* الهدف الأساسي من استخدام laplace حتى اسهل علي حل المعادلات

لهلأ انا حولها من Time domain (t) إلى ال Frequency (s) .

Time Function	Laplace
$x(t)$	$X(s)$
$x'(t) = \dot{x}(t)$	$sX(s) - x(0)$
$\ddot{x}(t) = x''(t)$	$s^2X(s) - sx(0) - \dot{x}(0)$

مهم

* حتى نقدر نجيب $x(0), \dot{x}(0)$ لازم امسك المعادلة الأصلية واعوّض فيها (0).

* انه دائماً ناقص.

Example ①:- $\dot{x}(t) + 2x(t) = r(t)$

$x(0) = \text{Zero}$

$\dot{x}(0) = \text{Zero}$

$\ddot{x}(0) = \text{Zero}$

$$[sX(s) - \underbrace{x(0)}_{\text{zero}}] + 2X(s) = R(s)$$

$$X(s)[s+2] = R(s)$$

$$\frac{\text{output} \leftarrow X(s)}{\text{input} \leftarrow R(s)} = \frac{1}{(s+2)}$$

Example ②:- $\dddot{x}(t) + 3\ddot{x}(t) + 7\dot{x}(t) + 5x(t) = \frac{d^2}{dt} r(t) + 4\frac{dr(t)}{d(t)} + 3r(t)$

$$[s^3X(s) - \cancel{s^2x(0)} - \cancel{s\dot{x}(0)} - \cancel{\ddot{x}(0)}] + 3[s^2X(s) - \cancel{sx(0)} - \cancel{x(0)}] + 7[sX(s) - \cancel{x(0)}] +$$

$$[5X(s)] = s^2R(s) + 4sR(s) + 3R(s)$$

$$s^3X(s) + 3s^2X(s) + 7sX(s) + 5X(s) = R(s)[s^2+4s+3]$$

$$X(s)[s^3 + 3s^2 + 7s + 5] = R(s)[s^2+4s+3]$$

$$\frac{\text{output} \leftarrow X(s)}{\text{input} \leftarrow R(s)} = \frac{s^2+4s+3}{s^3+3s^2+7s+5}$$

Example ③ mid :- $\ddot{X} + 2X = t$ Assume that $X(0) = 1$ $\dot{X}(0) = \text{Zero}$

$$[S^2X(S) - S\underbrace{X(0)}_{1} - \underbrace{\cancel{\dot{X}(0)}}_{\text{Zero}}] + 2X(S) = \frac{1}{S^2}$$

$$S^2X(S) - S + 2X(S) = \frac{1}{S^2}$$

$$X(S)[S^2+2] = \frac{1}{S^2} + S$$

$$\frac{X(S)[S^2+2]}{[S^2+2)} = \frac{1+S^3}{S^2 * (S^2+2)}$$

$$X(S) = \frac{(1+S^3)}{S^4+2S^2} \quad \text{or} \quad X(S) = \frac{1}{S^2(S^2+2)} + \frac{S}{S^2+2}$$

Example ④ slides :- $\ddot{X} + 3\dot{X} + 2X = 5\sin t$ Assume $X(0) = 1$, $\dot{X}(0) = \text{Zero}$

$$[S^2X(S) - S\underbrace{\cancel{X(0)}}_{1} - \cancel{\dot{X}(0)}] + 3[SX(S) - \underbrace{\cancel{X(0)}}_{1}] + 2X(S) = \frac{5}{S^2+1}$$

$$S^2X(S) - S + 3SX(S) - 3 + 2X(S) = \frac{S}{S^2+1}$$

$$X(S)[S^2+3S+2] \boxed{-S-3} = \frac{S}{S^2+1}$$

$$\frac{X(S)\,\cancel{[S^2+3S+2]}}{\cancel{S^2+3S+2}} = \frac{\frac{S}{S^2+1} + (S+3)}{S^2+3S+2}$$

$$X(S) = \frac{S}{(S^2+1)(S^2+3S+2)} + \frac{(S+3)}{S^2+3S+2}$$

* الخطوة الثانية بعد تحويل المعادلات على صيغة X(S) هي استخدام جداول الـ Laplace

في أغلب الحالات نستخدم طريقة partial Fraction في الحل مع الانتباه إلى أقسام [Roots] الجذر

[1] Real and Distinct Root. [2] Roots are Real and Repeated.

[3] Complex Root.

	$f(t)$	$F(s)$
impact Function or delta Function ←	$\delta(t)$	1
Step Function ←	$u(t)$	$\frac{1}{s}$
	e^{-at}	$\frac{1}{s+a}$
	$t e^{-at}$	$\frac{1}{(s+a)^2}$
	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
	t	$\frac{1}{s^2}$
	t^2	$\frac{n!}{s^{n+1}}$
	$\sin(wt)$	$\frac{w}{s^2+w^2}$
	$\cos(wt)$	$\frac{s}{s^2+w^2}$
	$e^{-at}\sin(wt)$	$\frac{w}{(s+a)^2+w^2}$
	$e^{-at}\cos(wt)$	$\frac{s+a}{(s+a)^2+w^2}$

1 Distinct and Real Root

جذور المقام مختلفة وحقيقية .

Example ①:- $F(s) = \dfrac{2}{(s+1)(s+2)} \longrightarrow \dfrac{A}{(s+1)} + \dfrac{B}{(s+2)}$

$2 = A(s+2) + B(s+1)$ توحيد مقامات

* انه المقام من الدرجة الاولى كل جذر لوحده من الدرجة الاولى اذن البسط لازم يكون بدرجة واحدة اقل من المقام عند الدرجة الصفرية .

$A = \dfrac{2}{(s+2)}\Big|_{s=-1} \longrightarrow 2$

$B = \dfrac{2}{(s+1)}\Big|_{s=-2} \longrightarrow -2$

$F(s) = \dfrac{2}{(s+1)} - \dfrac{2}{(s+2)}$

$F(t) = 2e^{-t} - 2e^{-2t}$ → من الجداول

* لما انك حولت من s الى t تسميه لهاي الحالة inverse of Laplace.

Example ②:- $F(s) = \dfrac{s^2+12s+44}{(s+2)(s+4)(s+6)}$

* انه ٣ اقواس في المقام درجته ٣ ككل ودرجة البسط ٢ اقل منه بدرجة ✓

$= \dfrac{A}{(s+2)} + \dfrac{B}{(s+4)} + \dfrac{C}{(s+6)} \longrightarrow$

* درجة كل قوس لحاله (١) ودرجة البسط (٥) ✓ ✓

$s^2+12s+44 = A(s+4)(s+6) + B(s+2)(s+6) + C(s+2)(s+4)$

$A = \dfrac{s^2+12s+44}{(s+4)(s+6)}\Big|_{s=-2} = 3$ $C = \dfrac{s^2+12s+44}{(s+2)(s+4)}\Big|_{s=-6} = -1$

$B = \dfrac{s^2+12s+44}{(s+2)(s+6)}\Big|_{s=-4} = -3$

$= \dfrac{2}{(s+2)} + \dfrac{5}{(s+4)} - \dfrac{8}{(s+6)}$

$f(t) = 2e^{-2t} + 5e^{-4t} - 8e^{-6t}$

example ③ :- $F(s) = \dfrac{32}{s^3+12s^2+32s}$

① درجة البسط اقل من درجة المقام ✓

② القانون العام لتحليل المقام

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$F(s) = \frac{32}{s(s^2+12s+32)}$$

$$s = \frac{-12 \pm \sqrt{12^2 - 4\times1\times32}}{2\times1}$$

$$F(s) = \frac{32}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+8)}$$

$s_1 = -4$

$s_2 = -8$

Roots : $(s+4)(s+8)$

$$32 = A(s+4)(s+8) + Bs(s+8) + Cs(s+4)$$

$$A = \left.\frac{32}{(s+4)(s+8)}\right|_{s=0} = 1 \qquad C = \left.\frac{32}{s(s+4)}\right|_{s=-8} = 1$$

$$B = \left.\frac{32}{s(s+8)}\right|_{s=-4} = -2$$

$$F(s) = \frac{1}{s} - \frac{2}{(s+4)} + \frac{1}{(s+8)}$$

$$f(t) = \underbrace{u(t)}_{1} - 2e^{-4t} + e^{-8t}$$

$\boxed{2}$ Roots are Real and Repeated :-

example ① :- $F(s) = \dfrac{2}{(s+1)\underbrace{(s+2)^2}_{\text{Repeated}}}$

Real and Repeated $\rightarrow s^2+4s+4 \rightarrow (s+2)$ *

$$F(s) = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$2 = A(s+2)^2 + B(s+1)(s+2) + C*(s+1) \rightarrow$$

دائماً قارن بالأصل

$\boxed{x=-2}$ $\quad 2 = -C \quad \boxed{C=-2}$

$\boxed{x=0}$ $\quad 2 = 4A + 2B - 2$

$\boxed{x=-1}$ $\quad 2 = A - 2 \longrightarrow \boxed{A=4}$

$$\frac{4}{(s+1)} - \frac{6}{(s+2)} - \frac{2}{(s+2)^2}$$

$$= 4e^{-t} - 6e^{-2t} - 2te^{-2t}$$

example ②:- $F(s) = \dfrac{2}{s^3(s+1)}$ Repeated التكرار (s^3)

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1}$$

$$2 = A(s+1)s^2 + B(s+1)s + C(s+1) + Ds^3$$

Assume that $\boxed{s=0}$ $\boxed{s=-1}$ $\boxed{s=1}$ $\boxed{s=2}$

$2 = C(1) \longrightarrow \boxed{C=2}$ ..... ①

$\boxed{-2 = D}$ ..... ②

$\cancel{2} = 2A + 2B + \cancel{2(2)} + \cancel{-2(1)} \longrightarrow 4 = 2A + 2B$ ..... ③

$2 = 4\times3A + 6B + 6 + 8\times-2$

$\cancel{2} = 12A + 6B \cancel{-10} \longrightarrow 12 = 12A + 6B$ ..... ④

$\boxed{A=4}$ $\boxed{B=-6}$

$$F(s) = \frac{4}{s} - \frac{6}{s^2} + \frac{2}{s^3} - \frac{2}{s+1} \longrightarrow f(t) = 4u(t) - 6t + t^2 - 2e^{-t}$$

---

example ③:- $F(s) = \dfrac{2}{s(s+1)^2(s+4)}$ $(s+1)^2$ Repeated.

$$\frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} + \frac{D}{(s+4)} \rightarrow 2 = A(s+1)^2(s+4) + B(s+1)(s+4)s + Cs(s+4) + Ds(s+1)^2$$

Assume that $\boxed{s=-1} \longrightarrow 2 = C(-1)(3) \longrightarrow C = \frac{-2}{3}$

$\boxed{s=0} \longrightarrow 2 = A(4) \longrightarrow A = \frac{1}{2}$

$\boxed{s=1} \longrightarrow 2 = 10 + 10B + \frac{-10}{3} + 4D \longrightarrow \frac{-14}{3} = 10B + 4D$

$\boxed{s=2} \longrightarrow 2 = 27 + 36B + -8 + 18D \longrightarrow -17 = 36B + 18D$

$B = \frac{-4}{9}$, $D = \frac{-1}{18}$

So :- $\dfrac{1}{2s} - \dfrac{4}{9(s+1)} - \dfrac{2}{3(s+1)^2} - \dfrac{1}{18(s+4)}$

$$f(t) = \frac{1}{2}u(t) - \frac{4}{9}e^{-t} - \frac{2}{3}te^{-t} - \frac{1}{18}e^{-4t}$$

example ④ Slides :- $y(s) = \frac{8(s+1)}{(s+2)^2}$ Repeated

$\frac{A}{(s+2)} + \frac{B}{(s+2)^2} \longleftrightarrow 8(s+1) = A(s+2) + B \longrightarrow$ لتقارب بالأصل

Assume $\boxed{s=-1} \longrightarrow 0 = A - 8$ $\boxed{A=8}$

Assume $\boxed{s=-2} \longrightarrow \boxed{-8 = B}$

So $\frac{8}{(s+2)} - \frac{8}{(s+2)^2} \longrightarrow y(t) = 8e^{-2t} - 8te^{-2t}$

example ⑤ slides :- $X(s) = \frac{8(s+3)(s+8)}{s(s+2)(s+4)}$ Real and Non-Repeated

$\frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+4)} \longrightarrow 8(s+3)(s+8) = A(s+2)(s+4) + B(s)(s+4) + C(s+2)(s)$

Assume $\boxed{s=0} \longrightarrow 192 = 8A$ $\boxed{A=24}$

Assume $\boxed{s=-4} \longrightarrow -32 = 8C$ $\boxed{C=-4}$

Assume $\boxed{s=-2} \longrightarrow 48 = -4B$ $\boxed{B=-12}$

So :- $\frac{24}{s} - \frac{12}{(s+2)} - \frac{4}{(s+4)} \longrightarrow X(t) = 24u(t) - 12e^{-2t} - 4e^{-4t}$

★ Transformation from Time domain to the frequency domain by Laplace and the inverse Laplace.

$\boxed{1}$ $f(t) = 5$

$F(s) = \frac{5}{s}$

$\boxed{2}$ $f(t) = 2e^{-at}$

$F(s) = \frac{2}{(s+a)}$

$\boxed{3}$ $F(s) = \frac{1}{(s+3)^2}$

$f(t) = te^{-3t}$

$\boxed{4}$ $F(s) = \frac{2}{s^2+5} \longrightarrow$ بيها شكل نتذكر من الجداول

$$= \frac{2 * \sqrt{5}}{(s^2 + (\sqrt{5})^2) * \sqrt{5}}$$

$$\frac{\omega}{s^2+\omega^2} = \sin(\omega t)$$

$$= \frac{2}{\sqrt{5}} * \frac{\sqrt{5}}{s^2 + (\sqrt{5})^2}$$

$$\frac{2}{\sqrt{5}} \sin(\sqrt{5}t)$$

$\boxed{5}$ $F(s) = \frac{s-3}{s^2+5}$ وزع

$$\frac{s}{s^2+5} - \frac{3}{s^2+5}$$

$$\downarrow \qquad \downarrow$$

$$\frac{s}{s^2+(\sqrt{5})^2} \ominus \frac{3}{\sqrt{5}} * \frac{\sqrt{5}}{s^2+(\sqrt{5})^2}$$

$\downarrow$ نتذكر $\cos(\omega t)$ $\qquad$ $\downarrow$ نتذكر $\sin(\omega t)$

$$\cos(\sqrt{5}t) - \frac{3}{\sqrt{5}} \sin(\sqrt{5}t)$$

* احياناً نحتاج إلى طريقة اكمال المربع لمّا المقام يكون لا تحلل بالطريقة التقليدية

example ①:- S^2+2S+5 $\longrightarrow$ $S_{1,2} = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$S_1 = -1 + j$ Complex !!

$S_2 = -1 - j$ Complex !!

