Weekly Notebook

1st Week

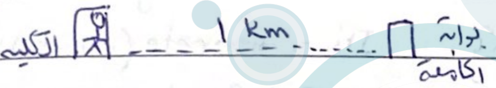
Stat 1

First Semester

2022

1. The role of statistics in Engineering

The field of statistics deals with the collection, presentation, analysis, and use of data to
- → Make decisions
- → solve problems
- → Design products and processes



X = Time ;
10 min
9.5
10.8
12
11

⇒ So we have an Experiment : go from one location to another.

⇒ out comes: {10, 9.5, 10.8, 12, 11 . . . . }

⇒ variable: X = time to arrive the main gait

بما انو التجربة بتطلعلي outcomes
واحنا مو قادرين نحدد شو هو exactly
للي بطلع كل مرة بشكل مختلف

Variability : successive observation of a system or phenomenon don't produce exactly the same result (different each time)

* I can't elimanite the variablity so we are going to
1- describe it
2- try to find the source of this variability

* since (X) → exhibits variability , so we can call (X) **random variable**

Three basic methods for collecting data:

- **retrospective study** using historical data like : weather
- **observational study**
  - like Data, presently collected, by a passive observer
- **designed experiment**
  - Data collected in response to process input changes

2. Probability

* Random (statistical) Experiment:

An experiment <with known outcomes> whose outcomes can't be predicted with certainty befor the experiment is run → The roll of dice
→ flipping the coin

* Types of models

- mechanistic model
  - physical system
  - measurement
  - model design
  - analysis
- Empirical model
  - Population
  - probability
  - sample
  - statistical inference

controlled variables
↓↓↓↓↓↓↓

input → $\boxed{\text{system}}$ → output

↑↑↑↑↑↑
noise variables

Sample Space:

- Set of all possible outcomes of a random experiment
- points of a sample space are called: "outcomes, elementary events or realization"

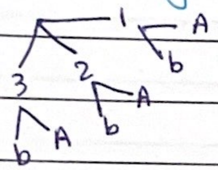
discrete: if it consists of finite or countable infinite set of outcomes

continuous: if it contains an interval (finite / infinite) of real numbers.

* Example: flipping a coin $S = \{H, T\}$

* 3 machine parts, each is classified as either (above or below) the target specifications.

a b



$S = \{AAA, AAB, ABB, BAA, BBA, ABA, BAB, BBB\}$

* Numbers of views recorded out at a higher view website in a day

$S = \{0, 1, 2, 3, 4, \ldots\ldots, n\}$ It's a discrete (countable infinite)

* selecting a ball from urn which has 3 balls: (Green, Red, Blue)

$S = \{G, R, B\}$

* Rain preceptation level

$S = R^+ \quad (0, \infty) = \{x \mid x > 0\}$

ماي

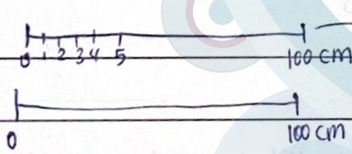
مايصير نحط فترة مغلقة [0,∞) لانو أصلاً لا يكون zero

لانو في rain.

* Two connectors measured

$S = R^+ * R^+$

$x = 100$ — outcome عدد ال — $y = 100$



لانو مثلاً لو طلع طول السلك 5cm التاني ما بنعرف ايش طولو

فحيطلع 100x100

مشان نطلع عدد ال (Posible out comes)

↳ in showing if connectors are confirming or not (yes, no)?

$S = \{yy, yn, ny, nn\}$

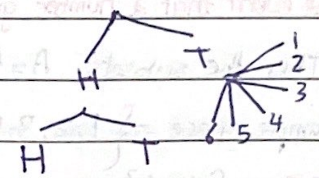
↳ number of confirming conectors?

$S = \{0, 1, 2\}$ how? $\{yy, yn, ny, nn\}$

$\downarrow$ 2 1 0

* An experiment consist of flipping a coin and then flipping it a second time if head occurs If tail occure on the first time, then a die is tossed once.

$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$



* Consider an experiment that selects a cell phone camera and records the recycle time of a flash (the time taken to ready the camera for anothe flash)

$S = R^{+} = \{x \mid x > 0\}$

- If it is Known that all recycle times are between 1.5 and 5 seconds, the sample space can be:

$S = \{x \mid 1.5 < x < 5\}$

* measuring thickness until first fail

$S = \{N, yN, yyN, yyyN, yyyyN, \ldots\}$ ⇒ I will Keep measuring until the first fail

Graphical Represntation

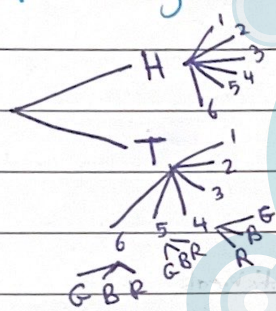
⇒ sample space is defined by a Tree diagram

we have an experiment is done in several steps

n_1 ways of 1st step ⇒ n_1 branchs of the tree

n_2 ways of 2nd step ⇒ n_2 branchs of each branch in n_1

Example: Tossing a coin, rolling a die and choosing a ball from urn (R,G,B)



$S = \{(H1R), (H2R), (H1B), (H1G), \ldots\}$

how to count the number of the ex? $2 \times 6 \times 3 = 36$

Basic of set theorem:-

An event is a subset of the Sample space of a random experiment.

