Choosing a Transportation Mode

	Capacity	Cost	Transit Time
Rail	400 units	\$1200/car load	20 days
Truck	100 units	\$700/truck load	3 days

origin: Plant

SS = 0

Production rate = 10 units/day

Plant space cost = \$20/unit*year

value of items = \$500/unit

destination: Warehouse

SS = 100 units

Demand rate = 10 units/day

Warehouse space cost = \$25/unit*year

value of items = \$512/unit

Inventory holding cost Rate = 25% of value per year held
(Holding interest Rate)

① Inventory space cost At origin $= \boxed{\text{Origin space cost} \times (q_{(unit)} + SS_{origin})}$

$= \frac{\$20}{(unit).year} \times q\ \cancel{(unit)} = \$20q\ /year$

② Holding cost at origin $= \boxed{\text{Inventory Holding cost Rate} \times \text{Value of items at origin} \times \left(\text{Average \# of units held in the inventory} + SS_{origin}\right)}$ ($\frac{q}{2}$)

$= \frac{.25}{year} \times \frac{\$500}{\cancel{unit}} \times \frac{q}{2}\ \cancel{unit} = \$62.5q\ /year$

*Inventory cost at origin = space cost at origin + Holding cost at origin

$= 20q + 62.5q = \$82.5q\ /year$

③ Inventory space cost at destination $= \boxed{\text{Destination space cost} \times (q + SS_{destination})}$

$= 25 \times (q + 100) = \$(2500 + 25q)\ /year$

④ Holding cost at destination $= \boxed{\text{Inventory Holding cost Rate} \times \text{value of items at destination} \times \left(\text{Average of units held in the inventory} + SS_{destination}\right)}$ ($\frac{q}{2}$)

$= .25 \times \$512 \times (\frac{q}{2} + 100) = \$(12800 + 64q)\ /year$

*Inventory cost at destination = space cost at destination + Holding cost at destination

$= 2500 + 25q + 12800 + 64q = \$(15300 + 89q)\ /year$

⑤ Pipline Inventory cost (for each mode) ← Not a Function of q

$= \boxed{\left(\text{Inventory Holding cost Rate} \times \text{value of items at transit}\right) \times \text{Demand Rate} \times \text{Duration of transit}}$

Pipline Holding cost

value of items at transit $= \frac{\text{Value@ origin} + \text{value destination}}{2} = \frac{500 + 512}{2} =$

الانتباه بين وحداتهم

PIC for Rail $= \frac{.25}{year} \times \frac{506}{unit} \times \frac{10\ \cancel{unit}}{\cancel{day}} \times 20\ \cancel{day} = \$25300\ /year$

PIC for Truck $= \frac{.25}{year} \times \frac{506}{\cancel{unit}} \times \frac{10\ \cancel{unit}}{\cancel{day}} \times 3\ \cancel{day} = \$3795\ /year$

⑥ Transportation cost (for each mode)
depends on → transportation mode → number of shipments

$= \boxed{\text{Cost for each mode} \times \text{number of shipments}}$

\# of Shipments $= \frac{\text{Demand Rate in year}}{q} = \frac{10\frac{units}{day} \times \frac{365\ days}{1\ year}}{q} = \frac{3650}{q}$ (shipments/year)

TC for Rail $= \$1200 \times \frac{3650}{q} = \frac{\$4380000}{q_{rail}}\ /year$

TC for Truck $= \$700 \times \frac{3650}{q} = \frac{\$2555000}{q_{truck}}\ /year$

* Note → if handling cost for truck is mentioned add it to the cost for truck
→ if there is more than 1 truck, just multiply the cost of 1 truck by the number of trucks

⑦ Total cost equation (for each mode) $= \boxed{\text{Inventory cost at origin} + \text{Inventory cost at destination} + \text{Pipeline inventory cost} + \text{Transportation cost}}$

Total cost → TC for Rail $= 82.5q_{rail} + 89q_{rail} + 15300 + 25300 + \frac{4380000}{q_{rail}} = 171.5q_{rail} + \frac{4380000}{q_{rail}} + 406$

TC for Truck $= 82.5q_{truck} + 89q_{truck} + 15300 + 3795 + \frac{2555000}{q_{truck}} = 171.5q_{truck} + \frac{2555000}{q_{truck}} + 190$