لاكمال المربع ① اجباري معامل $S^2 = 1$ ✓

$a^2 = 1$

$2ab = 2 * 1 * b = 2 \longrightarrow \boxed{b=1}$

$b^2 = 1$

② بعد ايجاد قيمة b^2 نجمعها ونطرحها من المعادلة الاصلية $\longrightarrow$ $S^2+2S+b^2-b^2+5$

$[S^2+2S+1]-1+5$

$\boxed{(S+1)^2+4}$ ✓

example ②:- S^2+2S+4

$a^2 = 1$

$2ab = 2 * a * b = 2$

$\boxed{b^2 = 1}$

$S^2+2S+1-1+4$

$\boxed{(S+1)^2+3}$ ✓

---

3 Complex unrepeated Roots $\longrightarrow$ سوف نلجأ لاكمال المربع

example ①:- slides

$$\frac{5.2}{S^2+2S+5} = \frac{AS+B}{S^2+2S+5}$$

* انتبه المقام تربيعي اذن البسط من الدرجة الاولى

* (S^2+2S+5)

$a^2 = 1$

$2ab = 2 * 1 * b = 2$ $\boxed{b=1}$

$b^2 = 1$

$S^2+2S+1-1+5$

$\boxed{(S+1)^2+4}$

$$\frac{AS}{(S+1)^2+4} + \frac{B}{(S+1)^2+4} = 5.2$$

$\boxed{AS+B = 5.2}$ Assume $S=0$

$\boxed{B = 5.2}$ Assume $S=1$

$\boxed{A = 0}$

$$F(S) = \frac{5.2}{(S+1)^2+4} \longrightarrow \frac{5.2}{2} * \frac{2}{(S+1)^2+(2)^2} \longrightarrow \frac{5.2}{2} e^{-t} \sin 2t$$

example ② slides :- $\frac{S+2}{(S+1)(S+3)^2(S^2+2S+5)} \longrightarrow$

$S_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$S_1 = -1+2j$
$S_2 = -1-2j$ } Complex!!

$$\frac{A}{(S+1)} + \frac{E}{(S+3)^2} + \frac{B}{(S+3)} + \frac{CS+D}{(S^2+2S+5)} = S+2$$

$$A(S+3)^2(S^2+2S+5) + B(S+1)(S^2+2S+5) + C(S+3)(S+1)(S^2+2S+5) + [DS+E](S+1)(S+3)^2 = S+2$$

Assume $\boxed{S=-1}$ $8A = 1$ $A = \frac{1}{16}$

Assume $\boxed{S=-3}$ $-4E = -1$ $E = \frac{1}{16}$

Assume $\boxed{S=0}$ $\rightarrow$ $\boxed{\frac{-9}{8} = 15B + 9E}$ ....①

Assume $\boxed{S=1}$ $\boxed{15B + 9E = \frac{-9}{8}}$ ......①

$\boxed{84B + 32D + 32E = -7}$ .....②

Assume $\boxed{S=2}$ $\rightarrow$

$\boxed{11 - 4B + 2D - E = 0}$ ....③

$B = \frac{-1}{48}$

$D = \frac{-25}{288}$

$C = \frac{1}{16}$

$A = \frac{1}{16}$ $E = \frac{-13}{144}$

$$F(S) = \frac{1}{16(S+1)} + \frac{-1}{48(S+3)^2} - \frac{117}{16(S+3)} + \frac{\frac{-25}{288}}{(S+1)^2+4} + \frac{-13}{144}\,\frac{1}{(S+1)^2+4}$$

$$f(t) = \frac{1}{16}e^{-t} + \frac{-1}{48}te^{-3t} - \frac{117}{16}e^{-3t} + \left[\frac{-25}{288}*\left(\frac{S+1-1}{(S+1)^2+4}\right)\right] + \frac{-13}{144*2}\,\frac{2}{(S+1)^2+(2)^2}$$

$$\frac{127}{960} * \frac{(S+1)}{(S+1)^2+(2)^2} - \frac{127}{960*2} * \frac{(2)}{(S+1)^2+(2)^2}$$

$\downarrow$

$$\boxed{\frac{127}{960} * e^{-t}\cos 2t - \frac{127}{960*2}e^{-t}\sin(2t)}$$

$$\boxed{\frac{13}{192 \times 2}e^{-t}\sin(2t)}$$

$$f(t) = \frac{e^{-t}}{8} + \frac{te^{-3t}}{4} - \frac{117e^{-3t}}{320} + \frac{127}{960}e^{-t}\cos(2t) - \frac{127}{1920}e^{-t}\sin(2t) + \frac{13}{384}e^{-t}\sin(2t)$$

initial value theorem	Final value theorem
$\lim\limits_{S\to\infty} S*F(S) =$	$\lim\limits_{S\to 0} S*F(S) =$
$\lim\limits_{t\to 0} f(t) = f(0)$	$\lim\limits_{t\to\infty} f(t) = f(\infty)$

Example ①:- $F(S) = \dfrac{(S+2)}{(S+1)^2+5^2}$ find initial value:-

Slides

$$\lim_{S\to\infty} S*F(S) \longrightarrow \lim_{S\to\infty} \frac{S(S+2)}{(S+1)^2+5^2} \longrightarrow \lim_{S\to\infty} \frac{S^2+2S}{S^2+2S+26}$$

$$\lim_{S\to\infty} \frac{1+\cancel{\frac{2}{\infty}}\,0}{1+\cancel{\frac{2}{\infty}}\,0+\cancel{\frac{26}{\infty}}\,0} = \boxed{1} \longleftarrow$$

* نقسم على اعلى قوة في البسط والمقام (S^2)

$0 = \dfrac{\text{عدد}}{\infty}$

Example ②:- $F(S) = \dfrac{(S+2)^2-3^2}{(S+2)^2+3^2}$ find $f(\infty)$:-

Slides

$$\lim_{S\to 0} \frac{S*[(S+2)^2-9]}{(S+2)^2+9} = 0$$

تعويض مباشر

* We Can use Matlab to Solve Laplace questions:-

example①:- use Matlab to find te^{-4t}.

```
Syms t, s
Laplace(t*exp(-4*t))
ans = 1/(s+4)^2
```

example②:- use Matlab to find inverse $F(s) = \dfrac{s(s+6)}{(s+3)(s^2+6s+18)}$

```
Syms s, t
ilaplace((s*(s+6))/((s+3)*(s^2+6*s+18)))
```

ans:- $\dfrac{s(s+6)}{(s+3)(s^2+6s+18)}$

$$= \frac{A}{(s+3)} + \frac{Bs+c}{s^2+6s+18}$$

$S(S+6) = A(S^2+6S+18) + [Bs+c](s+3)$

Assume $S=-3 \rightarrow -9 = -9A$ $\boxed{A=1}$

$S=0 \rightarrow 0 = 18 + 3c$ $\boxed{c=-6}$

$S=1 \rightarrow 7 = 25 + 4B - 24$

$\boxed{B = \frac{3}{2}}$

$$S_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$S_{1,2} = \frac{-6 \pm \sqrt{36-4(1)(18)}}{2(1)}$$

$S_1 = -3+3j$

$S_2 = -3-3j$] Complex !!

$a^2=1$

$2ab=6$ $\boxed{b=3}$

$\boxed{b^2=9}$

$(s^2+6s+9-9+18)$

$\boxed{(s+3)^2+9}$

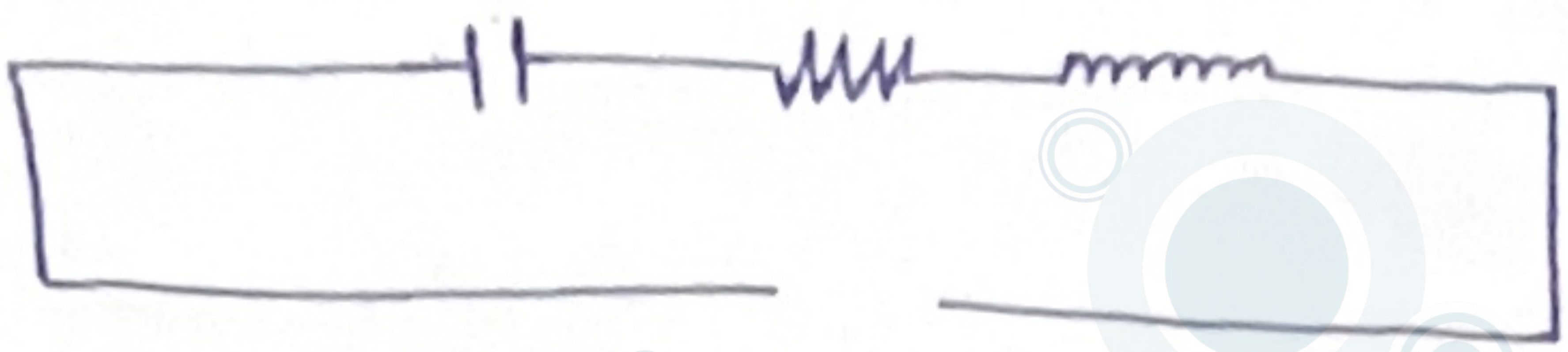
$$F(s) = \frac{1}{s+3} + \frac{3}{2}\frac{s+3}{(s+3)+9} + \frac{-3}{2}\frac{3}{(s+3)+3^2} + \frac{-6}{(s+3)^2+(3)^2}$$

$$f(t) = e^{-3t} + \frac{3}{2}e^{-3t}\cos(3t) - \frac{3}{2}e^{-3t}\sin(3t) - 2e^{-3t}\sin(3t)$$

Physical Systems :- ① Electrical. ② Mechanical.

[1] electrical System → * Resister —ww— R
* Capaciter —| |— C
* Inductor —mm— L

طالما يكون عندي electrical system واحد من هدول الـ Component أو أكثر من واحد.



→ لأنه ما بصير التعامل مع كل واحد لحال لهيك لازم احولهم كلهم بدلالة (Z) ← impedance

R	→ Z_R
$\frac{1}{CS}$	→ Z_C
LS	→ Z_L

* Series $Z_{eq} = Z_1 + Z_2 + Z_3 \ldots\ldots$

* Parallel $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \ldots\ldots$

* يلي تكون مطلوب عادة لهاد النوع من الأسئلة هو (Transver Function)

Transver Function (TF) $= \frac{\text{output}}{\text{input}}$

$V(s) = I(s) * Z$

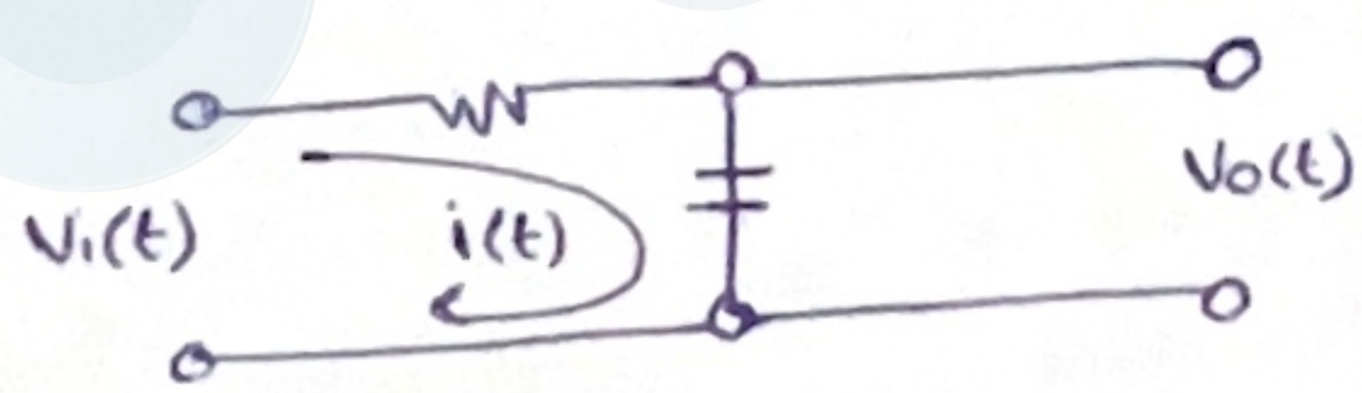
Voutput ← → Vinput

Voutput: لو خذنا العناصر الموصول في قطبي الدارة

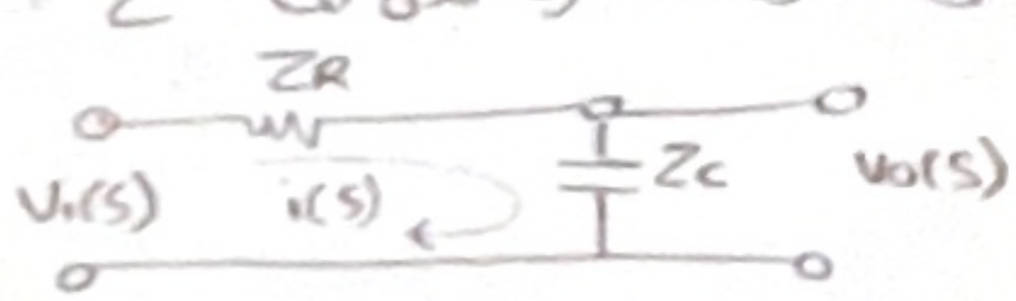
Vinput: لو خذنا كل العناصر يلي بالدارة.

وحدتها C : pF
R : ΩM

Example ① Slides :-



① حول الدارة بدلالة Z



② نسحب قيمة Vo(S) فقط مع المربوط فيها بشكل مباشر

$V_o(s) = I(s)\, Z_C$

③ نسحب قيمة Vi(S) فيها كل الدارة.

$V_i(s) = I(s)\, Z_{eq}$

→ Series So $Z_{eq} = Z_R + Z_C$

$Z_{eq} = R + \frac{1}{CS}$

$Z_{eq} = \frac{RCS+1}{CS}$

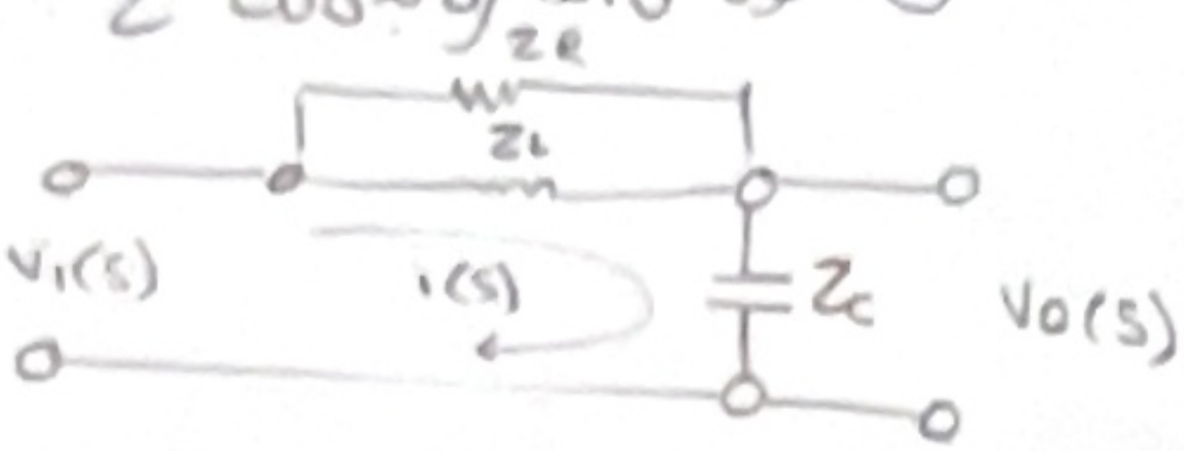
$TF = \frac{\cancel{I(s)} * \frac{1}{\cancel{CS}}}{\frac{\cancel{I(s)}\, RCS+1}{\cancel{CS}}} = \frac{1}{RCS+1}$

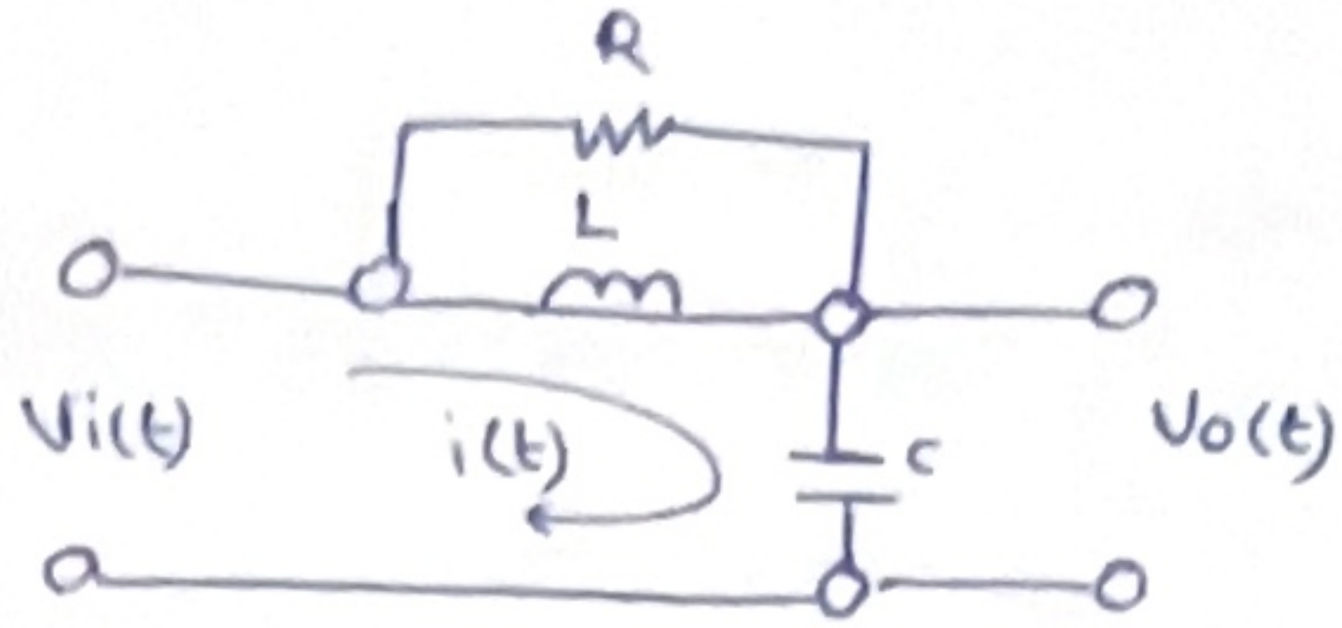
* pole $RCS+1=0$

$\boxed{S = \frac{-1}{RC}}$

example ② slides:-

① حوّل الدارة بدلالة Z





٢ Z_L and Z_R مشتركين مع بعض على التوازي parallel :-

$$\frac{1}{Z_{eq}} = \frac{1}{Z_R} + \frac{1}{Z_L}$$

$$\frac{1}{Z_{eq}} = \frac{1 * LS}{R * LS} + \frac{1 * R}{LS * R}$$

$$\frac{1}{Z_{eq}} = \frac{LS+R}{LSR} \longrightarrow \boxed{Z_{eq} = \frac{LSR}{LS+R}}$$

