Summary for Ch 2, 3, 4, 5, 6
نقل

Chapter (2)
crisp sets and Fuzzy sets

$\in$ → Belongs to

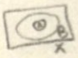
$\notin$ → Doesn't Belong to

* Definitions :-

- Universe of Discource (X) :- a collection of objects having the same characteristics.
- The elements (x) of a universe are Discrete or Continuous
- Cardinal Number (n_X) or (CN) :- the total number of elements in a universe.
  - → Discrete universes → finite (CN)
  - → Continuous universes → infinite (CN)
- Set : collections of elements within a universe
- Subset :- collections of elements within sets
- The whole Set :- collections of all elements within a set
- Null set (ϕ) :- The set that contains No Elements
- Power Set ($P(X)$) :- All possible sets of X

Special Topics in Manufacturing
Summary by Nada Ababneh

$A \subset B$ ⇝ A is Fully Contained in B (if $x \in A$, then $x \in B$)



$A \subseteq B$ ⇝ A is containd in or is equivalent to B

$A \leftrightarrow B$ ⇝ $A \subseteq B$ and $B \subseteq A$ (A is equivalent to B)

* ينطبق فقط على ال (crisp)

* Axiom of excluded middle
$$A \cup \bar{A} = X$$

* Axiom of contradiction
$$A \cap \bar{A} = \phi$$

Crisp Sets (operations)

Suppose A, B are crisp Sets in the universe X

Union (اتحاد): $A \cup B = \{x \mid x \in A \overset{(Max)}{\text{or}} x \in B\}$

Intersection (تقاطع): $A \cap B = \{x \mid x \in A \overset{(Min)}{\text{and}} x \in B\}$

complement: $\bar{A} = \{x \mid x \in A , x \in X\}$

← بقارن A مع ال universe

Difference: $A \mid B = \{x \mid x \in A \text{ and } x \notin B\}$

→ the set of elements in (A) and are Not in (B)

De Morgan's Principles :-
→ valid for Crisp sets, Fuzzy sets

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$
$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

Note for difference :- in Fuzzy

$$\underset{\sim}{A} \mid \underset{\sim}{B} = \underset{\sim}{A} \cap \bar{\underset{\sim}{B}}$$
$$\underset{\sim}{B} \mid \underset{\sim}{A} = \underset{\sim}{B} \cap \bar{\underset{\sim}{A}}$$

Mapping of Crisp Sets to Functions :-

Relating set theoretic forms to function theoretic terms

* Membership Function → is a mapping for a crisp set

membership degree

Union: $A \cup B$ ⇝ $\chi_{A \cup B}(x) = \chi_A(x) \vee \chi_B(x) = \max(\chi_A(x), \chi_B(x))$

Intersection: $A \cap B$ ⇝ $\chi_{A \cup B}(x) = \chi_A(x) \wedge \chi_B(x) = \min(\chi_A(x), \chi_B(x))$

Complement: $\bar{A}$ ⇝ $\chi_{\bar{A}}(x) = 1 - \chi_A(x)$

Containment: $A \subseteq B$ ⇝ $\chi_A(x) \leq \chi_B(x)$

All the operations are valid for Crisp sets and Fuzzy sets

Fuzzy Sets

→ contains elements that have varying degrees of membership

For Discrete Universe : $\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1}, \frac{\mu_{\underset{\sim}{B}}(x_2)}{x_2}, \ldots\ldots\ldots \right\}$

→ indication for fuzzy set

For Continuous Universe : $\underset{\sim}{A} = \left\{ \int \frac{\mu_{\underset{\sim}{A}}(x)}{x} \right\}$

Fuzzy powerset → ∞
Fuzzy cardinal # → ∞

Fuzzy Intersection ($\cap$) → t-no
Fuzzy Union ($\cup$) → t-conorm

chapter(3)

we can use the Crisp properties
- commutativity ✓
- associativity
- distributivity
- involution
- idempotecy

applicable ✓ → De Morgans principle

excluded middle Axiom × → $\underset{\sim}{R} \cup \underset{\sim}{\bar{R}} \neq E$

→ $\underset{\sim}{R} \cap \underset{\sim}{\bar{R}} \neq O$

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

* CN for Fuzzy sets → ∞

* CN (Cardinal number) for Fuzzy Relations → ∞

Fuzzy Relations

→ map elements of one universe to elements of another universe through (Cartesian Product)

→ the strength of the relation between ordered pairs of two universes is measured with a membership function

$(\mu_{\underset{\sim}{R}}(x,y))$

Operations
(for membership values)

R and S are Fuzzy Relations on the Cartesian space $(X \times Y)$

Union (max): $\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x,y) = \max(\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y))$ (membership degree)

Intersection (min): $\mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x,y) = \min(\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y))$

Complement: $\mu_{\underset{\sim}{\bar{R}}}(x,y) = 1 - \mu_{\underset{\sim}{R}}(x,y)$

Containment: $\underset{\sim}{R} \subset \underset{\sim}{S} \rightarrow \mu_{\underset{\sim}{R}}(x,y) \leq \mu_{\underset{\sim}{S}}(x,y)$

Same as before

Cartesian Product

A and B are Fuzzy sets on universes X and Y

The Cartesian Product → $\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R} \subset X \times Y$

Fuzzy Relation ($\underset{\sim}{R}$) → membership function: $\mu_{\underset{\sim}{R}}(x,y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x,y) = \min(\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y))$ (× → min)

Composition

R and S are fuzzy Relations on the cartesian space $(X \times Y)$ and $(Y \times Z)$ and $\underset{\sim}{T}$ is a fuzzy Relation on $(X \times Z)$

$\underbrace{X \rightarrow Y}_{R}$ $\underbrace{Y \rightarrow Z}_{S}$

- **Max-Min Composition**

$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S}$

$$\mu_{\underset{\sim}{T}}(x,z) = \overset{②\,\text{Max}}{\bigvee_{y \in Y}} (\mu_{\underset{\sim}{R}}(x,y) \overset{①\,\text{Min}}{\wedge} \mu_{\underset{\sim}{S}}(y,z))$$

*Fuzzy Composition is Not Commutative

$\underset{\sim}{R} \circ \underset{\sim}{S} \neq \underset{\sim}{S} \circ \underset{\sim}{R}$

- **Max-product composition**

يضرب وباخذ Maximum

$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S}$

$$\mu_{\underset{\sim}{T}}(x,z) = \overset{\text{Max}}{\bigvee_{y \in Y}} (\mu_{\underset{\sim}{R}}(x,y) \overset{\text{multiply}}{\bullet} \mu_{\underset{\sim}{S}}(y,z))$$

$T = R \circ S$

$$= \begin{array}{c|c} & A_1 \; A_2 \; A_3 \; A_4 \\ \begin{array}{c} D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} & \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix} \end{array} \circ \begin{array}{c|c} & w_1 \; w_2 \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix} \end{array}$$

Fuzzy Equivalence

Reflexivity: $\mu_{\underset{\sim}{R}}(x_i, x_i) = 1$

Symmetry: $\mu_{\underset{\sim}{R}}(x_i, x_j) = \mu_{\underset{\sim}{R}}(x_j, x_i)$

Fuzzy Tolerance Relation

يمكن التحويل by At most $(CN-1)$ compositions with itself $\underset{\sim}{R}_1^{n-1}$

Transitivity: $\mu_{\underset{\sim}{R}}(x_i, x_j) = \lambda_1$ and $\mu_{\underset{\sim}{R}}(x_j, x_k) = \lambda_2 \longrightarrow \mu_{\underset{\sim}{R}}(x_i, x_k) = \lambda$

where $\lambda \geq \min[\lambda_1, \lambda_2]$

Fuzzy Equivalence Relation

Crisp Relations (Equivalence Relation)

Reflexivity $(x_i, x_i) \in R$ or $\chi_R(x_i, x_i) = 1$

Symmetry $(x_i, x_j) \in R \rightarrow (x_j, x_i) \in R$

or $\chi_R(x_i, x_j) = \chi_R(x_j, x_i)$

Tolerance Relation

$\xrightarrow{\text{(at most } CN-1)}$ للتحويل

$R_1^{n-1} = R_1 \circ R_1 \circ \cdots \circ R_1 = R$

Equivalence Relation

Transitivity $(x_i, x_j) \in R$ and $(x_j, x_k) \in R \rightarrow (x_i, x_k) \in R$

or $\chi_R(x_i, x_j)$ and $\chi_R(x_j, x_k) = 1 \rightarrow \chi_R(x_i, x_k) = 1$

Value Assignment Methods

← in Fuzzy

- Cartesian Product
- Closed Form expression
- Lookup Table
- Linguistic Rules of Knowledge
- Classification
- Automated methods from input/output data
- [Similarity methods in data manipulation

Cosine Amplitude Method

(Similarity Method)

→ A Similarity metric that uses a collection of data Samples.

$$r_{ij} = \frac{\left| \sum_{k=1}^{m} x_{ik} x_{jk} \right|}{\sqrt{\left(\sum_{k=1}^{m} x_{ik}^2\right) \times \left(\sum_{k=1}^{m} x_{jk}^2\right)}}$$

where $i, j = 1, 2, \ldots, n$

نربع كل وحدة لحال

بعدين بجمعهم وبضرب بالموس الآخر

Max-Min Method

(Symmetric)

أسهل من ال cosine Amplitude

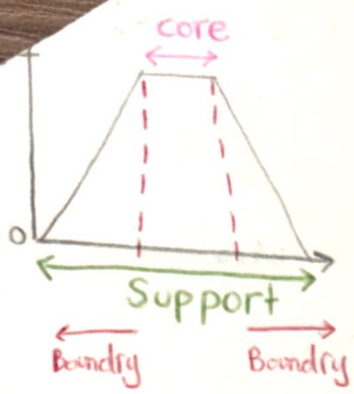
→ A Similarity metric that uses a collection of data Samples.