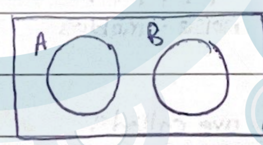
Two event A, B

(union) $A \cup B$ { A or B

(Intersection) $A \cap B$ { A and B

Venn diegram

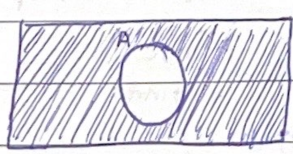
ϕ : empty set



A' { A^c : complement "not" $S \setminus A$

كل اشي برا ال A

⇓



A^c : $S - A$

If $A \cap B = \phi$

that's mean A & B are disjoint or exclusive

this implies if one event happened the other will not happen.

Example: Digital scale to provide wieghts to the nearest gram

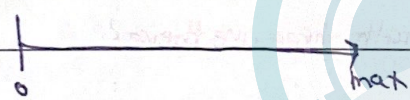
A: event that a weight exceeds 11 grams

B: event that a weight is less than or equal 15 grams

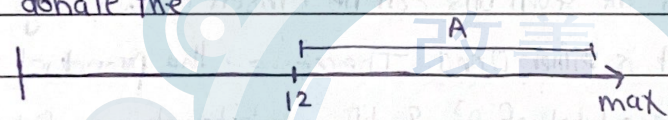
C: event that a weight is greater than or equal to 8 grams and less than 12 grams.

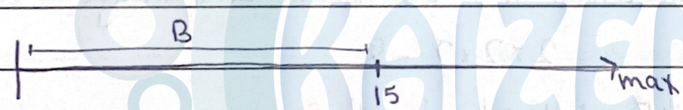
a. Sample Space

nonnegative integars from 0 to the Largest integr s that can be displayed by the scale



Let x donate the

A =  A = $x \geq 12$

B =  B = $x \leq 15$

C =  C = $12 > x \geq 8$

b. $A \cup B = S$

c. $A \cap B = \{12, 13, 14, 15\}$ → ما بصير نحط interval لانها بتاخد ارقام عشرية ~~$\{x \mid 12 \leq x \leq 15\}$~~

D. $A' = \{x \mid x \leq 11\}$

E. $A \cup B \cup C = S$

F. $(A \cup C)' = S - (A \cup C)$ ~~$S - \{x \mid x \geq 8\}$~~ ~~$\{x \mid x < 8\}$~~ $\{x \mid x \leq 7\}$

G. $A \cap B \cap C = \phi$

h. $B' \cap C = \phi$

K. $A \cup (B \cap C) = \{x \mid x \geq 8\}$

" Counting techniques "

how to count numbers of outcomes

1. Multiplication Rule

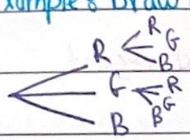
: Operation with K steps each step is done in a number of ways

Step 1 ⇒ with n_1 ways

Step 2 ⇒ with n_2 ways

The Total number of ways $= n_1 \times n_2 \times n_3 \times \dots \times n_k$

Example: Draw a marble twice from an urn which has 3 (G, B, R) ((with replacement))



1st draw we have 3 choises

2nd draw we have 3 choises

$3 \times 3 = 9$ ways

* Now, In how many ways can we select R distinct object from n distinct objects

2. Permutations

: Order of items arragment is important (GRB ≠ GBR)

Consider a set of element, such as $S = \{a, b, c\}$

A permutation of the elements is an ordered sequence of the elements, for example

{abc, cba, acb, bca, bac, cab} ⇒ are all the permutation of elements of S

$3 \times 2 \times 1 = 6$

(It's care about order " الترتيب ")

Total of ways $= \overset{1}{n} * (n-\overset{2}{1}) * (n-\overset{3}{2}) * (n-\overset{4}{3}) * \dots * (n-r+1)$

$(n-1+1) \quad (n-2+1) \quad (n-3+1) \quad (n-4+1) \quad (n-r+1)$

$_nP_r \; (P_r^n = \dfrac{n!}{(n-r)!}$ ⇝ Different Elements

$P_n^n = \dfrac{n!}{(n-n)!} =$

$P_1^n = \dfrac{n!}{(n-1)!} = \dfrac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

Example: 5 students, need to select prisedent, vice-president, secertary (A,B,C,D,F)

$P^5_3 = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 60$

5 students in 5 positions

$P^5_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 120$

Permutation ⇝ Similar Object

* Set of objects which contains K groups of identical items

number of ways of ordering n object $\frac{n!}{n_1! * n_2! * \cdots n_K!}$

Example: MISSISSIPPI

$n = 11$

$M = 1$

$I = 4$

$S = 4$

$P = 2$

number of words $= \frac{11!}{1! * 4! * 4! * 2!}$

3. Combinations

* number of ways of selecting r objects from n without regard to order

$C^n_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ * It's Don't care about sequence * means that GBA = GAB

Example: 4 cars (hounda, Toyota, Ford, BMW)

number of ways to select two of them

$C^4_2 = \frac{4!}{2! * 2!} = 6$ ways

Ex8 A group of 7 engineers and 5 statisciens
we need a committee of 5 people from the group
How many ways can we select the committee where we have 3 engineers and 2 statisclans.

$$\underline{E}\ \underline{E}\ \underline{E}\ \underline{S}\ \underline{S}$$

$$C^7_3 * C^5_2$$

$$\frac{7!}{3!\,4!} * \frac{5!}{2!\,3!}$$

Probability 8-

It's used to quantify the likelihood, or chance that an outcome will occur of a random experiment

The likelihood of an outcome is quantified by assigning a number from the interval [0,1] to the outcome (or a percentage from 0 to 100%)