٣

$$V_{output} = I(s) * Z_C$$

$$V_{output} = \frac{I(s)}{CS}$$

٤ Z_{eq} مشتركة مع Z_C على التوالي Series :-

$$Z_{eq_2} = \frac{CS * (LSR)}{CS * (LS+R)} + \frac{1 * (LS+R)}{CS * (LS+R)}$$

$$\boxed{Z_{eq} = \frac{CLRS^2 + LS + R}{CLS^2 + CSR}}$$

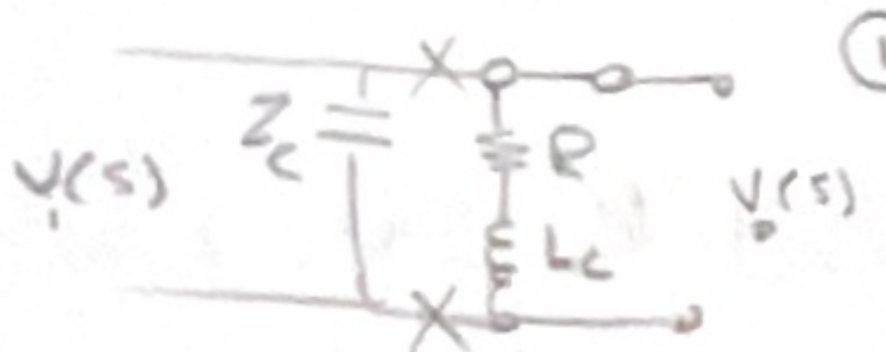
٥

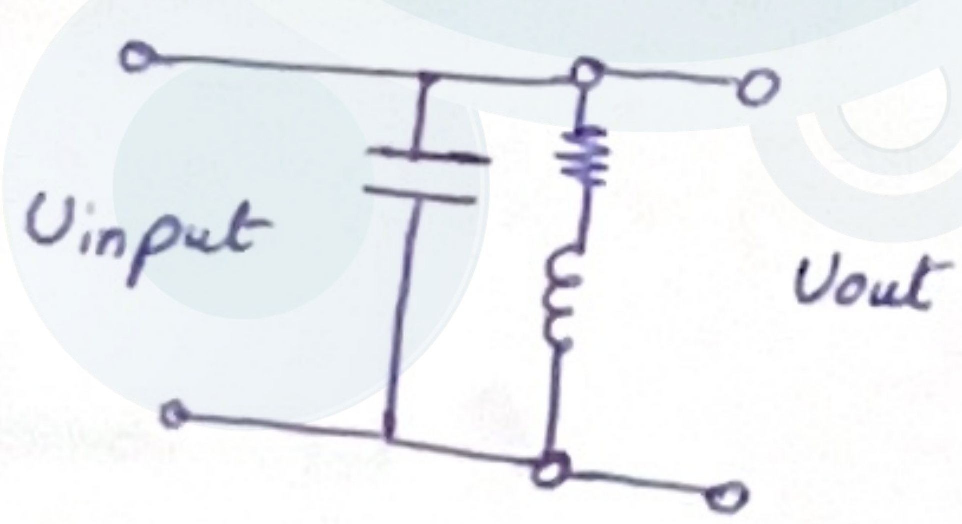
$$TF = \frac{V_{output}}{V_{input}} = \frac{\cancel{\frac{I(s)}{CS}}}{\cancel{\frac{I(s)\,[CLRS^2+LS+R]}{CS(LS+R)}}}$$

$$\boxed{TF = \frac{LS+R}{CLRS^2+LS+R}}$$

example ③ slides:-

①





$$TF = \frac{V_{out}}{V_{input}} = 1$$

٢

$$V_{output} = I(s) * Z_{eq_C}$$

* Z_{eq} series between R and L $\rightarrow Z_{eq} = R + LS$
* Z_{eq} is parallel with $Z_C \rightarrow \frac{1}{Z_{eq_2}} = \frac{1 * CS}{(R+LS) * CS} + \frac{1 * (R+LS)}{CS * (R+LS)}$

$$\frac{1}{Z_{eq}} = \frac{CS+R+LS}{RCS+CLS^2} \rightarrow Z_{eq} = \frac{RCS+CLS^2}{CS+R+LS}$$

* $V_{input} = I(s) * Z_{eq}$

2 mechanical System → * Spring
* mass
* Damper

المطلوب منك انك تجيب diff. eq
وعدد diffeq = عدد الـ X الموجودة في الـ System

النقاط M مع damper, النقاط F مع spring → المعادلات ←
النقاط M مع Spring ، النقاط F مع damper
لكل M معادلة قوة .

* Spring → Fixed → X
  → Free → x_1, x_2

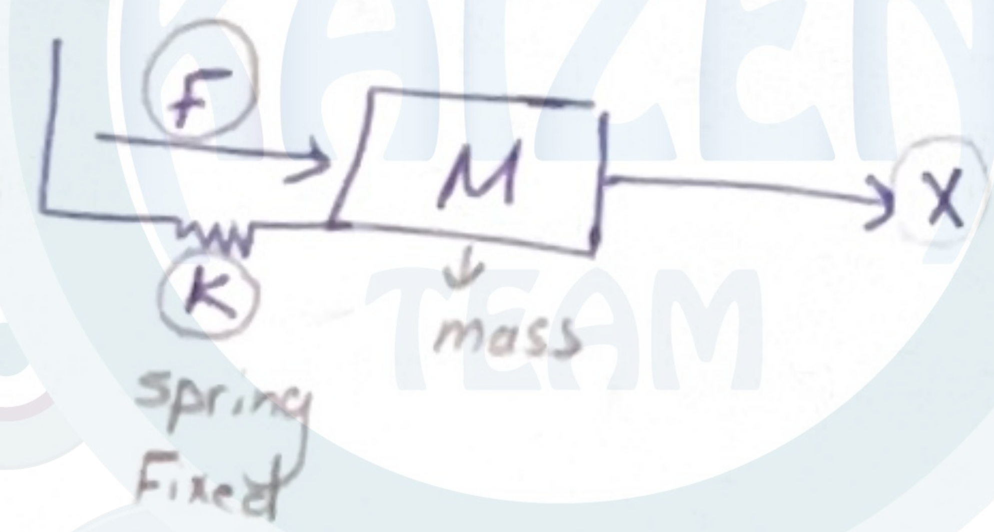
X واحدة فقط نؤثر عليه
$F_S = KX$

$F_S = K(x_1 - x_2)$ or $K(x_2 - x_1)$

* Damper → Fixed $F_D = D\dot{x}$
  → Free $F_D = D(\dot{x}_1 - \dot{x}_2)$ or $D(\dot{x}_2 - \dot{x}_1)$

* Mass → $\boxed{F_m = M\ddot{x}}$

example ① slides :-



① اتجاه الـ force الخارجية والباقي له عكس الاتجاه .

$x(0), \dot{x}(0), \ddot{x}(0) = 0$

$F = F_K + F_m$

$F = KX + M\ddot{x}$ → diffeq → $F(s) = M[s^2X(s) - \cancel{sX(0)} - \cancel{\dot{X}(0)}] + KX(s)$

$F(s) = X(s)\,[Ms^2 + K]$

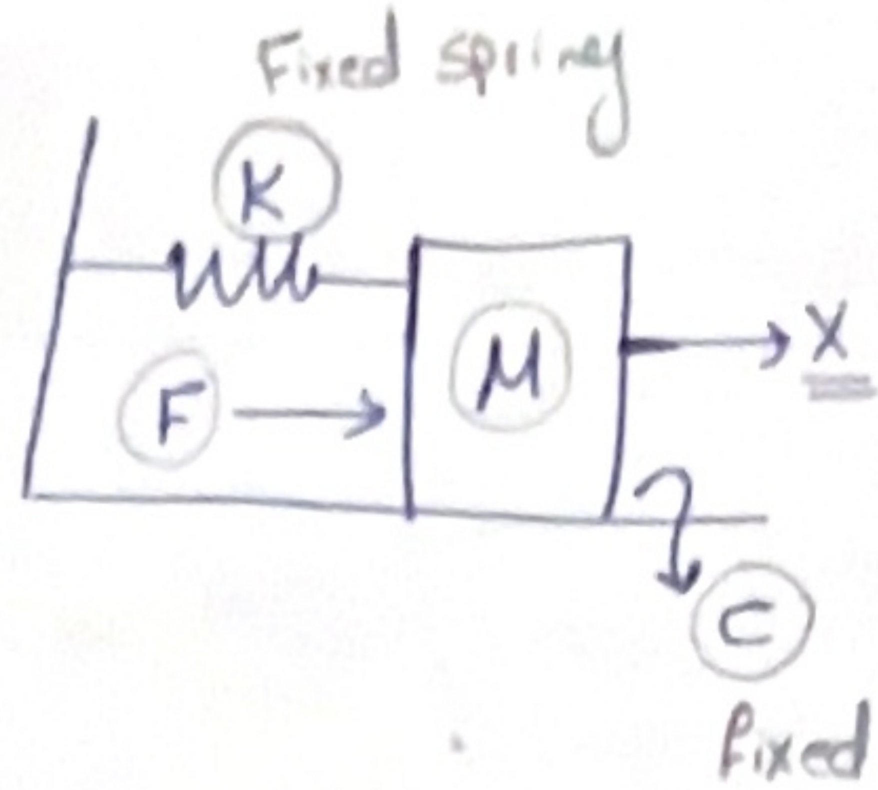
$TF = \frac{output}{input} = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K}$ if $M = 1000kg$ and $K = 2000\ Nm^{-1}$

$\frac{X(s)}{F(s)} = \frac{1/1000}{\frac{1000s^2}{1000} + \frac{2000}{1000}} = \frac{1/1000}{s^2 + 2}$

* Rotational Mechanical *

$T = K(\theta - \theta)$ spring
$T = C(\dot{\theta} - \dot{\theta})$ damper
$T = J\ddot{\theta}$ moment

Example ② slides :-



$$F = F_s + F_m + F_d$$

$$F = M\ddot{X} + C\dot{X} + KX$$

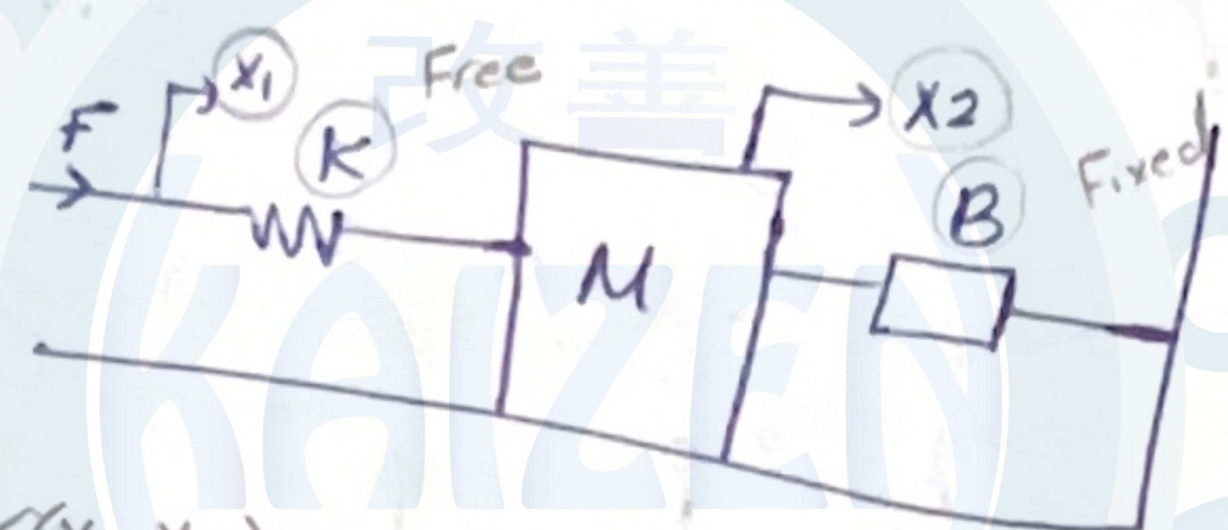
$$F(s) = M\left[S^2X(s) - \cancel{SX(0)} - \cancel{\dot{X}(0)}\right] + C\left[SX(s) - \cancel{X(0)}\right] + KX(s)$$

$$= MS^2X(s) + CSX(s) + KX(s) \longrightarrow X(s)\left[MS^2 + CS + K\right] = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{MS^2 + CS + K}$$ if $M = 1000$, $K = 2000$, $C = 1000$

$$TF = \frac{1}{1000S^2 + 1000S + 2000} \longrightarrow \frac{1/1000}{S^2 + S + 2}$$

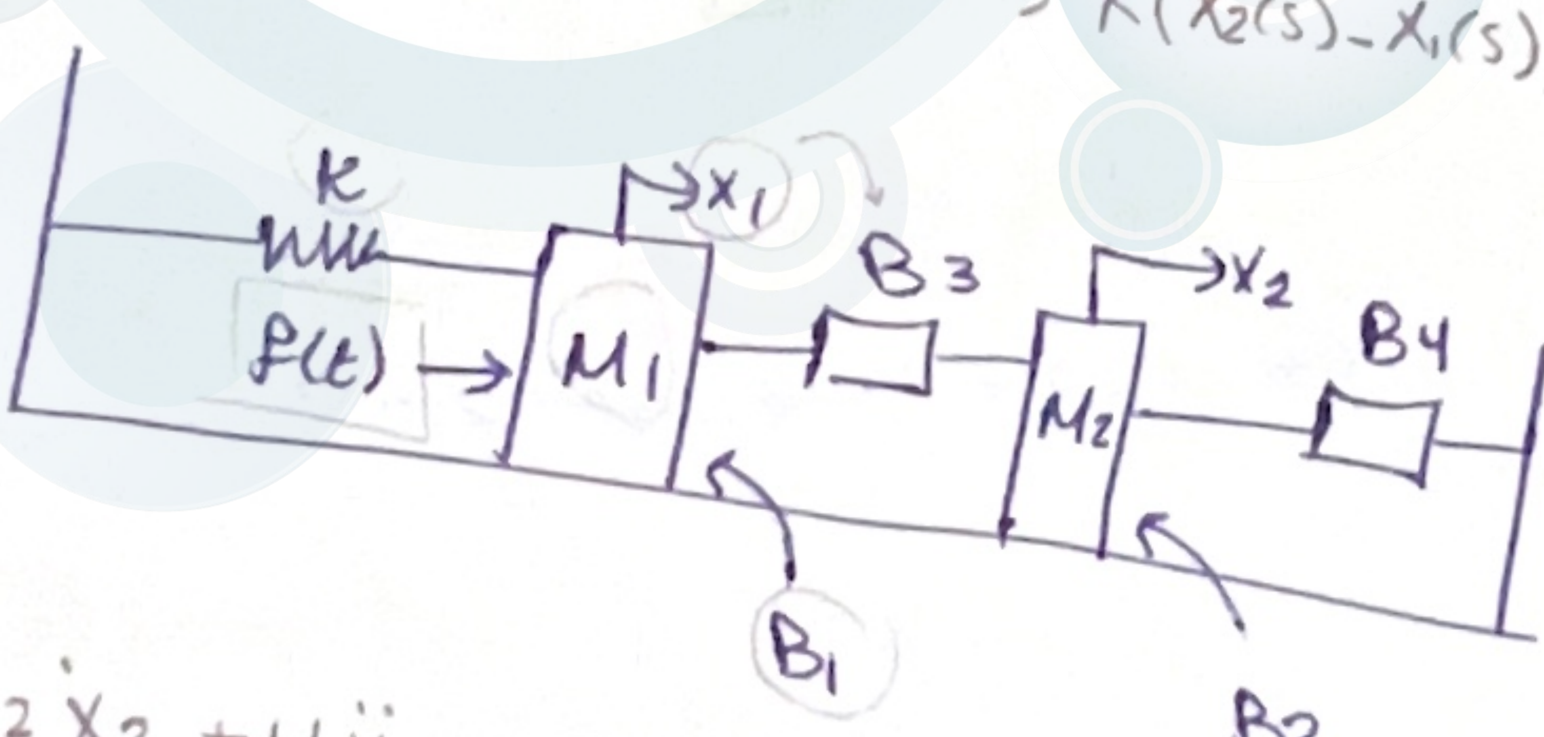
Example ③ Slides :-



$$F = F_s + F_m \longrightarrow F(s) = K(X_1 - X_2) \longrightarrow K(X_1(s) - X_2(s))$$

$$0 = F_d + F_m \longrightarrow 0 = K(X_2 - X_1) + M\ddot{X}_2 + B\dot{X}_2 \longrightarrow K(X_2(s) - X_1(s)) + MS^2X_2(s)$$

