University of Jordan
School of Engineering
Electrical Engineering Department

EE 204
Electrical Engineering Lab

EXPERIMENT 7
RESONANCE

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RESONANCE

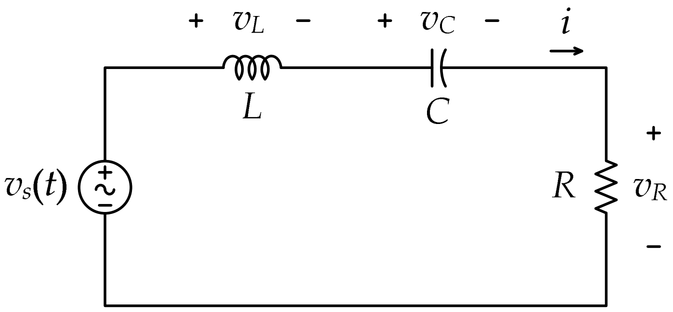
OBJECTIVE

This experiment investigates the voltage and current relationships in series and parallel resonant RLC circuits. Of primary importance is the establishment of the resonant frequency and the quality factor, or Q, of the circuit with relation to the values of the R, L, and C components.

DISCUSSION

Series Resonance

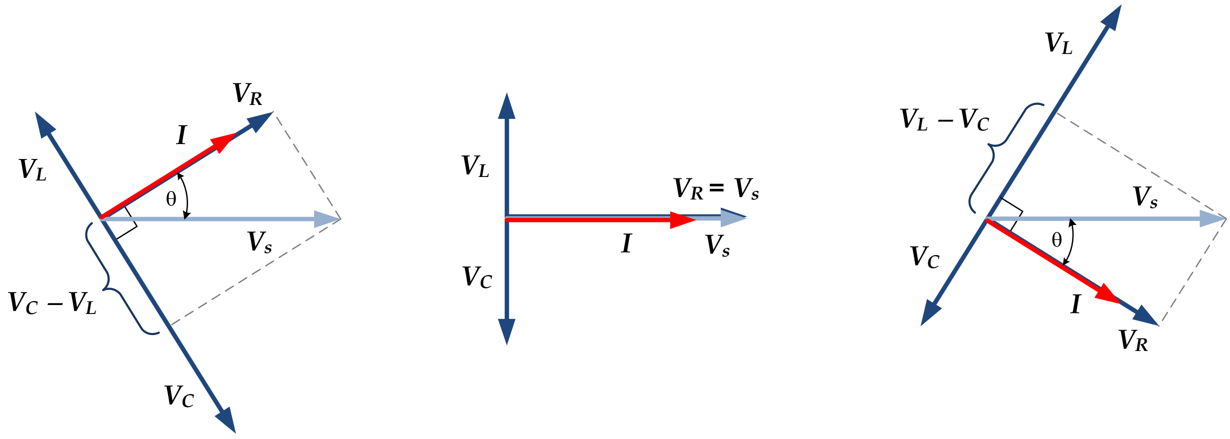
A series resonant circuit consists of a resistor, a capacitor, and an inductor in a simple loop as shown below.



At the resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$ the capacitive and inductive reactances will be of the same magnitude, and as they are 180 degrees in opposition, they effectively nullify each other. This leaves the circuit purely resistive, the source "seeing" only the resistive element. Consequently, the current will be at a maximum at the resonant frequency. At any higher or lower frequency, a net reactance (the difference between $\boldsymbol{Z_L}$ and $\boldsymbol{Z_C}$) must be added to the resistor value, producing a higher impedance and thus, a lower current.

