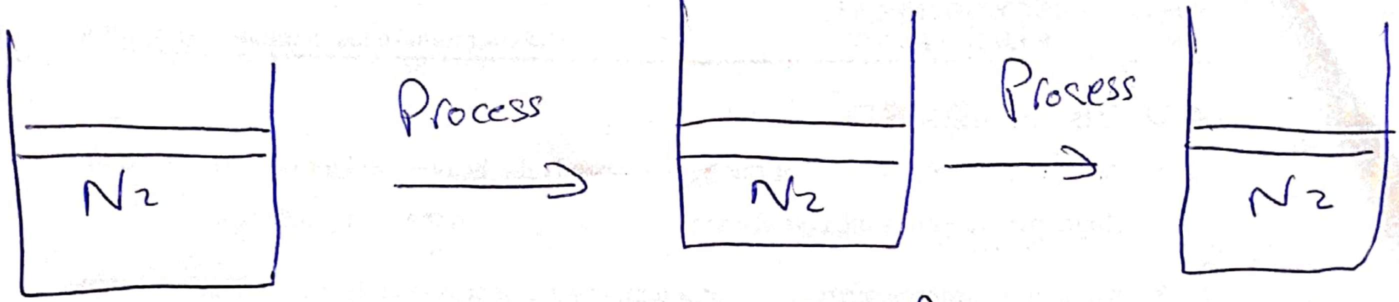
Q1) **(12 points)** Nitrogen gas is contained in a piston cylinder device at P_1 = 160 kPa, T_1 = 20°C, and initial volume, V_1 = 0.4 m^3. Nitrogen gas undergoes three successive processes as follows:

1) 1-2: a polytropic process such that P_2 = 420 kPa. (polytropic exponent, n = 1.4)
2) 2-3: an isometric process,
3) 3-1 : an isothermal process.

Assuming perfect and ideal gas behavior of Nitrogen, find:
(a) the total work done and total heat transfer.
(b) show the processes on a *P-v* or *T-v* diagram.

state 1:- state 2:- state 3



$P_1 = 160\,kPa$

$T_1 = 20^\circ C$

$\forall_1 = 0.4\,m^3$

$P_2 = 420\,kPa$

$P_1 \forall_1^n = P_2 \forall_2^n$

$\therefore n = 1.4$

$\forall_3 = \forall_2$

---

state 1

$\forall_2\ \forall_1$

$P = P_1$

$T = T_3 = T_1$

$$W_{Total} = W_{1\to2} + W_{2\to3} + W_{3\to1}$$

$$W_{1\to2} = \frac{P_2 \forall_2 - P_1 \forall_1}{1-n} \quad \text{(Polytropic process)}$$

$$P_1 \forall_1^n = P_2 \forall_2^n$$

$$160(0.4)^{1.4} = 420(\forall_2)^{1.4}$$

$$\forall_2 = 0.2008\,m^3$$

$$W_{1\to2} = \frac{420(0.2008) - 160(0.4)}{1-1.4} = -50.84\,kJ$$

$W_{2\to3} \equiv 0$ (isometric process).