⑧ optimal (q) for each mode = Derive TC(each q) equation and equal it by zero → $\boxed{q_{optimal} \xrightarrow{when} \frac{d(TC)}{dq} = 0}$

$q^*_{rail} \rightarrow 171.5 - \frac{4380000}{q^2} = 0 \rightarrow q^*_{rail} = 160$ units < 400 capacity

TC_{rail} with q^* $\rightarrow \frac{4380000}{(160)} + 171.5(160) + 40600 = \boxed{\$95415/year}$

$q^*_{truck} \rightarrow 171.5 - \frac{2555000}{q^{2*}} = 0 \rightarrow q^*_{truck} = 122$ units > 100 capacity so 2 trucks will be needed → so revert to $q^*_{truck} = 100$

TC_{truck} with q^* $\rightarrow 171.5(100) + \frac{2555000}{(100)} + 19095 = \boxed{\$61795/year}$

⑨ Most Economical Mode → is Truck with q^*

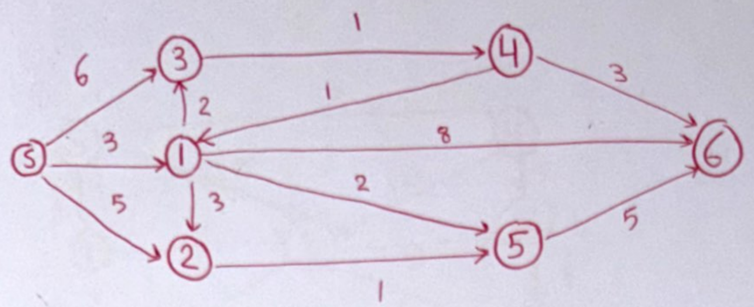
\# of shipments so \# of trucks needed $= \frac{3650}{100} = 36.5 \uparrow$ Roundup $= \boxed{37\ truck}$

Example (1)

Dijkstra's Algorithm

$n = 7 \longrightarrow$ *of iterations is $(n+1) = 8$

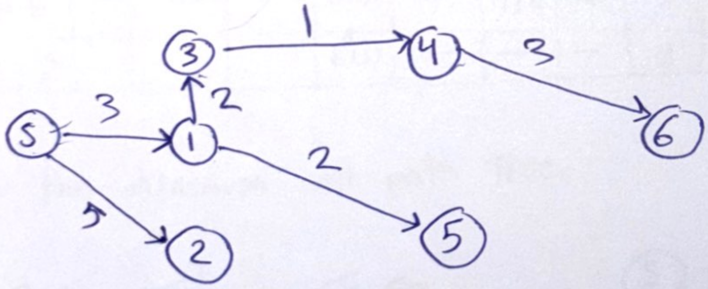
cost $\longrightarrow V(i)(\text{Pred}(i))$



initial Starting Node

	iteration	S	1	2	3	4	5	6	P (current node)	Perm (List of visited nodes)	$\overline{\text{Perm}}$ (List of unvisited nodes)
	0	0(0)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	—	φ	N
3	1	0(0)*	3(s)	5(s)	6(s)	∞(-1)	∞(-1)	∞(-1)	S	S	N/{s}
	2	—	3(s)*	5(s) ~~6(1)~~	5(1)	∞(-1)	5(1)	11(1)	1	S,1	N/{S,1}
5	3	—	—	5*(s)	5(1)	∞(-1)	~~6(2)~~ 5(1)	11(1)	2	S,1,2	N/{S,1,2}
5	4	—	—	—	5(1)*	6(3)	5(1)	11(1)	3	S,1,2,3	N/{S,1,2,3}
	5	—	—	—	—	6(3)	5(1)*	10(5)	5	S,1,2,3,5	4,6
	6	—	—	—	—	6*(3)	—	9(4)	4	S,1,2,3,5,4	6
	7	—	—	—	—	—	—	9*(4)	6	S,1,2,3,4,5,6	φ

*Draw the shortest path tree (minimum cost path Tree)



