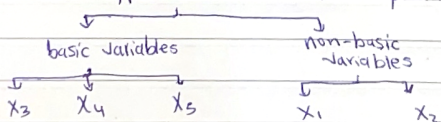
they called slack variable

$$(x_1, x_2) \geq 0$$

they called non basic variables

	2	x_1	x_2	x_3	x_4	x_5	RHS
R_0	1	3	-5	0	0	0	4
R_1	0	1	0	1	0	0	4
R_2	0	0	2	0	1	0	12
R_3	0	3	2	0	0	1	18

we have two types of variables in the simplex table



$$X_3 = 4, X_4 = 12, X_5 = 18$$

$X_1 = 0, X_2 = 0 \rightarrow$ always have zero value

3- optimality

in order to know if the solution we reach is optimal or not, we have to look at the objective row, then look at the most negative variable in this case $X_2 \Rightarrow$ entering variable

4- feasibility test

$$\frac{\text{RHS}}{\text{coef } X_2} \rightarrow \frac{4}{0} = \infty, \quad \frac{12}{2} = 6, \quad \frac{18}{2} = 9$$

↓
minimum

Z	X_1	X_2	X_3	X_4	X_5	RHS	
1	3	-5	0	0	0	4	$\rightarrow \frac{4}{-5}$
0	1	0	1	0	0	4	$\rightarrow \frac{4}{0} = \infty$
0	0	2	0	1	0	12	$\rightarrow \frac{12}{2} = 6$
0	3	2	0	0	1	18	$\rightarrow \frac{18}{2} = 9$

leaving variable: X_4 leader on the row.

entering var.

Z	X_1	X_2	X_3	X_4	X_5	RHS	
$5R_2 + R_1$	1	-3	0	2.5	0	30	Basic var
as it is	0	1	0	1	0	4	$X_3 = 4$
$\frac{R_2}{2}$	0	0	1	0	0.5	6	$X_2 = 6$
$+ 2R_2 + R_3$	0	3	0	0	-1	6	$X_5 = 6$

entering variable.

non basic var.
 $X_1 = 0, X_4 = 0$

	Z	x_1	x_2	x_3	x_4	x_5	RHS	
$R_3 + R_1$	1	0	0	0	1.5	1	36	Basic Variables
$-1R_3 + R_1$	0	0	0	1	0.33	-0.33	2	$x_3 = 2, x_2 = 6, x_1 = 2$
as it is	0	0	1	0	0.5	0	6	non basic Variables
R_3	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	$x_4, x_5 = 0$
3								

all values (coefficients) are non-negative in the obj. row

So that mean we are in the optimal solution.

$$Z = 36, \quad x_1 = 2, \quad x_2 = 6$$

Solving 4-3.4

$$\max Z - 4x_1 - 3x_2 - 6x_3 = 0$$

Subject to:

$$3x_1 + x_2 + 3x_3 + x_4 = 30$$

$$2x_1 + 2x_2 + 3x_3 + x_5 = 40$$

$$x_i \geq 0 \quad i=1,2,3,4,5$$

Z	x_1	x_2	x_3	x_4	x_5	RHS	RHS
1	-4	-3	-6	0	0	0	entering var
0	3	1	3	1	0	30	30/3
0	2	2	3	0	1	40	40/3

x_3 : entering variable, x_4 : leaving variable

	Z	x_1	x_2	x_3	x_4	x_5	RHS	
$6R_1 + R_0$	1	2	-1	0	2	0	60	entering var.
$R_1/3$	0	1	$1/3$	1	$1/3$	0	10	$10 \times 3 = 30$
$-3R_1 + R_2$	0	-1	1	0	-1	1	10	$10/1 = 10$

	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	1	0	0	1	1	70
$R_2 + R_1$	0	$4/3$	0	1	$2/3$	$-1/3$	$20/3$
	0	-1	1	0	-1	1	10

$$x_1 = 0, x_2 = 10, x_3 = \frac{20}{3}, x_4 = 0, x_5 = 0$$

$$\text{Obj} = 70$$

Recall the Partition therapy example

$$\min Z = 0.4x_1 + 0.5x_2$$

subject to:-

$$0.3x_1 + 0.1x_2 \leq 2.7$$

$$0.5x_1 + 0.5x_2 = 6$$

$$0.6x_1 + 0.4x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

First of all i have some problems!