$$W_{3\to1} = P\forall \ln \frac{\forall_1}{\forall_3}$$

$$P\forall = P_3\forall_3 = P_1\forall_1$$

$P_1 = 160 \text{ kPa}$

$\forall_1 = 0.4 \text{ m}^3$

$$W_{3\to1} = 160(0.4) \ln \frac{0.4}{0.2008}$$

$$= 44.106 \text{ kJ}$$

$$W_{Total} = -50.84 \text{ kJ} + 0 + 44.106 \text{ kJ}$$

$$= -6.734 \text{ kJ}$$

It's a net input work → $\boxed{W_{in} = 6.734 \text{ kJ}}$

OR $W_{net} = -6.734 \text{ kJ}$

☺

$$Q_{net} - W_{net} = \Delta U.$$

W_{net} ⊕ for W_{out}

W_{net} ⊖ for W_{in}

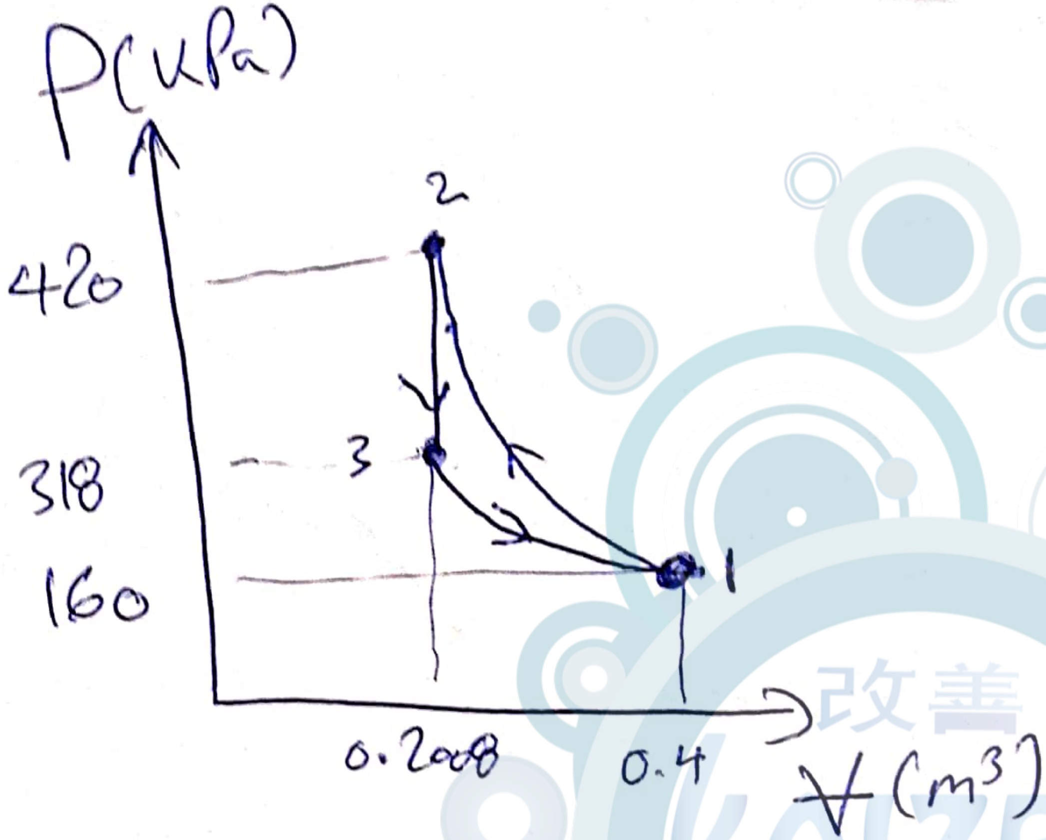
Q_{net} ⊕ for Q_{in}

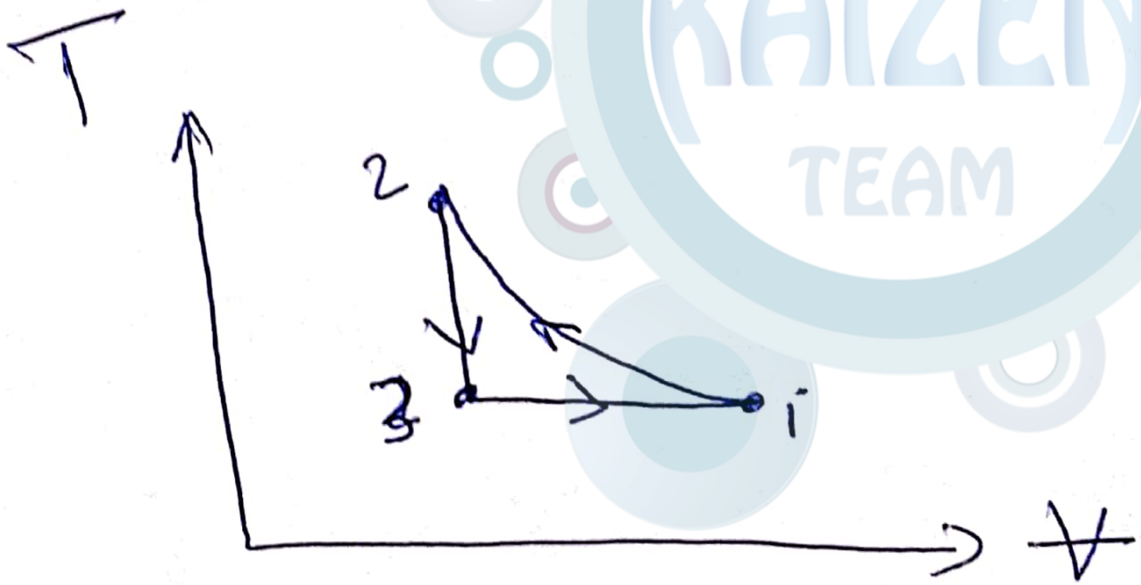
Q_{net} ⊖ for Q_{out}

$\Delta U = 0$ (cycle, initial state = final state)

$$Q_{net} - (-6.734) = 0$$

$$Q_{net} = -6.734 \text{ kJ}.$$


P(kPa)
420
318
160
2
3
1
0.2008
0.4
V (m³)


T
2
3
1
V

Q2) (6 points) A insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 4 kg of Argon at 600 kPa and 127° C, and the other part is evacuated. The partition is now removed, and Argon expands into the entire tank.

(a) The final pressure and temperature.

(b) the total work done and total heat transfer.

$P_1 = 600 kPa$

