

Electrical engineering

These notes were prepared to help students better understand the course. However, please note that they are not sufficient on their own. It is strongly recommended to practice the suggested questions provided by the instructor to fully grasp the material and prepare well for the exam.



Units & power Supply

* units:-

time \rightarrow sec
 Electrical current \rightarrow A
 voltage \rightarrow volt
 mass \rightarrow Kg
 length \rightarrow watt
 charge \rightarrow C

* preFixes

pico $\rightarrow 10^{-12}$	kilo $\rightarrow 10^3$
nano $\rightarrow 10^{-9}$	Mega $\rightarrow 10^6$
Micro $\rightarrow 10^{-6}$	giga $\rightarrow 10^9$
Milli $\rightarrow 10^{-3}$	

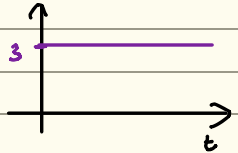
* power supply:-

a) types:

- 1) voltage Source
- 2) current Source

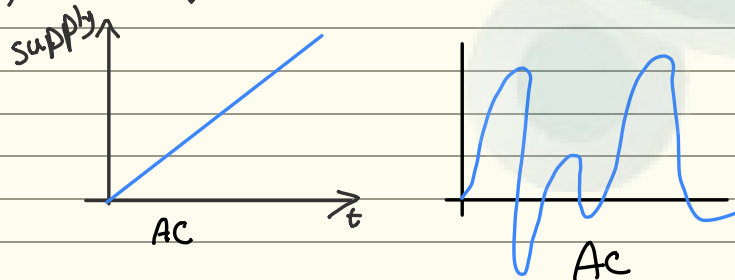
b) output signal

1) DC o/p signal \rightarrow Value of the supply is fixed with time



$V = 3V \rightarrow$ fixed
 $I = 2A \rightarrow$ fixed

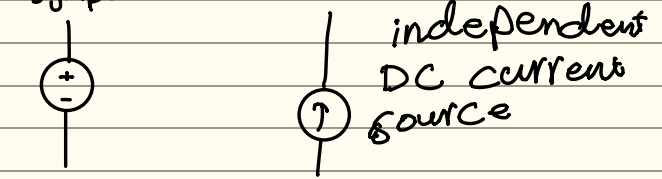
2) AC o/p signal \rightarrow It's value vary with time



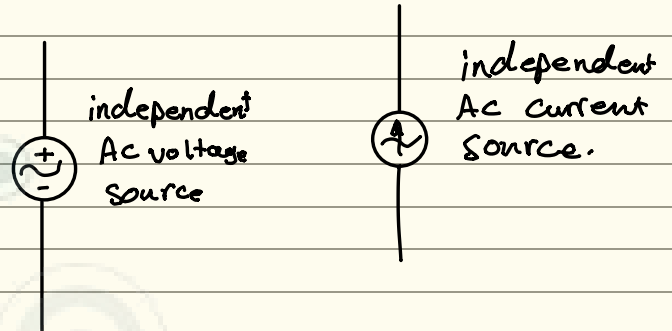
Ex:- $V(t) = 3 \sin(4t)$
 $i(t) = 2e^{-t}$

c) Dependancy:-

1) independent power supply
 does not depend on any element in the ckt
 symbol

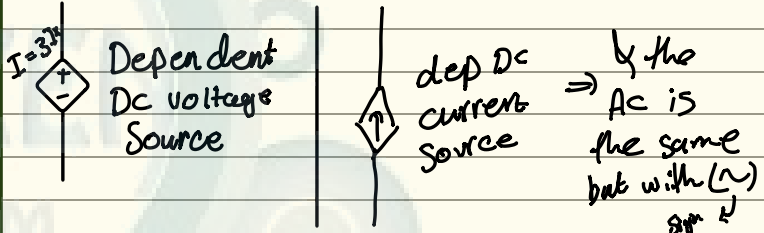


independant
 DC voltage
 Source



2) It's value depend on of function of voltage & current

symbol:-



* loads \rightarrow resistance R
 \rightarrow inductive L
 \rightarrow capacitive C

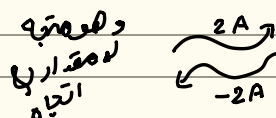
$\Phi \rightarrow Q$

$q_e = 1.6 \times 10^{-19} C \Rightarrow$ electron

* Electrical current: charges in motion in a wire
 for a closed loop

$I = \frac{dQ}{dt} \rightarrow$ slope of the current

unit \rightarrow Ampere $\rightarrow A, mA, \mu A$



$Q(t) = \int_{t_0}^t i(\tau) \cdot d\tau + \text{initial condition}$

for graphs
 it will be
 area under
 the curve

Ex: $Q(t) = 3e^{-t}$ C, find $i(t)$

$$i(t) = \frac{dQ}{dt} = -3e^{-t}$$

$$i(t) = -3e^{-t}$$

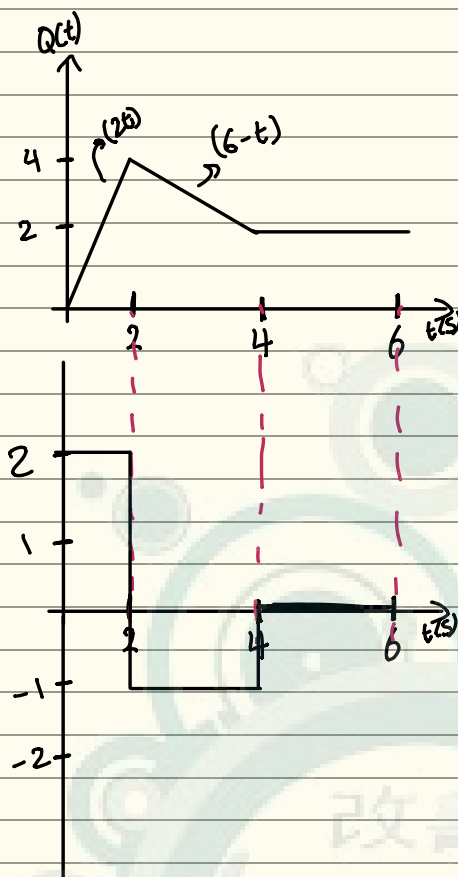
Ex: find $i(t)$

slope 1 = 2

Slope 2 = -1

Slope 3 = 0

$2 + (-1) + 0 = 1$
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تفاضل أكثر بأقل
العددي

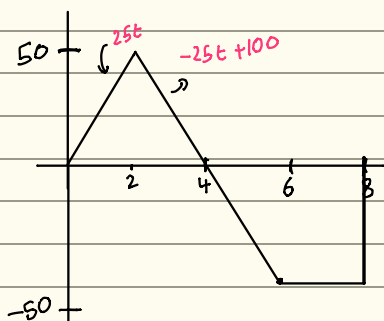


Ex: find the:

a) Total amount of charge $Q(t)$

b) total amount of charge up to $t=6$ sec

c) total amount of charge between $t=2$ sec to $t=8$ sec



a) $Q(t) \Rightarrow i(t) = 25t$, initial = 0 = $Q(0)$

$$0 \leq t < 2 \Rightarrow Q(t) = \int_0^t 25t = \frac{25t^2}{2} \Big|_0^t$$

$$= 12.5t^2$$

$$\downarrow 12.5 \times 4 = 50$$

$\Rightarrow 2 \leq t \leq 6$

$i(t) + Q \text{ initial} \rightarrow Q(2)$

$$\int_2^t -25t + 100 = \frac{-25t^2}{2} + 100t + Q(2) = 12.5t^2 + 100t + 50$$

$$(12.5t^2 + 100t) - (12.5 \times 4 + 200)$$

$$12.5t^2 + 100t - (50 + 200)$$

$$-12.5t^2 + 100t - 150 + 50$$

$$-12.5t^2 + 100t - 100$$

$$6 \leq t \leq 8$$

$$i(t) + Q(t)$$

$$\int_6^t -50 dt +$$

$$-50t \Big|_6^t + 50$$

$$-50t - (-50 \times 6) + 50$$

$$-50t + 300 + 50$$

$$-50t + 350$$

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طالب
In general

كجواب ورقه

b)

$$A = \frac{1}{2} \times 2 \times 50 + \frac{1}{2} \times (2) \times (-50)$$

$$100 + -50 = 50$$

b) amount charge up to 6 sec

$$Q = Q(6) - Q(0)$$

$$50 - 0 = 50$$

c) -100

* voltage :- work needed to move unit charge in a wire

$$V = \frac{\text{work}}{\text{charge}} \rightarrow \text{volt}$$

$U_A = 5V$ $U_B = 3V$
 A B voltage at ground is zero
 $U_{AB} = U_A - U_B$
 $= 5 - 3 = 2$

$$U_{BA} = U_B - U_A$$

$$= 3 - 5 = -2$$

$$U_{AB} = -U_{BA}$$

$$\Rightarrow V_{\text{a ground}} = V_A - U_{\text{ground}}$$

$$= 5 - 0 = 5V$$

* power: work needed per unit time.

$$P = \frac{\text{work}}{\text{time}} \Rightarrow \text{watt}$$

$$P = I \times V$$

* if sign of power is positive \Rightarrow power is dissipated consumed

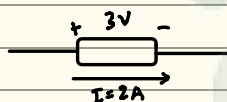
" " " " " Negative \Rightarrow power is generated supplied

* passive sign convention

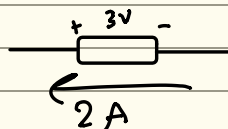
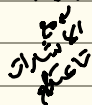
if the current from (+) sign $\Rightarrow P \Rightarrow (+)$

" " " " " (-) sign $\Rightarrow P \Rightarrow (-)$

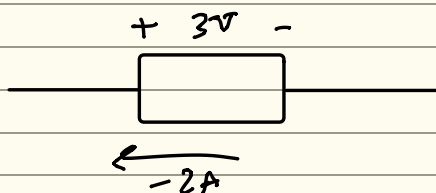
Ex:-



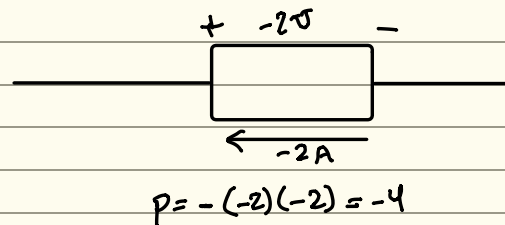
$$P = + (2)(3) = +6W \text{ consumed}$$



$$P = - (2)(3) = -6W \text{ generated}$$

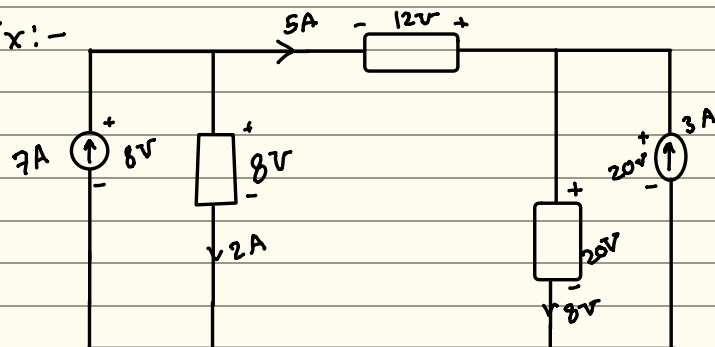


$$P = - (-2)(3) = +6W \rightarrow \text{consumed}$$



$$P = - (-2)(-2) = -4$$

Ex:-



find power for each element:-

$$- (8)(7) + (8)(2) + - (5)(2) + (8)(20) - (3)(20)$$

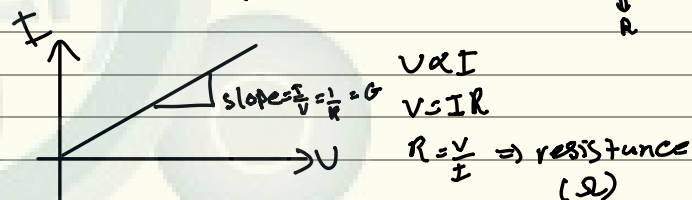
$$= \text{zero}$$

generated - consumed

$$176 - 176 = 0$$

* ohm Law:-

\Rightarrow current that passes in a wire face resistance



ohm's Law : voltage across an element is directly proportional to the current

* conductance: $G = \frac{1}{R}$ (S, V, S⁻¹)

$$P = I V$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

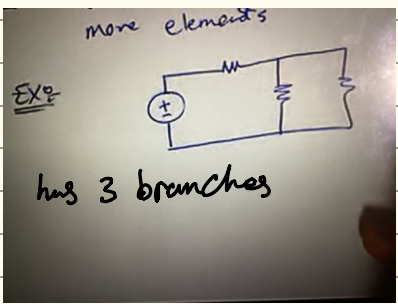
Ex: wire has resistance of 3Ω, pass a current of 2A, find the voltage across the wire

$$V = IR$$

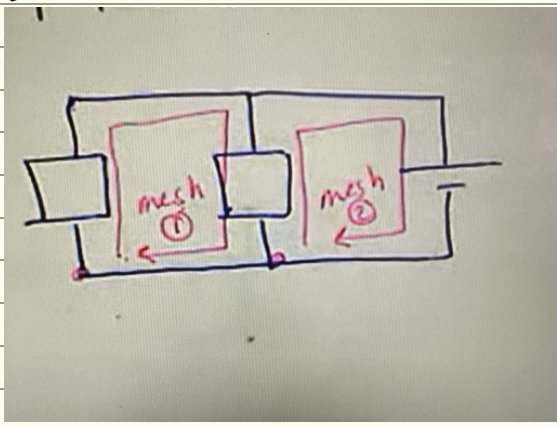
$$V = 3 \times 2 = 6V$$

Definitions of electrical circuit network

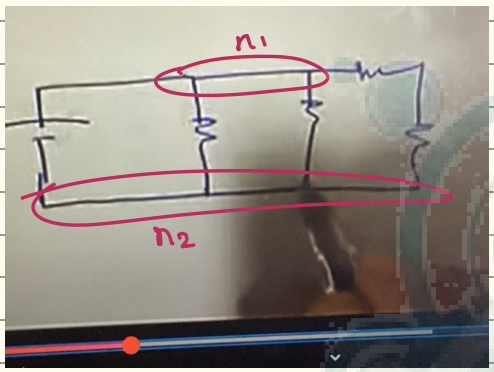
① Branch :- any part of the circuit that has one or more elements



loop that does not contain other loop.
Ex:-

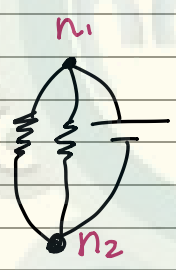
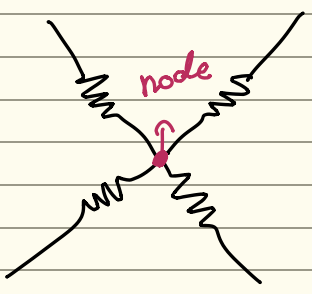


② Node: junction of two or more branches.



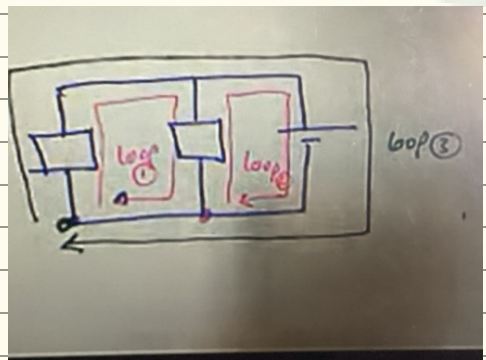
has 2 nodes

Note: If the connection bet nodes has no element \rightarrow consider as one node



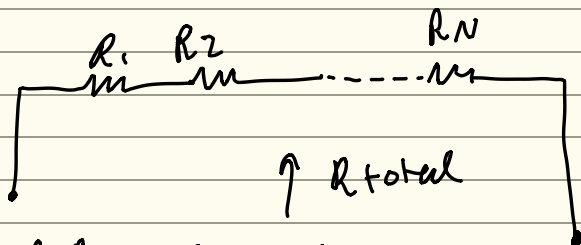
③ Loop: closed connection of Branches

Ex:-



So this ckt has 2 meshes.

* series connection of Resistors:-



$$R_{\text{Total}} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^N R_n$$

in series connection \Rightarrow same current pass through them

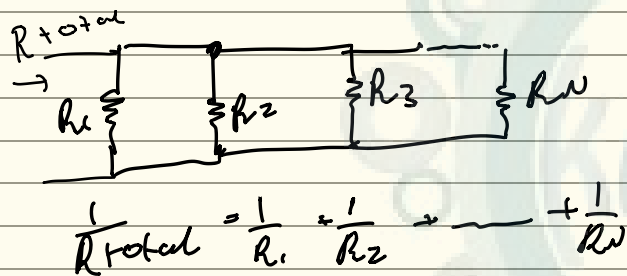
Voltage source $= I \times R_{\text{total}}$

\Rightarrow in series connection \Rightarrow Voltage is different for each element.

$$\begin{aligned} V_1 &= I \times R_1 \\ V_2 &= I \times R_2 \\ V_3 &= I \times R_3 \end{aligned} \quad \left. \begin{array}{l} \text{the current is} \\ \text{the same for} \\ \text{all of them} \end{array} \right\}$$

total = the voltage of the original power supply

* parallel connection of Resistors:-



$$\frac{1}{R_{\text{Total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

$$R_{\text{Total}} = \frac{R_1 R_2}{R_1 + R_2}$$

for two resistors only

④ for parallel connection \Rightarrow current divided between branches

the voltage is the same for all the branches

Resistance

in series

$$R_{\text{Total}} = R_1 + R_2 + \dots + R_N$$

in parallel

$$\frac{1}{R_{\text{Total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

current divides

in series
the same for all

in parallel
the current will divide

in series
divided

in parallel
the same for all

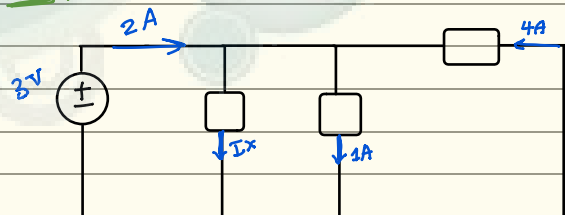
* Kirshoff current Law (Kcl):-

the sum of the current at a node must equal to zero

$$\sum I_{\text{node}} = 0$$

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Exo:



* How many branches this circuit consist of?

4 branches

* How many nodes?

2 nodes

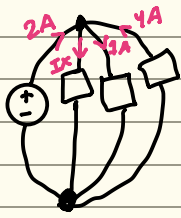
* Loops

6 Loops

* Meshes

3 Meshes

* apply KVL to find I_x



$$\sum I_{in} = \sum I_{out}$$

$$2 + 4 = 1 + I_x$$

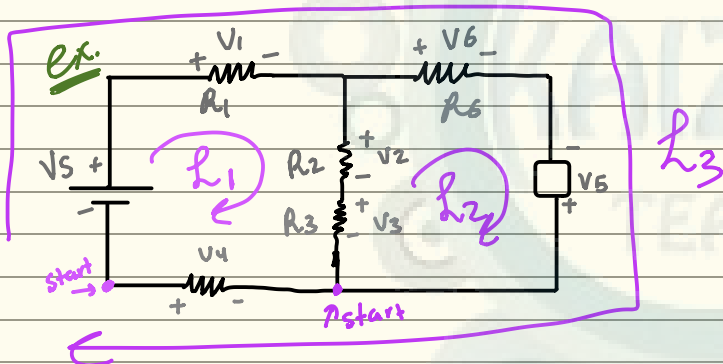
$$6 = 1 + I_x$$

$$I_x = 5A$$

Kirchhoff voltage Law (KVL):

the net voltage around a closed ckt sum is zero.

$$\sum V_{closed} = 0 \text{ loop}$$



write KVL equations:-

$$\sum V = 0$$

loop 1:-

$$-V_5 + V_1 + V_2 + V_3 - V_4 = 0$$

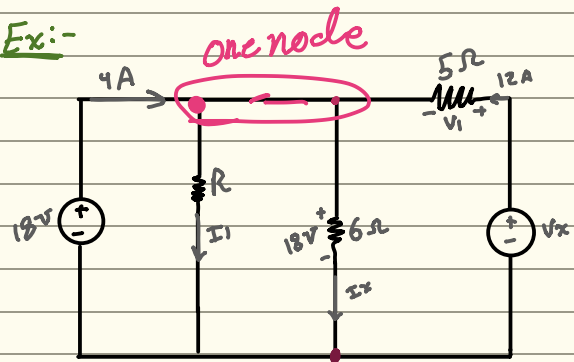
loop 2:-

$$-V_3 - V_2 + V_6 - V_5 = 0$$

loop 3:-

$$-V_5 + V_1 + V_6 - V_5 - V_4 = 0$$

Ex:-



find: V_1, I_x, V_x, R, I_1

we already say that resistance in parallel has the same voltage

applying ohm's Law to find I_x

$$V = IR$$

$$\frac{18}{6} = \frac{I_x (6)}{6} \Rightarrow I_x = 3A$$

applying KCL on n_1 :-

$$12 = I_x + I_0$$

$$12 = 3 + I_0$$

$$I_0 = 9A$$

applying KCL on n_2

$$I_0 + 4 = I_1$$

$$9 + 4 = I_1$$

$$I_1 = 13A$$

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أضيق
بالحل اي
تحت.

لا نو
node
و ان
فعل.

applying ohm's Law to find V_1

$$V = IR$$

$$V_1 = 12 \times 5$$

$$V_1 = 60V$$

use KCL at the upper node

$$\sum I_{in} = \sum I_{out}$$

$$4 + 12 = I_x + I_1$$

$$\frac{16}{3} = 3 + I_1 \Rightarrow I_1 = 13A$$

using ohm's Law to find R

$$V = IR$$

$$\frac{18}{13} = \frac{3 \times R}{13}$$

$$R = \frac{18}{13} \approx 1.4 \Omega$$

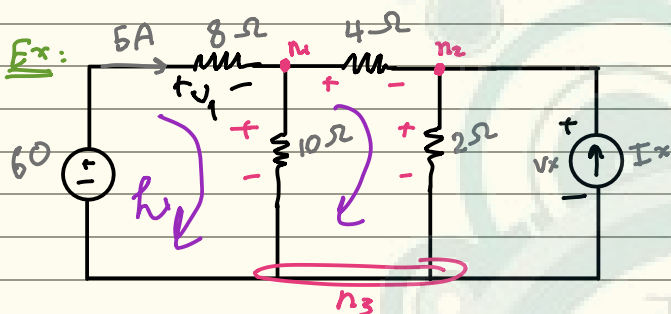
to find V_x we will use KVL:-

I will start from point •

$$-18 - 60 + V_x = 0$$

$$-78 + V_x = 0$$

$$V_x = 78 \text{ volt}$$



find: I_x , V_x & power of the current supply

applying ohm's Law on 8Ω

$$V = IR$$

$$V = 5 \times 8 = 40$$

$$V = 40$$

applying KVL on Loop 1

$$-60 + 40 + V_{10\Omega} = 0$$

$$-20 + V_{10\Omega} = 0$$

$$V_{10\Omega} = 20 \text{ volt}$$

now we will apply ohm's Law to find $I_{10\Omega}$

$$V = IR$$

$$\frac{20}{10} = \frac{I_{10\Omega} \times 10}{10}$$

$$I_{10\Omega} = 2 \text{ A}$$

applying KCL on node 1:-

$$5 = 2 + I_{4\Omega}$$

$$-2 \quad -2$$

$$I_{4\Omega} = 3 \text{ A}$$

applying ohm's Law to find $V_{4\Omega}$

$$V = IR$$

$$V = 3 \times 4 = 12$$

$$V_{4\Omega} = 12$$

applying KVL on Loop 2

$$-20 + 12 + V_{2\Omega} = 0$$

$$-8 + V_{2\Omega} = 0$$

$$V_{2\Omega} = 8 \text{ volt}$$

& node 2 & the lower wire are connected in parallel (have the same initial & final point)

$$\text{so } V_x = V_{2\Omega}$$

$$V_x = 8 \text{ volt}$$

applying KCL to find I_x on node 2:-

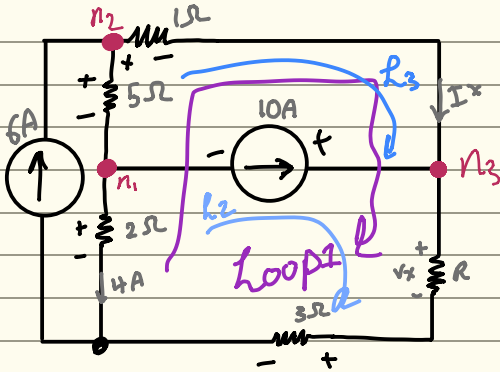
$$3 + I_x = \frac{8}{2}$$

$$\frac{3}{-3} + I_x = \frac{4}{-3}$$

$$I_x = 1 \text{ A}$$

so the power = IV

$$\underline{\underline{\text{so}}} \quad = -(8)(1) = -8 \text{ generated}$$



Find I_x , V_x , R

applying Kcl on n_1 :

$$I = 4 + 10$$

$$I_{5\Omega} = 14A$$

applying Kcl on n_2 :

$$6 = I_{5\Omega} + I_x$$

$$6 = 14 + I_x$$

$I_x = -8$ so I_x is in the opposite direction

applying Kcl for node 3:-

$$I_x + 10A = I_{3\Omega}$$

$$-8 + 10 = I_{3\Omega}$$

$$I_{3\Omega} = 2A$$

applying KVL for Loop 1

$$(-2 \times 4) + (-5 \times 14) + (1 \times -8) + V_x + (3 \times 2) = 0$$

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سالب عشاق بجاك لا يتقل

$$(-8) + (-70) + (-8) + V_x + (6) = 0$$

$$V_x = 80V$$



applying ohm's Law to find R

$$V = IR$$

$$\frac{80}{2} = \frac{2(R)}{2}$$

$$R = 40 \Omega$$

*we we did not use L_2 because we have current source & we don't find the voltage for it \rightarrow but if we have to we could

currents & voltage ال Loop2 و Loop3
Source

و بعد من بديع بعد Loop2 ال Kcl

Loop3:-

$$(-14 \times 5) + (1 \times -8) + V_{\text{current source}} = 0$$

$$V_{\text{current source}} = 78$$

finding V_x using Loop 2

$$(-4 \times 2) - V_1 + V_x + (2 \times 3) = 0$$

$$-8 - 78 + V_x + 6 = 0$$

$$V_x = 80V$$

$$V = IR$$

$$\frac{80}{2} = \frac{2R}{2}$$

$$R = 40 \Omega$$

(*) Idealization of Resistance:-

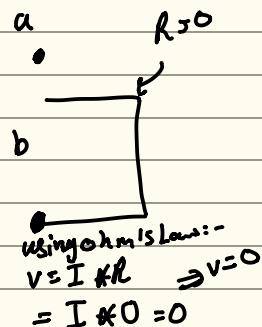
we have already said that current divides in parallel & the voltage is the same

& in series \rightarrow the current is the same for all elements & the voltage divides

but we have some Exeptions

① Short ckt $\Rightarrow R = 0 \Omega$

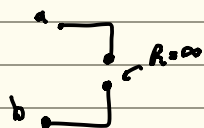
when $R = 0$ } regardless to $V = 0$ current



② open ckt:- $\Rightarrow R = \infty \Omega$

when $R = \infty \Rightarrow I = 0$

regardless to voltage across it



apply ohm's Law

$$V = IR$$

$$I = \frac{V}{R} = \frac{V}{\infty} = 0$$

so the current = zero A

★ Voltage divider:-

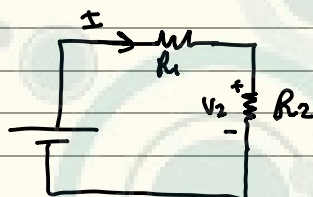
used in series connection

$$V_n = \frac{V_s \times R_n}{\sum R}$$

where

V_n : is the voltage for the resistor that I want to
 V_s : voltage for the supply
 R_n : the resistor I want

$\sum R$: sum of all resistors



$$V_2 = \frac{V_s \times R_2}{R_1 + R_2}$$

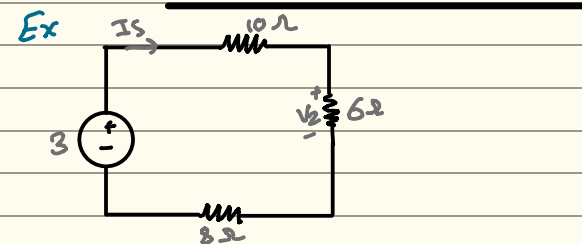
★ Current divider:-

used in parallel

$$I_n = I_s \times \frac{\frac{1}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_n}}$$



$$I_2 = I_s \times \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$



Find I_s , R_{eq} , V_2

to find R_{eq}

$$R_{eq} = 10 + 6 + 8 = 24 \Omega$$

to find I_s

$$V = IR$$

$$\frac{3}{24} = \frac{I_s \times 24}{24}$$

$$I_s = 0.125 A$$

to find V_2

$$V_2 = \frac{V_s \times R_2}{R_1 + R_2 + R_3}$$

$$V_2 = \frac{3 \times 6}{24}$$

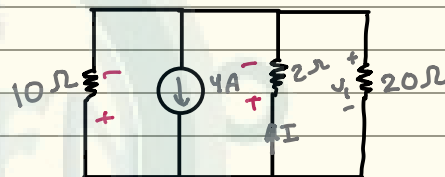
$$V_2 = \frac{18}{24} = 0.75 \text{ volt}$$

or I can use ohm's Law

$$V_2 = I_s \times 6$$

$$= 0.125 A \times 6 = 0.75 \text{ volt}$$

Ex:

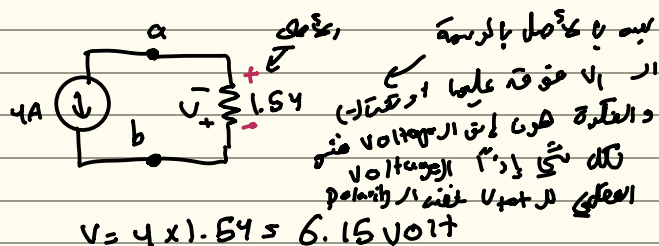


find R_{eq} , V_1 , I
 as seen by source

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{20} + \frac{1}{10} = \frac{10}{20} + \frac{1}{20} + \frac{2}{20} = \frac{13}{20}$$

$$R_{eq} = \frac{20}{13} = 1.54 \Omega$$

now the ckt looks like this



$$V = 4 \times 1.54 = 6.15 \text{ volt}$$

$$V_1 = -V$$

$$V = -V_1$$

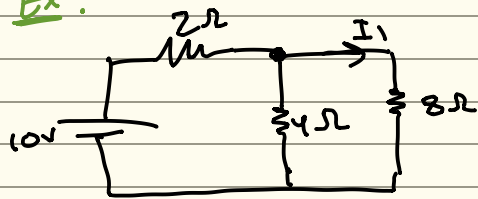
$$= -6.15 \text{ volt}$$

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 اي اننا ارادوا قتلنا او قتلنا
 والعتقوا كرمنا لاننا ارادوا قتلنا

$$V_2 = -V_1 = 6.15 \text{ volt}$$

$$I = \frac{V_2}{R} = \frac{6.15}{2} = 3.07 A$$

Ex:



Find I_1

$4\Omega || 8\Omega \Rightarrow$ if I find R_{eq} for them I can use voltage divider

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$

$$R_{eq} = \frac{8}{3} = 2.7$$

now we are going to use voltage divider

$$V_{eq} = \frac{V_s \times R_{eq}}{2 + 2.7} = \frac{10 \times 2.7}{4.7} = \frac{27}{4.7} = 5.74 \text{ volt}$$

& $4\Omega || 8\Omega$ so they have the same voltage

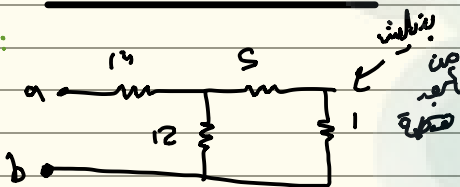
using ohm's law

$$V = I_1 \times R$$

$$\frac{5.74}{8} = \frac{I_1 \times 8}{8}$$

$$I_1 = 0.72 A$$

Ex:



find R_{eq} seen from a & b

$$R_{eq1} = 6 + 1 = 6\Omega$$

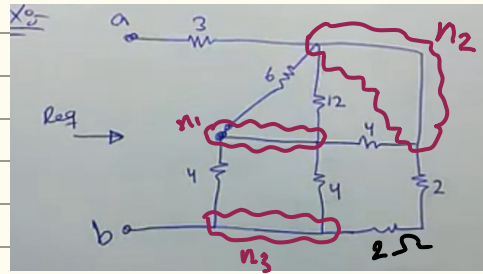
$$6\Omega || 12\Omega$$

$$\frac{1}{R_{eq2}} = \frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12}$$