*Give me the shortest path from S to 6

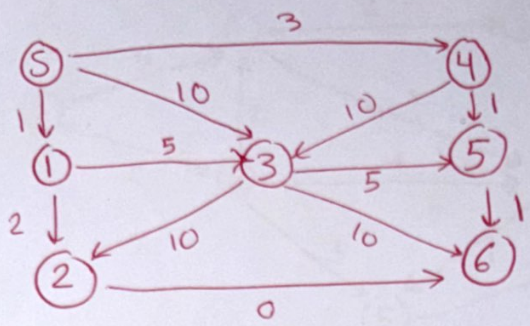
$$P_{S\,to\,6} = \{(S,1),(1,3),(3,4),(4,6)\}$$

and the cost for this path

$$3 + 2 + 1 + 3 = 9$$

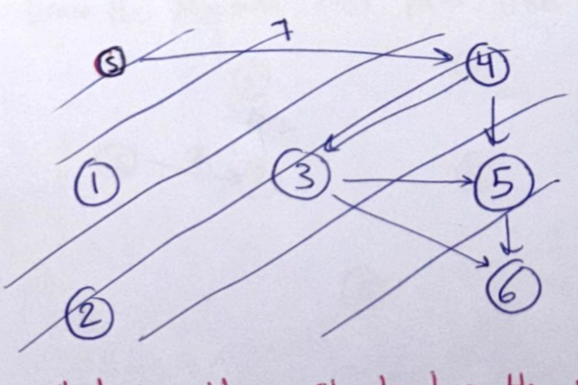
Example (2)

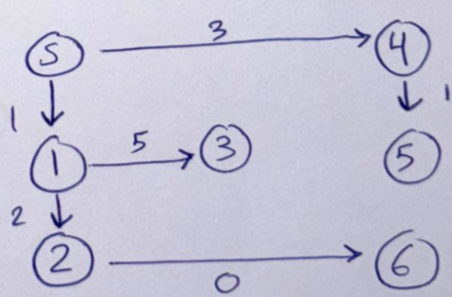
Dijkstra's Algorithm



Iteration	S	1	2	3	4	5	6	P current node	Perm visited	$\overline{\text{Perm}}$ unvisited
0	0(0)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	—	φ	N
1	$\overset{*}{0}$(0)	1(S)	∞(-1)	10(S)	3(S)	∞(-1)	∞(-1)	S	S	N/{S}
2	—	$\overset{*}{1}$(S)	3(1)	6(1)	3(S)	∞(-1)	∞(-1)	1	S, 1	N/{S,1}
3	—	—	$\overset{*}{3}$(1)	6(1)	3(S)	∞(-1)	3(2)	2	S, 1, 2	N/{S,1,2}
4	—	—	—	6(1) ~~3(4)~~	$\overset{*}{3}$(S)	4(4)	3(2)	4	S, 1, 2, 4	N/{S,1,2,4}
5	—	—	—	6(1)	—	4(4)	$\overset{*}{3}$(2)	6	S, 1, 2, 4, 6	N/{S,1,2,4,6}
6	—	—	—	6(1)	—	$\overset{*}{4}$(4)	—	5	S, 1, 2, 4, 5, 6	3
7				$\overset{*}{6}$(1)	—	—	—	3	N	φ

Draw the Minimum cost path Tree





What is the shortest path from (s to 6) and its cost

$P_{s\,to\,6} = \{(s,1),(1,2)(2,6)\}$

Cost = 3

Example (1)