$T_1 = 127^oC$

$Q_{in} = Q_{out} = 0$

$V_2 = 2V_1$

$P_1 V_1 = mRT_1$

$600 V_1 = 4(0.2081)(127+273)$

$V_1 = 0.555 \ m^3$

$V_2 = 1.11 \ m^3$

$\overset{0}{\cancel{Q}} - W = \Delta U$

Work = 0 (Ar didn't do any work).

$\Delta U = 0 \Rightarrow U_1 = U_2 \Rightarrow T_1 = T_2$

$\forall_2 = 1.11\ m^3, \quad T_2 = 127^\circ C$

$P_2 \forall_2 = m R T_2$

$P_2 = 300\ kPa$

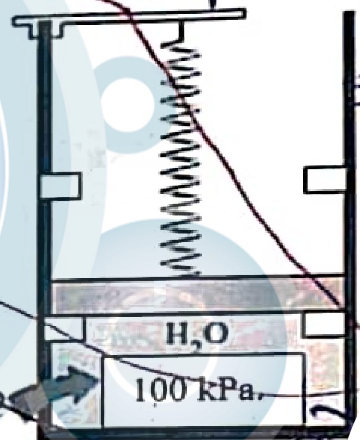
Q3) (12 points) 1 kg of water at 100 kPa is contained in a piston cylinder arrangement equipped with two sets of stops. The piston is frictionless, touches a linear spring and is resting on a set of stops as shown, where the initial volume is 0.27595 m^3. The mass of the piston is such that a pressure of 300 kPa is required to move it. When the piston hits the upper stops, the enclosed volume is 0.60582 m^3. Initially, the spring exerts no force on the piston. Heat is added until the piston starts moving upward such that the pressure is proportional to volume. If the pressure inside reached 450 kPa, find:

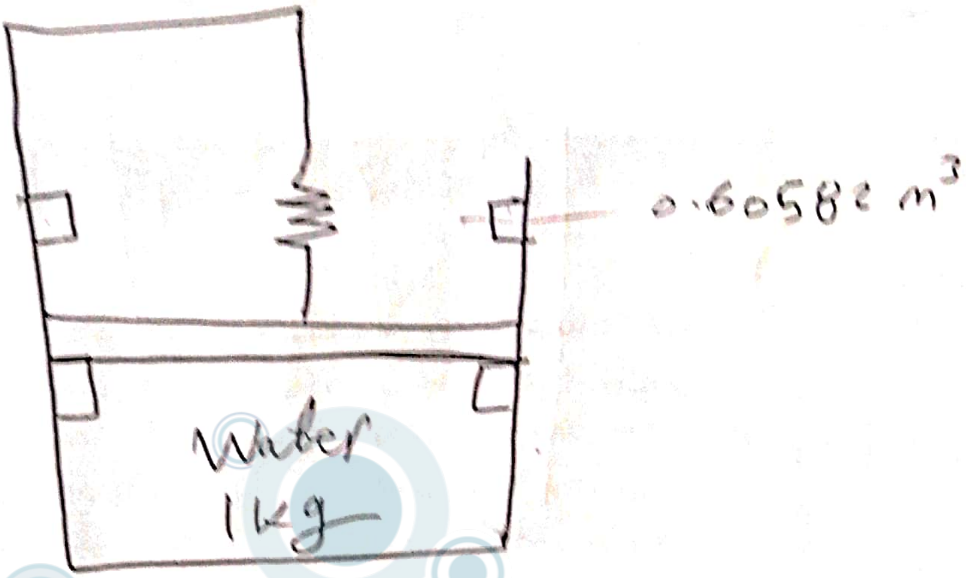
(a) The volume and temperature of water at all states.