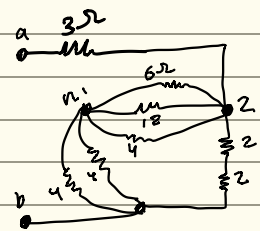
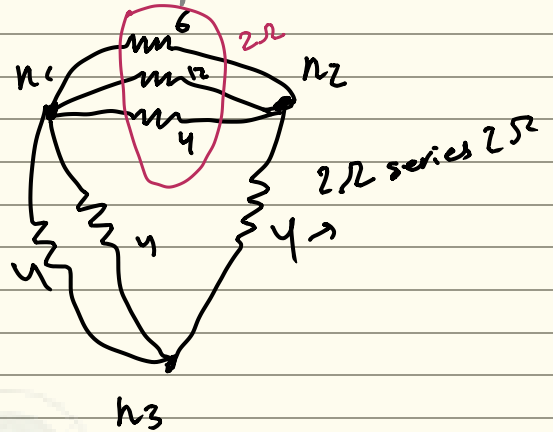
$$R_{eq2} = \frac{12}{3} = 4\Omega$$

$$14 + 4 = 18\Omega$$

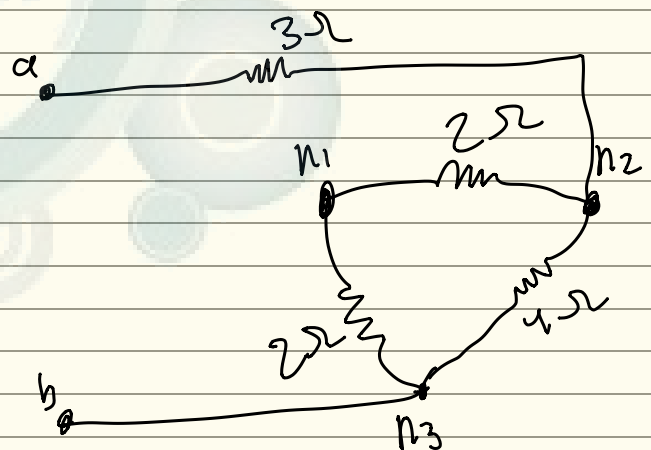
Ex



find R_{eq} seen from a & b



draw a simplified circuit



2Ω series with 2Ω

$$2\Omega + 2\Omega = 4\Omega$$

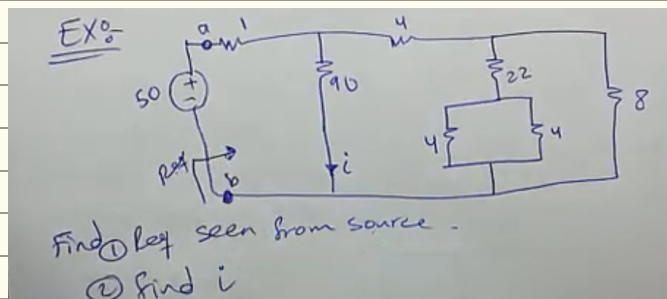
$$4\Omega || 4\Omega$$

$$\frac{1}{R_{eq1}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$R_{eq1} = 2\Omega$$

2Ω in series 3Ω

$$2 + 3 = 5\Omega$$



①
 $4\Omega \parallel 4\Omega$
 $\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad R_{eq} = 2$

2Ω & 22Ω series

$22 + 2 = 24\Omega$

8Ω || 24Ω

$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{24} = \frac{3}{24} + \frac{1}{24} = \frac{4}{24}$

$R_{eq} = \frac{24}{4} = 6\Omega$

4Ω series 6Ω

$4 + 6 = 10\Omega$

10Ω || 90Ω

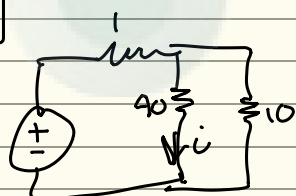
$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{90} = \frac{9}{90} + \frac{1}{90} = \frac{10}{90}$

$R_{eq} = 9\Omega$

1Ω series 9Ω = 1 + 9 = 10Ω

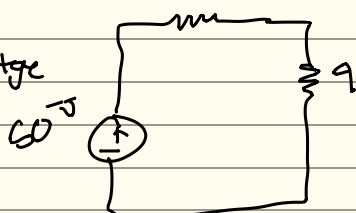
$R_{eq} = 10\Omega$

we will stop here
to find i



i is not related to 9Ω

but if I found the voltage
for 9Ω it's the
same voltage for
90 & 10Ω



$V_{9\Omega} = \frac{V_s \times R_{9\Omega}}{2R}$

$V_{9\Omega} = \frac{50(9\Omega)}{10}$

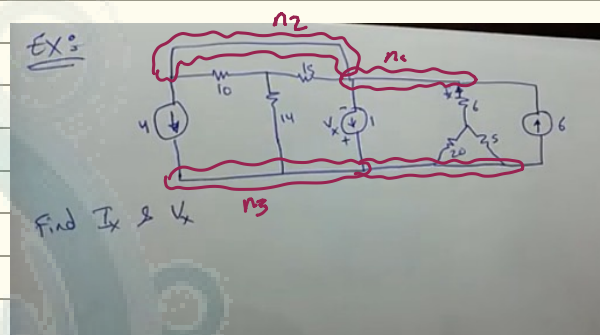
$V_{9\Omega} = 45V$

& it's the same for 90Ω & 10Ω
because they are connected in
parallel

$V = IR$

$\frac{45}{90} = \frac{I \times 90}{90}$

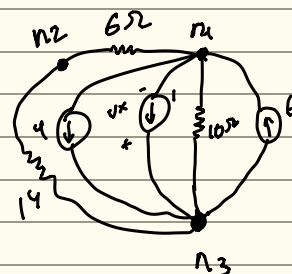
$I = 0.5A$



we can not use KCL / KVL / V/C divider

so I will simplify it

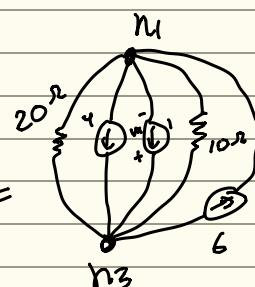
I will see how many nodes I have

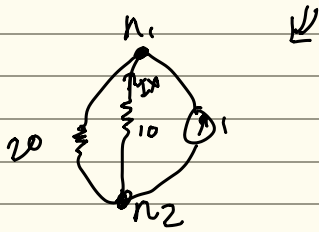


6Ω & 14Ω series

$6 + 14 = 20\Omega$

no current sources





now for I_x current division

$$I_x = \frac{I_s \frac{1}{R}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$I_x = \frac{(1)(10)}{2 \cdot \frac{1}{10} + \frac{1}{20}} = \frac{10}{\frac{2}{20} + \frac{1}{20}} = \frac{10}{\frac{3}{20}} = \frac{10 \times 20}{3} = \frac{200}{3}$$

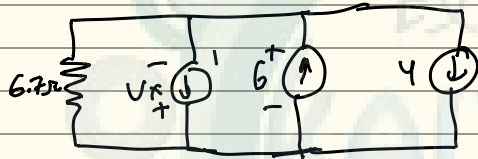
$$I_x = \frac{10}{\frac{3}{20}} = 10 \times \frac{20}{3} = \frac{200}{3}$$

$$I_x = -66.67 \text{ A}$$

I_x with
التيار
الذي داخل
المقاومة
معدني لـ V_x

عند V_x

now we must find R_{eq} & redrawing the supplies.



$$10 \Omega \parallel 20 \Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} = \frac{2}{20} + \frac{1}{20}$$

$$R_{eq} = \frac{20}{3} = 6.7 \Omega$$

Voltage for V_x is the same for the resistances & all current sources

المقاومة و التيار و V_x هي واحدة
التيار

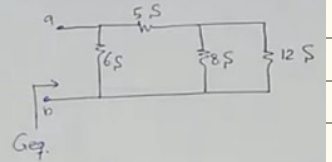
$$V = IR$$

$$V = (6.7)(1) = 6.7 \rightarrow$$

التيار
Polarity
المقاومة و التيار

$$V_x = -6.7 \text{ V}$$

Find G_{eq} between a & b.



$$G = \frac{1}{R} \text{ (V, } \Omega^{-1}, S)$$

جيبه R و G باقيا.

بجول كل G الى R

$$R_1 = \frac{1}{G} = \frac{1}{12} = 0.083 \Omega$$

$$R_2 = \frac{1}{8} = 0.125 \Omega$$

$$R_3 = \frac{1}{5} = 0.2 \Omega$$

$$R_4 = \frac{1}{6} = 0.167 \Omega$$

$$R_1 \parallel R_2$$

$$\frac{1}{0.083} + \frac{1}{0.125} = \frac{1664}{83}$$

$$R_{eq} = 0.05 \Omega$$

$$0.2 \text{ \& } 0.05 \text{ series}$$

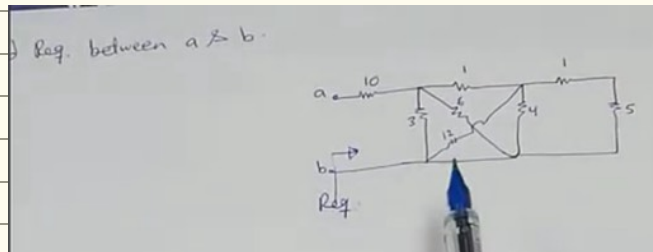
$$= 0.25 \Omega$$

$$0.167 \Omega \parallel 0.25$$

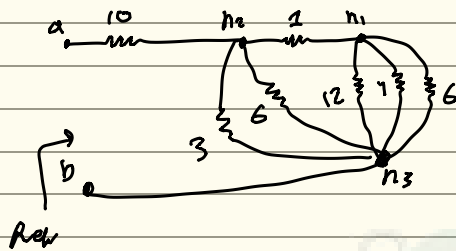
$$= \frac{1}{0.167} + \frac{1}{0.25}$$

$$R_{eq} = 0.1$$

$$G = \frac{1}{R} = \frac{1}{0.1} = 10 \text{ S}$$



معجزة بها نزل شدة الدارة :-

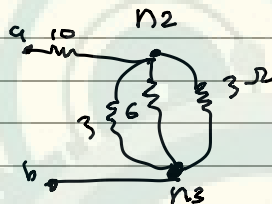


$$4\Omega \parallel 6\Omega \parallel 12\Omega$$

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{3}{12} + \frac{2}{12} + \frac{1}{12} = \frac{6}{12}$$

$$R_{eq} = 2\Omega$$

$$2\Omega \text{ \& \; } 1\Omega \text{ series} \\ 3\Omega$$



$$6\Omega \parallel 3\Omega$$

$$\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$$

$$R_{eq} = 2\Omega$$

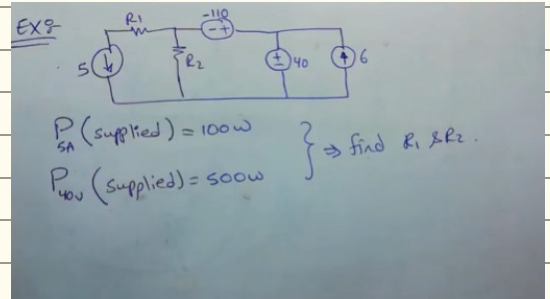
$$2\Omega \parallel 3\Omega$$

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$R_{eq} = 1.2$$

$$1.2 \text{ \& \; } 10 \text{ in series}$$

$$10 + 1.2 = 11.2\Omega$$



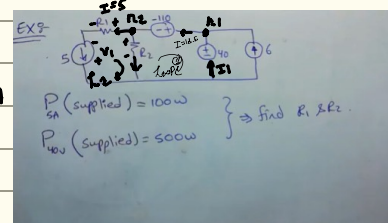
هون رتزي انو بركتيك supplied يعني به قدمين
(-) و رطبع من ادر (+) ادر ادر ادر ادر ادر

$$P_{40V} = 500 = IV$$

$$\frac{500}{40} = \frac{I \cdot 40}{40} \Rightarrow I = 12.5A$$

$$P_{5A} = 100 = IV$$

$$\frac{100}{5} = \frac{5V}{5} \Rightarrow V = 20V$$



now we are going to apply Kcl
for node 1

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + 6 = I_0$$

$$12.5 + 6 = I_0$$

$$I_0 = 18.5A$$

Kcl for n2

$$\sum I_{in} = \sum I_{out}$$

$$18.5 = \frac{V}{6} + x \rightarrow \frac{18.5}{6} + x = 18.5$$

$$I_{R2} = 13.5A$$

KVL for loop 1

$$-13.5 \times R_2 - 110 + 40 = 0$$

$$-13.5 R_2 + 110 + 40 = 0$$

$$+13.5 R_2 = +150$$

$$\frac{13.5 R_2 = 150}{13.5} \Rightarrow R_2 = 11.11\Omega$$

KVL for loop 2

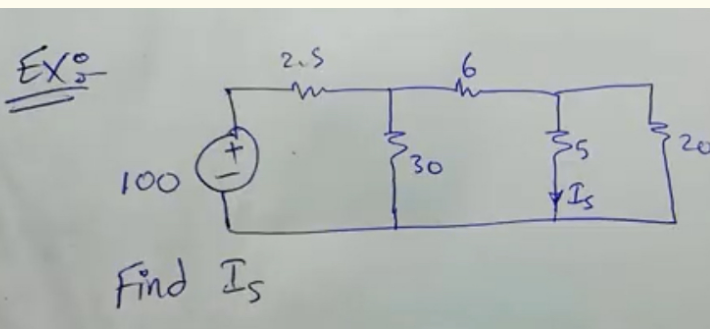
$$+20 - 5R_1 + 13.5 R_2 = 0$$

$$20 - 5R_1 + 150 = 0$$

$$170 - 5R_1 = 0$$

$$\frac{170}{5} = \frac{5R_1}{5}$$

$$R_1 = 34\Omega$$



we will find R_{eq}

$$\frac{1}{20} + \frac{1}{5} = \frac{1}{20} + \frac{4}{20} = \frac{5}{20}$$

$$R_{eq} = 4\Omega$$

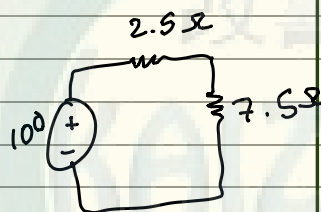
$$4 + 6 = 10\Omega$$

$$10\Omega \parallel 30\Omega$$

$$\frac{1}{10} + \frac{1}{30} = \frac{3}{30} + \frac{1}{30} = \frac{4}{30}$$

$$R_{eq} = \frac{30}{4} = 7.5$$

now applying voltage division

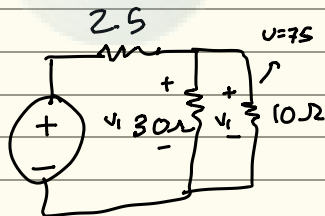


$$V_{7.5\Omega} = V_s \cdot \frac{7.5}{10}$$

$$= \frac{100 \times 7.5}{10} = 75V$$

& this is the same voltage for 30Ω & 10Ω because they are parallel

now we will consider 30Ω resistance as a supply & apply voltage division



$$V_2 = \frac{V_s \cdot 4}{10} = \frac{75 \cdot 4}{10} = 30V$$

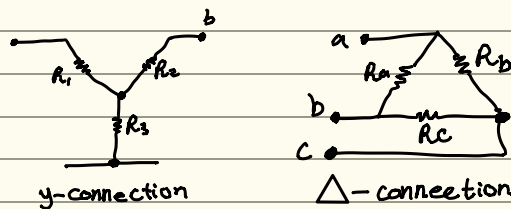
V_2 the same voltage for 5Ω & 20Ω

$$V = IR$$

$$\frac{30}{5} = \frac{I \cdot 5}{5}$$

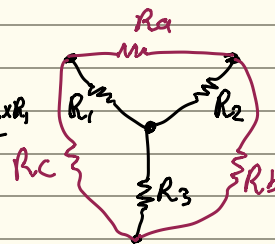
$$I_5 = 6A$$

Delta- π connection for resistors



conversion from y- Δ

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

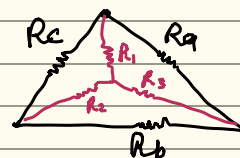


$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

conversion from Δ to y:-

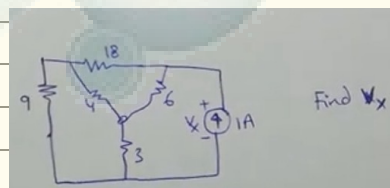
$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c}$$



$$R_2 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Ex:



V_x is the same for R_{eq} because they are connected in parallel

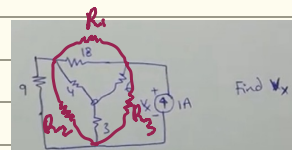
I will use y- Δ conversion

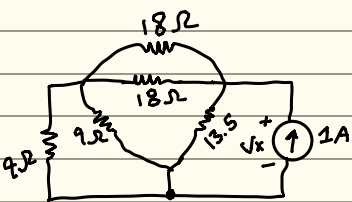
$$R_1 = \frac{(4 \times 6) + (4 \times 3) + (3 \times 6)}{3}$$

$$= \frac{54}{3} = 18\Omega$$

$$R_2 = \frac{54}{6} = 9\Omega$$

$$R_3 = \frac{54}{4} = 13.5\Omega$$





$$18\Omega \parallel 18\Omega$$

$$\frac{1}{18} + \frac{1}{18} = \frac{2}{18}$$

$$R_{eq1} = \frac{18}{2} = 9$$

$$9\Omega \parallel 9\Omega$$

$$\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$$

$$R_{eq} = \frac{9}{2} = 4.5$$

now the new one looks like

now we have 9Ω series with 4.5Ω

$$R_{eq} = 9 + 4.5 = 13.5\Omega$$

$$\frac{1}{13.5} + \frac{1}{13.5}$$

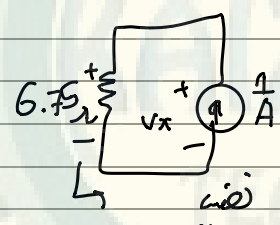
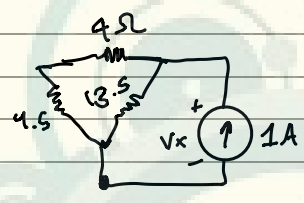
$$R_{eq} = 6.75\Omega$$

so

$$V = IR$$

$$V = (6.75)1 = 6.75$$

by this voltage is the same for (Vx)



pol/crit 1
w/ 1.2
- 2.2

* fundamental techniques for resistive circuit analysis:-

- ① Nodal Analysis
- ② Mesh Analysis
- ③ Super position
- ④ source transformation
- ⑤ Thevenin & Norton

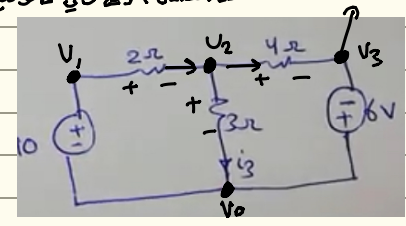
① Nodal analysis

Steps:-

- ① Determine the nodes
- ② assume one of the nodes as a reference
- ③ name the nodes using V_1, V_2, V_3
- ④ if we have voltage source between node & reference $\Rightarrow V_i$ at the node is known
- ⑤ Apply Kcl at each node

هاي و اي زيها عالشان بغير node عالشان 4 ريز تا 13.5 تا 9 تا 9 تا 1A

Ex:



find i_3 , $P(2\Omega) = ??$

using nodal analysis

كان الاسب قومه
و 9 تا 13.5 تا 9 تا 1A
reference = V3

$$V_1 = 10V, V_3 = -6V \rightarrow$$

apply Kcl at node 2

$$\sum I_{in} = \sum I_{out}$$

$$I_{2\Omega} = I_{4\Omega} + I_{3\Omega}$$

$$\frac{(V_1 - V_2)}{2} = \frac{(V_2 - V_3)}{4} + \frac{V_2}{3}$$

$$\frac{(10 - V_2)}{2} = \frac{(V_2 + 6)}{4} + \frac{V_2}{3}$$

$$5 - \frac{V_2}{2} = \frac{V_2}{4} + \frac{6}{4} + \frac{V_2}{3}$$

$$5 - \frac{6}{4} = \frac{V_2}{4} + \frac{V_2}{3} + \frac{V_2}{2}$$

$$\frac{12 \times \frac{7}{4}}{\frac{13}{3} \times \frac{2}{2}} = \frac{13}{12} V_2 \times \frac{12}{13}$$

$$V_2 = 3.24V$$

$$i_3 \Rightarrow V = IR$$

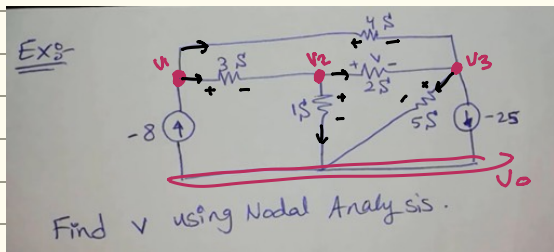
$$3.24 = i_3 \times 3$$

$$i_3 = 1.077$$

$$P(2\Omega) \Rightarrow V_{2\Omega} = V_1 - V_2 = 10 - 3.24$$

$$V_{2\Omega} = 6.76V$$

$$P_{2\Omega} = \frac{V^2}{R} = \frac{(6.76)^2}{2} = 22.85 \text{ watt}$$



find v using nodal analysis

$$G = \frac{1}{R}$$

$$I = V * G$$

writing nodal equation for node 1

$$-8 = (v_1 - v_2) \times 3 + (v_1 - v_3) \times 4$$

writing nodal equation for node 2

$$(v_1 - v_2) \times 3 = (v_2 - v_3) \times 2 + v_2 \times 1$$

writing nodal equation for node 3

$$4(v_1 - v_3) + 2(v_2 - v_3) = (v_3 \times 5) + -25$$

$$4v_1 - 4v_3 + 2v_2 - 2v_3 - 5v_3 = -25$$

$$7v_1 - 3v_2 - 4v_3 = -8$$

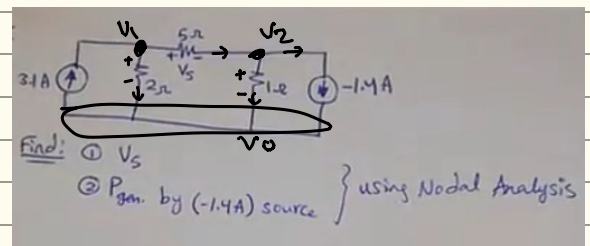
$$3v_1 - 6v_2 + 3v_3 = 0$$

$$4v_1 - 11v_2 - 4v_3 = -25$$

$$\begin{Bmatrix} 7 & -3 & -4 \\ 3 & -6 & 3 \\ 4 & -11 & -4 \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} -8 \\ 0 \\ -25 \end{Bmatrix}$$

$$v_1 = \frac{123}{121}, v_2 = \frac{211}{121}, v_3 = \frac{299}{121}$$

$$V = v_2 - v_3 = \frac{-8}{11}$$



at node 1 apply Kcl

$$\sum I_{in} = \sum I_{out}$$

$$3.1 = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$3.1 = \frac{v_1}{2} + \frac{v_1}{5} - \frac{v_2}{5}$$

$$3.1 = 0.7v_1 - 0.2v_2$$

at node 2 apply Kcl

$$\left(\frac{v_1 - v_2}{5}\right) = -1.4 + \frac{v_2}{2}$$

$$\frac{v_1}{5} - \frac{v_2}{5} = -1.4 + \frac{v_2}{2}$$

$$\frac{v_1}{5} - \frac{6}{5}v_2 = -1.4$$

$$0.2v_1 - 1.2v_2 = -1.4$$

$$\begin{Bmatrix} 0.7 & -0.2 \\ 0.2 & -1.2 \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} 3.1 \\ -1.4 \end{Bmatrix}$$

$$v_1 = 5V, v_2 = 2V$$

$$v_0 = v_1 - v_2 = 5 - 2 = 3V$$

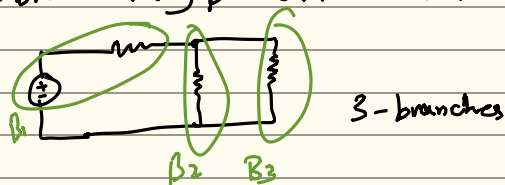
$$P(-1.4) = I(2)$$

$$= +(-1.4 \times 2) = -2.8 \text{ watt}$$

$$P_{generated} = 2.8 \text{ watt.}$$

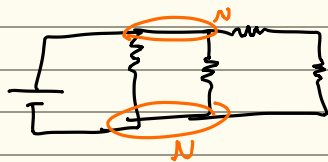
* Definitions of electrical ckt Networks

① Branch: any part of the ckt that has one or more elements

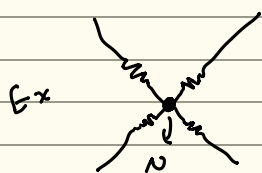


② Node: junction of two or more branches

there is no elements in this wire we consider them as one node



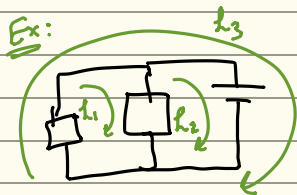
⇒ has two nodes



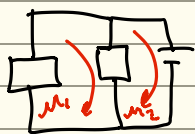
Ex



③ Loop: closed connection of Branches



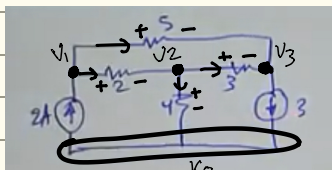
④ Mesh: loop that does not contain other loop.



has 2 meshes

Super node

Ex



writing nodal analysis for node 1:-

$$2 = \frac{V_1 - V_2}{2} + \left(\frac{V_1 - V_3}{5} \right)$$

$$2 = \frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{5} - \frac{V_3}{5} \Rightarrow 2 = \frac{7}{10} V_1 - \frac{V_2}{2} - \frac{V_3}{5} \sim 0$$

writing nodal equation for node 2

$$\left(\frac{V_1 - V_2}{2} \right) = \left(\frac{V_2 - V_3}{3} \right) + \frac{V_2}{4}$$

$$\frac{1}{2} V_1 - \frac{1}{2} V_2 = \frac{V_2}{3} - \frac{V_3}{3} + \frac{V_2}{4}$$

$$\frac{1}{2} V_1 = \frac{V_2}{3} + \frac{V_2}{4} + \frac{1}{2} V_2 - \frac{V_3}{3}$$

$$0 = \frac{13}{12} V_2 - \frac{1}{3} V_3 - \frac{1}{2} V_1$$

writing nodal equation for node 3

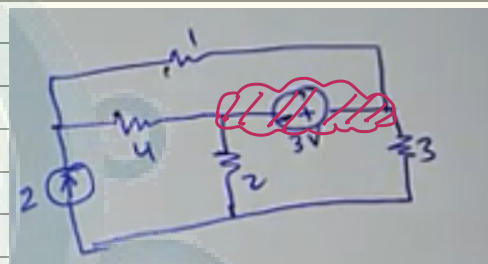
$$\left(\frac{V_1 - V_3}{5} \right) + \left(\frac{V_2 - V_3}{3} \right) = 3$$

$$\frac{V_1}{5} - \frac{V_3}{5} + \frac{V_2}{3} - \frac{V_3}{3} = 3$$

$$\frac{V_1}{5} - \frac{8}{15} V_3 + \frac{V_2}{3} = 3$$

↳ now we are solving the Matrix on calculator

Ex:



if I do the regular solution I will face problem in the second node.