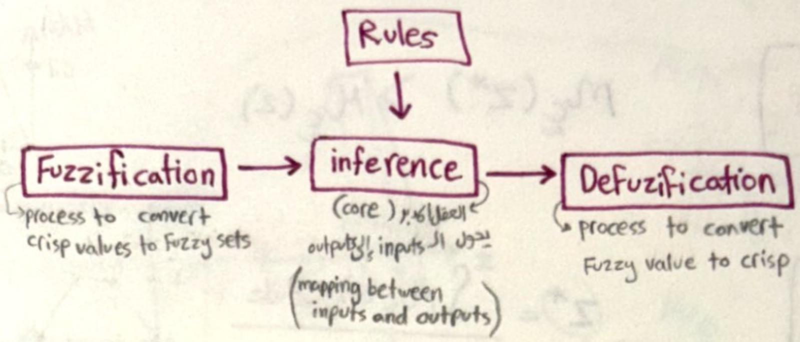
$$r_{ij} = \frac{\sum_{k=1}^{m} \min(x_{ik}, x_{jk})}{\sum_{k=1}^{m} \max(x_{ik}, x_{jk})}$$

where $i, j = 1, 2, \ldots, n$

Chapter (4)

Properties of Membership Functions (Fuzzification and Defuzification)





- **Membership Function :-** The values assigned to elements of a universal set fall within a specified range.
  - it describes the information contained in a fuzzy set
  - Larger values indicates higher degrees of membership

  can be: symmetrical / asymmetrical; 1D / nD

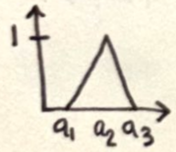
- **The Core :-** The region (element) of a universe that is characterized by full membership in a set.
- **The Support :-** The regoin (element) of a universe that is characterized by Nonzero membership in a set.
- **Prototype :-** It is an element (only one element) that has a membership value that is equal to one.

* The height of a fuzzy set is the maximum value of the membership function

Membership Function Types (Fuzzification)

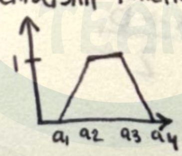
(Fuzzification) → represents the process of mapping crisp values to fuzzy sets

① Triangular Membership Function



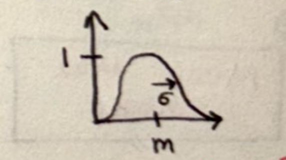
$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u-a_1}{a_2-a_1} & a_1 \leq u < a_2 \\ \dfrac{a_3-u}{a_3-a_2} & a_2 \leq u < a_3 \\ 0 & u \geq a_3 \end{cases}$$

② Trapezoidal Membership Function



$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u-a_1}{a_2-a_1} & a_1 \leq u < a_2 \\ 1 & a_2 \leq u < a_3 \\ \dfrac{a_4-u}{a_4-a_3} & a_3 \leq u < a_4 \\ 0 & u \geq a_4 \end{cases}$$

③ Gaussian Member ship Function



$$\mu_A(u) = e^{-\left(\frac{u-m}{\sigma}\right)^2}$$

④ Singleton

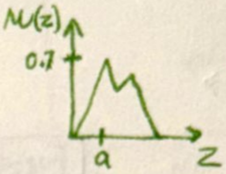
trapezoidal area $= \frac{1}{2} \times$ (مجموع القاعدتين) $\times h$

Deffuzzification

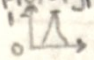
→ represents the process of mapping fuzzy values to crisp values

ⓐ Max Membership (the height method)

$$\mu_{\tilde{C}}(z^*) \geq \mu_{\tilde{C}}(z)$$



- ussually we use it with prototype
- (restricted to prototype)



ⓑ Centroid Method (centre of area)

(After Aggregation)

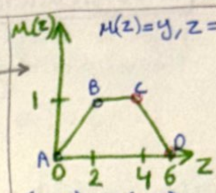
$$z^* = \frac{\int \mu_{\tilde{C}}(z) \cdot z \, dz}{\int \mu_{\tilde{C}}(z) \, dz}$$

Membership Function

بعطيني x Calculator

z^* → centre of the area covered by the Membership Function

$\mu_{\tilde{C}}$ → in fuzzy set c



$\mu(z) = y, \; z = x$

$(x, y)_A = (0, 0)$

$(x, y)_B = (2, 1)$

$\boxed{y = mx + c}$

$0 = m0 + c \rightarrow c = 0$

$1 = m \times 2 + 0 \rightarrow m = \frac{1}{2}$

$\boxed{y = \mu(z) = \frac{1}{2} z + 0}$

$(x, y)_C = (4, 1)$

$(x, y)_D = (6, 0)$

$\boxed{y = mx + c}$

$\boxed{y = (\mu(z)) = -\frac{1}{2} z + 3}$

$$z^* = \frac{\int_0^2 \left(\frac{z}{2}\right) \cdot z \, dz + \int_2^4 (1) \cdot z \, dz + \int_4^6 \left(-\frac{z}{2} + 3\right) \cdot z \, dz}{\int_0^2 \frac{z}{2} \, dz + \int_2^4 (1) \, dz + \int_4^6 \left(-\frac{z}{2} + 3\right) dz}$$

$z^* = 3$

$\frac{\Delta y}{\Delta x} = m = \text{slope}$ ①

تعويض Point لايجاد C ②

$\mu(z) = \boxed{m} \times z + \boxed{c}$ ③

نعوض قيمهم بالمعادلة

ⓒ Weighted Average Method

(restricted to symmetrical outputs) Membership functions

بفترض انها symmetric !!

$$z^* = \frac{\sum \mu_{\tilde{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\tilde{C}}(\bar{z})}$$

$\bar{z}$ → centroid of each symmetric function

$= \frac{\sum \text{Member Fuction} \times \text{centroid}}{\sum \text{Member Function}}$

→ here the fuzzy set C is the union of M fuzzy sets

ⓓ Centre of Sum

$$z^* = \frac{\sum_{k=1}^{n} \mu_{\tilde{C}_k}(z) \int_z \bar{z} \, dz}{\sum_{k=1}^{n} \mu_{\tilde{C}_k}(z) \int_z dz}$$

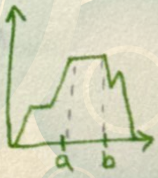
$\sum$ بناخد الـ center في كل رسمة و بنضربه بالـ Area تبعها و مجموعهم بالنهاية

تقسيم مجموع الـ Areas

ⓔ Mean Max Membership

$$z^* = \frac{a + b}{2}$$

$\frac{a+b+c}{3}$

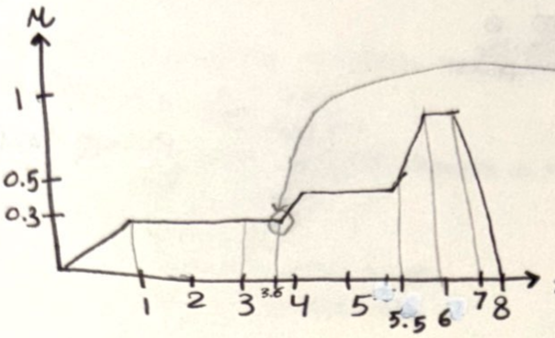


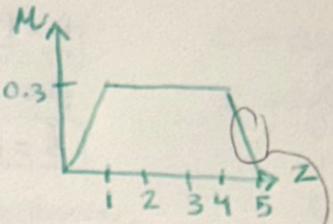
بنروح لأعلى قيمة Membership و باخد الـ Mean