→ A 0 indicates an ~~another~~ outcome will not occur

→ A prob of 1 indicates that an out come will occur with certainty

Math definition of probability is a function that assigns to each event $A \subset S$ a real number $P(A) \in [0,1]$, such that

i) $P(A) \geqslant 0$

ii) $P(S) = 1$

iii) If $A_1, A_2, \dots\dots A_r$ are disjoint, then probability of $P(\cup A) = \sum P(A)$

$\sum P(A) = P(A_1) + P(A_2) + P(A_3) + \dots\dots$

Three interpretations for probability

1- Degree of belief : common sense (weather, personal judgement)

2- Frequency : how many times the event occur in a very large number of sample

3- Equally likely : If we have N outcomes $\Rightarrow P(\text{each outcome}) = \frac{1}{N}$

example : Urn contains 30 red marbles, 20 green marbles, 50 blue marbles

$P(\text{of choosing any marble}) = \frac{1}{100}$

$P(\text{of choosing red marble})$

$P(\text{red marble } 1) + P(\text{red marble } 2) + \text{---------} = \frac{30}{100}$ / 0.3

for a discrete sample space

$$P(E) = \sum_{1}^{n} P(\text{all the outcomes in event } E)$$

Simple properties of probability A . B

i) $P(\phi) = 0$

ii) if $\underset{\text{subset}}{A \subset B} \Rightarrow P(A) \leq P(B)$

iii) $0 \leq P(A) \leq 1$

iv) $P(A^c) = 1 - P(A)$

v) If $A \cap B = \phi \Rightarrow P(A \cup B) = P(A) + P(B)$

vi) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Total Propability rule

Example: Batch contains 6 parts $\{a, b, c, d, e, f\}$
2 parts are selected randomly without replacment & what is the probability that (f) is in the sample?

E = f is in the sample

$$P(E) = \frac{\text{\# of sample that contains f}}{\text{Total possible \# of samples}}$$

Total # of possible

samples: is choosing 2 out of 6 (order is not important)

$$= C^6_2 = \frac{6!}{2! * 4!} = \frac{6 \times 5}{2} = 15$$

Prob 2 - p61

Nickel battery, charge state probility for each state

Nickel charge	Prob found
0	0.17
+2	0.35
+3	0.33
+4	0.15

a. probability that a cell has at least one of the positive charges

E: the event that a cell has at least one of the

$E = \{+2, +3, +4\}$

$P(E) = P(+2) + P(+3) + P(+4) = 0.83$

b. what is the prob that a cell has not a charge greater than +3 ?

$E = \{\text{not} > +3\} \Rightarrow E = \{\text{charge} \leq +3\}$

$= \{0, +2, +3\}$

$P(E) = 0.85$

Equally Likely Outcomes:- (defult)

- S Is a sample space, E is an event
- $E \subseteq S$

$$P(E) = \frac{\text{\# of outcomes in event } (E)}{\text{Total \# of out comes in sample space } (S)} = \frac{n(E)}{n(S)}$$

$$1 \geq P(E) \geq 0$$

Not Equally likely Outcomes:-

Ex1 A random experiment can result in one of the outcomes $\{a, b, c, d\}$ with probability 0.1, 0.3, 0.5 and 0.1, respectively. Let A denote the event $\{a, b\}$, B the event $\{b, c, d\}$, and C the event $\{d\}$. Then,

$P(A) = 0.1 + 0.3 = 0.4$
$P(B) = 0.3 + 0.5 + 0.1 = 0.9$
$P(C) = 0.1$

Ex2 A dice is loaded in such a way that an even number is twice as likely to occure as an odd number. If E is the event that a number less than 4 occurs on a single toss of the dice, find P(E)

$S = \{\overset{w}{1}, \overset{2w}{2}, \overset{w}{3}, \overset{2w}{4}, \overset{w}{5}, \overset{2w}{6}\}$

$E = \{1, 2, 3\}$

$P(E) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

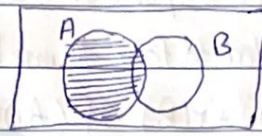
how to think

we assign a probability of w to each odd number and a prob of $2w$ to each even #.
Since the sum of the prob = 1
we have $9w = 1$ so $w = 1/9$

Addition rules

for any two events A, B

$P(A \cup B) = P(A) + P(B \cap A^c)$



$A \cup B = A \cup (B \cap A^c)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$B = (B \cap A) + (B \cap A^c)$

for any three events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C) - P(A \cap B) - P(C \cap B) + P(A \cap B \cap C)$

If $A_1, A_2, \ldots, A_n$ are mutually exclusive, then

$P(A_1 \cup A_2 \cup \ldots A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

Example: what is the prob of getting a total of 7 $\boxed{\text{or}}$ 11 when a pair of dice is tossed

$\boxed{\text{or}} = \cup$

$S = 36$

$P(A) = \frac{6}{36}$, $P(B) = \frac{2}{36}$

the two events ← are mutually exclusive, since a total of 7 and 11 can't both occur on the same toss

$P(A \cup B) = P(A) + P(B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{4}{9}$

مشان هيك ماحكينا $-P(A \cap B)$

Example: In a suburb, 60% at all houseolds subscribe to the news-paper published in a nearby city, 80% sub scribe into the local news paper, and 50% subsribe to both news papers

A = { household subscribed to nearby city paper }

B = { house hold subscribed to local paper }

$P(A) = 0.6 \quad P(B) = 0.8 \quad P(A \cap B) = 0.5$

a) the house hold subs to at least one of the two newspapers

$P(A \cup B) = 0.6 + 0.8 - 0.5 = 0.9$

b) subs to exactly one of the two papers?

