

Fuzzy VS Crisp

Fuzzy
↓
Time limits

Crisp
yes no

* Vague
مضبوط

* Fuzzy
see you in a few minutes

وجود نسبي = 1

(X) Universe
(x) element

Set → universe
subset → sets



* $x \in X$

* $x \notin A$ doesn't belong to A

Power set = $2^{CN} = 2^4 = 16$ (of)

cardinal number = 4

$X = \{1, 2, 3, 4\}$

Power set

- $\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}$

[16 subsets]

* Operations: $A \cup B = \{x | x \in A \text{ or } x \in B\}$

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

$$\bar{A} = \{x | x \notin A, x \in X \text{ universe}\}$$

$$A \cap \bar{B} = A \setminus B = \{x \in A \text{ and } x \notin B\}$$

* Properties:

①

تبادل

$$A \cup A = A$$

②

توزيع

$$A \cap A = A$$

③

الترابط

$$A \cap X = A, A \cup \emptyset = A$$

$$A \subseteq B \text{ and } B \subseteq C \implies A \subseteq C$$

only Crisp not fuzzy: $A \cup \bar{A} = X$
 $A \cap \bar{A} = \emptyset$

De Morgan: $\overline{A \cap B} = \bar{A} \cup \bar{B}$
 $\overline{A \cup B} = \bar{A} \cap \bar{B}$



المجموعة
الكلية

Union $A \cup B: \max(\mu_A(x), \mu_B(x))$

Intersection $A \cap B: \min(\mu_A(x), \mu_B(x))$

Complement $\bar{A} \rightarrow \mu_{\bar{A}}(x) = 1 - \mu_A(x)$

membership degree

every member
ship is [1]

[green=1]

Cardinal fuzzy relationship = ∞

strength of membership

* Fuzzy Relationship ($\mu_R(x, y)$)

Same operation and properties

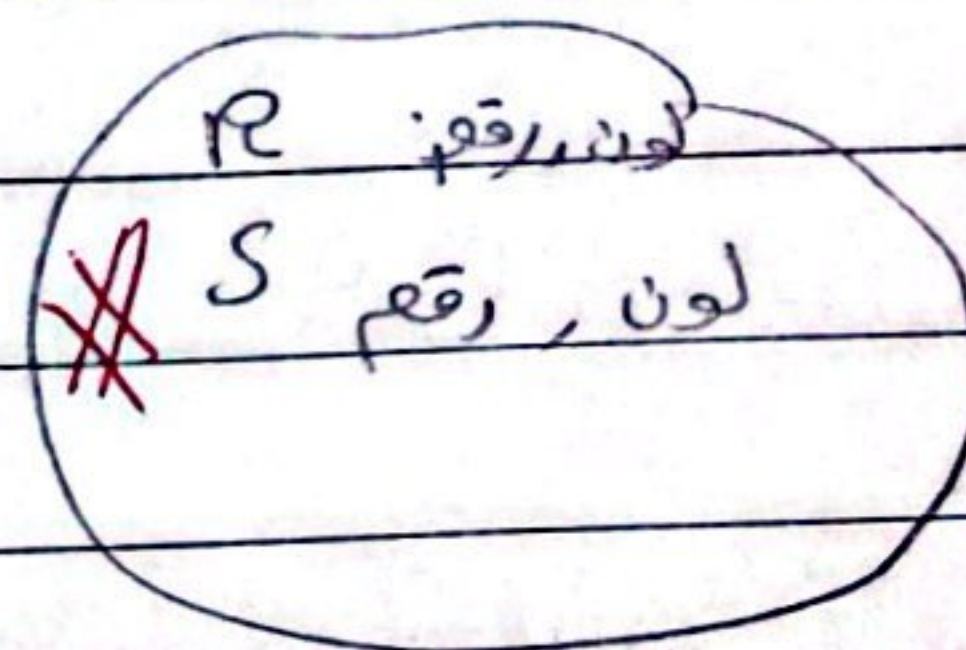
union: max
intersection: min

$$\mu_R(x, y) = \mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y))$$

ex

$$A = \frac{0.2}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3} \quad B = \frac{0.3}{y_1}, \frac{0.9}{y_2}$$

$$A \times B = \begin{matrix} & y_1 & y_2 \\ x_1 & 0.2 & 0.2 \\ x_2 & 0.3 & 0.5 \\ x_3 & 0.3 & 0.9 \end{matrix}$$



* Fuzzy relationship: Composition

$$T = R \circ S$$

max-min

$$T = R \cdot S$$

max-product

$$x \rightarrow y \quad y \rightarrow z$$

R S

$$T \quad x \rightarrow z$$

الترتيب

when you don't have direct relation

$$0.2, 0.1 = 0.1$$

$$0.2, 0.2 = 0.2 \checkmark \text{max}$$

① * max-min

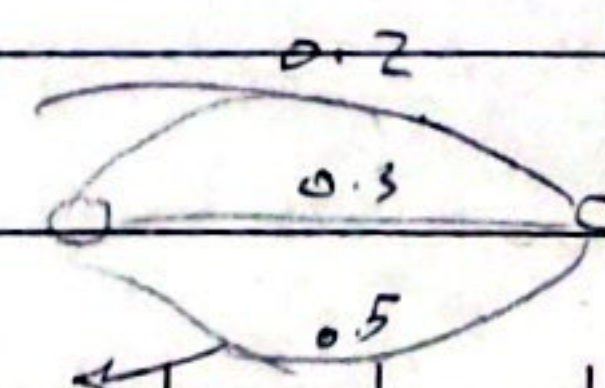
$$R = \begin{matrix} & y_1 & y_2 \\ x_1 & .2 & .2 \\ x_2 & .3 & .5 \\ x_3 & .3 & .9 \end{matrix} \quad 3 \times 2$$

$$R \circ S = \begin{matrix} & z_1 & z_2 & z_3 & z_4 \\ x_1 & 0.2 & 0.2 & 0.2 & 0.2 \\ x_2 & 0.2 & 0.3 & 0.4 & 0.5 \\ x_3 & 0.2 & 0.3 & 0.4 & 0.5 \end{matrix}$$

$$S = \begin{matrix} & z_1 & z_2 & z_3 & z_4 \\ y_1 & .1 & .2 & .3 & .4 \\ y_2 & .2 & .3 & .4 & .5 \end{matrix} \quad 2 \times 4$$

$$.3, .4 = .3$$

$$.9, .5 = .5 \checkmark \text{max}$$



max

Five Apple

$$\left[\begin{array}{l} R = B \times T \\ S = (T \times U) \\ W = R \times S \end{array} \right] \rightarrow \left[\begin{array}{l} \text{في هذه الحالة } R \text{ صفه} \\ \text{row و } T \text{ عمود} \\ \text{column} \\ \text{لتسريع الحساب} \end{array} \right]$$

في حال اقل الجمع

$$\begin{array}{l} 0.1 \times 0.1 = 0.01 \\ 0.2 \times 0.2 = 0.04 \end{array} \quad \text{max}$$

②

max product :- R.S

	z_1	z_2	z_3	z_4
x_1	0.04	0.06	0.08	0.1
x_2	0.1	0.15	0.2	0.25
x_3	0.18	0.27	0.36	0.45

example slide

max-min

	z_1	z_2	z_3
x_1	0.7	0.6	0.5
x_2	0.8	0.6	0.4

دقة

$$\max(\min(0.7, 0.9), \min(0.5, 0.1))$$

$$\max(0.7, 0.5)$$

$$(x_1, z_1) = 0.7$$

product

	z_1	z_2	z_3
x_1	0.63	0.42	0.25
x_2	0.72	0.48	0.2

سرعة

CH3:

crisp

⑧ Equivalence $\rightarrow$ reflex

$$x_R(x_i, x_i) = 1$$

الذات

$\rightarrow$ symmetry

$$(x_i, x_j) \in R \rightarrow (x_j, x_i) \in R$$

$\rightarrow$ transitivity

$$(x_i, x_j) \wedge (x_j, x_k) \rightarrow (x_i, x_k)$$

⑧ Tolerance $\rightarrow$ reflex

Tolerance $\rightarrow$ Eq. 15

$\rightarrow$ symmetry

ليس التكرار

maximum # of combination = CN

(CN-1)

$$R_1^{n-1} = R_1 \circ R_2 \circ \dots = R$$

(x_i, x_i)

Fuzzy ① Reflexivity $\mu_R(x_i, x_i) = 1$

② Symmetry $\mu_R(x_i, x_j) = \mu_R(x_j, x_i)$

③ Transitivity $\mu_R(x_i, x_j) = z_1$ $(x_2, x_5) = 0.9$
 $\mu_R(x_j, x_k) = z_2$ $(x_5, x_3) = 0$
 $\mu_R(x_i, x_k) = z$ $(x_2, x_3) = (0.4)$

$$z \geq \min[z_1, z_2]$$

$$* z_2 \geq z \geq z_1$$

(3.11) reflexive ✓ dialog

Symmetric ✓

Transitive [x]

$$r_{ij} = \frac{\sum_{k=1}^m x_{ik} x_{jk}}{\sqrt{(\sum_{k=1}^m x_{ik}^2)(\sum_{k=1}^m x_{jk}^2)}} \quad * \text{Cosine Method}$$

$$r_{ij} = \frac{\min(x_{ik}, x_{jk})}{\max(x_{ik}, x_{jk})} \quad * \text{Max-Min Method}$$

$$r_{13} = \frac{0.3 \times 0.1 + 0.6 \times 0.6 + 0.1 \times 0.3}{(0.3^2 + 0.6^2 + 0.1^2)(0.1^2 + 0.6^2 + 0.3^2)} = 0.913$$