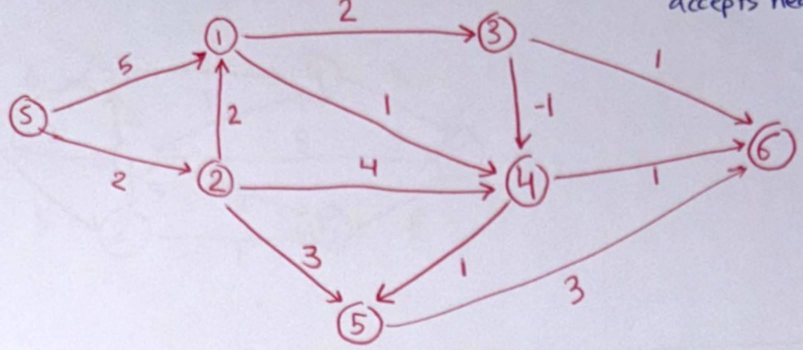
Bellman Ford Algorithm

Just for Directed + accepts negative cost

Modification Rules

if j has never been in the List

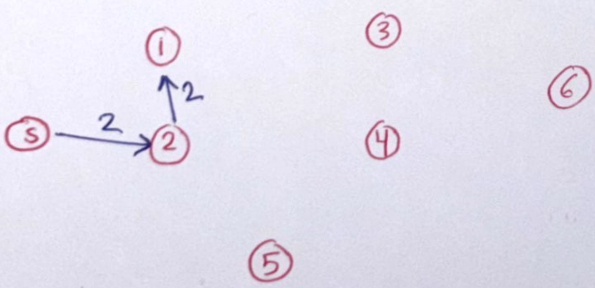
j ∈ List



\# of iterations is unknown

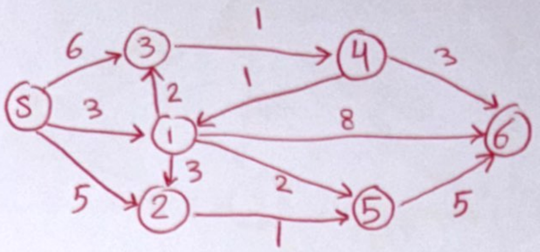
iterations	s	1	2	3	4	5	6	P current	List
0	0(0)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	—	s
1	*0(0)	5(s)	2(s)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	s	1, 2
2	0(0)	*5(s)	2(s)	7(1)	6(1)	∞(-1)	∞(-1)	1	2, 3, 4
3	0(0)	4(2)	*2(s)	7(1)	6(1)	5(2)	∞(-1)	2	1, 3, 4, 5
4	0(0)	*4(2)	2(s)	6(1)	5(1)	5(2)	∞(-1)	1	3, 4, 5
5	0(0)	4(2)	2(s)	*6(1)	5(1)	5(2)	7(3)	3	4, 5, 6
6	0(0)	4(2)	2(s)	6(1)	*5(3)	5(2)	6(4)	4	5, 6
7	0(0)	4(2)	2(s)	6(1)	5(3)	*5(2)	6(4)	5	6
8	0(0)	4(2)	2(s)	6(1)	5(3)	5(2)	*6(4)	6	ϕ

Draw the Minimum Cost path Tree



Example (2)

Bellman Ford Algorithm

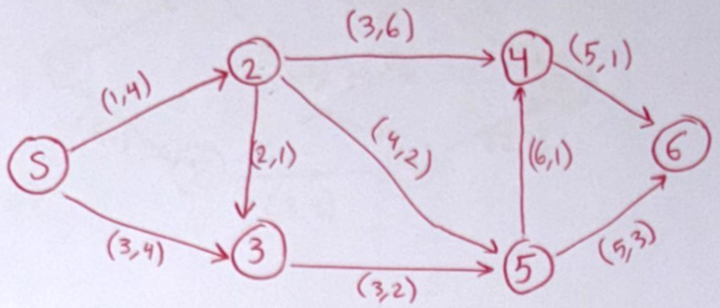


Interation	S	1	2	3	4	5	6	P	List
0	0(0)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	∞(-1)	—	S
1	0*(0)	3(S)	5(S)	6(S)	∞(-1)	∞(-1)	∞(-1)	S	1, 2, 3
2	0(0)	3*(S)	5(S)	5(1)	∞(-1)	5(1)	11(1)	1	2, 3, 5, 6
3	0(0)	3(S)	5*(S)	5(1)	∞(-1)	~~6(2)~~ 5(1)	11(1)	2	3, 5, 6
4	0(0)	3(S)	5(S)	5*(1)	6(3)	5(1)	11(1)	3	5, 6, 4
5	0(0)	3(S)	5(S)	5(1)	6(3)	5*(1)	10(5)	5	6, 4
6	0(0)	3(S)	5(S)	5(1)	6(3)	5(1)	10*(5)	6	4
7	0(0)	3(S)	5(S)	5(1)	6*(3)	5(1)	9(5)	4	6
8	0(0)	3(S)	5(S)	5(1)	6(3)	5(1)	9*(5)	6	Φ

Draw Min Cost Tree

Example (1)

