Maximize $Z = 54x_1 - 9x_1^2 + 78x_2 - 13x_2^2$,

subject to $x_1 \leq 4$

$2x_2 \leq 12$

Introduction to

Operations Research

Tenth Edition

Question 1. Circle the correct answer.

1. In simplex method, we add ———— variables in the case of '='

A. Slack Variable	B. Surplus Variable	C. Artificial Variable	D. None of the above

2. If the feasible region of a LPP is empty, the solution is ———— no Solution

A. Infeasible	B. Unbounded	C. Alternative	D. None of the above

3. A minimization problem can be converted into a maximization problem by changing the sign of coefficients in the ————

A. Constraints	B. Objective Functions	C. Both A and B	D. None of the above

4. If in a LPP, the solution of a variable can be made infinity large without violating the constraints, solution is ————

A. Infeasible	B. Unbounded	C. Alternative	D. None of the above

5. In maximization cases, ———— are assigned to the artificial variables as their coefficients in the objective function

A. +M	B. –M	C. 0	D. None of the above

6. Alternative solution exist in a linear programming problem when

A. one of the constraint is redundant	B. objective function is parallel to one of the constraints	C. two constraints are parallel	D. all of the above

7. Constraints in an LP model represents

A. Limitations	B. Requirements	C. balancing limitations and requirements	D. all of above

8. To convert ≥ inequality constraints into equality constraints, we must

A. add a surplus variable	B. subtract an artificial variable	C. subtract a surplus variable and an add artificial variable	D. add a surplus variable and subtract an artificial variable

9. If for a given solution, a slack variable is equal to zero, then

A. the solution is optimal	B. the solution is infeasible	C. there exist no solution	D. None of the above

10. In the optimal simplex table $z_j - c_j = 0$ value indicates

A. unbounded solution	B. degenerate	C. alternative solution	D. None of these

Primal

max $z = 3X_1 + 5X_2$

subject to

$X_1 \le 4 \longrightarrow y_1$

$2X_2 \le 12 \longrightarrow y_2$

$3X_1 + 2X_2 \le 18 \longrightarrow y_3$

$X_1, X_2 \ge 0$

$\boxed{*}$ num of constraints in primal problem = desicion variables in dual problem

$\boxed{*}$ obj in Primal is oposite than the object in dual

Dual:

min $w = 4y_1 + 12y_2 + 18y_3$

subject to:

$1y_1 + 0y_2 + 3y_3 \ge 3$

$0y_1 + 2y_2 + 2y_3 \ge 5$

$y_1, y_2, y_3 \ge 0$

$\boxed{*}$ obj for dual:

step 1: do the opposit (min, max)

step 2: RHS of constraints in primal * y for each const

$\boxed{*}$ subject for dual

$\square y_1 + \square y_2 + \square * y_3 \ge$ coef of X_1 in obj of primal

coef of X_1 in each constraint in primal

Ex: primal

Max $z = 4X_1$

subject to

$X_1 + 2X_2 \le 5 \to y_1$

$X_2 \le 3 \to y_2$

$X_1, X_2 \ge 0$

obj dual

min $w = 5y_1 + 3y_2$

Subject to

$1y_1 + 0y_2 \ge 4$

$2y_1 + 1y_2 \ge 0$

$y_1, y_2 \ge 0$

Ex primal

Max $z = 3X_1 + 4X_2 + X_3$

subject to

$X_1 + X_3 \le 3 \to y_1$

$2X_2 + 3X_3 \le 9 \to y_2$

$X_1, X_2, X_3 \ge 0$

dual

obj

min $w = 3y_1 + 9y_2$

Subject to

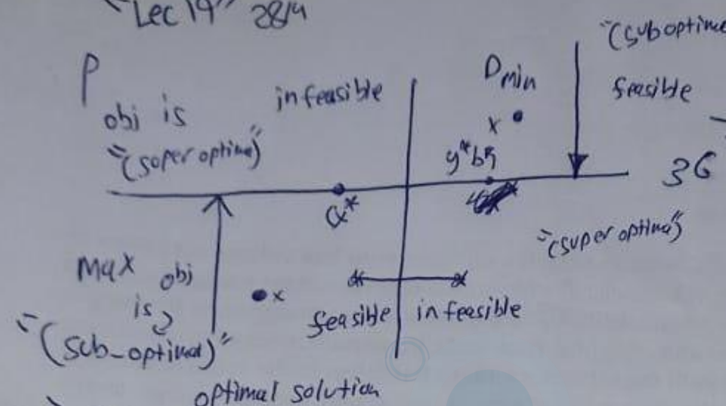
$1y_1 + 0y_2 \ge 3$

$0y_1 + 2y_2 \ge 4$

$1y_1 + 3y_2 \ge 1$

$y_1, y_2, y_3 \ge 0$

"Lec 19" 28/4



→ If x is a feasible solution for the primal problem and y is a feasible solution for the dual problem then $cx \le yb$