$$\max -Z = -0.4x_1 - 0.5x_2 + Mx_{a1} - Mx_{a2}$$

$$\max -Z + 0.4x_1 + 0.5x_2 = 0$$

$$0.3x_1 + 0.1x_2 + x_3 = 2.7$$

$$0.5x_1 + 0.5x_2 + x_{a1} = 6$$

$$0.6x_1 + 0.4x_2 + x_4 + x_{a2} = 6$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0.4	0.5	0	0	M	M	0
0	0.3	0.1	1	0	0	0	2.7
0	0.5	0.5	0	0	1	0	6
0	0.6	0.4	0	-1	0	1	6

Restoring G.F:-

$$-MR_2 + R_0$$

$$-MR_3 + R_0$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS	
-1	0.4-1M	0.5-9M	0	+M	-M	0	-12M	
0	0.3	0.1	1	0	0	0	2.7	$\rightarrow 9$
0	0.5	0.5	0	0	1	0	6	$\rightarrow 12$
0	0.6	0.4	0	-1	0	1	6	$\rightarrow 10$

$\frac{R_1}{0.3}$
 $-0.5R_1 + R_2$
 $-0.6R_1 + R_3$
 $(0.4+1.M)R_1 + R_0$

- This table is not optimal $\rightarrow$ we have -ve value

- This table is not feasible $\rightarrow x_{a1} = 6, x_{a2} = 6$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	$\frac{11}{30} - \frac{8}{15}M$	$-\frac{4}{3} + \frac{11}{3}M$	M	0	0	$-3.6 - 2.1M$
0	1	$1/3$	$10/3$	0	0	0	9
0	0	$1/3$	$-5/3$	0	1	0	$3/2$
0	0	$1/5$	-2	-1	0	1	$3/5$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	$\frac{7}{3} - \frac{5}{3}M$	$\frac{11}{6} - \frac{5}{3}M$	0	$-\frac{11}{6} + \frac{8}{3}M$	$-\frac{47}{10} - \frac{1}{2}M$
0	1	0	$20/3$	$+5/3$	0	$-5/3$	8
0	0	0	$5/3$	$5/3$	1	$-5/3$	$1/2$
0	0	1	-10	-5	0	5	3

$$-\frac{1}{3}R_3 + R_2$$

$$-\frac{1}{3}R_3 + R_1$$

$$\left(-\frac{11}{30} + \frac{8}{15}M\right)R_3 + R_4$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	0.5	0	$-\frac{11}{10} + M$	M	-5.25
0	1	0	5	0	1	0	$15/2$
0	0	0	1	1	$3/5$	-1	$3/10$
0	0	1	-5	0	3	0	$9/2$



as long as
the artificial values
taking values equal to zero
then this table is feasible

Solving 3.4-5

$$\text{Min } Z = 3x_1 + 2x_2$$

Subjected to:-

$$x_1 + 2x_2 \leq 12$$

$$2x_1 + 3x_2 \leq 12$$

$$2x_1 + x_2 \geq 8$$

$$x_1, x_2 \geq 0$$

$$\text{Max } -Z = -3x_1 - 2x_2$$

$$\text{Max } -Z + 3x_1 + 2x_2 + Mx_{a1} + Mx_{a2} = 0$$

$$x_1 + 2x_2 + x_3 = 12$$

$$2x_1 + 3x_2 + x_{a1} = 12$$

$$2x_1 + x_2 + x_4 + x_{a2} = 8$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	3	2	0	0	M	M	0
0	1	2	1	0	0	0	12
0	2	3	0	0	1	0	12
0	2	1	0	-1	0	1	8

Restoring G.F. $\rightarrow ((R_2 + R_3)(-M)) + R_0$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	$3-4M$	$2-4M$	0	M	0	0	$-20M$
0	1	2	1	0	0	0	12
0	2	3	0	0	1	0	12
0	2	1	0	-1	0	1	8

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	$\frac{5-4M}{3}$	0	0	M	$-\frac{2}{3} + \frac{4}{3}M$	0	$-8-4M$
0	$-1/3$	0	1	0	$-2/3$	0	4
0	$2/3$	1	0	0	$1/3$	0	4
0	$4/3$	0	0	-1	$-1/3$	1	4

$((-2+4M)(R_2)) + R_0$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	0	$\frac{5}{4}$	$-\frac{1}{4}+M$	$-\frac{5}{4}+M$	-13
0	0	0	1	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	5
0	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	2
0	1	0	0	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	3

OPTIMAL + FEASIBLE



Recall the Radiation therapy example

First phase obj $\rightarrow$ min the art. var.

second phase obj $\rightarrow$ the original var. that we do have in the original prob.