Example ④ slides :-



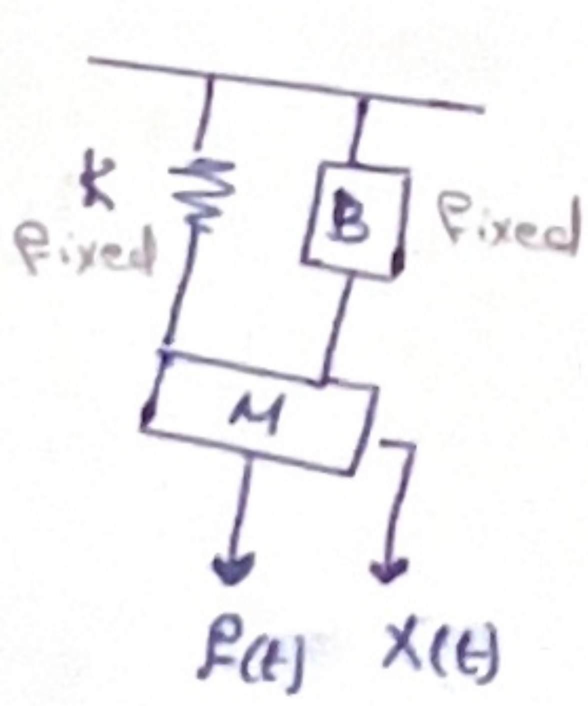
$$F(s) = KX_1 + M_1\ddot{X}_1 + B_1\dot{X}_1 + B_3(\dot{X}_1 - \dot{X}_2)$$

$$0 = B_3(\dot{X}_2 - \dot{X}_1) + B_4\dot{X}_2 + B_2\dot{X}_2 + M\ddot{X}_2$$

$$\longrightarrow F(s) = KX_1(s) + M_1S^2X_1(s) + B_1SX(s) + B_3S(X_1(s) - X_2(s))$$

$$\longrightarrow 0 = B_3S(X_2(s) - X_1(s)) + B_4SX_2(s) + B_2SX_2(s) + MS^2X(s)$$

Example ⑤ slides :-



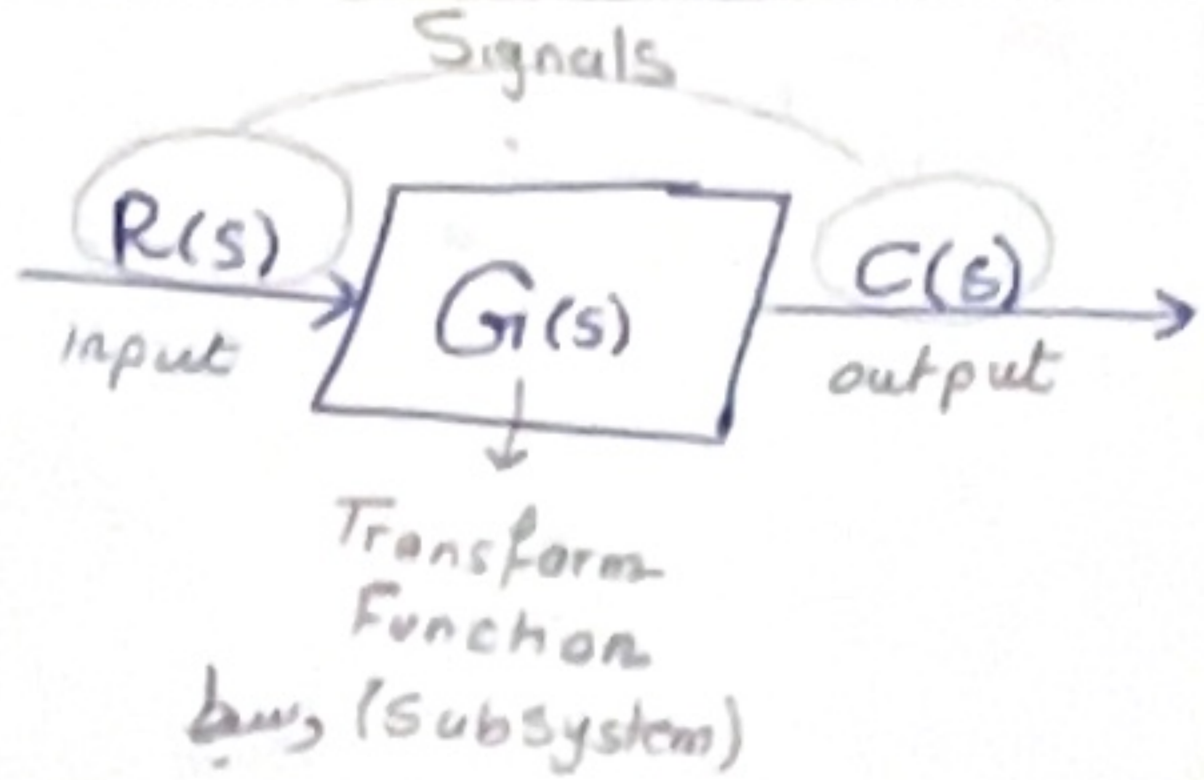
$$F(s) = KX + B\dot{X} + M\ddot{X}$$

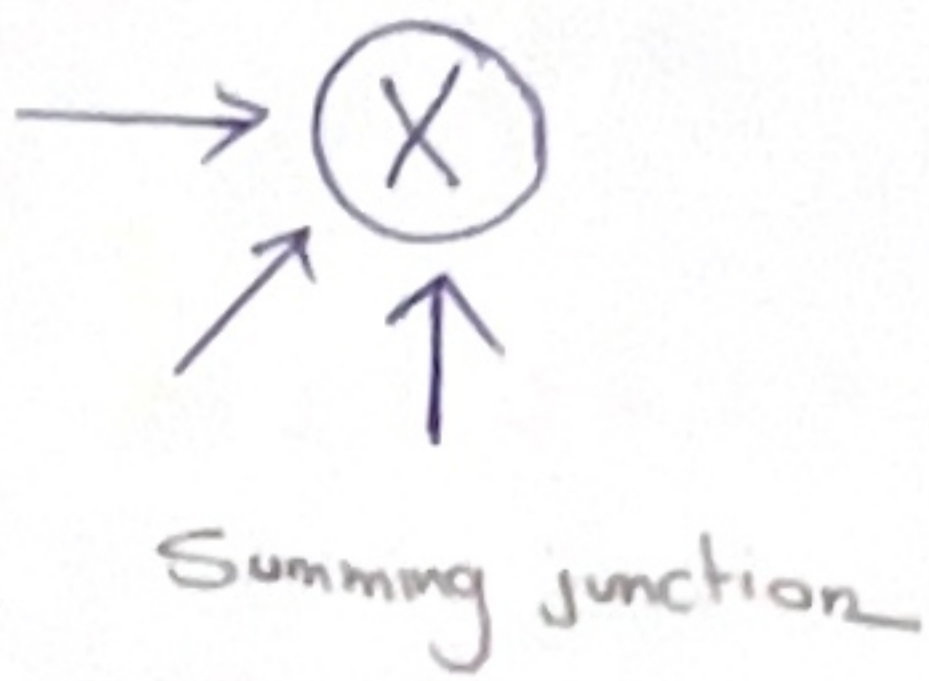
$$F(s) = KX(s) + BSX(s) + MS^2X(s)$$

$$X(s)\left[K + SB + MS^2\right] = F(s)$$

$$\frac{X(S)}{F(S)} = \frac{1}{MS^2 + BS + K}$$

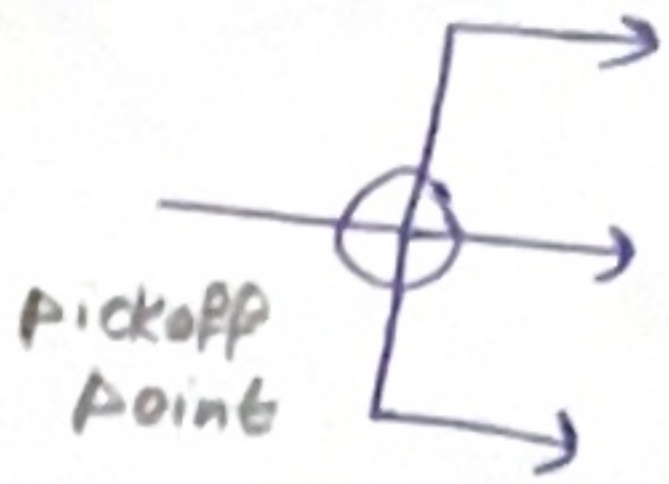
Block diagram:



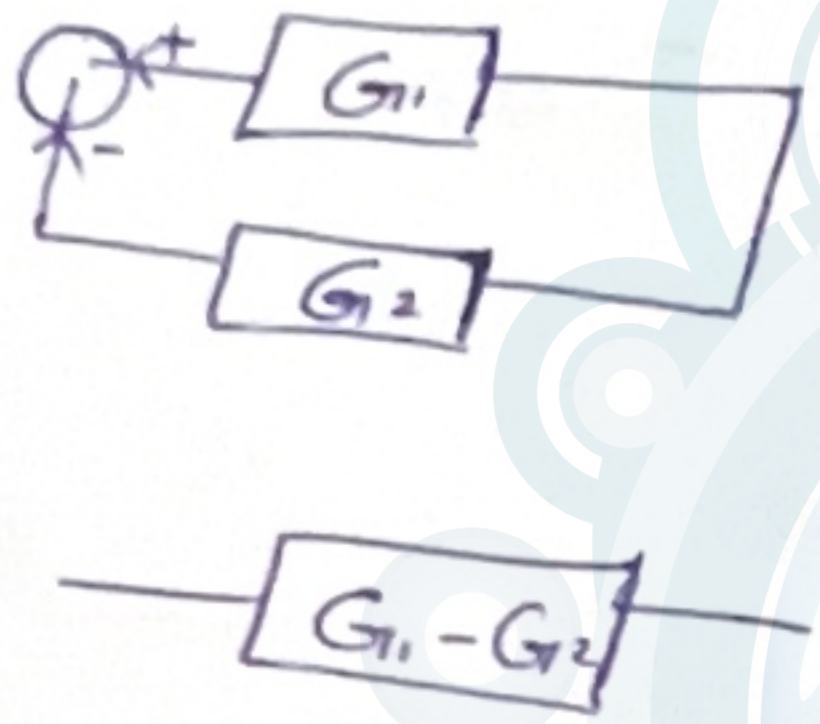


① Series

G_1G_2



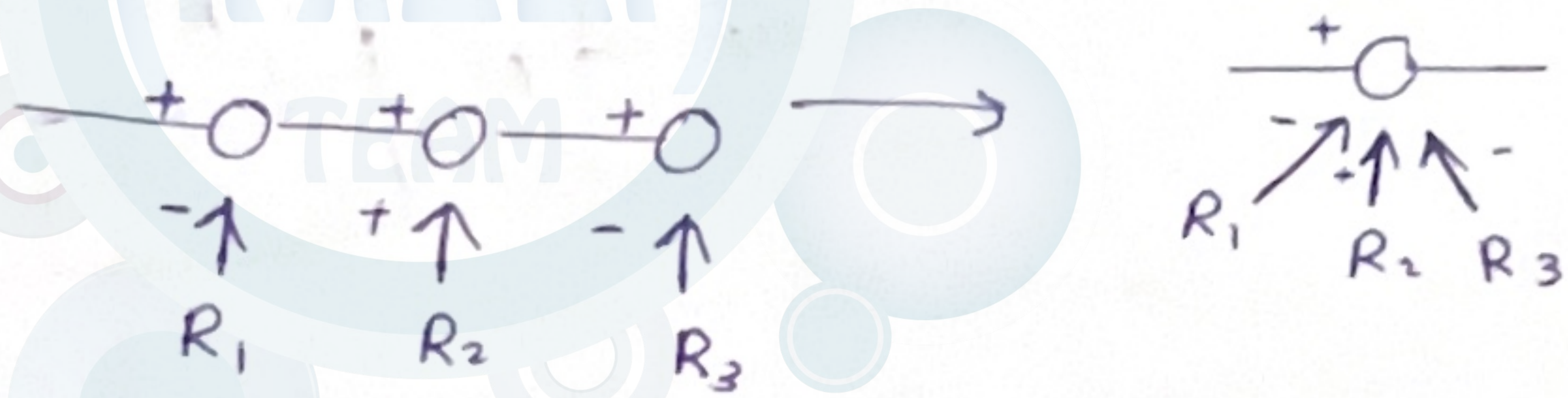
② parallel



$G_1 - G_2$

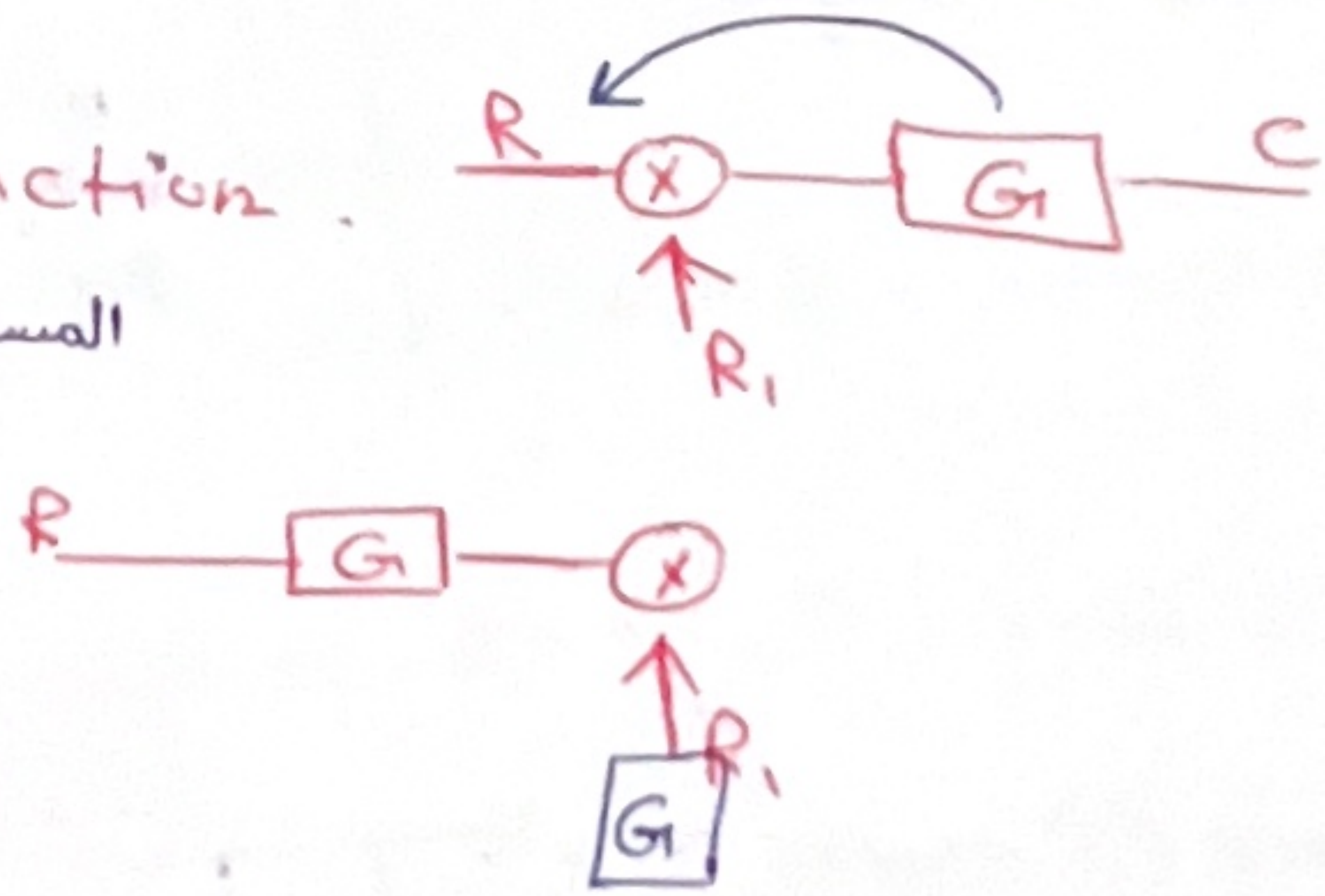
③ Collection of junctions

$-R_1 + R_2 - R_3$

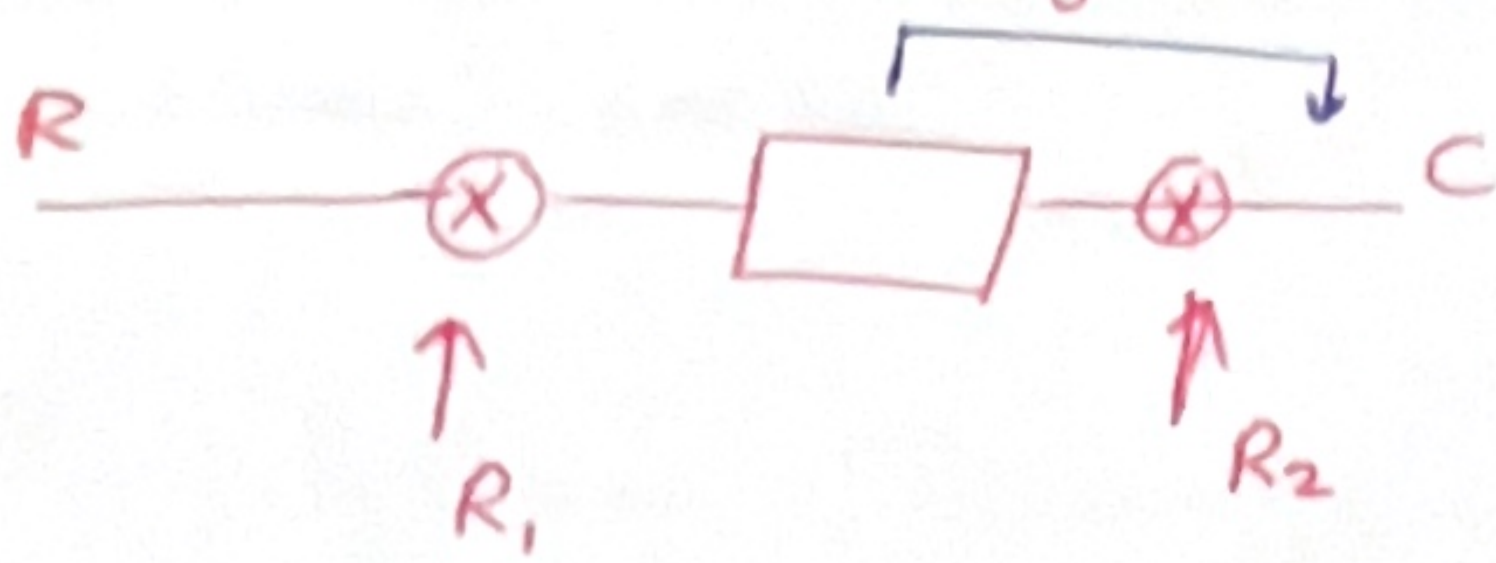


④ Moving Block to the Left Summing junction.

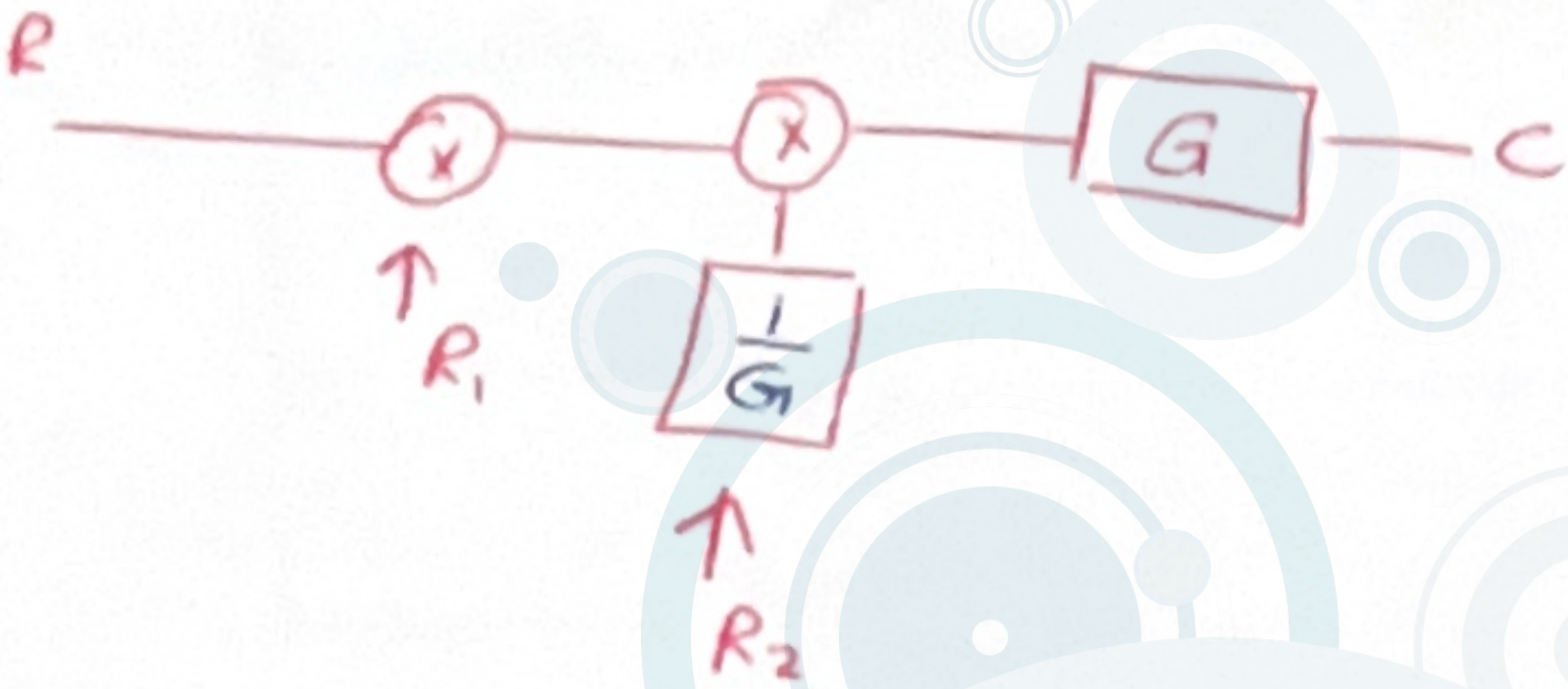
المسار الفرعي يجب ضربه بـ G.



٥ Moving Block to the right Summing junction.



المسار الفرعي بجيب ضربه بـ $\frac{1}{G}$

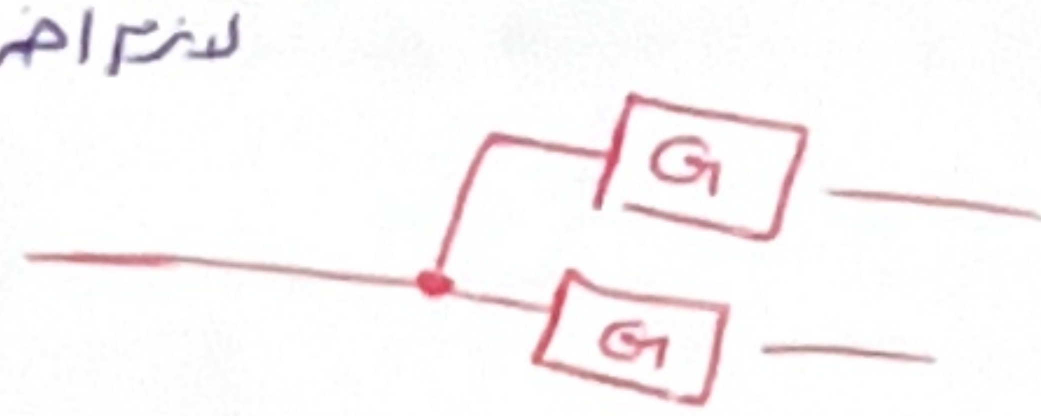


٦ Moving Block to the Left pickoff point.

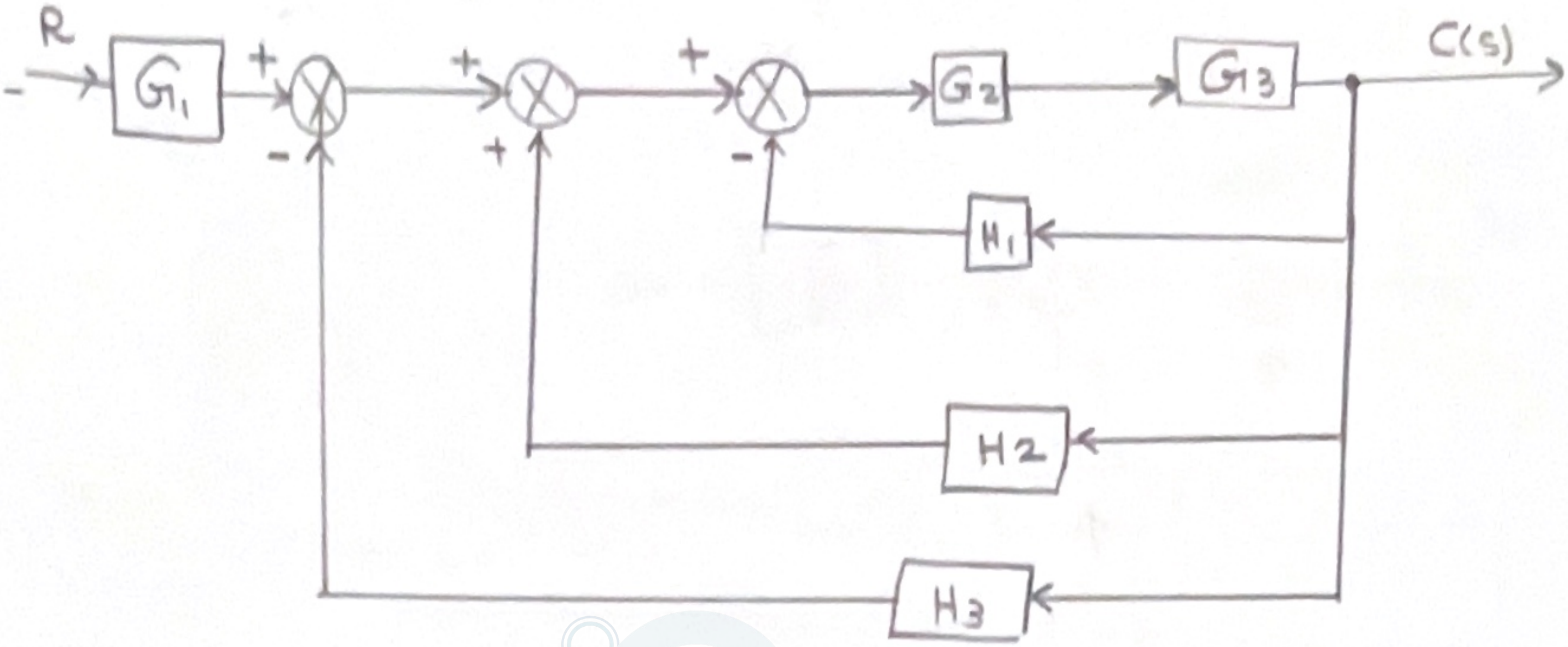
لازم اضرب المسار الفرعي بـ $\frac{1}{G}$

٧ Moving Block to the right pickoff point.

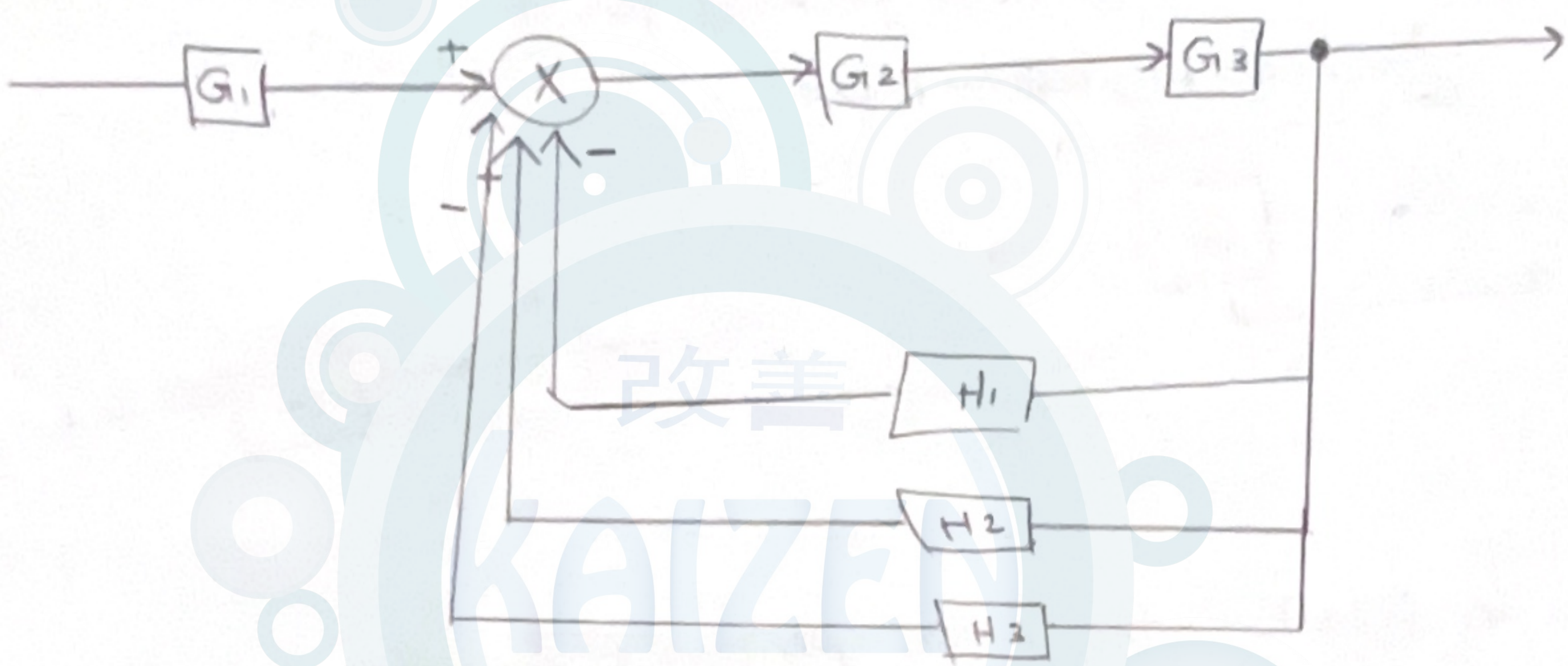
لازم اضرب المسار الفرعي بـ G



Example ①:-



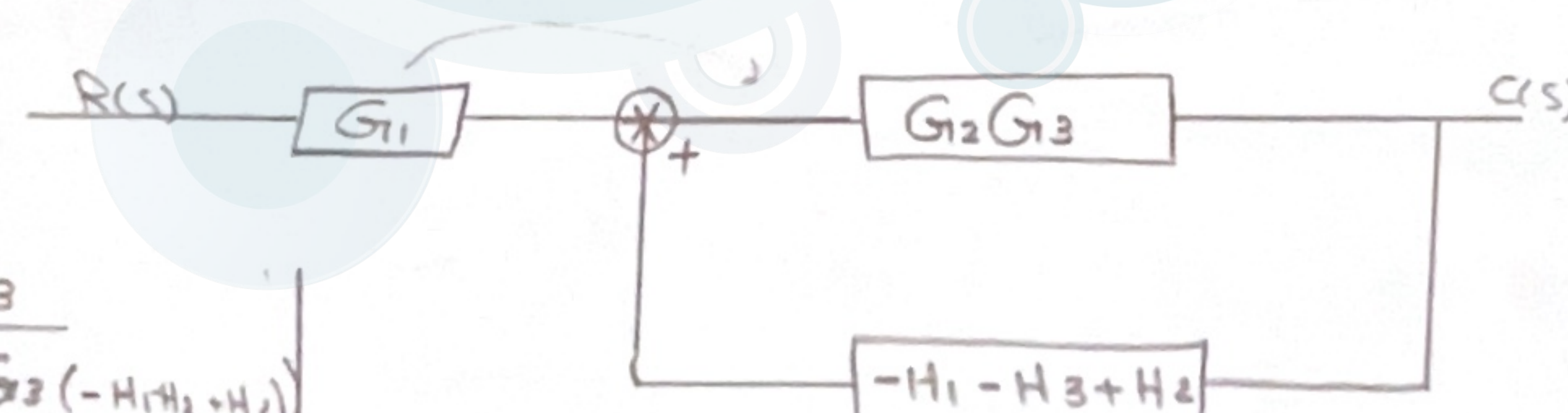
* انا عندي 3junction وراح نعوض بنفس الإشارة بدل ما نعمل سهم اي اشي لقدام خليهم 1junction