Multi-Label Algorithm

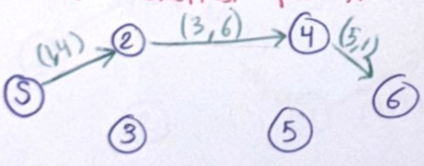


Labels

S	2	3	4	5	6	List
$(0,0)^{s}_{1}$	$(1,4)^{2}_{2}$	$(3,4)^{3}_{3}$	$(4,10)^{4}_{5}$	$(5,6)^{5}_{6}$	$(9,11)^{6}_{8}$	$(0,0)^{s}_{1}$ X
Pred(1)=0	Pred(2)=1	Pred(3)=1	Pred(5)=2	Pred(6)=2	Pred(8)=5	$(1,4)^{2}_{2}$ X
		$(3,5)^{3}_{4}$ X	$(11,7)^{4}_{10}$	$(6,6)^{5}_{7}$ X	$(10,9)^{6}_{9}$	$(3,4)^{3}_{3}$ X
		Pred(4)=2	Pred(10)=6	Pred(7)=3	Pred(9)=6	$(3,5)^{3}_{4}$ X
					$(16,8)^{6}_{11}$	$(4,10)^{4}_{5}$ X
					Pred(11)=10	$(5,6)^{5}_{6}$ X
						$(6,6)^{5}_{7}$ X
						$(9,11)^{6}_{8}$ X
						$(10,9)^{6}_{9}$ X
						$(11,7)^{4}_{10}$ X
						$(16,8)^{6}_{11}$ X
						ϕ

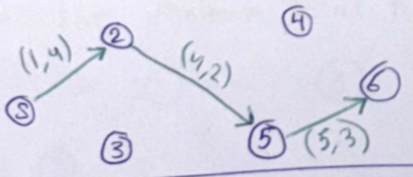
what is the Shortest path from (s to 6)?

Path 1



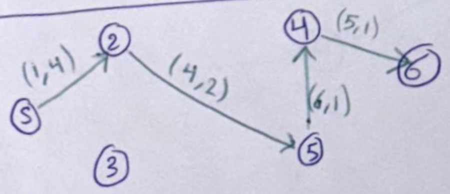
$(9,11)^{6}_{8}$ ✓ $P=\{(s,2),(2,4),(4,6)\}$

path 2



$(10,9)^{6}_{9}$ $P=\{(s,2),(4,2),(5,3)\}$

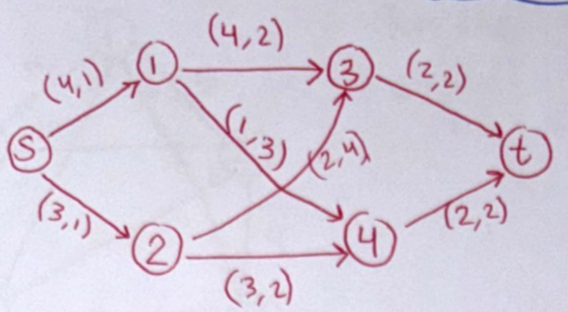
Path 3



$(16,8)^{6}_{11}$ $P=\{(s,2),(2,5),(5,4),(4,6)\}$

Example (2)

Multi-Label Algorithm



LABELS

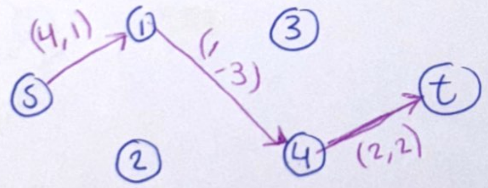
S	1	2	3	4	t
$(0,0)^{s}_{1}$ Pred(1)=0	$(4,1)^{1}_{2}$ Pred(2)=1	$(3,1)^{2}_{3}$ Pred(3)=1	$(8,3)^{3}_{4}$ Pred(4)=2	$(5,4)^{4}_{5}$ Pred(5)=2	$(10,5)^{t}_{8}$ Pred(8)=4
			$(5,5)^{3}_{6}$ Pred(6)=3 No Domination	$(6,3)^{4}_{7}$ Pred(7)=3 No Domination	$(7,6)^{t}_{9}$ Pred(9)=5
					$\cancel{(7,7)^{t}_{10}}$ ~~Pred(10)=6~~ X
					$(8,5)^{t}_{11}$ Pred(11)=7

List

- $(0,0)^{s}_{1}$ X
- $(4,1)^{1}_{2}$ X
- $(3,1)^{2}_{3}$ X
- $(8,3)^{3}_{4}$ X
- $(5,4)^{4}_{5}$ X
- $(5,5)^{3}_{6}$ X
- $(6,3)^{4}_{7}$ X
- $(10,5)^{t}_{8}$ X
- $(7,6)^{t}_{9}$ X
- $(7,7)^{t}_{10}$ X
- $(8,5)^{t}_{11}$ X