$$\text{Min } Z = 0.4x_1 + 0.5x_2 + Mx_{a1} + Mx_{a2}$$

sub to

$$\begin{cases} 0.3x_1 + 0.1x_2 \leq 2.7 \\ 0.5x_1 + 0.5x_2 = 6 \\ 0.6x_1 + 0.4x_2 \geq 6 \end{cases}$$

$$\text{Max } -Z + 0.4x_1 + 0.5x_2 + Mx_{a1} + Mx_{a2} = 0$$

$$0.3x_1 + 0.1x_2 + x_3 = 2.7$$

$$0.5x_1 + 0.5x_2 + x_{a1} = 6$$

$$0.6x_1 + 0.4x_2 + x_4 + x_{a2} = 6$$

PHASE I: $\text{Min } Z = Mx_{a1} + Mx_{a2}$

PHASE II: $\text{Min } Z = 0.4x_1 + 0.5x_2 \rightarrow \text{Max } -Z + 0.4x_1 + 0.5x_2 = 0$

PHASE I

$$\text{Max } -Z + x_{a1} + x_{a2} = 0$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	0	0	1	1	0
0	0.3	0.1	1	0	0	0	2.7
0	0.5	0.5	0	0	1	0	6
0	0.6	0.4	0	-1	0	1	6

$$(\text{Restore G.F.}) \rightarrow -1(R_2 - R_3) + R_0$$

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	-1.1	-0.9	0	1	-0	-0	-12
0	0.3	0.1	1	0	0	0	2.7
0	0.5	0.5	0	0	1	0	6
0	0.6	0.4	0	-1	0	1	6

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	-8/15	11/3	1	0	0	-2.08 ←
0	1	1/3	10/3	0	0	0	9
0	0	1/3	-5/3	0	1	0	3/2
0	0	1/5	-2	-1	0	1	3/5

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	-5/3	-5/3	0	5/3	-1/2
0	1	0	20/3	5/3	0	-5/3	8
0	0	0	5/3	5/3	1	-5/3	1/2
0	0	1	-10	-5	0	5	3

Z	x_1	x_2	x_3	x_4	x_{a1}	x_{a2}	RHS
-1	0	0	0	0	1	1	0
0	1	0	0	-5	-4	5	6
0	0	0	1	1	3/5	-1/5	3/10
0	0	1	0	5	6	-5	6

$$x_1 = 6, x_2 = 6$$

PHASE II

Z	x_1	x_2	x_3	x_4	RHS
-1	0.4	0.5	0	0	0
0	1	0	0	-5	6
0	6	0	1	1	0.3
0	0	1	0	5	6

$$\leftarrow \text{Max } -Z + 0.4x_1 + 0.5x_2 = 0$$

(Restoring G.F) $\rightarrow (-0.4R_1 - 0.5R_3) + R_0$

Z	x_1	x_2	x_3	x_4	RHS
-1	0	0	0	-0.5	-5.4
0	1	0	0	-5	6
0	0	0	1	1	0.3
0	0	1	0	5	6

It's feasible but not optimal.

Z	x_1	x_2	x_3	x_4	RHS
-1	0	0	0.5	0	-5.25
0	1	0	5	0	7.5
0	0	0	1	1	0.3
0	0	1	-5	0	4.5

example 4.1-7 (solving two phase method)

$$\text{Min } Z = 5x_1 + 7x_2$$

Sub to:

$$2x_1 + 3x_2 \geq 42$$

$$3x_1 + 4x_2 \geq 60$$

$$x_1 + x_2 \geq 18$$

$$x_1 \geq 0, x_2 \geq 0$$

$$\text{Max } -Z = -5x_1 - 7x_2 \quad Mx_1 = -x_{a1} \quad Mx_2 = -x_{a2} \quad Mx_3 = -x_{a3}$$

$$\longrightarrow 2x_1 + 3x_2 + x_3 + x_{a1} = 42$$

$$\longrightarrow 3x_1 + 4x_2 - x_4 + x_{a2} = 60$$

$$\longrightarrow x_1 + x_2 - x_5 + x_{a3} = 18$$

PHASE I:-

$$\text{Min } Z = x_{a1} + x_{a2} + x_{a3}$$

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	0	0	0	0	0	1	1	1	0
0	2	3	-1	0	0	1	0	0	42
0	3	4	0	-1	0	0	1	0	60
0	1	1	0	0	-1	0	0	1	18

Restoring G.F $-(R_1 + R_2 + R_3) + R_0$

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	-6	-8	1	1	1	0	0	0	-120
0	2	3	-1	0	0	1	0	0	42
0	3	4	0	-1	0	0	1	0	60
0	1	1	0	0	-1	0	0	1	18

$$R_1/3, 8R_1 + R_0, -4R_1 + R_2, -R_1 + R_3$$

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	$-2/3$	0	$-5/3$	1	1	$8/3$	0	0	-8
0	$2/3$	1	$-1/3$	0	0	$1/3$	0	0	14
0	$1/3$	0	$4/3$	-4	1	$-5/3$	1	0	4
0	$4/3$	0	$1/3$	0	-1	$-1/3$	0	1	4