(b) The total work done.

(c) The total heat transfer.

Show the process (es) on a *P-v* or *T-v* diagram.



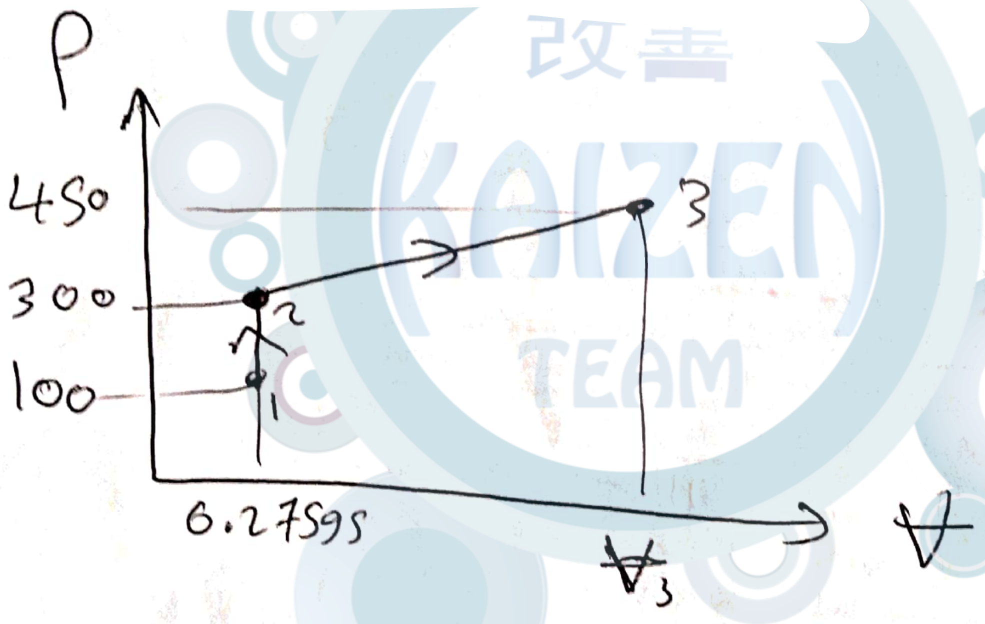


$P_1 = 100\ kPa$ $P_2 = 450\ kPa$

$V_1 = 0.27595\ m^3$

Did the piston reach the upper stops??

If yes, $V_{final} \equiv 0.60582$



$$\frac{450}{300} = \frac{\forall_3}{0.27595} \Rightarrow \forall_3 = 0.414\ m^3$$

Work ≡ Area under P–V diagram

$$W = +51.77 \text{ kJ}$$

$$Q - W = m\Delta U_{1\to 3}$$

$$\Delta U = U_3 - U_1$$

~~at~~ state 1 is mixture

$$u_1 = u_f + x u_{fg}$$

$$v_1 = v_f + x v_{fg} \implies x = 0.1624$$

$$u_1 = 756.52 \text{ kJ/kg}$$

state 3 is ~~mixture~~ sat. vapor

~~$v_3 = v_f + x v_{fg} \implies x_3$~~

$$u_3 = u_g @ 450\text{kPa} = 2557.1 \text{ kJ/kg}$$

$$Q - 51.77 = 1(2557.1 - 756.52)$$

$$\implies Q = +1852.35 \text{ kJ}$$

Temperatures :-

For state 1 :-

$P_1 = 100 kPa$, $v_1 = 0.27595 \, m^3/kg$

From table A-5 :-

$v_f < v_1 < v_g \Rightarrow$ mix ☺

$\therefore T_1 = T_{sat} @ P_1 \equiv 99.61 ^\circ C$

state 2 :-

$P_2 = 300 kPa$, $v_2 = 0.27595 \, m^3/kg$

mix.