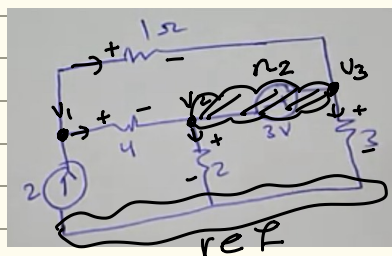
we can not find the current for the voltage supply here

* إذا كان الجهد مجهولاً يكون عتبة voltage source بين 2 nodes

solution: we will consider this wire as super node

Example
in the
second page

Ex



at node 1

$$2 = \left(\frac{V_1 - V_2}{1} \right) + \left(\frac{V_1 - V_3}{3} \right) \dots \textcircled{1}$$

at node 2 \Rightarrow here I face problem

$$\left(\frac{V_1 - V_2}{1} \right) + \left(\frac{V_1 - V_3}{3} \right) = \frac{V_3}{2} + \frac{V_2}{2} \dots \textcircled{2}$$

I have 3 variables I can not find them using only 2 equations so I will use this equation for super node

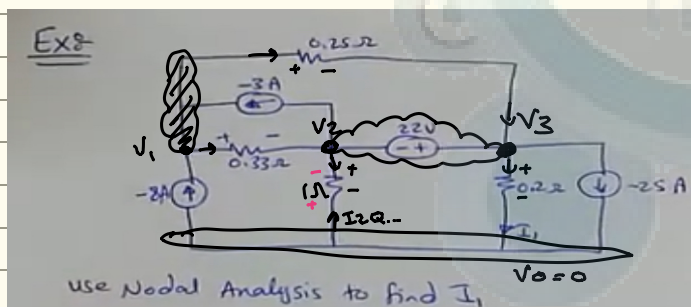
Super node :-

solving them using the calculator

$$V_3 - V_2 = 3 \dots \textcircled{3}$$

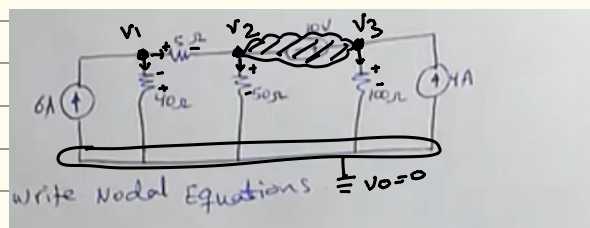
So

Super node: when there is a voltage source between nodes, we can not know the current as a function of voltage



if I asked to find I_2

$$\frac{0 - V_2}{1} \text{ polarity}$$



writing nodal equation for node 1

$$6 = \left(\frac{V_2 - V_1}{5} \right) + \frac{V_1}{40} \dots \textcircled{1}$$

writing nodal equation for super node

$$\left(\frac{V_1 - V_2}{5} \right) + 4 = \frac{V_2}{50} + \frac{V_3}{100}$$

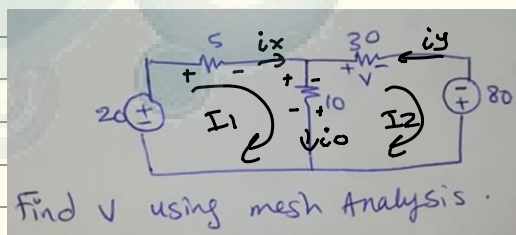
Super node equation

$$V_3 - V_2 = 10 \Rightarrow \text{هذا يعني ان فرق الجهد بين النودتين هو 10 فولت}$$

2 Mesh analysis

steps:-

- ① Define meshes
- ② assign mesh current (I_m) for all meshes
- ③ apply KVL for each mesh.
- ④ Solve equations



Find V using mesh Analysis.

writing nodal equation for node 1

$$(-8 + -3) = \left(\frac{V_1 - V_2}{0.33} \right) + \left(\frac{V_1 - V_3}{0.25} \right)$$

writing nodal equation for the super node

$$\left(\frac{V_1 - V_2}{0.33} \right) + \left(\frac{V_1 - V_3}{0.25} \right) = -3 + \frac{V_2}{1} + -25 + \frac{V_3}{0.2}$$

super node

solving them using the calculator

$$V_3 - V_2 = 22 \quad I_1 = \frac{V_3}{0.2}$$

at mesh 1

$$-20 + 5I_1 + 10I_1 - 10I_2 = 0 \dots \textcircled{1}$$

at mesh 2

$$10I_2 - 10I_1 + 30I_2 - 80 = 0 \dots \textcircled{2}$$

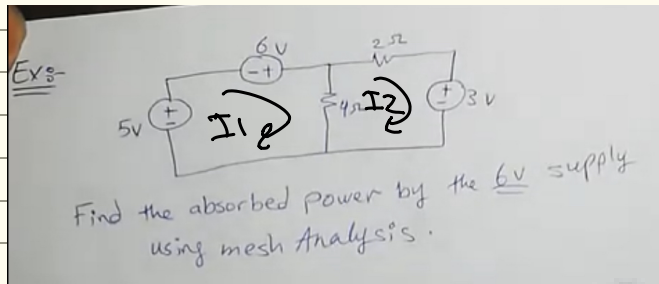
solving them using calculator

$$V = I_2 \times 30$$

يمكن تحديد السؤال أوجه من يكون :-
 $i_o = I_1 - I_2 \Rightarrow$ التيار يكون
 (في ما يشعرون)

إذا طلب مني i_x هي نفس I_1

إذا طلب مني i_y هي $-I_2$ لأن i_y في اتجاه



at mesh 1

$$-5 - 6 + 4I_1 - 4I_2 = 0 \quad \text{--- (1)}$$

at mesh 2

$$4I_2 - 4I_1 + 2I_2 + 3 = 0 \quad \text{--- (2)}$$

$$I_2 = 0.8, I_1 = 1.95$$

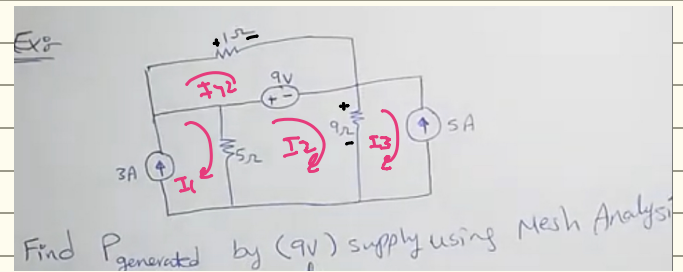
حاليا إذا بي P_{absorb} لا نرمسها في خط الاتجاه الموجبة
 so I need the same current but in the opposite direction (-)

$$P_{\text{abs}} = (-I_1)(V) \\ = (-1.95)(6) = -11.7$$

Note:

\Rightarrow power absorbed

\Rightarrow power generated



already we can know that

$$I_1 = 3A$$

$$I_3 = -5A$$

at mesh 2

$$5I_2 - 5I_1 + 9 + 4I_2 - 4I_3 = 0$$

$$5I_2 + 4I_2 = -39 \quad \text{--- eq (1)}$$

$$I_2 = -2.78571 \text{ so it is ccw}$$

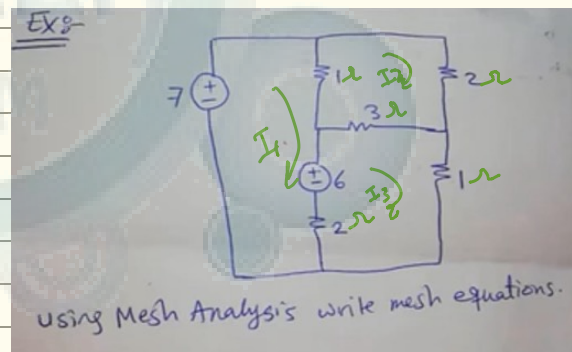
at mesh 4

$$I_4 - 4 = 0$$

$$I_4 = 4$$

$$P = (I_4 - I_2) \times V \\ = 106.0714286$$

هون هو I_4 بي generated
 إذا هو I_2 بي generated
 فبدي أدخله من السالب وأعتبر
 حاد هو التيار (لوجبة أمارة)



at mesh 1

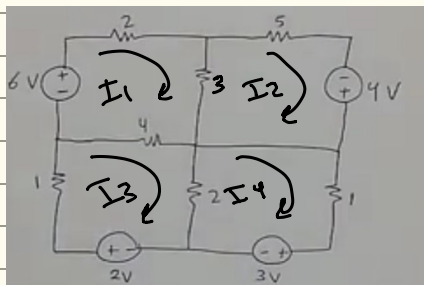
$$-7 + I_1 - I_2 + 6 + 2(I_1) - 2I_3 = 0 \quad \text{--- (1)}$$

at mesh 2

$$I_2 - I_1 + 2I_2 = 0 \quad \text{--- (2)}$$

at mesh 3

$$2I_3 - 2I_1 - 6 + 3I_3 - 3I_2 + I_3 = 0 \quad \text{--- (3)}$$



write mesh equation?

at mesh 1

$$-6 + 2I_1 + 3I_1 - 3I_2 + 4I_1 - 4I_3 = 0 \quad \text{--- (1)}$$

at mesh 2

$$3I_2 - 3I_1 + 5I_2 - 4 = 0 \quad \text{--- (2)}$$

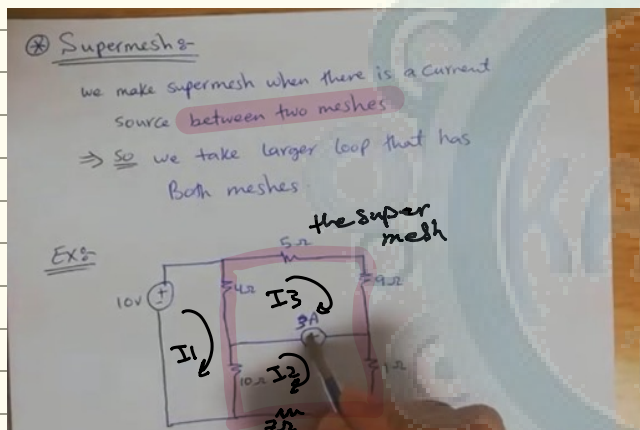
at mesh 3

$$I_3 + 4I_3 - 4I_1 + 2I_3 - 2I_4 - 2 = 0 \quad \text{--- (3)}$$

at mesh 4

$$2I_4 + I_4 + 3 - 2I_3 = 0 \quad \text{--- (4)}$$

super mesh



Find the P_{gen} by the 10V supply using mesh analysis

at mesh 1

$$-10 + 4I_1 - 3I_2 + 10I_1 - 10I_2 = 0$$

I have a current source between 2 meshes so I am going to apply **super mesh**

$$+10I_2 - 10I_1 + 4I_3 - 4I_1 + 6I_3 + 9I_3 + I_2 + 7I_2 = 0$$

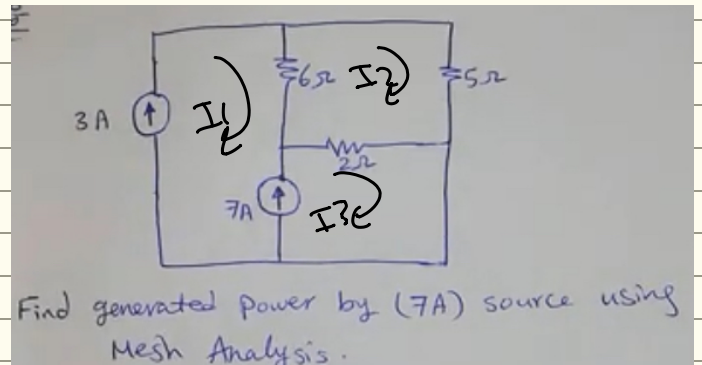
by the current source equation

$$I_3 - I_2 = 3$$

$$P = -(I_1) \times 10$$

$$P_{gen} = (I_1 \times 10) = \dots \text{ watt}$$

$$P_{abs 10V} = -P_{gen}$$



Find generated power by (7A) source using Mesh Analysis.

$$I_1 = 3$$

at mesh 2

$$6I_2 - 6I_1 + 5I_2 + 2I_2 - 2I_3 = 0$$

for the current source

$$I_3 - I_1 = 7$$

$$I_3 = 10$$

Back to equation 1

$$I_2 = \frac{33}{13}$$

$$P_{gen} = I \times V$$

$$I_3 \text{ already } = 10$$

$$V = -10V$$

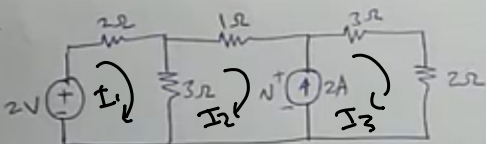
$$\text{loop 3}$$

$$-V + (10 - 4)2 = 0$$

$$-V + 12 = 0$$

$$V = 12$$

$$P = -12 \times 7 =$$



using Mesh Analysis find ① V

② $P_{gen.}$ by (2A) source.

③ $P_{abs.}$ by (1Ω) resistor.

④ $P_{gen.}$ by (2V) source.

at mesh 1:-

$$-2 + 2I_1 + 3I_1 - 3I_2 = 0$$

I have current source between 2 mesh

so

$$3I_2 - 2I_1 + I_2 + 3I_3 + 2I_3 = 0$$

super mesh

$$I_3 - I_2 = 2$$

Solving them

$$\begin{Bmatrix} 5 & -3 & 0 \\ -2 & 4 & 5 \\ 0 & -1 & 1 \end{Bmatrix} \begin{Bmatrix} I_1 \\ I_2 \\ I_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 0 \\ 0 \end{Bmatrix}$$

$$I_1 = \frac{6}{13}, I_2 = \frac{4}{39}, I_3 = \frac{4}{39}$$

① V

$$-V + 3I_3 + 2I_3 = 0$$

$$-V + \frac{20}{39} = 0 \Rightarrow V = \frac{20}{39}$$

$$② P_{gen.} (2A) = -(I_3 - I_2) \times \frac{20}{39}$$

$$-3 \times \frac{20}{39} = -\frac{20}{13}$$

$$P_{gen} = \frac{20}{13}$$

③ P_{obs} by 1Ω resistor

$$P = (I_2)^2 (1) = 1.49 \text{ watt}$$

$$④ P_{gen} = -2(-0.33) = 0.66 \text{ W}$$

$$P_{obs} = -P_{ge}$$

$$= 0.66$$

3 Superposition

$$P = I^2 R \text{ \& ino calculation}$$

③ used for linear circuits & linear quantities (I & V)

when there is multiple sources in the circuit

steps:

① we will choose one supply & short circuit the other sources as follow:

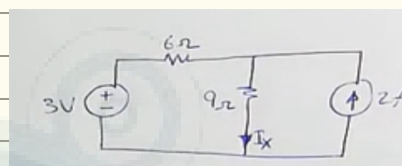
1) current source \Rightarrow open circuit

2) voltage \Rightarrow short circuit

② Find the currents & voltages in the circuit

③ repeat steps (1 \rightarrow 2) for other supplies

④ the Total current or voltage is the sum of values founded in previous steps.



Find I_x using superposition

2 sources \Rightarrow I am going to solve it twice

* leave voltage source & kill the current source



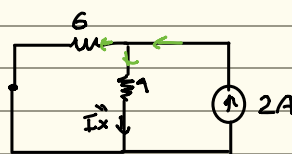
using Kvl

$$-3 + 6I_x' + 9I_x' = 0$$

$$-3 + 15I_x' = 0$$

$$\frac{15I_x'}{15} = \frac{3}{15} \Rightarrow I_x' = 0.2 \text{ A}$$

* leave the current source & kill the voltage source



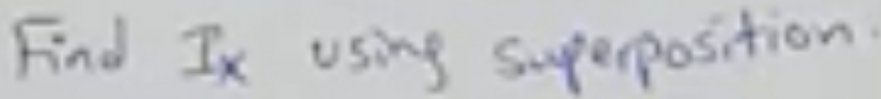
So

using current division

$$I_x = I_x' + I_x'' = 0.2 + 0.8 = 1 \text{ A}$$

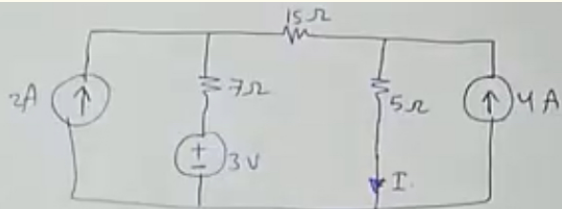
$$I_x'' = \frac{2 \cdot \frac{1}{9}}{\frac{1}{6} + \frac{1}{9}}$$

$$I_x'' = 0.8 \text{ A}$$



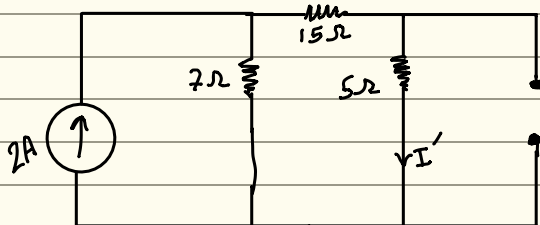
Scientific class

Ex 8



Find I using superposition.

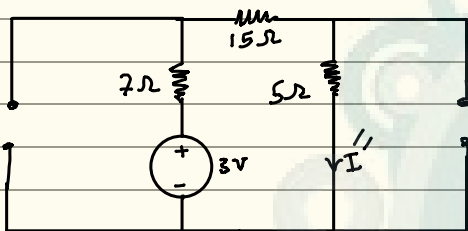
* Keep (2A) & Kill (3V, 4A)



$$R_{eq} = 20$$

$$I' = \frac{2 \times \frac{1}{20}}{\frac{1}{20} + \frac{1}{5}} = I' = 0.52$$

* Keep (3V) & Kill (2A, 4A)



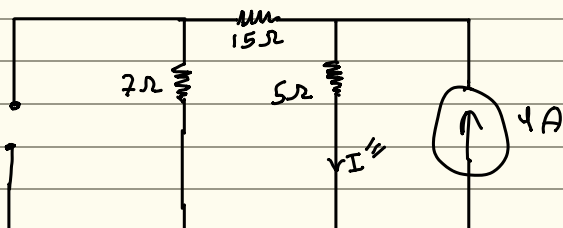
$$-3 + 7I'' + 15I'' + 5I'' = 0$$

$$-3 + 27I'' = 0$$

$$27I'' = 3$$

$$I'' = 0.111A$$

* Keep (4A) & Kill (2A, 3V)

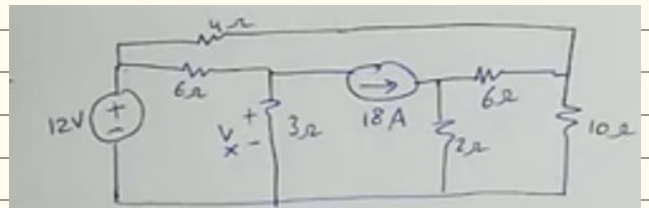


$$I''' = 4 \left(\frac{1}{5} \right) = \frac{88}{\frac{1}{5} + \frac{1}{22}} = \frac{88}{27}$$

$$I''' = 3.26A$$

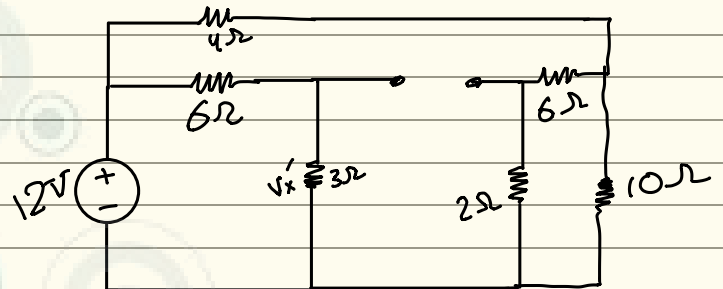
$$I = I' + I'' + I'''$$

$$I = 3.89A$$



Find V_x using Superposition.

* Remove 12V & Kill (18A)



$$-12 + 6I_x + 3I_x = 0$$

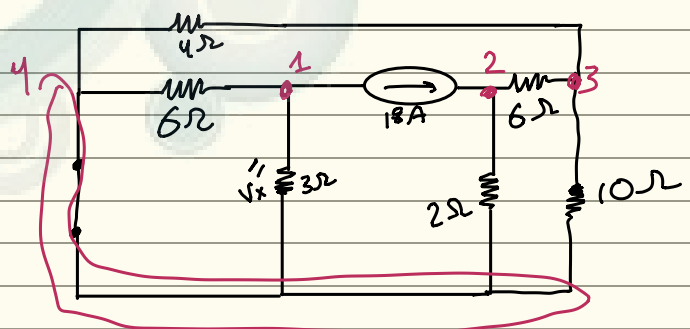
$$9I_x - 12 = 0$$

$$I_x = 1.33A$$

$$V_x = I_x \cdot R$$

$$V_x' = (1.33)(3)$$

$$V_x' = 3.99 \approx 4V$$



* By current division

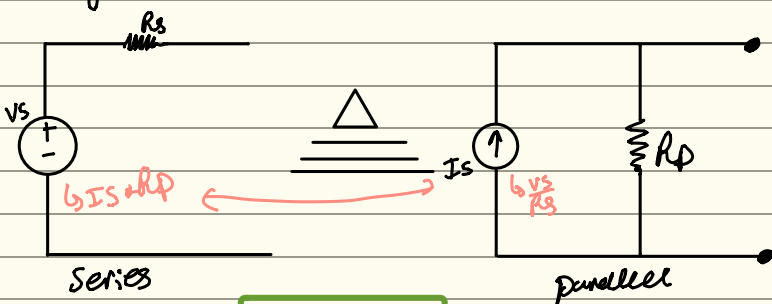
$$I' = \frac{18 \cdot \frac{1}{3}}{\frac{1}{6} + \frac{1}{3}} = -12A$$

$$V_x'' = -12 \times 3 = -36$$

$$V_x = V_x' + V_x'' = 4 + -36 = -32$$

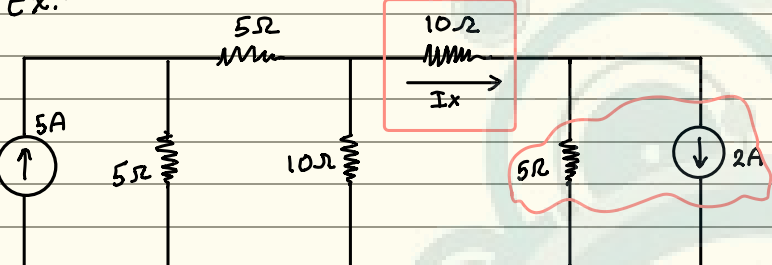
4 Source transformation:-

In some ckt replacing current sources with voltage sources or visa versa make the ckt simple & easy to solve



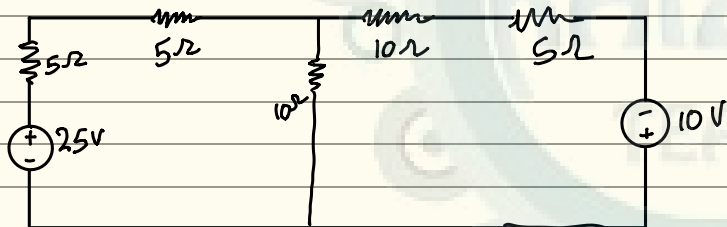
$$\begin{aligned} R_s &= R_p \\ V_s &= I_s R_s \\ I_s &= \frac{V_s}{R_s} \end{aligned}$$

Ex:-

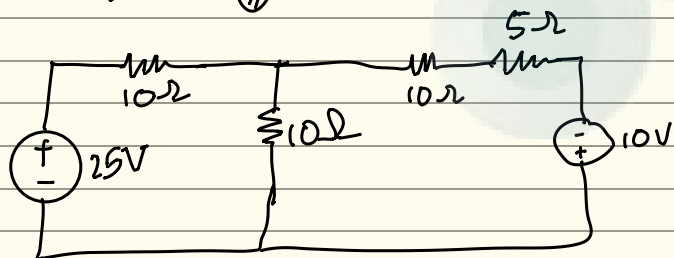


Find I_x using source transformation.

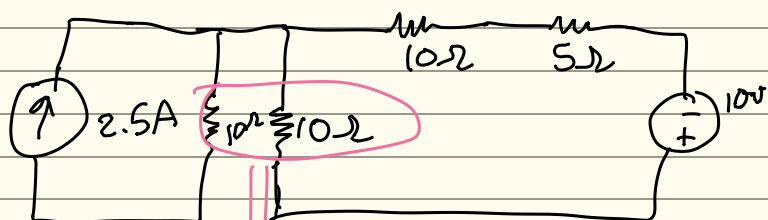
* branch الذي طالعنا فيه قيمة التيار ما نشتغل فيه



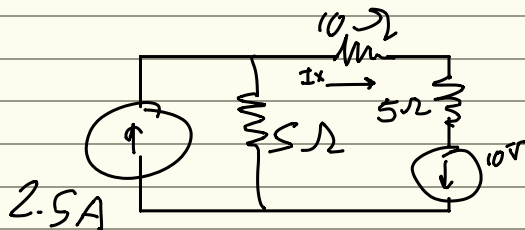
now



now



$$\frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 10}{20} = \frac{100}{20} = 5\Omega$$



now this ckt is very simple (one loop)

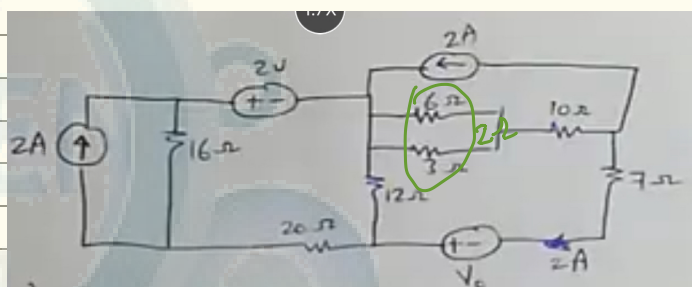
* applying KVL

$$-12.5 + 5I_x + 10I_x + 5I_x - 10 = 0$$

$$-22.5 + 20I_x = 0$$

$$20I_x = 22.5$$

$$I_x = 1.125A$$

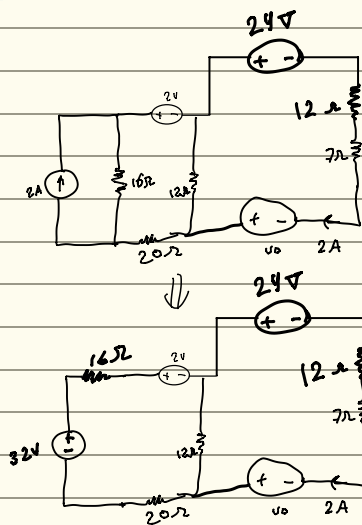


find V_o by source transformation.

Sol

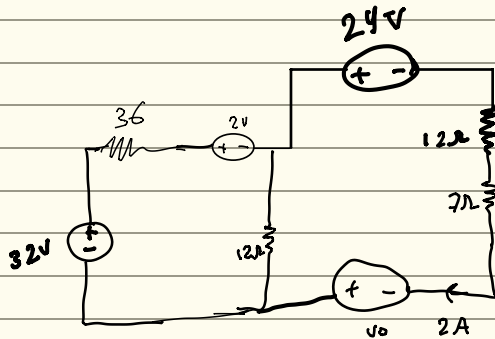
$$\begin{aligned} 6\Omega \parallel 3\Omega \\ \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\Omega \end{aligned}$$

$$\begin{aligned} 2\Omega \text{ series } 10\Omega \\ 2 + 10 = 12\Omega \end{aligned}$$



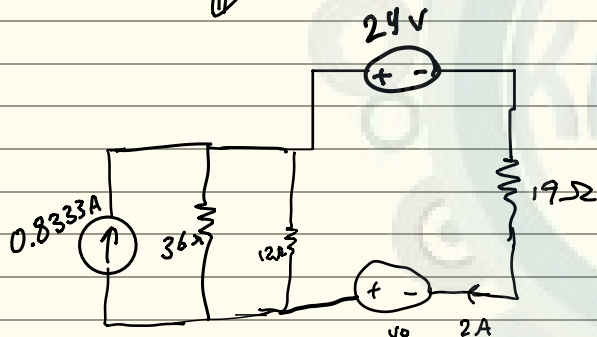
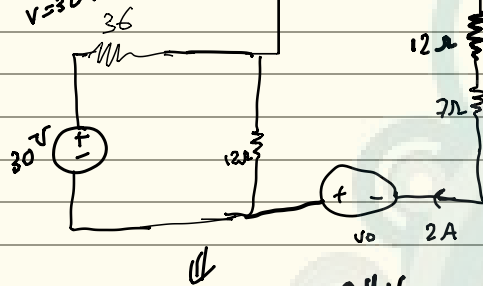
$$16 + 20 = 36$$

$$\text{series } 16\Omega \text{ \& } 20\Omega \text{ is } 36\Omega$$

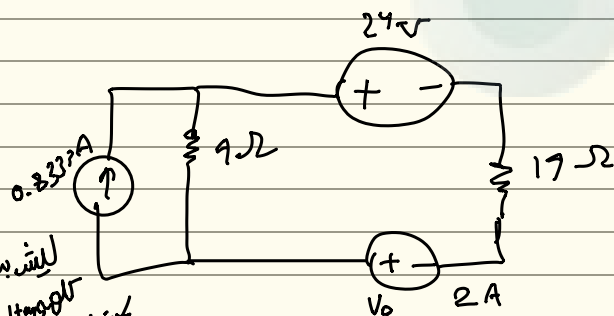


هنا عندي 2 voltage source
 واتجاهاتهم على رجة شواتجاه ال
 الجديد ولم تغير

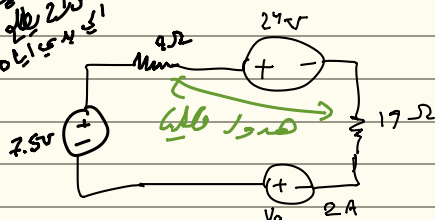
طب عندي أناس بقر اتجاه ال polarity
 بخط polaris كذا
 loop ال
 في الأساس ال
 في polarity
 $-V - 2 + 32 = 0$
 $-V + 30 = 0$
 $V = 30V$



$$12\Omega || 36\Omega \Rightarrow 9\Omega$$



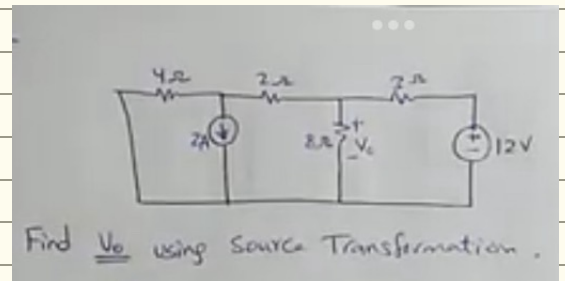
لش متعلق
 ال voltage
 في ال
 ال



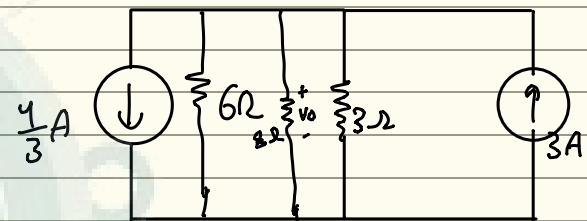
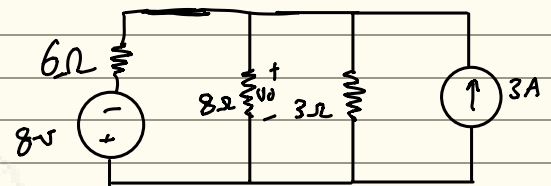
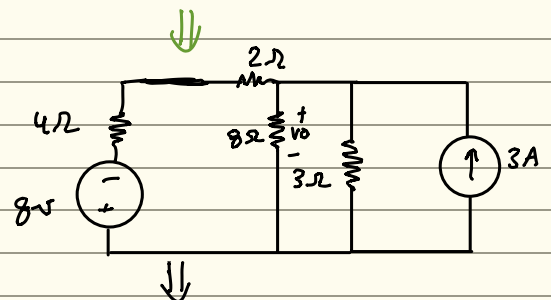
$$-2.45 + 2(4 + 17) + 24 - 2 - V_o = 0$$

$$70.5 = V_o \Rightarrow$$

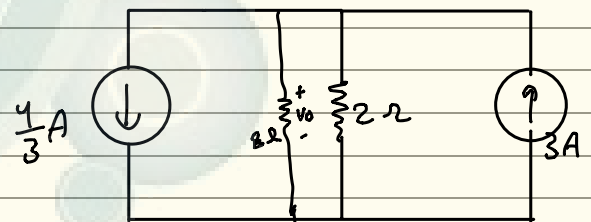
طب
 $V_o = 70.5$
 $V_o = 70.5$



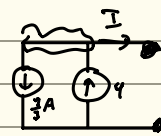
Find V_o using Source Transformation.



$3\Omega || 6\Omega$ like



now we are having 2 current sources

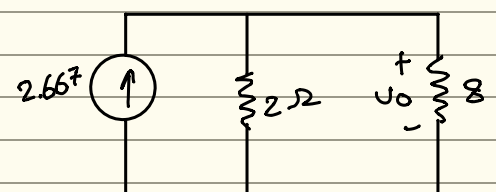


$$Kcl$$

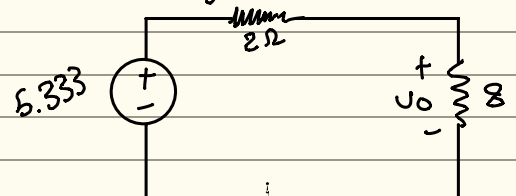
$$I + 1.33 = 4$$

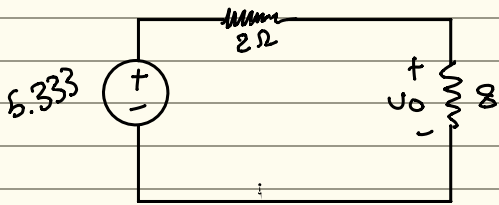
$$-1.33 - 1.33$$

$$I = 2.6667$$



simplify it





now applying KVL

$$-5.333 + 2I_0 + 8I_0 = 0$$

$$10I_0 = 5.333$$

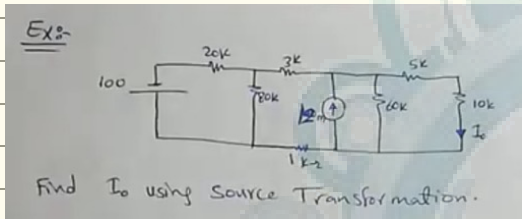
$$I_0 = 0.5333 \text{ A}$$

$$V_0 = I_0 \times R_0$$

(voice)

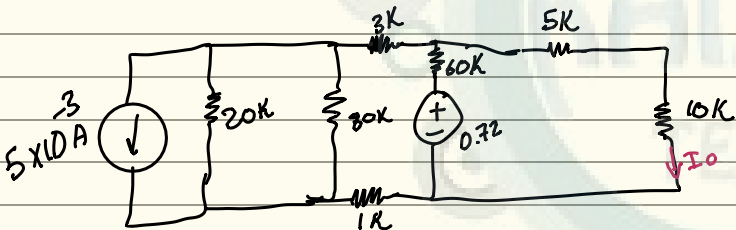
$$V_0 = 4.266672 \text{ V}$$

already they have the same polarity so we don't have (-) sign



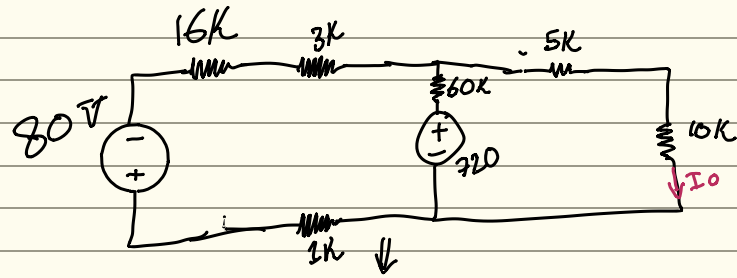
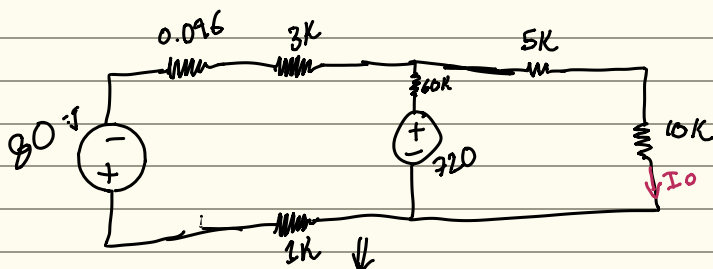
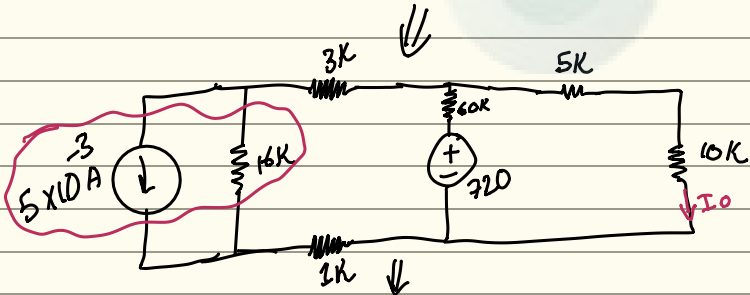
Find I_0 using Source Transformation.