ϕ

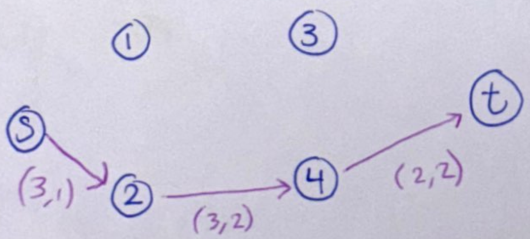
* Draw the Minimum distance path from (s to t)



$(7,6)^{t}_{9}$

Pred (9) = 5

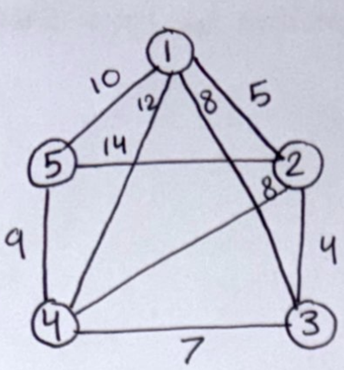
Draw the Minimum total time path from (s to t)



$(8,5)^{t}_{11}$

Pred (11) = 7

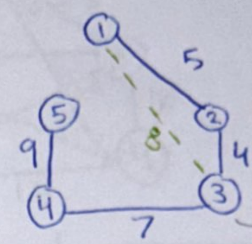
Find Lower Bound (LB → MST+Arc, → 1-Tree$_i$) for the TSP



Method 1

$$LB = C(MST) + C^*_{ij} = 25+8 = 33$$

C^*_{ij} → minimum cost Arc not in the MST



$C(MST) = 25$

$C^*_{ij} = 8$

→ Not a feasible tour (there is a subtour)

4
5
7
8
8
9

Method 2 1-Tree$_i$

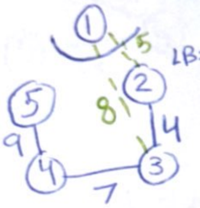
$|N| = n-1$

$|A| = n-2 \to 5-2 = 3$ Arcs

$$LB = MST_i + C^*_{(i,k)} + C^*_{(i,j)}$$

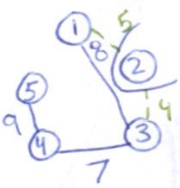
→ Minimum two Arcs from omitted node

1-Tree$_1$



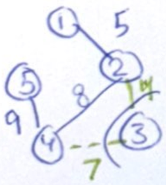
$LB = MST + C^*_{ik} + C^*_{ij}$
$= 20 + 8 + 5$
$= 33$

1-Tree 2



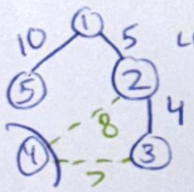
$LB = 24 + 4 + 5$
$= 33$

1-Tree 3



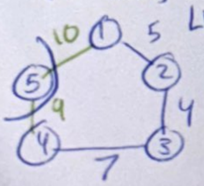
$LB = 22 + 4 + 7$
$= 33$

1-Tree 4



$LB = 19 + 7 + 8$
$= 34$

1-Tree 5

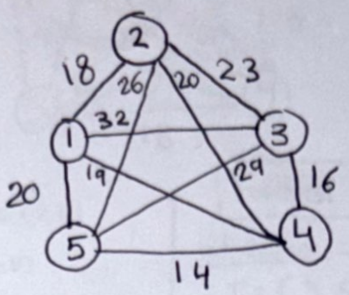


$LB = 16 + 9 + 10$
$= 35$ → يختار الأعلى من ال Lower Bound لأنه عم أقرب لل optimal

TSP heuristic based on MST with Performance guaranteed

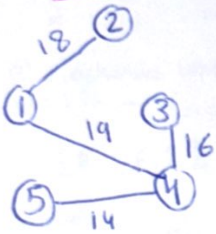
→ 2 MST based heuristic → Twice Around MST heuristic ($\alpha=2$)
→ Christified heuristic ($\alpha=1.5$)
→ optimal heuristic
→ approximation heuristic

14
16
18
19
20



Twice Around MST Heuristic ($\alpha=2$)

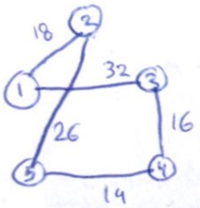
$\boxed{1}$ MST



$\boxed{2}$ List
~~(1,2)~~
~~(1,2)~~
~~(1,4)~~
~~(1,4)~~
~~(3,4)~~
~~(3,4)~~
~~(4,5)~~
~~(4,5)~~