$\frac{3}{4} R_2$
 $-\frac{1}{3} R_2 + R_3$

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	$-1/4$	0	0	$-1/4$	1	1	$5/4$	0	-3
0	$3/4$	1	0	$-1/4$	0	0	$1/4$	0	15
0	$1/4$	0	1	$-3/4$	0	-1	$3/4$	0	3
0	$5/4$	0	0	$1/4$	-1	0	$-1/4$	1	3

$R_3 \times 4$
 $\frac{3}{4} R_3 + R_2$
 $-\frac{1}{4} R_3 + R_1$
 $-\frac{1}{4} R_3 + R_4$

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	1	0	0	0	0	1	1	1	0
0	2	1	0	0	-1	0	0	1	18
0	4	0	1	0	-3	-1	0	3	12
0	5	0	0	1	-4	0	-1	4	12

Z	x_1	x_2	x_3	x_4	x_5	x_{a1}	x_{a2}	x_{a3}	RHS
-1	0	0	0	0	0	1	1	1	0
0	0	1	0	-1	3	0	1	-3	6
0	0	0	1	-1	1	-1	1	-1	0
0	1	0	0	1	-4	0	-1	4	12

PHASE II $\rightarrow \text{Max } Z + 5x_1 + 7x_2 = 0$

Z	x_1	x_2	x_3	x_4	x_5	RHS
-1	5	7	0	0	0	0
0	0	1	0	-1	3	6
0	0	0	1	-1	1	0
0	1	0	0	1	-4	12

Restoring G.F $\rightarrow -5R_3 - 7R_1 + R_0$

Z	x_1	x_2	x_3	x_4	x_5	RHS
-1	0	0	0	2	-1	-102
0	0	1	0	-1	3	6
0	0	0	1	-1	1	0
0	1	0	0	1	-4	12

If we have zero there's no need to do this step.

Z	x_1	x_2	x_3	x_4	x_5	RHS
-1	0	0	1	1	0	-102
0	0	1	-3	2	0	6
0	0	0	1	-1	1	0
0	1	0	4	-3	0	12

$$x_1 = 12, x_2 = 6$$

If we have

Z	x_1	x_2	x_3	x_4	RHS
-1	-0.25	1	0	-0.25	-3
5	1	0	0	1	5

both have the same negative value

so I can choose whatever I want

TIE FOR ENTERING

& I can select any of them

If we have

Z	x_1	x_2	x_3	x_4	RHS
1	-0.25	1	0	0	-3
1	0.25	0	1	0	3
1	0.25	0	0	-1	3

When we multiply

$$\frac{3}{0.25} = 12$$

$$\frac{3}{0.25} = 12$$

RHS the same
Coeff value

TIE FOR LEAVING VAR.

If we have

Z	x_1	x_2	x_3	x_4	RHS
1	-5	-2	0	0	0
0	-1	2	1	0	2
0	0	1	0	1	4

& I can select any of them

UNBOUNDED

both of them
I should ignore them

It's feasible tableau because I don't have art. var.

If we increase x_1 to $\infty \Rightarrow$ both station will satisfy
the obj. fun is not bounded.

$$\text{Max } z = 5x_1 + 2x_2$$

s.t.

$$-x_1 + 2x_2 \leq 2$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Multioptimal Sol.

Z	x_1	x_2	x_3	x_4	x_5	RHS	
1	0	0	0	0	1	18	
0	1	0	1	0	0	4	$z = 18$
0	0	0	3	1	-1	6	$\begin{cases} x_1 = 4 \\ x_2 = 3 \\ x_4 = 6 \end{cases}$
0	0	1	-1.5	0	0.5	3	

non basic Var.

coef for $x_3 = 0$

So I can take it as entry var.

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	0	0	0	1	18
0	1	0	0	-1/3	1/3	2
0	0	0	1	1/3	-1/3	2
0	0	1	0	0.5	0	6

$$\begin{cases} x_1 = 2 \\ x_2 = 6 \\ x_3 = 2 \end{cases}$$

$$z = 18$$

$P_1 = (4, 3)$ $\rightarrow$ all the points between these (∞)
 $P_2 = (2, 6)$ will give us 18 (obj)

$$P_n = vP_1 + (1-v)P_2$$

$$= 0.3(4, 3) + (1-0.3)(2, 6)$$

$$v = (0, 1)$$

lets take $v = 0.3$

If it take 0 then the point will be 2

If it take value 1 then the point will be P_1

$$P = (2.6, 5.1)$$

If we take P & put it on the obj it will give us 18.