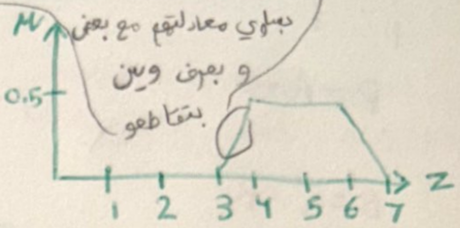
ⓕ First Maximum

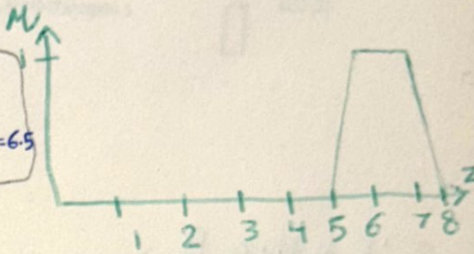
→ The Value of the domain with maximized membership degree

ⓖ Last Maximum









ⓐ Max Membership $\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z)$

X not aplicable

- first Maximum → 6
- last Maximum → 7
- Average Maximum → $\frac{6+7}{2} = 6.5$

(بشرط ال convex)

ⓑ Centroid Method

بتفرج عالرسمة بعد التجميع

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz}$$

deffuzified value

$$z^* = \frac{\int_0^1 (0.3z)\, z\,dz + \int_1^{3.6} (0.3) z\,dz + \int_{3.6}^{4} \left(\frac{z-3}{2}\right) z\,dz + \int_4^{5.5} (0.5) z\,dz + \int_{5.5}^{6} (z-5) z\,dz + \int_6^7 z\,dz + \int_7^8 (8-z) z\,dz}{\int_0^1 (0.3z)\,dz + \int_1^{3.6} (0.3)\,dz + \int_{3.6}^{4} \left(\frac{z-3.6}{2}\right) dz + \int_4^{5.5} (0.5)\,dz + \int_{5.5}^{6} \left(\frac{z-5.5}{2}\right) dz + \int_6^7 dz + \int_7^8 \left(\frac{7-z}{2}\right) dz} = 4.9 \text{ m}$$

ⓒ Weighted Average Method

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})}$$

$$z^* = \frac{(2.5 \times 0.3) + (5 \times 0.5) + (6.5 \times 1)}{0.3 + 0.5 + 1} = 5.41 \text{ m}$$

$$= \frac{\sum (\text{Membership Function of each} \times \text{Centroid of each})}{\sum \text{membership Function}}$$

ⓓ Mean Max Membership

ⓔ Centre of Sum

$$z^* = \frac{\sum_{K=1}^{n} \mu_{\underset{\sim}{C}_K}(z) \int_z \bar{z}\, dz}{\sum_{K=1}^{n} \mu_{\underset{\sim}{C}_K}(z) \int_z dz}$$

$$= \frac{\text{Area الـ تبعها} \times \text{centroid باخر في كل رسمة}}{\text{مجموع الـ Areas}}$$

$$z^* = \frac{2.5 \times \left(\frac{1}{2} \times (5+3) \times 0.3\right) + 5 \times \left(\frac{1}{2} \times (4+2) \times 0.5\right) + 6.5 \times \left(\frac{1}{2} \times (3+1) \times 1\right)}{\left(\frac{1}{2} \times (5+3) \times 0.3\right) + \left(\frac{1}{2} \times (4+2) \times 0.5\right) + \left(\frac{1}{2} \times (3+1) \times 1\right)} = 5 \text{ m}$$

$(y) = m(x) + C$

$\mu(z) = m(z) + C$

Centroid Method / Centre of Area / Centre of Gravity

Defuzzified value z* is the centre of the area covered by the membership function

$$\boxed{z^* = \frac{\int \mu_{\tilde{A}}(z) \cdot z\,dz}{\int \mu_{\tilde{A}}(z)\,dz}}$$

$\mu(z) = y \ , \quad z = x$

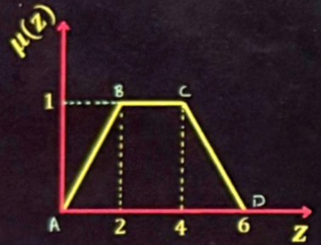
$(x,y)_A = (0,0) \ , \ (x,y)_B = (2,1)$

$y = mx + c$

$0 = m \times 0 + c \Rightarrow c = 0$

$1 = m \times 2 + \cancel{c}^{0} \Rightarrow m = 1/2$

AB: $y = \frac{x}{2}$ or $\mu(z) = z/2$



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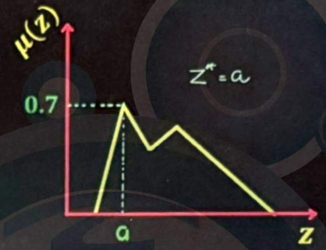
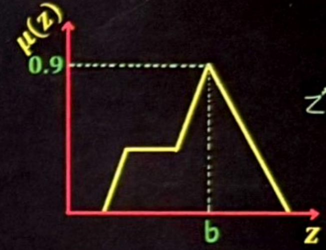
Max Membership Principle

As per this method, the defuzzified value or scalar value z* is chosen for a fuzzy set $\tilde{A}$ such that,

$$\boxed{z^* | \mu_{\tilde{A}}(z^*) \geq \mu_{\tilde{A}}(z) \quad \forall z \in \tilde{A}}$$



$z^* = a$



$z^* = b$

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Weighted Average Method

Fuzzy set $\tilde{A}$(which is the output of the fuzzy controller/fuzzy inference engine) is the union of M fuzzy sets.

This method is restricted to symmetrical output membership functions

$$z^* = \frac{\sum_{i=1}^{M} \mu_{\tilde{A}}(\bar{z}_i) \cdot \bar{z}_i}{\sum_{i=1}^{M} \mu_{\tilde{A}}(\bar{z}_i)}$$

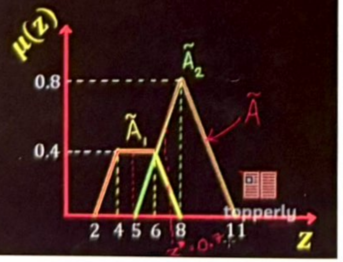
$\sum$ represents algebraic sum

$\bar{z}_i$ respresents centroid of each symmetric function

$\bar{z}_{A_1} = 5$ and $\mu_{\tilde{A}}(5) = 0.4$

$\bar{z}_{A_2} = 8$ & $\mu_{\tilde{A}}(8) = 0.8$

$z^* = \frac{(0.4 \times 5) + (0.8 \times 8)}{0.4 + 0.8} = \underline{\underline{7}}$



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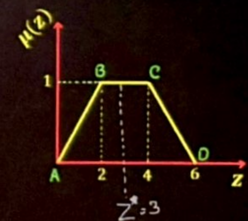
$z^* = \frac{\overset{\mu}{\text{Membership Function}} \times \overset{z}{\text{Centroid}} + \text{Membership Function} \times \text{centroid} + \dots}{\sum \text{membership Functions}}$

Centroid Method / Centre of Area / Centre of Gravity

Defuzzified value z* is the centre of the area covered by the membership function

$$\boxed{z^* = \frac{\int \mu_{\tilde{A}}(z) \cdot z\,dz}{\int \mu_{\tilde{A}}(z)\,dz}}$$



$z^* = 3$

$$z^* = \frac{\int \mu_{\tilde{A}}(z) \cdot z\,dz}{\int \mu_{\tilde{A}}(z) \cdot dz}$$

$$= \frac{\int_0^2 \left(\frac{z}{2}\right) \cdot z\,dz + \int_2^4 (1) \cdot z\,dz + \int_4^6 \left(-\frac{z}{2} + 3\right) \cdot z\,dz}{\int_0^2 \left(\frac{z}{2}\right) dz + \int_2^4 1 \cdot dz + \int_4^6 \left(-\frac{z}{2} + 3\right) dz} = \underline{\underline{3}}$$

AB: $y = \frac{x}{2}$ or $\mu_{\tilde{A}}(z) = \frac{z}{2}$

BC: $y = 1$ or $\mu_{\tilde{A}}(z) = 1$

CD: $y = \frac{-x}{2} + 3$ or $\mu_{\tilde{A}}(z) = \frac{-z}{2} + 3$

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entre of Sums

efuzzified value is given by,

$$z^* = \frac{\sum_i A_i \cdot \bar{z}_i}{\sum_i A_i}$$

$$A_i = \int_z \mu_{\tilde{C}_i}(z)\,dz \quad \text{(Area of } \tilde{C}_i\text{)}$$

$$\bar{z}_i = \frac{\int_z \mu_{\tilde{C}_i}(z)\cdot z\,dz}{\int_z \mu_{\tilde{C}_i}(z)\,dz} \quad \text{(Centroid of } \tilde{C}_i\text{)}$$