$\boxed{3}$ walk = {5, 4, 3, 4, 1, 2, 1, 4, 5}

$\boxed{4}$ Tour = {5, 4, 3, 1, 2, 5}



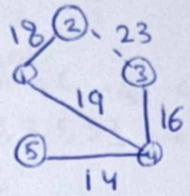
worse case

$C^H = 100 < 2 \times C^*$

Christofides Heuristic ($\alpha=1.5$)

best

$\boxed{1}$ MST



$\boxed{2}$ S = {2, 3, 4, 5}

|S| is even

Nodes with odd degrees in MST

$\boxed{3}$ to find Perfect node matching W

least cost

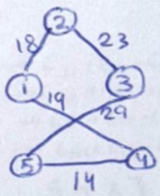
2-3 and 4-5 → $C_{23} + C_{45} = 23 + 14 = 37 \rightarrow w^*$

2-4 and 3-5 → $C_{24} + C_{35} = 20 + 29 = 49$

2-5 and 3-4 → $C_{25} + C_{34} = 26 + 16 = 42$

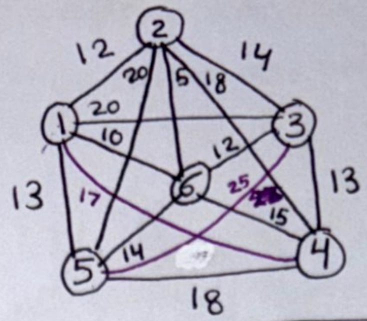
$\boxed{4}$ walk = {5, 4, 1, 2, 3, 4, 5}

$\boxed{5}$ Tour = {5, 4, 1, 2, 3, 5}



worse case

$C^H = 103 < 1.5 \times C^*$



Nearest insertion heuristic

Initialization → we start with min cost arc (2,6)

$T_{Tour} = \{2, 6, 2\} \rightarrow C(T) = 5 \times 2 = 10$

iteration 1

step 1 To determine which node to insert

$j \notin T$	$i \in T$	c_{ij}
1	6	10 → min cost
3	6	12
4	6	15
5	6	14

(insert node 1)

2 To determine where to insert this node

btw 2-6 → $C_{21} + C_{16} - C_{26} = 12 + 10 - 5 = \boxed{17}$ ← so btw 2-6

$T = \{2, 1, 6, 2\} \rightarrow C(T) = 10 + 17 = 27$

iteration 2

$j \notin T$	$i \in T$	c_{ij}
3	6	12 → min
4	6	15
5	1	13

(insert node 3)

insertion cost

btw 2,1 → $C_{23} + C_{31} - C_{21} = 22$

1,6 → $C_{13} + C_{36} - C_{16} = 22$

6,2 → $C_{63} + C_{32} - C_{62} = \boxed{21}$ → btw 6,2

$T = \{2, 1, 6, 3, 2\} \rightarrow C(T) = 27 + 21 = 48$

iteration 3

$j \notin T$	$i \in T$	c_{ij}
4	3	13 → choose any
5	1	13

(insert node 4)

insertion cost

btw 2,1 → $C_{24} + C_{41} - C_{21} = 23$

1,6 → $C_{14} + C_{46} - C_{16} = 22$

6,3 → $C_{64} + C_{43} - C_{63} = \boxed{16}$ → (btw 6,3)

3,2 → $C_{34} + C_{42} - C_{32} = 17$

$T = \{2, 1, 6, 4, 3, 2\} \rightarrow C(T) = 48 + 16 = 64$

iteration 4

$j \notin T$	$i \in T$	c_{ij}
5		→

(insert node 5)

incertion cost

btw 2,1 → $C_{25} + C_{51} - C_{21} = 21$

1,6 → $C_{15} + C_{56} - C_{16} = \boxed{17}$ →

6,4 → $C_{65} + C_{54} - C_{64} = \boxed{17}$ →

4,3 → $C_{45} + C_{53} - C_{43} = 22$

3,2 → $C_{35} + C_{52} - C_{32} = 23$

Last iteration so we put 2 possible Tours

$T = \{2, 1, 5, 6, 4, 3, 2\}$
$C(T) = 64 + 17 = 81$

$T = \{2, 1, 6, 5, 4, 3, 2\}$
$C(T) = 64 + 17 = 81$

Farthest insertion heuristic

Initialization → we start with max cost Arc (3-5) →

$T = \{3, 5, 3\} \rightarrow C(T) = 25 \times 2 = 50$

iteration 1

$j \notin T$	$i \in T$	c_{ij}
1	5	13
2	3	14 → max cost
4	3	13
6	3	12

(insert node 2)

insertion cost

btw (3,5) → $C_{32} + C_{25} - C_{35} = \boxed{9}$ (btw 3-5)