↓

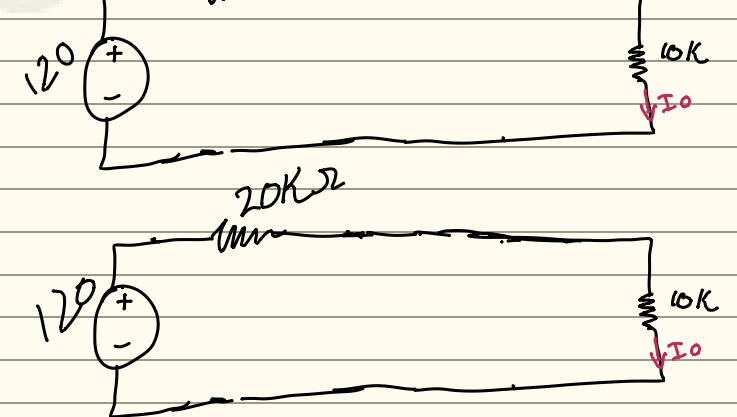
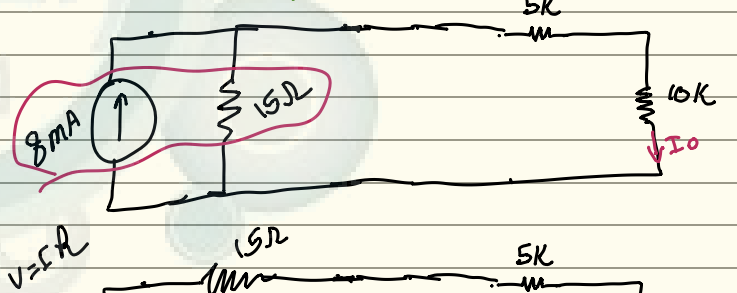
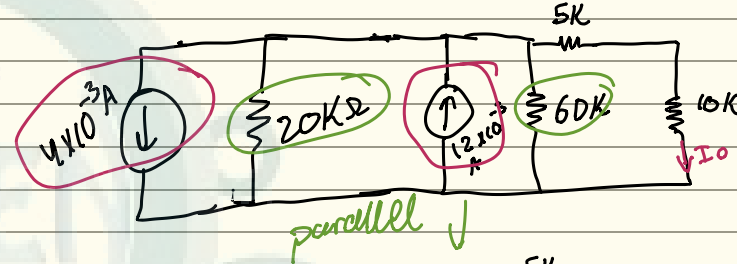
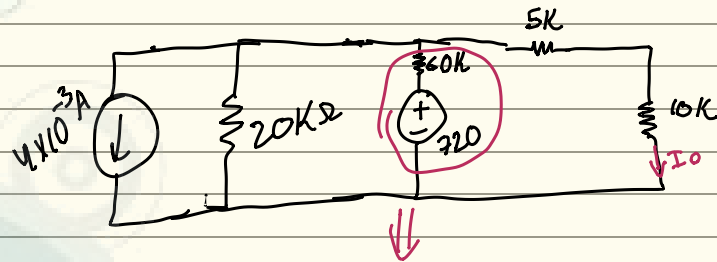
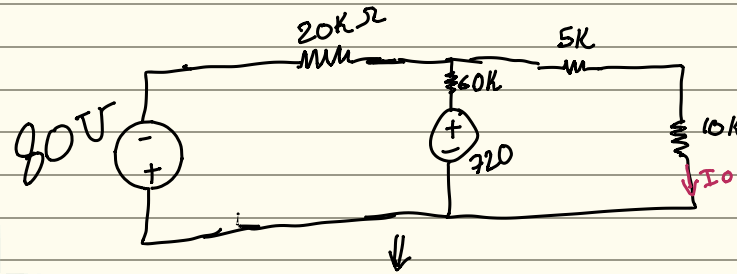


$$20 \text{ k} \parallel 80 \text{ k}$$

$$= 16 \text{ k} \Omega$$

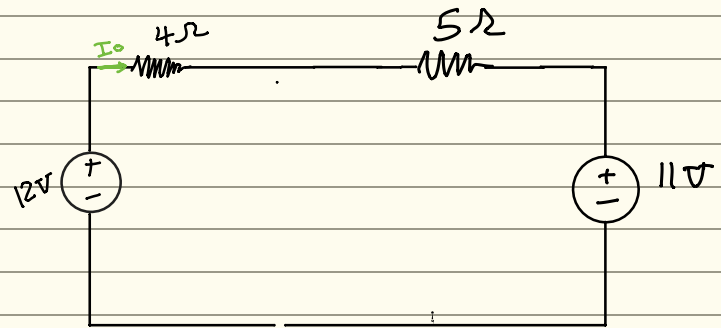
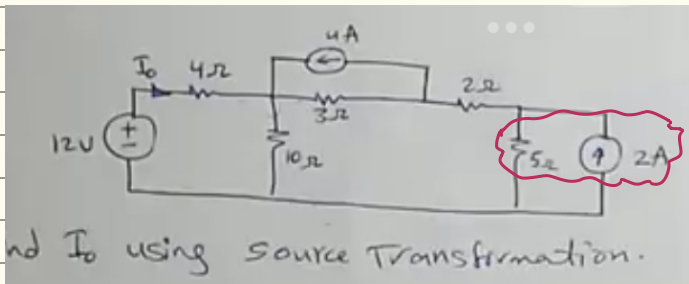


16k & 3k & 1k → Series



$$-120 + 20 \times 10^3 \times I_0 + 10 \times I_0 = 0$$

$$I_0 = 4 \times 10^{-3} \text{ A}$$



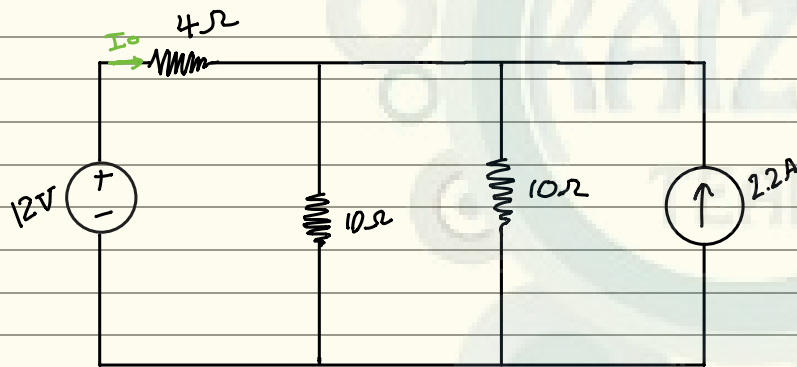
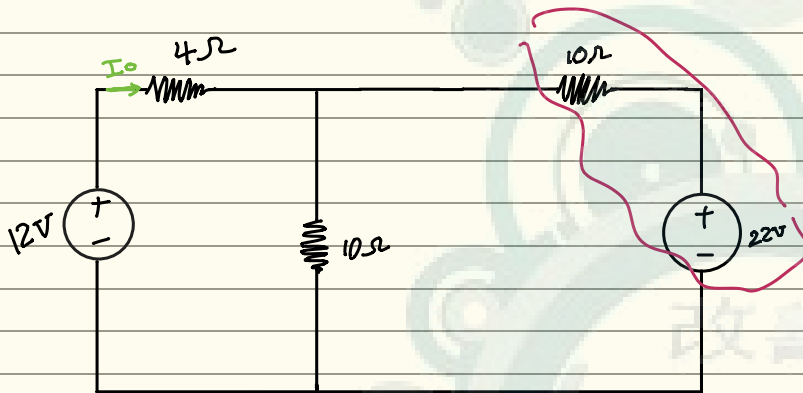
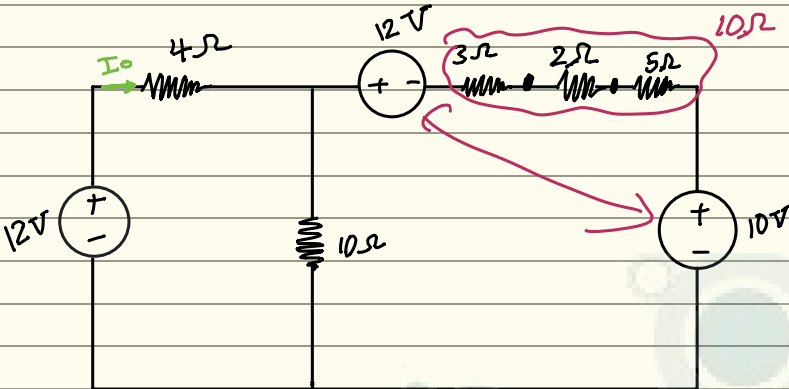
applying KVL

$$-12 + 4I_0 + 5I_0 + 11 = 0$$

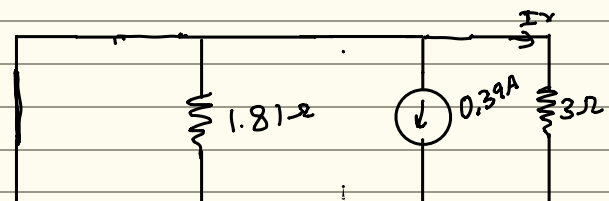
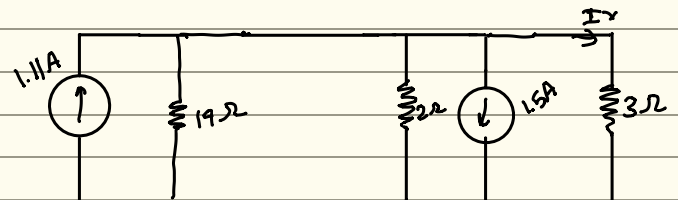
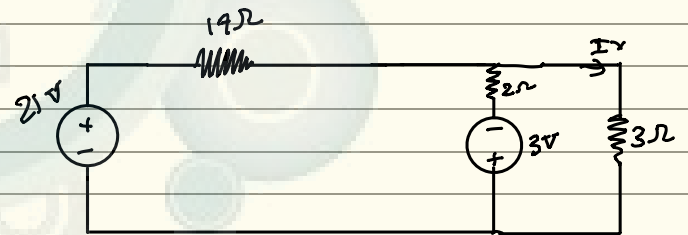
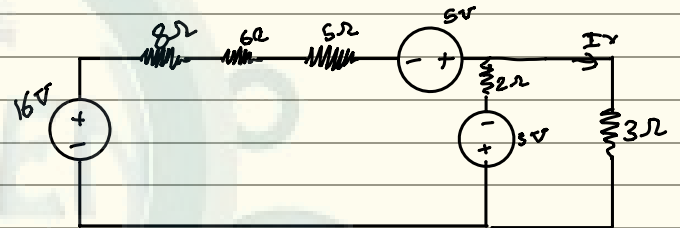
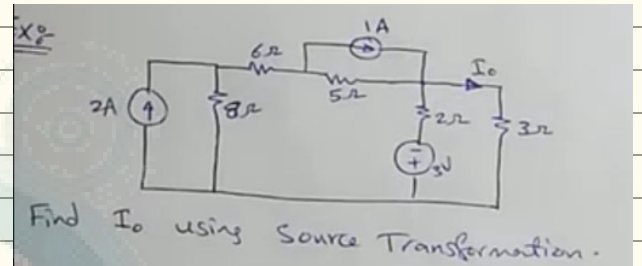
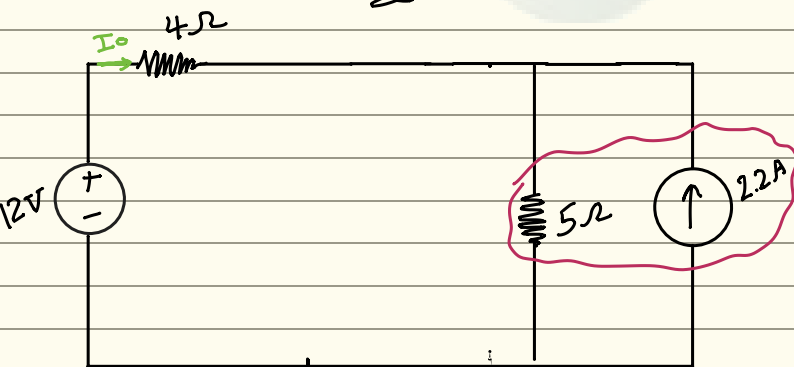
$$-1 + 9I_0 = 0$$

$$I_0 = \frac{1}{9}$$

$$I_0 = 0.1111A$$



$$10\Omega \parallel 10\Omega = \frac{10\Omega}{2} = 5\Omega$$



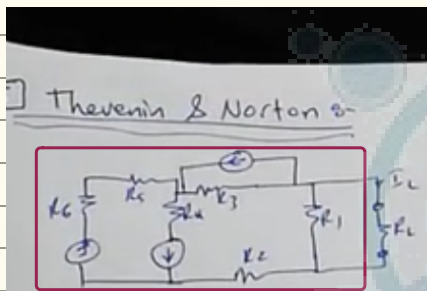


$$I_x = I_s \cdot \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{1.81}} = 0.1468 A$$

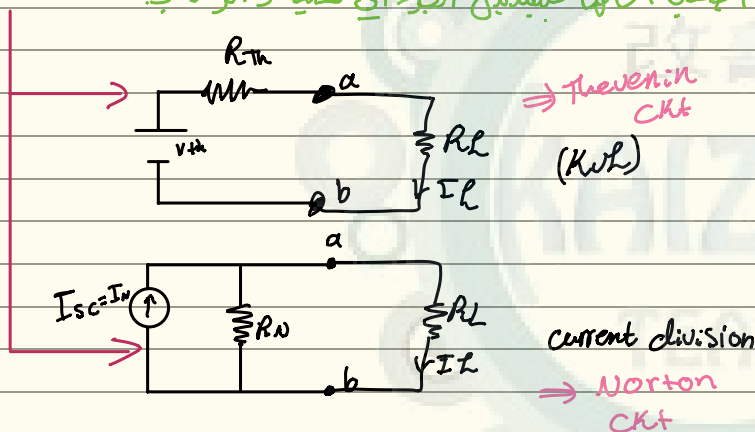
⑤ Thevenin & Norton:-

۳) بلحاظ انسانی اعتبار سے تمام انسانوں کو بالدرتہ و انسانی کے طور پر سمجھنا۔

Ex:



أصعب أهلها فليست بيل الجزع الي عليه دائرة!



Steps to find ($R_{th} = R_N$)

$$R_{th} = \frac{V_{OC}}{I_{SC}}$$

OC: open CKG

SC = short ckt

or

- 1) Remove R_L
- 2) Kill all Ckt

} Voltage source \Rightarrow short

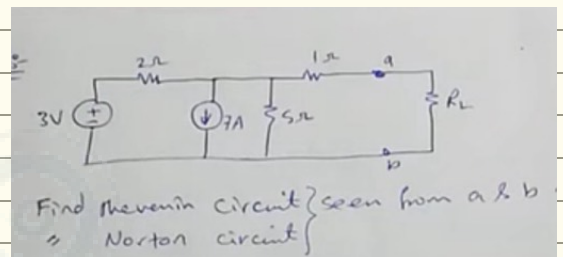
} Current source \Rightarrow open

③ Find R_{eq} seen at the terminals
place of R_L

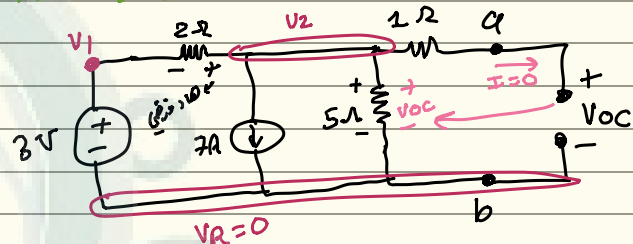
Note:-

$$V_{th} = I_N * R_{th}$$

$$I_{SC} = \frac{V_{th}}{R_{th}}$$



→ Find $V_{th} = V_{OC}$



we will solve it by nodal method

$$V_1 = 3V$$

cb node 2

$$\left(\frac{V_2 - V_1}{2}\right) + 7 + \frac{V_2}{5} = 0$$

$$\frac{\sqrt{2}}{2} - \frac{3}{2} + 7 + \frac{\sqrt{2}}{5} = 0$$

$$5.5 + 0.7v_2 = 0$$

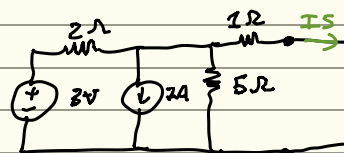
$$V_2 = -7.86 \text{ V}$$

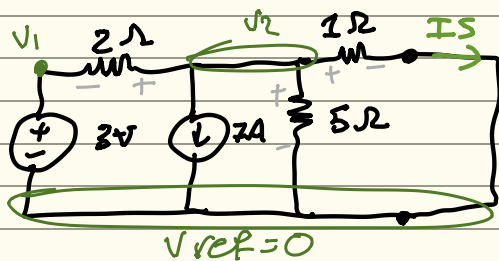
هياكل بنو اؤويدنا
 $V_{OC} = V_2$
 $V_{OC} = -7.86V$ V_{th}

Steps to find ($I_N = I_{sc}$)

- ① Remove R_L
- ② put short ckt in the place of R_L
- ③ Find current in that short ckt (short circuit)
 $I_{sc} = I_N$

* find $I_{SC} = I_N$





$$V_1 = 3V$$

using nodal method

at V_2

$$\left(\frac{V_2 - V_1}{2}\right) + 7 + V_2 = 0$$

$$\frac{V_2}{2} - \frac{3}{2} + 7 + V_2 = 0$$

$$\frac{3}{2} V_2 + 5.5 = 0$$

$$V_2 = -6.5 \times \frac{2}{3}$$

$$V_2 = -3.6667$$

V for $I_S \neq V_2$

$$V = IR$$

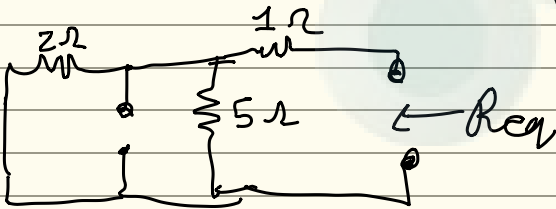
$$-3.667 = I \times 1$$

$$I_{CS} = -3.67A$$

To Find R_{th}

$$① R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-7.86}{-3.67} = 2.14169 \Omega$$

or ②



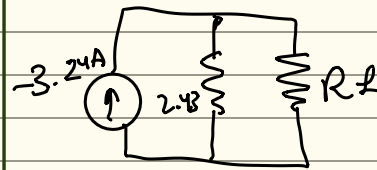
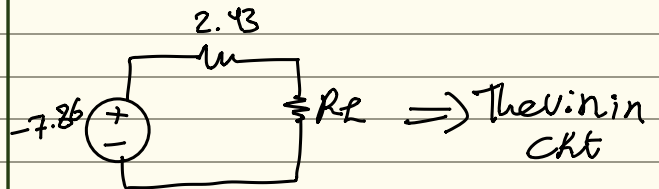
$$2\Omega \parallel 5\Omega$$

$$\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10} \Rightarrow R_{eq} = \frac{10}{7} = 1.43 \Omega + 1 = 2.43 \Omega$$

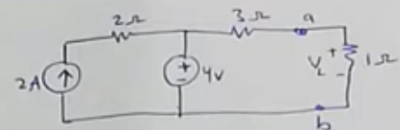
$$R_{eq} = \frac{10}{7} = 1.43 \Omega + 1 = 2.43 \Omega$$

Thevenin Ckt
& norton ckt

المطلوب بالسؤال ١١

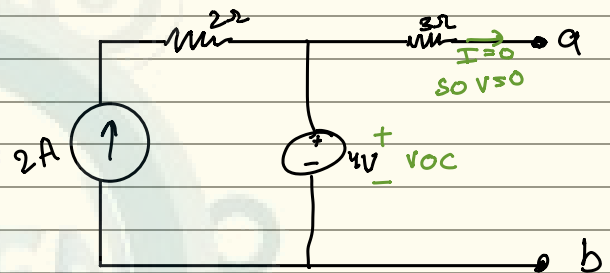


Ex 6



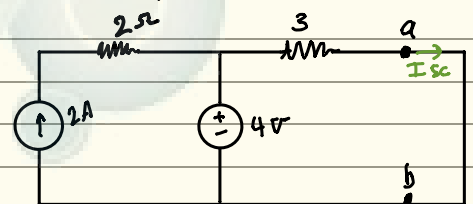
Find V_L using Thevenin & Norton Method.

* Find $V_{th} = V_{OC}$



$$V_{OC} = 4V$$

* Find $I_{sc} = I_N$



what is the relation between the right 3Ω & 4V they are parallel to each other so they have the same voltage

$$V_{3\Omega} = 4V$$

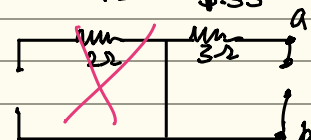
$$V_{sc} = 4V$$

$$V = IR$$

$$\frac{4}{3} = \frac{I \times 3}{3} \Rightarrow I_{sc} = \frac{4}{3} A = 1.33A$$

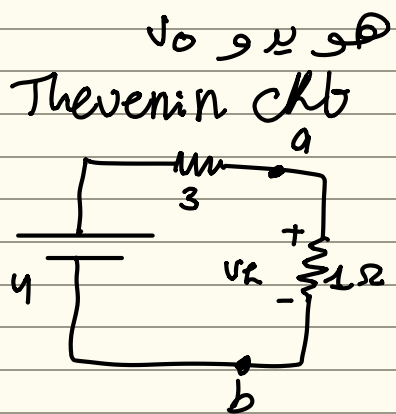
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{4}{1.33} = 3 \Omega$$

or
cancelled by
open ckt



$$R_{eq} = 3\Omega$$

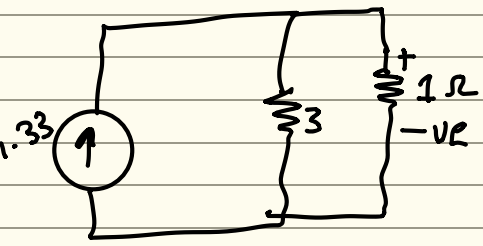
$$R_{th} = 3\Omega$$



$$\begin{aligned}
 -4 + 3 \times I + 1 \times I &= 0 \\
 -4 + 4I &= 0 \\
 I &= 1A \\
 V &= IR \\
 V &= 1 \times 1 = 1V
 \end{aligned}$$

or voltage division

Norton ckt

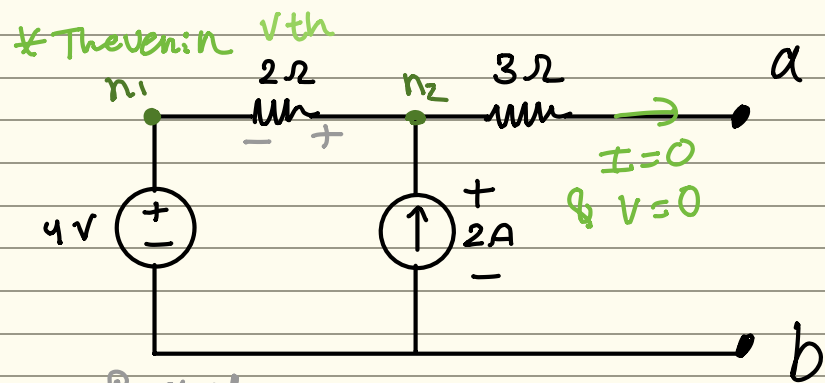
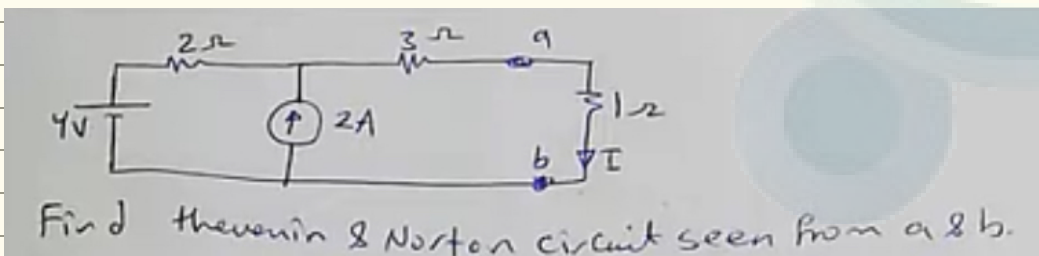


by c.d

$$I = \frac{1.33 \times \frac{1}{3}}{\frac{1}{3} + \frac{1}{1}} = 1$$

$$\begin{aligned}
 V &= IR \\
 V &= 1 \times 1 \\
 V &= 1V
 \end{aligned}$$

Ex:-



$$-2 \times R + 2 \times 2 + 4 = 0$$

$$-2 \times R + 4 + 4 = 0$$

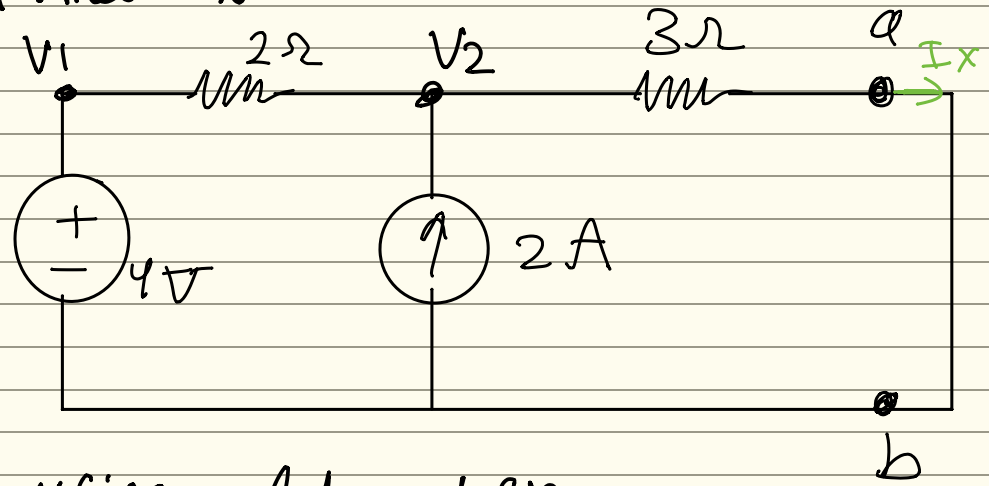
$$-2R + 8 = 0$$

$$\frac{2R}{2} = \frac{8}{2}$$

$$\begin{aligned}
 R &= 4\Omega \Rightarrow \\
 V &= IR \\
 V &= 4 \times 2 \\
 V &= 8V
 \end{aligned}$$

$$V_{OC} = 8V$$

* Find I_N



using nodal analysis

$$V_1 = 4V$$

for n_2

$$\left(\frac{V_1 - V_2}{2}\right) + 2 = \frac{V_2}{3}$$

$$2 - \frac{V_2}{2} + 2 = \frac{V_2}{3}$$

$$4 = \frac{2V_2}{3} + \frac{3V_2}{3}$$

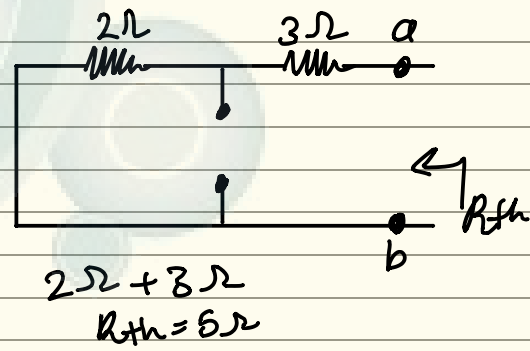
$$\frac{6}{3} \times 4 = \frac{5V_2}{3} \Rightarrow V_2 = 4.8V$$

$$I_x = \frac{V_2 - 0}{3} = \frac{4.8}{3} = 1.6A$$

* Find R_{th}

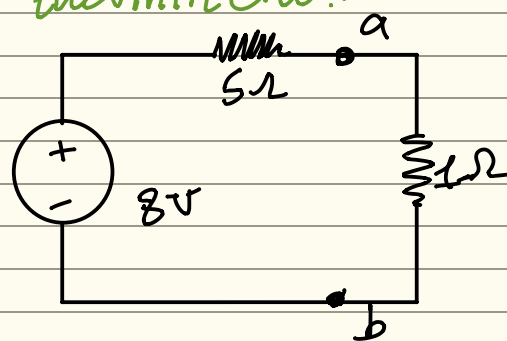
$$R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{8}{1.6} = 5\Omega$$

or

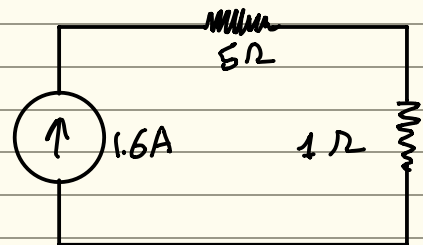


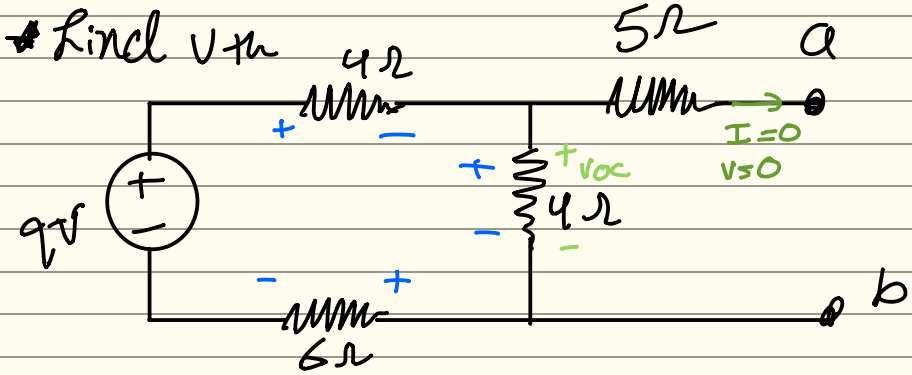
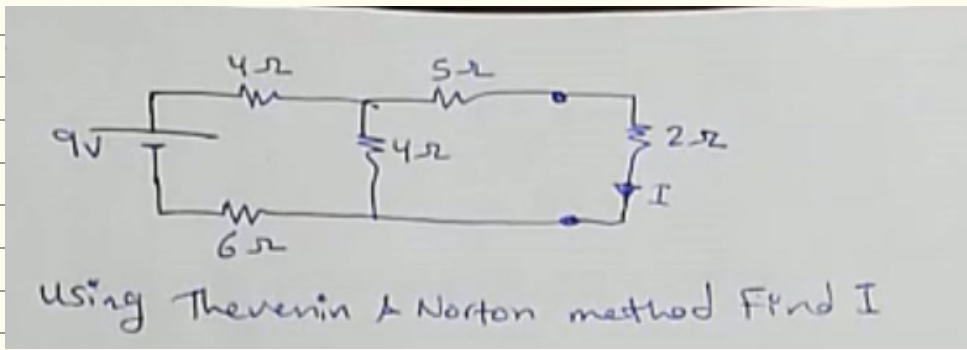
So

thevenin ckt:-



norton ckt





$$-9 + 4I + 4I + 6I = 0$$

$$-9 + 14I = 0$$

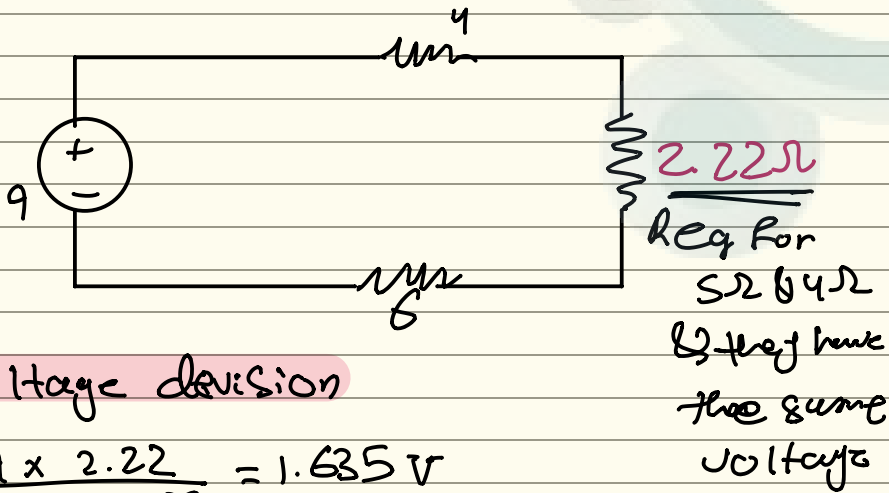
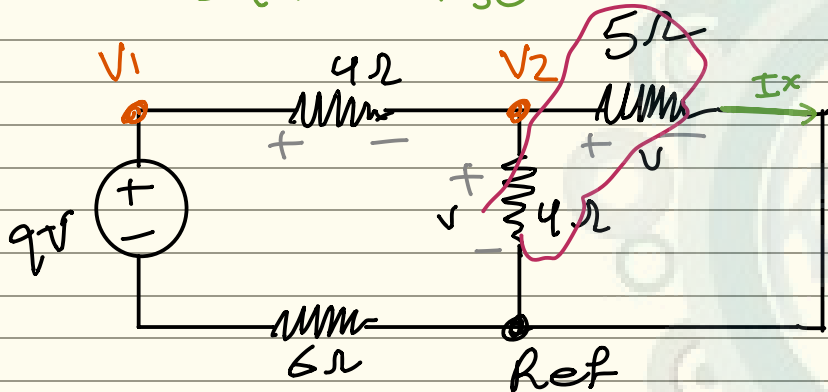
$$\frac{14I}{14} = \frac{9}{14} \Rightarrow I = 0.643A$$

$$V = IR$$

$$V = 0.643 \times 4$$

$$V = 2.57V$$

* Find $I_{norton} \Rightarrow I_{sc}$



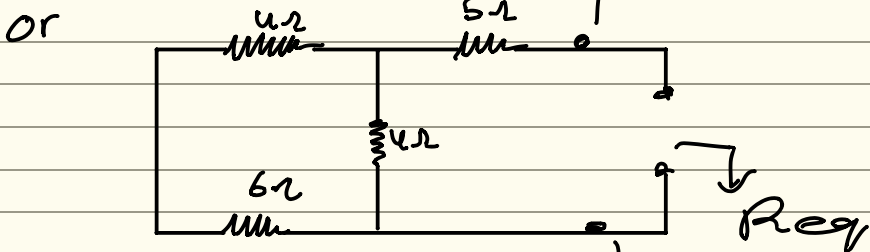
By voltage division

$$V = \frac{9 \times 2.22}{4 + 6 + 2.22} = 1.635V$$

$$I_N = \frac{V}{R} = \frac{1.635}{5} = 0.33$$

to find R_{th}

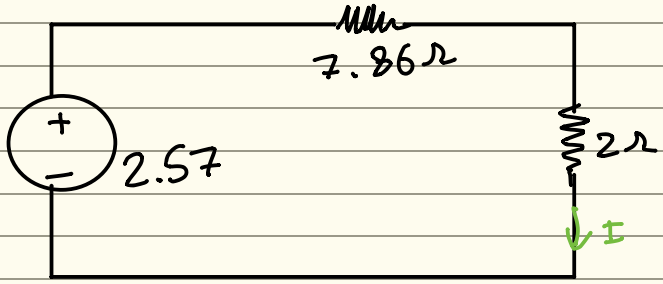
$$R_{th} = \frac{V_{OC}}{I_{SC}} = 7.86$$



$$4 + 6 = 10 \rightarrow 10 \parallel 4 = 2.857 \rightarrow 2.857 + 5 = 7.86\Omega$$

* now how to find I?