$$(0.3^2 + 0.6^2 + 0.1^2)(0.1^2 + 0.6^2 + 0.3^2)$$

strength of

membership degree

for relation

$$\text{Max-Min: } r_{12} = \frac{0.2 + 0.4 + 0.1}{0.3 + 0.4 + 0.4} = 0.538$$

CH4

rules

fuzzification

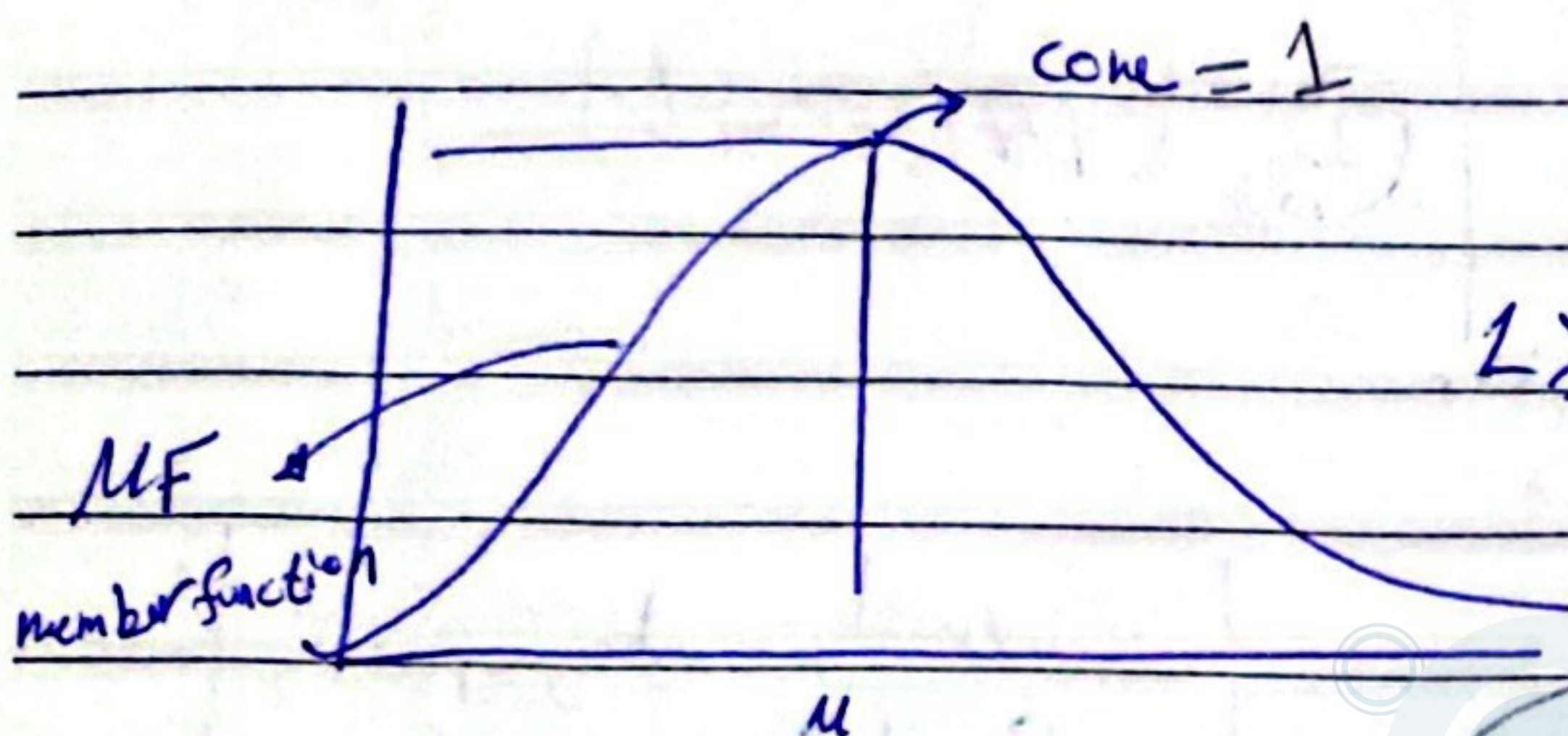
inference

defuzzification

$C \rightarrow f$

$f_{in} \rightarrow f_{out}$

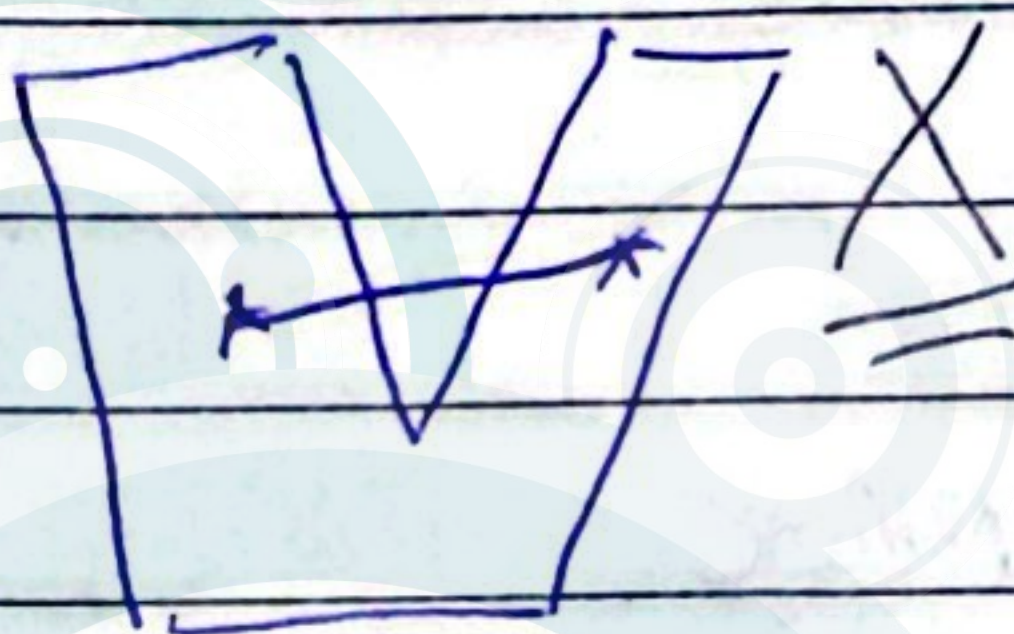
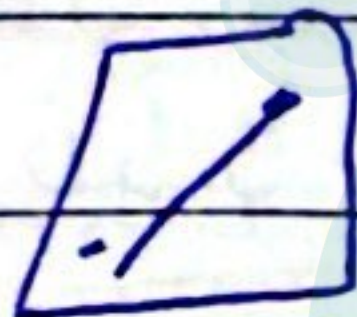
Value



$1 \geq \text{Support} \geq 0$

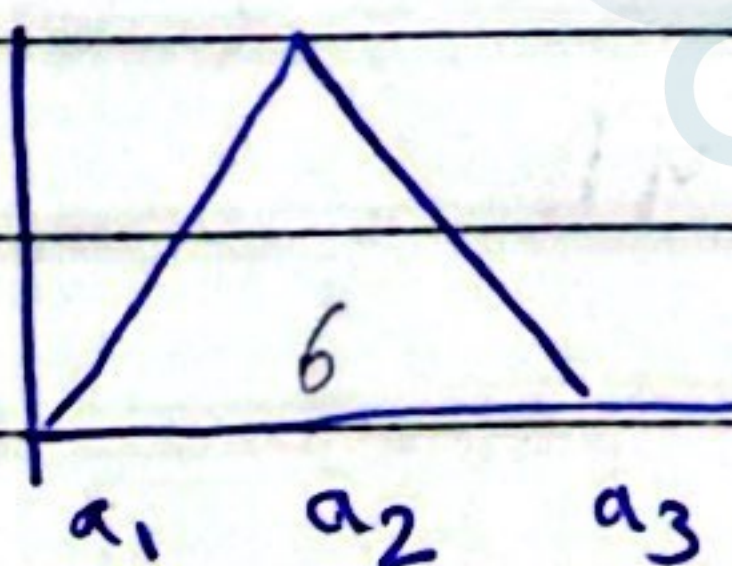
normal; at least
@ prototype: only one

Convex



* جشی مع اقل error

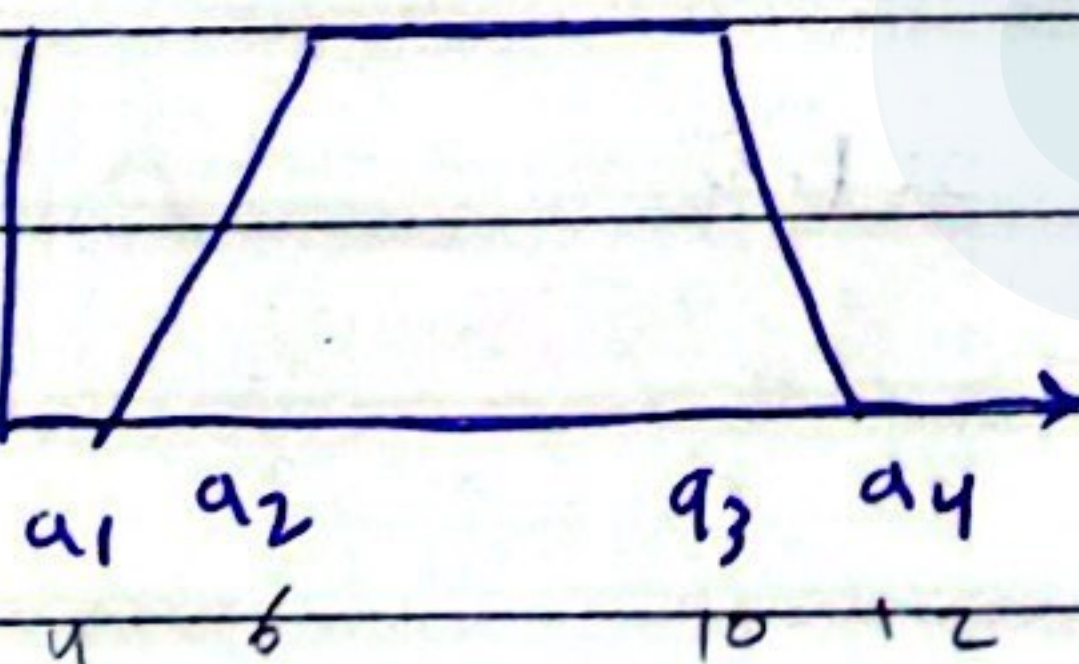
① triangular



$$\mu_A = \begin{cases} 0 & u < a_1 \\ \frac{u-a_1}{a_2-a_1} & a_1 \leq u \leq a_2 \\ \frac{a_3-u}{a_3-a_2} & a_2 \leq u < a_3 \\ 0 & u \geq a_3 \end{cases}$$

$\mu = 1$

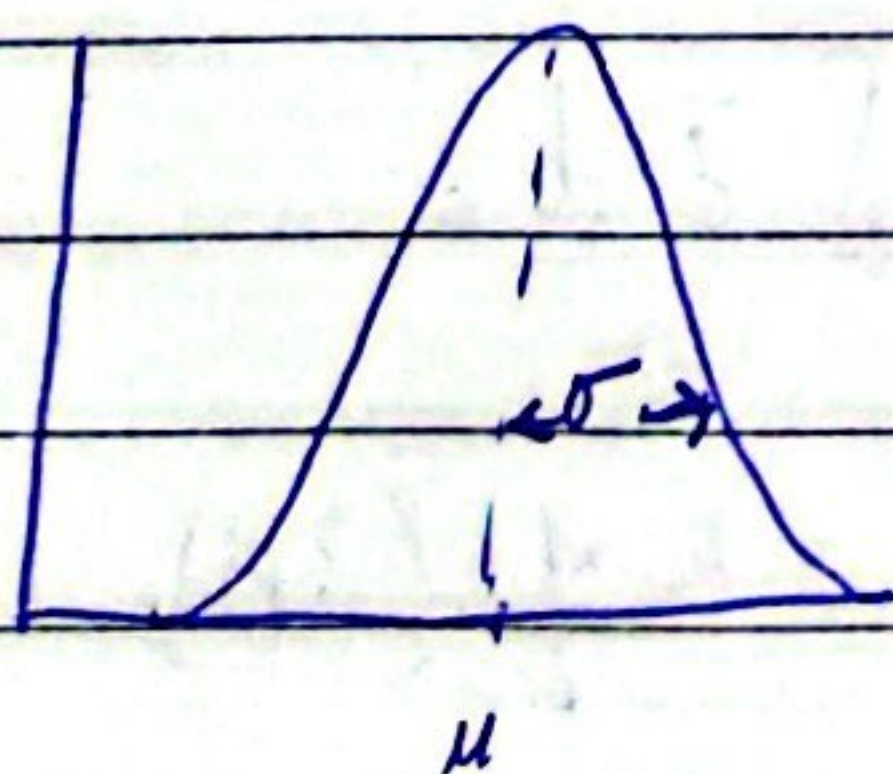
②



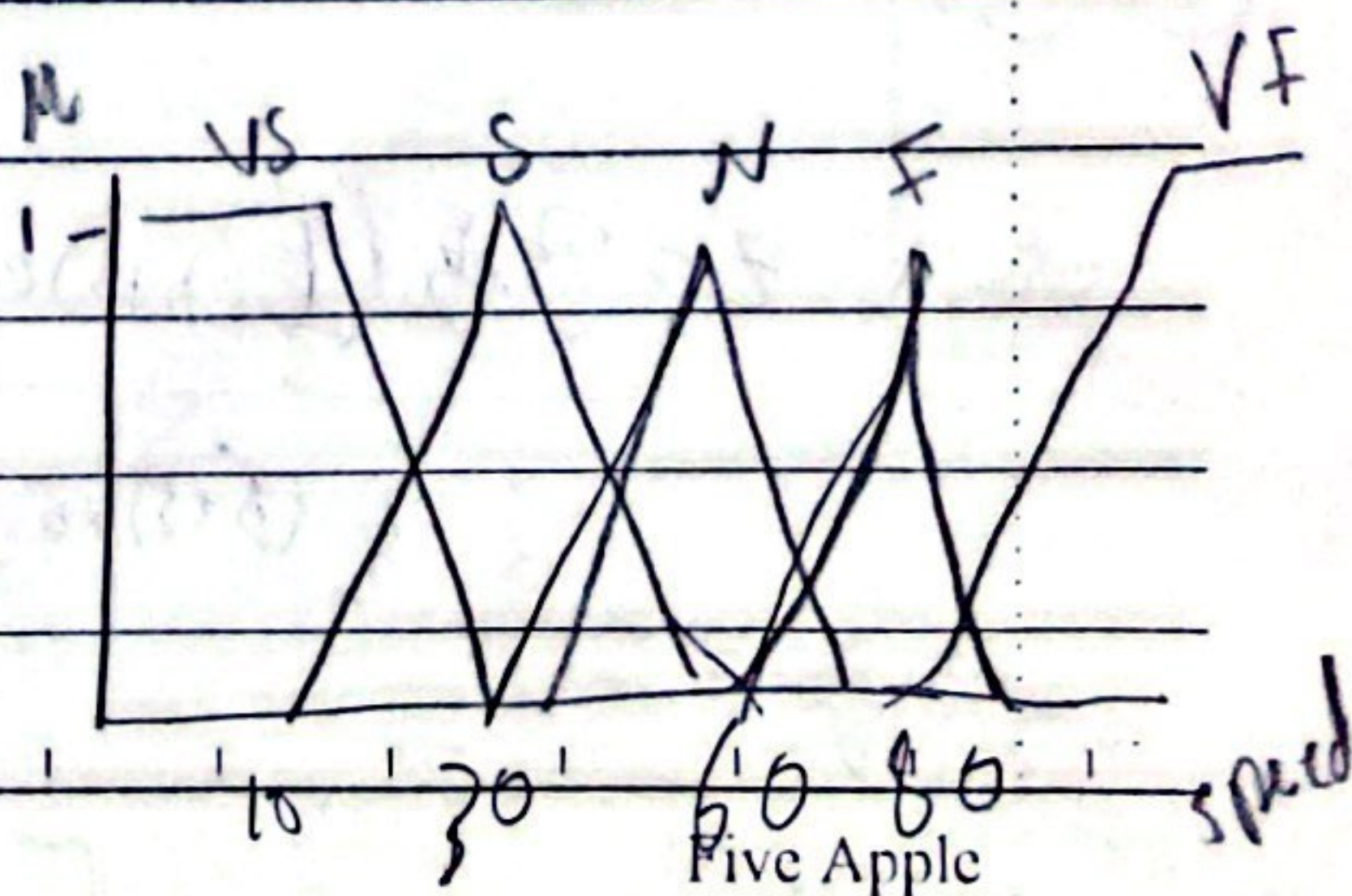
$\therefore 1 \quad a_2 \leq u \leq a_3$

$$\frac{a_1-u}{a_1-a_2} = 1 = \frac{12-10}{12-10} = \frac{1}{2}$$

③



$$\mu_A = e^{-\frac{(u-m)^2}{\sigma^2}}$$



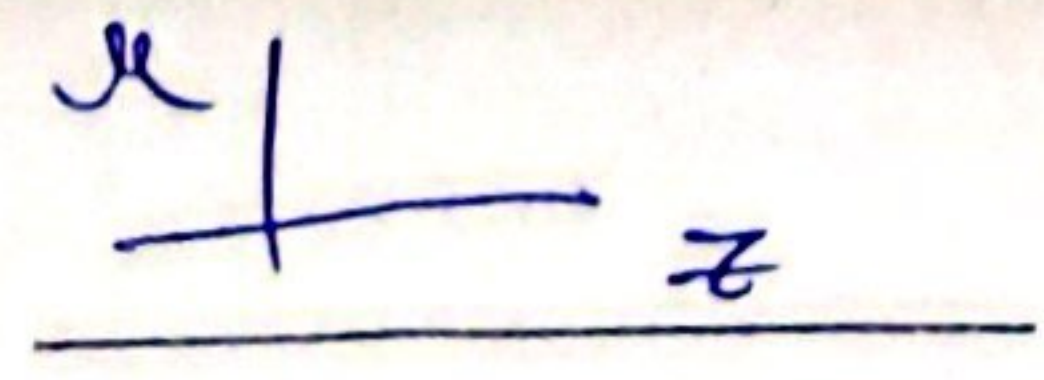
VS $\leq$ 30
10 $\leq$ S $\leq$ 60
30 < N $\leq$ 80
60 $\leq$ F $\leq$ 120

VF $\geq$ 80

4.14

Defuzzy

الافتقار



* From Fuzzy to crisp

① Max membership الأكبر

[0-5] width

① membership degree

① slope of z

prototype

الافتقار

② (intercept)

① union

② Centroid method :

②

$$z^* = \frac{\int \mu_c(z) \cdot z \, dz}{\int \mu_c(z) \, dz}$$

[centroid لازم نزل first]

Defuzzy

الافتقار (2.00 0.50 0.50)

③ Weighted average.