{at least 1} = {exactly 1} ∪ {both}

$$\underbrace{P(A \cup B)}_{\substack{\text{at least 1} \\ 0.9}} = P(A) + P(B) - \underbrace{P(A \cap B)}_{\substack{\text{both} \\ 0.5}}$$

$$P(\text{exactly}) = P(A \cup B) - P(A \cap B)$$
$$= P(A) + P(B) - 2P(A \cap B)$$

mutually exclusive → The event have nothing in common and might not include all the samplespace

mutually exclusive and collectively exhaustive

↳ meaning that the events have nothing in common and include (cover) the Sample(S).

Example: A card is drawn from a standard deck of 52 playing cards. What is the prob that card will be a spade or a seven? Express your answer as a fraction or a decimal number rounded to four decimal places.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = 0.307$$

$P(\text{spade})$ or $P(\text{heart})$

$= P(\text{spade} + \text{heart})$

$$= \frac{13}{52} + \frac{13}{52} - \frac{26}{25} = 0.5$$

conditional probability

when probabilities change as additional information becomes available

⇒ Knowledge of the out come of an $\underbrace{\text{event } A}_{\text{condition}}$ will change the prob of othe $\underbrace{\text{event } B}_{\text{conditional}}$

* It's all about narrowing down the sample

$P(B \setminus A)$ "conditional probability of B given A"

Ex: flipping two coins $S = \{HT, TT, HH, TH\}$

E_1 = at least one head $S = \{HT, HH, TH\}$

E_2 = 2 heads $S = \{HH\}$

what is the probability of 2 heads flipped given at least one head?

$$P(E_2 \setminus E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{1/4}{3/4} = \frac{1}{3}$$

Ex: 100 dicks are analyzed for shock resistance and scratch resistance.

		shock	
		high	low
scratch	high	70	9
	low	16	5

A: event that dick has high shock

B: event that " " " scratch

$P(A) = \frac{86}{100}$ $P(B) = \frac{79}{100}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{70/100}{79/100} = \frac{70}{79}$$

$$P(B \setminus A) = \frac{P(B \cap A)}{P(A)} = \frac{70/100}{86/100} = \frac{70}{86}$$

Ex: A die is rolled twice. What is the probability that the sum equal 10 (A), if you know (= same / the given that) that the 1st element equal 6? (B)

$S = \{11, \ldots, 66\} = 36$

$A = \{46, 55, 64\}$

$B = \{61, 62, 63, 64, 65, 66\}$

$$P(A \mid B) = \frac{\overset{\text{equally likely}}{\boxed{P(A \cap B)}}}{P(B)} = \frac{1/36}{6/36} = \frac{1}{6}$$

حل اخر

$\because$ $(A \cap B)$ is equally likely $\dfrac{n\,\text{event}(E)}{n(S)}$

so $\dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{6}$ that's It

Multiplication and total prob rule :-

$$P(A\backslash B) = \frac{P(A\cap B)}{P(B)} \quad , P(B) > 0$$

$$P(B\backslash A) = \frac{P(B\cap A)}{P(A)} \quad , P(A) > 0$$

$\because A\cap B = B\cap A$

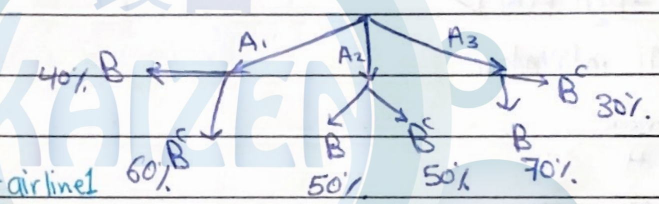
$\therefore P(A\cap B) = P(A\backslash B)\,P(B) = P(B\backslash A)\,P(A)$

Ex. Three air lines (A_1, A_2, A_3), Late take off $A_1 = 40\%$, $A_2 = 50\%$, $A_3 = 70\%$

Event A : choosing an airline

Event B : It being Late

$P(A_1) = \frac{1}{3} = P(A_2) = P(A_3)$



a) what is the prob that I select airline1 and I am Late ?

$P(A_1\cap B) = 1/3 \times 40/100 = 4/30$

b) Prob that I am Late taking off ?

$P(B) = P(A_1\cap B) + P(A_2\cap B) + P(A_3\cap B)$

$= \frac{4}{30} + \frac{5}{30} + \frac{7}{30} = \frac{16}{30} = \frac{8}{15}$

c) Given that I was Late what is the prob that I selected airline 1 ?

$$P(A_1\backslash B) = \frac{P(A_1\cap B)}{P(B)} = \frac{4/30}{16/30} = \frac{4}{16} = \frac{1}{4} \; (25\%)$$

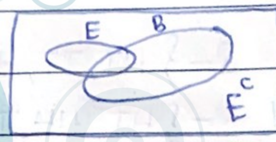
Law of total Probability

We use it to calculate prob of an indiviual event give under several conditions

- Let $E_1, E_2, \dots E_n$ be a collection of mutually exclusive and collectively exhaustive events then for event B

$$P(B) = \sum_{i=1}^{n} P(B \setminus E_i) * P(E_i)$$ ⟸ for a one condition

$$P(B) = P(B \setminus E)P(E) + P(B \setminus E^c)P(E^c)$$



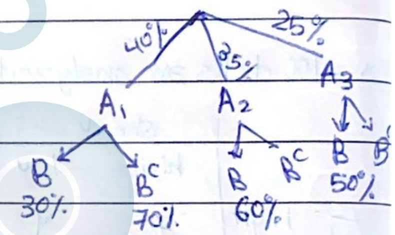
⟸ each one is complement for each other

IMP ex: At a gas station, 40% of the customers use regular gas, 25% use plus gas (A_2) and 25% are premium gas. of those customers using regular gas, only 30% fill their tank, of those using plus gas 60% fill tanks & 50% of premuim customers fill tank