The Simplex Method in Matrix Form

$$\text{Max } Z = 3x_1 + 5x_2$$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 18$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

big M & II Phase Method

→ Max Z

→ ~~4~~ +ve

→ $x_1, x_2 \geq 0$

1- min Z instead of Max Z

2. constraints

≤ no.

> take out surplus ^(-x_u) & add Art. var. & the negative effect on obj.

= Add art. var (x_{ai})

3. $x_1 \leq 0$

$$\text{Max } Z = 3x_1 + 5x_2$$

Sub to :-

$$x_1 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 18$$

$$x_1 \leq 0$$

$$x_2 \geq 0$$

$$x_p = -x_1 \rightarrow x_p \geq 0$$

$$x_1 = -x_p$$

$$\text{Max } Z = -3X_{\text{prime}} + 5X_2$$

Sub to

$$-X_{\text{prime}} \leq 4$$

$$-X_{\text{prime}} + X_2 \leq 6$$

$$-X_{\text{prime}} + 3X_2 \leq 8$$

$$-X_{\text{prime}} \leq 0$$

$$\rightarrow X_{\text{prime}} \geq 0$$

$$X_2 \geq 0$$

at the end $\rightarrow$ If $X_{\text{prime}} = 5$
 $X_1 = -5$

$$\text{Max } Z = 3(X_p - 5) + 5X_2$$

Sub to

$$(X_p - 5) \leq 5$$

$$2(X_p - 5) + 4X_2 \leq 10$$

$$X_1 \geq -5 \rightarrow X_p = X_1 + 5 \geq 0 \rightarrow X_1 = X_p - 5$$

$$X_2 \geq 0$$

$$X(\text{var}) \geq -ve$$

$$\text{Max } Z = -3X_1 + 4X_2$$

Sub to

$$3X_1 + 2X_2 \leq 10$$

$$3X_1 + X_2 \geq 5$$

X_1 is not constrained $(-\infty, \infty)$

$$X_2 \geq 0$$

we should assume that X have two terms

$$X_1 = X_{1p} - X_{1n} \quad \begin{matrix} \text{(pos)} \\ \text{(negative)} \end{matrix}$$

so that

$$\text{Max } Z = 3(X_{1p} - X_{1n}) + 4X_2$$

Sub to

$$3(X_{1p} - X_{1n}) + 2X_2 \leq 10$$

$$3(X_{1p} + X_{1n}) + X_2 \geq 5$$

Wyndor Glass Co.

$$\text{Max } Z = 3x_1 + 5x_2$$

subject to

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} \begin{matrix} \} 0 \\ \} 1 \end{matrix}$$

↓

change it to Matrix form

$$\text{Max } Z = CX$$

sub to

$$AX = b$$

$$x \geq 0$$

parameters

$$C = [3 \quad 5]$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

* Start solving the problem

First thing we look at the constraints & add the slack variables needed.

$$\text{Max } Z = 3x_1 + 5x_2$$

subject to

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

for the first (initial) tableau

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	-C			0		0
0	A		I			b

for each iteration

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	$C_B B^{-1} A - C$		$C_B B^{-1}$			$C_B B^{-1} b$
0	$B^{-1} A$		B^{-1}			$B^{-1} b$

solving 5.1-4

$$\text{Max } Z = 2x_1 - x_2 + x_3$$

Subject to:

$$3x_1 + x_2 + x_3 \leq 60$$

$$x_1 - x_2 + 2x_3 \leq 10$$

$$x_1 + x_2 - x_3 \leq 20$$

$$\begin{array}{l} x_1 \\ x_2 \\ x_3 \end{array} \Bigg| 0$$

the constant Matrices

$$C = [2 \quad -1 \quad 1]$$

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 60 \\ 10 \\ 20 \end{bmatrix}$$

$$\text{Max } Z = 2x_1 - x_2 + x_3$$

Sub to

$$3x_1 + x_2 + x_3 + x_4 = 60$$

$$x_1 - x_2 + 2x_3 + x_5 = 10$$

$$x_1 + x_2 - x_3 + x_6 = 20$$

So the initial tableau is

Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	-2	+1	-1	0	0	0	0
0	3	1	1	1	0	0	60
0	1	-1	2	0	1	0	10
0	1	1	-1	0	0	1	20