→ If x^* is an optimal solution for the primal problem and y^* is an optimal solution for the dual problem, then $cx^* = y^*b$

(Strong duality problem)

→ AT each iteration, the simplex method simultaneously identifies a CPF solution x for the primal problem and a complementary solution y for the dual problem

"complementary solution"

$cx = yb$

Note : $y_1 / y_2 / y_3$ ~~Value~~ is the coef of the slack variables in obj

Ex x_3 x_4 x_5 — 0 0 0 $y_1 = 0$ $y_2 = 0$ $y_3 = 0$

→ "Symmetry property"

For Any primal problem and its dual problem

All relationships between them must be symmetric

→ Duality theorem

- Identifies the only possible relationships between the primal and dual prob

- If one problem has feasible solutions and a bounded obj function, then so does the other problem.

  - both weak and strong duality properties apply

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The Essence of Duality Theory

- Duality theorem (cont'd.)
  - If one problem has feasible solutions and an unbounded objective function, then the other problem has no feasible solutions
  - If one problem has no feasible solutions, then the other problem either has no feasible solutions or an unbounded objective function

.

The Essence of Duality Theory

- Applications
  - Dual problem can be solved directly by the simplex method to identify an optimal solution for the primal problem
    - Can be useful if one of the problems has fewer functional constraints
  - Evaluation of a proposed solution for the primal problem
  - Economic interpretation of the dual problem
    - Insights for the primal problem

TABLE 6.10 Classification of basic solutions

		Satisfies Condition for Optimality?	
		Yes	**No**
Feasible?	**Yes**	Optimal	Suboptimal
	No	Superoptimal	Neither feasible nor superoptimal

TABLE 6.11 Relationships between complementary basic solutions

Primal Basic Solution	Complementary Dual Basic Solution	Both Basic Solutions: Primal Feasible?	Both Basic Solutions: Dual Feasible?
Suboptimal	Superoptimal	Yes	No
Optimal	Optimal	Yes	Yes
Superoptimal	Suboptimal	No	Yes
Neither feasible nor superoptimal	Neither feasible nor superoptimal	No	No

Adapting to Other Primal Forms

- Sensible-odd-bizarre (SOB) method for determining the form of constraints in the dual
  - Formulate the primal problem in either maximization or minimization form
    - Dual problem will be in other form
  - Label the different forms of the functional and variable constraints as being sensible, odd, or bizarre
    - See Table 6.14 for guidance

SOB method

Max

constraints

$\le \rightarrow$ Sensible

$= \rightarrow$ odd

$\ge \rightarrow$ Bizarre

DV

$x \ge 0 \rightarrow$ Sensible

x unconstrained $(-\infty, \infty) \rightarrow$ odd

$x \le 0 \rightarrow$ Bizarre

min

constraints

$\le \rightarrow$ Bizarre

$= \rightarrow$ odd

$\ge \rightarrow$ Sensible

DV

$x \ge 0 \rightarrow$ Sensible

x unconstrained $\rightarrow$ odd

$x \le 0 \rightarrow$ Bizarre

Max $z = 3x_1 + 4x_2 - x_3$

subject to

$x_1 + 2x_2 + x_3 \ge 10 \xrightarrow{B} y_1 \xrightarrow{B} y_1 \le 0$

$x_1 + x_2 + x_3 \le 20 \xrightarrow{S} y_2 \xrightarrow{S} y_2 \ge 0$

$x_2 + 3x_3 = 15 \xrightarrow{O} y_3 \xrightarrow{O} y_3 =$ un const

~~max~~ $x_1 \ge 0 \xrightarrow{S}$

$x_2 \le 0 \xrightarrow{B}$

x_3 is unconstrained

dual

min $\Rightarrow W = 10y_1 + 20y_2 + 15y_3$

subject to:

$1y_1 + 1y_2 + 0y_3 \ge 3$

$2y_1 + y_2 + y_3 \le 4$

$1y_1 + y_2 + 3y_3 = -1$

$y_1 \le 0$

$y_2 \ge 0$

y_3 is unconstrained

Min $z = 3x_1 + 4x_2 - x_3$

Subject to :