$A_1 = \frac{1}{2} \times 4 \times 1 = 2$ units

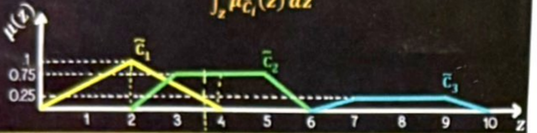
$\bar{z}_1 = 2$

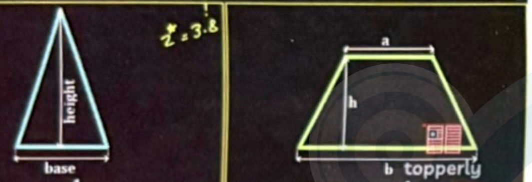
$A_2 = \frac{1}{2} \times (2+4) \times 0.75 = 2.25$ units

$\bar{z}_2 = 4$

$A_3 = 0.75$, $\bar{z}_3 = 8$

$$z^* = \frac{(2\times2) + (2.25\times4) + (0.75\times8)}{2 + 2.25 + 0.75} = 3.8$$



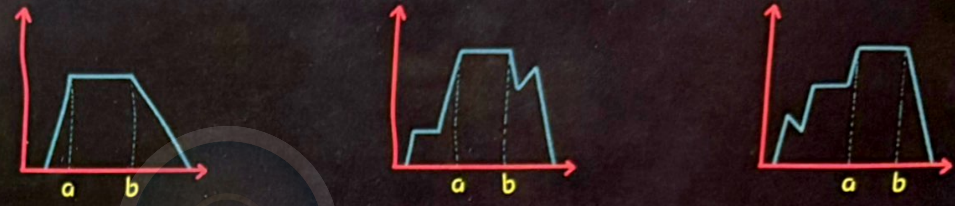


Area of triangle $= \frac{1}{2} \times base \times height$ | *Area of trapezoid* $= \frac{1}{2} \times (a+b) \times h$

Center of Sums $z^* = \dfrac{\overset{\text{Area}}{A_1} \times \overset{\text{centroid}}{\bar{z}_1} + \overset{\text{Area}}{A_2} \times \overset{\text{centroid}}{\bar{z}_2} + \overset{\text{Area}}{A_3} \times \overset{\text{centroid}}{\bar{z}_3}}{\sum \text{Areas}}$

Mean-Max Membership Method

Applicable only to membership functions which has a plateau of maximum membership



$$z^* = \frac{a+b}{2}$$

Best method for defuzzification?

Continuity: A small change in the output fuzzy set should not produce a large change in the defuzzfied value z*

Disambiguity: Defuzzified value shouldn't be ambiguous

Plausibility: z* should respresent ouput fuzzy set from an intuitive point of view
> z* should lie approximately in the middle of the support of output fuzzy set
> z* should have high degree of membership in the output fuzzy set

Computational Simplicity: The more time consuming a method is, the less value it should have in computational system

Weighting method

First(or last) of maxima

Step 1: If the output of fuzzy controller is comprised of M fuzzy sets, take the union of all of them to form a single fuzzy set $\tilde{A}_k$ such that,

$$\tilde{A}_k = \tilde{A}_1 \cup \tilde{A}_2 \cup \ldots \cup \tilde{A}_M = \bigcup_{i=1}^{M} \tilde{A}_i$$

Step 2: The height of the union($\tilde{A}_k$) is determined.

$$hgt(\tilde{A}_k) = \max\{\mu_{\tilde{A}_k}(z)\}$$

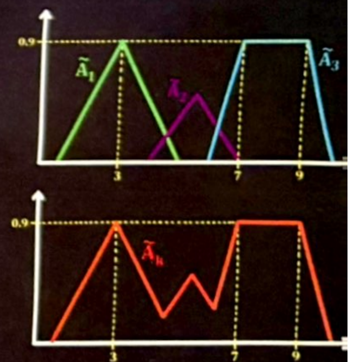
Step 3: Find the set Z_{hgt} such that,

$$Z_{hgt} = \{z \mid \mu_{\tilde{A}_k}(z) = hgt(\tilde{A}_k)\}$$

Step 4: Find z* such that,

First of maxima : $z^* = \min(Z_{hgt}) = \min\{3,[7,9]\} = 3$

Last of maxima : $z^* = \max(Z_{hgt}) = \max\{3,[7,9]\} = 9$



$hgt = 0.9$

$Z_{hgt} = \{3,[7,9]\}$

$\underset{\sim}{R} = \underset{\sim}{P} \times \underset{\sim}{S}$

given P' what is S'

$\boxed{S' = P' \circ R}$

composion: ② Max or ① Min; Max product

P	Q
0	0
0	1
1	0
1	1

$R = A \times B$

$B' = A' \circ R$

composition: max min; max product

And $\cap$ min conjunctive

OR $\cup$ Max Disjunctive

implication Rule

$P \rightarrow Q = \bar{P} \vee Q$

$\bar{Q} \rightarrow \bar{P} = T(Q \vee \bar{P})$

عكس الأول

واتحاد الثاني

$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \underset{\sim}{\bar{B}}$

$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \underset{\sim}{\bar{A}}$

if A then B

$R = (A \times B) \overset{max}{\cup} (\bar{A} \times Y)$

Y: universe for B

Crisp Sets (Ch5)

Tautologies → always True

Modus ponens $\xrightarrow[\text{chaining}]{\text{Forward}}$ if (A) then (B)

premisis: (A) — conclusion: (B)

Antecedent $\xrightarrow{\text{leads to}}$ Consequent

$$(A \wedge (A \rightarrow B))$$

$$= (A \underset{min}{\wedge} (\bar{A} \overset{max}{\vee} B))$$

Modus Tollens $\xleftarrow[\text{chaining}]{\text{backward}}$ if (B) didn't happen so (A) didn't happen

$$(\bar{B} \wedge (A \rightarrow B))$$

$$(\bar{B} \wedge (\bar{A} \vee B))$$

Contradictions → Always False

Exclusive or (XOR) → وحدة منهم فقط لازم تصير

→ it does not involve the intersection

$(P)\,XOR\,(Q) \rightsquigarrow (P \vee Q) \wedge (\overline{P \wedge Q})$

→ or the negation for the equivelance

Exclusive nor $(\overline{XOR})$ → الثنين لازم يكونو متشابهين حتى يكون True

$\overline{(P)\,XOR\,(Q)}$

Fuzzy Sets

* the implication can be modelled in Rule-based form $(P \rightarrow Q)$ if x is $\underset{\sim}{A}$, then y is $\underset{\sim}{B}$.

$$R = (\underset{\sim}{A} \times \underset{\sim}{B}) \overset{Max}{\cup} (\bar{\underset{\sim}{A}} \times Y)$$

if x is $\underset{\sim}{A}$, then y is $\underset{\sim}{B}$, Else y is C

$$R = (\underset{\sim}{A} \times \underset{\sim}{B}) \overset{Max}{\cup} (\bar{\underset{\sim}{A}} \times \underset{\sim}{C})$$

Same dimension

بالآخر بقارن بينهم وبختار الـ Maximum

Mamdani systems (Method 2)

And → $\cap$ → min
OR → $\cup$ → max

Max - Product Inference Method

Consider a simple 2 rule system where each rule has 2 antecents and one consequent as follows:

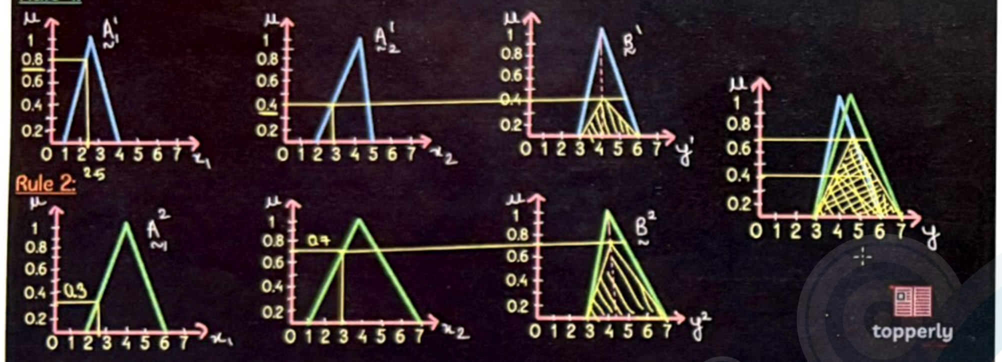
Rule 1: IF x1 is $\tilde{A}_1^1$ and x2 is $\tilde{A}_2^1$, THEN y^1 is $\tilde{B}^1$