يكتبوا بوحدة من الشكليات النهائية
وبحسب ادا

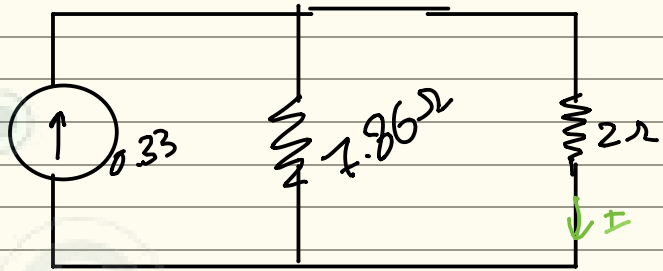


$$-2.57 + 7.86I + 2 \times I = 0$$

$$-2.57 + \frac{64}{7}I = 0$$

$$\frac{7}{64} \times \frac{64}{7} I = 2.57 \times \frac{7}{64}$$

$$I = 0.261A$$

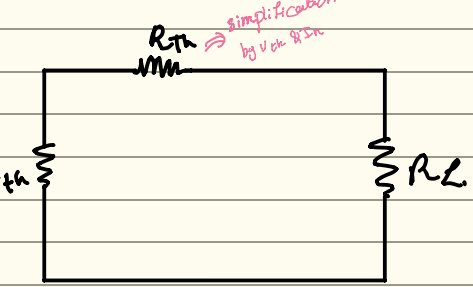


using C.d

$$I = 0.33 \times \frac{1}{\frac{1}{7.86} + \frac{1}{2}} = 0.29A$$

{Maximum power transfer}

* we concern about power consumption by load after simplification of the ckt



Maximum power P_L is the power consumed by the load R_L when the load resistance is equal to the Thevenin resistance R_{th} .

$$P_{load} = I^2 R_L \quad (1)$$

$$I = \frac{V_{th}}{R_{th} + R_L} \quad (2)$$

put (2) in (1)

$$P_L = \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 R_L \quad (3)$$

for max P_L

$$\frac{dP_L}{dR_L} = 0$$

Result $R_L = R_{th}$

for any ckt to get max power at the load $\Rightarrow R_L = R_{th}$

put (4) in (3)

$$P_{Lmax} = \frac{(V_{th})^2}{4R_{th}} = \frac{I_N^2 R_L}{4}$$

EX:-

Find Power at load (P_L) when

- $R_L = 2\Omega$
- $R_L = 8\Omega$
- $R_L = 4\Omega$

① when $R_L = 2\Omega$

$$V_L = \frac{12 \times 2}{6} = 4V \quad I = 2A \quad P_L = \frac{V^2}{R} = \frac{16}{2} = 8W$$

② when $R_L = 8\Omega$

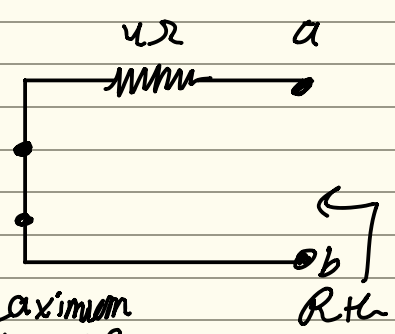
$$V_L = \frac{12 \times 8}{12} = 8V \quad P_L = \frac{V^2}{R} = \frac{64}{8} = 8W$$

③ when $R_L = 4\Omega$

$$V_L = \frac{12 \times 4}{8} = 6V \quad P_L = \frac{36}{4} = 9W \quad \text{max } P$$

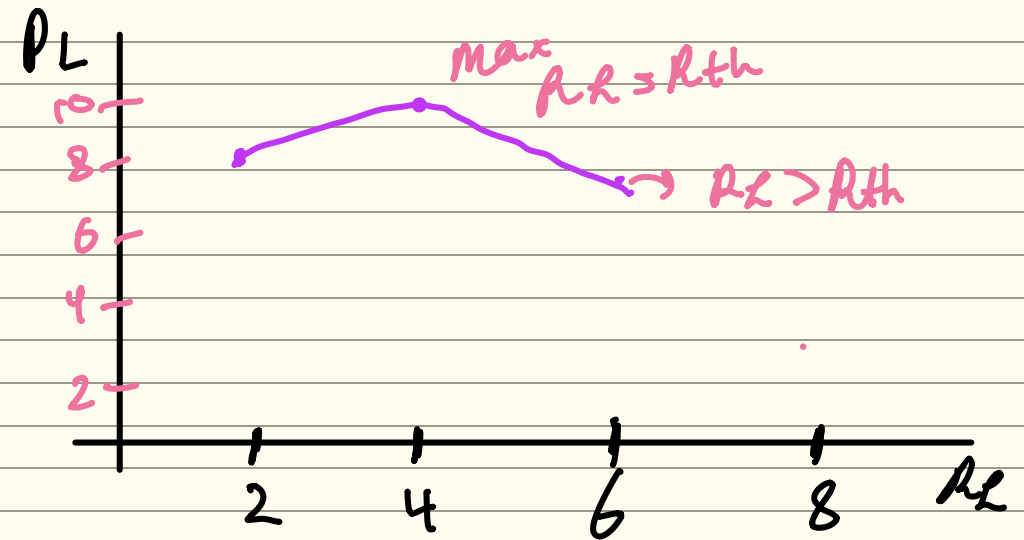
* refer to ckt \Rightarrow find R_{th}

$$R_{th} = 4\Omega$$

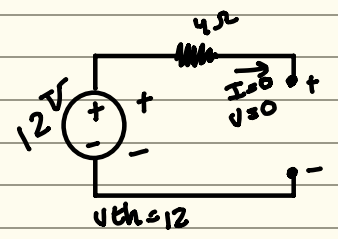


when the $R_L = R_{th} = 4\Omega$ was the maximum power transfer

when we upgrade it the power it minimize



$P_{max} \Rightarrow$ when $R_L = R_{th}$

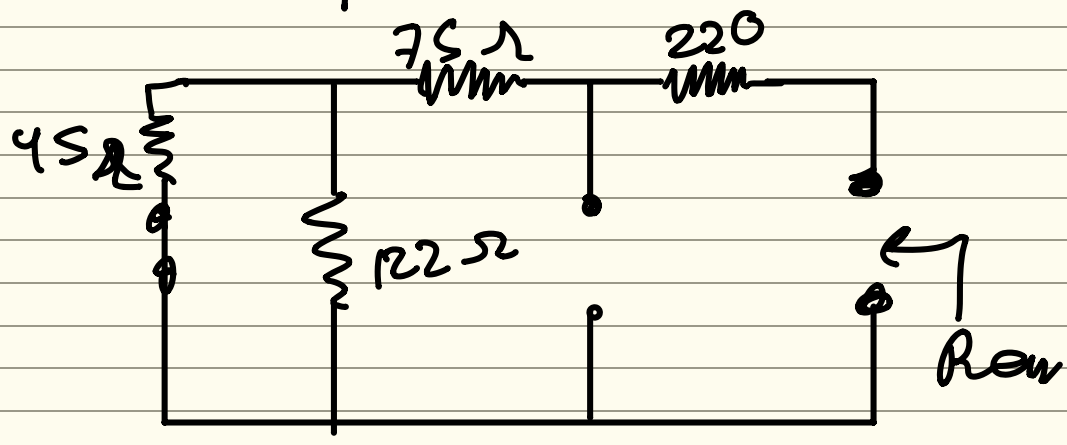


$$P_L (max) = \frac{(V_{th})^2}{4R_{th}} = \frac{(12)^2}{4 \times 4} = \frac{144}{16} = 9W$$

EX:-

Find: ① R_L to get max. Power.
② $P_L (max)$.

\Rightarrow for max power $\Rightarrow R_L = R_{th}$



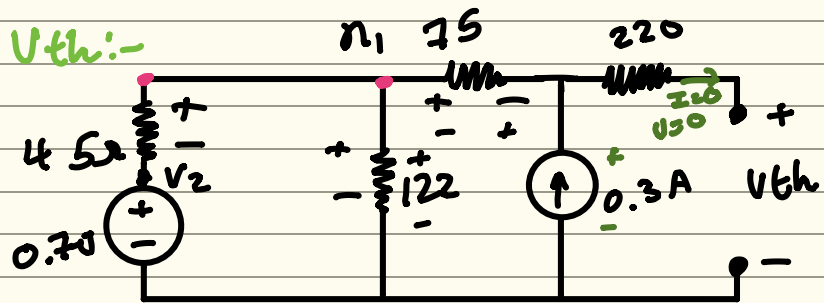
$$R_{eq} = 327.9\Omega$$

for max power transfer

$$R_L = 327.9\Omega$$

$$P_{Lmax} = \frac{(V_{th})^2}{4R_{th}}$$

To find V_{th} :-



$$V_2 = 0.7 \text{ V}$$

$$\sum I_{in} = \sum I_{out}$$

$$0.3 = \frac{V_1 - 0.7}{45} + \frac{V_1}{122}$$

$$V_1 = 10.45$$

By KVL

V_{th} is the voltage across the 220 ohm resistor.

$$-10.45 + 75(0.3) + V_{th} = 0$$

$$V_{th} = 32.45 \text{ V}$$

$$P_{L(max)} = 0.842 \text{ W}$$

{ Chapter 6 }

* AC - Network analysis:-

power absorption $\Rightarrow V = IR$
 $P = I^2 R$

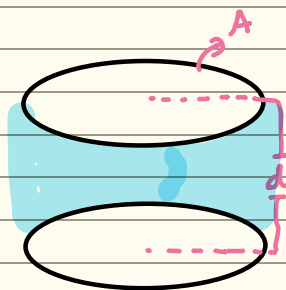
* we will talk about energy storage elements

Capacitor
 Inductor

1) Capacitor

\Rightarrow device (element) store energy

\Rightarrow consists of two parallel plates
 each plate has cross sectional area (A)
 & have distance (d) between them



the separation filled with dielectric material has certain permittivity (ϵ)

$$C = \frac{\epsilon A}{d}$$

\Rightarrow unit of the capacitance \Rightarrow Farad (F)

\Rightarrow Symbol in electric ckt capacitor

\Rightarrow charge on plates of capacitor

$$Q = CV \Rightarrow \text{fixed voltage} \Rightarrow \text{fixed charge}$$

$$q(t) = C v(t) \Rightarrow \text{variable voltage} \Rightarrow \text{variable charge}$$

$$\Rightarrow i(t) = \frac{dq(t)}{dt} \Rightarrow i_c(t) = C \frac{dv_c(t)}{dt}$$

$$\Rightarrow v_c(t) = \left[\frac{1}{C} \int_{t_0}^t i_c(\tau) d\tau \right] + v_c(t_0)$$

\hookrightarrow initial condition

$$\Rightarrow i_c(t) = C \cdot \frac{dv_c(t)}{dt}$$

if $V_c(t) = \text{DC value - fixed}$

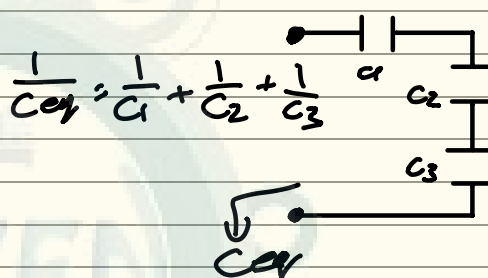
$$\Rightarrow i_c(t) = 0$$

\hookrightarrow means that capacitor act like open ckt in DC-ckt
 لا يعمل capacitor بال DC values like open ckt
 في تيار

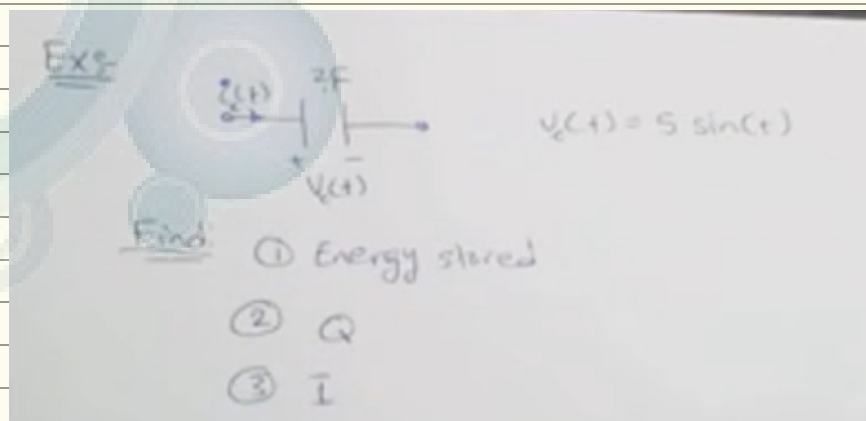
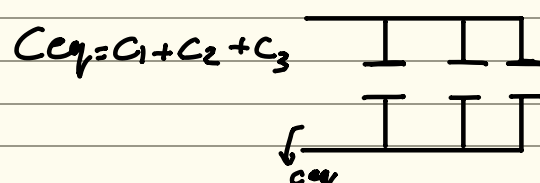
\Rightarrow Energy stored in capacitor

$$E = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C} \Rightarrow \text{unit: joule}$$

in series



in parallel



① energy stored

$$E = \frac{1}{2} C V^2$$

$$E = \frac{1}{2} (2) \times (5 \sin t)^2$$

$$E = 25 \sin^2 t \text{ joule}$$

② Q

$$Q = C \times V$$

$$= 2 \times (5 \sin t)$$

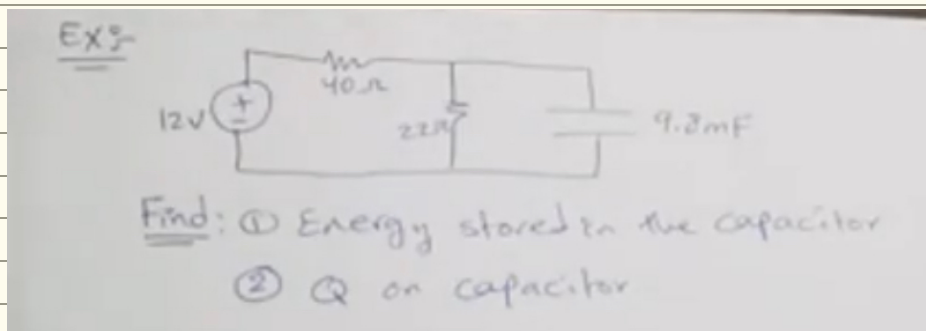
$$= 10 \sin t \text{ coulomb}$$

③ I

$$i_c(t) = C \frac{dv(t)}{dt}$$

$$i_c(t) = 2 \times 5 \cos t$$

$$i(t) = 10 \cos t \text{ A}$$



first of all I notice that this supply is DC supply so

Capacitor act like open CK.

In DC-CKT, the capacitor act like open CK. $I_C = 0$

$$① E = \frac{1}{2} C U^2$$

now finding V

$$-12 + 40I + 22I = 0$$

$$-12 + 62I = 0$$

$$62I = 12$$

$$I = 0.19A$$

$$V = IR$$

$$V = 0.19 \times 22$$

$$V = 4.258V$$

$$\text{so } E = \frac{1}{2} C U^2$$

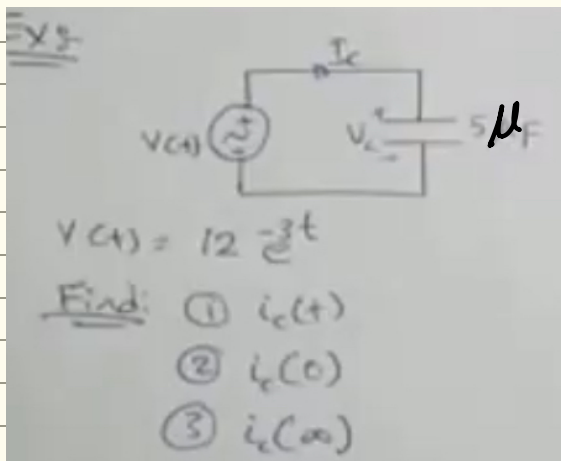
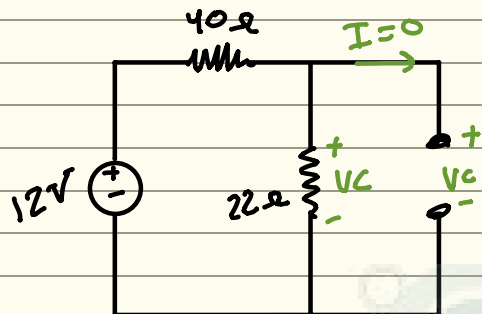
$$= \frac{1}{2} \times 9.8 \times 10^{-9} \times (4.26)^2$$

$$E = 88.92 \text{ mJoules}$$

now finding Q

$$Q = CV$$

$$= 4.8 \times 10^{-3} \times 4.26 = 0.042 \text{ coulomb}$$



voice.

$$V(t) = 12e^{-3t}$$

Find:-

$$① i_C(t)$$

$$② i_C(0)$$

$$③ i_C(\infty)$$

$$V_C(t) = V(t) = 12e^{-3t}$$

(parallel)

$$i_C(t) = C \frac{dV}{dt}$$

$$= 5 \times 10^{-6} \times -36e^{-3t}$$

$$= -180 \times 10^{-6} e^{-3t}$$

$$= -180e^{-3t} \mu A$$

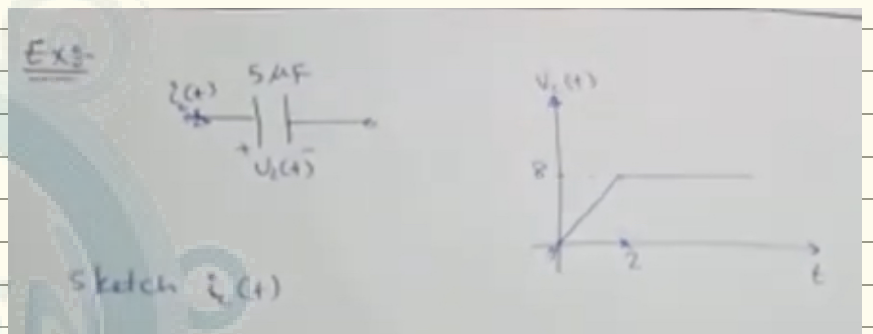
$$② i_C(0) = -180 \mu A$$

$$③ i_C(\infty) = (-180e^{-\infty}) \mu A$$

$$i_C(\infty) = 0A$$

$$e^{-\infty} = 0$$

$$e^{\infty} = \infty$$



$$i_C(t) = \frac{C dV(t)}{dt}$$

① for $0 \leq t \leq 2$

$$V_C(t) = 4t$$

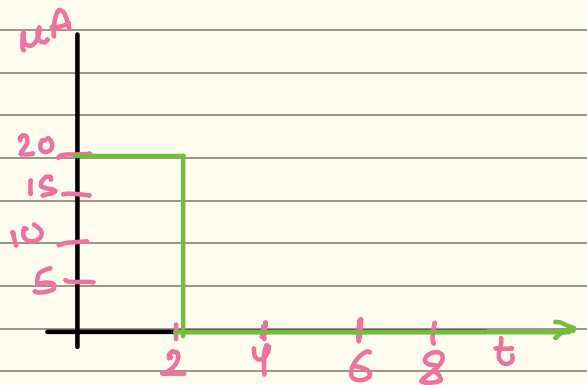
$$i_C(t) = 5 \times 10^{-6} \times 4 = 20 \times 10^{-6} A$$

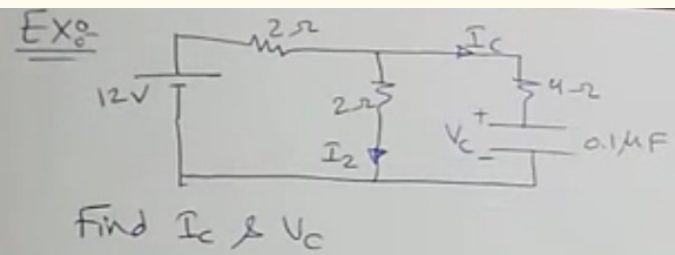
② for $2 \leq t < \infty$

$$V_C = 8V$$

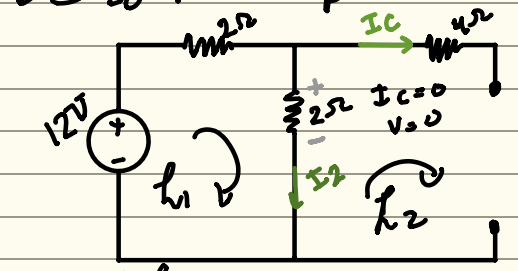
$$i_C = 5 \times 10^{-6} \times 0 = 0 A$$

Sketch





DC so the Capacitor act like open ckt



By KVL l_1

$$-12 + I_2 \cdot 2 + 2 \cdot I_2 = 0$$

$$-12 + 4I_2 = 0$$

$$\frac{4I_2}{4} = \frac{12}{4} \Rightarrow I_2 = 3A$$

$V = IR$
 $= 3 \cdot 2 = 6V$

$V_c = 6V$

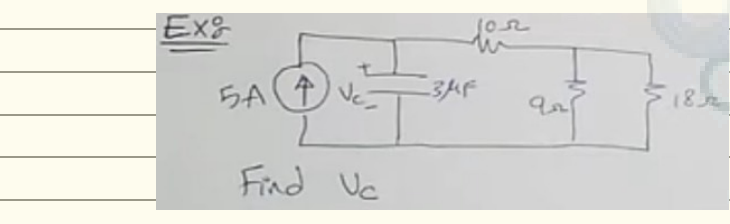
By KVL l_2

$$-V + V_c = 0$$

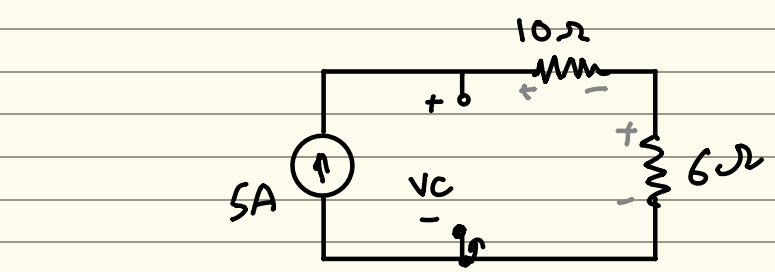
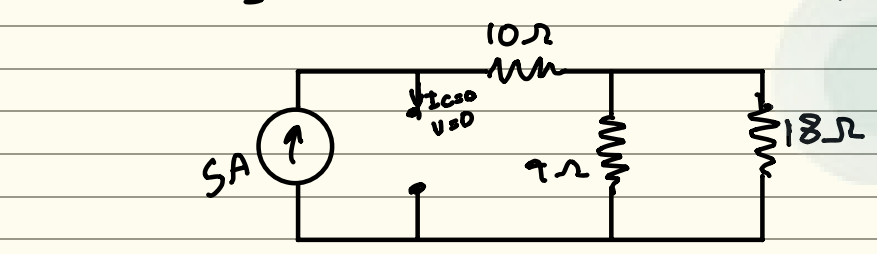
$$-6 + V_c = 0$$

$V_c = 6$

في كل وقت اي فرق



DC supply so capacitor will act like open ckt



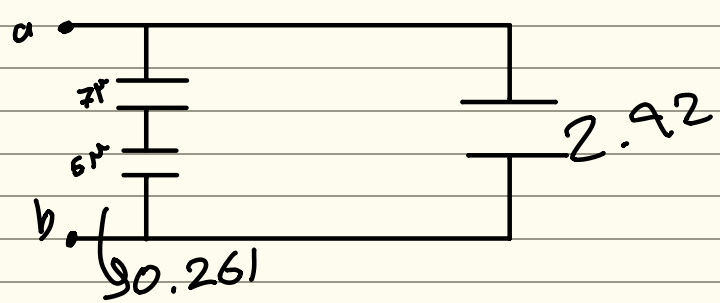
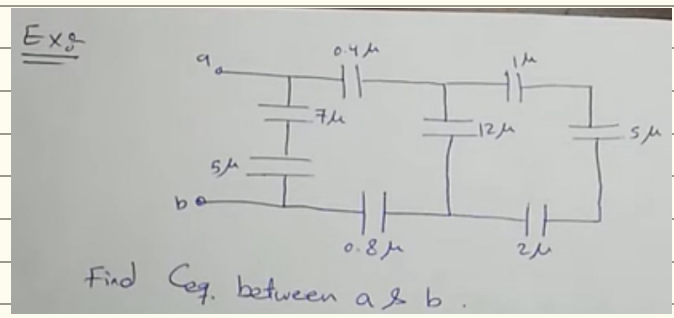
Starting KVL from \uparrow

$$-V_c + 10 \cdot 5 + 6 \cdot 5 = 0$$

$$-V_c + 50 + 30 = 0$$

$$-V_c + 80 = 0$$

$V_c = 80V$



$C_{eq} = 2.92 + 0.261 = 3.18 \mu F$

4 inductor

- ⇒ Store energy into
- ⇒ made of winding coil of wire around a core
- core: could be insulator or ferromagnetic material
- ⇒ Symbol $i_L(t)$ \rightarrow $\frac{L}{\mu}$ \rightarrow permeability

⇒ unit: Henry $\Rightarrow H, mH, \mu H$

$$V_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = \left[\frac{1}{L} \int_{t_0}^t V_L(\tau) d\tau \right] + i_L(t_0)$$

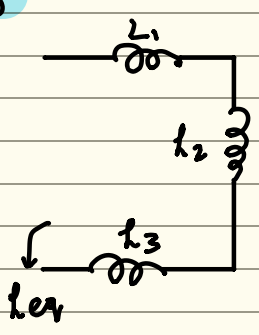
⇒ Energy stored :-

$E = \frac{1}{2} L I^2$ Joule

⇒ In DC-ckt, the inductor act like short ckt $V_L = 0$

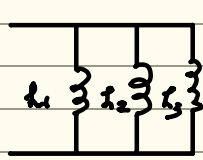
in series

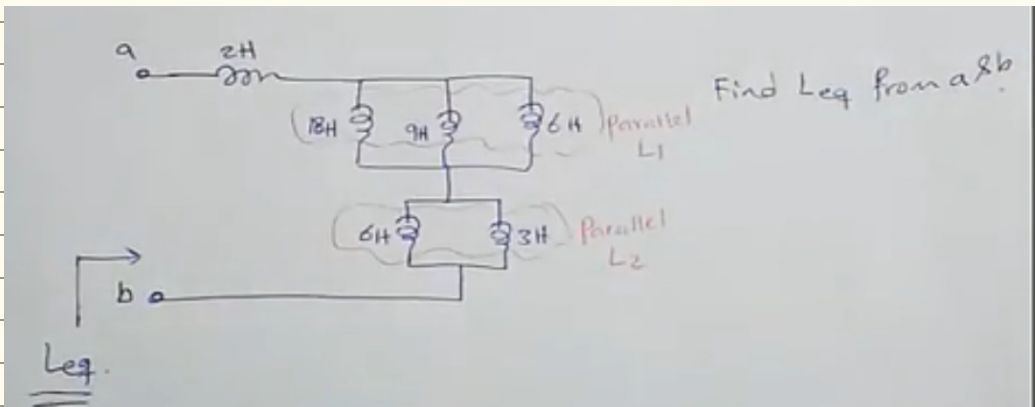
$L_{eq} = L_1 + L_2 + L_3$



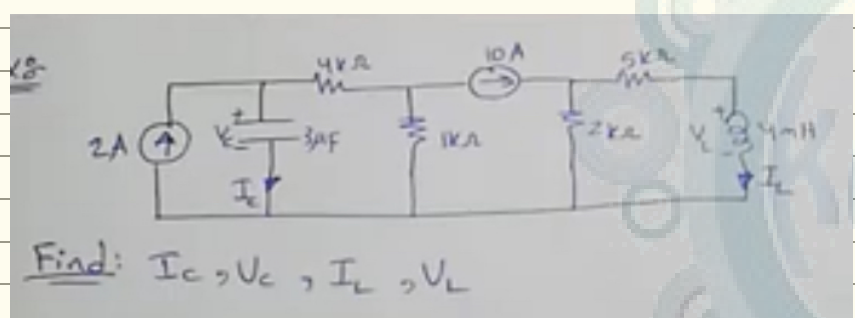
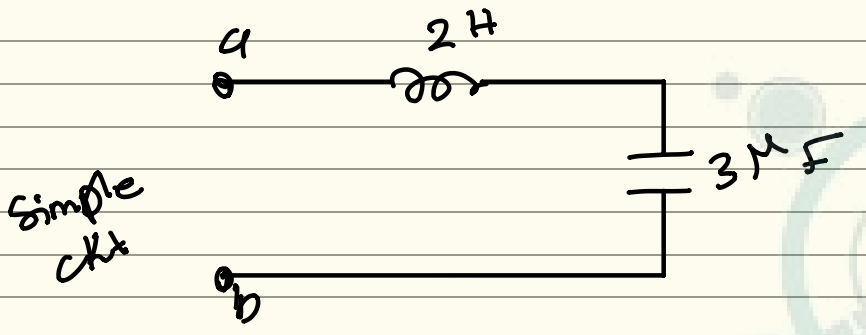
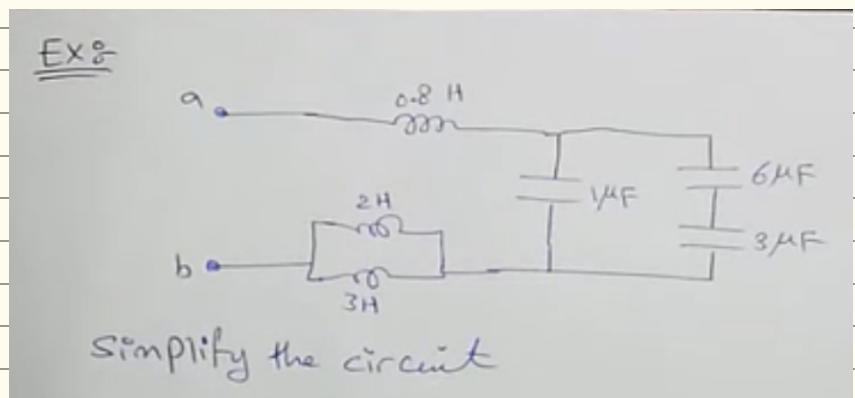
in parallel

