Final quiz

Sunday, January 12, 2025 9:49 PM

- RANDOM NUMBER GENERATION
- RANDOM VARIATE GENERATION
- CHAPTER 10

1. RANDOM NUMBER GENERATION (RNG)

↳ Given $\underline{a = 17}$, $\underline{c = 43}$, $\underline{m = 100}$, $\underline{X_0 = 27}$

$\boxed{\text{Solution}}$

$$X_i = [a * X_{i-1} + c] \bmod m$$

$$X_i = [17 * X_{i-1} + 43] \bmod 100$$

$$X_1 = [17 * X_0 + 43] \bmod 100 = [17 * 27 + 43] \bmod 100 = 2 \rightarrow R_1 = \frac{2}{m} = \frac{2}{100} = 0.02$$

$$X_2 = [17 * X_1 + 43] \bmod 100 = [17 * 2 + 43] \bmod 100 = 77 \rightarrow R_2 = \frac{77}{m} = \frac{77}{100} = 0.77$$

$$X_3 = [17 * X_2 + 43] \bmod 100 = [17 * 77 + 43] \bmod 100 = 52 \rightarrow R_3 = \frac{52}{m} = \frac{52}{100} = 0.52$$

2. RANDOM VARIATE GENERATION

↳ Given the Random numbers from above $\{0.02, 0.77, 0.52\}$

Find Expo(5)

$\boxed{\text{Solution}}$

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* $\mu = 5 \rightarrow \lambda = \frac{1}{\mu} = \frac{1}{5} = 0.2$

* $X_i = \frac{-1}{\lambda} * \ln\left[1 - R_i\right]$

$$-\frac{1}{\lambda}\ln(1 - R_i)$$

$$X_i = \frac{-1}{(1/5)} * \ln\left[1 - R_i\right]$$

$$X_i = -5 * \ln\left[1 - R_i\right]$$

$$X_1 = -5 * \ln\left[1 - 0.02\right] = 0.1010$$

$$X_2 = -5 * \ln\left[1 - 0.77\right] = 7.3484$$

$$X_3 = -5 * \ln\left[1 - 0.52\right] = 3.6698$$

continuous

Uniform Distribution [UN(a,b)] (X = a + (b-a)R)

$UN(\overset{a}{1}, \overset{b}{3})$ يعطيني مثلاً

$X = 1 + (3-1)R$

$X = 1 + 2R$

$$\left\{\begin{aligned} X_1 &= 1+2(0.02) = 1.04 \\ X_2 &= 1+2(0.77) = 2.54 \\ X_3 &= 1+2(0.52) = 2.04 \end{aligned}\right.$$

- A Discrete Uniform Distribution $\quad X = \text{roundup}\ (kR)$

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EXAMPLE:

For example, consider the generating of a random variate X that is uniformly distributed on $\{1, 2, \ldots, 10\}$. The variate, X, might represent the number of pallets to be loaded onto a truck. Using Table A.1 as a source of random numbers R and using Equation (8.16) with $k = 10$ yields

$R_1 = 0.78 \quad X_1 = \lceil 7.8 \rceil = 8 \rightarrow X_1 = \lceil R_1 K \rceil = \lceil 0.78 * 10 \rceil = \lceil 7.8 \rceil = 8$

$R_2 = 0.03 \quad X_2 = \lceil 0.3 \rceil = 1 \rightarrow X_2 = \lceil R_2 K \rceil = \lceil 0.03 * 10 \rceil = \lceil 0.3 \rceil = 1$

$R_3 = 0.23 \quad X_3 = \lceil 2.3 \rceil = 3 \rightarrow X_3 = \lceil R_3 K \rceil = \lceil 0.23 * 10 \rceil = \lceil 2.3 \rceil = 3$

$R_4 = 0.97 \quad X_4 = \lceil 9.7 \rceil = 10 \rightarrow X_4 = \lceil R_4 K \rceil = \lceil 0.97 * 10 \rceil = \lceil 9.7 \rceil = 10$