Rule 2: IF x1 is $\tilde{A}_1^2$ or x2 is $\tilde{A}_2^2$, THEN y^2 is $\tilde{B}^2$

$\boxed{x1 = 2.5 \quad x2 = 3}$

Rule 1:

Rule 2:



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output لل scale بيسوي عهنا

Mamdani Systems (Method 1)

→ After Aggregation (Defuzzification by 7 methods (any method))

Max-Min Inference Method

Consider a simple 2 rule system where each rule has 2 antecents and one consequent as follows:

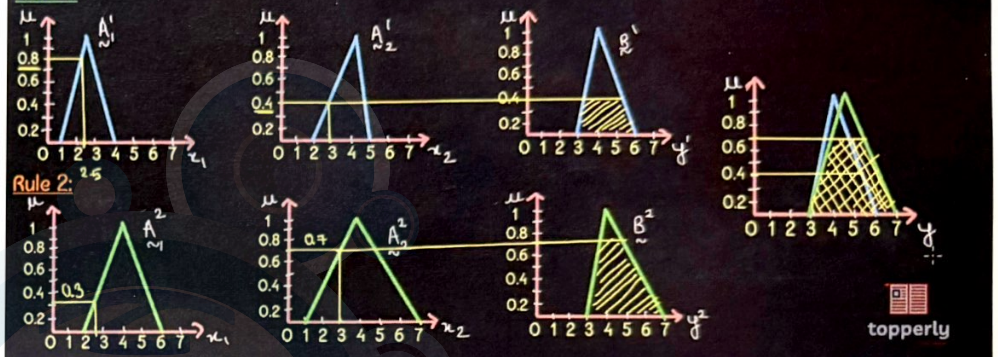
Rule 1: IF x1 is $\tilde{A}_1^1$ and x2 is $\tilde{A}_2^1$, THEN y^1 is $\tilde{B}^1$

Rule 2: IF x1 is $\tilde{A}_1^2$ or x2 is $\tilde{A}_2^2$, THEN y^2 is $\tilde{B}^2$

$\boxed{x1 = 2.5 \quad x2 = 3}$

Rule 1:

Rule 2:



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Consider a two input - single output Sugeno model with 4 rules as:

Rule 1 : IF x1 is small and x2 is small, THEN y1 = (-x1) + x2 + 1

Rule 2 : IF x1 is small and x2 is large, THEN y2 = (-x2) + 3

Rule 3 : IF x1 is large and x2 is small, THEN y3 = (-x1) + 3

Rule 4 : IF x1 is large and x2 is large, THEN y4 = (-x1) + x2 + 2

Find the output when x1 = 1.5 and x2 = 2.5.

The graphs for small and large fuzzy sets for x1 and x2 is given as:

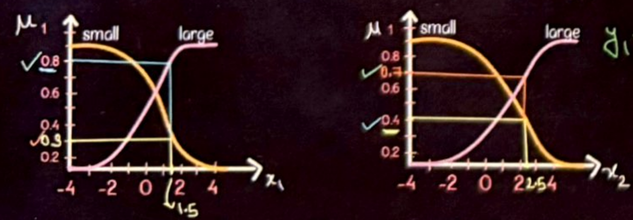
$w_{y_1} = \min(w_{x_1}, w_{x_2}) = \min(0.3, 0.4)$

$w_{y_1} = 0.3$

$w_{y_2} = \min(0.3, 0.7) = 0.3$

$w_{y_3} = \min(0.8, 0.4) = 0.4$

$w_{y_4} = \min(0.8, 0.7) = 0.7$



$y_1 = (-x_1) + x_2 + 1 = (-1.5) + 2.5 + 1$

$y_1 = 2$

$y_2 = 0.5$

$y_3 = 1.5$

$y_4 = 6$

$$y^* = \frac{w_{y_1} \cdot y_1 + w_{y_2} \cdot y_2 + w_{y_3} \cdot y_3 + w_{y_4} \cdot y_4}{w_{y_1} + w_{y_2} + w_{y_3} + w_{y_4}} = \frac{(0.3 \times 2) + (0.3 \times 0.5) + (0.4 \times 1.5) + (0.7 \times 6)}{(0.3) + (0.3) + (0.4) + (0.7)}$$

$y^* = 3.264$

$y^* \approx 3.26$

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Sugeno Systems → output is a fuction

SUGENO SYSTEMS

Mamdani Systems : IF x1 is $\tilde{A}_1$ and x2 is $\tilde{A}_2$ THEN y is $\tilde{B}$

Sugeno Systems : IF x1 is $\tilde{A}_1$ and x2 is $\tilde{A}_2$ THEN y is y = f (x1, x2)

where y = f (x1, x2) is a crisp function in the consequent.

$$\boxed{y^* = \frac{(w_{y_1} \cdot y_1) + (w_{y_2} \cdot y_2)}{w_{y_1} + w_{y_2}}}$$

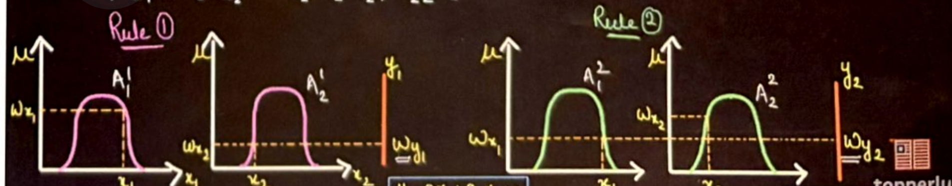
The overall aggregated output will be obtained through weighted average defuzzification method.

Rule 1: IF x_1 is $\tilde{A}_1^1$ and x_2 is $\tilde{A}_2^1$ THEN y_1 is $y_1 = px_1 + qx_2 + r_1$

Rule 2: IF x_1 is $\tilde{A}_1^2$ and x_2 is $\tilde{A}_2^2$ THEN y_2 is $y_2 = px_1 + qx_2 + r_2$

Rule ①

Rule ②



$\boxed{y_1 = p_1 x_1 + q_1 x_2 + r_1}$

$\boxed{y_2 = p_2 x_1 + q_2 x_2 + r_2}$

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$$y^* = \frac{\left(w_{y_1} \text{ membership function} \cdot y_1 \text{ الدالة function}\right) + \left(w_{y_2} \cdot y_2\right)}{(w_{y_1}) + (w_{y_2})}$$

Tsukamoto Systems → The output is ~~...~~ Monotonic Membership Function + output is crisp

TSUKAMOTO MODEL

Mamdani Systems : IF x1 is $\underset{\sim}{A}_1$ and x2 is $\underset{\sim}{A}_2$ THEN y is $\underset{\sim}{B}$

Sugeno Systems : IF x1 is $\underset{\sim}{A}_1$ and x2 is $\underset{\sim}{A}_2$ THEN y is $y = f(x1, x2)$

Tsukamoto Systems : If x1 is $\underset{\sim}{A}_1$ and x2 is $\underset{\sim}{A}_2$ THEN y is $\underset{\sim}{B}$

where the fuzzy set $\underset{\sim}{B}$ has a monotonic membership function.

A monotonic function, also called as a shoulder function, is the one whose successive values are increasing, decreasing or constant.

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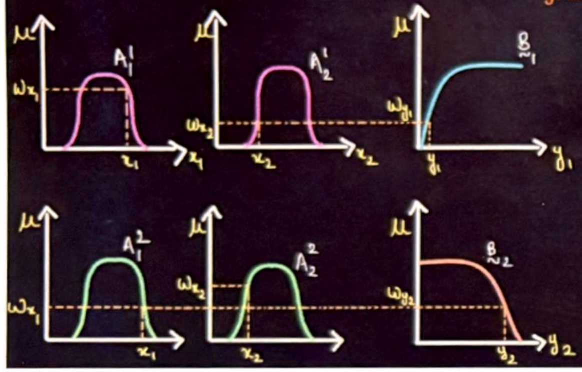
The inferred output of each rule is defined as a crisp value induced by the membership value coming from the antecedent of the rule.

