7.2-6

Max $Z = 2x_1 - x_2 + x_3$

$3x_1 - 2x_2 + 2x_3 \le 15 \quad x_4$

$-x_1 + x_2 + x_3 \le 3 \quad x_5$

$x_1 - x_2 + x_3 \le 4 \quad x_6$

$x_{1,2,3} \ge 0$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	R
	1	0	0	2	1	1	0	18
x_2	0	0	1	5	1	3	0	24
x_6	0	0	0	2	0	1	1	7
x_1	0	1	0	4	1	2	0	21

y^* (over x_4, x_5, x_6)

s^* (under x_4, x_5, x_6)

(a) $b_{new} = \begin{bmatrix} 10 \\ 4 \\ 2 \end{bmatrix}$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	R
	1	0	0	2	1	1	0	14
	0	0	1	5	1	3	0	22
	0	0	0	2	0	1	1	6
	0	1	0	4	1	2	0	18

basic ✓

$x_1 = 18$

$x_2 = 22$ $\quad Z = 14$

feasibility ✓ $\quad x_3 = 0$

optimality ✓

(b) Coefficient of x_3 in objective

$C_{new} = [2 \ -1 \ 2]$

$[1 \ 1 \ 0] \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - [2]$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	R
	1	0	0	1	1	1	0	18
	0	0	1	5	1	3	0	24
	0	0	0	2	0	1	1	7
	0	1	0	4	1	2	0	21

basic ✓

$x_1 = 21$

feasibility ✓ $\quad x_2 = 24$

$x_3 = 0$

optimality ✓ $\quad Z = 18$

(c) Coefficient of x_1 in objective

$[1\ 1\ 0]\begin{bmatrix}-3\\-1\\1\end{bmatrix}-[3]$

$C_{new} = [\,3\ \ -1\ \ 1\,]$

z	x_1	x_2	x_3	x_4	x_5	x_6	R
1	−1	0	2	1	1	0	18
0	0	1	5	1	3	0	24
0	0	0	2	0	1	1	7
0	1	0	4	1	2	0	21

basic dv ✗

$R_0 + R_3$ restore gf

1	0	0	6	2	3	0	39
0	0	1	5	1	3	0	24
0	0	0	2	0	1	1	7
0	1	0	4	1	2	0	21

$x_1 = 21$

$x_2 = 24$ $\quad z = 39$

$x_3 = 0$

(d) Coefficients of x_3

$C_{new} = [\,2\ \ -1\ \ 4\,]$

$A_{new} = \begin{bmatrix}3 & -2 & 3\\ -1 & 1 & 2\\ 1 & -1 & 1\end{bmatrix}$

z	x_1	x_2	x_3	x_4	x_5	x_6	R
1	0	0	1	1	1	0	18
0	0	1	9	1	3	0	24
0	0	0	3	0	1	1	7
0	1	0	7	1	2	0	21

basic ✓

feasibility ✓

optimality ✓

$\begin{bmatrix}1 & 3 & 0\\ 0 & 1 & 1\\ 1 & 2 & 0\end{bmatrix}\begin{bmatrix}3\\2\\1\end{bmatrix}$ $\quad [1\ \ 1\ \ 0]\begin{bmatrix}3\\2\\1\end{bmatrix}-[4]$