$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$

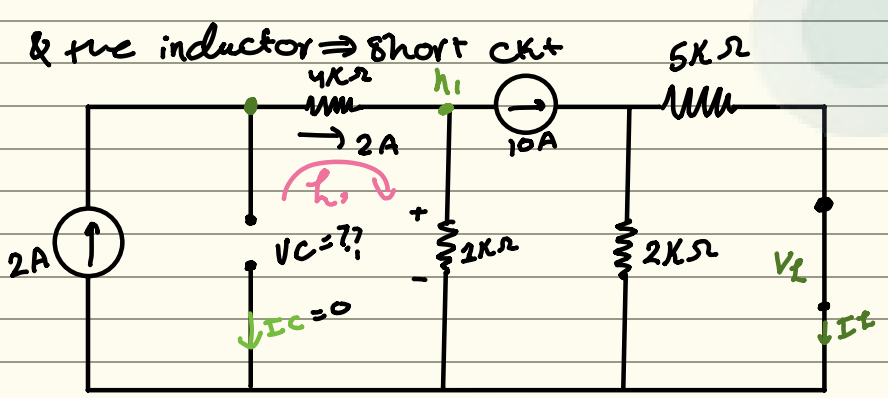




$L_{eq} = 7 \text{ H}$



DC supply \Rightarrow DC-Ckt the capacitor will acts as open ckt



now we have $I_C = 0$ & $V_L = 0$

applying Kcl for n_1

$2 = I_{1k\Omega} + 10$
 $I_{1k\Omega} = -8 \text{ A}$

applying Kvl for loop 1

$-V_C + 8 \times 10^3 + 1 \times -8 \times 10^3 = 0$

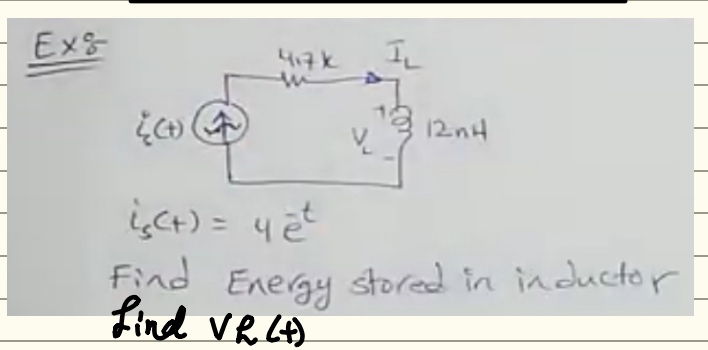
$-V_C = 0$
 $V_C = 0$
 $I_C = 0$
 open ckt

now finding I_L

ملاحظة: Current Source \Rightarrow Current division \Rightarrow same fork

$I_{5k\Omega} = \frac{10 \times (\frac{1}{5})}{(\frac{1}{2} + \frac{1}{5}) \times 10^3} = \frac{20}{7} = 2.857 \text{ A}$

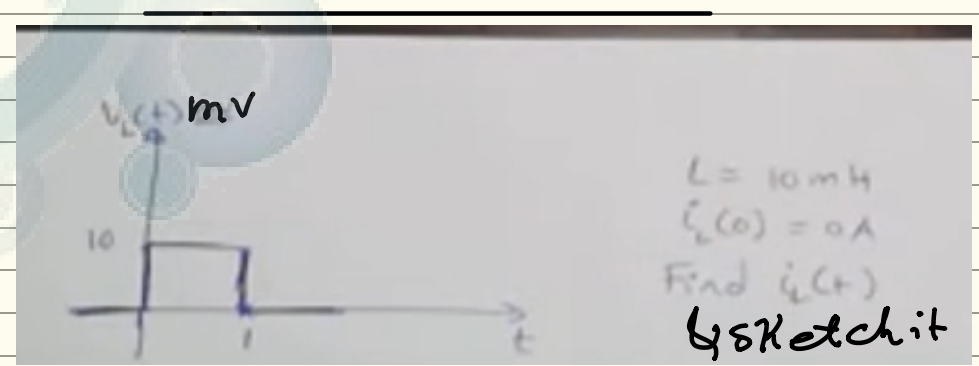
$V_L = 0 \Rightarrow$ because its short ckt



$E = \frac{1}{2} L I_L^2$
 $E = \frac{1}{2} \times 12 \times 10^{-9} \times (4e^{-t})^2$
 $E = 96 \times 10^{-9} e^{-2t}$
 $E = 96 e^{-2t} \text{ nJ}$

$V_L(t) = L \frac{di_L(t)}{dt}$

$V_L(t) = 12 \times 10^{-9} \times -4e^{-t}$
 $V_L(t) = -48 e^{-t} \text{ nV}$



$i_L(t) = \left(\frac{1}{L} \int_{t_0}^t v_L(\tau) d\tau \right) + i_L(t_0)$

for $t < 0$

$V = 0 \Rightarrow i_L(t) = 0$

for $0 < t < 1$, $i_L(0) = 0$

$V_L(t) = 10 \text{ mV}$

$i_L(t) = \left\{ \frac{1}{10 \times 10^{-3}} \int_0^t (10 \text{ m}) d\tau \right\} + 0$

$i_L(t) = \frac{1}{10 \times 10^{-3}} \times 10 \times 10^{-3} t + 0$

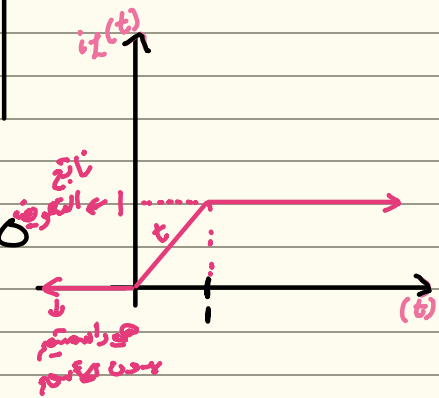
$i_L(t) = t$

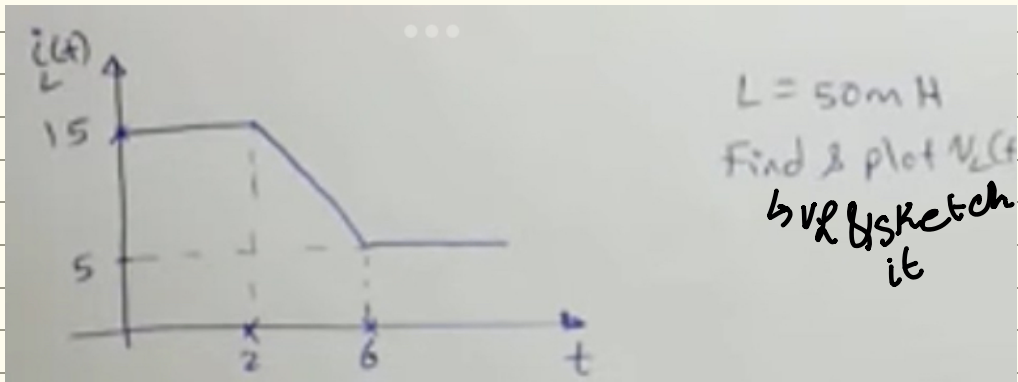
for $t > 1$

$V_L(t) = 0$

$i_L(t) = \int_{t_0}^t v_L(\tau) d\tau + i_L(t_0)$

$i_L(t) = 0 + 1 = 1 \text{ A}$





$$V_L(t) = L \cdot \frac{di(t)}{dt}$$

$$= 50 \times 10^{-3} \times 0 = 0$$

starting from finding the slope

$$\text{slope} = \frac{15-5}{2-6} = -\frac{10}{4}$$

$$y-5 = -2.5(x-6)$$

$$y = -2.5x + 15 + 5$$

$$y = -2.5x + 20$$

so $V_L(t) = \frac{di(t)}{dt} \cdot L$

$$V_L(t) = (-2.5) \cdot 50 \times 10^{-3}$$

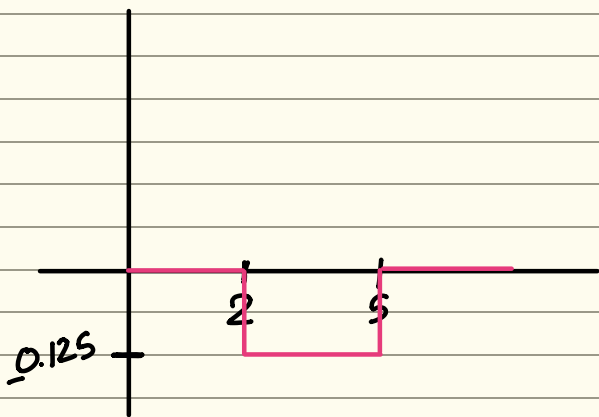
$$V_L(t) = -0.125$$

for $t \geq 6$

$$i(t) = 5$$

$$V_L(t) = 0$$

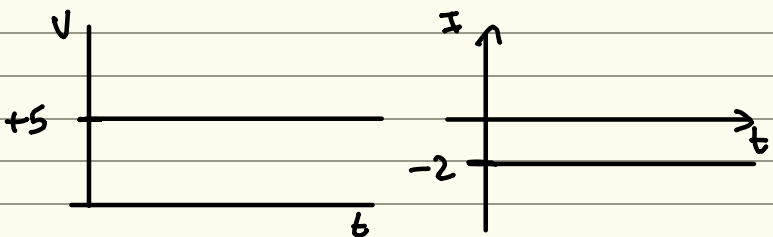
the sketch



initial condition

Time Dependent signal sources

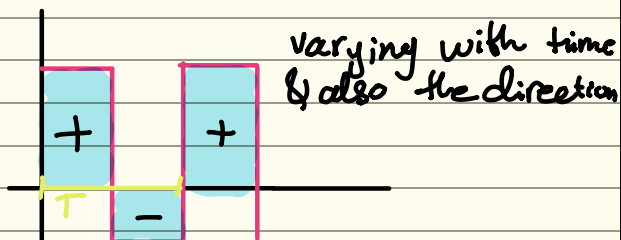
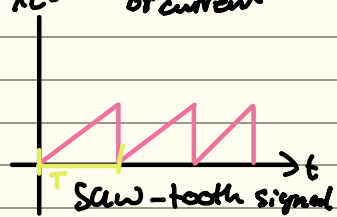
Dc - sources \Rightarrow fixed value & direction



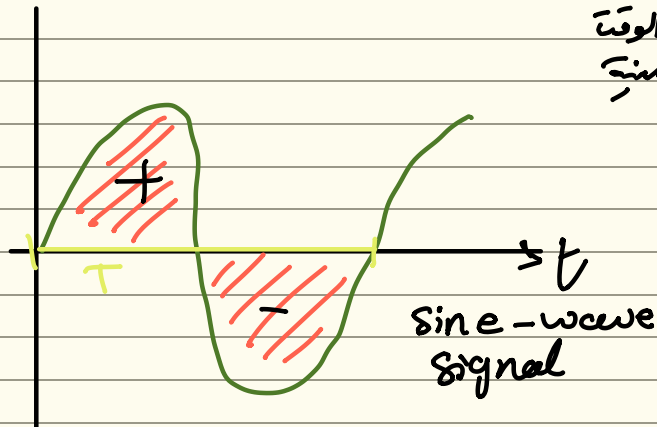
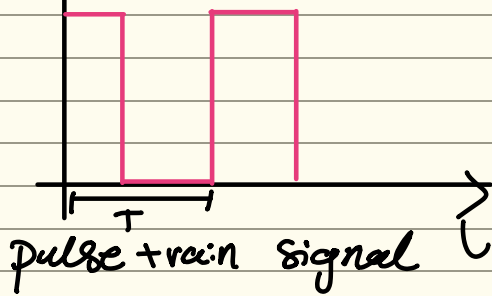
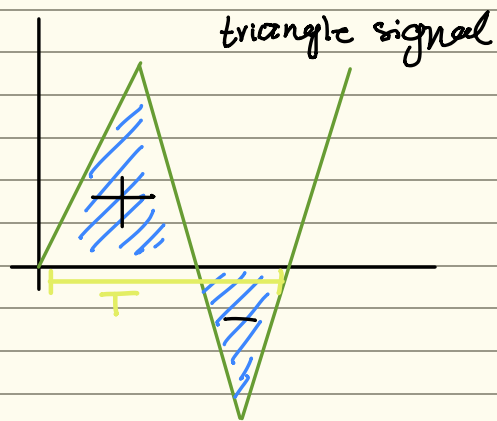
Ac - sources

generate voltage or current varying with time.

$x(t) \rightarrow$ voltage or current



varying with time & also the direction

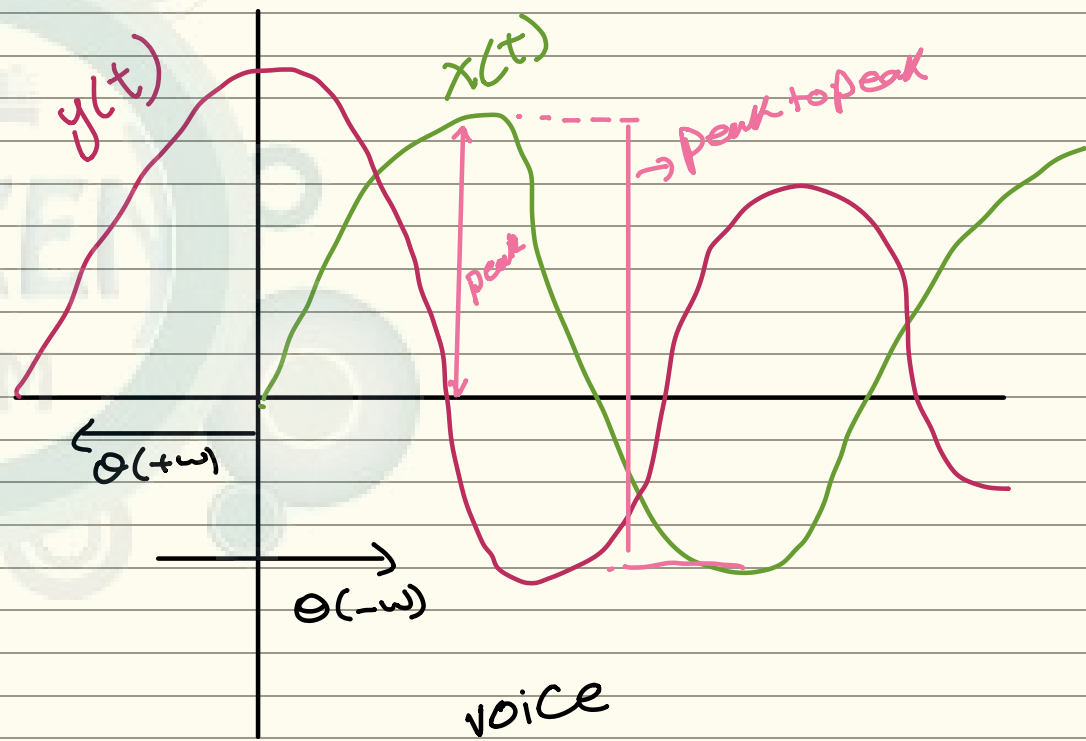


بتنظيم الوقت وبنفس الوقت
بشئ فني إشارة موجة جيبية
قابلة للتغير مع (t)

T: الزمن الى يحتاجو عشان يكرر نفسو عايرسة.

* periodic - signal Repeat it self after one periode (T)

random signal لا يتكرر



$$x(t) = A \cos(\omega t - \theta)$$

$$y(t) = B \cos(\omega t)$$

A: amplitude of signal \Rightarrow peak value

$$\text{peak to peak} = 2 * \text{peak} = 2A$$

ω : radian Freq \Rightarrow unit Rad/s

$$\omega = 2\pi f$$

where:-

f: Natural Freq \Rightarrow unit Hz

$$T = \frac{1}{f} \text{ sec}$$

$$\theta: \text{phase shift} \Rightarrow$$

distance between two signal in sec

$$\theta = \frac{\Delta T * 360}{T}$$

period

$$\sin(\omega t) = \cos(\omega t - 90^\circ)$$

قد يثبت الزاوية
التي تحركها

$$-\sin(\omega t) = \sin(\omega t \pm 180)$$

$$-\cos(\omega t) = \cos(\omega t \pm 180)$$

فعل بال

Ex $i(t) = 10 \cos(377t + 30^\circ)$

find: I_{peak} , $I_{peak to peak}$, ω , f , period θ , $i(0.1)$ sec, graph this function

$$I_{peak} = 10 A$$

$$I_{peak to peak} = 2 \times 10 = 20 A$$

$$\omega = 377$$

$$\omega = 2\pi f$$

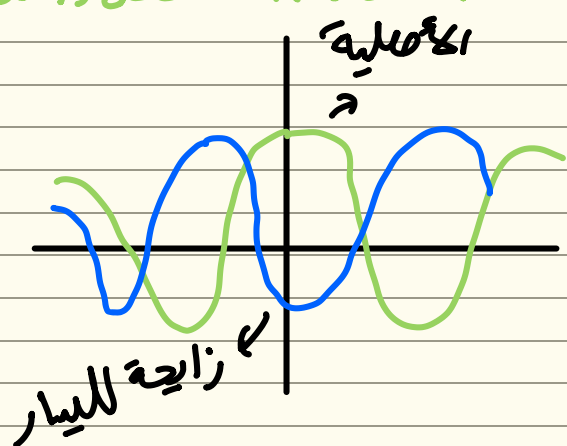
$$\frac{377}{2\pi} = \frac{2\pi f}{2\pi} \Rightarrow f = \frac{377}{2\pi} = 60 \text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{60} = 0.016 \text{ sec}$$

$$\theta = 30^\circ$$

الزاوية حركتها
Radian

$$i(0.1) \text{ sec} = 4.657 A$$



* Complex numbers:-

$$j \text{ or } i = \sqrt{-1}$$

$$(j)^2 = -1$$

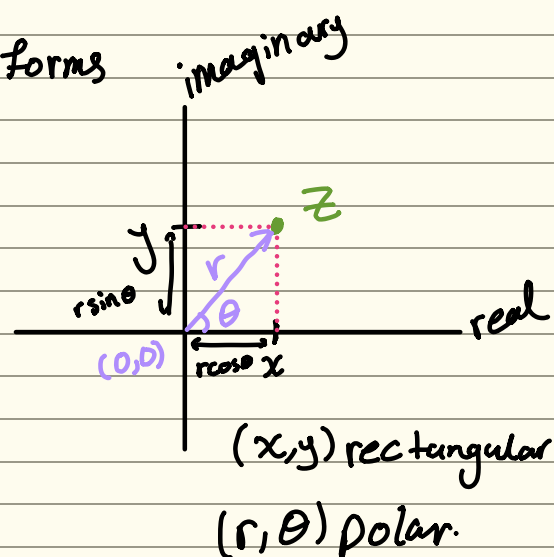
Complex number has two forms

① Rectangular Form

$$Z = x + jy$$

② polar Form:

$$Z = r \angle \theta$$



* Conversion from Rect. to polar

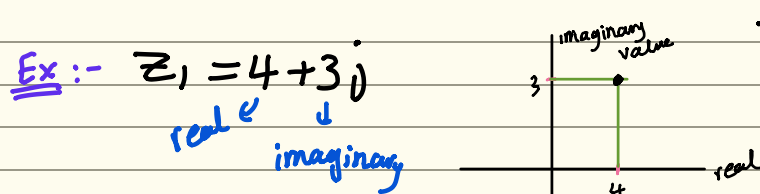
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow \text{الزاوية}$$

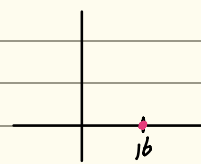
* Conversion from polar to Rect

$$x = r \cos(\theta)$$

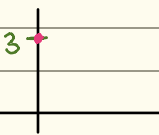
$$y = r \sin(\theta)$$



$$Z_2 = 16$$



$$Z_3 = 3j$$



* Mathematical

$$X_1 = a + jb, X_2 = c + jd$$

* addition

$$X_1 + X_2 = (a + c) + (b + d)j$$

* Subtraction

$$X_1 - X_2$$

$$a + jb - c - jd$$

$$(a - c) + (b - d)j$$

* multiplication:-

$$X_1 * X_2 = (a + jb) * (c + jd)$$

$$= ac + adj + cbj - bd$$

$$(ac - bd) + (ad + cb)j$$

④ Division

$$\frac{X_1}{X_2} = \frac{a + jb}{c + jd} * \frac{c - jd}{c - jd}$$

$$= \frac{ac - adj + cbj + bd}{c^2 - d^2}$$

$$= \frac{ac + bd}{c^2 - d^2} + \left(\frac{cb - ad}{c^2 - d^2}\right)j$$

that's when we are doing mathematic for complex numbers in rectangular form

In polar form

$$X_1 = r_1 \angle \theta_1, X_2 = r_2 \angle \theta_2$$

$$X_1 * X_2 = (r_1 \angle \theta_1) * (r_2 \angle \theta_2)$$

$$= r_1 * r_2 (\theta_1 + \theta_2)$$

$$\frac{X_1}{X_2} = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} (\theta_1 - \theta_2)$$

$$X_1 \pm X_2 = r_1 \angle \theta_1 \pm r_2 \angle \theta_2$$

→ convert to rectangular then compute

Using calculator to Solve complex numbers:-

$$x = 3 + 2j$$

$$y = 5 + j$$

$$\Rightarrow x + y = (3 + 2j) + (5 + j)$$

$$= 13 + 3j$$

$$= 13\sqrt{2} \angle \frac{\pi}{4}$$

$$\Rightarrow x \div y = \frac{(3 + 2j)}{(5 + j)} = \frac{17}{26} + \frac{7}{26}j \Rightarrow \frac{\sqrt{2}}{2} (0.39)$$

but if It is already in polar form

$$3 \angle 20 + 4 \angle -15$$

$$= 6.68 - 4.2 \times 10^{-3}j$$

$$6.68 \angle 0.08$$

$$\frac{3 \angle 20}{4 \angle 15} = 0.614 + 0.43j$$

$$\frac{3}{4} \angle 35$$

2 signals ω is in

each one of them has their own (ω) we can not combine them in one phasor diagram.

So if I have $\begin{cases} z(t) = A \cos(\omega t + \theta_1) \\ z(\omega) = A e^{j\theta_1} = A \angle \theta_1 \end{cases}$ we could not draw them in the same diagram

4/5/2025

Phasor diagram

different can't be drawn together because of different ω
every ω has a diff phasor diagram convert

$$x(t) = A \sin(2\pi(50)t + \pi/2) \rightarrow x(t) = A \cos(2\pi(50)t - \pi/2 + \pi/2)$$

same can be drawn

$$y(t) = B \cos(2\pi(50)t + \pi/4)$$

but sin should become cos to be drawn

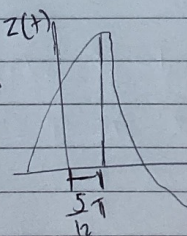
Same frequency & can be drawn together

$$z(t) = A \cos(2\pi(50)t - \frac{5\pi}{12})$$

$$z(\omega) = A e^{-j\frac{5\pi}{12}} = A \angle -\frac{5\pi}{12}$$

1) draw 2 ω is in 1 place
(2) draw 1 ω in 1 place
Phasor diagram

time domain



Review of complex arithmetics:

- addition: add real # together and imaginary together

* Sinusoidal signals in freq domain:-

Time domain $\rightarrow x(t) = A \cos(\omega t + \theta)$

Frequency domain $\rightarrow X(j\omega) = A \angle \theta \rightarrow$ phase shift + referenced to cosine signal
peak value

Ex: $V_1(t) = 15 \cos(377t + \frac{\pi}{4})$ V
 $V_2(t) = 15 \cos(377t + \frac{\pi}{12})$ V

Find $V_s(t) = V_1(t) + V_2(t)$

Sol

الحل باستخدام المتطابقات المثلثية كثير صعب فنبينهم (complex)

\Rightarrow frequency domain \rightarrow هاد الحل اليه بجاو هسا اسهل frequency domain
قد صاؤوذ الزاوية بتا در (cos)

$V_1(j\omega) = 15 \angle \frac{\pi}{4} = 10.61 + j10.61$

$V_2(j\omega) = 15 \angle \frac{\pi}{12} = 14.489 + j3.88229j$

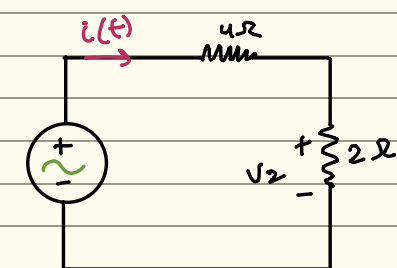
$V_s(j\omega) = V_1(j\omega) + V_2(j\omega)$

$= 25.0988 + j14.492j$

$= 28.9824 \angle 0.5236416933$

$V_s(t) = 28.98 \cos(377t + \frac{\pi}{6})$
الهم فتن frequency فبكتبو

Ex:- \sin wave $\Leftarrow AC \Leftarrow$ supply \rightarrow مجرد صاؤوذ ال supply



$V(t) = 12 \cos(377t + 40^\circ)$ V

Find $i(t)$, $V_2(t)$

Sol

we already took this type of ckt & we solve it by voltage division

now we are going to solve it using voltage division
لما سمعنا بل صاؤوذ رفق هسا هار رفق وازاوة

$V_1(j\omega) = 12 \angle \frac{2\pi}{4}$

$V_2(j\omega) = \frac{(12 \angle \frac{2\pi}{4}) \cdot 2}{2+4} = 4 \angle \frac{2\pi}{4}$

$V_2(t) = 4 \cos(377t + \frac{2\pi}{4})$ V
 $4 \angle \frac{2\pi}{4}$

$i(j\omega) = \frac{4 \angle \frac{2\pi}{4}}{2} = 2 \angle \frac{2\pi}{4}$ A

هو بجاو
time domain

$i(t) = 2 \cos(377t + \frac{2\pi}{4})$

* Impedance: $Z(j\omega)$

open ckt \Leftarrow capacitor \rightarrow DC supply
short ckt \Leftarrow inductor

\Rightarrow we will describe the ckt elements (R, L, C)

in AC-CKT \Rightarrow they will have impedance

1 Resistor

if $V(t) = V_m \cos(\omega t + \theta)$ \rightarrow $i(t)$ \rightarrow $V(t)$

$V(j\omega) = V_m \angle \theta$

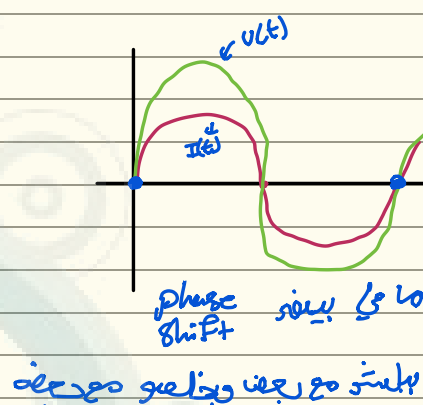
to find $I(j\omega) = \frac{V(j\omega)}{R} = \frac{V_m \angle \theta}{R} = \frac{V_m}{R} \angle \theta$

$\Rightarrow i(t) = \frac{V_m}{R} \cos(\omega t + \theta)$

$\Rightarrow Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{V_m \angle \theta}{\frac{V_m}{R} \angle \theta} = R \angle 0$
 \rightarrow so there is no phase shift

$Z_R(j\omega) = R \Omega$

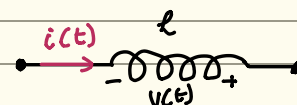
إذا صافي تغيير بتا طوع
المساوية زي صافي



يعني لو بدى أرسم
no phase shift because it is Resistance.