Rule 1: IF x_1 is $\underset{\sim}{A}_1^1$ and x_2 is $\underset{\sim}{A}_2^1$ THEN y_1 is $\underset{\sim}{B}_1$

Rule 2: IF x_1 is $\underset{\sim}{A}_1^2$ and x_2 is $\underset{\sim}{A}_2^2$ THEN y_2 is $\underset{\sim}{B}_2$



The overall output will be calculated by the weighted average of each rule's output i.e weighted average of y1 and y2 as:

$$\boxed{y^* = \frac{\omega_{y_1} \cdot y_1 + \omega_{y_2} \cdot y_2}{\omega_{y_1} + \omega_{y_2}}}$$

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$\omega_{y_1}, \omega_{y_2}$ و محور الصادات للـ output

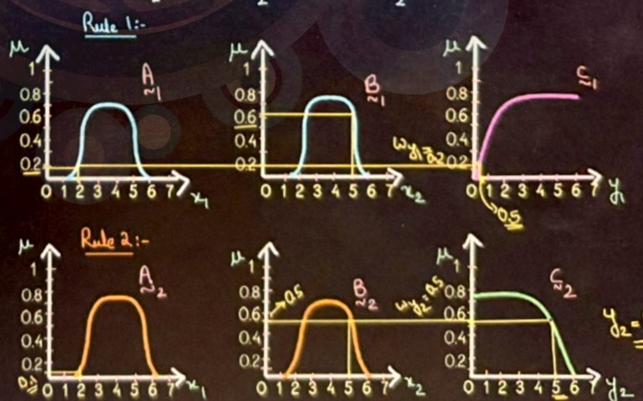
y_1, y_2 هنا محور السينات للـ output

Q]

Rule 1 : IF x1 is $\underset{\sim}{A}_1$ and x2 is $\underset{\sim}{B}_1$, THEN y1 is $\underset{\sim}{C}_1$

Rule 2 : IF x1 is $\underset{\sim}{A}_2$ or x2 is $\underset{\sim}{B}_2$, THEN y2 is $\underset{\sim}{C}_2$

$\boxed{x1 = 2 \quad x2 = 5}$



$$y^* = \frac{\omega_{y_1} \cdot y_1 + \omega_{y_2} \cdot y_2}{\omega_{y_1} + \omega_{y_2}}$$

$$= \frac{0.2 \times 0.5 + 0.5 \times 5}{0.2 + 0.5}$$

$$= 3.714$$

$$y^* \approx \underline{3.71}$$

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Advantages:

Tsukamoto model avoids the time consuming process of defuzzification since each rule infers a crisp output and the overall output can be calculated using weighted average method.

Disadvantages:

Since the output membership function should be monotonic in nature, its not as useful as a general approach and hence can only be used in specific situations.

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Chapter 6

Development of Membership Functions

* Membership Value Assignment Methods

① **Intuition** → can be derived from the capacity of humans to develop them through their own innate understanding.
(الرأي الشخصي حسب تجربتنا)

② **Inference** → Knowledge is utilised to perform deductive reasoning
(مبني على التفكير الاستنتاجي أو الاستنباطي)

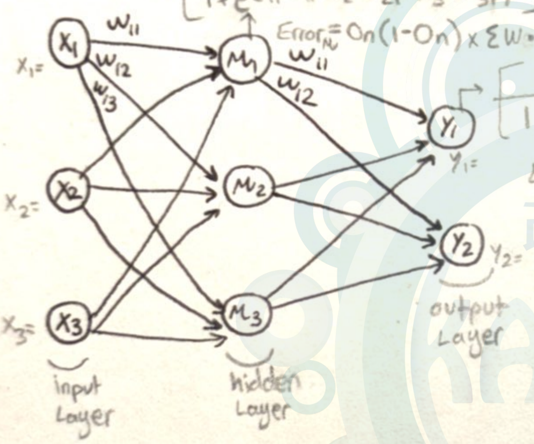
③ **Rank Ordering** → Prefrence is determined by pairwise comparisons and these determine the ordering of the Membership

④ **Neural Networks** → is a technique that builds an intelligent system by simulating the biological Neural Network
(there is an iput and according to it there will be an output)

$O_n = \left[\frac{1}{1+e^{-(x_1 \cdot w_{11} + x_2 \cdot w_{21} + x_3 \cdot w_{31})}} \right]$

*we will use sigmoid function

$Error_{N} = O_n (1 - O_n) \times \sum w \cdot Error_y$



$\left[\frac{1}{1+e^{-(M_1 \cdot w_{11} + M_2 \cdot w_{21} + M_3 \cdot w_{31})}} \right] = \text{value}$

$Error_y = y_1 - \text{value}$

to calculate new weights → $W_{new} = W_{old} + \alpha \cdot E \cdot (\text{input for it})$

α: Learning factor
E: Error Neuron for next Layer

* بظل اعيد ال process لحتى اوصل لل Error in output (أقل من 5%)

lets Assume we want 2 Rules (2 clusters, K=2)

(Clustering Method)

$$\text{Distance} = \sqrt{(e_1 - e_{c1})^2 + (e_2 - e_{2c})^2 + \dots}$$

Continue .... (until the Cluster centers doesn't change)

$$D_{c1} = \sqrt{(e_1 - .2)^2 + (e_2 - 150)^2 + (e_3 - 10)^2}$$

بختار الأقل بين D_{c1} و D_{c2} (اقرب على اي cluster)

# of exp	Depth of Cut	Speed	Surface Roughness	D_{c1}	D_{c2}	choose cluster	D_{c1}	D_{c2}	choose cluster	D_{c1}	D_{c2}	choose cluster
1	.2	100	5	50.2	2	2	150.03	1	2	166.6	16.7	2
2	.3	200	7	50.08	100	1	50.01	100	1	66.6	83.4	1
3	.2	150	10	0	50.09	1	100.01	50.15	2	116.6	33.5	2
4	.15	400	8	250	300	1	150	300	1	133.4	283	1
5	.21	300	7	150.03	200	1	50.01	200	1	33.41	183	1
6	.17	200	6	50.01	100	1	50.04	100	1	66.6	83.4	1
7	.25	100	7	50.09	0	2	150	1	2	166.6	16.6	2
8	.31	200	10	50	100.05	1	50.03	100.08	1	66.6	84.4	1
9	.27	300	9	150	200.01	1	50	200	1	33.4	183	1

3 اخترنا → $C_1 = (.2, 150, 10)$
7 اخترنا → $C_2 = (.25, 100, 7)$

$C_1 = (.23, 250, 8.14)$
$C_2 = (.225, 100, 6)$

$C_1 = (.235, 266.6, 7.83)$
$C_2 = (.22, 116.6, 7.33)$

$C_1 = (.235, 266.6, 7.83)$
$C_2 = (.22, 116.6, 7.33)$

stopping criteria

The centers didn't change so we stop

For Assigning new centers

$$C_{2\,\text{new}} = \frac{[(.2, 100, 5) + (.25, 100, 7)]}{2} = (.225, 100, 6)$$

as vectors

Prefer عدد ال cluster 2

this is One Pass Method

$\mu_{gaussian}(x;m,\sigma) = e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$

step ① → **Ranges:-**

DOC → (.15-.31)

SP → (100-400)

SR → (5-10)

Final Answer

Centervalue for output — we assumed the standard deviation (the numbers are incorrect) — س للفهوم

$C_1 = (.24, 266.6, 7.8) \rightarrow \sigma = (.1, 50, 2)$

$C_2 = (.21, 116.6, 7.3) \rightarrow \sigma = (.12, 30, 1.5)$

بعد ما ال centers بطلو يتغيرو

for $Exp_1 = (\underset{x_1}{.2}, \underset{x_2}{100}, \underset{y_{actual}}{5})$ — y_{actual} mean for output

---

$\mu_{mean} = e^{\left(-\frac{1}{2}\frac{(x-m)^2}{(\sigma)^2}\right)}$

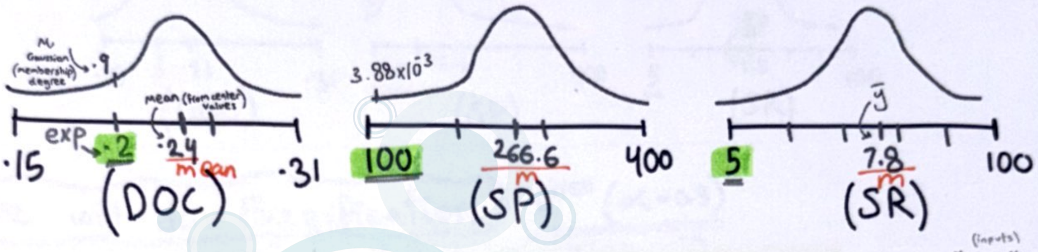
Rule ①

step ② $C_1 = (.24, 266.6, 7.8)$ — DOC SP SR — رأس ال Gaussian بتكون عنده

$\mu_{DOC} = e^{-.5\left(\frac{.2-.24}{.1}\right)^2} = 0.92$ ($x_1 - m$)

$\mu_{SP} = e^{-.5\left(\frac{100-266.6}{50}\right)^2} = 3.88\times10^{-3}$ ($x_2 - m$)