$P(B \setminus A_1) = 30\%$

$P(B \setminus A_2) = 60\%$

$P(B \setminus A_3) = 50\%$



a) what is the prob that the next customer will fill his tank?

$$P(B) = P(B \setminus A_1)P(A_1) + P(B \setminus A_2)P(A_2) + P(B \setminus A_3)P(A_3)$$

$$= 0.3 \times 0.4 + 0.6 \times 0.35 + 0.35 \times 0.5 = 0.455$$

Ex: 2004 election

	Bush	Kerry
no collage degree (62%)	50%	50%
collage degree (38%)	54%	46%

- What is the prob a randomly selected respondent voted for bush?

A: event that respondent a college degree / B: event that voted for bush

$$P(B) = P(B \setminus A)P(A) + P(B \setminus A^c)P(A^c)$$

$$0.54 \times 0.38 + 0.5 \times 0.62 = 0.514$$

Independente

occurance of an event does not effect probabity of a another event

$\Downarrow$

two events are independent

$P(B \backslash A) = P(B)$

$P(A \cap B) = P(B \backslash A) \cdot P(A) = P(B) \cdot P(A)$

$P(A \backslash B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$

* To check if two events are independence $\Rightarrow$ if satisfied one of three condition

Examples:-

for $E_1, E_2, \ldots\ldots E_n$ independent events

$P\left(\bigcap_{i=1}^{n} E_i\right) = P(E_1 \cap E_2 \cap \ldots\ldots E_n)$

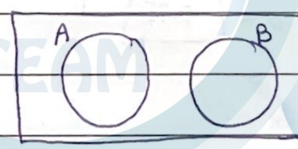
$= \prod_{i=1}^{n} P(E_i) = P(E_1) \cdot P(E_2) \cdot P(E_3) \ldots\ldots P(E_n)$

Explinations:-

* $P(A \cap B) = \phi = 0$

  $P(A) \cdot P(B) > 0$

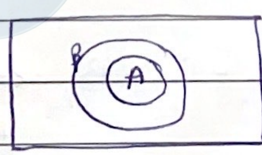
  So they are not-Independente



* $A \subset B$

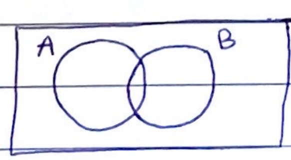
  $P(A \cap B) = P(A)$

  $\neq P(A) \cdot P(B)$



*

They may or may not be Idependent



Independence can be:

1- As an assumption

⇒ coin tossing $P(H) = P(T) = \frac{1}{2}$

$P(\underbrace{HHT \ldots\ldots TH}_{K}) = (\frac{1}{2})^K$

2- As a property (depends on data)

⇒ tossing a fair dice $= S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ $P(A) = \frac{1}{2}$

$B = \{1, 2, 3, 4\}$ $P(B) = \frac{4}{6} = \frac{2}{3}$

$P(A \cap B) = \{2, 4\} = \frac{2}{6} = \frac{1}{3}$

$P(A \cap B) \overset{?}{=} P(A) \cdot P(B)$ ✓

So It's independent

Ex ⇒

		shock H	shock L
scratch	H	70	9
scratch	L	16	5

A: event that a disk has high a high scratch

B: event that a disk has high scratch

A & B are independent?

$P(A \cap B) = \frac{70}{100}$ $\frac{86 \times 79}{100 \times 100} \overset{?}{=} \frac{70}{100}$

$P(A) = \frac{86}{100}$

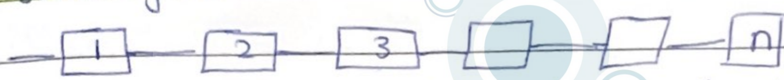
$P(B) = \frac{79}{100}$

So they are not independent

Application of probability concepts (Reliability)

Reliability: is the prob that a system or product will perform its intended function properly

1- series system



- each component works / fails independently than others.
- system will work if there is a path (all of them working)

A_i = event that component i works

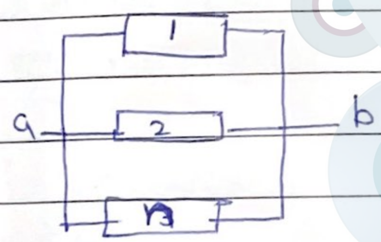
P_i = probability that the event A will happen $P(A_i)$

$\therefore P(\text{system works}) = P(\text{comp}_1 \text{ works and comp}_2 \text{ works} \ldots\ldots n)$

$P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots A_n)$

$= P(A_1) . P(A_2) . P(A_n)$

2- Parallel system



$P(\text{system work}) = P(A_1 \cup A_2 \cup A_3 \cup \ldots A_n)$

$= 1 - P(\text{system fail})$

$P(\text{system does not work})$

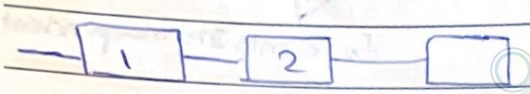
$= P(A_1^c \cap A_2^c \cap \ldots\ldots \cap A_n^c)$