$x_1 = 21$

$x_2 = 24$

$z = 18$

(e) Coefficients of x_1 / x_2

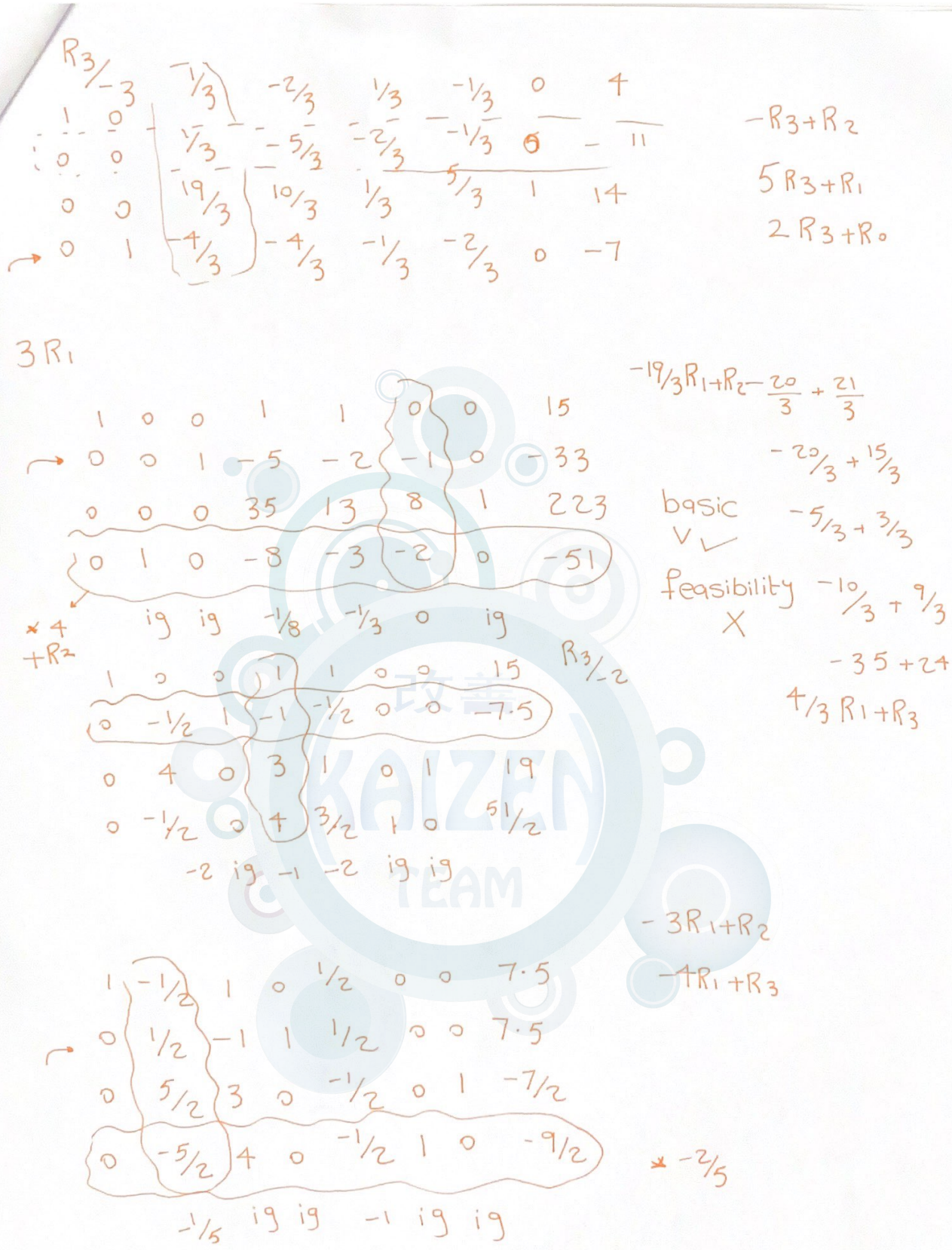
$C_{new} = [\,1\ \ -2\ \ 1\,]$

$A_{new} = \begin{bmatrix}1 & -2 & 2\\ -2 & 3 & 1\\ 3 & 2 & 1\end{bmatrix}$

z	x_1	x_2	x_3	x_4	x_5	x_6	R
1	−2	3	2	1	1	0	18
0	−5	7	5	1	3	0	24
0	1	5	2	0	1	1	7
0	−3	4	4	1	2	0	21

$\begin{bmatrix}1 & 3 & 0\\ 0 & 1 & 1\\ 1 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & -2\\ -2 & 3\\ 3 & 2\end{bmatrix}$ $\quad [1\ \ 1\ \ 0]\begin{bmatrix}1 & -2\\ -2 & 3\\ 3 & 2\end{bmatrix}-[1\ \ -2]$

restore gf


R3/-3
-R3+R2
5R3+R1
2R3+R0
3R1
-19/3R1+R2
basic
feasibility
X
-3R1+R2
-1R1+R3
-2/5
-1/5

0	0	1/5	0	3/5	-1/5	0	42/5
0	0	-1/5	1	2/5	1/5	0	33/5
0	0	7	0	-1	1	1	-8
0	1	-8/5	0	1/5	-2/5	0	9/5
		ig	ig	ig	-3/5	ig	ig

$-1/2R_3+R_2$

$1/2R_3+R_0$

0	0	22/5	0	0	2/5	3/5	18/5
0	0	13/5	1	0	3/5	2/5	17/5
0	0	-7	0	1	-1	-1	8
0	1	-1/5	0	0	-1/5	1/5	1/5