As this is a simple series loop, the resistor's voltage will be proportional to the current. Consequently, the resistor voltage should be a maximum at the resonant frequency and decrease as the frequency is either increased or decreased (see phasor diagram below). At resonance, the resistor value sets the maximal current and also has a major effect on the "tightness" of the voltage versus frequency curve: The smaller the resistance, the tighter the curve and the higher the voltage seen across the capacitor and inductor. The quality factor, Q, of the circuit can be defined as the ratio of the resonant reactance to the circuit resistance, $Q = \frac{X}{R} = \frac{\omega_r L}{R} = \frac{1}{\omega_r RC}$, which also corresponds to the ratio of the resonant frequency to the circuit bandwidth, $Q = \frac{f_r}{B}$ where the bandwidth is given by $B = \frac{R}{2\pi L}$. The bandwidth is the interval between the half-power cut-off frequencies, which are the frequencies at which the current is $1/\sqrt{2}$ of its maximum value. The cut-off frequencies for the series RLC circuit are given by:

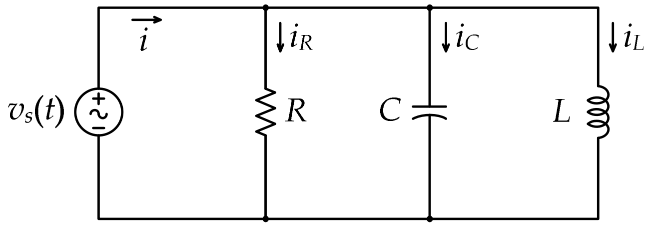
$$\omega_{1,2} = \omega_r \left[\mp \frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = \mp \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$f < f_r$ $f = f_r$ $f > f_r$

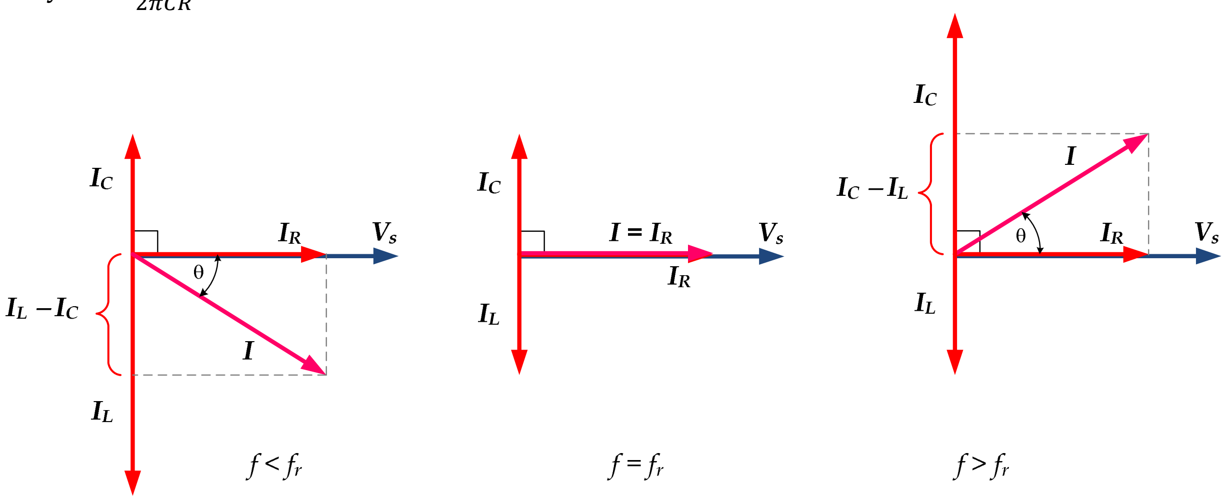
Parallel Resonance

A parallel resonant circuit consists of a resistor, a capacitor, and an inductor in parallel as shown below.



At the resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$ the capacitive and inductive reactances will be of the same magnitude, and as they are 180 degrees in opposition, they effectively nullify each other. This leaves the circuit purely resistive, the source "seeing" only the resistive element. At any lower or higher frequency the inductive or capacitive reactance will shunt the resistance, producing a lower impedance, and thus a higher current.

Consequently, in a parallel RLC circuit, minimum current happens at resonance (see the following phasor diagram). The quality factor, Q, of the circuit is defined as the ratio of the resonant susceptance to the circuit conductance, $Q = \frac{B}{G} = \frac{R}{\omega_r L} = \omega_r RC$, which also corresponds to the ratio of the resonant frequency to the circuit bandwidth, $Q = \frac{f_r}{B}$ where the bandwidth is given by $B = \frac{1}{2\pi CR}$.