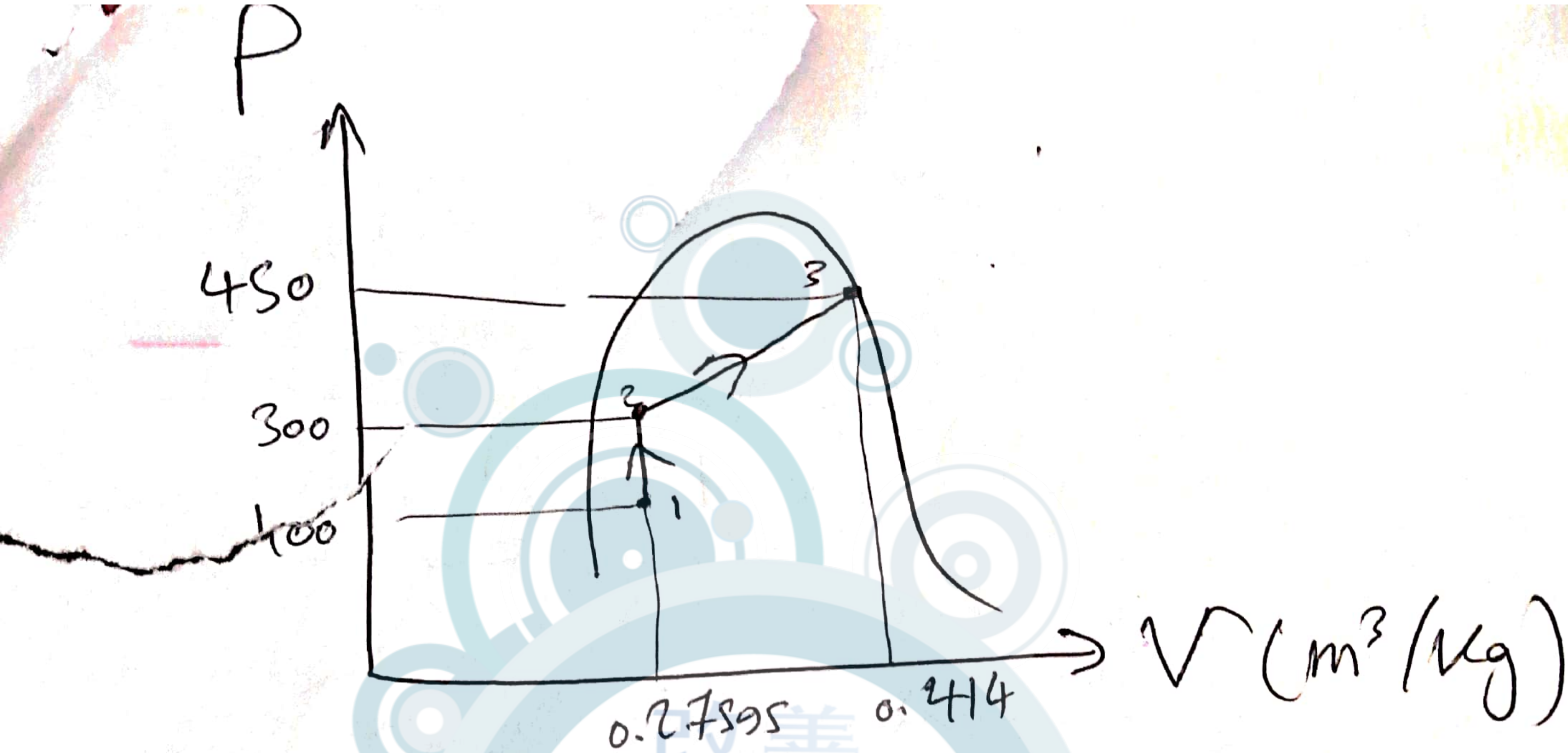
$T_2 = T_{sat} @ P_2 = 133.52 ^\circ C$

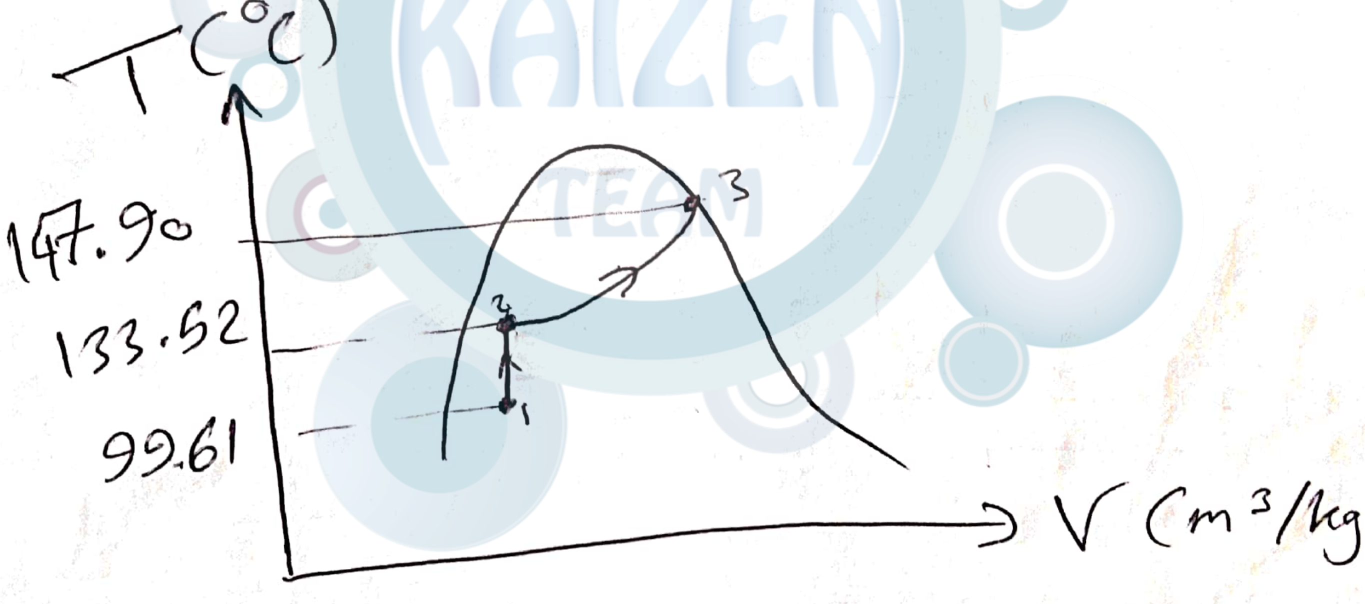
state 3 :-

$P_3 = 450 kPa$, $v_3 = 0.414 \, m^3/kg$

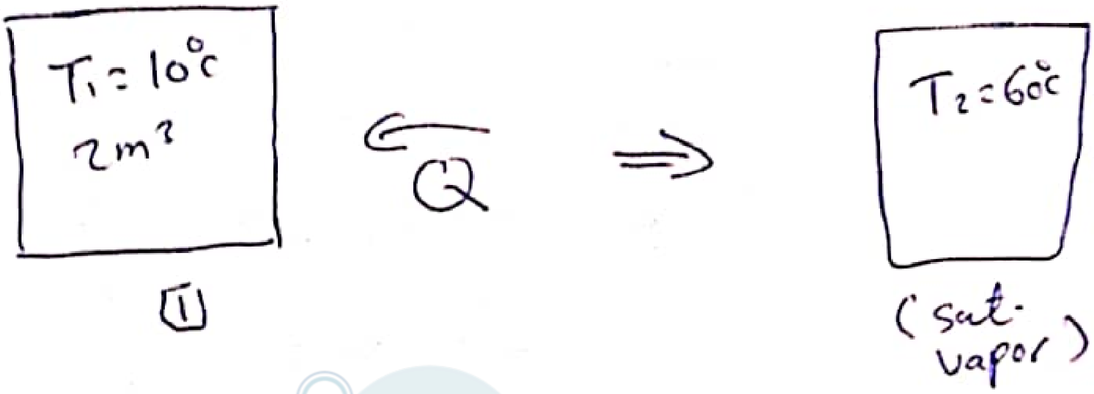
sat. vapor ☺

$T_3 = T_{sat} @ P_3 = 147.90 ^\circ C$


P
450
300
100
1
2
3
0.27595
0.414
v (m³/kg)


T (°C)
147.90
133.52
99.61
1
2
3
v (m³/kg

Q.1 (10 points) A 2- m^3 rigid container contains a wet mixture of *R-134a at 10 °C.* It is heated to a temperature of *60 °C*, at which the phase is saturated vapor. *Find the* work done, heat transfer, and sketch the process on a *P-V* or *T-v* diagram.