$x_1 + 2x_2 + x_3 \geq 10 \xrightarrow{S} y_1 \geq 0$

$x_1 + x_2 + x_3 \leq 20 \xrightarrow{B} y_2 \leq 0$

$x_2 + 3x_3 = 15 \xrightarrow{0} y_3$ unconstraind

$x_1 \geq 0 \longrightarrow S$

$x_2 \leq 0 \longrightarrow B$

x_3 is unconstraind

dual :

Max $w = 10y_1 + 20y_2 + 15y_3$

Subject to :

$1y_1 + 1y_2 + 0y_3 \leq 3$

$2y_1 + 1y_2 + 1y_3 \geq 4$

$1y_1 + 1y_2 + 3y_3 = -1$

$y_1 \geq 0$

$y_2 \leq 0$

y_3 unconstrained

$$\text{Max } z = x_1 + 0x_2 + 3x_3$$

Subject to

$$x_1 + x_2 + x_3 \geq 10 \xrightarrow{B} y_1 \leq 0$$

$$x_1 + 2x_2 + x_3 \leq 12 \xrightarrow{S} y_2 \geq 0$$

$$x_1 \geq 0$$

x_2 uncastrained

$$x_3 \geq 0$$

Dual

$$\text{Min obj } w = 10y_1 + 12y_2$$

subject to:

$$1y_1 + 1y_2 \geq 1$$

$$y_1 + 2y_2 = 0$$

$$1y_1 + 1y_2 \geq 3$$

$$y_1 \leq 0$$

$$y_2 \geq 0$$

Q2: For the following linear programming problem, use SOB method to contract its dual problem

Max $z = 3x_1 + 4x_2$

Subject to: $4x_1 + 3x_2 \geq 4 \xrightarrow{B} y_1 \leq 0$

$2x_1 + 3x_2 = 6 \xrightarrow{O} y_2$ un constraind

$4x_1 + x_2 \leq 4 \xrightarrow{S} y_3 \geq 0$

$x_2 \leq 0$

$\cancel{x_1 \geq 1}$ $x_1 \geq 0$

$x_1 \geq 1 \longrightarrow y_4 \leq 0$

Dual: obj

Min $w = 4y_1 + 6y_2 + 4y_3 + 1y_4$

subject to

$4y_1 + 2y_2 + 4y_3 + 1y_4 \geq 3$

$3y_1 + 3y_2 + 1y_3 + 0y_4 \leq 4$

$\cancel{0y_1 + 0y_2 + 0y_3 + 0y_4 \geq 0}$

$\cancel{0y_1 + 0y_2 + 0y_3 + 0y_4 \geq 0}$

$y_1 \leq 0$

y_2 un constrained

$y_3 \geq 0$

$y_4 \leq 0$

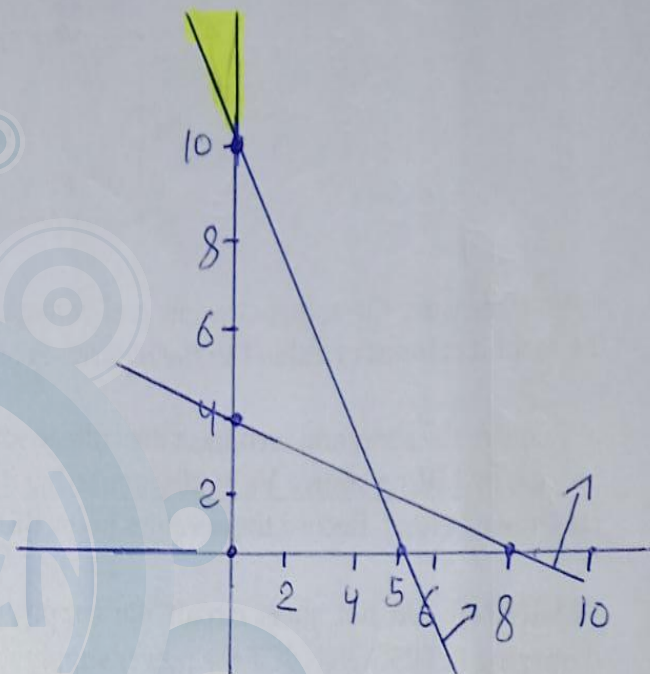
Q18 Solve the following linear program graphically showing the objective function, all constraints, and the feasible region, and marking all basic feasible solutions