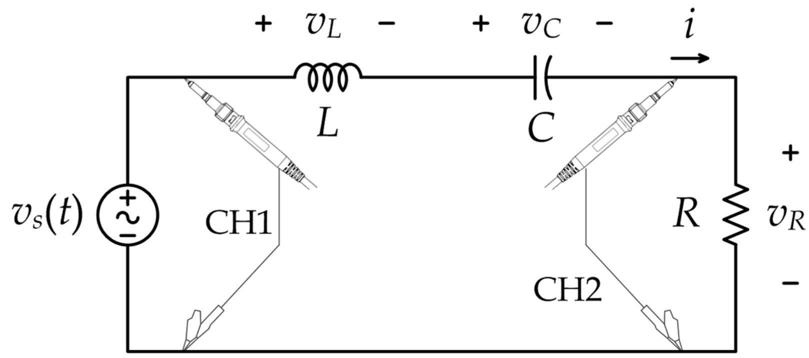
$f < f_r$ $f = f_r$ $f > f_r$

The bandwidth is again the interval between the half-power cut-off frequencies, which are the frequencies at which the current is $\sqrt{2}$ of its minimum value. The cut-off frequencies for the parallel RLC circuit are given by:

$$\omega_{1,2} = \omega_r \left[\mp \frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = \mp \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

PROCEDURE A – SERIES RESONANCE

1. Construct the circuit shown below. Assume that R = 820 Ω, L = 10 mH, C = 10 nF.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 6000 Hz, and *peak voltage* of $V_p = 4$ V.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift. Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the voltages $\boldsymbol{V_R}$, $\boldsymbol{V_L}$, $\boldsymbol{V_C}$ and current $\boldsymbol{I}$ in the circuit at the different frequencies in Tables 1, 2 and 3. Make sure to evaluate both magnitude and phase for each complex quantity, and record the expected period T of the $\boldsymbol{V_S}$ signal in microseconds. Record the answers in the three tables?

4. What is the equation for the resonant frequency of a series RLC circuit?

. . . .

5. Use that above equation to find the resonant frequency in this experiment.

. . . .

6. What is the equation for the quality factor Q, and bandwidth B of a series RLC circuit?

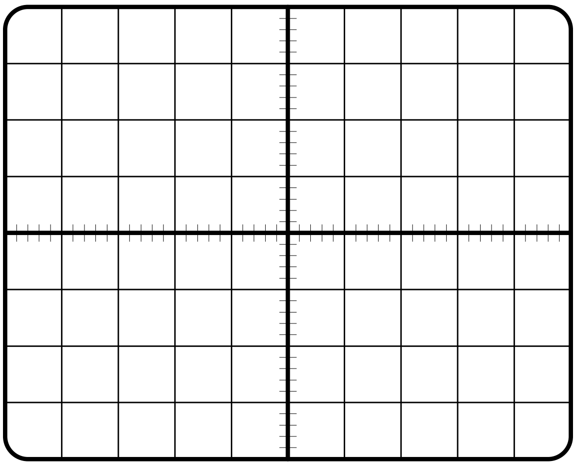
. . . .

7. Use the proper equations to find the frequencies f_1 and f_2, at which the current amplitude is approximately 0.707 times the resonant current (i.e., the half-power points).

8. Use the oscilloscope to measure the peak values of the voltages $\boldsymbol{V_S}$ and $\boldsymbol{V_R}$ and the phase shift of $\boldsymbol{V_R}$ compared to $\boldsymbol{V_S}$**.** Remember that you can change the horizontal sweep setting of the oscilloscope to make more accurate measurements of the phase. Record the measurements in Table 1?