$$B = \begin{bmatrix} x_4 & x_5 & x_6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_B = [0 \quad 0 \quad 0]$$

$$C_B B^{-1} = [0 \quad 0 \quad 0]$$

$$C_B B^{-1} A - C = [-2 \quad 1 \quad -1]$$

x_1 is entering variable

x_5 is leaving variable

$$B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C_B = [0 \quad 2 \quad 0]$$

$$B^{-1} = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	x_5	x_6	RH
1	0	-1	3	0	2	0	20
0	0	4	-5	1	-3	0	30
0	1	-1	2	0	1	0	10
0	0	2	-3	0	-1	1	10

$$C_B B^{-1} = [0 \quad 2 \quad 0]$$

$$B^{-1} A = \begin{bmatrix} 0 & 4 & -5 \\ 1 & -1 & 2 \\ 0 & 2 & -3 \end{bmatrix}$$

$$C_B B^{-1} A - C = [2 \quad -2 \quad 4] - [2 \quad -1 \quad 1] = [0 \quad -1 \quad 3]$$

$$B^{-1} b = \begin{bmatrix} 30 \\ 10 \\ 10 \end{bmatrix}$$

$$C_B B^{-1} b = 20$$

x_2 is entering variable & x_6 is leaving variable.

$$B = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C_B = [0 \quad 2 \quad -1]$$

$$B^{-1} = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	0	0	1.5	0	1.5	0.5	25
0	0	0	0	1	-1	-2	10
0	1	0	0.5	0	1/2	1/2	15
0	0	1	-1.5	0	-1/2	1/2	5

$$C_B B^{-1} = [0 \quad 1.5 \quad 0.5]$$

$$C_B B^{-1} A - C = [2 \quad 2 \quad 2.5] - [2 \quad -1 \quad 1] = [0 \quad 0 \quad 1.5]$$

$$B^{-1} A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0.5 \\ 0 & 1 & -1.5 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 10 \\ 15 \\ 5 \end{bmatrix} \rightarrow \begin{matrix} x_4 \\ x_1 \\ x_2 \end{matrix}$$

$$C_B B^{-1} b = 25$$

value of $x_3 = 0$

value of $x_2 = 5$

value of $x_1 = 15$

optimal value of $z = 25$

solving S.1-15

$$\text{Max } Z = 3x_1 + 4x_2 + 2x_3$$

subject to

$$x_1 + x_2 + x_3 \leq 20$$

$$x_1 + 2x_2 + x_3 \leq 30$$

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \Bigg| 0$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C_B = [0 \quad 0] \quad b = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

$$\text{Max } Z = 3x_1 + 4x_2 + 2x_3$$

subject to

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$x_1 + 2x_2 + x_3 + x_5 = 30$$

$$C = [3 \quad 4 \quad 2]$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

iteration 1, the entering basic var. is x_2 & the leaving var. is x_5

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \quad C_B = [0 \quad 4]$$

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	$C_B B^{-1} A - C$		$C_B B^{-1}$		$C_B B^{-1} b$	
0	$B^{-1} A$		1	1	$B^{-1} b$	
0			0	2		

in iteration 2, entering var is x_1 & leaving var. x_4

$$B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad C_B = [3 \quad 4]$$

$$B^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	0	1	2	1	70
0	1	0	1	1	1	10
0	0	1	0	1	2	10

$$C_B B^{-1} = [2 \quad 1]$$

$$C_B B^{-1} A - C = [3 \quad 4 \quad 3] - [3 \quad 4 \quad 2] = [0 \quad 0 \quad 1]$$

$$B^{-1} A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad B^{-1} b = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \quad C_B B^{-1} b = 70$$

Solving 5.1-14

$$\text{Max } Z = 2x_1 + 2x_2 + 3x_3$$

Subject to

$$2x_1 + x_2 + 2x_3 \leq 4$$

$$x_1 + x_2 + x_3 \leq 3$$

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \geq 0$$

Let x_4 & x_5 be slack var.

1st iteration the entering basic var. is x_3 & leaving var. is x_4

2nd iteration the entering var. is x_2 & leaving var. is x_5

$$C = [2 \quad 2 \quad 3]$$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$B = \begin{bmatrix} x_1 & x_5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_B = [0 \quad 0]$$

$$\text{1st iteration} \Rightarrow B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad C_B = [3 \quad 0]$$

$$\text{2nd iteration} \Rightarrow B = \begin{bmatrix} x_3 & x_2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad C_B = [3 \quad 2], \quad B^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$C_B B^{-1} = [1 \quad 1]$$

$$C_B B^{-1} A - C = [3 \quad 2 \quad 3] - [2 \quad 2 \quad 3] = [1 \quad 0 \quad 0]$$

$$B^{-1} A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C_B B^{-1} b = 7$$

the coefficient of the slack var. $\rightarrow$ Shadow price ($C_B B^{-1}$)

exam Q : what are the constraints that gives us the value of the corner point (optimal sol.)?