$W = ??$, $Q = ??$

$W = 0$ (no boundary moves & thus no any types of work done on the system).

first law:-

$$Q - \overset{0}{\cancel{W}} = \Delta U$$

$$Q = m(u_2 - u_1)$$

$$u_2 = u_g @ T_2 = 60°C = 2529.23 \text{ kJ/kg}$$

(table A-11)

* for state 1, we just have the temperature ($T_1 = 10°C$), so we need a second property

* It's a rigid tank, so

$$v_1 = v_2$$

$$v_2 = v_g @ 60°C = 0.011434 \text{ m}^3/\text{kg}$$

$$= v_1$$

(a) 10°C & 0.011434 m^3/kg

, the substance is in the mixture state !!!

$(v_f < v_1 < v_g)$

$v_1 = v_f + x v_{fg}$ at 10°C

$0.011434 = 0.00079229 + x(0.049466 - 0.00079229)$

$x = 0.219$

$m = \frac{V}{v} = \frac{2}{0.011434} = 174.92\ kg$

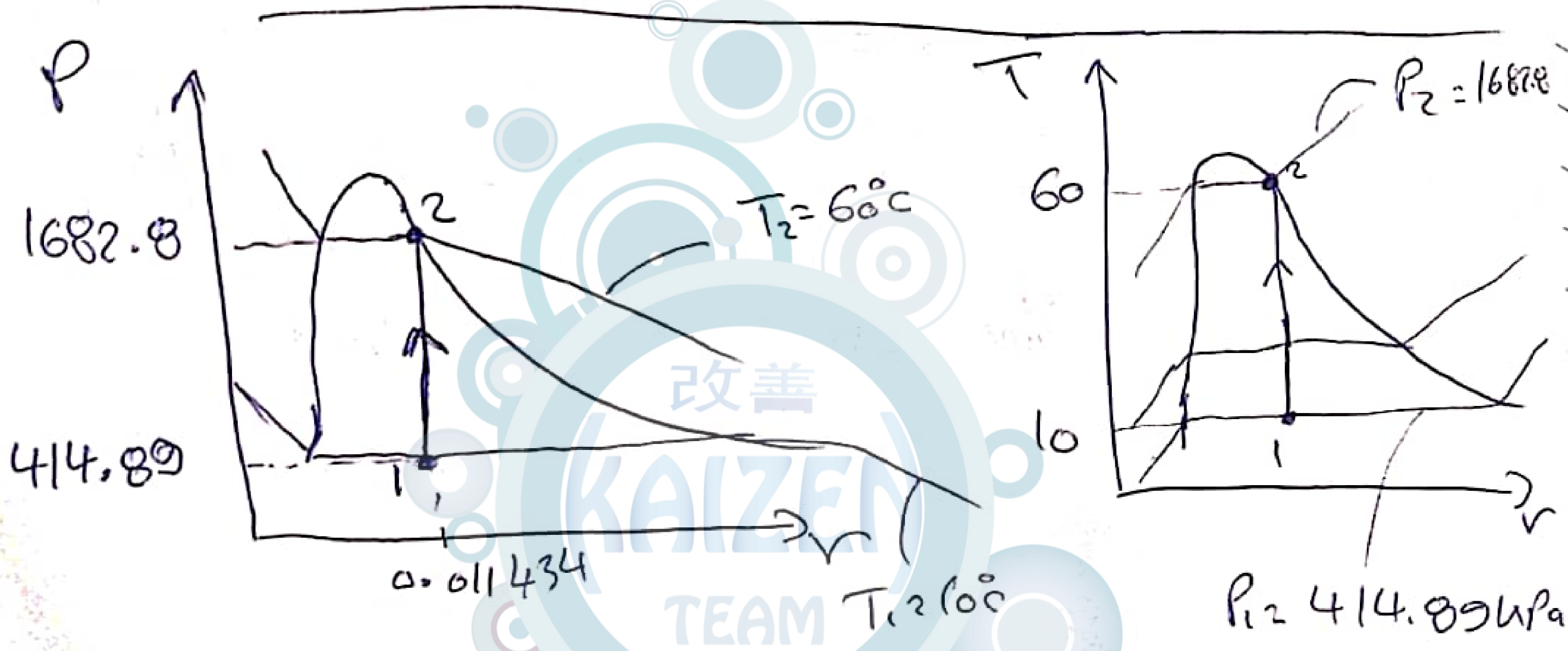
$u_1 = u_f + x u_{fg}$

$= 65.09 + 0.219(170.61)$

$= 102.454\ kJ/kg.$

$Q = 174.92(259.23 - 102.454)$

$= 28207.138\ kJ.$

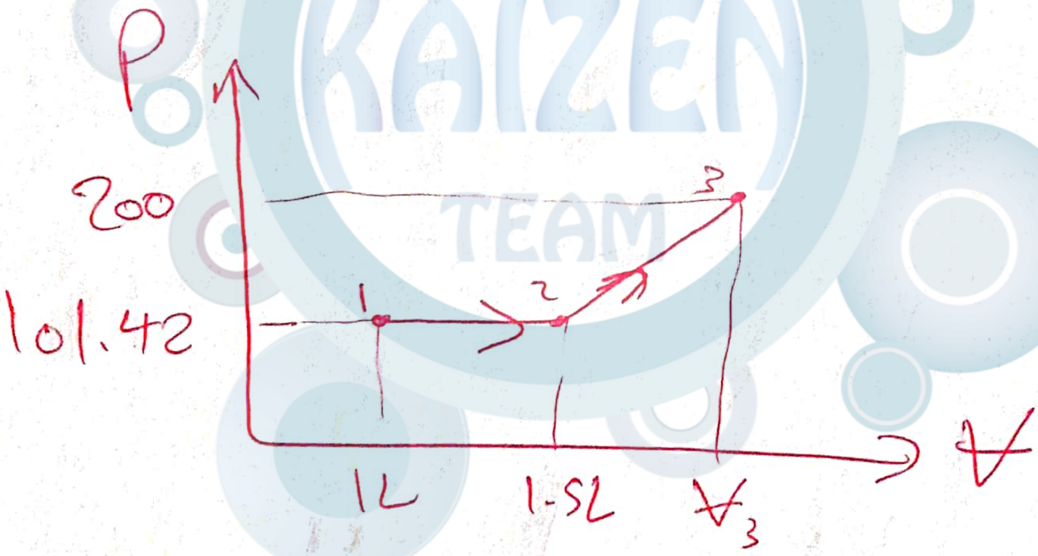


P
1682.8
414.89
2
1
$T_2 = 60°C$
0.011434
$T_1 = 10°C$
v
T
60
10