$= (1-P_1)(1-P_2) \ldots\ldots (1-P_n)$

$= 1 - [(1-P_1)(1-P_2) \ldots (1-P_n)]$

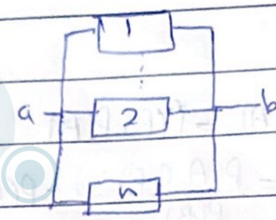
series

$P(\text{system works}) = \prod_{i=1}^{n} p_i$



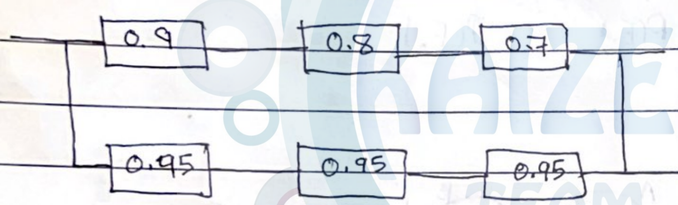
parallel

$p(\text{system works}) = 1 - \prod_{i=1}^{n} (1 - p_i)$



Examples:-

A circuit operates if there is a functional path from left to right
assum that devices fail & function independetly & what is the prob that the circuit operate



A: upper side works

B: lower side works

$P(A) = 0.9 \times 0.8 \times 0.7 = 0.504$ " series "

$P(B) = 0.95 \times 0.95 \times 0.95 = 0.8574$

$1 - \left[(1 - 0.504)(1 - 0.8574)\right] = 0.9293$ ⟸ that's the prob that the circuit operate

Bayes' Theorem

↳ Information is often available in terms of conditional probability

$P(A \cap B) = \boxed{P(B \backslash A)}\, P(A) = P(A \backslash B) P(B)$

I want this

$$P(B \backslash A) = \frac{P(A \backslash B) P(B)}{P(A)}$$

If $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive & collectively exhaustive and B is any event

$$P(E_j \backslash B) = \frac{P(B \backslash E_i) P(E_i)}{P(B)}$$

ex An Inspector at a prodction line

99% correctly identifying a defective Item

0.5% in correctly classyfing a good item as defective

0.9% of items are defective

a) what is the prob that the item is defective

$P(D) = P(D \backslash G) P(G) + P(D \backslash G') P(G')$

$= 0.005 \times (1 - 0.009) + 0.99 \times (0.009)$

$= 0.013865$

D : Item is defective

D' : Item is good

G : classfication as good

G' : classfy as defective

B) If the item is classified as non defective, what is the probability that It's indeed good

$$P(G \backslash D') = \frac{P(D' \backslash G) P(G)}{P(D')} =$$

$$= \frac{(1 - 0.005)(1 - 0.009)}{1 - 0.013865} = 0.999$$

3. Discrete Random Variables

Random variable: A function defined on the sample space or A function that assigns a real number to each outcome in the sample space of random experiment.

Denoted by an uppercase Letter such as X

Random variables

- Discrete — finite or countable infinite
- continous — finite or interval

– A Discrete Random variable

- It's random variable with a finite (countable infinite) range
- The possible values of X may be listed as $x_1, x_2, x_3 \ldots$

Ex. Throw 2 4-face dice. Let X is the sum of values of the two dice

$S = \{(1,1), (1,2), \ldots\ldots (4,4)\}$

1 \ 2	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$X = \{2, 3, 4, 5, 6, 7, 8\}$

Prob of each value of X:-

$P(x=2) = 1/16$

$P(X=3) = 2/16$

$P(X=4) = 3/16$

$P(X=5) = 4/16$

$P(X=6) = 3/16$

$P(X=7) = 2/16$

$P(X=8) = 1/16$

Ex. 8 Batch of 500 machined parts, contains 10 defective parts

number of parts in a sample of 5 parts that are defective

$X = \{0, 1, 2, 3, 4, 5\}$

$\text{Sample} = C_1^{10} * C_4^{490}$

$$P(X=0) = \frac{\#\text{ of samples that contain no defictive}}{\text{Total number of samples from } 500}$$

$$= \frac{C_5^{490}}{C_5^{500}}$$

$$P(X=1) = \frac{10 \times C_4^{490}}{C_5^{500}}$$

For a discrete random variable X with possible values $x_1, x_2, \dots, x_n$

the prob mass function is $f(x_i)$

1- $f(x_i) \geq 0$

2- $\sum^n f(x_i) = 1$

3- $f(x_i) = P(X = x_i)$

Ex $f(x) = \frac{8}{7} * \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3$

What is the prob of:

a) $P(X \leq 1) = P(X=1) = \frac{4}{7} = f(1)$

B) $P(X>1) = P(X=2) + P(X=3) = 1 - P(X \leq 1) = \frac{3}{7}$

C) $P(2 < X < 6) = P(X=3) = \frac{8}{7} \times \frac{1}{8} = \frac{1}{7} = f(3)$

D) $P(X \leq 1 \text{ or } X > 1) = f(1) + f(2) + f(3) = 1$

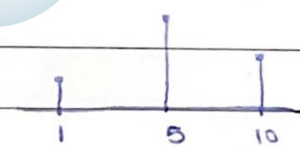
Ex A new instrument for soil andysis → successful , modertly successful , unsuccessful

successful ↓ 0.5 , modertly successful ↓ 0.65 , unsuccessful ↓ 0.05

Prob distribution for anual revenue? Revenue → 10 M , 5 M , 1 M

X: annual Revenue

$P(X=1) = 0.05$, $P(X=5) = 0.65$, $P(X=10) = 0.3$



Ex A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the prob distribution for the number of defective