1st constraint $\rightarrow 2x_1 + x_2 + 2x_3 \leq 4 \rightarrow x_4 = 0$

2nd constraint $\rightarrow x_1 + x_2 + x_3 \leq 3 \rightarrow x_5 = 0$

$x_1 \gg 0 \rightarrow x_1 = 0$

Z	x_1	x_2	x_3	x_4	x_5	RHS
1	1	0	0	1	1	7
0	1	0	1	1	-1	1
0	0	1	0	-1	2	2

↓

$x_1 = 0$

$x_2 = 2 \rightarrow x_5 = 0$

$x_3 = 1 \rightarrow x_4 = 0$

Solving prob 5.1-1

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \leq 6$$

$$x_1 + 2x_2 \leq 6$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} \geq 0$$

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$2x_1 + x_2 + x_3 = 6$$

$$x_1 + 2x_2 + x_4 = 6$$

$$C = [3 \quad 2]$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$B = \begin{bmatrix} x_3 & x_4 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_B = [0 \quad 0]$$

initial tableau:

Z	x_1	x_2	x_3	x_4	RHS
1	-3	-2	0	0	0
0	2	1	1	0	6
0	1	2	0	1	6

x_1 is entering variable & x_3 is leaving variable

$$B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad C_B = [3 \quad 0], \quad B^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}$$

$$C_B B^{-1} = [1.5 \quad 0] \quad \text{coefficients of slack variables in Obj.}$$

$$C_B B^{-1} A - C = [3 \quad 1.5] - [3 \quad 2] = [0 \quad -0.5]$$

$$B^{-1} A = \begin{bmatrix} 1 & 1/2 \\ 0 & 1.5 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	RHS
1	0	-0.5	1.5	0	9
0	1	1/2	1/2	0	3
0	0	1.5	-1/2	1	3

x_2 is entering var. & x_4 is leaving var.

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad C_B = [3 \quad 2], \quad B^{-1} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$C_B B^{-1} = [4/3 \quad 1/3]$$

$$C_B B^{-1} A - C = [3 \quad 2] - [3 \quad 2] = [0 \quad 0]$$

$$B^{-1} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$C_B B^{-1} b = 10$$

Z	x_1	x_2	x_3	x_4	RHS
1	0	0	4/3	1/3	10
0	1	0	2/3	-1/3	2
0	0	1	-1/3	2/3	2

$$x_1 = 2$$

$$x_2 = 2$$

$$\text{optimal obj}(z) = 10$$

Chapter 6 "Duality"

every linear programming problem has an associated problem called (dual) the original one $\rightarrow$ primal. & Both of them will lead to the same optimal sol.

Back to Wyndor Glass Co. problem.

primal:

$$\text{Max } Z = 3x_1 + 5x_2$$

Subject to:-

$$x_1 \leq 4 \rightarrow y_1$$

$$2x_2 \leq 12 \rightarrow y_2$$

$$3x_1 + 2x_2 \leq 18 \rightarrow y_3$$

$$\left. \begin{array}{l} x_1 \\ x_2 \\ x_3 \end{array} \right\} 0$$

dual:

$$\text{Min } W = 4y_1 + 12y_2 + 18y_3$$

Sub to:-

$$1y_1 + 3y_3 \geq 3$$

$$2y_2 + 2y_3 \geq 5$$

$$\left. \begin{array}{l} y_1 \\ y_2 \\ y_3 \end{array} \right\} 0$$

because it's written in the std. form

* no. of constraints in the primal prob. = no. decision variables in dual prob.

* every constraint in the primal problem is correspond to a d.v in dual problem

Example

$$\text{Max } Z = 4x_1$$

Sub to:

$$x_1 + 2x_2 \leq 5$$

$$x_2 \leq 3$$

$$\left. \begin{array}{l} x_1 \\ x_2 \end{array} \right\} 0$$

$$\text{Min } W = 5y_1 + 3y_2$$

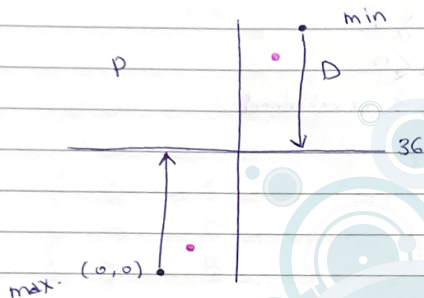
Sub to:

$$y_1 \geq 4$$

$$2y_1 + 1y_2 \geq 0$$

Summary of primary-dual relationships

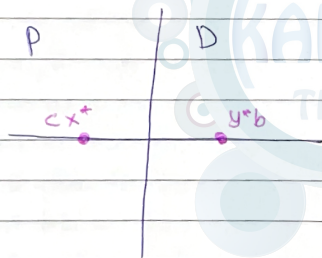
- weak duality property.