$P_2 = 1682.8$
$P_1 = 414.89 kPa$

Q.2 (10 points) A piston-cylinder arrangement contains water at *100°C* and *85%* quality with a total volume of 1 lit. Heat is added until the piston touches a linear spring at the instant the volume is 1.5 lit. Heating continues until the pressure reached 200 kPa. If the piston diameter is 150 mm and the spring constant is 100 kN/mm, calculate the total work done and total heat transfer. Sketch the *P-V* or *T-v*

$X = 85\%$

$T_1 = 100°C$



$$\frac{P_3}{P_2} = \frac{\forall_3}{\forall_2} \Longrightarrow \forall_3 = 2.958L$$

Work ≡ Area P-V

$W = +72.06\ kJ$

$Q - W = \Delta U$

$\Delta U = m(u_3 - u_1)$

$u_1 = u_f + x u_{fg}$

$u_1 = 2193.01\ kJ/kg$

$P_3 = 200\ kPa$

$v_3 = \frac{V_3}{m}$

$m = \frac{V_1}{v_1} = \frac{1 \times 10^{-3}}{v_f + x v_{fg}} = \frac{10^{-3}}{0.753}$

$= 1.33 \times 10^{-3}\ kg$

$v_3 = \frac{2.958 \times 10^{-3}}{1.33 \times 10^{-3}} = 2.2241 \times 10^{-3}\ m^3/kg$

Mixture