Max $Z = 2x_1 + 7x_2 - 3x_3$

7.2-4

Sub to

$x_1 + 3x_2 + 4x_3 \leq 30 \quad x_4$

$x_1 + 4x_2 - x_3 \leq 10 \quad x_5$

$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$

	Z	x_1	x_2	x_3	x_4	x_5	R
	1	0	1	1	0	2	20
x_4	0	0	-1	5	1	-1	20
x_1	0	1	4	-1	0	1	10

y^* over x_4, x_5; s^* under x_4, x_5

(a) $b_{new} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

	Z	x_1	x_2	x_3	x_4	x_5	R
	1	0	1	1	0	2	60
	0	0	-1	5	1	-1	-10
	0	1	4	-1	0	1	30
		ig	-1	ig	ig	-2	

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

basic ✓

feasibility ✗

super optimal

$-4R_1 + R_2$

1	0	0	6	1	1	50
0	0	1	-5	-1	1	10
0	1	0	19	4	-3	-10

1	1/3	0	37/3	7/3	0	140/3
0	1/3	1	4/3	1/3	0	20/3
0	-1/3	0	-19/3	-4/3	1	10/3

basic ✓

feasibility ✓

optimality ✓

$x_1 = 0$

$x_2 = 20/3$

$x_3 = 0$

Coefficients of x_3

$C_{new} = [2 \ 7 \ -2]$

$A_{new} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & -2 \end{bmatrix}$

Z	x_1	x_2	x_3	x_4	x_5	R
	0	1	-2	0	2	20
1						
0	0	-1	5	1	-1	20
0	1	4	-2	0	1	10

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$[0 \ 2] \begin{bmatrix} 3 \\ -2 \end{bmatrix} - [-2]$

1	0	3/5	0	2/5	8/5	28
0	0	-1/5	1	1/5	-1/5	4
0	1	18/5	0	2/5	3/5	18

$2R_1 + R_0$

$2R_1 + R_2$

b Variables ✓

feasible ✓

optimal ✓

$x_1 = 18$ $Z = 28$

$x_2 = 0$

$x_3 = 4$

(C) Coefficients of x_1

$C_{new} = [4 \ 7 \ -3]$

$A_{new} = \begin{bmatrix} 3 & 3 & 4 \\ 2 & 4 & -1 \end{bmatrix}$

$x_1 = 5$

$x_2 = 0$ $Z = 20$

$x_3 = 0$

Z	x_1	x_2	x_3	x_4	x_5	R
	0	1	1	0	2	20
1						
0	1	-1	5	1	-1	20
0	2	4	-1	0	1	10

Restore gf $R_{3/2}$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$[0 \ 2] \begin{bmatrix} 3 \\ 2 \end{bmatrix} - [4]$

1	0	1	1	0	2	20
0	0	-3	11/2	1	-3/2	15
0	1	2	-1/2	0	1/2	5

basic ✓ feasible ✓

optimal ✓

2

new Variable x_6

$C_{new} = [2\ 7\ -3\ -3]$

$A_{new} = \begin{bmatrix} 1 & 3 & 4 & 1 \\ 1 & 4 & -1 & 2 \end{bmatrix}$

	x_1	x_2	x_3	x_6	x_4	x_5	R
z	0	1	1	7	0	2	20
1							
0	0	-1	5	-1	1	-1	20
0	1	4	-1	2	0	1	10

basic ✓
feasible ✓
optimal ✓

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $[0\ 2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} - [-3]$

$x_1 = 10$ $x_6 = 0$
$x_2 = 0$ $z = 20$
$x_3 = 0$

(e) Change obj $C_{new} = [1\ 5\ -2]$

	x_1	x_2	x_3	x_4	x_5	R
z	1	3	0	0	2	20
1						
0	0	-1	5	1	-1	20
0	1	4	-1	0	1	10

restore gf →

1	0	-1	1	0	1	10
0	0	-1	5	1	-1	20
0	1	4	-1	0	1	10

$[0\ 2] \begin{bmatrix} 1 & 3 & 4 \\ 1 & 4 & -1 \end{bmatrix} - [1\ 5\ -2]$

1	1/4	0	3/4	0	5/4	25/2
0	1/4	0	19/4	1	-19/4	105/2
0	1/4	1	-1/4	0	1/4	5/2

$x_1 = 0$ $x_3 = 0$
$x_2 = 5/2$ $z = 25/2$

(f) $3x_1 + 2x_2 + 3x_3 \leq 25$

$x_1 = 10$ $30 \leq 25$
$x_2 = 0$ infeasible
$x_3 = 0$ So we continue

	x_1	x_2	x_3	x_4	x_5	x_6	R
z	0	1	1	0	2	0	20
1							
0	0	-1	5	1	-1	0	20
0	1	4	-1	0	1	0	10
0	3	2	3	0	0	1	25

restore gf
$-3R_2 + R_3$ 3

1	0	1	1	0	2	0	20
0	0	-1	5	1	-1	0	20
0	1	4	-1	0	1	0	10
0	0	-10	6	0	-3	1	-5

ig $-1/10$ ig ig $-2/3$ ig

$$\begin{array}{cccccc} & & & 8/5 & 0 & 17/10 & 1/10 & 39/2 \\ 1 & 0 & 0 & & & & & \\ & & & 22/5 & 1 & -7/10 & -1/10 & 41/2 \\ 0 & 0 & 0 & & & & & \\ & & & & & -1/5 & 2/5 & 8 \\ 0 & 1 & 0 & 7/5 & 0 & & & \\ 0 & 0 & 1 & -3/5 & 0 & 3/10 & -1/10 & 1/2 \end{array}$$

$-4R_3+R_2$

$x_1 = 8 \quad x_3 = 0$

$x_2 = 1/2 \quad z = 39/2$

$2/5 \quad -5/5$

$-6/5 \quad +5/5$

(g) $A_{new} = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 2 \end{bmatrix}$

$b_{new} = \begin{bmatrix} 30 \\ 35 \end{bmatrix}$

z	x_1	x_2	x_3	x_4	x_5	R
1	0	−3	7	0	2	70
0	0	1	2	1	−1	−5
0	1	2	2	0	1	35
	ig	ig	ig	ig	=2	

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 35 \end{bmatrix} \quad [0\ 2] \begin{bmatrix} 30 \\ 35 \end{bmatrix} \quad [0\ 2] \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 2 \end{bmatrix} - [2\ 7\ -3]$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 4 \\ 2 & 2 & 2 \end{bmatrix}$

$$\begin{array}{cccccc} & & & 11 & 2 & 0 & 60 \\ 1 & 0 & -1 & & & & \\ & & & -2 & -1 & 1 & 5 \\ 0 & 0 & -1 & & & & \\ & & & 4 & 1 & 0 & 30 \\ 0 & 1 & 3 & & & & \end{array}$$

$31 - 2R_1 + R_0$

$$\begin{array}{cccccc} & & & 37/3 & 7/3 & 0 & 70 \\ 1 & 1/3 & 0 & & & & \\ & & & -2/3 & -2/3 & 1 & 15 \\ 0 & 1/3 & 0 & & & & \\ & & & 4/3 & 1/3 & 0 & 10 \\ 0 & 1/3 & 1 & & & & \end{array}$$

$x_1 = 0$

$x_2 = 10$

$x_3 = 0$

$z = 70$