③

$$z^* = \frac{\sum \mu_c(\bar{z}) \cdot \bar{z}}{\sum \mu_c(\bar{z})}$$

$$z^* = \frac{0.3 \times 2.5 + 0.5 \times 5 + 1 \times 6.5}{0.3 + 0.5 + 1}$$

$$z^* = 5.41$$

④ Mean Max membership

بين اقله اثنين (4)

(Maximum height)
↓ (peak)

$$z^* = \frac{a+b}{2} = \frac{6+7}{2} = 6.5m$$

⑤ Center of sum

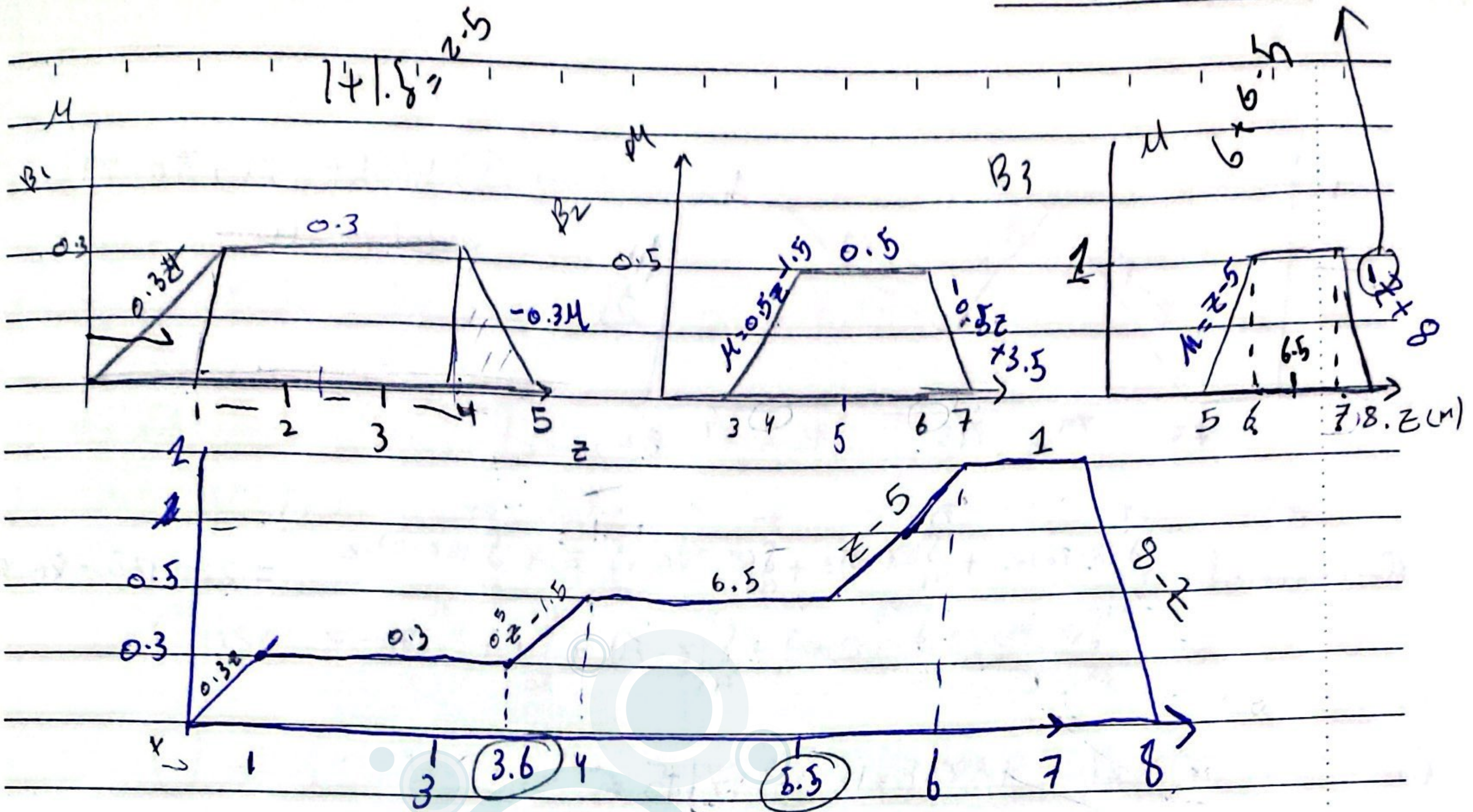
$$z^* = \frac{\sum \mu_c(\bar{z}) \int \bar{z} \, dz}{\sum \mu_c(\bar{z}) \int dz}$$

$$z = \frac{0.3 \left(\frac{1}{2} (5+3) \cdot 0.3 \right) + 5 \times \frac{1}{2} (4+2) \cdot 0.5 + 6.5 \times \frac{1}{2} (3+1)}{\frac{1}{2} (5+3) \cdot 0.3 + \frac{1}{2} (4+2) \cdot 0.5 + \frac{6.5}{2} (3+1)} = 5.0m$$

first=6
last=7

[نص مجموع القادرين * الارتفاع]

انتبه الحيل



من قبله

$$\mu = 0.3 = 0.5z - 1.5 \quad z = 3.6$$

$$0.5 = z - 5 \quad z = 5.5$$

$$z^* = \int_0^1 (0.3z)z + \int_{3.6}^4 0.3z + \int_{3.6}^4 (0.5z - 1.5)z + \int_4^{5.5} 0.5z + \int_{5.5}^6 (z - 5)z + \int_6^7 1z + \int_7^8 (8 - z)z$$

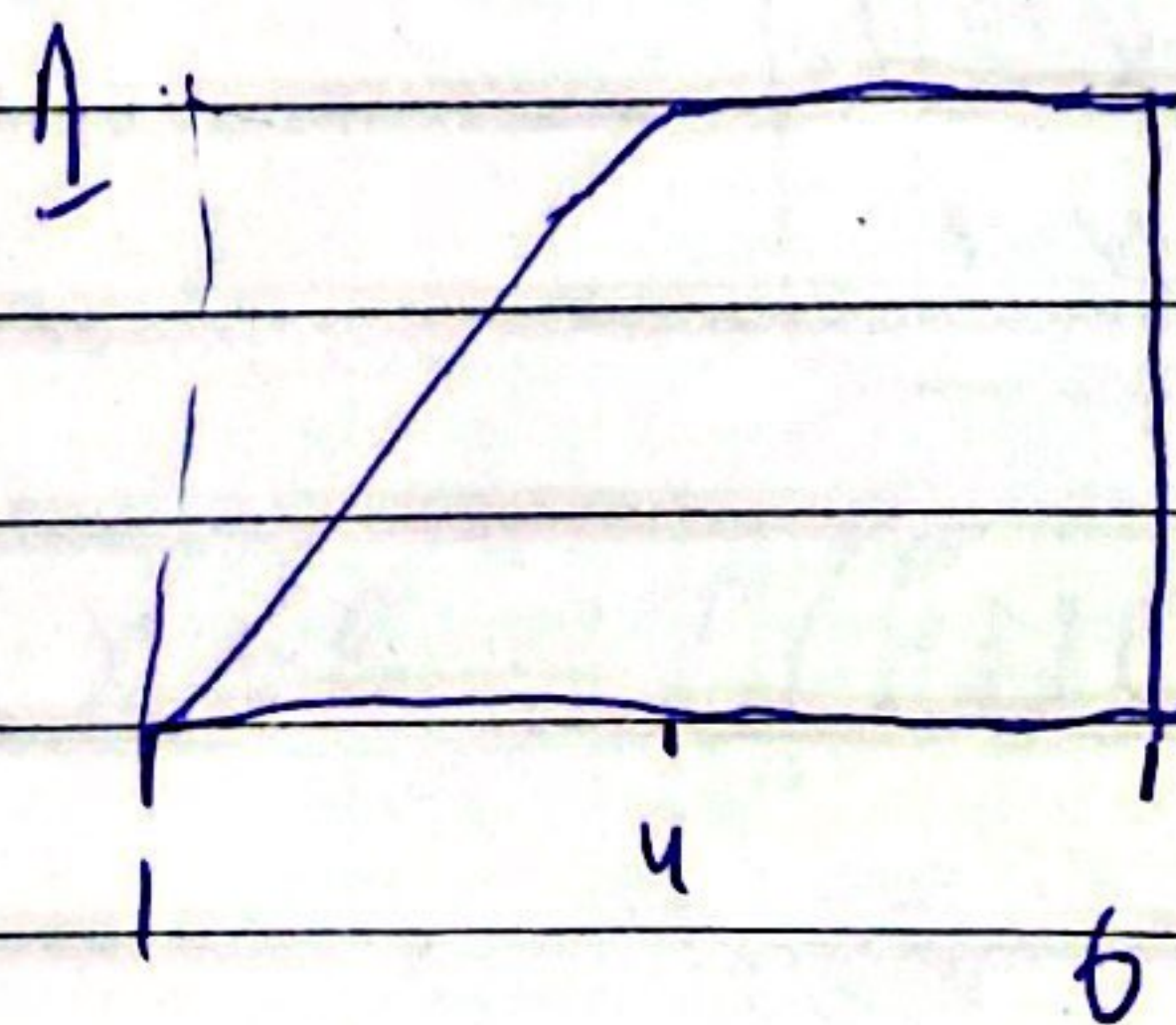
$$\int_0^1 0.3z + \int_{3.6}^4 0.3 + \int_{3.6}^4 0.5z - 1.5 + \int_4^{5.5} 0.5z + \int_{5.5}^6 (z - 5) + \int_6^7 1 + \int_7^8 8 - z$$

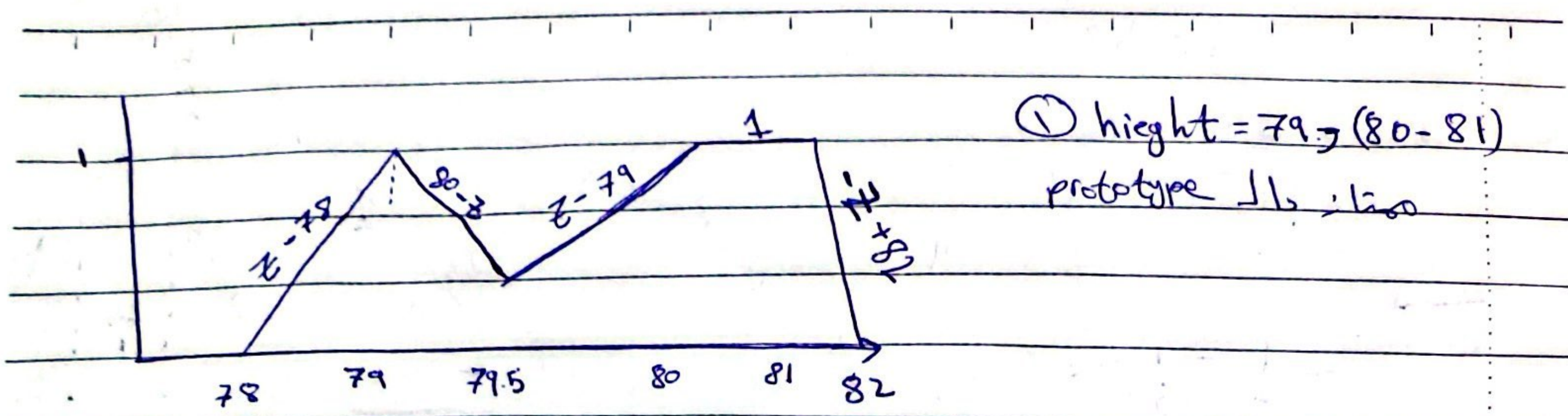
$$z^* = 4.9m$$

Defuzzy slope
eq

$$z^* = \frac{0.3 \times 2.5 + 0.5 (5) + 1 \times 6.5}{0.3 + 0.5 + 1} = 5.41m$$

(q) 4.14





① height = 79.5 (80-81)
prototype 1, 1, 1, 1

② Centroid =
$$\frac{\int_{78}^{79} (z-78)z + \int_{79}^{79.5} (80-z)z + \int_{79.5}^{80} (z-79)z + \int_{80}^{81} z + \int_{81}^{82} (82-z)z}{\int_{78}^{79} (z-78) + \int_{79}^{79.5} (80-z) + \int_{79.5}^{80} (z-79) + \int_{80}^{81} 1 + \int_{81}^{82} 82-z} = 220.125 = 80.05$$

③
$$z = \frac{79 \times 1 + 80.5 \times 1}{1+1} = 79.75^\circ$$

Weight

mean max

④
$$z = \frac{79 + 80 + 81}{3} = 80^\circ$$

⑤ Center of sum =
$$\frac{79 \times (\frac{1}{2} \times 2 \times 1) + 80.5 \times \frac{1}{2} (3+1) \times 1}{(\frac{1}{2} \times 2 \times 1) + \frac{4}{2}} = 80^\circ$$

Function also 1, Area

First = 79

last = 81 not exist

• human capacity to reason.

proposition $\rightarrow$ sentences

✓ interpolative : between true and false.