Let x : the possible numbers of defective computers purchased by the school

$X = \{0, 1, 2\}$

$f(0) = P(X=0) = \frac{\binom{3}{0}\binom{17}{2}}{{}_{20}C_2} = \frac{136}{190}$

$f(1) = P(X=1) = \frac{{}_3C_1 * {}_{17}C_1}{{}_{20}C_2} = \frac{51}{190}$

$f(2) = P(X=2) = \frac{{}_3C_2 * {}_{17}C_0}{{}_{20}C_{20}} = \frac{3}{190}$

- The cumulative distribution function (cdf)

If X is ~~descet~~ discrete

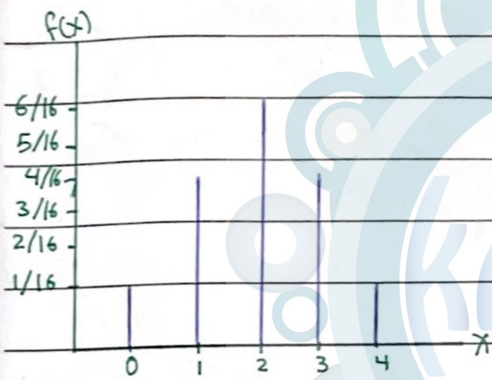
$F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$

(مثل قصة المعدل التراكمي)

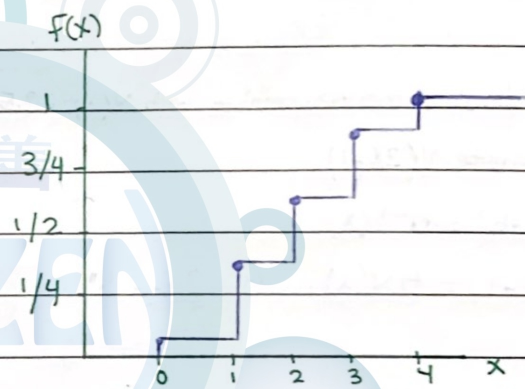
بأول فصل بالجامعة ما في معدل تراكمي

الفصل الثاني بكون معدل الفصل + الي قبله

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16
$F(x) = P(X \le x)$	1/16	5/16	11/16	15/16	16/16



prob mass function plot.



Discrete cumulative distribution function

Problem 3-35

x	−2	−1	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

Determine the Cdf

$F(-2) = \frac{1}{8}$, $F(-1) = \frac{3}{8}$, $F(0) = \frac{5}{8}$, $F(1) = \frac{7}{8}$, $F(2) = \frac{8}{8} = 1$

a) $P(X \le 1.25) = 7/8$

b) $P(X \le 2.2) = 1$

c) $P(-1.1 < X \le 1) = P(X \le 1) - P(X < -1.1) = \frac{7}{8} - \frac{1}{8} = \frac{6}{8}$

d) $P(X > 0) = 1 - P(X \le 0) = 1 - \frac{5}{8} = \frac{3}{8}$

Mean and variance of discrete R.V.

mean and variance are two measures used to summarize a prob dist

mean $\mu = E(X)$ "expected value of a discrete R.V. / is a measure of the center or middle of the probability distribution" $= \sum_x x f(x)$

Variance $\sigma^2 = V(X) = E(x^2) - (E(x))^2$

standard deviation $= \sigma = \sqrt{\sigma^2}$

Ex a lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components, A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

$X = 0, 1, 2, 3$

x	0	1	2	3
$f(x)$	$1/35$	$12/35$	$18/35$	$4/35$

$P(X=0) = \frac{{}_3C_3 * {}_4C_0}{{}_7C_3} = \frac{1}{35}$

$P(X=2) = \frac{\binom{4}{2}\binom{3}{1}}{\binom{7}{3}} = \frac{18}{35}$

$P(X=1) = \frac{\binom{3}{2}\binom{4}{1}}{\binom{7}{3}} = \frac{12}{35}$

$P(X=3) = \frac{\binom{4}{3}\binom{3}{0}}{\binom{7}{3}} = \frac{4}{35}$

$\mu = E(X) = 0 \times \frac{1}{35} + \left(1 \times \frac{12}{35}\right) + \left(2 \times 18/35\right) + \left(3 \times \frac{4}{35}\right) = \frac{12}{7} = 1.7$

$V = E(x^2) - (E(x))^2$

$E(x^2) = \frac{1}{35} + \left(1 \times 12/35\right) + \left(4 \times \frac{18}{35}\right) + \left(9 \times \frac{4}{35}\right) = 3.43$

$V(X) = 3.43 - (1.7)^2 = 0.54$

$\sigma = \sqrt{\sigma^2} = \sqrt{0.54} = 0.74$

For any constant a & B :

Mean:

1- $E(a) = a \quad , a \in \mathbb{R}$

2- $E(aX + b) = aE(X) + b \quad , a, b \in \mathbb{R}$

Variance:

1- $V(a) = 0 \quad , a \in \mathbb{R}$ "لان مقدار التشتت ما بنقدر نجيبو الا اذا كان اكتر من قيمة"

2- $V(aX+b) = a^2 V(X)$

Ex A discrete random variable with $V(X) = 2.5$

Evalute $V(2X+1)$

$V(aX+b) = a^2 V(X)$

$V(2X+1) = 4V(X) = 4 \times 2.5 = 10$

Ex Let X is a random variable with mean 6 and variance 100.

Consider another random variable Y such that $Y = 3X + 6$

evalute the mean and variance?