Consider the discrete uniform distribution on $\{1, 2, \ldots, k\}$ with pmf and cdf given by

$$p(x) = \frac{1}{k}, \quad x = 1, 2, \ldots, k$$

and

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{k}, & 1 \le x < 2 \\ \frac{2}{k}, & 2 \le x < 3 \\ \vdots & \vdots \\ \frac{k-1}{k}, & k-1 \le x < k \\ 1, & k \le x \end{cases}$$

Let $x_i = i$ and $r_i = p(1) + \cdots + p(x_i) = F(x_i) = i/k$ for $i = 1, 2, \ldots, k$. Then, from Inequality (8.13), it can be seen that, if the generated random number R satisfies

$$r_{i-1} = \frac{i-1}{k} < R \le r_i = \frac{i}{k} \tag{8.14}$$

then X is generated by setting $X = i$. Now Inequality (8.14) can be solved for i:

$$\begin{aligned} i - 1 &< Rk \le i \\ Rk &\le i < Rk + 1 \end{aligned} \tag{8.15}$$

Let $\lceil y \rceil$ denote the smallest integer $\ge y$. For example, $\lceil 7.82 \rceil = 8$, $\lceil 5.13 \rceil = 6$, and $\lceil -1.32 \rceil = -1$. For $y \ge 0$, $\lceil y \rceil$ is a function that rounds up. This notation and Inequality (8.15) yield a formula for generating X, namely

$$X = \lceil Rk \rceil \tag{8.16}$$

For example, consider the generating of a random variate X that is uniformly distributed on $\{1, 2, \ldots, 10\}$. The variate, X, might represent the number of pallets to be loaded onto a truck. Using Table A.1 as a source of random numbers R and using Equation (8.16) with $k = 10$ yields

- Normal
  - if I had 2 RANDOM NUMBERS "R_1" & "R_2"
    - ① standard normal random variate $[\mu = 0, \sigma = 1]$
    - ② random normal variate $[\mu =$ أي رقم $, \sigma =$ أي رقم $]$

① standard random normal variate

$$Z_1 = \left[\sqrt{-2 * \ln(R_1)}\right] \left[\cos(2\pi R_2)\right]$$

$$Z_2 = \left[\sqrt{-2 * \ln(R_1)}\right]\left[SIN(2\pi R_2)\right]$$

mutually independent. Combining Equations (8.26) and (8.27) gives a direct method for generating two independent standard normal variates, Z_1 and Z_2, from two independent random numbers, R_1 and R_2:

$$Z_1 = (-2\ln R_1)^{1/2}\cos(2\pi R_2)$$
$$Z_2 = (-2\ln R_1)^{1/2}\sin(2\pi R_2) \qquad (8.28)$$

FOR EXAMPLE:- $R_1 = 0.1758$ & $R_2 = 0.1489$

$$Z_1 = \left[\sqrt{-2 * \ln(0.1758)}\right]\left[COS(2\pi * 0.1489)\right] = 1.11$$

$$Z_2 = \left[\sqrt{-2 * \ln(0.1758)}\right]\left[SIN(2\pi * 0.1489)\right] = 1.50$$

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To illustrate the generation scheme, consider Equation (8.28) with $R_1 = 0.1758$ and $R_2 = 0.1489$. Two standard normal random variates are generated as follows:

$$Z_1 = [-2\ln(0.1758)]^{1/2}\cos(2\pi 0.1489) = 1.11$$
$$Z_2 = [-2\ln(0.1758)]^{1/2}\sin(2\pi 0.1489) = 1.50$$

(2) Random normal variate

$$X_i = \mu + \sigma Z_i$$

For Example; $\mu = 10$, $\sigma^2 = 4$ [$\sigma = 2$]

$$X_1 = 10 + 2(Z_1) = 10 + 2(1.11) = 12.22$$

$$X_2 = 10 + 2(Z_2) = 10 + 2(1.50) = 13$$

To obtain normal variates X_i with mean μ and variance σ^2, we then apply the transformation

$$X_i = \mu + \sigma Z_i \qquad (8.29)$$

to the standard normal variates. For example, to transform the two standard normal variates into normal variates with mean $\mu = 10$ and variance $\sigma^2 = 4$, we compute

$$X_1 = 10 + 2(1.11) = 12.22$$
$$X_2 = 10 + 2(1.50) = 13.00$$

(3.) CHAPTER 10