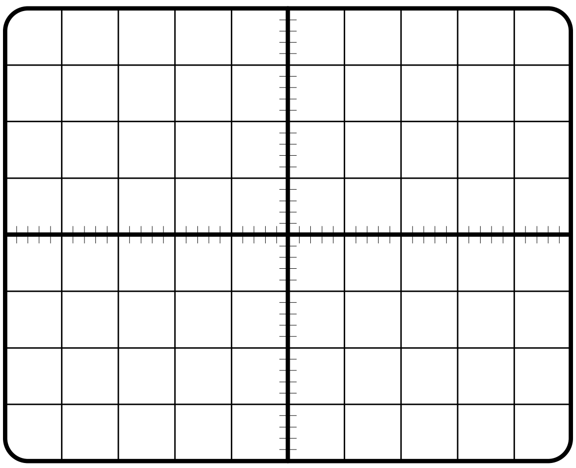
CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

9. At the frequency of 6000 Hz, draw what you see on the oscilloscope screen.



10. Remember that the voltage $\boldsymbol{V_R}$ across the resistor is directly proportional to (and has the same phase shift as) the current $\boldsymbol{I}$ in the circuit. Adjust the frequency of the function generator in small increments, up and down, to find the frequency at which $\boldsymbol{V_S}$ and $\boldsymbol{V_R}$ (i.e, the current $\boldsymbol{I}$) have the same phase shift. This is the *experimental resonance frequency*. Record this frequency value in Table 1.

11. Draw what you see on the oscilloscope screen at the resonant frequency.



12. Notice that the voltage $\boldsymbol{V_R}$ has the maximum possible value at the resonance frequency. Record this peak voltage value in Table 1.

13. Now start changing the frequency of the function generator above and below the resonance frequency until the experimental frequencies f_1 and f_2 are found. These will occur at a voltage amplitude of approximately 0.707 times the resonant voltage $\boldsymbol{V_R}$ (i.e., the half-power points). Record such frequencies and the corresponding voltage values in Table 1.

Table 1

AC Source Frequency (Hz)	$\lvert\boldsymbol{V_S}\rvert$ **(peak) (V)**		$\boldsymbol{V_S}$ **period** $\boldsymbol{T}$ **(µs)**		$\lvert\boldsymbol{V_R}\rvert$ **(peak) (V)**		$\measuredangle\boldsymbol{V_R}$ **with** $\boldsymbol{V_s}$ **(degrees)**	
	Theory	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**
6000								
9000								
14400								
16800								
30000								
48000								
$f_r =$ ______								
$f_1 =$ ______								
$f_2 =$ ______								

14. Use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_C}$ and its phase shift compared to $\boldsymbol{V_S}$. To do that, swap the locations of C and R in the circuit while keeping the oscilloscope connections unchanged, allowing you to measure $\boldsymbol{V_C}$. Record the measurements for all frequencies in Table 2.

15. Use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_L}$ and its phase shift compared to $\boldsymbol{V_S}$. To do that, swap the locations of L and C in the circuit while keeping the oscilloscope connections unchanged, allowing you to measure $\boldsymbol{V_L}$. Record the measurements for all frequencies in Table 2.

Table 2

AC Source Frequency (Hz)	$\lvert\boldsymbol{V_C}\rvert$ **(peak) (V)**		$\measuredangle\boldsymbol{V_C}$ **with** $\boldsymbol{V_s}$ **(degrees)**		$\lvert\boldsymbol{V_L}\rvert$ **(peak) (V)**		$\measuredangle\boldsymbol{V_L}$ **with** $\boldsymbol{V_s}$ **(degrees)**	
	Theory	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**
6000								
9000								
14400								
16800								
30000								
48000								
$f_r =$ ______								
$f_1 =$ ______								
$f_2 =$ ______								

16. What is the relationship between the capacitor voltage $\boldsymbol{V_C}$ and inductor voltage $\boldsymbol{V_L}$ at resonant frequency?

. . . .

17. Consider the capacitor voltage amplitude $|\boldsymbol{V_C}|$. Which one is higher $|\boldsymbol{V_C}|$ at resonant frequency or $|\boldsymbol{V_C}|$ below resonant frequency?

. . . .

18. Consider the inductor voltage phase $\measuredangle \boldsymbol{V_L}$. Which one is higher $\measuredangle \boldsymbol{V_L}$ at resonant frequency or $\measuredangle \boldsymbol{V_L}$ below resonant frequency? Note: In phase, consider the positive or negative signs.

. . . .

19. Using the values in Tables 1 and 2, evaluate the current $\boldsymbol{I}$ and total impedance $\boldsymbol{Z}$ of the series *RLC* circuit using Ohm's law, and record them in Table 3. Remember that the total impedance $\boldsymbol{Z}$ is a complex number, so you need to find both its magnitude and phase.