$-1/5R_2+R_3$

$-2/5R_2+R_1$

$-3/5R_2+R_0$

$x_1 = 1/5 \quad x_2 = 0 \quad x_3 = 17/5 \quad R = 18/5 = 7$

(f) $C_{new} = [5 \;\; 1 \;\; 3]$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	R
	1	-3	-2	0	1	1	0	18
②	0	0	1	5	1	3	0	24
	0	0	0	2	0	1	1	7
③	0	1	0	4	1	2	0	21

restore gf

$2R_1+3R_3+R_0$

$$[1 \;\; 1 \;\; 0]\begin{bmatrix} 3 & -2 & 2 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} - [5 \;\; 1 \;\; 3]$$

1	0	0	22	6	13	0	129
0	0	1	5	1	3	0	24
0	0	0	2	0	1	1	7
0	1	0	4	1	2	0	21

$x_1 = 21$

$x_2 = 24$

$x_3 = 0$

$Z = 129$

3)

$$A_{new} = \begin{bmatrix} 2 & -1 & 4 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$b_{new} = \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$$

Z	x_1	x_2	x_3	x_4	x_5	x_6	R
1	-1	1	4	1	1	0	15
0	-1	2	7	1	3	0	21
0	0	0	2	0	1	1	7
0	0	1	6	1	2	0	18

→ restore of how?

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 4 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$$

$-R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 4 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}$$

1	0	0	3	1	0	0	12
0	0	1	6	1	2	0	18
0	0	0	2	0	1	1	7
0	1	-1	-1	0	-1	0	-3

→ $R_1 + R_3$

	x_1	x_2	x_3	x_4	x_5	x_6	
	1	0	0	3	1	0	12
	0	0	1	6	1	2	0	18
	0	0	0	2	0	1	1	7
	0	1	0	5	1	1	0	15

Multi optimal ?

$x_4 = 0$
$x_5 = 0$
$x_6 = 7$

$x_1 = 15 \quad x_2 = 18 \quad x_3 = 0 \quad Z = 12$

g) Continue...

1	0	0	3	1	0	0	12
0	0	1	2	1	0	-2	4
0	0	0	2	0	1	1	7
0	1	0	3	1	0	-1	8

$x_1 = 8$ $x_3 = 0$ All the points

$x_2 = 4$ on the line

Joining

(8,4) & (15,18)

(h)

$2x_1 + x_2 + 3x_3 \leq 60$

$x_1 = 21$ $66 \leq 60$ infeasible

$x_2 = 24$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	R	
	1	0	0	2	1	1	0	0	18	
	0	0	1	5	1	3	0	0	24	~~$-2R_1+R_3$~~
(-1)	0	0	1	2	0	1	1	0	7	
	0	0	0	4	1	2	0	0	21	$-2R_3-R_1$
(-2)	0	1	0	4	1	2	0	0	21	$+R_4$
	0	2	1	3	0	0	0	1	60	

1	0	0	2	1	1	1	0	0	18
0	0	1	5	1	1	3	0	0	24
0	0	0	2	6	0	1	0	0	7
0	1	0	4		1	2	0	0	21
0	0	0	-10	-3		-7	0	-2	-6

ig ig $-1/5$ $-1/3$ $-1/7$ ig ig

$$\begin{array}{ccccc|c} 1 & 0 & 0 & 4/7 & 4/7 & 120/7 \\ 0 & 0 & 1 & 5/7 & 1/7 & 150/7 \\ 0 & 0 & 0 & 4/7 & -3/7 & 43/7 \\ 0 & 1 & 0 & 8/7 & 1/7 & 135/7 \\ 0 & 0 & 0 & 10/7 & 3/7 & 6/7 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & 1 & 1/7 & 120/7 \\ 0 & 0 & 0 & 3/7 & 150/7 \\ 0 & 1 & 0 & 1/7 & 43/7 \\ 0 & 0 & 0 & 2/7 & 135/7 \\ 0 & 0 & 0 & -1/7 & 6/7 \end{array}$$

$-2R_4 + R_3$

$-30/7 + 35/7 \quad 3/7$

$-6/7 + 7/7 \quad -18/7 + 24$

$-20/7 + 28/7$

$-6/7 + 7/7$

$-12/7 + 21$

$-3R_4 + R_1$

$x_1 = 135/7$

$x_2 = 150/7$

$x_3 = 0$

$x_4 = 120/7$