If both of • are feasible

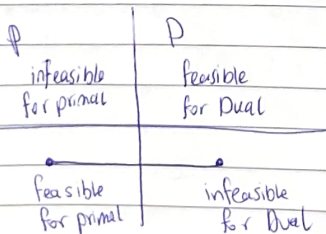
$$\underbrace{cx}_{\text{value of objective in the primal}} \leq \underbrace{by}_{\text{value of obj in dual}}$$

- strong duality property



If the objective in the primal is equal to the objective in the dual then we conclude to $cx^* = y^*b$

- Complementary solutions property



• Symmetry Property

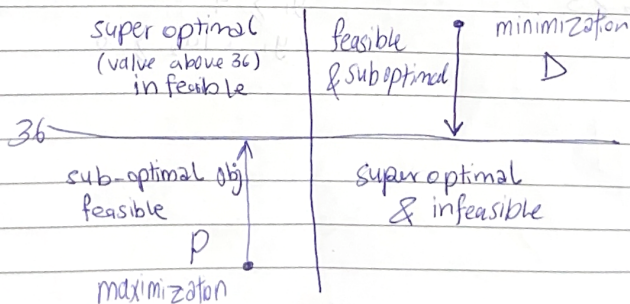
All relationships between the primal & the Dual must be symmetric

If one problem has feasible solutions and an unbounded (there is no optimal solution), then the other problem has no feasible solutions

If one problem has no feasible solutions, then the other problem either has no feasible solution or unbounded objective function.

If one problem has feasible solutions and an optimal sol. (bounded objective func.), then so does the other problem.

NOTE THAT: the values of y 's in the dual represent the shadow prices (economic interpretation of the dual)



		satisfy cond of optimality	
		Yes	No
Feasible	Yes	optimal	suboptimal
	No	Super optimal	Neither feasible nor super optimal.

Sensible Odd-Bizarre (SOB)

Maximization		Minimization	
constraints	$\leq$	S	$\leq$ B
	$=$	O	$=$ O
	$\geq$	B	$\geq$ S
decision var	≥ 0	S	
	unconstrained	O $\rightarrow (-\infty, \infty)$	
	≤ 0	B	

examples:

$$\text{Max } Z = 3x_1 + 4x_2 - x_3$$

Subject to:

$$x_1 + 2x_2 + x_3 \geq 10 \quad B \rightarrow y_1 \leq 0$$

$$x_1 + x_2 + x_3 \leq 20 \quad S \rightarrow y_2 \geq 0$$

$$x_2 + 3x_3 = 15 \quad O \rightarrow y_3 \text{ unconstrained}$$

$$x_1 \geq 0 \quad S$$

$$x_2 \leq 0 \quad B$$

$$x_3 \text{ unconstrained} \rightarrow O$$

dual:

$$\text{Min } W = 10y_1 + 20y_2 + 15y_3$$

Sub to:

$$y_1 + y_2 \geq 3$$

$$2y_1 + y_2 + y_3 \leq 4$$

$$y_1 + y_2 + 3y_3 = -1$$

we look for these to know which sign we will use!

$$\text{Min } Z = 3x_1 + 4x_2 - x_3$$

Sub to

$$x_1 + 2x_2 + x_3 \geq 10 \quad S \rightarrow y_1 \geq 0$$

$$x_1 + x_2 + x_3 \leq 20 \quad B \rightarrow y_2 \leq 0$$

$$x_2 + 3x_3 = 15 \quad O \rightarrow y_3 \text{ is unconstrained}$$

$$x_1 \geq 0 \quad S$$

$$x_2 \leq 0 \quad B$$

$$x_3 \text{ is unconstrained} \quad O$$

$$\text{Max } w = 10y_1 + 20y_2 + 15y_3$$

Sub to -

$$y_1 + y_2 \leq 3$$

$$2y_1 + y_2 + y_3 \geq 4$$

$$y_1 + y_2 + 3y_3 = -1$$

$$y_1 \geq 0$$

$$y_2 \leq 0$$

$$y_3 \text{ is unconstrained}$$