2 Inductor:-

$V(t) = L \frac{di(t)}{dt}$



if $i_L(t) = I_m \cos(\omega t + \theta)$

$I_L(j\omega) = I_m \angle \theta$
نستعمل $\cos \rightarrow -\sin$

so $V(t) = I_m(\omega L) (-\sin(\omega t + \theta))$
بتا نعمل $\sin \rightarrow \cos$
 $= I_m(\omega L) \cos(\omega t + \theta + 90^\circ)$

$V(j\omega) = I_m(\omega L) \angle \theta + 90^\circ$

$Z_L(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{I_m(\omega L) \angle \theta + 90^\circ}{I_m \angle \theta}$

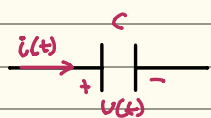
$Z_L(j\omega) = (\omega L) \angle \theta + 90^\circ - \theta$

$Z_L(j\omega) = \omega L \angle 90^\circ = j\omega L \Omega$

$Z_L(j\omega) = j\omega L \Omega$

$\omega = 2\pi f$
& that means that $Z_L(j\omega)$ is frequency dependent

3) Capacitor



$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$\text{if } v_C(t) = V_m \cos(\omega t + \theta) \Rightarrow v_C(j\omega) = V_m \angle \theta$$

$$i_C(t) = V_m (C\omega) (-\sin(\omega t + \theta))$$

$$= (C\omega) V_m \cos(\omega t + \theta + 90^\circ)$$

$$I_C(j\omega) = (C\omega) V_m \angle \theta + 90^\circ$$

$$Z_C(j\omega) = \frac{V_C(j\omega)}{I_C(j\omega)} = \frac{V_m \angle \theta}{C\omega V_m \angle \theta + 90^\circ}$$

$$\frac{1}{C\omega} \angle \theta - \theta - 90^\circ$$

$$Z_C(j\omega) = \frac{1}{C\omega} \angle -90^\circ = \frac{-j}{\omega C} \Omega$$

$$Z_C(j\omega) = \frac{-j}{\omega C} \Omega$$

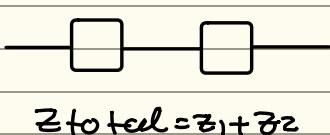
Review :-

$Z_R(j\omega) = R \rightarrow$ Fixed with frequency variation

$$Z_L(j\omega) = j\omega L = \omega L \angle 90^\circ$$

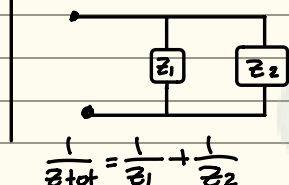
$$Z_C(j\omega) = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

Series



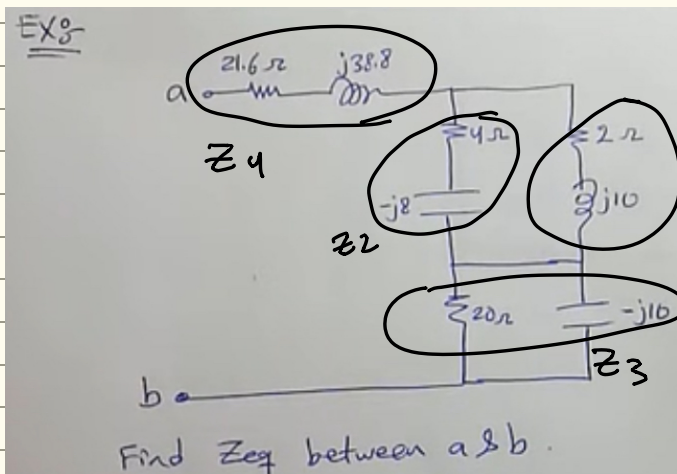
$$Z_{total} = Z_1 + Z_2$$

parallel



$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

EX:-



$$Z_1 = 2 + j10$$

$$Z_2 = 4 - j8$$

$$Z_3 \Rightarrow \frac{1}{Z_3} = \frac{1}{20} + \frac{1}{-j10}$$

$$Z_3 = 4 - j8$$

$$Z_4 = 21.6 + j38.8$$

for Z_1 & Z_2

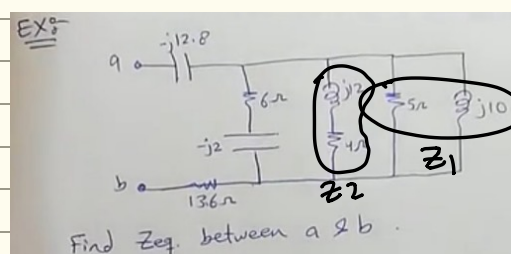
$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2 + j10} + \frac{1}{4 - j8}$$

$$Z_{eq} = 14.4 - j0.8$$

$$Z_{total eq} = (4 - j8) + (21.6 + j38.8) + (14.4 - j0.8)$$

$$Z_{eq} = 40 + j30$$

means that we have only resistor and inductor



$$Z_1 = \frac{1}{5} + \frac{1}{j10}$$

$$Z_1 = 4 + j2$$

$$Z_2 = 12j + 4$$

$$Z_1 \parallel Z_2$$

$$\frac{1}{4 + j2} + \frac{1}{12j + 4} = 2.76923 + 2.15384j$$

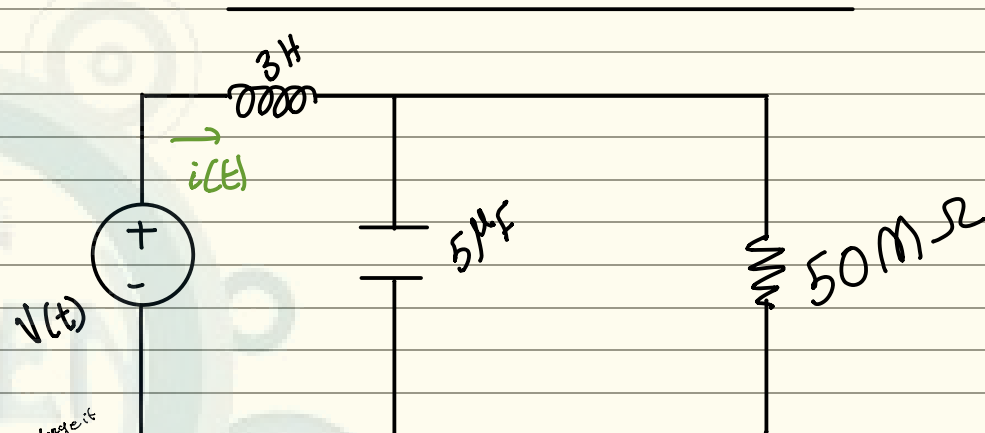
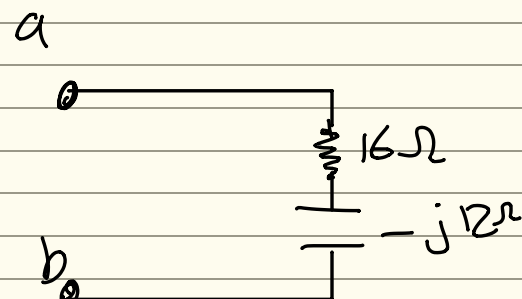
$$Z_3 \parallel (2.76923 + 2.15384j)$$

$$Z_4 = 2.4 + j0.8$$

$$Z_{eq} = 2.4 + j0.8 - 12.8j + 13$$

$$Z_{eq} = 16 - j12$$

that mean that we have



we will change it to cos because it is our reference

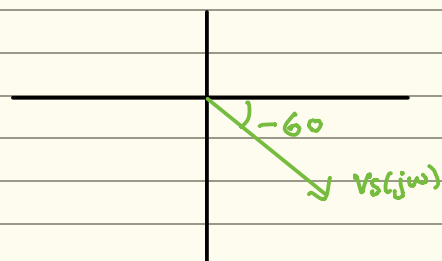
$$V_L(t) = 5 \sin(2\pi 50t + \frac{\pi}{6}) \text{ volts}$$

$$Z_C = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ = \frac{-j}{2\pi 50 \times 5 \times 10^{-6}}$$

$$Z_L = j\omega L = j \times 3 \times 2\pi \times 50 = 942.42j \Omega$$

$$Z_R = 50 \times 10^6$$

$$V_S(j\omega) = 5 e^{-j\frac{2\pi}{6}} = 5 \angle \frac{-2\pi}{6} \text{ volts}$$



$$Z_{eq1} = 8.1056 \times 10^{-3} - j636.62$$

$$Z_{eq2} = 305.8520 \angle 89.49848$$

دائماً أحسن أنك تختار

rectangular impedance

* notes on chapter 4

$$\Rightarrow Z_L(j\omega) = j\omega L = \omega L \angle 90^\circ$$

$$X_L = \omega L \Rightarrow \text{reactance}$$

$$\omega = 2\pi f$$

$$X_L = (2\pi f)L$$

so that reactance depends on frequency

① at $f=0 \Rightarrow$ at DC

$X_L = 0 \Omega \Rightarrow$ because of this we said that the conductor acts like short ckt in DC supply

② at $f=\infty \Rightarrow X_L = \infty \Omega \Rightarrow$ open ckt

$$\Rightarrow Z_C(j\omega) = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

$$X_C = \frac{1}{\omega C} \Rightarrow \text{reactance}$$

① at $f=0$ (DC) $\Rightarrow X_C = \infty \Omega$

capacitor acts like open ckt
DC

② at $f=\infty \Rightarrow X_C = 0$

\Rightarrow like short ckt

$$\Rightarrow Z_R(j\omega) = R \Rightarrow \text{independent of freq}$$

$$\Rightarrow Z = R + jX$$

Resistance \hookrightarrow Reactance

a) if $Z = R$ i.e.

assume $V = V_m \angle \theta$

$$\text{so } I = \frac{V_m \angle \theta}{R} = \frac{V_m}{R} \angle \theta$$

same angle so there is no phase between voltage & current across resistor

phase shift between V & $I = 0$

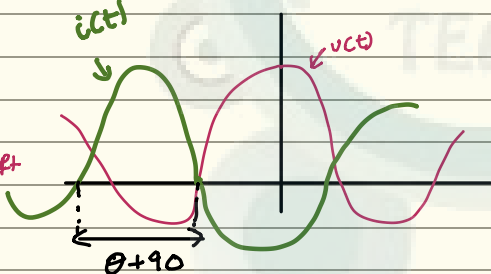
b) if $Z = -jX_C \Rightarrow$ pure capacitive

assume $V = V_m \angle \theta$

$$\text{so } I = \frac{V_m \angle \theta}{X_C \angle -90^\circ} = \frac{V_m}{X_C} \angle \theta + 90^\circ$$

shift to left

\Rightarrow current I leads voltage by 90°
or V lags I by 90°



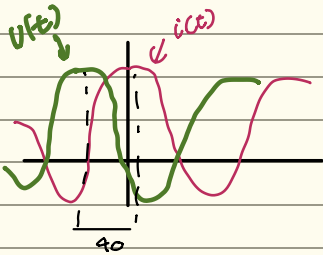
c) if $Z = jX_L \Rightarrow$ pure inductive

assume $V = V_m \angle \theta$

$$\text{so } V = V_m \angle \theta$$

$$I = \frac{V_m \angle \theta}{X_L \angle 90^\circ} = \frac{V_m}{X_L} \angle -90^\circ$$

shift to right



$\Rightarrow V$ leads I by 90°

$\Rightarrow I$ lags the V by 90°

if $Z = R + jX_L \Rightarrow$ behave like inductive (pure inductor)

so here $0 < \theta < 90^\circ$

if $Z = R - jX_C \Rightarrow$ behave like capacitive (pure capacitor)

$-90^\circ < \theta < 0$

* Admittance

$$Z = R + jX$$

impedance \hookrightarrow Resistance \hookrightarrow Reactance (X_L or X_C)

$$Y = G + jB$$

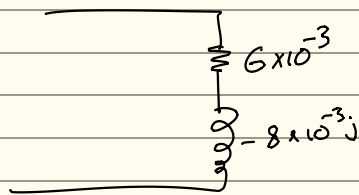
admittance \hookrightarrow conductance \hookrightarrow susceptance \hookrightarrow which means that $G \neq \frac{1}{R}$ $B \neq \frac{1}{X}$

Example:- $Z = 100 \angle 53^\circ$, find Y

$$\Rightarrow Y = \frac{1}{Z} = \frac{1}{100 \angle 53^\circ} = \frac{6.02 \times 10^{-3}}{\angle 53^\circ} = 6.02 \times 10^{-3} \angle -53^\circ$$

G

B_C (the negative value of B)



* The Average value

\hookrightarrow method measuring the ^{average} mean of the voltage or current over a period of time

المتوسط
المتوسط

$$\langle X(t) \rangle = \frac{1}{T} \int_0^T x(\tau) d\tau \quad \text{integral of the signal (wave form) over period of time}$$

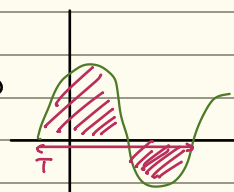
Example: $x(t) = A \cos(\omega t + \theta)$

Find $\langle x(t) \rangle$

$$\langle x(t) \rangle = \frac{1}{T} \int_0^T A \cos(\omega \tau + \theta) d\tau = 0$$

$$= \frac{A}{T} \sin(\omega t + \theta) \Big|_0^T$$

$$= \frac{A}{T} \sin(\omega t + \theta)$$

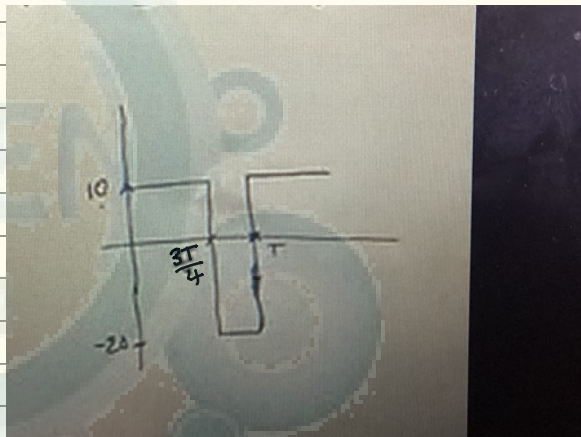


تكون متوسط المدة تحت
المتوسط (المتوسط من فترة 0 الى T)

تحت واحد (متوسط واحد على واحد)

Note: the average of the sinusoidal signal is zero independent of amplitude or frequency

Example:-



$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(\tau) d\tau$$

$$= \frac{1}{T} \left[\int_0^{3T/4} 10 d\tau + \int_{3T/4}^T -20 d\tau \right]$$

$$= \frac{1}{T} \left[\left[10\tau \right]_0^{3T/4} + \left[-20\tau \right]_{3T/4}^T \right]$$

$$= \frac{1}{T} \left[10 \left(\frac{3T}{4} \right) + -20 \left(T - \frac{3T}{4} \right) \right]$$

$$= \frac{1}{T} \left[\frac{15}{2} T + -20T + 15T \right]$$

$$= \frac{1}{T} \left[\frac{5}{2} T \right]$$

$$= \frac{5}{2} = 2.5$$

* Root mean square (RMS):-

RMS: effective value of varying voltage or current

$$X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

= $x_{effective}$
effective value of
current → التيار الفعّال

التيار الفعّال التيار الفعّال

Ex: $x(t) = A \cos(\omega t)$

$$X_{rms} = \sqrt{\frac{1}{T} \int_0^T A^2 \cos^2(\omega t) dt}$$

$$X_{rms} = \sqrt{\frac{A^2}{T} \int_0^T \cos^2(\omega t) dt}$$

$$X_{rms} = \sqrt{\frac{A^2}{T} \int_0^T \frac{1}{2} (1 + \cos(2\omega t)) dt}$$

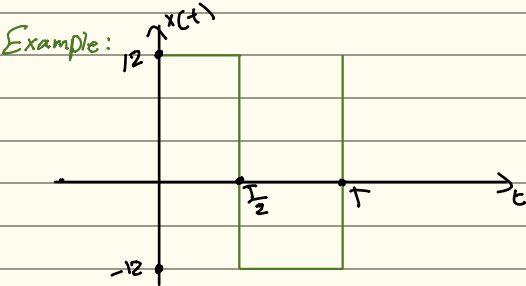
$$X_{rms} = \sqrt{\frac{A^2}{2T} [T + 0]}$$

$$X_{rms} = \sqrt{\frac{A^2}{2T} \cdot T}$$

$$X_{rms} = \frac{A}{\sqrt{2}}$$

* Note: for sinusoidal signal

$X_{rms} = \frac{\text{Peak value}}{\sqrt{2}}$



$$x(t) = \begin{cases} 12 & , 0 \leq t \leq \frac{T}{2} \\ -12 & , \frac{T}{2} \leq t \leq T \end{cases}$$

$$X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

$$= \sqrt{\frac{1}{T} \left[\int_0^{\frac{T}{2}} (12)^2 dt + \int_{\frac{T}{2}}^T (-12)^2 dt \right]}$$

$$= \sqrt{\frac{1}{T} \left[144t \Big|_0^{\frac{T}{2}} + 144t \Big|_{\frac{T}{2}}^T \right]}$$

$$= \sqrt{\frac{1}{T} \left[\frac{144T}{2} + \left(144T - \frac{144T}{2} \right) \right]}$$

$$= \sqrt{\frac{1}{T} [72T + 144T - 72T]}$$

$$= \sqrt{\frac{1}{T} [144T]}$$

$$= \sqrt{144} = 12$$

Note: ① For square wave signal

$X_{rms} = A \leftarrow \text{max value}$

② For triangular signal

$X_{rms} = \frac{A}{\sqrt{3}}$

③ For Dc-signal

$X_{rms} = Dc$

Example $V_{rms} = 220 \text{ Volt}$
 $V_{peak} = 220 \times \sqrt{2}$
 $= 311 \text{ Volt}$

* Note:- لو كان فيه دc و اسي

$V(t) = V_1 \cos(\omega t) + V_2 \sin(\omega t) + \dots + V_0$
↓
Dc value

$$V_{rms} = \sqrt{(V_{DC})^2 + \frac{\sum_{i=1}^N V_i^2}{2}}$$

$$R_{ms} (total) = \sqrt{(R_{ms1})^2 + (R_{ms2})^2 + (R_{ms3})^2}$$

Example: $x(t) = 3 + 2 \cos(\omega t)$

$X_{rms} = \sqrt{9 + \left(\frac{2}{\sqrt{2}}\right)^2}$

$= \sqrt{9 + 2} = \sqrt{11}$

3 → Dc
 $X_{rms} = 3$
 $2 \cos(\omega t) \rightarrow \text{sinusoidal}$
 $\frac{A}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

Example: $v(t) = 4 \cos(100t) + 12 \sin(100t) - 6 \cos(300t) + 5$

$$X_{rms} = \sqrt{\left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + 5^2}$$

$X_{rms} = \sqrt{123} = 11.1$

For sinusoidal لا بنية موجي
 $\frac{4}{\sqrt{2}}$
 $\frac{12}{\sqrt{2}}$
 $\frac{6}{\sqrt{2}}$
for Dc 5 sin و cos

* Ac-analysis steps:-

- التردد او Frequency مقادير المعادلات

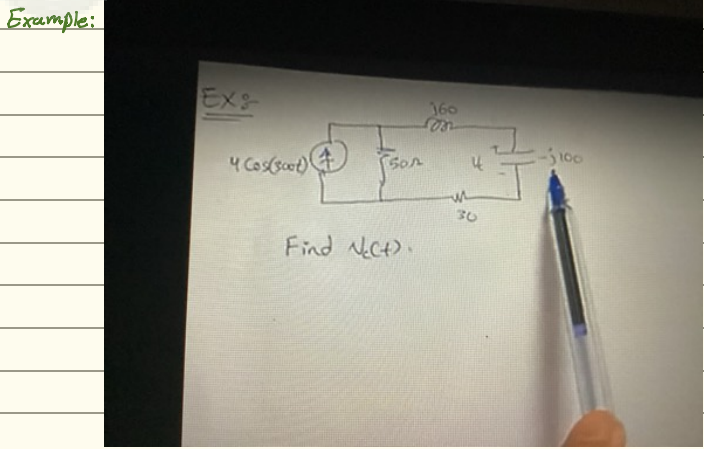
① Note the frequency of the sinusoidal excitation

② Convert all sources to phasor form (Phy domain) ولا يفرج عن cos و sin

③ Convert each ckt element to impedance form

④ Solve the resulting phasor ckt using any method (mesh, nodal, superposition, source transform, Norton & Thevenin, voltage Division, current division, Kcl, Kvl, Ohm's Law)

⑤ Convert to tim domain

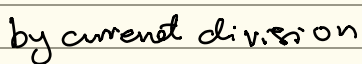


* Step 1: Find ω impedance مقادير المعادلات
 $\omega = 300 \text{ rad/s}$ $\omega = 2\pi f =$
 $\frac{300 = 2\pi f}{2\pi} \rightarrow f = 47.746$

Step 2: Impedance المقادير المعادلات

Step 3: phasor sources المقادير المعادلات
Frequency domain
 $4 \cos(300t) \Rightarrow 4 \angle 0^\circ \text{ A}$
المقادير المعادلات

$Z_1 = 60j - 100j + 30$
 $Z_1 = 30 - 40j$



$$I_{21} = \frac{(4 \angle 0) \cdot \frac{1}{30 - 40j}}{\left(\frac{1}{50}\right) + \left(\frac{1}{30 - 40j}\right)} = 2 + j$$

$$V_C = I \times -j100$$

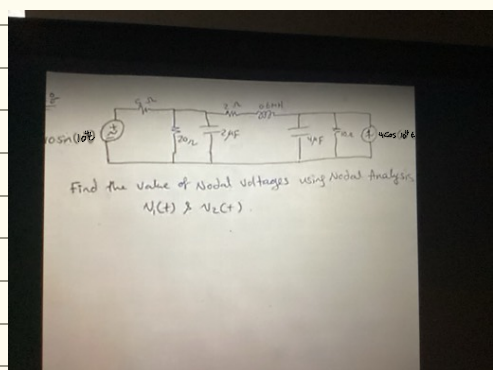
$$= 2 + j \times -j100$$

$$V_C = 223.6 \angle -63.4^\circ \text{ V}$$

نبي لازم اطلع الجواب بالآخر باا time domain

$$V_C = 223.6 (\cos(300t - 63.4))$$

Example:



لا يمكن توفير الترددات من قبل الموردين

Step 1 $\Rightarrow \omega = 10^4$

⇒ step 2 Frequency Domain Sources → DJ Voice

$$\frac{10 \sin(10^4 t) = 10 \sin(10^4 - 90)}{= 10 \angle -90^\circ}$$

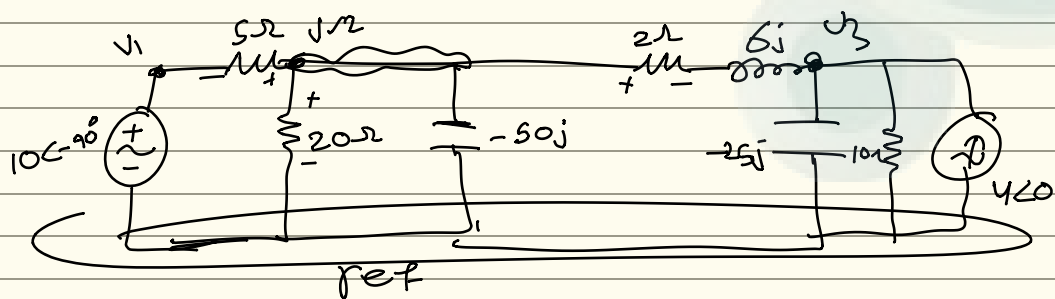
$$4 \cos(10^4 t) = 4 \angle 0$$

⇒ step 3: impedance (CK + elements) نچول آل

$$2 \mu F \Rightarrow Z = \frac{-j}{\omega C} = \frac{-j}{10^4 \times 2 \times 10^{-6}} = -50j$$

$$0.6 \text{ mH} \Rightarrow Z = j\omega L = j \times 0.6 \times 10^{-3} \times 10^4 = 6j$$

$$4 \mu F = \frac{-j}{\omega C} = \frac{-j}{4 \times 10^{-6} \times 10^4} = -25j \Omega$$



$$v_1 = 10 \angle -90^\circ$$

For node 2

$$\frac{V_2 - 10\angle -90^\circ}{5} + \frac{V_2}{20} + \frac{V_2}{-50j} + \frac{V_2 - V_3}{2 + 6j}$$

$$\frac{V_2}{5} - 2i + \frac{V_2}{20} + \frac{V_2}{-50j} + \frac{V_2}{2+6j} - \frac{V_3}{2+6j}$$

$$0.83 \angle -23.43^\circ \sqrt{2} - 2i - \frac{\sqrt{3}}{2+6i} = 0$$

$$0.33 \angle -23.43^\circ + 0.158113883 \angle 108.435^\circ = 2i$$

for node 3

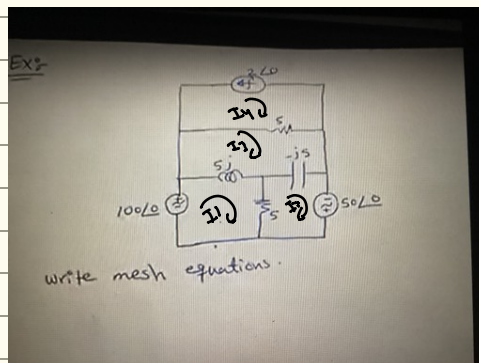
$$\frac{V_2 - V_3}{2 + 6j} + 4 \angle 0 = \frac{V_3}{-25j} + \frac{V_3}{10}$$

$$0.158 \angle -71.565^\circ V_2 - 0.158 \angle -71.565^\circ V_3 + 0.04 \angle 90^\circ V_3 - 0.1 V_3 = -4 \angle 0^\circ$$

$$V_1 = 14.28 \angle 54.8^\circ \text{ V} \Rightarrow 14.28 \cos(10^4 t - 54.8^\circ)$$

$$v_2 = 17.33 \angle 1.99 \text{ volt} \quad 17.33 \cos(10^4 t + 1.99)$$

Example



\Rightarrow Step 1 done

⇒ step 2 done

⇒ step 3 done

قبلمش حال د عزی

$$I_4 = -2 \angle 0^\circ \text{ A}$$

for loop 1

$$-100 < 0 + 5j(I_1 - I_3) + 5(I_1 - I_2) = 0$$

Hot Loop2

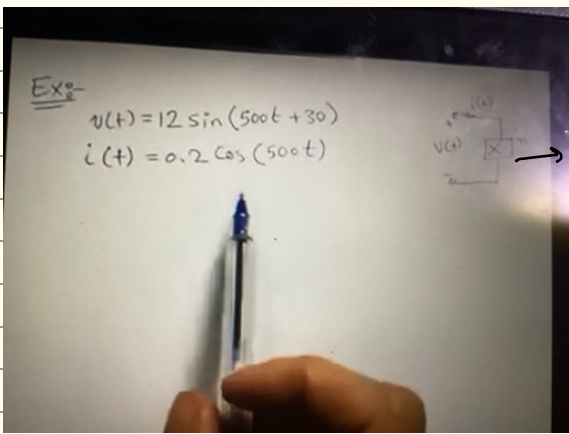
$$5(I_2 - I_1) - 5j(I_2 - I_3) - 50 \angle 0^\circ$$

For loop 3

$$5(I_3 - I_4) - 5j(I_3 - I_2) + 5j(I_3 - I_1) = 0$$

بالعادة الحل هيلك عال Calculator كثير صعب فما بنطلبه من

Example:-



where this device is unknown how to find it?

في ١٠ هرتز في التردد

$$V(t) = 12 \cos(500t - 60^\circ)$$

$$V(t) = 12 \angle -60^\circ \text{ volt}$$

$$i(t) = 0.2 \cos(500t)$$

$$X = \frac{V}{i} = \frac{12 \angle -60^\circ}{0.2 \angle 0^\circ}$$

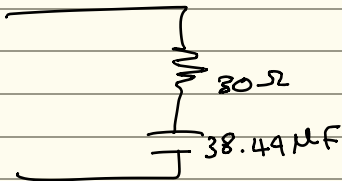
$$X = 60 \angle -60^\circ \Omega$$

$$X = 30 - 51.96j \Omega$$

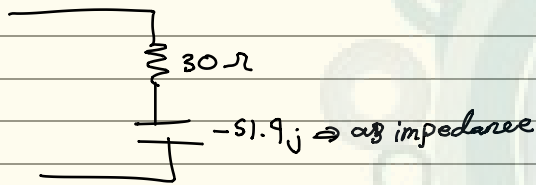
$$X_C = \frac{1}{\omega C}$$

$$51.961 = \frac{1}{500 \times C}$$

$$C = 38.49 \mu F$$



or



* Max power transfer in AC-ckt is:-

Value of load to find max power transfer

$$Z_L = Z_{th}^*$$

$$\text{Ex: } Z_1 = 3 + 2j$$

$$Z_1^* = 3 - 2j \text{ conjugate}$$

only change the sign of the imaginary part

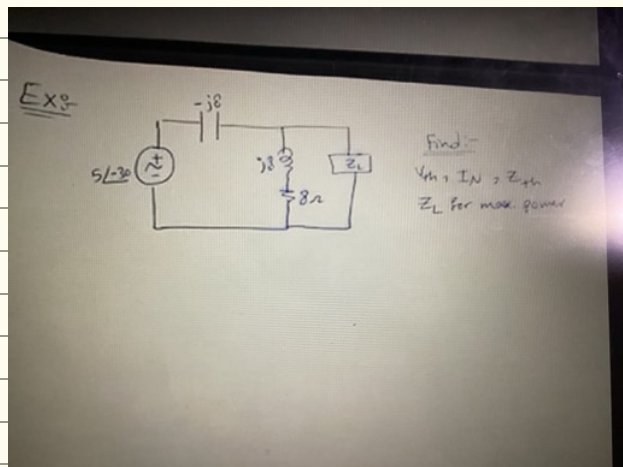
$$\text{Ex: } Z_1 = 12 \angle -60^\circ$$

$$Z_1^* = 12 \angle 60^\circ$$

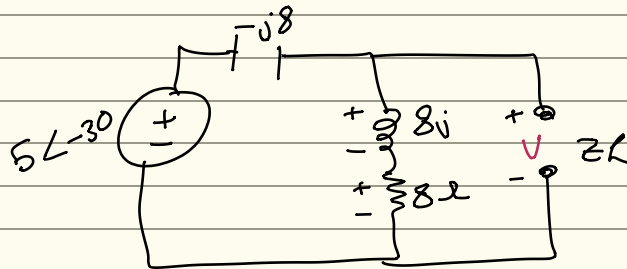
only change the sign

imaginary في إشارة triangular
polar في إشارة الزاوية

Example



find $V_{th} = V_{OC}$ impedances are in series, so $V = V_{th}$



by voltage division

$$V_{8\Omega} = \frac{5 \angle -30^\circ \times 8}{8} = 5 \angle -30^\circ$$

$$V_{8j} = \frac{5 \angle -30^\circ \times 8j}{8} = 5 \angle 60^\circ$$

$$-5 \angle -30^\circ - 5 \angle 60^\circ + V = 0$$

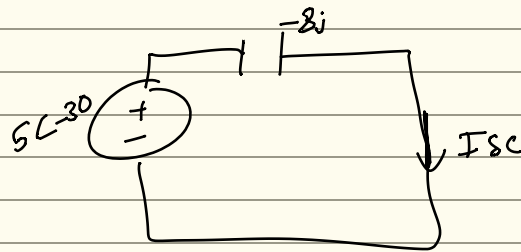
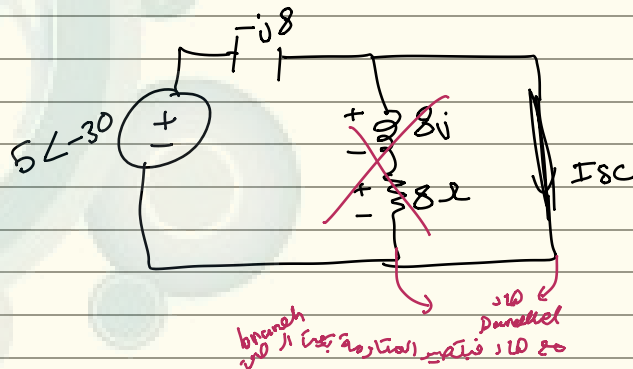
$$\left(\frac{5\sqrt{3}}{2} - \frac{5}{2}j\right) - \left(\frac{5}{2} + \frac{5\sqrt{3}}{2}j\right) + V = 0$$

$$7.07 \angle -75^\circ + V = 0$$

$$V_1 = -(A \sin \omega t)$$

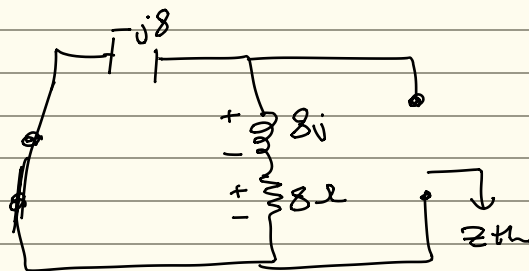
$$V_{th} = 7.07 \angle 15^\circ$$

find I_N



$$I_{SV} = \frac{V}{R} = \frac{5 \angle -30^\circ}{-8j} = 0.625 \angle 60^\circ A$$

find Z_{th}



$$Z_{th} = 8 - 8j \Omega$$

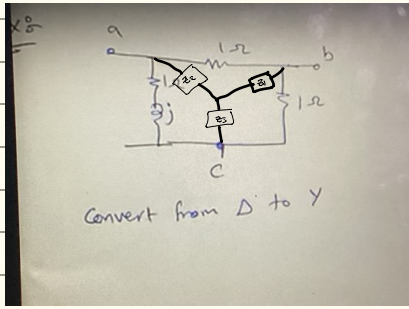
the value for Z_{load} for maximum power transfer

$$Z_L = 8 + 8j \Omega$$

delta & wye in Ac

نفس الطريقة بتعتبر DC

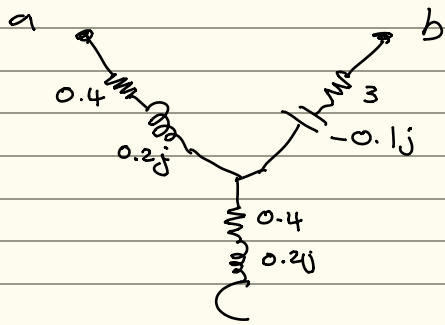
Example



$$Z_1 = \frac{(1)(1)}{1+1+1} = \frac{1}{3} = 0.3 - 0.1j \Omega$$

$$Z_2 = \frac{(1)(1+j)}{3+j} = 0.4 + 0.2j \Omega$$

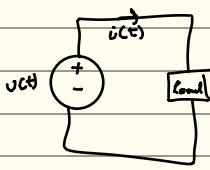
$$Z_3 = \frac{1+j}{3+j} = 0.4 + 0.2j \Omega$$



power in Ac ckt

1 Instantaneous power:- $p(t)$

if $v(t) = V_m \cos(\omega t + \theta_v)$
Peak value Phase



if $i(t) = I_m \cos(\omega t + \theta_i)$
Peak value Phase

$$p(t) = v(t) \cdot i(t)$$

$$= V_m \cos(\omega t + \theta_v) \cdot I_m \cos(\omega t + \theta_i)$$

$$\cos \theta_1 \cdot \cos \theta_2$$

$$\frac{1}{2} \cos(\theta_1 + \theta_2) + \frac{1}{2} \cos(\theta_1 - \theta_2)$$

$$p(t) = \frac{V_m \cdot I_m}{2} \cdot \cos(2\omega t + \theta_v + \theta_i) + \frac{I_m \cdot V_m}{2} \cos(\theta_v - \theta_i)$$

oscillating signal with double frequency (متذبذب بتردد مضاعف)
constant value (average power) (قيمة ثابتة - القدرة المتوسطة)

Graph $p(t)$



Average power

$$P_{average} = \frac{1}{T} \int_0^T p(t) \cdot dt$$

$$= \frac{1}{T} \int_0^T \left[\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \right] dt$$

$$= \frac{V_m I_m}{2T} \int_0^T \cos(2\omega t + \theta_v + \theta_i) dt + \int_0^T \cos(\theta_v - \theta_i) dt$$