$\min Z = 5x_1 + 4x_2$

subject to: $2x_1 + x_2 \geq 10$

$x_1 + 2x_2 \geq 8$

$x_1 \leq 0, x_2 \geq 0$



1) $2x_1 + x_2 = 10$

$x_1 = 0 \quad x_2 = 10 \quad (0, 10)$

$x_1 = 5 \quad x_2 = 0 \quad (5, 0)$

2) $x_1 + 2x_2 = 8$

$x_1 = 0 \quad x_2 = 4 \quad (0, 4)$

$x_1 = 8 \quad x_2 = 0 \quad (8, 0)$

* corner point $(0, 10)$

$\min \rightarrow (0, 10) \quad Z = 40$

If it is max => optimum point (∞, ∞)

"un-bounded"

Q2: For the Sollowing linear programming problem use SOB method to construct its dual problem

min $z = 5x_1 + 4x_2$

subject to $2x_1 + x_2 \geq 10 \xrightarrow{S} y_1 \geq 0$

$x_1 + 2x_2 \geq 8 \xrightarrow{\cancel{R}S} y_2 \cancel{\geq} 0$

x_1 is unconstrained

$x_2 \leq 0$

dual

obj: max $w = 10y_1 + 8y_2$

subject to

$2y_1 + 1y_2 = 5$

$1y_1 + 2y_2 \geq 4$

$y_1 \geq 0$

$y_2 \cancel{\geq} 0$

Question 3: The University of Jordan (UoJ) produces 3D printed medical parts in The Industrial Engineering Department (IED). Such parts can be sterilized either at the IED or the Jordan University Hospital (JUH), this leads to different sterilization cost. Four different department at the JUH have already placed orders for the next year. The table provided shows the number of parts required by each department, in addition to the number of parts that can be sterilized in the IED or the JUH. This table also provides the unit profit for the parts to be sold for each department. The UoJ wishes to know the number of parts to be sterilized in the IED and the JUH and then sold to these four departments. Formulate a LINEAR programming model for this case.

Sterilization place	Unit Profit				
	Department 1	Department 2	Department 3	Department 4	Sterilized parts
IED	$80	$70	$60	$90	1000
JUH	$60	$70	$50	$100	600
Ordered parts	400	500	200	500	

X_{ij} : num of parts to be sterilized in i and sold to department j

obj: max

$$Z = 80X_{11} + 70X_{12} + 60X_{13} + 90X_{14} + 60X_{21} + 70X_{22} + 50X_{23} + 100X_{24}$$

Subject to:

$$X_{11} + X_{21} = 400$$
$$X_{12} + X_{22} = 500$$
$$X_{13} + X_{23} = 200$$
$$X_{14} + X_{24} = 500$$

$$X_{11} + X_{12} + X_{13} + X_{14} \le 1000$$
$$X_{21} + X_{22} + X_{23} + X_{24} \le 600$$

$X_{ij} \ge 0$ for all i and j

Question 4: WHSmith is a food company that produces two types of freeze dried food: Dragon fruit and Strawberry fruit. They used special treated sugar at a maximum rate 200kg per month. Each Dragon fruit requires 0.1kg of sugar and each Strawberry fruit requires 0.2kg of sugar. All the other ingredients are in plentiful. The WHSmith has currently hired 2 employees to work 60 hours per month. Each Dragon fruit requires 20 minutes of working hours, and each Strawberry fruit requires 30 minutes of working hours. Although Dragon fruit that yields a profit of $5 per unit is more expensive, the Strawberry fruit is more profitable, where the profit of a unit of Strawberry fruit is twice the profit of Dragon fruit. The WHSmith wants to know how many units of Dragon fruit and Strawberry fruit should produce each month so as to achieve the highest possible profit.

x_1 : number of units of dragon fruit to be produced per month

x_2 : " " " " strawberry " " " " "

Max obj : $Z = 5x_1 + 10x_2$

Subject to : $0.1x_1 + 0.2x_2 \le 200$

$20x_1 + 30x_2 \le 2*60*60$

$x_1, x_2 \ge 0$

Q5: mid

For the following linear programming model, use the matrices form to find the optimal simplex ~~method~~ tableau whose basic variables are x_1 and x_2. find the value by which the optimal value will increase by when the RHS of the first constraint is increased one unit.

max $z = 5x_1 + 4x_2$ subto: $2x_1 + x_2 \le 10$

$x_1 + 2x_2 \le 8$

$x_1 \le 0$, $x_2 \ge 0$

$-x_1 \ge 0$ $(x_1' = -x_1) \ge 0$

$z = -5x_1' + 4x_2$

subject to:

$2(-x_1') + x_2 \le 10$

$-x_1' + 2x_2 \le 8$

$x_1' \ge 0$ $x_2 \ge 0$

$C = [-5 \ \ 4]$ $b = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$

$x = \begin{bmatrix} x_1' \\ x_2 \end{bmatrix}$ $A = \begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}$ $C_B = [-5 \ \ 4]$

$B = A$

BCz the basic variable is x_1 and x_2 so c is coefficients of the decision variables in the obj and C_B is the coefficients of the basic variables in the obj. So $C_B = C$

$C_B * B^{-1} * A - C = [0 \ \ 0] \rightarrow$ optimal

$C_B * B^{-1} = [2 \ \ 1] \rightarrow$ shadow price

$\rightarrow$ 2-units will increase

$\rightarrow$ RHS: $B^{-1} * b = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$ → so it is infeasible

$$\boxed{C_B * B^{-1} * b = 28}$$

$B^{-1} * A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow$

$\rightarrow$ B^{-1} $\begin{bmatrix} -0.6667 & 0.333 \\ -0.333 & 0.6667 \end{bmatrix}$ coefficients of the slack variables in ~~the~~ constraints.

Q6: Using the 2-Phase method, find the optimal solution for the following linear programming model.

max: $z = 5x_1 + 4x_2$

subject to: $2x_1 + x_2 \le 10$

$x_1 + 2x_2 = 8$

$x_1 \le 0 \quad x_2 \ge 0$

~~$\max z = -5x_1' + 4x_2 - Mx_a$~~

$\Rightarrow \max z = -5x_1' + 4x_2$

$\Rightarrow -2x_1' + x_2 + x_3 = 10$

$-x_1' + 2x_2 + x_{A1} = 8$

$\underbrace{-x_1}_{x_1'} \ge 0 \quad x_2 \ge 0$

Phase 1: objective

Min $z = x_{a1}$ $\Rightarrow \max -z = -x_{a1}$ $\quad -z + x_{a1} = 0$

	z	x_1	x_2	x_3	x_{A1}	RHS
R_0	-1	0	0	0	1	0
R_1	0	-2	1	1	0	10
R_2	0	-1	2	0	1	8

restoring $\Rightarrow$ $-R_2 + R_0$

	z	x_1	x_2	x_3	x_{a1}	RHS	SR
R_0	-1	1	-2	0	0	-8	x
R_1	0	-2	1	1	0	10	10
R_2	0	-1	2	0	1	8	4

x_2: is entering

x_{a1}: is leaving

	z	x_1	x_2	x_3	x_{a1}	RHS
R_0	-1	0	0	0	1	0
R_1	0	-1.5	0	1	-1/2	6
R_2	0	-1/2	1	0	1/2	4

1) $R_2/2$

2) $-R_2 + R_1$

3) $2R_2 + R_0$

Phase 2: objective

max $z = -5x_1 + 4x_2$ $\quad z + 5x_1 - 4x_2 = 0$

~~$-z + 5x_1 + 4x_2 = 0$~~

	z	x_1	x_2	x_3	RHS
R_0	+1	+5	-4	0	0
R_1	0	-1.5	0	1	6
R_2	0	-0.5	1	0	4

Restoring $\Rightarrow$ $+4R_2 + R_0$

	z	x_1	x_2	x_3	RHS
R_0	1	3	0	0	16
R_1	0	-1.5	0	1	6
R_2	0	-0.5	1	0	4

So It is optimal and feasdble

Q2: consider the following problem **use Big M-Method**

maximize $Z = 4X_1 + 2X_2 + 3X_3 + 5X_4$

subject to: $2X_1 + 3X_2 + 4X_3 + 2X_4 = 300 \rightarrow 2X_1 + 3X_2 + 4X_3 + 2X_4 + X_{a1} \leq 300$

$8X_1 + X_2 + X_3 + 5X_4 = 300 \rightarrow 8X_1 + X_2 + X_3 + 5X_4 + X_{a2} \leq 300$