✓ tautologies always true ✓ statement

✓ Contradictions always false.

Crisp logic

Fuzzy logic

$U \rightarrow (0,1)$

yes or no

* $U = \{\text{blue, yellow, red, white}\}$

* $P \rightarrow$ red belong to U

$T(P) = \begin{cases} 1 & \text{if } x \in U \\ 0 & \text{if } x \notin U \end{cases}$ ✓

start with (b)

Truth set = $\{\text{bl, blo}\}$

Falsity set = $\{\text{red, white, green}\}$

$\bar{P}$	P	Q	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \leftrightarrow Q$	$Q \rightarrow P$
0	1	1	1	1	1	1	1
0	1	0	1	0	0	0	1
1	0	1	1	0	0	1	0
1	0	0	0	0	1	1	1

P	Q	T	C
1	1	1	0
1	0	1	0
0	1	1	0
0	0	1	0

* disjunction $\cup$ max

$\sqrt{A} = \{1, 2, 3, 4, 5, 6\}$
 $\sqrt{B} = \{3, 4, 6, 8, 9, 10\}$

* Conjunction $\cap$ min

$F(3) =$

* negation $\bar{A}$

$P \rightarrow Q$

* implication $\rightarrow$

$x \in A, x \in B$

$T(\bar{P} \cup Q)$

equivalence $\leftrightarrow$

only two cases.

$T(U) = 1$
 $T(\emptyset) = 0$

Five Apple

$$\bar{B} \cap (A \cup B)$$

بعضوا بعض

$T(P) \cap T(Q) = \emptyset$ mutually exclusive propositions.

8 All human are mammals.

①

إذا ما رزق طلع

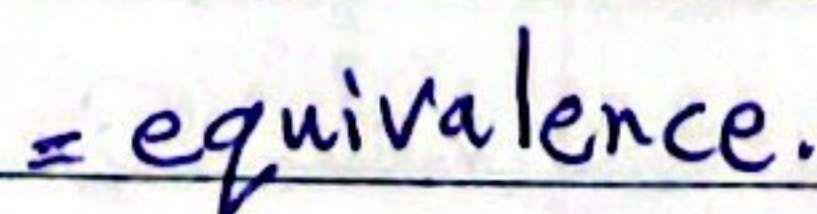
النسبة المئوية

② • Modus Tollens ($\neg B \wedge (A \rightarrow B)$)
(back)

$$\frac{A \quad B \quad A \rightarrow B}{A \wedge (A \rightarrow B)}$$

* Contradiction:

$\left[\begin{array}{l} \text{prime number} \\ \text{multiple of 4} \end{array} \right]$ always zeros



التناقضات : المركبة الخطية

بعض النظر عن المقترحات الفردية

(و) تغیر ۱ نقاط

example:-

* $U = 1, 2, 3, 4, 5, 6, 7, 8$

$P \leftrightarrow Q$

$P = 2, 4, 6, 8$

$Q = 3, 4, 5, 7$

both T $\rightarrow$ Both F

$P \leftrightarrow Q = \{1, 4\}$

equivalence

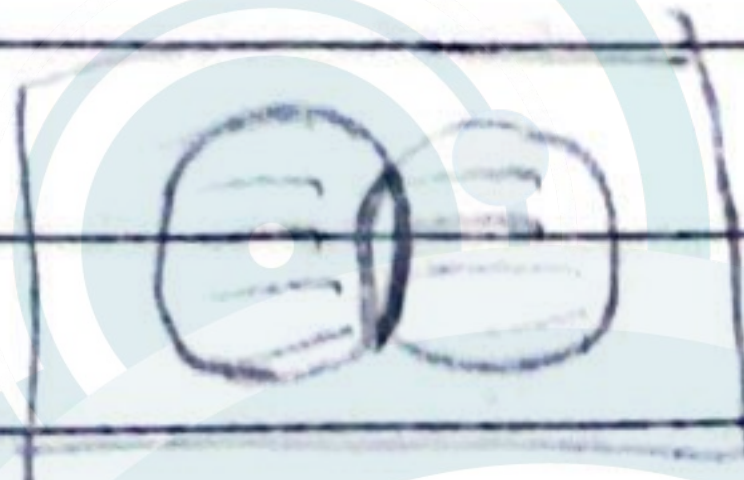
* Exclusive

الباقي

both X

P Q XOR

1	1	0
1	0	1
0	1	1
0	0	0



	P	Q
1	0	0
2	1	0
3	0	1
4	1	1
5	0	1
6	1	0
7	0	1
8	1	0

$XOR = (P \vee Q) \wedge (P \vee Q)$

Exclusive (NOR)

equivalence



$\bar{P} \wedge Q \vee P \wedge \bar{Q}$

$\bar{P} \wedge \bar{Q} \rightarrow Q$

* Dective inference

الباقي

→ membership

Fuzzy logic: $T(P) = \mu_A(x)$ $0 \leq \mu_A \leq 1$

Implication $T(P \rightarrow Q) = T(\bar{P} \vee Q) = \max(T(\bar{P}), T(Q))$

equivalent $(A \times B) \cup (\bar{A} \times Y) \rightarrow$ universe
 all dimension $\downarrow$ (if x is A, then y is B)

*example

$$A = \left\{ \frac{0.6}{2}, \frac{1}{3}, \frac{0.2}{4} \right\}$$

$$Y = \{1, 2, 3, 4, 5, 6\}$$

universe

$$B = \left\{ \frac{0.4}{2}, \frac{1}{3}, \frac{0.8}{4}, \frac{0.3}{5} \right\}$$

$$C = \left\{ \frac{0.3}{1}, \frac{0.5}{2}, \frac{0.6}{3}, \frac{0.6}{4}, \frac{0.5}{5}, \frac{0.3}{6} \right\}$$

$$A \times B = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0.4 & 0.6 & 0.6 & 0.3 \\ 0.4 & 1 & 0.8 & 0.3 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{matrix}$$

$$\bar{A} \times Y = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix} \end{matrix}$$

dimension

$$R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.4 & 0.6 & 0.6 & 0.4 & 0.4 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix} \end{matrix}$$

C-R ©

COR = K

If x is A Then y is B else C $R = (A \times B) \cup (\bar{A} \times C)$

* Approximate Reasoning

الإستدلال التقريبي

* Imprecise propositions

اقتراح غير دقيقة

* مقولات غير دقيقة أو غير كاملة

If we have new product

(new not prime)

If x is A' Then y is B

$$(B' = A' \text{ OR } B)$$

Finding B

$$B' = \left\{ \frac{0.5}{1}, \frac{0.5}{2}, \frac{0.6}{3}, \frac{0.6}{4}, \frac{0.5}{5}, \frac{0.5}{6} \right\} \text{ market size for new invention}$$

Other techniques (5)

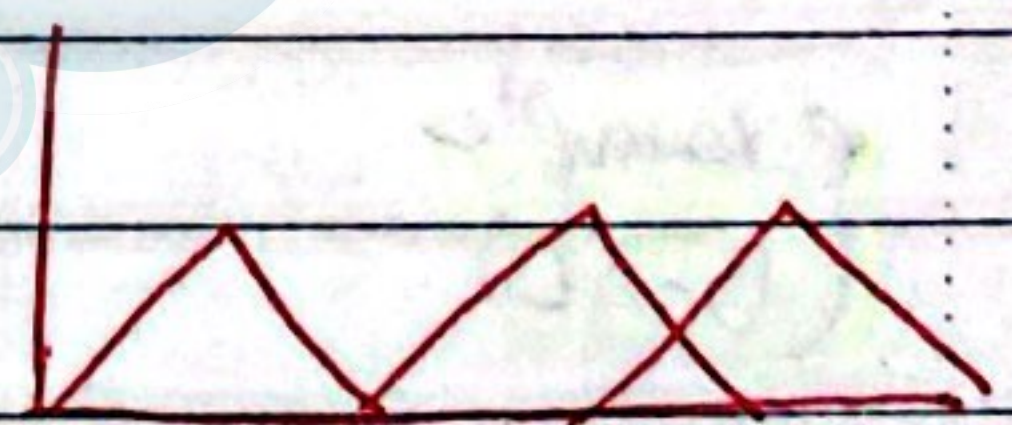
$$\mu_R(x, y) = \textcircled{1} \max[\mu_B(y), 1 - \mu_A(x)]$$

$$\textcircled{2} \min(A, B)$$

$$\textcircled{3} \min(1, (1 - \mu_A(x) + \mu_B(y)))$$

$$\textcircled{4} A \cdot B$$

$$\textcircled{5} \begin{cases} 1 & A \leq B \\ \mu_B(y) & \text{otherwise} \end{cases}$$



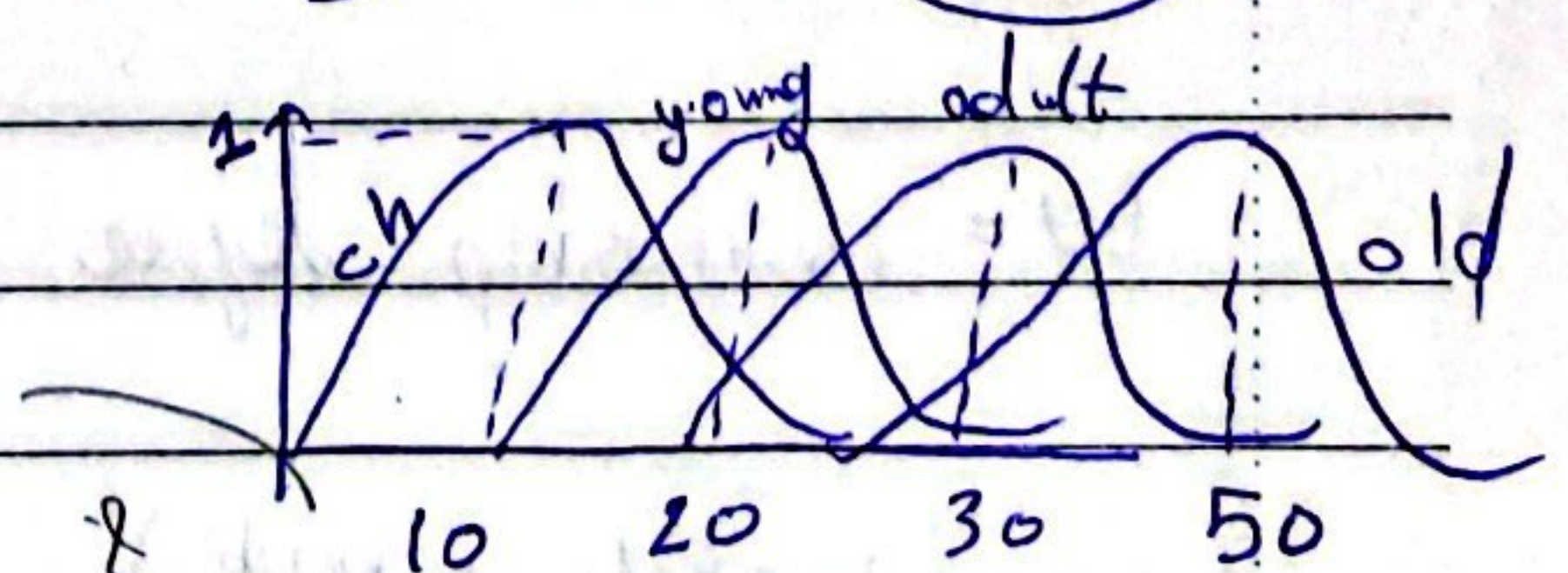
* Fuzzy system: To modeling

α or β : max

α and β : min

$$\alpha = \bar{\alpha} = 1 - \alpha$$

وصف الفئات العمرية



* it's time
yours

fuzzy day

can be partly cloudy

Five Apple

لا يختلف
لما

different antecedents
sedants

تجميع
[أنا، انتم، الطوبى]
حنا أخذ الحاضرة

(رجل اخطي الثلاث واحد)

If antecedent the consequent.

If then Rules:- one rule A then B

and min $\cap$

or max $\cup$

* Aggregation: connect all consequent

1- Conjunctive (min) and $\cap$

2- Disjunctive (max) or $\cup$

(out go in و)

* إزالة الضبابية

* Inference: Graphical Techniques: مخرج بياك (أ) مدلول

out fuzzy
membership
function

Mamdani

B = membership function

(max-min)

max product

مثال

example
(5-32)

Takagi Sugeno

Tsukamoto

union find

output function
in input

* Takagi Sugeno: x is A the z is $z = f(x, y)$
y is B

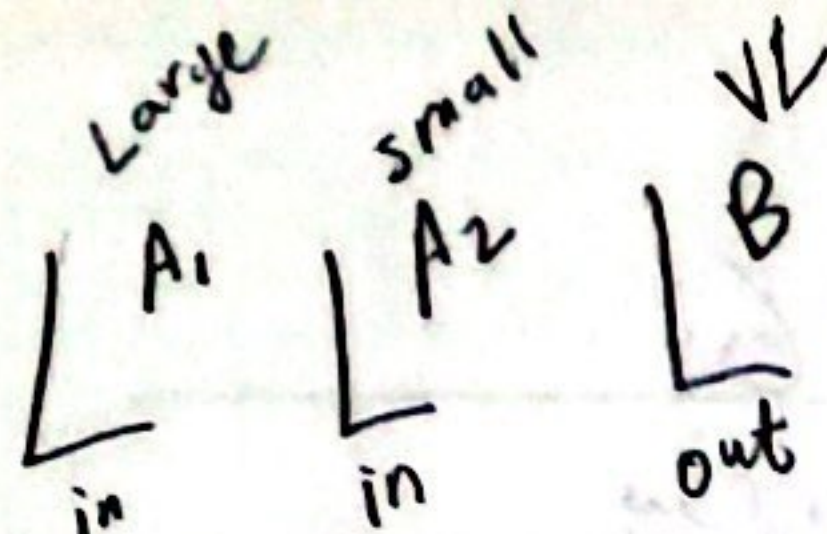
W = membership degree for input.