$\Phi = \text{Firing Level} = \mu_y \text{ (for Output)} = (.9)\times(3.88\times10^{-3}) = 3.5\times10^{-3}$ (inputs) $\mu_{DOC} \times \mu_{SP}$



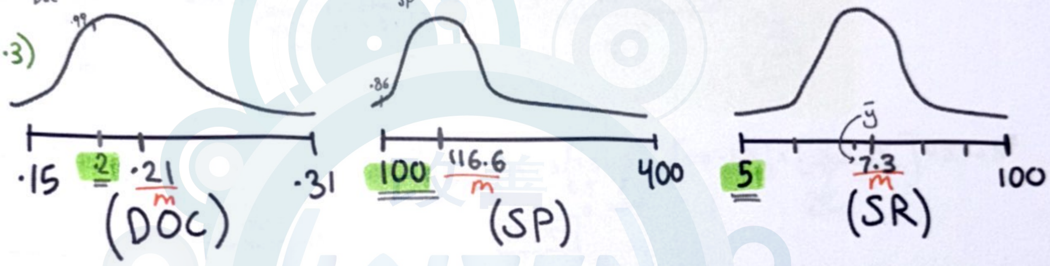
Rule ②

$C_2 = (.21, 116.6, 7.3)$

$\mu_{DOC} = e^{-.5\left(\frac{.2-.21}{.12}\right)^2} = .99$

$\mu_{SP} = e^{-.5\left(\frac{100-116.6}{30}\right)^2} = 0.86$

$\Phi = \text{Firing Level} = \mu_y \text{ (for Output)} = (.99)\times(.86) = 0.85$ (inputs) $\mu_{DOC} \times \mu_{SP}$



Linguisticly:- when DOC is medium and speed is high then SR is high

$Pr\ DO$ = Predicted Deffuzified Value for output $= \dfrac{\sum(\mu_y \text{ membership degree for output} \times \text{center value for output})}{\mu_y \text{ membership degree for output (1)} + \mu_y \text{ membership degree for output (2)}} = \dfrac{[3.5\times10^{-3}\times7.8]+[0.85\times7.3]}{3.5\times10^{-3}+0.85} = 7.3$ ← The Predicted Output

$C_1 = (.24, 266.6, 7.8) \rightarrow \sigma = (.1, 50, 2)$

$C_2 = (.21, 116.6, 7.3) \rightarrow \sigma = (.12, 30, 1.5)$

[for $Exp_1 = (.2, 100, 5)$] y_{actual}

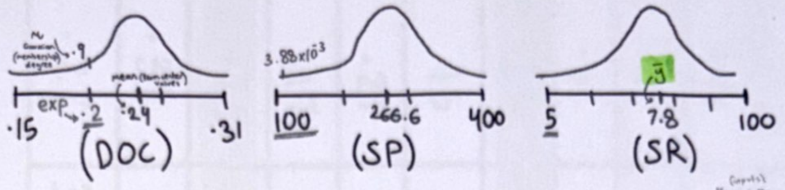
Predicted Deffuzified output (for exp 1) for R1, R2:

$$= \sum \frac{\left(\mu_{y\,\text{for output}} \times \text{center value for output}\right)}{\mu_{y\,\text{for output1}} + \mu_{y\,\text{for output2}}} = \frac{(3.5\times10^{-3}\times7.8)+(.85\times7.3)}{(3.5\times10^{-3})+(.85)} = 7.3$$

$$\mu = e^{-.5\left(\frac{x-m}{\sigma}\right)^2}$$

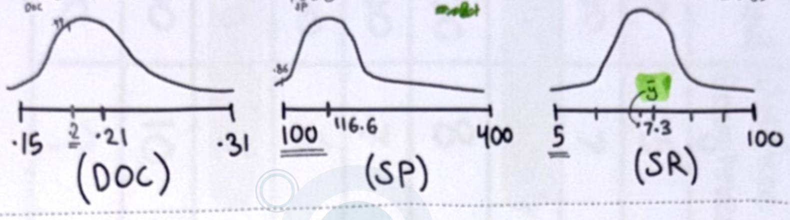
Rule ①

$\mu = e^{-.5\left(\frac{.2-.24}{.1}\right)^2} = 0.92$ $\quad \mu = e^{-.5\left(\frac{100-266.6}{50}\right)^2} = 3.88\times10^{-3}$ $\quad \Phi = \text{Firing Level} = \mu_y \text{ (for output)} = (\mu_{doc}\times\mu_{sp}) = (.9)\times(3.88\times10^{-3}) = 3.5\times10^{-3}$



Rule ②

$\mu_{doc} = e^{-.5\left(\frac{.2-.21}{.12}\right)^2} = .99$ $\quad \mu_{sp} = e^{-.5\left(\frac{100-116.6}{30}\right)^2} = 0.86$ $\quad \Phi = \text{Firing Level} = \mu_y \text{ (for output)} = (.99)\times(.86) = 0.85$



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* Now we will do Fuzzification

given $(\alpha = 0.3)$

R1 → ① $\mu_{new\ Doc}$, ② $\mu_{new\ SP}$
R2 → ③ $\mu_{new\ Doc}$, ④ $\mu_{new\ SP}$

(for input mean only)

$$\mu_{new} = \mu_{old} - \alpha_{(\text{learning factor})}\left[P_r - y\right]\left[\bar{y} - P_r\right] \times \frac{\left[X_{(input)} - \mu_{old}\right]}{\sigma^2} * \Phi$$

(P_r: predicted deffuzified value; y: actual; $\bar{y}$: mean for output; Φ: firing level يعني (Rule ال))

$Exp = (.2, 100, 5)$ (5: output) → $P_r = 7.3$ (Predicted output for first Exp (for Rule 1st and 2nd))

كل الحسابات Exp1 ال, Rule1 ال, Rule2

DOC R1: $m_{new} = .24 - .3\left[7.3 - 5\right]\times\left[7.8 - 7.3\right]\times\left[\frac{.2 - .24}{.1^2}\right]\times 3.5\times10^{-3} = \boxed{0.244}$ (input for Doc)

SP R1: $m_{new} = 266.7 - .3\left[7.3-5\right]\times\left[7.8-7.3\right]\times\left[\frac{100-266.7}{50^2}\right]\times3.5\times10^{-3} = \boxed{266.7}$

SP R2: $m_{new} = 116.6 - .3\left[7.3-5\right]\times\left[7.3-7.3\right]\times\left[\frac{100-116.6}{30^2}\right]\times 0.85 = 116.6 - 0 = \boxed{116.6}$

$\bar{y}_{new}$ → R1, R2

$$\bar{y}_{new} = \bar{y}_{old} - \alpha * \left[P_r - y_{actual}\right] * \Phi$$

($\bar{y}$: mean for output; Φ: Firing level للـ Rule يعني)

for R1: $\bar{y}_{new} = 7.8 - .3\times\left[7.3-5\right]\times3\times10^{-3} = 7.79$

for R2: $\bar{y}_{new} = 7.3 - .3\times\left[7.3-5\right]\times.85 = 6.71$

σ_{new} → R1: σ_{Doc}, σ_{SP}; R2: σ_{Doc}, σ_{SP}

$$\sigma_{new} = \sigma_{old} - \alpha\left[P_r - y_{actual}\right] * \left[\bar{y} - P_r\right] * \frac{\left[X - m\right]^2}{\sigma^3} * \Phi_{\text{Firing Level}}$$

(باخذ يلي ما عدلتها)

DOC R1: $\sigma_{new} = .1 - .3\left[7.3-5\right]*\left[7.8-7.3\right]*\frac{\left[.2-.24\right]^2}{.1^3}\times3.5\times10^{-3} = .098$

SP R1: $\sigma_{new} = 50 - .3\left[7.3-5\right]*\left[7.8-7.3\right]*\frac{\left[100-266.7\right]^2}{50^3}\times3.5\times10^{-3} = 49.9$

DOC: $\sigma_{new} = .12 - .3\left[7.3-5\right]\times\left[7.3-7.3\right]\times\frac{(.2-.21)^2}{.12^3}\times.85 = .12$

Multi Objective Desision Making

(our Goal is to Find the best Alternative)