$X_{1,2,3,4} \geq 0$

$Z = 4X_1 + 2X_2 + 3X_3 + 5X_4 - MX_{a1} - MX_{a2}$

obj $Z - 4X_1 - 2X_2 - 3X_3 - 5X_4 + MX_{a1} + MX_{a2}$

	Z	X_1	X_2	X_3	X_4	X_{a1}	X_{a2}	RHS
R_0	1	-4	-2	-3	-5	M	M	0
R_1	0	2	3	4	2	1	0	300
R_2	0	+8	+1	1	+5	0	1	300

$-MR_1 + R_0$

$-MR_2 + R_0$

restoring $\Rightarrow$

	Z	X_1	X_2	X_3	X_4	X_{a1}	X_{a2}	RHS	ST
R_0	1	$-4-10M$	$-2-4M$	$-3-5M$	$-5-7M$	0	0	$-600M$	
R_1	0	2	3	4	2	1	0	200	
R_2	0	8	1	1	5	0	1	300	300/8

* BV: X_{a1} / X_{a2}

* artificial variables: X_{a1}, X_{a2}

* pivot: 8

* entering: X_1

* leaving: X_{a2}

Q1: Solve the following linear program graphically, showing the objective function, all constraints and the feasible region, and marking all basic feasible solutions.

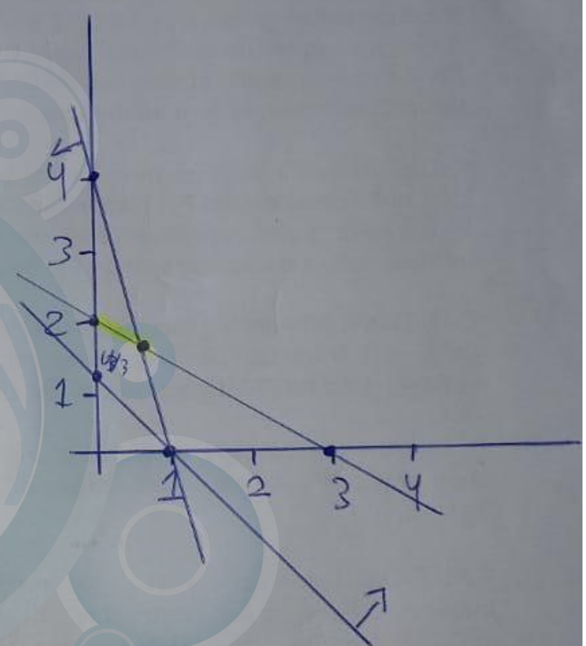
Minimize $Z = 3x_1 + 4x_2$

Subject to: $4x_1 + 3x_2 \geq 4$ → $(x_1 = 0, x_2 = 4/3)$ $(x_1 = 1, x_2 = 0)$

$2x_1 + 3x_2 = 6$ → $(x_1 = 0, x_2 = 2)$ $(x_1 = 3, x_2 = 0)$

$4x_1 + x_2 \leq 4$ → $(x_1 = 0, x_2 = 4)$ $(x_1 = 1, x_2 = 0)$

$x_1 \geq 0, x_2 \geq 0$



corner points:

(0,2) (0.6,1.6)

$2x_1 + 3x_2 = 6$ and $4x_1 + x_2 = 4$

$2x_1 + 3x_2 = 6$

$-3(4x_1 + x_2 = 4)$

$2x_1 + 3x_2 = 6$ +

$-12x_1 - 3x_2 = -12$

$-10x_1 = -6$

$x_1 = \frac{6}{10} = 0.6$ and $x_2 = 1.6$

min → $3(0) + 4(2) = 8$ optimal point is (0,2)

$3(0.6) + 4(1.6) = 8.2$

Question 3. A company produces precision medical diagnostic equipment at two factories. Three medical centers have placed orders for this month's production output. The table below shows what the cost would be for shipping each unit from each factory to each of these customers. Also shown are the number of units that will be produced at each factory and the number of units ordered by each customer.

From \ To	Unit Shipping Cost			
	Customer 1	Customer 2	Customer 3	Output
Factory 1	$600	$800	$700	400 units
Factory 2	$400	$900	$600	500 units
Order size	300 units	200 units	400 units	

A decision now needs to be made about the shipping plan for how many units to ship from each factory to each customer.

Formulate a linear programming model for this problem. DO NOT SOLVE

X_{ij}: number of units to be shipped from factory i to each customer j

Objective: Max

$$Z = 600X_{11} + 800X_{12} + 700X_{13} + 400X_{21} + 900X_{22} + 600X_{23}$$

Subject to:

$$X_{11} + X_{12} + X_{13} \le 400$$
$$X_{21} + X_{22} + X_{23} \le 500$$

$$X_{11} + X_{21} = 300$$
$$X_{12} + X_{22} = 200$$
$$X_{13} + X_{23} = 400$$

$X_{ij} \ge 0$ for all i and j

改善

KAIZEN

TEAM