(weight average)

(5-31)

✓

P2-5.13



Union

(2)

(1)

(A) * Mamdani

Defuzzification

$$M(A_1) = 0.4 \rightarrow (1.8) \rightarrow \text{and, min is } 0.4$$

and, min is 0.4
OR, max

↑ interpretability ↓
and
 x_1 is A_1 x_2 is A_2
then y is B



$$\text{fuzzy output} = \text{Area}$$

for each rule

(B) * Takagi Sugeno :

output

(4)

if x_1 is A_1 then y is $f(x_1, x_2)$

* output direct

x_2 is A_2

* linear

* Rules

$$y = px_1 + qx_2 + b$$

no need to Defuzzy

output

x_1, x_2 input

p, q, b constant.

(weighting)

$$Z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

$$w_1 + w_2$$

* w : minimum

rules used
Small, small
Large, Large
S, L
L, S

(C) * Tsukamoto : (mix mamdani and Sugeno)

[Shoulder membership function]

from w we can know Z (because no close area)

then weight without function.

$$(5-31) \left[\frac{5 \times 0.5 + 3.2 \times 0.4 + 3.5 \times 0.25}{0.5 + 0.4 + 0.25} = \frac{4.655}{1.15} = 4.04 \right]$$

0.5

Object: Define membership function

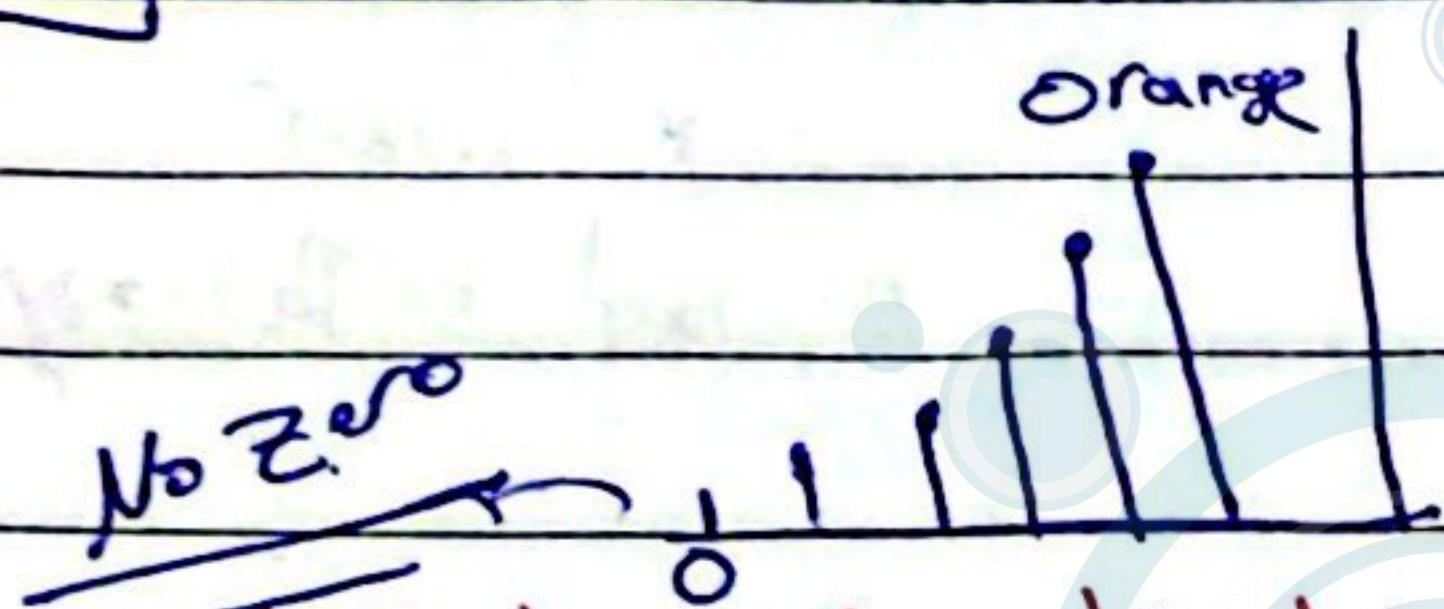
CH 6:

intuitive: بديهي

* Intuition: human capacity ^{الحق} experience

* Inference: according your knowledge
Equation or equations

* Rank Order:-
الوقت



مدى تفضيل الناس للون البرتقالي
لا يمكن معادلات للاعتقاد عليها

تخير لوني كالمصرة
فضل احدهم على الآخر

Prefer.

تختار
Membership degree

* Neural Network:-
biological

3 layers in - hidden - out

$$S = 1 \times 0.2 + 2 \times 0.2 + 0.1 \times 1$$

20 points

70 training

Validation (eliminate zigzag line) 15%

test 15% (hidden for testing)

function

sigmoid function

$$\frac{1}{1 + e^{-x}}$$

all zeros we use threshold.

x_1	x_2	x_3	y_1	y_2
1	2	1	3	4

$$y_1 = \frac{1}{1 + e^{-(0.668 \times 0.3 + 0.668 \times 0.5 + 0.768 \times 0.1)}}$$

$$y_1 = 0.648$$

$$3 - 0.648 = \text{error}_1 = 2.352$$

Five Apple

Training

forward
backward
epoch

Back

error for hidden

$$\alpha = 0.3$$

$$E_n = O_n (1 - O_n) * \sum w_{e \text{ error}_1, \text{ error}_2}$$

$$\mu_1 = 0.668 (1 - 0.668) * (0.3 * 2.352 + 0.4 * 3.314) = 0.413 \quad H_1 \text{ new error}$$

$$\mu_2 = 0.668 (1 - 0.668) * (0.5 * 2.352 + 0.2 * 3.314) = 0.407$$

$$\mu_3 = 0.768 (1 - 0.768) * (0.1 * 2.352 + 0.5 * 3.314) = 0.337$$

$$W_{new} = W_{old} + \alpha e_{(in \text{ next layer})} * X$$

$$\mu_1 \rightarrow 0.3$$

$$= 0.2 + 0.3 * 0.413 * 1 = 0.3239$$

$$0.3 + 0.3 * 0.407 * 1 = 0.4221$$

$$0.1 + 0.3 * 0.337 * 1 = 0.2011$$

$$\textcircled{1} \xrightarrow{0.3} \mu_1$$

$$\mu_1 \xrightarrow{0.77} y_1$$

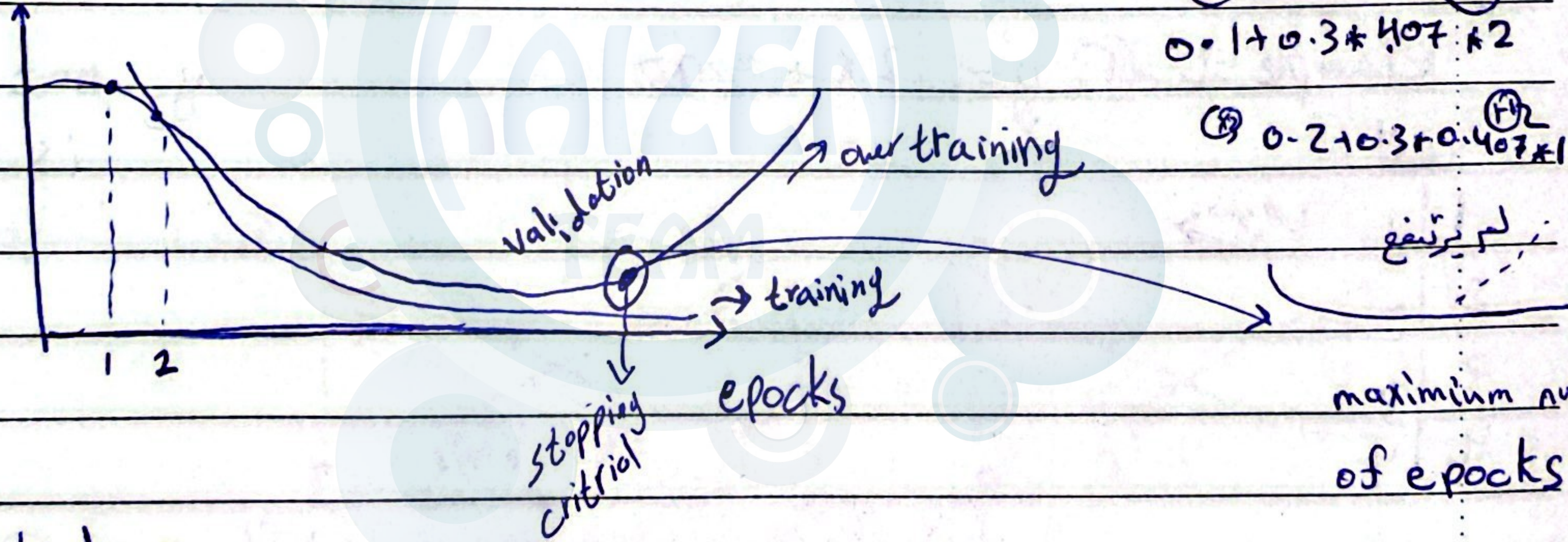
$$0.3 + 0.3 * 2.325 * 0.668 = 0.76593$$

$$\textcircled{2} \xrightarrow{0.407} \mu_2$$

$$0.1 + 0.3 * 4.07 * 2$$

$$\textcircled{3} 0.2 + 0.3 * 0.407 * 1$$

MSE



* black box.

* Normalization Data = [0, 1]

One : ١

63 = 111111

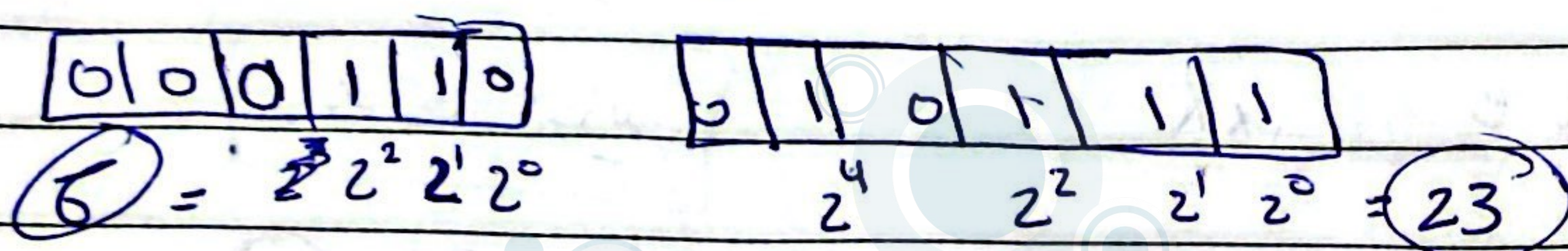
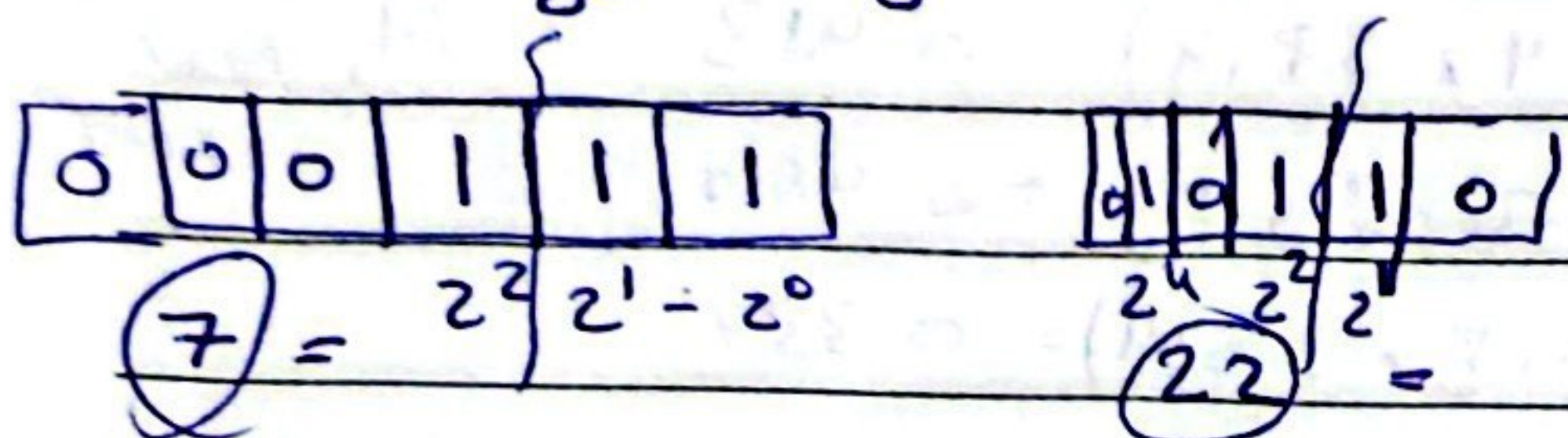
3// steps in slides