Example

$O_1 = \{ \frac{.4}{MSE}, \frac{1}{Conc}, \frac{.1}{Gab} \}$ cost

Alternatives

$O_2 = \{ \frac{.7}{MSE}, \frac{.8}{Conc}, \frac{.4}{Gab} \}$ Maintability

$O_3 = \{ \frac{.2}{MSE}, \frac{.4}{Conc}, \frac{1}{Gab} \}$ Standard

$O_4 = \{ \frac{1}{MSE}, \frac{.5}{Conc}, \frac{.5}{Gab} \}$ Enviromental

$MSE \rightarrow a_1$, $Conc \rightarrow a_2$, $Gab \rightarrow a_3$

[in First scenario the prefrences (wieghts) are

b_1, b_2, b_3, b_4

$.8 \quad .9 \quad .7 \quad .5$]

weight for First objective

$\bar{b}_1 = .2 \quad \bar{b}_2 = .1 \quad \bar{b}_3 = .3 \quad \bar{b}_4 = .5$

For Evaluating every Alternative

$$\boxed{D(a_i) = (\bar{b}_1 \overset{Max}{\cup} O_1) \underset{Min}{\cap} (\bar{b}_2 \overset{Max}{\cup} O_2) \underset{Min}{\cap} (\bar{b}_3 \overset{Max}{\cup} O_3) \underset{Min}{\cap} (\bar{b}_4 \overset{Max}{\cup} O_4)}$$

$$D(MSE) = (.2 \overset{Max}{\cup} .4) \underset{Min}{\cap} (.1 \cup .7) \cap (.3 \cup .2) \cap (.5 \cup 1)$$
$$= (.4) \cap .7 \cap \underset{Min}{.3} \cap 1$$
$$= \boxed{.3}$$

$$D(Conc) = (.2 \cup 1) \cap (.1 \cup .8) \cap (.3 \cup .4) \cap (.5 \cup .5)$$
$$= \boxed{.4}$$

$$D(Gab) = (.2 \cup .1) \cap (.1 \cup .4) \cap (.3 \cup 1) \cap (.5 \cup .5)$$
$$= \boxed{.2}$$

To choose the optimum Alternative (Desision)

$$\boxed{D^* = \max(D(a_1), D(a_2), D(a_3))}$$
$$= \max(.3, .4, .2)$$
$$= \boxed{.4}$$

Fuzzy Bayesian Decision Method

it depends on the probability of something taking place in the future

Ordering

1) $I_1 = \{\frac{1}{3}, \frac{.8}{5}, \frac{.5}{9}\}$, $I_2 = \{\frac{.7}{4}, \frac{1}{6}\}$, $I_3 = \{\frac{.8}{2}, \frac{1}{4}, \frac{.5}{8}\}$

$I_1 \geq I_2 = \underset{x_1 \geq x_2}{Max}(Min(.8, .7), (.5, .7), (.5, 1)) = .7$

$I_2 \geq I_1 = Max(Min(.7, 1)(1, 1) = 1$

$I_1 \geq I_3 = Max(Min(1, .8)(.8, .8), (.8, 1)\, \cancel{(.8,.5)}\, (.5, .8)(.5, 1)(.5, .5)) = .8$

$I_3 \geq I_1 = Max(Min(1, 1) = 1$

$I_2 \geq I_3 = Max(Min(.7, .8)(.7, 1)(1, .8)(1, 1)(1, .5) = 1$

$I_3 \geq I_2 = Max(Min(1, .7)(1, .8) = .7$

$I_1 \geq I_2, I_3 = Min(I_1 \geq I_2, I_1 \geq I_3) = Min(.7, .8) = .7$

$I_2 \geq I_1, I_3 = 1$

$I_3 \geq I_1, I_2 = .7$

Synthetic Evaluation

9.2) $a = [.1 \quad .35 \quad .4 \quad .15]$ ← weight factor

evaluated for the following parameters

$$e = \begin{array}{c} \\ \text{color} \\ \text{consistency} \\ \text{BN} \\ \text{FP} \end{array} \begin{array}{c} \text{Exeelont} \quad \text{Very Good} \quad \text{Good} \\ \begin{bmatrix} .3 & .4 & .3 \\ .1 & .5 & .4 \\ .5 & .4 & .1 \\ .4 & .3 & .3 \end{bmatrix} \end{array} = [.4 \quad .4 \quad .35]$$

Max-Min Composition

Nontrastive Ranking

9.6)

	x_1	x_2	x_3	x_4
x_1	1	.6	.5	.9
x_2	.5	1	.7	.8
x_3	.9	.8	1	.5
x_4	.3	.2	.3	1

Develop a Comparison Matrix (Relativity Values Matrix)

column بدل Row الـ

$$f(x/y) = \frac{f_y(x)}{Max(f_y(x), f_x(y))}$$

C =	x_1	x_2	x_3	x_4	Min	Rank ordering
x_1	1	.83	1	.33	.33	3
x_2	1	1	1	.25	.25	4
x_3	.56	.88	1	.6	.56	2
x_4	1	1	1	1	1	1

$\frac{.5}{Max(.5, .9)}$

$\frac{.7}{Max(.7, .8)}$

Fuzzy Preference and consensus

9.9) what is the distance M_1^* consensus

$0 \times 0 + .5 \times .5 + .2 \times .8 + .6 \times .4$

$$R \times R = \begin{bmatrix} 0 & .5 & .8 & .4 \\ .5 & 0 & .9 & .2 \\ .2 & .1 & 0 & .1 \\ .6 & .8 & .9 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & .5 & .8 & .4 \\ .5 & 0 & .9 & .2 \\ .2 & .1 & 0 & .1 \\ .6 & .8 & .9 & 0 \end{bmatrix} = \begin{bmatrix} \boxed{.65} & & & \\ & \boxed{.5} & & \\ & & \boxed{.34} & \\ & & & \boxed{.49} \end{bmatrix}$$

Reciprocal Relation (given)

$tr(R^2)$ = مجموع Diagonal trace

$$F(R) = \frac{tr(R^2)}{n(n-1)/2} = \frac{.65 + .5 + .34 + .49}{4 \times (4-1)/2} = .33$$

$C(R) = 1 - F(R) = 1 - .33 = .67$

$m(R) = 1 - (2\,C(R) - 1)^{\frac{1}{2}} = 1 - (2 \times .67 - 1)^{\frac{1}{2}}$

$M_1^*(m) = 1 - (2/n)^{\frac{1}{2}} = 1 - (2/4)^{\frac{1}{2}} = .293$

Distance from M_1^* consensus is $m(R) - M_1^*(m)$

$= .417 - .293$

$= 0.124$

$M_2^*(m) = 0$

$$\text{Distance to type 1} = \frac{1 - m(R)}{1 - M_1^*(m)}$$

$$\text{Distance to type 2} = 1 - m(R)$$

Multiobjective Desicion Making

$O_1 = \left\{ \frac{.8}{a_1}, \frac{1}{a_2}, \frac{.7}{a_3} \right\}$

$O_2 = \left\{ \frac{.9}{a_1}, \frac{.5}{a_2}, \frac{.5}{a_3} \right\}$

$O_3 = \left\{ \frac{.8}{a_1}, \frac{.7}{a_2}, \frac{.4}{a_3} \right\}$

$O_4 = \left\{ \frac{.4}{a_1}, \frac{0.8}{a_2}, \frac{0.9}{a_3} \right\}$

$$\begin{array}{ccccc} & b_1 & b_2 & b_3 & b_4 \\ P = \{ & .8, & .8, & .6, & .5 \} \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \bar{b} & .2 & .2 & .4 & .5 \end{array}$$

$$D(a_i) = (\bar{b}_1 \overset{Max}{\cup} O_1) \underset{Min}{\cap} (\bar{b}_2 \overset{Max}{\cup} O_2) \underset{Min}{\cap} (\bar{b}_3 \overset{Max}{\cup} O_3) \underset{Min}{\cap} (\bar{b}_4 \overset{Max}{\cup} O_4)$$

$D(a_1) = (.2 \vee .8) \cap (.2 \vee .9) \cap (.4 \vee .8) \cap (.5 \vee .4) = .5$

$D(a_2) = (.2 \vee 1) \cap (.2 \vee .5) \cap (.4 \vee .7) \cap (.5 \vee .8) = .5$

$D(a_3) = (.2 \vee .7) \cap (.2 \vee .5) \cap (.4 \vee .4) \cap (.5 \vee \underset{Max}{.9}) = .4$

$$D^* = \max(D(a_1), D(a_2), D(a_3))$$

There is a Tie between Alternatives (a_1) and (a_2)

So we will use Tie breaking method

$D(a_1) = .8 \wedge .9 \wedge .8 \wedge \cancel{.5} = .8 \xrightarrow{Max}$ the best choice is (a_1)

$D(a_2) = 1 \wedge \cancel{.5} \wedge .7 \wedge .8 = .7$

Bayesian Desision Making

$$E(u_j) = \sum_{i=1}^{n} u_{ji} \cdot p(s_i)$$

$$E(u^*) = Max\ E(u_j)$$