$\boxed{-H_1 - H_3 + H_2}$

* H_1, H_2, H_3 بيبلشوا بنفس النقطة وبخلصوا عند junction ← parallel

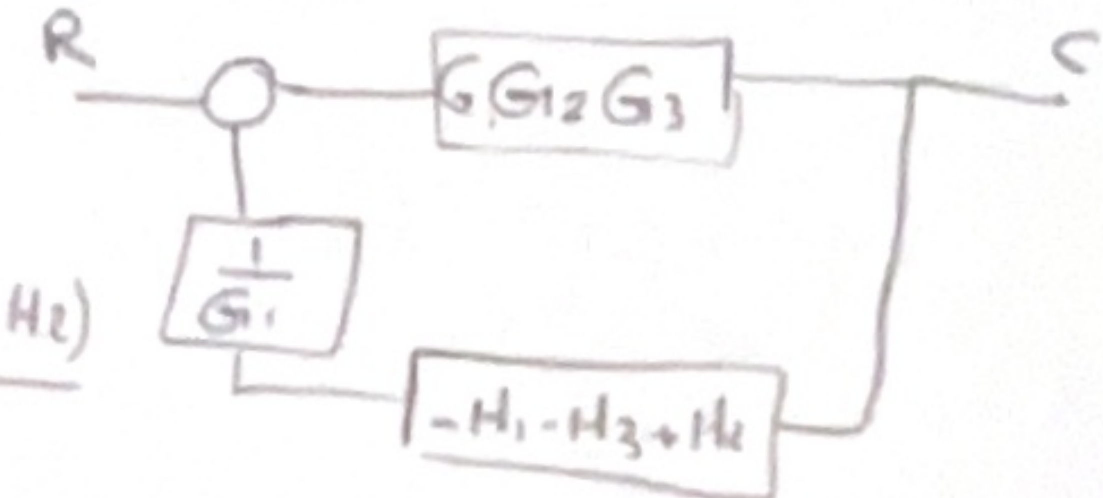
* G_2, G_3 (Series) ← $\boxed{G_2G_3}$



closed Loop: $\frac{G}{1-GH} \rightarrow \frac{G_2G_3}{1-(G_2G_3(-H_1-H_3+H_2))}$

$$\frac{G_2G_3\,G_1}{1-(G_2G_3)(-H_1-H_3+H_2)}$$

Block to the right junction →



$$\frac{G}{1-GH} = \frac{G_1G_2G_3}{1-\frac{(\cancel{G_1}G_2G_3)(-H_1-H_3+H_2)}{\cancel{G_1}}}$$

$$\frac{G_1G_2G_3}{1-(G_2G_3)(-H_1-H_3+H_2)}$$

$$\frac{C(S)}{R(S)} = \frac{\sum_{i=1}^{n} P \, \Delta K}{\Delta_i}$$

① pathes لي بيوصلك من البداية وحتى النهاية

② n = # of pathes

③ loop gain → Loop مسكرة

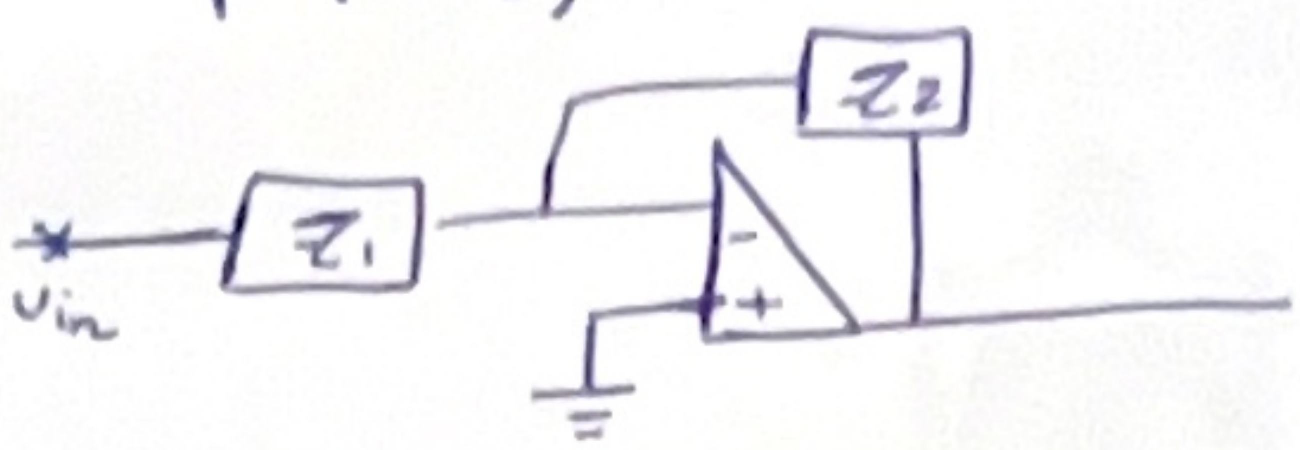
④ Non touching loops → loop غير مشتركين مع بعض

⑤ $\Delta = 1 - \sum \text{loop gain} + \sum \text{non-touching loop gain}$

⑥ $\Delta_K = 1 - \sum$ loop gain don't touch Forward path.

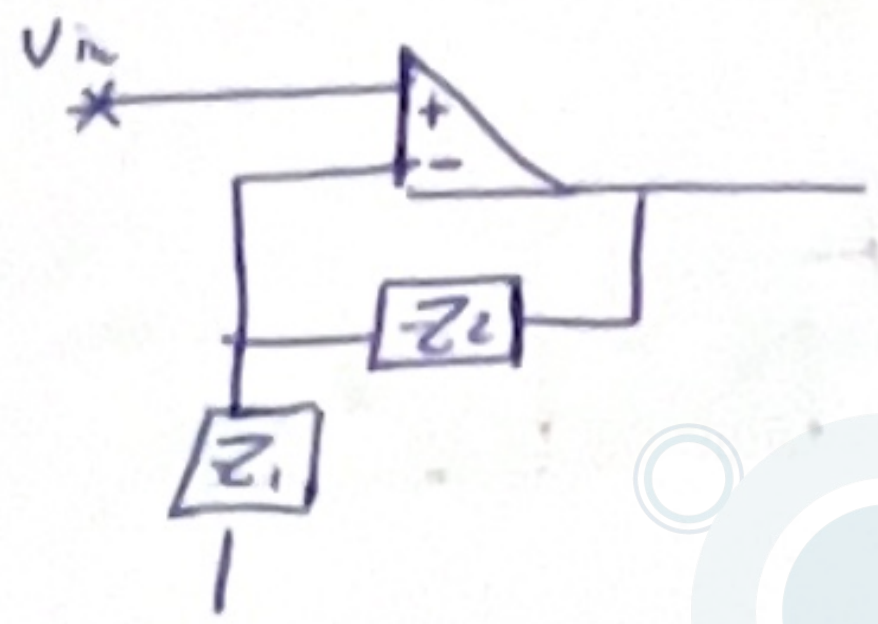
Electronics (Amplifiers)

1 Inverting



* يهمني مكان القطب السالب ومكان القطب الموجب .

2 Non-inverting



Inverting $TF = \frac{V_{output}}{V_{input}} = \frac{-Z_2}{Z_1}$

non inverting $TF = \frac{V_{out}}{V_{input}} = 1 + \frac{Z_2}{Z_1}$

* احياناً بكون دمج بين 2system

$$TF = \frac{V_{out}(1)}{V_{in}(1)} * \frac{V_{out}(2)}{V_{in}(2)}$$

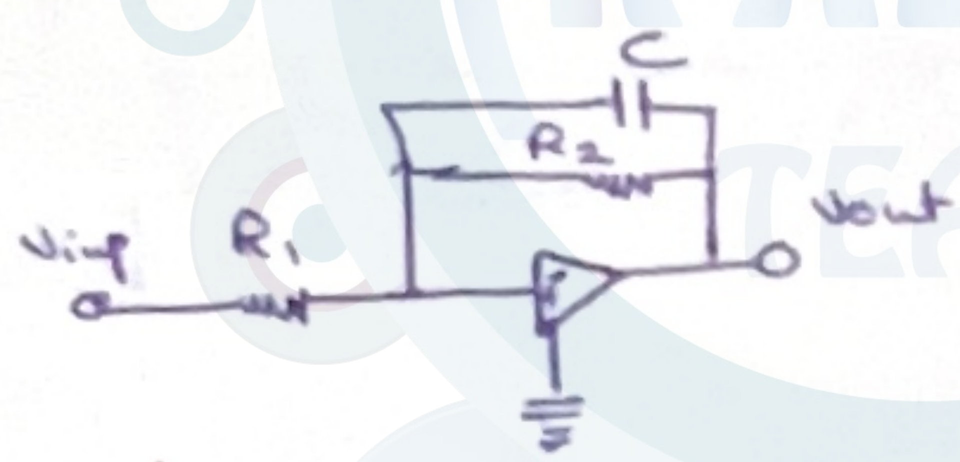
* يهمنا طريقة التوصيل

Series $Z_{eq} = Z_1 + Z_2 + Z_3$

Parallel $= \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$

R	Z_R
$\frac{1}{Cs}$	Z_C
Ls	Z_L

* Example 1 :-



inverting

$$Z_2 \text{ (parallel)} = \frac{1}{Z_{eq}} = \frac{1}{\frac{1}{Cs}} + \frac{1}{R_2}$$

$$\frac{1}{Z_{eq}} = Cs + \frac{1}{R_2}$$

$$\boxed{\frac{1}{Z_{eq}} = \frac{CsR_2 + 1}{R_2}}$$

$$Z_{eq} = \frac{R_2}{CsR_2 + 1}$$

$\boxed{Z_1 = R_1}$

$$TF = \frac{-Z_2}{Z_1} = -\frac{\frac{R_2}{CsR_2+1}}{R_1} \Rightarrow \frac{-(CsR_2+1)(R_1)}{R_2} = \frac{-R_2}{(CsR_2+1)R_1}$$

Tanks

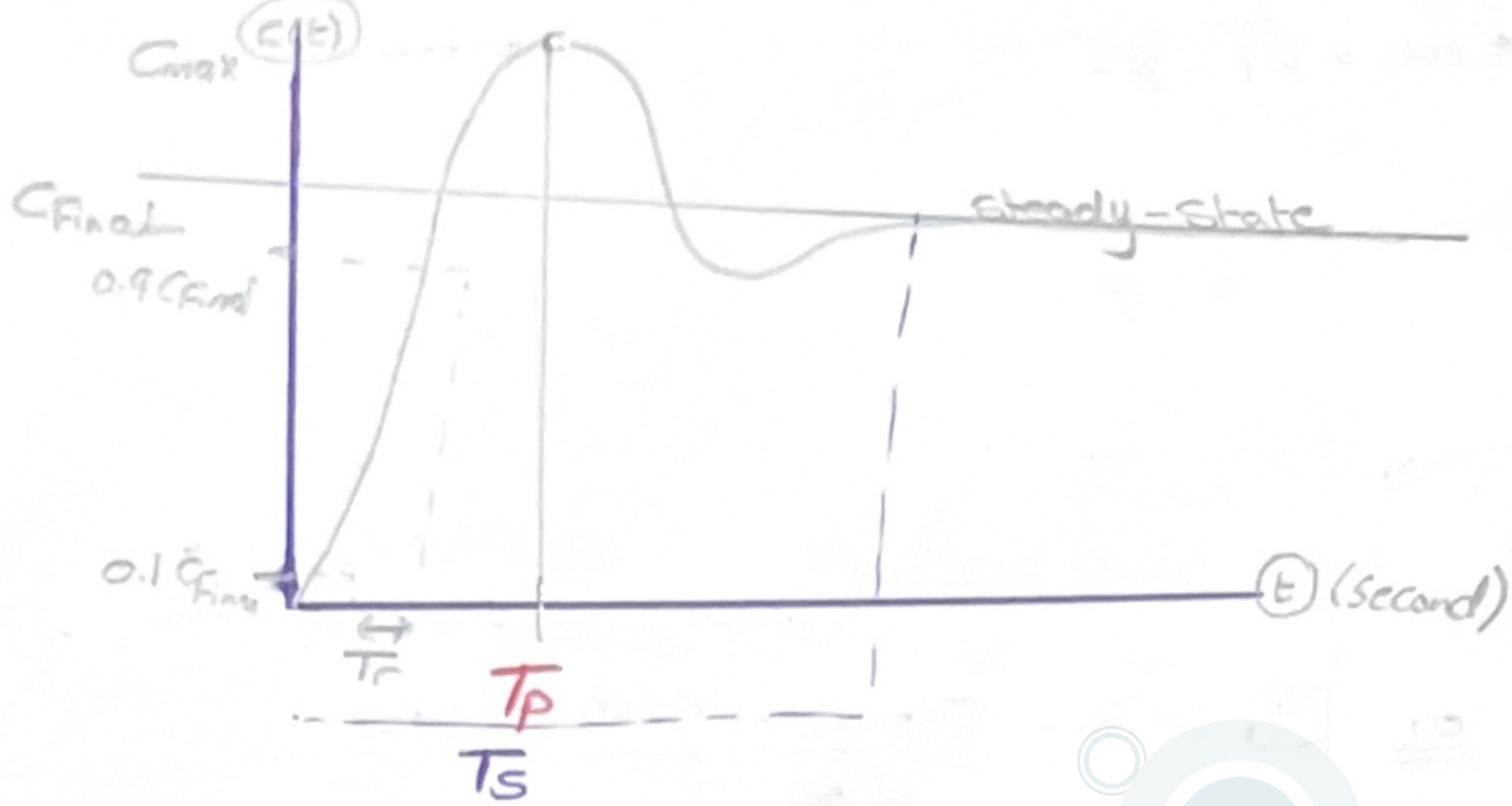
capacitance

$\boxed{1}$ $C\frac{dh}{dt} = \underline{q_1} - \underline{q_2}$ $\Rightarrow$ tank عندكل

q_1: مضخة تدخل المي

q_2: مضخة تطلع المي

$\boxed{2}$ $R = \frac{h(\text{بيدخل المي}) - h(\text{بطلع المي})}{q \text{ عندكل}}$ $\Rightarrow$ عندكل مضخة q



$$\frac{\text{output } C(s)}{\text{input } R(s)} = \frac{1}{\tau s + 1}$$

$$C(t) = KA\,(1 - e^{-t/\tau})$$

KA ↓ Final Value

معامل الـ step function.

$$\left[\frac{1}{\tau} e^{-\frac{t}{\tau}}\right]$$

(1) Rise Time ⟶ T_r — 10% — 90% CFinal

$T_r = 0.9\,C_{Final} - 0.1\,C_{Final}$ $\boxed{T_r = 2.2\tau}$

(2) Settling Time ⟶ T_s (2%) from final value

كم بدها وقت حتى توصل Steady state

$\boxed{T_s = 4\tau}$

(3) $T_c = \tau$ ⟶ 63% final value.