كل مرة بطول يحدل اءفعل

* Genetic Algorithms :- simulation human genetic

أخذنا أول قطعين

Coding strength $\rightarrow$ crossover



② * reproduction : take as it

من التحويل

③ * Mutation: 1 $\leftrightarrow$ 0

ااكس

$$= -2 + 18 + (5 - -2)$$

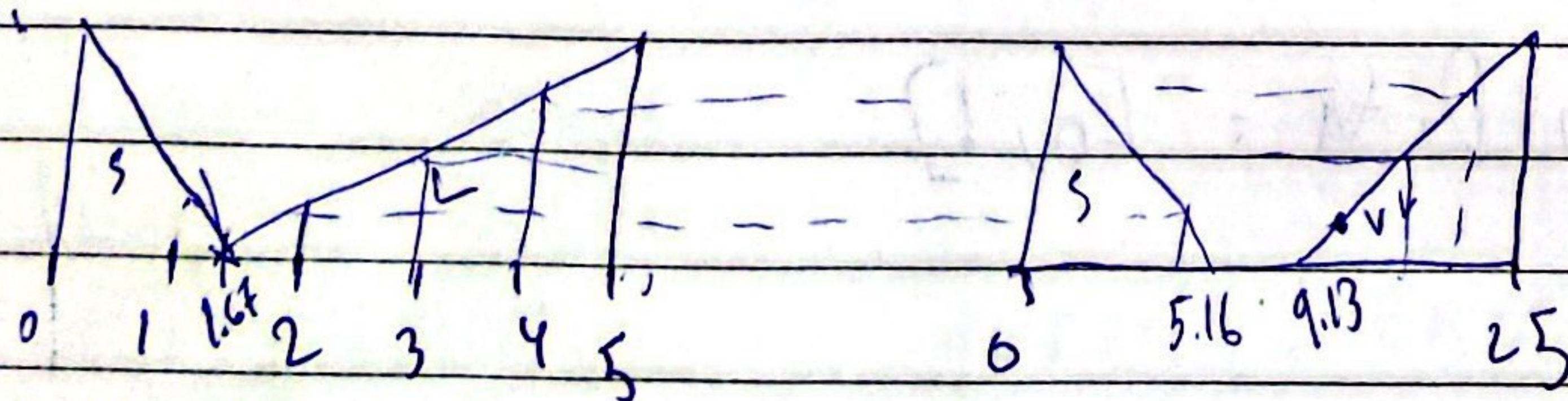
$$\begin{matrix} 26-1 \\ \downarrow \\ 110010 \end{matrix}$$

Example $\Rightarrow$ ① $y = -1.22x + 0.22$

y_3 not 5
is 4 (3.99)

x	y
1	1
2	2
3	3
4	4

نعوض $\rightarrow$



1	2	3	4	5
3.1	10.72	15.96	20.9	25

Large wali 5 wali

Targets minimizing

* 4 solutions. (X X y y)

$$* C_i = C_{min} + \frac{b}{2^k - 1} (C_{max} - C_{min})$$

First 2 columns

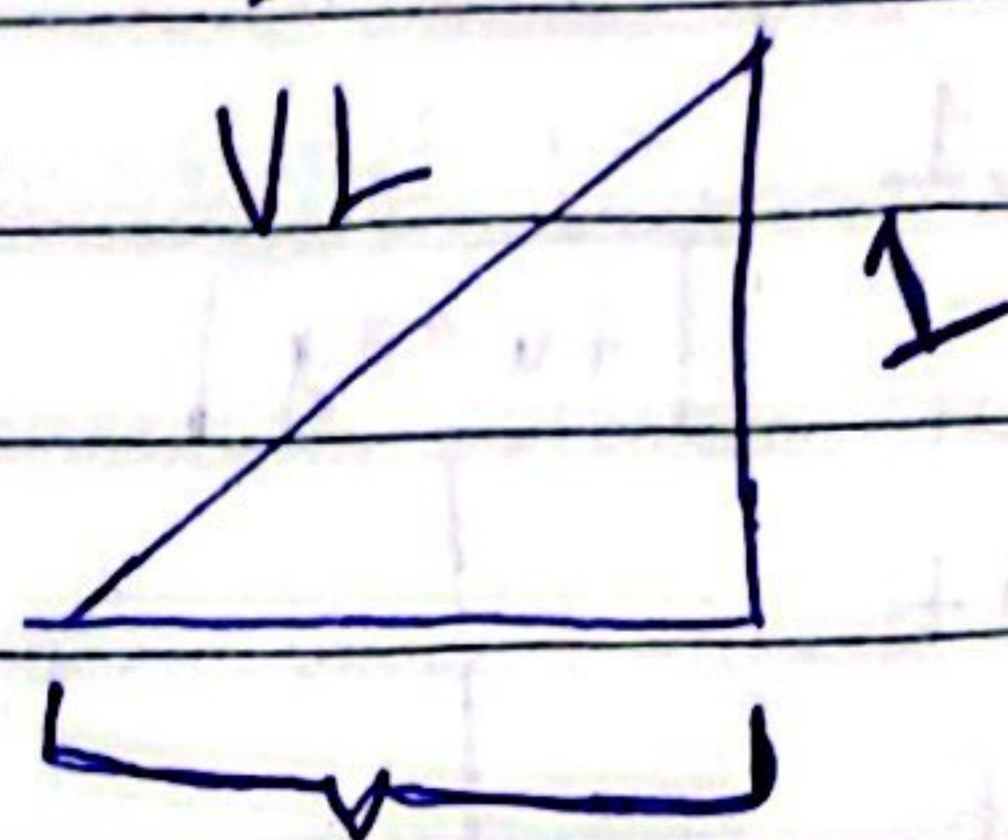
$$2^2 - 1$$

$$C_i = 0 + \frac{7}{2^2 - 1} (5 - 0) = 1.1667$$

Second 2 columns

$$2^6 - 1$$

$$C_i = 0 + \frac{22}{2^6 - 1} (25 - 0) = 8.73$$

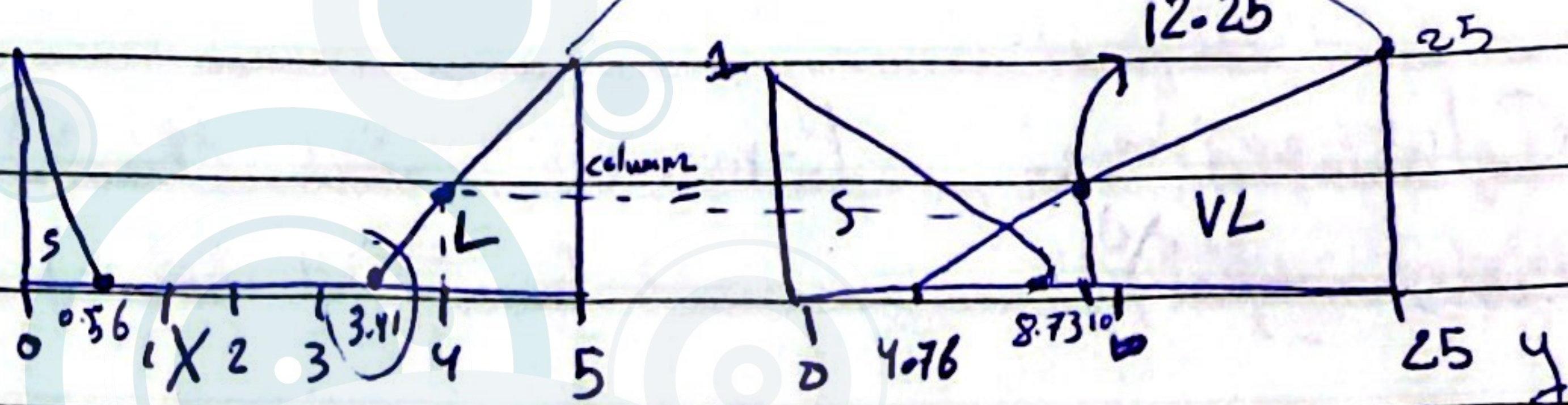


parameter

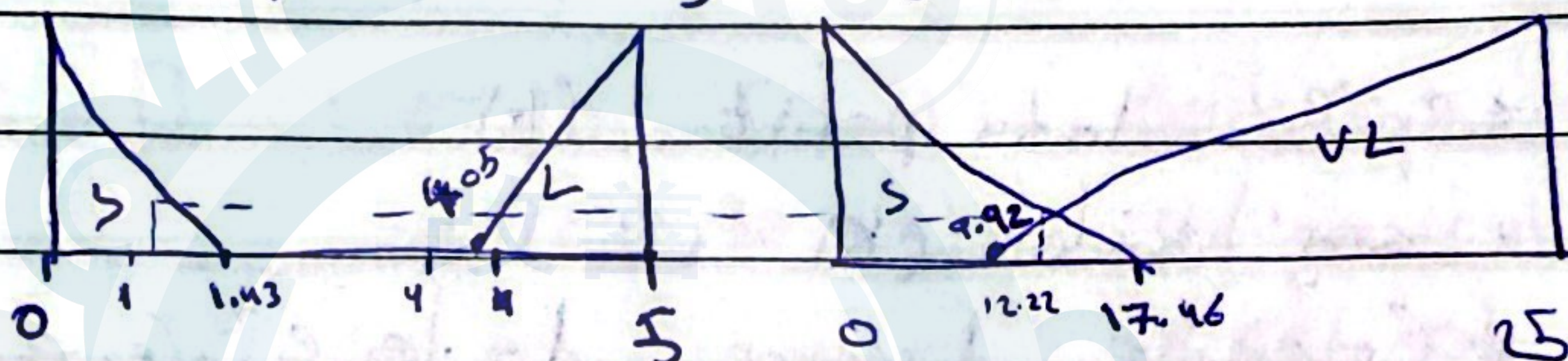
parameter, wali 1/2 *

1 2 3 4 5
0 0 0 12.25 25

(1)

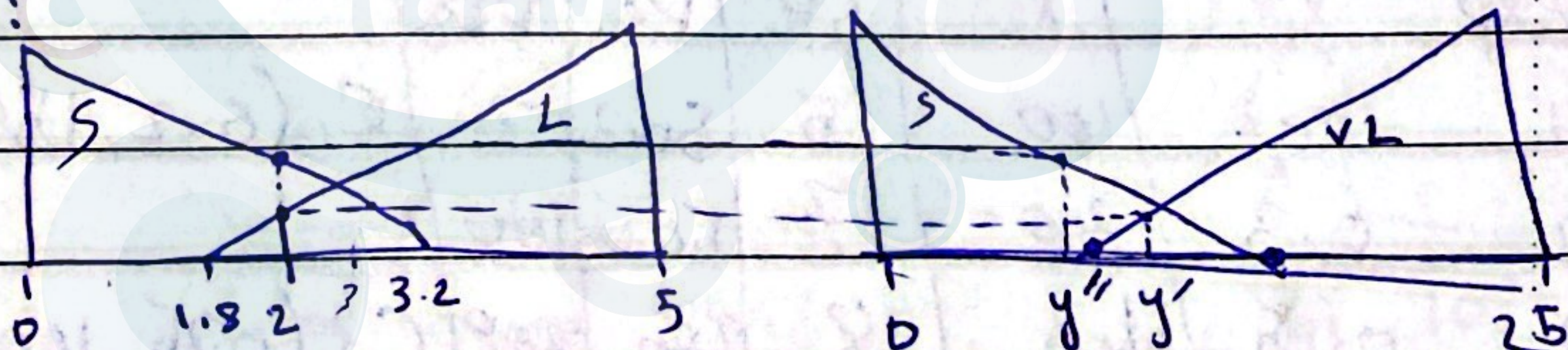


(2)



1 2 3 4 5
12.25 0 0 0 25

* Special case:



* Defuzzification: $\frac{y' + y''}{2}$
↳ weighted average.

* Same thing for number
3

col 15 = (10-14) with Original table

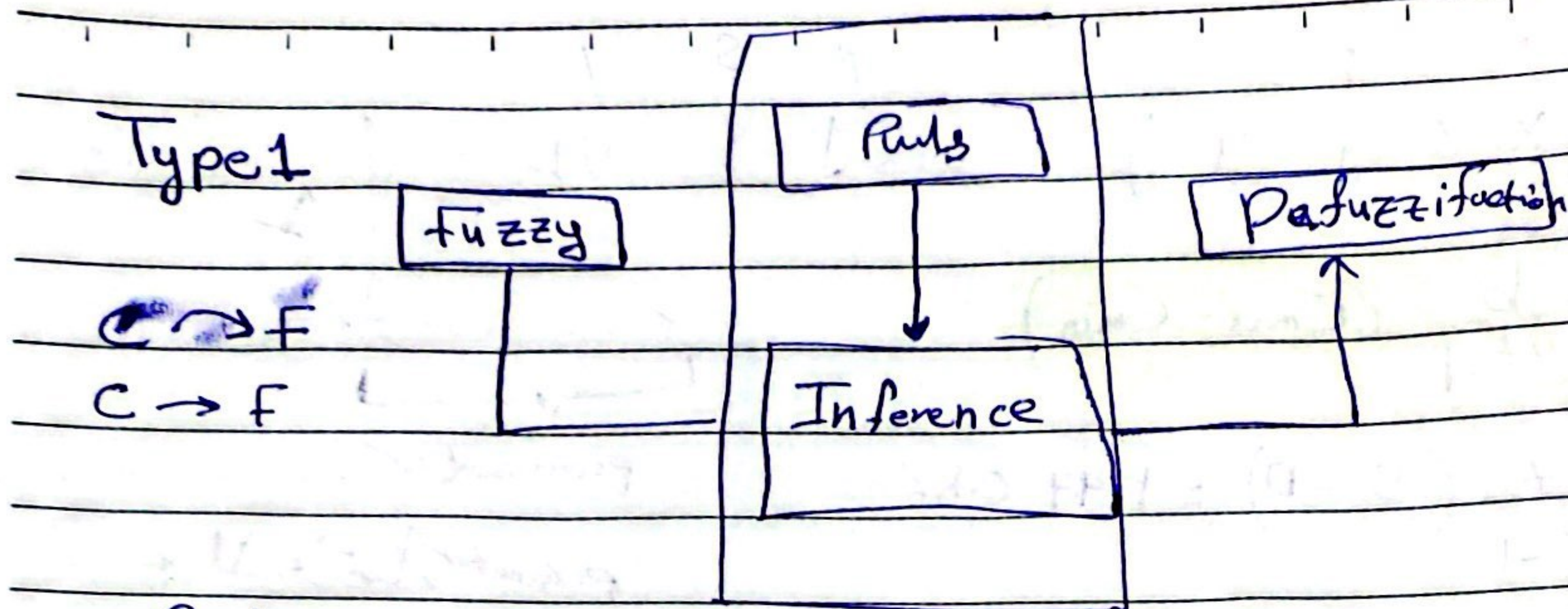
add with

randomly

$$\sum (y - y_i)^2$$

$$\sigma(x) = e^{-\frac{1}{2} \left(\frac{x-m}{\sigma} \right)^2}$$

CH 7



* Rules

↳ If x_1 is A_1 and x_n is A_n , Then y is B

✓ Clustering:- k-mean
(rules in $\bar{x}$)