$T = \{3, 2, 5, 3\} \rightarrow C(T) = 50 + 9 = 59$

iteration 2

$j \notin T$	$i \in T$	c_{ij}
1	2	12
4	3	13 → max cost
6	2	5

(insert node 4)

incertion cost

btw 3,2 → $C_{34} + C_{42} - C_{32} = 17$

2,5 → $C_{24} + C_{45} - C_{25} = 16$

5,3 → $C_{54} + C_{34} - C_{53} = \boxed{6}$ → (btw 5,3)

$T = \{3, 2, 5, 4, 3\} \rightarrow C(T) = 59 + 6 = 65$

iteration 3

$j \notin T$	$i \in T$	c_{ij}
1	2	12 → max
6	2	5

(insert node 1)

incertion cost

btw 3,2 → $C_{31} + C_{12} - C_{32} = 18$

2,5 → $C_{21} + C_{15} - C_{25} = \boxed{5}$ → btw (2,5)

5,4 → $C_{51} + C_{14} - C_{54} = 12$

4,3 → $C_{41} + C_{13} - C_{43} = 24$

$T = \{3, 2, 1, 5, 4, 3\} \rightarrow C(T) = 65 + 5 = 70$

iteration 4

$j \notin T$	$i \in T$	c_{ij}
6		

(insert node 6)

incertion cost

btw 3,2 → $C_{36} + C_{62} - C_{32} = 3$ →

2,1 → $C_{26} + C_{61} - C_{21} = 3$ →

1,5 → $C_{16} + C_{65} - C_{15} = 11$

5,4 → $C_{56} + C_{46} - C_{54} = 11$

4,3 → $C_{46} + C_{63} - C_{43} = 14$

Last iteration so we put 2 possible Tours

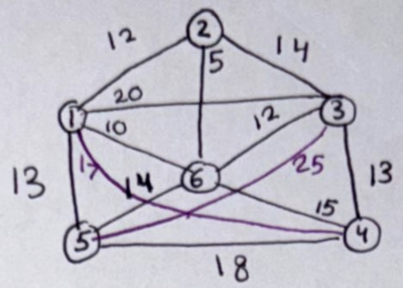
$T = \{3, 6, 2, 1, 5, 4, 3\}$
$C(T) = 70 + 3 = 73$

$T = \{3, 2, 6, 1, 5, 4, 3\}$
$C(T) = 70 + 3 = 73$

Practical TSP Heuristic
(No Performance guarantee)

→ ① Construction Approach → Nearest Neighbor / → Nearest incertion cost / → Furthest incertion cost

→ ② Improvement Approach

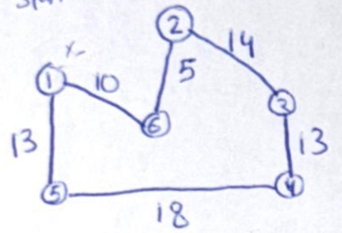


Solve by

Nearest Neighbor → start with → lowest cost Arc / or → any node

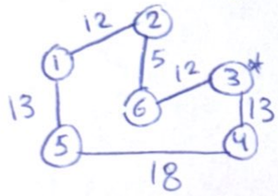
No unique solution

Start with lowest cost Arc $(\overset{Arc}{2-6}) = 5$



$\rightarrow C^H = 91$

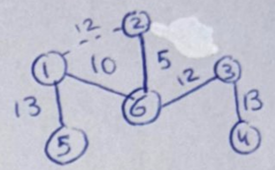
Start with Arbitrary Node → Node 3



$C^H = 73$

Solve for LB

$C(MST) + C_{ij}^* = 53 + 12$
$= 65$



LB ≤ Optimal ≤ Hueristic ≤ worse case

Not in practical heuristic