Table 3

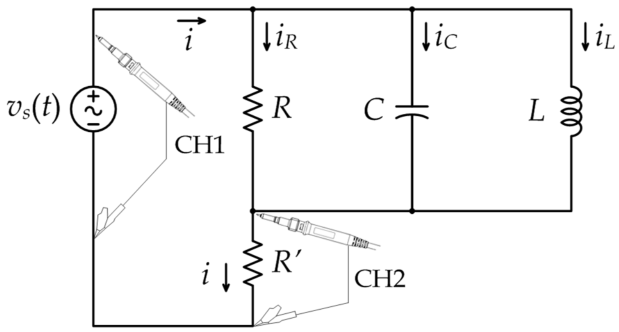
AC Source Frequency (Hz)	$\|\boldsymbol{I}\|$ **(peak) (mA)** $= \boldsymbol{V_R}/R$		$\measuredangle \boldsymbol{I}$ **with** $\boldsymbol{V_s}$ (= $\measuredangle \boldsymbol{V_R}$ **with** $\boldsymbol{V_s}$)		$\|\boldsymbol{Z}\| = \|\boldsymbol{V_S}\|/\|\boldsymbol{I}\|$ **(peak/peak) (kΩ)**		$\measuredangle \boldsymbol{Z} = \measuredangle \boldsymbol{V_S} - \measuredangle \boldsymbol{I}$ **(degrees)**	
	Theory	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**
6000								
9000								
14400								
16800								
30000								
48000								
$f_r =$ ______								
$f_1 =$ ______								
$f_2 =$ ______								

20. Using the measured values in Table 3, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $|\boldsymbol{Z}|$ versus source frequency; (2) $\measuredangle \boldsymbol{Z}$ versus source frequency; (3) $\boldsymbol{I}$ versus source frequency.

21. For the above plots, state your conclusions under the plot? Also identify the resonant frequency and bandwidth in each plot.

PROCEDURE B –PARALLEL RESONANCE

1. Construct the circuit shown below. Assume that R = 820 Ω, L = 10 mH, C = 10 nF, and R' = 10 Ω. Make sure you use the correct resistor values.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 6000 Hz, and *peak voltage* of $V_p = 4$ V.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift. Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the current $\boldsymbol{I}$ and the voltage $\boldsymbol{V_R}$ in the circuit at the different frequencies in Tables 4 and 5. Make sure to evaluate both magnitude and phase for each complex quantity, and record the expected period T of the $\boldsymbol{V_S}$ signal in microseconds. Record the answers in the two tables. The small resistor R' was placed as a convenient way to measure the current $\boldsymbol{I}$ using the oscilloscope. It has a small resistance compared to other resistors in the circuit, which means you can neglect it in theoretical calculations.

4. What is the equation for the resonant frequency of a parallel RLC circuit?

. . . .

5. Use that above equation to find the resonant frequency in this experiment.

. . . .

6. What is the equation for the quality factor Q, and bandwidth B of a parallel RLC circuit?

. . . .

7. Use the proper equations to find the frequencies f_1 and f_2, at which the current amplitude is approximately 1.414 = 1/0.707 times the resonant current (i.e., the bandwidth limits).

. . . .

8. Use the oscilloscope to measure the peak values of the voltages $\boldsymbol{V_S}$ and $\boldsymbol{V_{R'}}$ and the phase shift of $\boldsymbol{V_{R'}}$ compared to $\boldsymbol{V_S}$. Remember that you can change the horizontal sweep setting of the oscilloscope to make more accurate measurements of the phase. Record the measurements in Table 4?

CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

9. Remember that by Ohm's law, the voltage $\boldsymbol{V_{R'}}$ across the resistor is directly proportional to (and has the same phase shift as) the current $\boldsymbol{I}$ in the circuit. Adjust the frequency of the function generator in small increments, up and down, to find the frequency at which $\boldsymbol{V_S}$ and $\boldsymbol{V_{R'}}$ (i.e, the current $\boldsymbol{I}$) have the same phase shift. This is the experimental resonance frequency. Record this frequency value in Table 4.