* Specific data point (k)

* any random (k)

* Calculate distance $d = \sqrt{(e_1 - c_1)^2 + (e_2 - c_2)^2}$

* Calculate mean

K=2

Exp	POC x_1	speed x_2	SR	P_{c1}	P_{c2}	Cluster	Count
1	0.2	100	5	50.2	2	$C_1 (0.2, 150, 10)$	2
2	0.3	200	7	50.04	100		1
3	0.15	400	8	250	300	$C_2 (0.25, 100, 7)$	1
4	0.21	300	7	150.03	200		1
5	0.17	200	6	50.01	100		1
6	0.25	100	7	50.04	0		2
7	0.31	200	10	50	100.05		1
8	0.27	300	9	150	200.01		1
9	0.2	150	10	0	500		1

PC1

$$d = \sqrt{(e_1 - 0.2)^2 + (e_1 - 150)^2 + (e_1 - 10)^2} = \sqrt{(0.2 - 0.2)^2 + (100 - 150)^2 + (5 - 10)^2} = 50.2$$

$$PC2 = \sqrt{(e_1 - 0.25)^2 + (e_2 - 100)^2 + (e_3 - 7)^2}$$

$$C_1 = \frac{\sum C_1}{7}, \quad C_2 = \frac{(0.2, 100, 5) + (0.25, 100, 7)}{2} = (0.225, 100, 6)$$

$$C_1 = (0.23, 250, 8.14)$$

finally: $C_1 (0.235, 266.7, 7.8)$

$C_2 (0.216, 116.7, 7.3)$

linguistic

Med, high, high

Med, low high

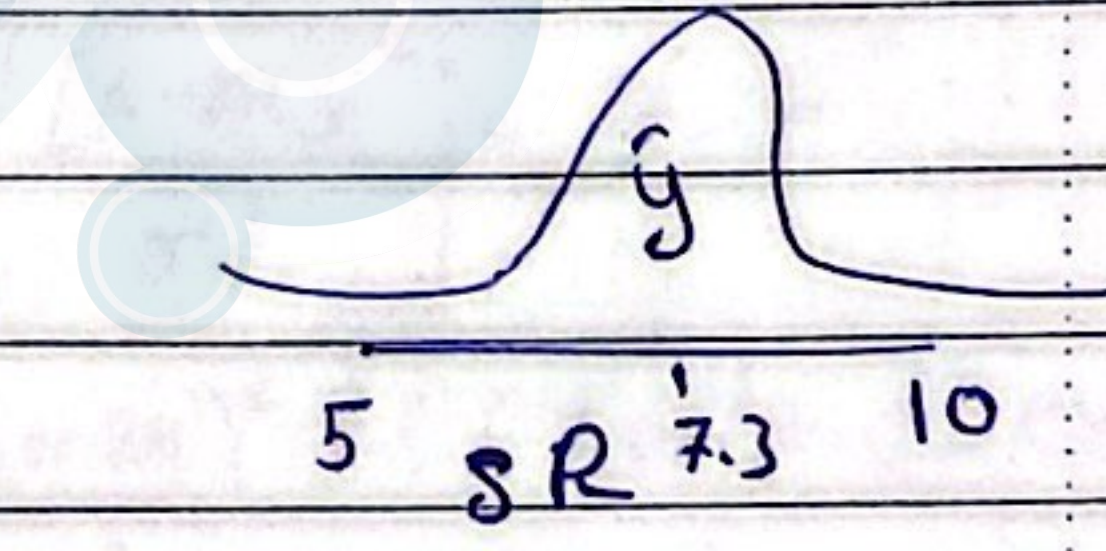
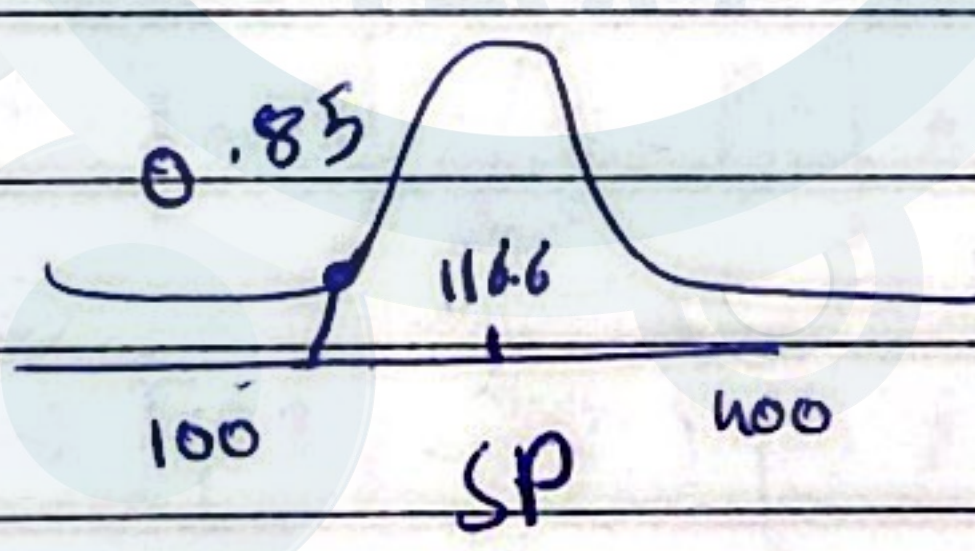
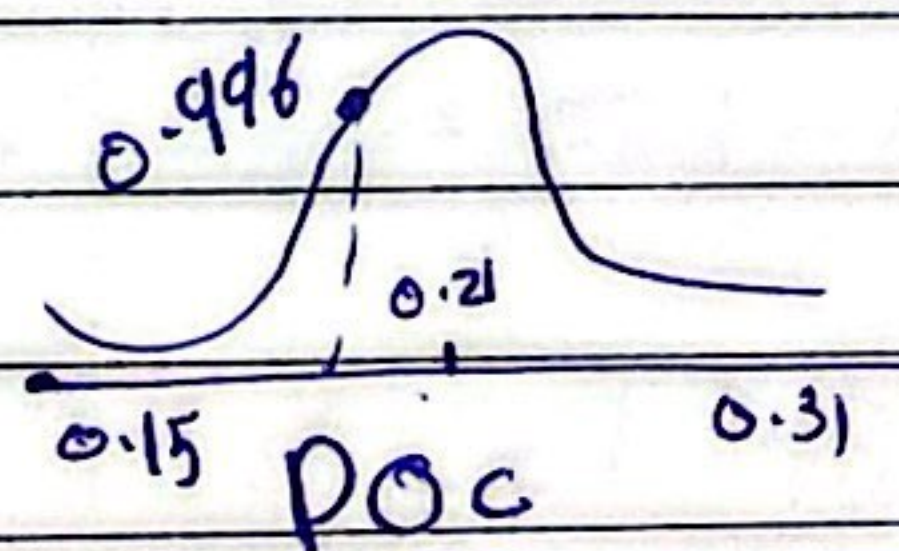
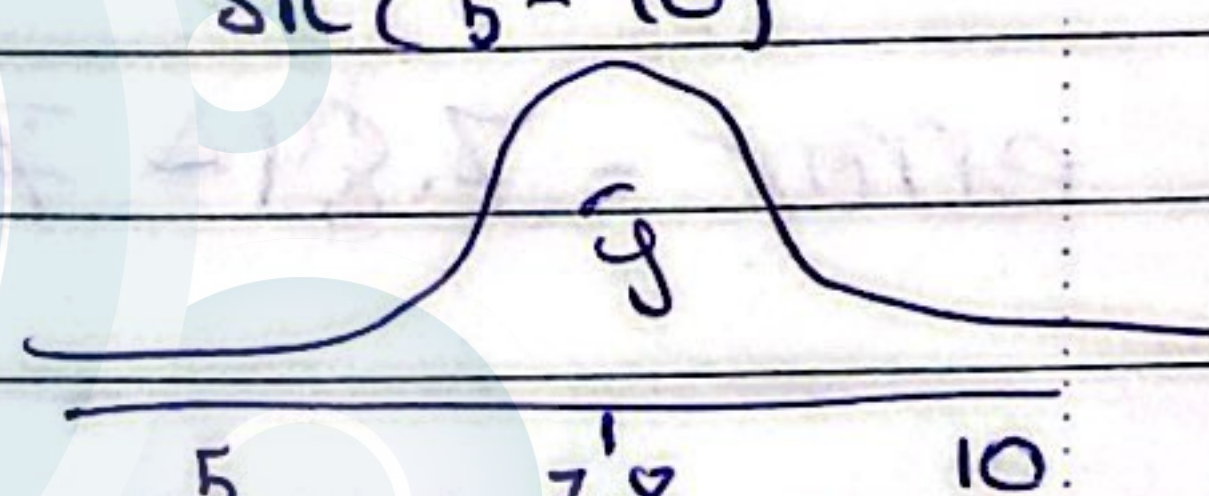
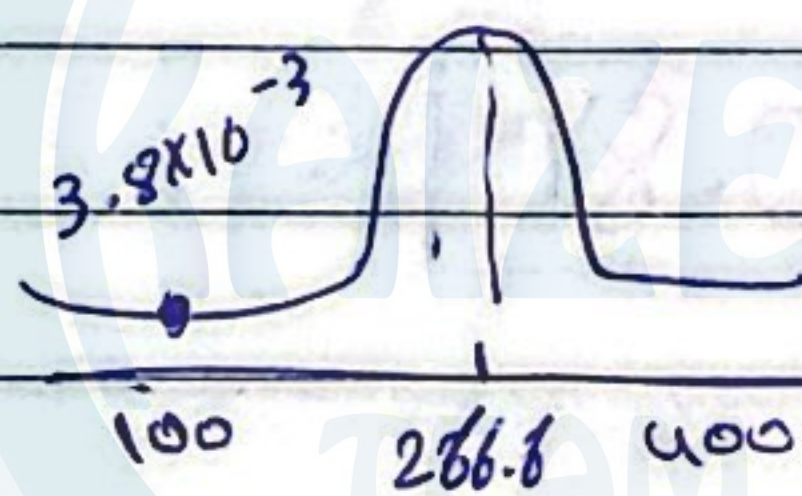
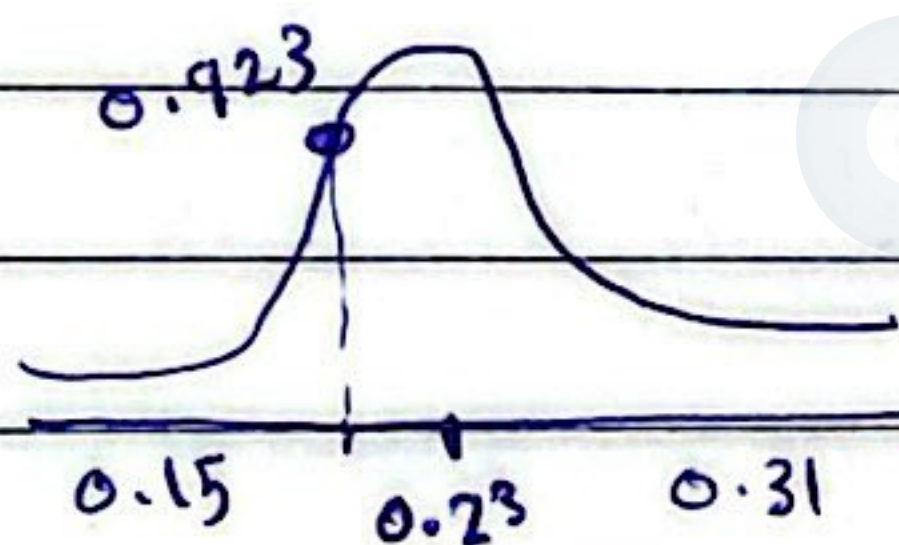
* Calculate SD : $\sigma_1 (0.1, 50, 2)$ POC (0.15 - 0.31)

3 time

$\sigma_2 (0.12, 30, 1.5)$

SP (100 - 400)