(4) initial slope $= \frac{1}{\tau} = a$

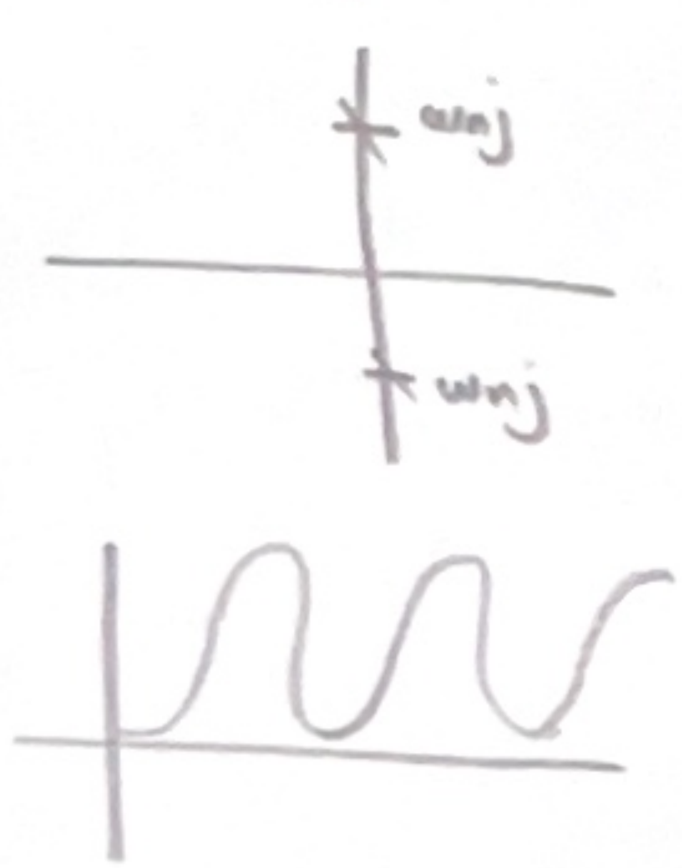
Second order ⟶ $\frac{\text{out } C(s)}{\text{inp } R(s)} = \frac{K\,w_n^2}{S^2 + 2\xi w_n S + w_n^2}$

w_n : Natural Frequency
ξ : damping ratio
k : Static gain (DC)

$$S_{1,2} = -\xi w_n \pm w_n\sqrt{\xi^2 - 1}$$ Roots

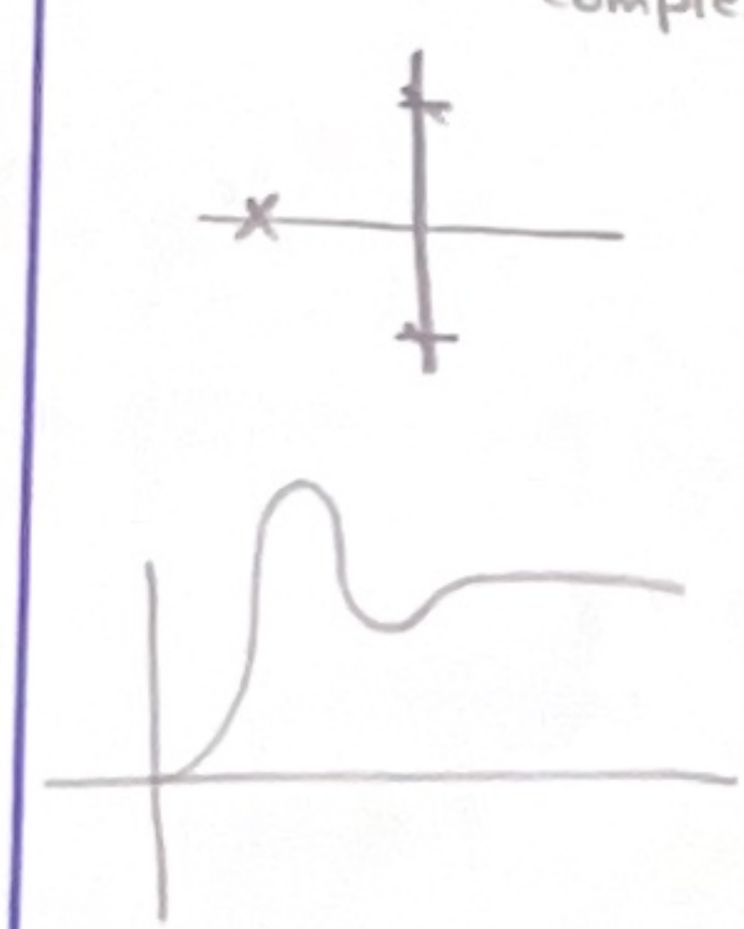
(1) No-damping $\xi = 0$

$S_{1,2} = \pm w_n j$



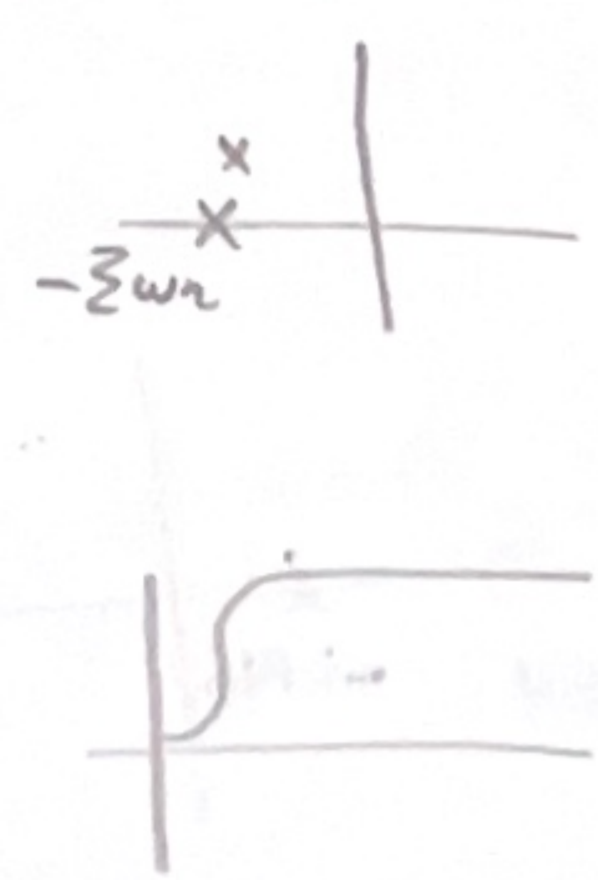
(2) underdamped $\xi < 1$

$\underbrace{-w_n\xi}_{\text{real}} \pm \underbrace{w_n\sqrt{\ }}_{\text{Complex}}$



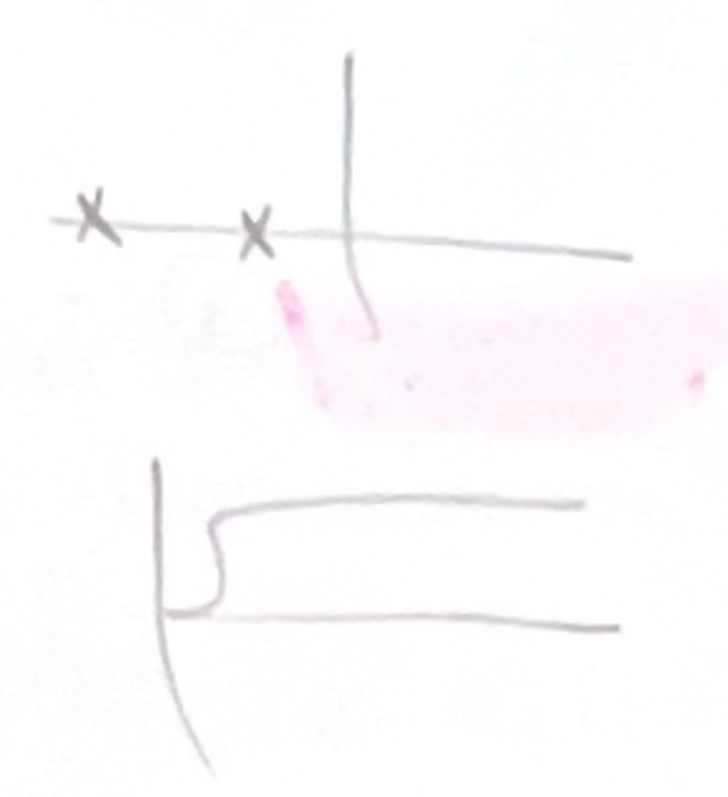
(3) critically damped $\xi = 1$

$S_{1,2} = -w_n$ جذرين

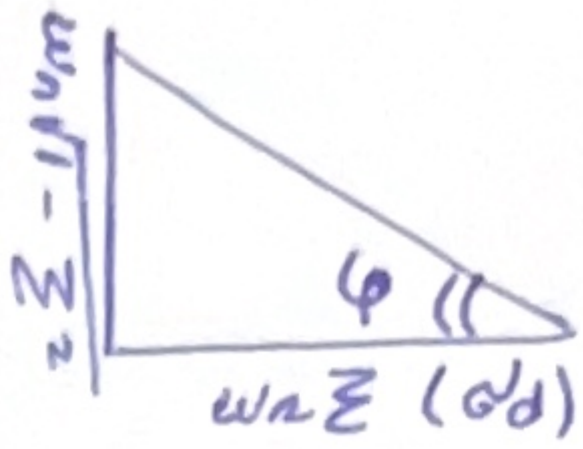


(4) overdamped $\xi > 1$

$S_{1,2} = \underbrace{-\xi w_n}_{\text{real}} \pm \underbrace{w_n\sqrt{\xi^2 - 1}}_{\text{real}}$



١ $\varphi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) * \frac{\pi}{180}$



٢ $\varphi = \cos^{-1}(\zeta) * \frac{\pi}{180}$

٣ ω_d (damping Frequency) $= \omega_n * \sqrt{1-\zeta^2}$

٤ peak Time $= \frac{\pi}{\omega_d}$

٥ Rise Time $= \frac{\pi - \varphi}{\omega_d}$

٦ Time Constant $= \frac{1}{\omega_n \zeta}$

٧ Time Settling (2%) $= 4T_c$

٨ $OS\% = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} * 100\%$ <u>or</u> ٩ $OS\% = \frac{C(max) - C(\infty)}{C(\infty)} * 100\%$

---

example ①:- slides

$G(s) = \frac{36}{S^2 + 4.2S + 36}$ Find ω_n, ζ

$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$* \ \omega_n^2 = 36$ $\boxed{\omega_n = 6}$

$* \ 2\zeta\omega_n = 4.2$

$2(\zeta)(6) = 4.2$ $\boxed{\zeta = 0.35}$

example ②:- slides

$G(s) = \frac{100}{s^2 + 15s + 100}$ T_P, T_S, $OS\%$.

$\omega_n^2 = 100 \rightarrow \boxed{\omega_n = 10}$

$2\zeta\omega_n = 15$

$2(10)\zeta = 15 \rightarrow \boxed{\zeta = 0.75}$

$T_P = \frac{\pi}{\omega_d}$

$\boxed{T_P = 0.475 \text{ second.}}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$= 10\sqrt{1-(0.75)^2}$

$= 6.614$

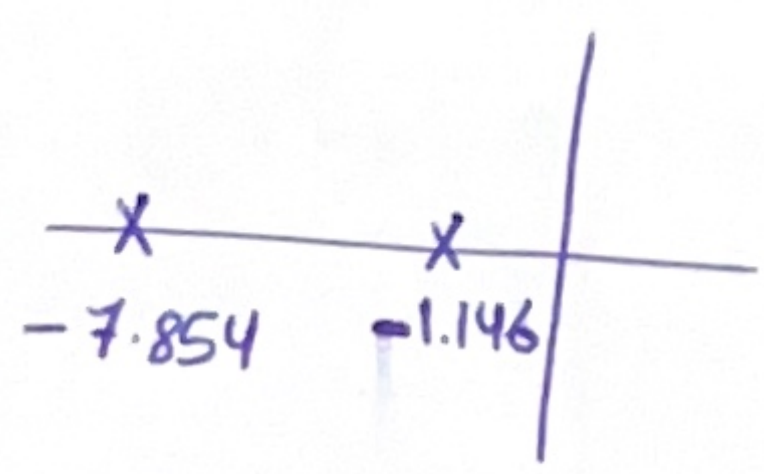
$T_S = 4T_c$

$T_S = T_c = \frac{4}{(0.75)(10)} = \boxed{0.533 \text{ second}}$

$OS\% = \frac{-\pi\zeta}{e^{\sqrt{1-\zeta^2}}} * 100\% = \boxed{2.837\%}$

example ③:- slides

overdamped.



$\omega_n^2 = 9$

$\boxed{\omega_n = 3}$

$2\zeta\omega_n = 9$

$2(3)\zeta = 9$

$\zeta = \frac{9}{6} = 1.571$

$(S+7.854)(S+1.146)$

$S^2 + 1.146S + 7.854S + 9$

$\boxed{S^2 + 9S + 9}$

example ④ :-

$$\frac{C(s)}{R(s)} = \frac{9}{s^2+3s+9}$$

① $\omega_n^2 = 9$ $\boxed{\omega_n = 3}$

② $2\zeta\omega_n = 3$

$2(3)\zeta = 3$

$\boxed{\zeta = 0.5}$ <1 under damping

$$S_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$S_1 = -1.5 + 2.598j$

$S_2 = -1.5 - 2.598j$

example ⑤ :-

$$\frac{C(s)}{R(s)} = \frac{9}{s^2+6s+9}$$

$\omega_n^2 = 9$ $\omega_n = 3$

$2\zeta\omega_n = 6$

$\boxed{\zeta = 1}$

$S_1 = -3$

$S_2 = -3$

* **Steady state error**

e(feedback) $= R - C$ (unity, $H=1$)

e (Non-feedback) $= R - HC$ (unity, $H \neq 1$)

$$\text{error} = \lim_{s \to 0} S * \text{error}$$

	N=0	N=1	N=2
Step	$\frac{1}{1+k}$	∞	∞
Ramp	∞	$\frac{1}{k}$	∞
parabola	0	0	$\frac{1}{k}$

$$\text{error} \ \frac{G}{1+G} \longrightarrow \frac{R}{1+G}$$

* الحد الثابت لازم يساوي ①

example ① :- Slides

$$G = \frac{3(s+1)}{s(s+2)}$$

$r = (2+3t).ut$

$$r = \frac{2}{s} + \frac{3}{s^2}$$

$$r = \frac{2s+3}{s^2}$$

$$\text{error} = \lim_{s \to 0} \frac{\cancel{S} * \frac{2s+3}{s^{\cancel{2}}}}{1 + G(s)}$$

$$= \lim_{s \to 0} \frac{2s+3}{s + sG(s)}$$

$$= \frac{3}{\lim_{s \to 0} sG(s)} \longrightarrow \frac{3}{2}$$

$$= \boxed{2}$$

Stability

Roots
على جهة اليسار
So Stable.

$*$ Marginally Stable System.

$*$ unstable

* خطوات الحل :- ① اكتب المعادلة على صيغة $\frac{output}{input}$
② رتب المعادلة وساوي المقام بالصفر
③ اعمل (Routh Table) لازم توصل فيه الى S^0.
④ حدد الاشارات (+) or (-) اي تغير بالاشارة معناته صار النظام (unstable)
عدد مرات التغير = عدد الجذور على جهة اليمين ليس انتبه عدد الجذور الكلي = العدد (القوة) لاعلى $\underline{\underline{S}}$

* حالات خاصّة *

① لوكان عندك صف فيه اول عنصر = 0 وباقي الصف — فيه ارقام / فارغ
نهاي الحالة نشيل 0 ونحط مكانها ε وهي قيمة صغيرة موجبة.

$$\boxed{\varepsilon = 1\times10^{-3}}$$

* حتى اقرر بهاي الحالة هل هو Stable أو Unstable *

A نتطلع على Row يلي فوقه ويلي تحته لو ما صار تغير في الاشارة هاد معناته انه في عنا جذرين على $\boxed{j\omega}$ ما بنقدر احكي عنه Stable

B نتطلع على Row يلي فوقه ويلي تحته لو صار تغير في الاشارة هاد معناته انه اولًا unstable وما عندي جذور على $\boxed{j\omega}$.