$E(X) = E(aX+b) = aE(X) + b$

$= E(3X+6) = 3E(X) + 6 = 3(6) + 6 = 24$

$V(X) = 9V(X) = 9(100) = 900$

1- Discrete uniform distribution

each outcome has equal probability (pmf)

$f(x_i) = \frac{1}{n}$

DU (Discrete uniform) = (i, j) → $f(x) = \begin{cases} \frac{1}{(j-i+1)} \\ 0 & \text{othewise} \end{cases}$

i = start point, j = end point

"اي شي خارج الرينج"

$\mu = E(X) = \frac{j+i}{2}$

$V = \sigma^2 = \frac{(j-i+1)^2-1}{12}$

The bernoulli trial

1- The experiment consists of repeated trials and each trial called bernolli trial

2- Each trial in an outcome that may be classified as a success or a failure (two outcomes → yes, no { Head, tail { True, false)

3- The probability of success denoted by $\boxed{p}$ ⇒ Constant from trial to trial "ما بتتغير"

4- The repeated trial are independent

Examples:-

* flipping a coin 10 times $P = 0.5$
* A Multiple-choise test contains 10 questions, each with 4 ~~answer~~ choices and you guess at each question. Let X = number of questions answered correctly $P = 0.25$
* In the next 20 biths at a hospital, let X = number of female births $P = 0.5$

$p(\text{success}) = p$ { $p(\text{faliar}) = 1 - P$

Ex. Let Y be R.V. that represent bernoulli experiment with values at 1 = success, 0 = fail

$f(1) = f(y=1) = p$

$f(0) = f(y=0) = 1-p$

$E(X) = \sum_{x} y f(y) = P$

$V(X) = \sum y^2 f(y) - \mu^2 = P(1-P)$

2- Binomial Distribution $b(n,p)$

عدد ال trials

series of bernoulli trials

The random variable X that equals the number of trials that result a success

with parameters $0<p<1$ and finit $n=1,2,3,\ldots$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x=0,1,2,\ldots,n$$

$$\mu = E(x) = np$$

$$V(x) = \sigma^2 = np(1-p)$$

Ex flip a coin 10 times, what is the probability that 3 heads occurs?

$n=10$, x

- Bernoulli trial $\rightarrow$ flip a coin
- Number of trials $\rightarrow$ $(n=10)$
- success $\rightarrow$ head occurs $(p=0.5)$
- $X=0,1,2,3,\ldots,10$

$$f(x) = \binom{10}{x}(0.5)^x(0.5)^{10-x}$$

Ex A traffic control engineer reports that 75% of the vehicles passing through a check point are from within the state. What is the prob that fewer than 4 of the next 9 vehicles are from out of state?

$n=9$

success $\rightarrow$ vehicle is from out of state $(p=0.25)$

$X=0,1,2,\ldots,9$

$$f(x) = \binom{9}{x}(0.25)^x(0.75)^{9-x}$$

4- Geometric Distribution

It's a special case from negative binomial Dist $\rightsquigarrow K=1$

- If repeated independent trials can result in a success with probability p, then the prob dist of the random variable X, the number of trials on which the first success occurs

$$f(x) = p\,(1-p)^{x-1}$$

$$\mu = E(X) = \frac{1}{p} \qquad \sigma^2 = V(X) = \frac{1-p}{p^2}$$

Ex 3-101 geometric dist, $p = 0.2$

a) what is the expected value of trials to obtain first success

$$E(x) = \frac{1}{p} = \frac{1}{0.2} = 5$$

b) after the 8^{th} success occurs, what is the expected number of trials to obtain the ninth success?

because it is independent we don't look at the ~~ninth~~ eighth so the μ is still $5 \Rightarrow \frac{1}{0.2}$ " because the lack of memory "

5- Negative Binomial Distribution

- If repeted independent trials can result in a success with probability p, then the prob dist of the random variable X, the number of the trials on which the K_{th} success occurs, is:

$$f(x) = \binom{x-1}{K-1} p^K (1-p)^{x-K}$$

$$\mu = E(X) = \frac{K}{p} \qquad \sigma^2 = V(X) = \frac{K(1-p)}{p^2}$$

Ex. In a clinical study, volunteers are tested for a gene

this gene → disease , prob that a person carried the gene is 0.1

a) what is the prob that 4 or more people will have to be tested befor two with the gene be detected.

X = number of people who are tested

$P(X \geq 4) = 1 - P(X < 4) = 1 - P(X=2) + P(X=3)$

$1 - \left[\binom{1}{1}(1-0.1)^0(0.1)^2 + \binom{2}{1}(1-0.1)^1(0.1)^2\right] = 0.972$

b) How many people an expected to be tested befor two with the gene on detected?

$E(X) = \frac{K}{P} = \frac{2}{0.1} = 20$

Ex. I an NBA championship series, the teams that wins four games out of seven is the winer.

Suppose that team A and B face each in the championship games and that team A has probability 0.55 of winning a game over team B.

a) what is the prob that team A will win the series in 6 games?

$P = 0.55$, $X = 6$, $K = 4$

$f(x) = \binom{5}{3}(0.55)^4(1-0.55)^{6-4} = 0.1853$

b) what is the prob that team A will win the series?

$P = 0.55$, $K = 4$

1- $X = 4$, $K = 4$ $\quad x = 4, 5, 6, 7$

2- $X = 5$, $K = 4$

3- $X = 6$, $K = 4$

4- $X = 7$, $K = 4$

$f(4) + f(5) + f(6) + f(7) = 0.6083$