SR (5 - 10)



for exp 1: (0.2, 100, 5)

calculate for 2 rules

[1]

$$\mu_1 = e^{-0.5 \left(\frac{0.2 - 0.235}{0.1} \right)^2} = 0.923$$

$$\mu_2 = e^{-0.5 \left(\frac{100 - 266.7}{50} \right)^2} = 3.8 \times 10^{-3}$$

[2]

$$\mu = e^{-0.5 \left(\frac{0.2 - 0.21}{0.12} \right)^2} = 0.996$$

$$\mu = e^{-0.5 \left(\frac{100 - 116.6}{30} \right)^2} = 0.85$$

$$[3] FL_1 = 0.923 \times 3.8 \times 10^{-3} = 3.5 \times 10^{-3} \quad [4] W_A = \frac{7.3 \times 0.85 + 3.5 \times 10^{-3} \times 7.8}{7.3 + 7.8}$$

$$FL_2 = 0.996 \times 0.85 = 0.85$$

$$Pr = 7.3$$

$$\rightarrow error_1 = 7.3 - 5 = 2.3$$

Five Apple

$$C_1(0.24, 266.6, 7.8)$$

$$C_2(0.21, 116.6, 7.3)$$

forward

another one

$$\exp^2(0.3, 200, 7)$$

$$POC_1 = e^{-0.5 \left(\frac{0.3 - 0.24}{0.1} \right)^2} = 0.835$$

$$\sigma_1 = (0.2, 50, 2)$$

$$SP_1 = e^{-0.5 \left(\frac{200 - 266.6}{50} \right)^2} = 0.411$$

$$\sigma_2(0.12, 30, 1.5)$$

$$POC_2 = e^{-0.5 \left(\frac{0.21 - 0.3}{0.12} \right)^2} = 0.754$$

$$SP_2 = e^{-0.5 \left(\frac{200 - 116.6}{30} \right)^2} = 0.0209$$

$$FL_1 = 0.835 \times 0.411 = 0.34$$

$$FL_2 = 0.754 \times 0.0209 = 0.0157$$

$$dO = \frac{7.8 \times 0.34 + 7.3 \times 0.0157}{7.3 + 7.8} = (7.81)$$

$$\text{error} = 7.81 - 7 = \boxed{0.81}$$

one pass method

Backward

$$* \text{error} = \frac{1}{2} [\text{pred} - \text{out}]^2$$

$$M = \exp\left(-0.5 \frac{(x - \mu)^2}{\sigma^2}\right)$$

انتبه للتغير

$$\kappa = 0.3$$

(for input)

$$\checkmark \mu_{\text{new}} = \mu_{\text{old}} - \alpha [Pr - y] [\bar{y} - Pr] \left[\frac{x - \mu_{\text{old}}}{\sigma^2} \right] * \phi \rightarrow \boxed{FL}$$

$$\mu_{\text{new}_1} = 0.24 - 0.3 [7.3 - 5] [7.8 - 7.3] * \frac{(0.2 - 0.24)}{0.1^2} * 3.5 \times 10^{-3} = 0.244$$

$$\mu_{\text{new}_2} = 266.6 - 0.3 [7.3 - 5] [7.8 - 7.3] * \frac{100 - 266.6}{50^2} * 3.5 \times 10^{-3} = 266.7$$

$$\checkmark \bar{y}_{\text{new}} = \bar{y}_{\text{old}} - \alpha [Pr - y_{\text{act}}] \phi$$

$$y_1 = 7.8 - 0.3 [7.3 - 5] 3.5 \times 10^{-3} = 7.79$$

$$y_2 = 7.3 - 0.3 [7.3 - 5] 3.5 \times 10^{-3} = 7.29$$

$$\checkmark \sigma_{\text{new}} = \sigma_{\text{old}} - \alpha [Pr - \bar{y}_{\text{act}}] * [\bar{y} - Pr] * \left[\frac{[x - \mu]^2}{\sigma^3} \right] * \phi$$

$$= 0.1 - 0.3 (7.3 - 5) (7.8 - 7.3) \left(\frac{(0.2 - 0.24)^2}{0.1^3} \right) * 3.5 \times 10^{-3} = 0.098$$

$$= 50 - 0.3 (7.3 - 5) (7.8 - 7.3) \left(\frac{(100 - 266.6)^2}{50^3} \right) * 3.5 \times 10^{-3} = 49.9997$$

Pr = Weight

y_{act} = 3 dimension

y = SR

▼ Decision Making with Fuzzy Information.

✓ Decision → Binary

* Marks not clear cut.

✦ There is difference between a good decision and good outcomes.

* decision → Operational: near to you.

↳ Strategic: prepare

① Fuzzy Synthetic Evaluation: Numerical evaluation
مثال: (تقييم الحاضرة)

حساب بعد بأخر membership degree

* $WOR = \left[\frac{0.1}{e}, \frac{0.3}{s}, \frac{0.4}{a}, \frac{0.2}{c} \right]$ هذا التقييم ✓

② Fuzzy Ordering :-

✓ Ordering

تقييم المشاريع بشكل عام
مراتب تقييم بناء على أستاذ واحد
* 2 groups

③ lightly hood.

possible
① $T(I_1 \geq I_2) = 0.8$ ✓
[max-min]

← غفلة من حقيقة

$T(I_2 \geq I_1) = \max(1, 1) = 1$

$(I_1 \geq I_3) = \max(0.8, 0.8, 0.8) = 0.8$

$(I_3 \geq I_1) = \max(1, 0.5, 0.5) = 1$

$I_1 = \frac{1}{3}, \frac{0.8}{7}$

$I_2 = \frac{0.7}{4}, \frac{1}{6}$

$I_3 = \frac{0.8}{2}, \frac{1}{4}, \frac{0.5}{8}$

(min)
 $I_1 > I_2, I_3 = 0.8$

$I_2 > I_1, I_3 = 1$

$I_3 > I_2, I_1 = 0.7$

(max) ترتيب
 I_2, I_1, I_3

$(I_2 \geq I_3) = \max(0.7, 0.7, 0.8, 1) = 1$

$(I_3 \geq I_2) = (0.7, 0.5, 0.5) = 0.7$
Five Apple

* Nontransitive Ranking :

* مثال تقييم المسابقة

pairwise function

* برتب المسابقات

row column
 $f(x|y) = f_y(x)$

* نغير unverse

$$\max [f_y(x), f_x(y)]$$

$f(x|y)$ x في y

$$f(x_1|x_2) = f_{x_2}(x_1) = \frac{0.7}{\max(0.7, 0.5)} = 1$$

$$\max [f_{x_2}(x_1), f_{x_1}(x_2)]$$

بالنسبة

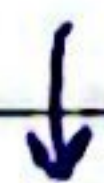
preference

$$f(x_1|x_3) = \frac{f_{x_3}(x_1)}{f_{x_1}(x_3)} = \frac{0.5}{0.5/0.3} = 1$$

$$C = \begin{matrix} & x_1 & x_2 & x_3 & x_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.71 & 1 & 0.375 & 0.11 \\ 0.6 & 1 & 1 & 0.43 \\ 0.67 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$

$$f(x_1|x_4) = \frac{0.3}{\max(0.3, 0.2)} = 1$$

$$C = x_1, x_4, x_3, x_2$$



take the min in the rows

1, 0.11, 0.43, 0.67

بالترتيب 1, 0.67, 0.43, 0.11

$$f(x_2|x_1) = \frac{12}{2.1} = \frac{0.5}{0.7} = \frac{0.5}{0.7}$$

$$f(x_2|x_3) = \frac{3.2}{3.2, 2.3} = \frac{0.3}{0.8}$$

$$f(x_2|x_4) = \frac{0.1}{\max(0.1, 0.9)} = 0.11$$

* Preference and Consensus
 (chq) (18-25)

(self reading)

Type 1

reciprocal relation

علاقة متبادلة

tr trace $\rightarrow \sum_{i=1}^n r_{ii}$ (الوتر)
 τ transpose

reciprocal relation : $r_{ii} = 0 \quad 1 \leq i \leq n$

$r_{ij} + r_{ji} = 1 \quad i \neq j$

$r_{ij} \rightarrow$ preference x_i relative to x_j

$F(R) = \frac{\text{tr}(R^2)}{n(n-1)/2}$ average fuzziness

$F(R) + C(R) = 1$

$C(R) = \frac{\text{tr}(RR^T)}{n(n-1)/2}$ average certainty

$I = \frac{1 - n(C)}{1 - n}$

distance $m(R) = 1 - \sqrt{2C(R) - 1}$

Type 1 الحفاضة, التقييم
 ↓
 Type 2 الحفاضة

Consensus

cardinality

Type I : clear alternative

(n) 2

* أكثر عاودته أصغار
 هو الأكثر تفضيل

Type II : one clear choice
 other level unknown

$(2^{(n^2-3n+2)/2})$
 (n)

Column 2

Type III Fuzzy: تنوع سهل اختيار

(∞)

التفصيل لكن الباقي

you can find final answer

(فوضي)
 (مفوض)

fuzzy

الاختيار بينهم

2, 1, 4, 3

all most the same

[row التفصيل]

$I = II$

when $n=2$

Movie (A, B, C, D) I : Movie (A, B, C, D) clear

الباقى
 تساهي

II : selecting city clear

الباقى بينهم تبين

الباقى
 لا ترتيب

Fuzziness: one choose, other

ميكلة القوي

Five Apple

الباقى
 ليس ترتيب
 احدهم

(اتفاق جزئي, وعدم يقين)

0.5, 0.8, 0.4
c s w

Alternative بديل A Laptop
Objc أهداف O - حرة

[الشراء لا يتقرب]

* Multi-Objective Decision Making

✓ μ تعكس [رغبتك أو لهذا الشيء] أهمية بالنسبة لي

$$D = O_1 \cap O_2 \quad (\text{Decision Function})$$

$$D = \bigcap_{i=1}^n (\bar{b} \cup O_i) \quad (\text{max-min}) \quad (2) \quad (1)$$

Example: 3 wall

4 Obj $\rightarrow$ O

b

Objective

0.2	6.8	$O_1 = \left[\frac{0.4}{MSE}, \frac{1}{con}, \frac{6.1}{Gab} \right]$
0.1	0.9	$O_2 = \left[\frac{0.7}{MSE}, \frac{0.8}{con}, \frac{0.4}{Gab} \right]$
0.3	0.7	$O_3 = \left[\frac{0.2}{MSE}, \frac{0.4}{con}, \frac{1}{Gab} \right]$
0.5	0.5	$O_4 = \left[\frac{1}{MSE}, \frac{0.5}{con}, \frac{0.5}{Gab} \right]$

① $\bar{b} \cup O_1 = 0.4$
 ② $\bar{b} \cup O_2 = 0.7$
 $\bar{b} \cup O_3 = 0.3$
 $\bar{b} \cup O_4 = 1$

$\Rightarrow \min \Rightarrow 0.3$
 First alternative

1	0.2
6.8	0.4
0.4	1 $\Rightarrow 0.2$
0.5	0.5

③ max = 0.4

Second Scenarios :-

slide 35

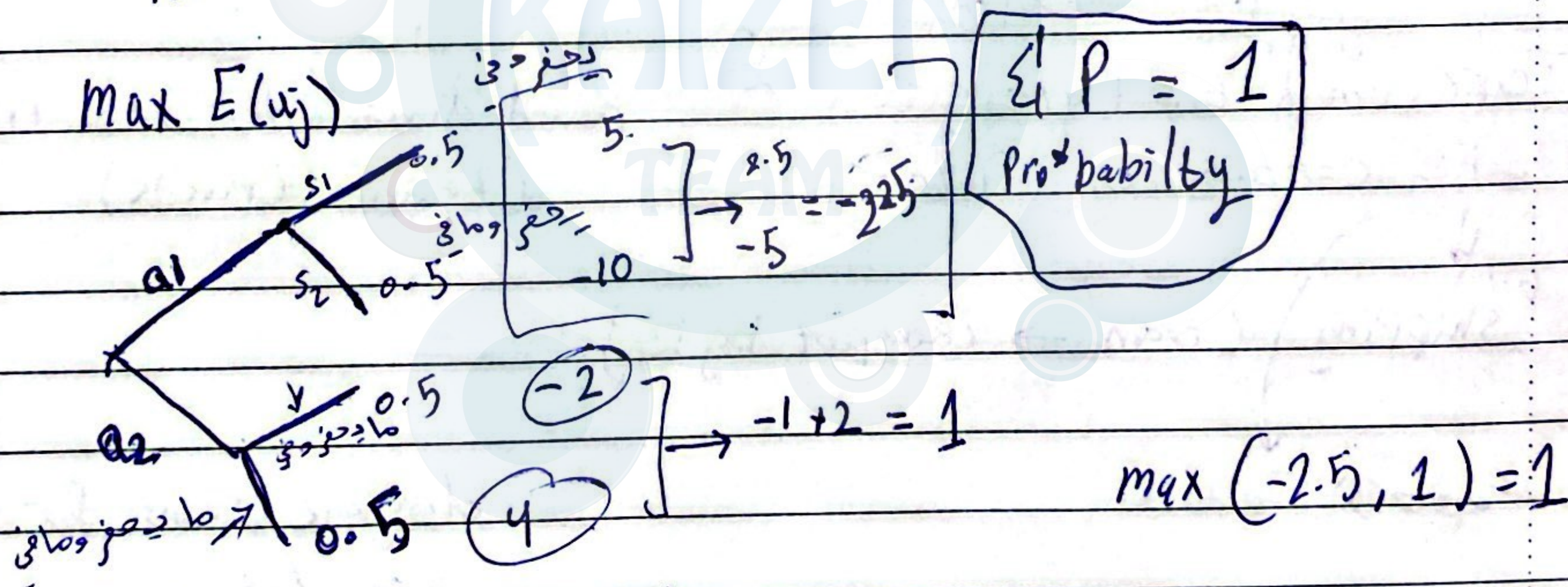
b	G_{ab}	U	min
0.5	0.1	0.5	
0.3	0.4	0.4	0.4
0.2	1	1	
0.3	0.5	0.5	

Tie breaking: $\otimes$ (min) $\otimes$ (x)

* Fuzzy Bayesian Decision Method:

$$E = \sum_{i=1}^n U_{ji} P(S)$$

utility value U_{ji}



finally = not drill.

Slide 43