الأول = 0 لأن \cos له متوسط 0 على فترة كاملة

$$= \frac{V_m I_m}{2T} \int_0^T \cos(\theta_v - \theta_i) dt$$

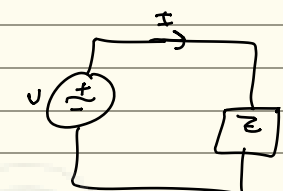
$$= \frac{V_m I_m}{2T} \cdot (\cos(\theta_v - \theta_i) t) \Big|_0^T$$

$$= \frac{V_m I_m}{2T} \cos(\theta_v - \theta_i) T$$

$$= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

القدرة المتوسطة = Source

القدرة المتوسطة = V_m و I_m في الجهد والتيار



$$I = I_m \angle \theta_i$$

$$V = V_m \angle \theta_v$$

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$|Z| = \frac{V_m}{I_m}, \theta_Z = \theta_v - \theta_i$$

$$P_{average} = \frac{1}{2} I_m^2 |Z| \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \frac{(V_m)^2}{|Z|} \cos(\theta_v - \theta_i)$$

Power for the load average

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

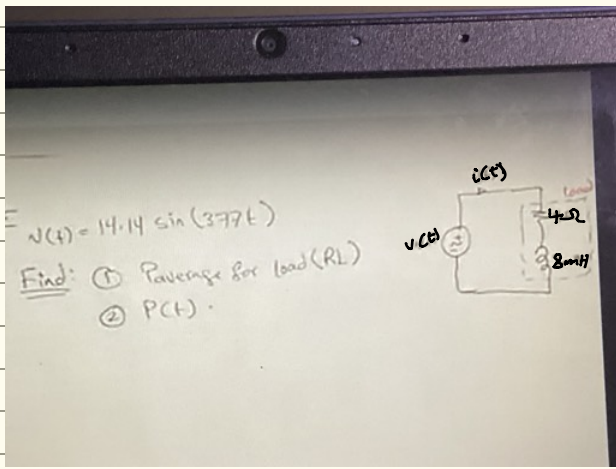
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$P_{average} = V_{rms} \times I_{rms} \cos(\theta_v - \theta_i)$$

$$= I_{rms}^2 |Z| \cos(\theta_v - \theta_i)$$

$$= \frac{V_{rms}^2}{|Z|} \cos(\theta_v - \theta_i)$$

* Example



$$v(t) = 14.14 \cos(377t - 90) \Rightarrow v(t) = 14.14 \angle -90^\circ \text{ V}$$

$$\omega = 377$$

$$\hookrightarrow V_m = 14.14$$

$$8 \text{ mH} \rightarrow Z = j\omega L = j \times 8 \times 10^{-3} \times 377 = 3j \Omega$$

$$\theta_V = -90$$

$$Z_{\text{load}} = R + jX_L$$

$$= 4 + 3j = 5 \angle 36.87^\circ \Omega$$

$$|Z| = 5, \theta_Z = 36.9$$

$$\hookrightarrow \theta_V - \theta_i$$

$$\frac{V}{R} = \frac{I R}{R}$$

$$I = \frac{14.14 \angle -90}{5 \angle 36.9} = 2.83 \angle -126.9$$

$$5 \angle 36.9$$

$$\hookrightarrow I_m = 2.83$$

$$\theta_i = -126.9$$

$$P_{\text{average}} = \frac{1}{2} I_m V_m \cos(\theta_V - \theta_i)$$

$$= \frac{1}{2} \times 2.83 \times 14.14 \times \cos(-90 + 126.9)$$

$$P_{\text{average}} = 16 \text{ watt}$$

$$\text{or } P_{\text{average}} = \frac{1}{2} I_m |Z| \cos(\theta_V - \theta_i) = 16 \text{ watt}$$

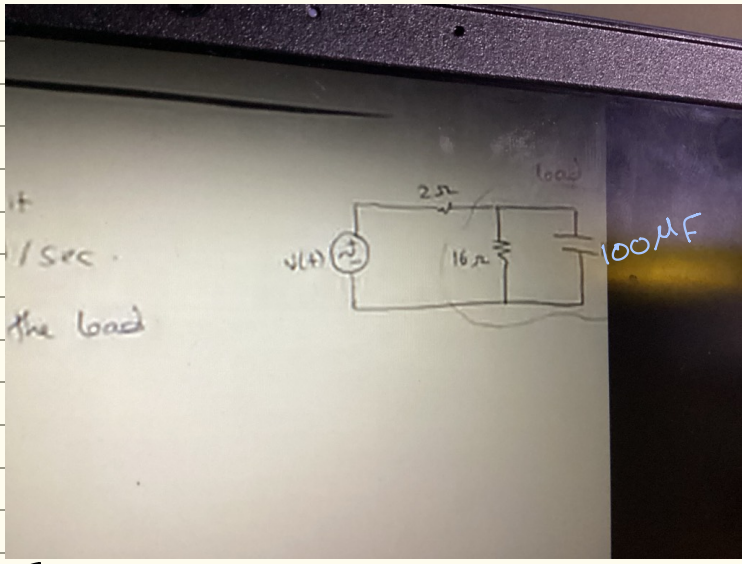
$$\text{or } P_{\text{avg}} = \frac{1}{2} \frac{V_m^2}{|Z|} \cos(\theta_V - \theta_i) = 16 \text{ watt}$$

$$p(t) = v(t) * \underline{i(t)} \rightarrow \text{بر مخرج (cc)}$$

$$(14.14 \cos(377t - 90)) \times 2.83 \cos(377t - 126.9)$$

$$\frac{1}{2} 14.14 \times 2.83 [\cos(377t - 216.9) + \cos(377t + 36.9)]$$

Example:



مثال
 $V = 110 \angle 0^\circ \text{ volt}$
 $\omega = 377 \text{ rad/sec}$

Find power for the load

$$Z = j\omega \frac{1}{C} = \frac{-j}{100 \times 10^{-6} \times 377} = -26.5252j$$

$$Z_{\text{load}} = 11.732 - 7.076j$$

$$= 13.7 \angle -31.0983^\circ$$

$$\downarrow \quad \downarrow$$

$$|Z| = 13.7 \quad \theta_Z = -31.0983^\circ$$

$$\theta_V - \theta_i$$

$$V = IR$$

by voltage division

$$P_{\text{avg}} = \frac{1}{2} \frac{V_m^2}{|Z|} \cos(\theta_V - \theta_i) \Rightarrow V_L = \frac{(110 \angle 0^\circ) \times -26.5252j}{13.7}$$

$$= \frac{1}{2} \frac{(97.65)^2}{13.7} \cos(-31.0983^\circ) \quad V_L = 97.65 \angle -3.85^\circ$$

$$\downarrow \quad \downarrow$$

$$V_m \quad \theta_V$$

$$= 296.8 \text{ watt}$$

* complex power

product of the rms voltage and the complex conjugate of the rms current

⇒ if voltage ⇒ $V_{rms} \angle \theta_v$
current ⇒ $I_{rms} \angle \theta_i$

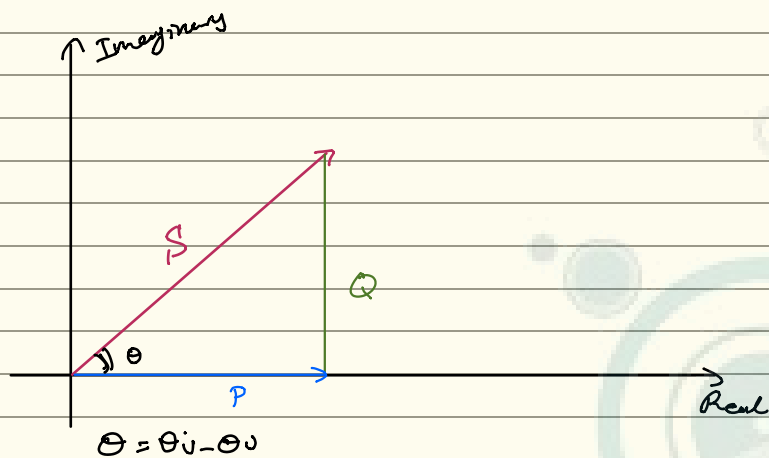
⇒ $S = (V_{rms} \angle \theta_v) \times (I_{rms}^*)$ ↗ conjugate
complex power
 $= (V_{rms} \angle \theta_v) \times I_{rms} \angle -\theta_i$

$S = V_{rms} \times I_{rms} \angle \theta_v - \theta_i$

↗ polar form

$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_P + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_Q$$

Apparent power (V.A) Real power (average power) (W) Dissipated power reactive power (VAR) (goes to V.A.R) Volt-Ampere-Reactive



$$S = I_{rms} V_{rms} \cos(\theta_v - \theta_i) + j I_{rms} V_{rms} \sin(\theta_v - \theta_i)$$

$$S' = V_{rms} I_{rms}^* \Rightarrow P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \omega$$

$$S' = P + jQ \quad Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \text{ var}$$

بنا شرط پابند

$$Z = R + jX$$

$$Z = \frac{V_{rms}}{I_{rms}}$$

$$\Rightarrow V_{rms} = Z I_{rms}$$

$$S' = I_{rms}^* (Z \times I_{rms})$$

$$S' = |I_{rms}|^2 \times Z$$

$$S' = |I_{rms}|^2 \times (R + jX)$$

$$S' = \underbrace{|I_{rms}|^2 R}_P + j \underbrace{|I_{rms}|^2 X}_Q$$

↓ dissipated power measure the energy exchange between the source & the reactive impedances

Summary

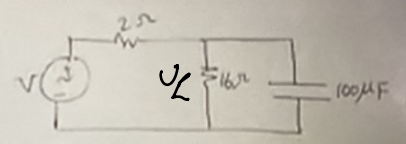
$$S' = V_{rms} I_{rms}^*$$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j I_{rms} V_{rms} \sin(\theta_v - \theta_i)$$

$$= |I_{rms}|^2 R + j |I_{rms}|^2 X$$

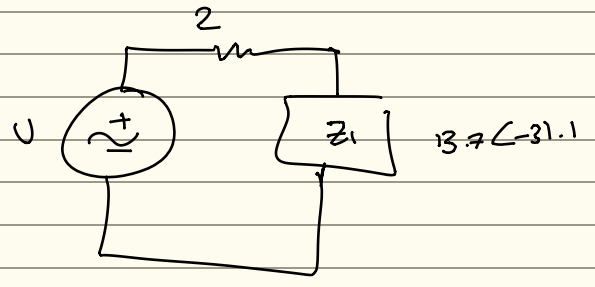
Ex 8

$V = 110 \angle 0^\circ$, $\omega = 377 \text{ rad/s}$
 I_{peak}



Find S , P , Q , PF

$$100 \times 10^{-6} \Rightarrow Z = \frac{-j}{377 \times 10^{-6} \times 100} = -26.53j$$



$$I = \frac{V}{Z_1 + 2} = \frac{7.12 \angle 27.26^\circ}{I_{\text{peak}}}$$

$$V_{\text{rms}} = \frac{110 \angle 0^\circ}{\sqrt{2}} = 77.782$$

$$I_{\text{rms}} = \frac{7.12 \angle 27.26^\circ}{\sqrt{2}} =$$

$$I_{\text{rms}} = 5.035 \angle 27.26^\circ$$

$$S^* = I_{\text{rms}}^* V_{\text{rms}}$$

$$(5.035 \angle -27.26^\circ) \times 77.782$$

$$S^* = 391.63 \angle -27.26^\circ$$

$$S^* = 348.13 - j179.37 \text{ var}$$

So that

$$P = 348.13 \text{ watt}$$

$$Q = -j179.37 \text{ var}$$

→ here $Q < 0$
 so this means
 that this capacitive
 load but not pure

و هنا
 Q < 0
 يعني
 حمل
 سعتي

$$V_L = \frac{V \times Z_1}{2 + Z_1} = 97.5 \angle -3.8^\circ$$

$$Q = \frac{|V_L|^2}{X_C} = \frac{\left(\frac{97.5}{\sqrt{2}}\right)^2}{26.5} = 179.4 \text{ var}$$

$$P_{\text{total}} = P_{2\Omega} + P_{\text{load}}$$

$$P_{2\Omega} = I_{\text{rms}}^2 (2) = \left(\frac{7.116}{\sqrt{2}}\right)^2 \times 2 = 50.64 \text{ W}$$

$$P(16 \Omega) = \frac{|V_{\text{rms}}|^2}{16} = \frac{\left(\frac{97.5}{\sqrt{2}}\right)^2}{16} = 297.1 \text{ watt}$$

$$P_{\text{total}} = 297.1 + 50.64 = 347.7 \text{ watt}$$

$$PF = \cos(\theta_v - \theta_i)$$

$$PF = \cos(0 - 27.35^\circ)$$

$$PF = 0.89 \text{ leading}$$

→ power factor

عكس
 Capacitive

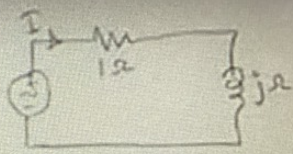
التي
 لها
 سعة

معامل

Ex:

assume rms values $\Rightarrow 10/30$

بفرض قيم الجهد والتيار
rms value \Rightarrow



Find S, P, Q, PF

$$Z = 1 + j = 1.41 \angle 45^\circ$$

$$I = \frac{V}{Z} = \frac{10 \angle 30^\circ}{1.41 \angle 45^\circ} = 7.07 \angle -15^\circ \text{ A rms}$$

$$S = V_{rms} \times I_{rms}^* = 10 \angle 30^\circ \times 7.07 \angle 15^\circ$$

$$S = 50 + 50j \rightarrow \begin{matrix} \downarrow & \downarrow \\ P & Q \end{matrix} \rightarrow \text{as inductive}$$

$$PF = \cos(\theta_V - \theta_I) \quad \begin{matrix} P = 50 \text{ W} \\ Q = 50 \text{ var} \end{matrix} \quad j \text{ is } \omega L$$

$$PF = \cos(30^\circ + 15^\circ)$$

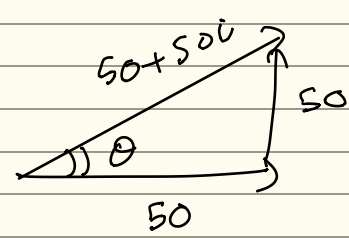
$$PF = 0.707 \rightarrow \text{Lag} \quad \left\{ \begin{matrix} \text{lag} \Rightarrow \text{as } \omega L \\ \text{inductive} \end{matrix} \right.$$

$$P = (I_{rms})^2 R$$

$$P = (7.07)^2 \times 1 = 50 \text{ W}$$

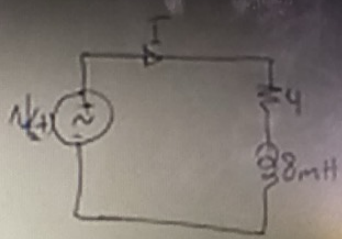
$$Q = (I_{rms})^2 \times (1) = 50 \text{ W}$$

so my answer is correct



inductive load.

$V(t) = 14.14 \sin(377t)$
Find S, P, Q, PF at source.



$$Z = j\omega L = j \times 8 \times 10^{-3} \times 377$$

$$Z = 3.016j$$

$$V = IR$$

$$P = V$$

$$V(t) = 14 \cos(377t - 90)$$

peak $\rightarrow 14 \angle -90$

$$I = \frac{V(t)}{3.016j + 4} = 2.8 \angle -127.02$$

\downarrow
peak

$$P = |I_{rms}|^2 \times R$$

$$= (1.98 \angle -127.02)^2 \times 4$$

$$V_{rms} = \frac{14 \angle -90}{\sqrt{2}} = 9.9 \angle -90$$

$$I_{rms} = \frac{2.8 \angle -127.02}{\sqrt{2}}$$

$$I_{rms} = 1.98 \angle -127.02$$

$$S = I_{rms}^* \times V_{rms}$$

$$S = (9.9 \angle -90) \times 1.98 \angle -127.02$$

$$S = 19.8 \angle -217.02$$

\downarrow
P

$$S = 15.65 + j11.8$$

\downarrow
Q

P = 15.65
Q = 11.8 Var

\rightarrow act as inductor

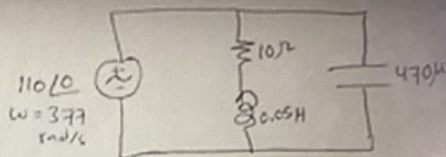
$$PF = 0.79 \rightarrow \text{Lags}$$

Ex 8-

assume RMS values:

Find:

- ① Dissipated power
- ② Total reactive power
- ③ Complex power
- ④ PF



Dissipated power
is R is 0.37

$$470\mu F \rightarrow Z = \frac{-j}{470 \times 10^{-6} \times 377} = -j5.644\Omega$$

$$0.05 H \rightarrow Z = j\omega L = j \times 377 \times 0.05 = 18.85j$$

$$Z = 1.16 - j7.17\Omega$$

$$Z = 7.27 \angle -80^\circ$$

$$I = \frac{V}{Z} = \frac{110 \angle 0^\circ}{1.16 - j7.17j}$$

$$I_{rms} = 0.946 \angle 3.537^\circ$$

$$V = 110 \angle 0^\circ$$

to find dissipated

$$V_R = \frac{110 \angle 0^\circ \times 10}{10 + 18.85j}$$

$$V_R = 51.55 \angle -62.05^\circ \text{ volt}$$

$$P = \frac{(V_R)^2}{R} = \frac{(51.55)^2}{10} = 265.74 \text{ W}$$

② Reactive power
capacitor is -ve
inductor

$$Q_L = \frac{(V_L)^2}{Z}$$

$$V_L = \frac{110 \angle 0^\circ (18.85)}{18.85 + 10}$$

$$V_L = 97.1727 \angle -62.05^\circ$$

$$Q_L = \frac{(V_L)^2}{Z} = \frac{(97.1727)^2}{18.85} = 501 \text{ var}$$

parallel supply

$$Q_C = \frac{(V_C)^2}{Z} = \frac{(110)^2}{5.6} = 2160.7 \text{ var}$$

$$Q_{total} = 501j - 2160j$$

$$= -1659 \text{ VAR}$$

$$\text{Complex power} = 2.6574 - 1659j$$

$$PF = \cos(\theta_V - \theta_I)$$

$$PF = \cos(0 - 80^\circ)$$

$$PF = 0.173648 \rightarrow \text{leading}$$

$$\theta_Z = 0$$

$$\theta_I = \angle 80^\circ$$

angle
leading

power factor

$$PF = \cos(\theta_v - \theta_i)$$

$$0 < PF < 1$$

$$S = \frac{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}{P} + V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$\downarrow$$

$$PF$$

$$PF = \frac{P}{V_{rms} I_{rms}} = \frac{P}{|S|}$$

$$|S| = \sqrt{P^2 + Q^2}$$

* Cases for power factor

① When $PF = 1 \Rightarrow$ pure resistive load

$$\theta_v - \theta_i$$

$$\cos(\theta_v - \theta_i) = \cos(0) = 1$$

$$\sin(\theta_v - \theta_i) = 0$$

$$\downarrow$$

$$Q = 0 \rightarrow \text{كموازلة صافية على مقادير}$$

② When $PF = 0$

\Rightarrow either inductive load

$$Q > 0$$

$$PF \rightarrow \text{lag}$$

$$PF = 0 \Rightarrow \cos(\theta_v - \theta_i) = \cos(90) = 0$$

special case

& only in pure inductive load

pure capacitive load

$$Q < 0$$

$$PF = \cos(\theta_v - \theta_i) = \cos 90 = 0$$

③ inductive load (pure ind) $PF \rightarrow \text{lag}$

④ Capacitive load (pure cap) $PF \rightarrow \text{leading}$

Example:

$$PF = 0.8 \text{ Lag}, |S| = 1000 \text{ VA}$$

inductive load

$$S = |S| \angle +\cos^{-1}(PF)$$

$$\cos(\theta_v - \theta_i) = PF$$

$$\theta_v - \theta_i = \cos^{-1}(PF)$$

Lagging $\Rightarrow \theta_s$ is positive

Example:

$$|S| = 1000 \text{ VA}, PF = 0.8 \text{ lead}$$

$$\theta_v - \theta_i = \cos^{-1}(PF) \text{ capacitive}$$

so $Q < 0$

$$S = |S| \angle -\cos^{-1}(PF)$$

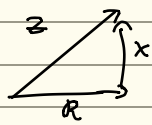
\hookrightarrow because it's lead.

leading $\Rightarrow \theta_s \Rightarrow$ is negative

Important Facts related to complex power:-

impedance triangle

$Z = R + jX$



$R = |Z| \cos \theta$

$X = |Z| \sin \theta$

$\theta = \tan^{-1}(\frac{X}{R})$

$|Z| = \sqrt{R^2 + X^2}$

power triangle

$|Z|^2 = R^2 + X^2$

Power triangle:

$S = P + jQ$

$P = |S| \cos \theta$

$Q = |S| \sin \theta$

$\theta = \tan^{-1}(\frac{Q}{P})$

$|S| = \sqrt{P^2 + Q^2}$

$S = P - jQ$

Resistive load.

Resistive load:

$Z = R$

assume $V_s = V \angle 0^\circ$

$\Rightarrow I = \frac{V_s}{Z} = \frac{V \angle 0^\circ}{R} = \frac{V}{R} \angle 0^\circ$

You can see $\Rightarrow \theta_i = \theta_v$

$\Rightarrow \theta_v - \theta_i = 0^\circ$

the voltage and the current across resistive load are in phase

$PF = \cos(\theta_v - \theta_i)$

$= \cos(0)$

$\Rightarrow PF = 1$

$Q = |S| \sin(\theta_v - \theta_i)$

$= |S| \sin(0)$

$\Rightarrow Q = 0$

Real Power / no phase shift and zero reactive power

capacitive load

Capacitive load:

$Z = R - jX = |Z| \angle -\tan^{-1}(\frac{X}{R})$

assume $V_s = V \angle 0^\circ$

$\Rightarrow I = \frac{V_s}{Z} = \frac{V \angle 0^\circ}{|Z| \angle -\tan^{-1}(\frac{X}{R})} = \frac{V}{|Z|} \angle \tan^{-1}(\frac{X}{R})$

$\Rightarrow I = \frac{V}{|Z|} \angle \theta_v + \tan^{-1}(\frac{X}{R}) \rightarrow$ positive value

the current lead the voltage

$PF = \cos(\theta_v - \theta_i)$

$= \cos(\theta_v - (\theta_v + \tan^{-1}(\frac{X}{R})))$

$= \cos(-\tan^{-1}(\frac{X}{R}))$

$PF < 1 \rightarrow$ leading

$S = V_{rms} I_{rms}^*$

$= (V \angle 0^\circ) (\frac{V}{|Z|} \angle (\theta_v + \tan^{-1}(\frac{X}{R})))$

$= \frac{V^2}{|Z|} \angle (-\tan^{-1}(\frac{X}{R})) \rightarrow$ the angle is negative

$= \frac{V^2}{|Z|} \cos(-\tan^{-1}(\frac{X}{R})) + j \frac{V^2}{|Z|} \sin(-\tan^{-1}(\frac{X}{R}))$

\Downarrow

$Q < 0 \Rightarrow$ negative

inductive load:-

wer_3

Inductive load:

$Z = R + jX = |Z| \angle \tan^{-1}(\frac{X}{R})$

assume $V_s = V \angle 0^\circ$

$\Rightarrow I = \frac{V_s}{Z} = \frac{V \angle 0^\circ}{|Z| \angle \tan^{-1}(\frac{X}{R})} = \frac{V}{|Z|} \angle -\tan^{-1}(\frac{X}{R})$

$\Rightarrow I = \frac{V}{|Z|} \angle \theta_v - \tan^{-1}(\frac{X}{R}) \rightarrow$ the angle negative value

the current lag the voltage

$PF = \cos(\theta_v - \theta_i)$

$= \cos(\theta_v - (\theta_v - \tan^{-1}(\frac{X}{R})))$

$= \cos(\tan^{-1}(\frac{X}{R}))$

$PF < 1 \Rightarrow$ lag

$S = V_{rms} I_{rms}^*$

$= (V \angle 0^\circ) (\frac{V}{|Z|} \angle -\theta_v + \tan^{-1}(\frac{X}{R}))$

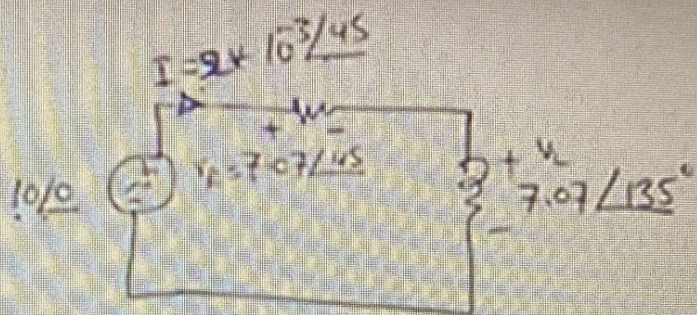
$= \frac{V^2}{|Z|} \angle \tan^{-1}(\frac{X}{R}) \rightarrow$ angle is positive

$= \frac{V^2}{|Z|} \cos(\tan^{-1}(\frac{X}{R})) + j \frac{V^2}{|Z|} \sin(\tan^{-1}(\frac{X}{R}))$

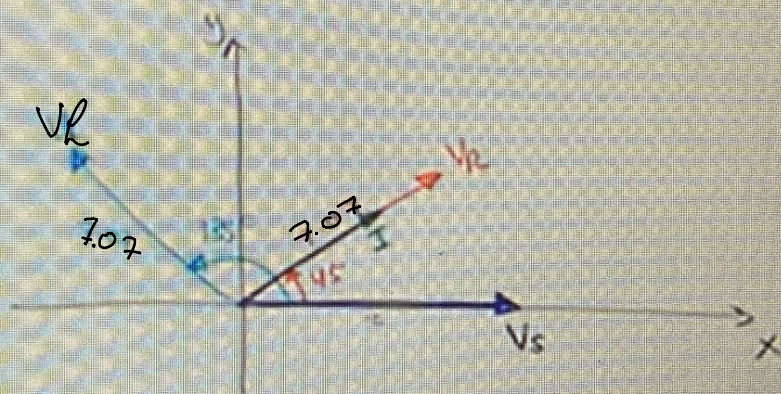
\Downarrow

$Q > 0 \Rightarrow$ inductive positive

Ex:



Draw phasor Diagram.



۱۔ اولیٰ پھیلاؤ کے لیے V_L و I پھیلاؤ ۱ phase

۲۔ دوسری پھیلاؤ کے لیے V_L

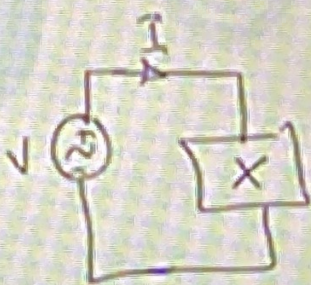
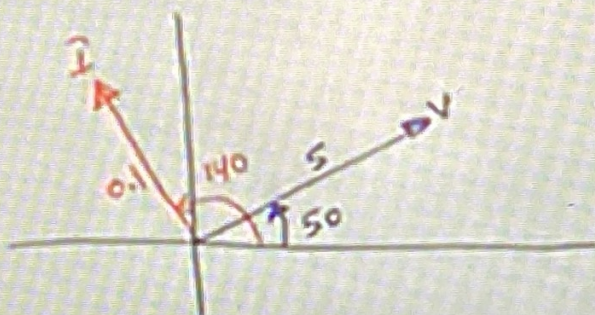
current lags the voltage with 90°

inductor

Supply $\Rightarrow V_s$ سے بائیں

current lead V_s with 45°

Ex:



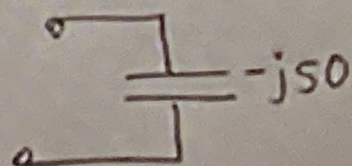
$$\left. \begin{aligned} V &= 5 \angle 50 \\ I &= 0.1 \angle 140 \end{aligned} \right\}$$

$$X = \frac{V}{I} = \frac{5 \angle 50}{0.1 \angle 140}$$

$$X = 50 \angle -90$$

$$X = -j50$$

It's pure capacitive.



Power factor correction:-

$$PF = \cos(\theta_v - \theta_i)$$

$$PF = \frac{P}{|S|}$$

$$S = P + jQ = |S| \angle \theta$$

Ex: $S = 600 + j800 = 1000 \angle 53.13^\circ$

لهذا يعني أننا نحتاج
إلى زيادة القدرة
نحتاجها

PF is poor ($\cos 53.13 = 0.6$)

if $PF = 1 \Rightarrow P = |S|$
 Ideal case
 لا نحتاج زيادة القدرة
مما نحتاجه الطاقة التي
نحتاجها

يعني أننا نعدل مقدار Q بحيث نصل إلى قيمة 0

Power factor correction: a method to reduce the lagging power reactive in the inductive load by fixing a high value capacitor

because it has negative power reactive which cancel the positive reactive in the inductive load

$$\Rightarrow Q_{total} \approx 0$$

$$PF \approx 1$$

⇒ Capacitor added in parallel with the load

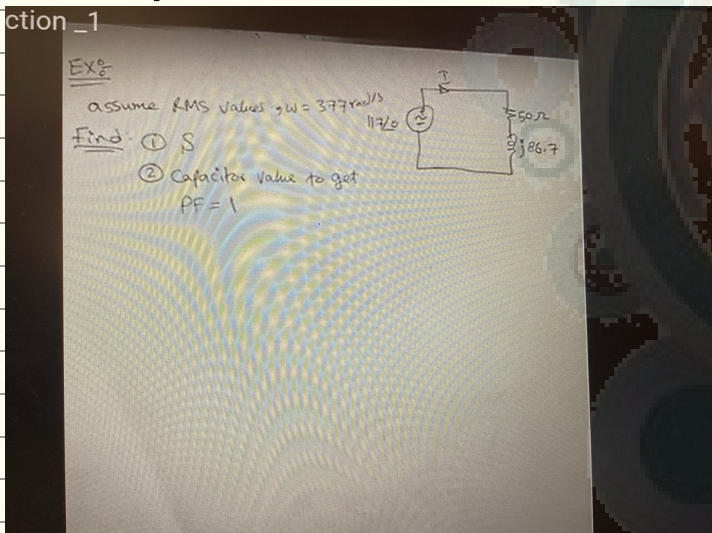
Capacitor added in parallel with the load

$$C = \frac{P}{\omega} (\tan(\theta_1) - \tan(\theta_2))$$

$$\theta = \frac{S}{P}$$

$$\frac{W}{V_{rms}^2}$$

Example



$$I = \frac{V}{Z} = \frac{112 \angle 0}{50 + j86.7} = 1.169 \angle -60.0279$$

$$① S = I_{rms}^* \times V_{rms}$$

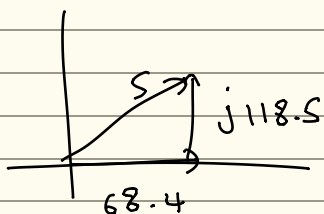
$$1.169 \angle -60.0279 \times V_{rms}$$

$$= 68.3288 + j118.5 = 136.7 \angle 60$$

$$② PF = \cos(\theta_v - \theta_i) = \cos(0 + 60.0279)$$

PF is 0.5 lagging

$$(\theta_v - \theta_i)_{old} = 60.03$$



$$\Rightarrow \text{need } PF = 1 \Rightarrow (\theta_v - \theta_i)_n$$

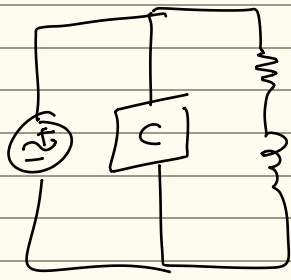
need to add capacitor
with $Q = -j118.5$
such that $Q_{total} = 0$

$$118.5 = (I_{rms})^2 X_C$$

$$X_C = \frac{118.5}{(1.17)^2} = 11522$$

$$11522 = \frac{1}{\omega C}$$

$$C = 23.1 \mu F$$



$$C = \frac{P_r (\tan(\theta_{before}) - \tan(\theta_{after}))}{\omega (V_{rms})^2}$$

± 1000000
5/4

$$C = \frac{P_r (\tan(60.03) - \tan(0))}{377 \times (117 \angle 0)}$$

$$C = \frac{(68.4) (\tan(60.03) - \tan(0))}{(377) (117)^2}$$

$$C = 23.1 \mu F$$