10. Notice that the voltage $\boldsymbol{V_{R'}}$ has the minimum possible value at the resonance frequency. Record this peak voltage value in Table 4.

11. Now start changing the frequency of the function generator above and below the resonance frequency until the experimental frequencies f_1 and f_2 are found. These will occur at a voltage amplitude of approximately 1.414 times the resonant voltage $\boldsymbol{V_{R'}}$. Record such frequencies and the corresponding voltage values in Table 4.

Table 4

AC Source Frequency (Hz)	V_S (peak) (V)		V_S period T (μs)		$V_{R'}$ (peak) (V)		$\measuredangle V_{R'}$ with V_s (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
6000								
9000								
14400								
16800								
30000								
48000								
f_r = ______								
f_1 = ______								
f_2 = ______								

12. What is the relationship between the capacitor current $\boldsymbol{I_C}$ and inductor current $\boldsymbol{I_L}$ at resonant frequency?

. . . .

13. Consider the capacitor current phase $\measuredangle \boldsymbol{I_C}$. Which one is higher $\measuredangle \boldsymbol{I_C}$ at resonant frequency or $\measuredangle \boldsymbol{I_C}$ above resonant frequency? Note: In phase, consider the positive or negative signs.

. . . .

14. Consider the inductor current amplitude $|\boldsymbol{I}_L|$. Which one is higher $|\boldsymbol{I}_L|$ at resonant frequency or $|\boldsymbol{I}_L|$ above resonant frequency?

. . . .

15. Using the values in Table 4, evaluate the current $\boldsymbol{I}$ and total admittance $\boldsymbol{Y}$ of the parallel *RLC* circuit using Ohm's law, and record them in Table 5. Remember that the total admittance $\boldsymbol{Y}$ is a complex number, so you need to find both its magnitude and phase.

Table 5

AC Source Frequency (Hz)	$\boldsymbol{I}$ (peak) (mA) $= \boldsymbol{V}_{R'}/R'$		$\measuredangle\boldsymbol{I}$ with $\boldsymbol{V}_s$ ($= \measuredangle\boldsymbol{V}_{R'}$ with $\boldsymbol{V}_s$)		$\lvert\boldsymbol{Y}\rvert = \lvert\boldsymbol{I}\rvert/\lvert\boldsymbol{V}_S\rvert$ (peak/peak) (mS)		$\measuredangle\boldsymbol{Y} = \measuredangle\boldsymbol{I} - \measuredangle\boldsymbol{V}_S$ (degrees)	
	Theory	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**
6000								
9000								
14400								
16800								
30000								
48000								
$f_r =$ ______								
$f_1 =$ ______								
$f_2 =$ ______								

16. Using the *measured* values in Table 5, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $|\boldsymbol{Y}|$ versus source frequency; (2) $\measuredangle\boldsymbol{Y}$ versus source frequency; (3) $\boldsymbol{I}$ versus source frequency.

17. For the above plots, state your conclusions under the plot? Also identify the resonant frequency and bandwidth in each plot.

18. Using the *measured* values of $\boldsymbol{V}_S$ and $\boldsymbol{I}$ in Table 5, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 6 below. Also find the power factor (PF) and state whether it is leading or lagging.

19. Using the values in Table 6, plot (**by hand**) the following figure using the graph paper attached at the end of the report: $\boldsymbol{P}$ and $\boldsymbol{Q}$ on the same plot versus source frequency.

20. For the above plot, state your conclusions under the plot?

Table 6

AC Source Frequency (Hz)	$\lvert S \rvert$ (mVA)	$\measuredangle S$ (degrees)	P (mW)	Q (mVAR)	PF value	PF lead or lag
	Measured	Measured	Measured	Measured	Measured	Measured
6000						
9000						
14400						
16800						
30000						
48000						
$f_r =$ ______						
$f_1 =$ ______						
$f_2 =$ ______						

21. At what frequency the real power $\boldsymbol{P}$ is minimum? Why?

. . . .

22. At what frequency the magnitude of the reactive power $\lvert \boldsymbol{Q} \rvert$ is maximum? Why?

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**** End ****