② لوكان عندك صف كامل كلّه = 0 نتطلع على الصف يلي فوقه ونكتب معادلته ونعملها اشتقاق الارقام يلي طلعت بعد الاشتقاق نعوضها مكان (0) ونكمل حل عادي بعدين نرجع كانه مرة للمعادلة قبل الاشتقاق ونحلها ونجيب اصفارها.

example ①:- $\frac{G(S)}{R(S)} = \dfrac{\frac{K}{(S+1)(S^2+4S+20)}}{1+\frac{K}{(S+1)(S^2+4S+20)}} = \frac{K}{(S+1)(S^2+4S+20)+K}$

then : $S^3+\underline{4S^2}+\cancel{20S}+\cancel{S^2}+\cancel{4S}+20+K$

$S^3+5S^2+24S+(20+K)=0$

			Signs
S^3	1	24	
S^2	5	$(20+K)$	+
S^1	$\frac{100-K}{5}$	0	+
S^0	$20+K$		

$x = \frac{5\times24-(20+K)}{5} = \frac{120-20-K}{5} = \frac{100-K}{5}$

$y = \frac{(20+K)\left(\frac{100-K}{5}\right)-0}{\left(\frac{100-K}{5}\right)} = 20+K$

$\frac{100-K}{5} > 0 \quad 100-K>0$
$100>K \rightarrow \boxed{K<100}$

$20+K>0 \quad K>-20 \ \cup\ K>0$

$\boxed{0<K<100}$

example ②:- $\frac{C(s)}{R(s)} = \frac{\frac{k}{(s+1)(s^2+4s+10)}}{1+\frac{k}{(s+1)(s^2+4s+10)}}$

$\frac{k}{(s+1)(s^2+4s+10)+k} \rightsquigarrow s^3+4s^2+10s+s^2+4s+10+k$

$s^3+5s^2+14s+(10+k)=0$

s^3	1	14	+
s^2	5	$(10+k)$	+
s^1	$\frac{60-k}{5}$	0	
s^0	$10+k$		

$X = \frac{5\times 14-(10+k)}{5} = \frac{60-k}{5}$

$y = 10+k$

$\frac{60-k}{5} > 0$

$60 > k$

$k < 60$

$10+k>0$

$k>-10$ X $\underline{k>0}$

$0<k<60$

example ③:- $\frac{C(s)}{R(s)} = \frac{\frac{15k}{s(s+5)(s+7)}}{1+\frac{15k}{s(s+5)(s+7)}}$

$s(s+5)(s+7)$

$(s^2+5s)(s+7)$

$s^3+7s^2+5s^2+35s$

$s^3+12s^2+35s+15k$

$= \frac{15k}{s^3+12s^2+35s+15k} \rightsquigarrow s^3+12s^2+35s+15k=0$

$X = \frac{12*35-15k}{12} = \frac{420-15k}{12}$

			Signs
s^3	1	35	+
s^2	12	$15k$	+
s^1	$\frac{420-15k}{12}$	0	
s^0	$15k$		

$\frac{420-15k}{12} > 0$

$420 > 15k$ $\boxed{28>k}$

$15k>0$ $\boxed{k>0}$

$0<k<28$

example ④

S^4	1	3	1
S^3	3	2	0
S^2	7/3	1	
S^1	A		
S^0	1		

$$A = \frac{((7/3)*2)-3}{7/3} = \frac{5}{7}$$

example ⑤ :- $S^3+S^2+2S+k=0$

S^3	1	2
S^2	1	k
S^1	2-k	
S^0	k	

$$X = \frac{2-k}{1}$$

$$y = k$$

$2-k>0$ $\boxed{2>k}$ $\boxed{k>0}$

$\boxed{0<k<2}$

example ⑥ :-

S^4	1	3	2
S^3	3	2	0
S^2	7/3	2	
S^1	A		
S^0	2		

$$A = \frac{(7/3)\times 2-6}{(7/3)}$$

$$= -4/7$$

example ⑦ :- $S^4+3S^3+3S^2+2S+k=0$

S^4	1	3	k
S^3	3	2	0
S^2	$\frac{7}{3}$	k	0
S^1			
S^0	k		

$$X = \frac{9-2}{3} =$$

$$y = \frac{3k-0}{3} = k$$

$$R = \frac{(7/3)\times 2-3k}{(7/3)}$$

$$\frac{14/3-3k}{(7/3)} > 0$$

$$14/3 > 3k$$

$\boxed{\frac{14}{9} > k}$ $\boxed{k>0}$

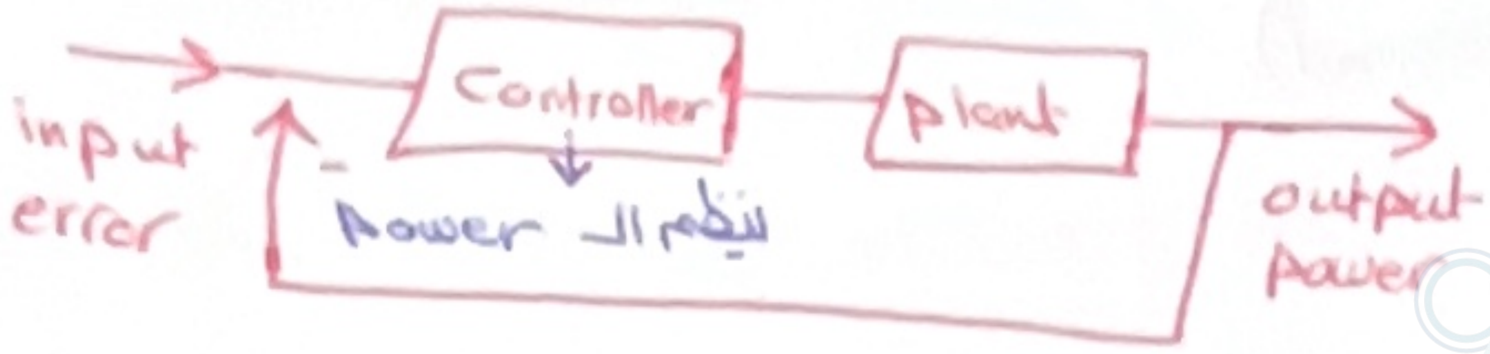
$\boxed{0<k<\frac{14}{9}}$

PID :

P → proportional (سرعة الوصول (البداية

I → integral (Steady State) آخر شي

D → Derivative ↑damping، OS% قلل



Kp →
1. decreasing Rise Time.
2. decreasing error
3. increasing OS%
4. Small change in Settling Time.

Ki →
1. decreasing Rise Time.
2. Zero Error.
3. increasing OS%.
4. increasing in Settling Time.

KD →
1. Small change Rise Time.
2. Small change Error.
3. decreasing in OS%.
4. decreasing Settling Time.

---

* $\boxed{\text{Power} = \text{error} * \text{Constant}}$

P controller: power $C_P(s) = e(s) * K_P$

PD controller: power $C_{PD}(s) = e(s) * (K_P + K_d s)$

PI controller: power $C_{PI}(s) = e(s) * (K_P + \frac{K_i}{s})$

PID controller: power $C_{PID}(s) = e(s) * (K_P + \frac{K_i}{s} + K_d s)$ → Parallel

$\frac{C(s)}{e(s)}$ when using P only → $C_P(s) = K_P$

$\frac{C(s)}{e(s)}$ when using P and d → $C_{Pd}(s) = K_P(1 + \frac{T_d s}{T_d s + 1})$ مهم

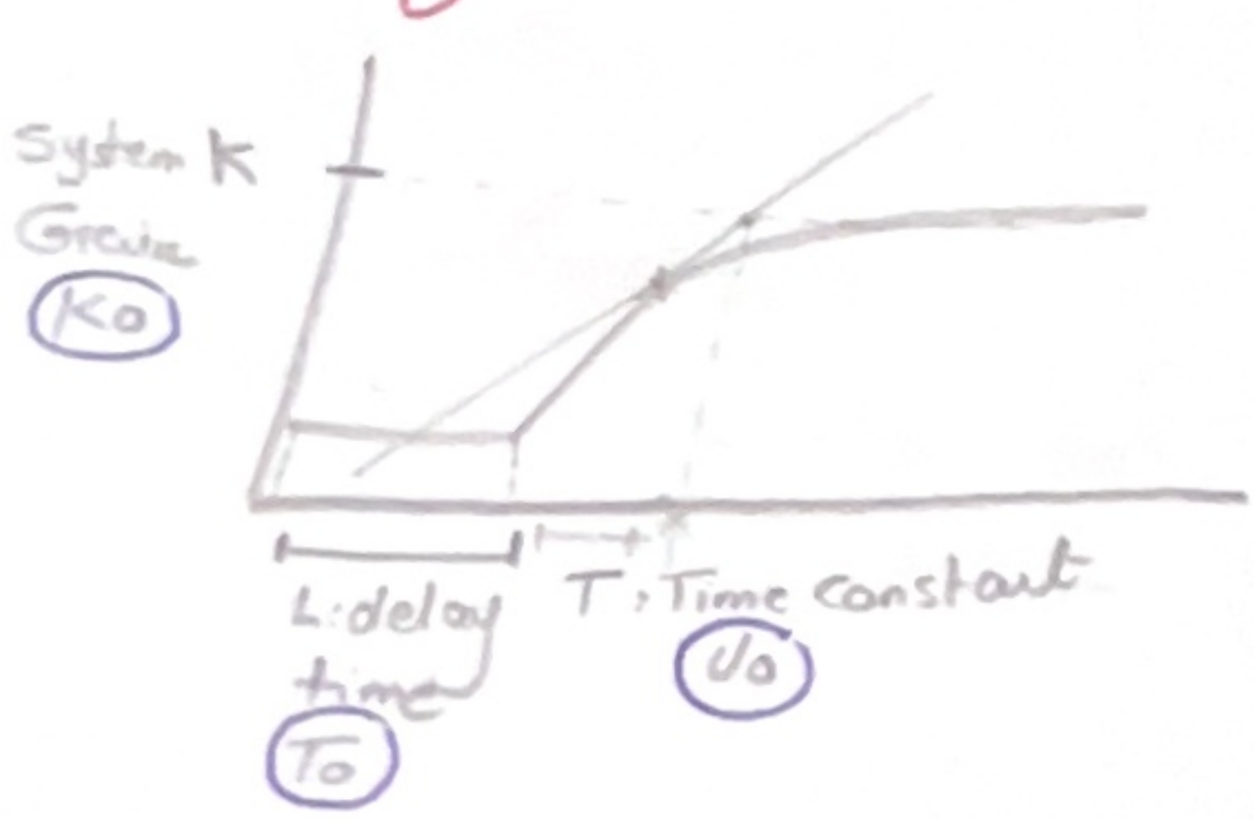
$\frac{C(s)}{e(s)}$ when using P and I → $C_{PI}(s) = K_P(1 + \frac{1}{T_i s})$

$\frac{C(s)}{e(s)}$ when using P and I and d → $C_{PID}(s) = K_P(1 + \frac{1}{T_i s} + \frac{T_d s}{T_d s + 1})$

---

1 Zeigler-Nichol's PID Tunning :- [open loop and step Function input]

H = 0
No Feedback



controller	Kp	Ti	TD
P	$\frac{v_o}{K_o T_o}$	∞	0
PI	$\frac{0.9 v_o}{K_o T_o}$	$3T_o$	0
PID	$\frac{1.2 v_o}{K_o T_o}$	$2T_o$	$0.5 T_o$

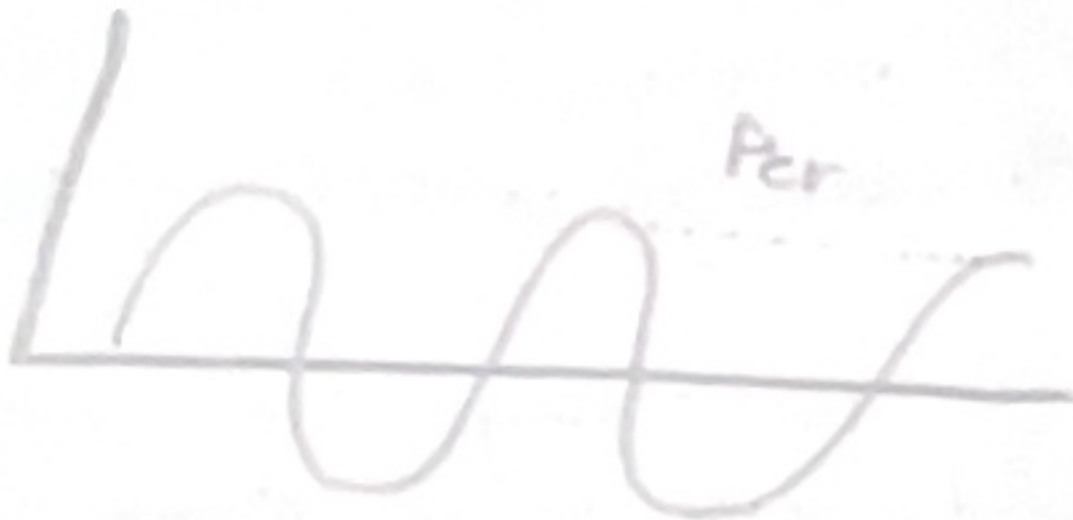
مهم

$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}$ / $T_i = \frac{K_P}{K_i}$ / $T_d = \frac{K_d}{K_P}$

$a = \frac{K_o T_o}{v_o}$

* The overshoot % is the Least when we use P controller.

2 Zeigler - Nichol's Second Method :- closed Loop $H \neq 0$



$$P_{cr} = \frac{1}{F_{cr}} \text{ (Second)}$$

$$P_{cr} = \frac{2\pi}{W_{cr}} \text{ (Second)}$$

$$W_{cr} = 2\pi F_{cr}$$

Controller	Kp	Ti	Td
P	0.5 Kcr	∞	0
PI	0.45 Kcr	$\frac{P_{cr}}{1.2}$	0
PID	0.6 Kcr	0.5 Pcr	0.125 Pcr

[illegible]

* خطوات الحل :-

① ايجاد $\frac{C(S)}{R(S)}$

② المقام = صفر وترتيبهم لعمل
ايجاد Routh Table

③ احسب قيمة Kcr.

④ من صف $\underline{\underline{S^2}}$ عوّض قيمة Kcr ولجذرية
Wcr.

⑤ احسب قيمة Pcr ومن الجدول ايجاد
المجاهيل وتعويضهم في معادلة ال power.

Ex:- S^2+5

S^2+0S+5

105 المعاملات

[1] S=Tf('s')

$g = \frac{y}{4}(4*s+3)/(s^2+6*s+5)$

[2] num = [4 3]; → coefficient (بسط)

den = [1 6 5]; → coefficient (مقام)

sys = tf (num, den)

[3] Zero - pole - gain Model (ZPK)

Ze = جذور البسط

Po = جذور المقام

K = gain $\frac{\text{معامل اعلى قوة بسط}}{\text{معامل اعلى قوة المقام}}$

Sys1 = ZPK(,[ ],)

لوكان عندك اكثر من واحد.

* Time Response *

- Impulse Response (impulse) → unit function R=1
- step Response (step)
- General time Response (lsim)
- Polynomial multiplication (conv) → الضرب
- Polynomial division (deconv) → قسمة طولية
- partial Fraction Expanison (residue) → ترسيمة لبعطيك اياها بصيغة complex
- Root locus (rlocus)

LTI ⇒ impulse
step
lsim

- Series / - Feed back
- sisotool → ليعطيك رسمة منها Freq ومنها Pm

* عدد المعاملات = الدرجة +1

حتى لو الحقيقة صفر المفروض تكتبها

* حرف التعامل مع * Matlabe

[E] State Space Model

عادة لتستخدم لوكان عندي معادلة ليست من الدرجة الأولى وبناءا لها التكامل

[A] اكتب TF

[B] $\frac{Y}{R} = \frac{\text{البسط}}{\text{المقام}} = \frac{1}{\text{مقام}}$ * البسط

$\frac{1}{\text{مقام}}$ * البسط $= \frac{X}{R} * \frac{Y}{X}$

[C] حلي المعادلات بدلالة y, R

[D] $X_1, X_2 \Rightarrow \dot{X}_1 = X_2$

$\ddot{X}_1 = \dot{X}_2$

مكان كل $\dot{X}$ عوضنا X_2 وعندك

عادية X عوضي X_1

* رح يطلع عندك Matrix

[E] sys = ss([ ],[ ],[ ],[ ])

⇓

* rss, drss → Random SSM
* ss2 → transform
* Canon → Conical forms
* ctrb → controllability
* obsv → observability
* ssbal → Diagonal balance.
* sminreal → minimal realization. (structurally)
* minreal → minmal realization or Pole/Zero
* modred → Model reduction.
* balreal → Gramian (input-output)
* gram → Controllability + observibility.

MatLabe (ODE)

① احنا ما نقدر ندخل المعادلات Second order فنضطر نحولها لـ First order عن طريقة اني افرض 2variable (x_1 ، x_2)

② عوّض المعادلات الي فرضتها بالمعادلات من السؤال

$$\dot{x}_1 = x_2$$
$$\ddot{x}_1 = \dot{x}_2$$

③ اي اشي فيه (.) نحطه على طرف والباقي ع طرف ثاني .

④

```
Function dx = Rama (t, x)
[m n] = Size(x);
dx = Zeros (m, n);
dx(1) = ---- ;
dx(2) = --- ;
tspan [ , ]; → معطى في السؤال
X0 = [ , ]; → فرض منه السؤال
[t x]= ode45(@Rama, tspan, X0)
```

$\boxed{\dot{x} + ax = b}$ $a=2$ $b=4$

* احيانا ممكن يعطيني معادلات (First order) وقيم a, b ثابتة لا تتغير

```
Function dx = Rama (t, x)
a = 2;
b=4;
dx = b - a*x;
tspan[ ];
X0 = [ ];
[t x]= ode45(@Rama, tspan, X0)
plot(t, x)
```

* افرض حسبك Firstorder وحكيتلك انه فيهم a,b تتغيروا .

```
Function dx = Rama(t,X)
a = pram (1);
b = pram(2);
dx = b - a * X;

tspan = [ , ];
Xo = [ , ]
pram = [ , ];
[t x] = ode45 (@Rama, tspan, Xo    pram]
```

Symbolic :- ماعنا مشكلة سواء First / Second او اي شي

① لازم نحدد الـSymboles يلي بدك تستخدمهم

$$\ddot{X} y + y = \log(t)$$

$x(t)$, $y(t)$

```
Syms x(t) y(t)

eqn = [diff(x,t,2) * y + ------- ];
dx = diff (x,t);
dy = diff(y,t);
Cond = [ x(0) == 0 , . ------ ]
Sol = dsolve (eqn, Cond)
```