University of Jordan
School of Engineering
Electrical Engineering Department

EE 204
Electrical Engineering Lab

EXPERIMENT 5
CAPACITIVE REACTANCE

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CAPACITIVE REACTANCE

OBJECTIVE

Capacitive reactance will be examined in this experiment. In particular, its relationship to the AC source frequency will be investigated, including a plot of capacitive reactance versus frequency. In addition, AC power and power factor calculations will be introduced.

DISCUSSION

Impedance, Reactance, Admittance and Susceptance

The AC current-voltage characteristic of a capacitor, known as capacitive *reactance* X_C, is inversely proportional to frequency. Hence, the *impedance* of a capacitor (in units of Ohm) is:

$$\boldsymbol{Z_C} = jX_C = -j\frac{1}{\omega C}$$

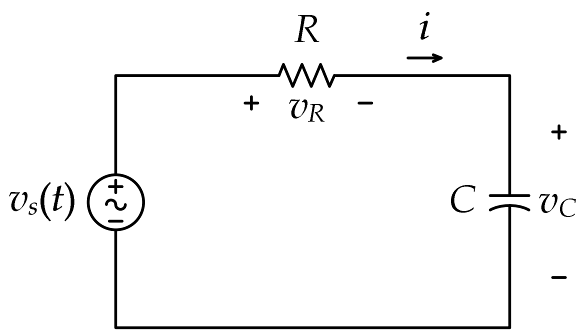
The inverse of the impedance is *admittance* $\boldsymbol{Y} = 1/\boldsymbol{Z} = G + jB$, which has the real part G called *conductance*, and the imaginary part B called *susceptance*. All three quantities have units of Siemens (S).

The capacitive reactance may be determined experimentally by applying a known AC voltage across the capacitor, measuring the resulting current, and dividing the two. This process may be repeated across a range of frequencies in order to obtain a plot of capacitive reactance versus frequency.

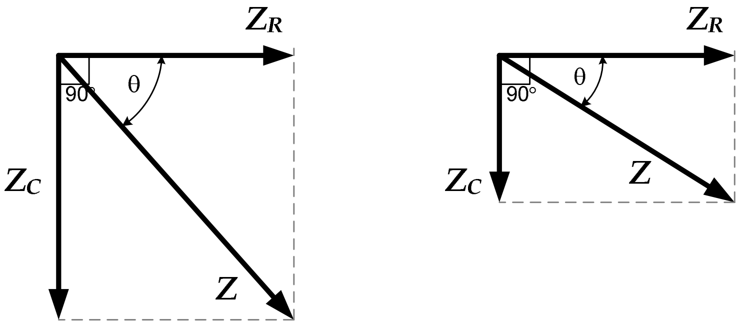
AC-excited series RC circuit

For the series RC circuit shown below, the total impedance seen by the AC source is given by:

$$\boldsymbol{Z} = \boldsymbol{Z_R} + \boldsymbol{Z_C} = R - j\frac{1}{\omega C} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \measuredangle - \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

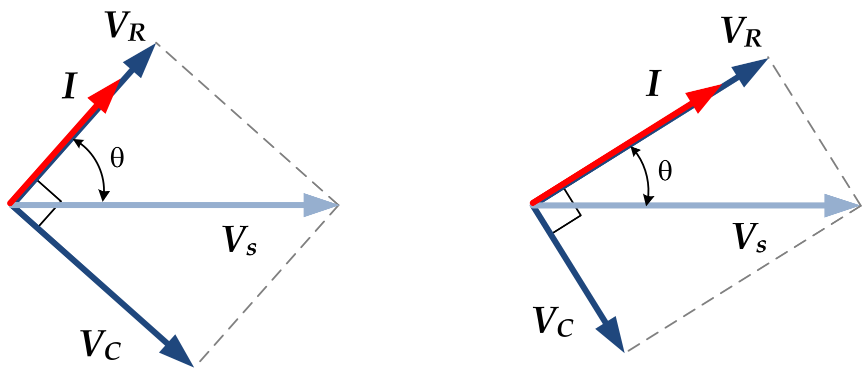


Since the capacitor impedance $\boldsymbol{Z_C}$ changes with the frequency of the source, the total impedance $\boldsymbol{Z}$ changes with frequency as well, as shown in the following phasor diagram. Notice how the resistor impedance remains constant with frequency.



f_1 $\qquad$ $f_2 > f_1$

The following phasor diagram shows how the above impedance change affects the capacitor voltage $\boldsymbol{V_C}$ and resistor voltage $\boldsymbol{V_R}$, both of which now change with frequency for a constant $\boldsymbol{V_S}$ due to the voltage divider rule. The current $\boldsymbol{I}$ also changes along with its phase shift compared to the source voltage $\boldsymbol{V_S}$, but the current remains leading compared to the source voltage. Notice that complex numbers are added like vectors, not like scalars.

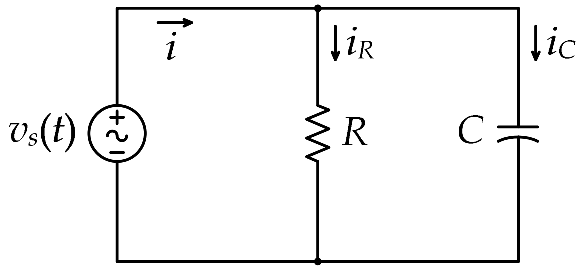


f_1 $\qquad$ $f_2 > f_1$

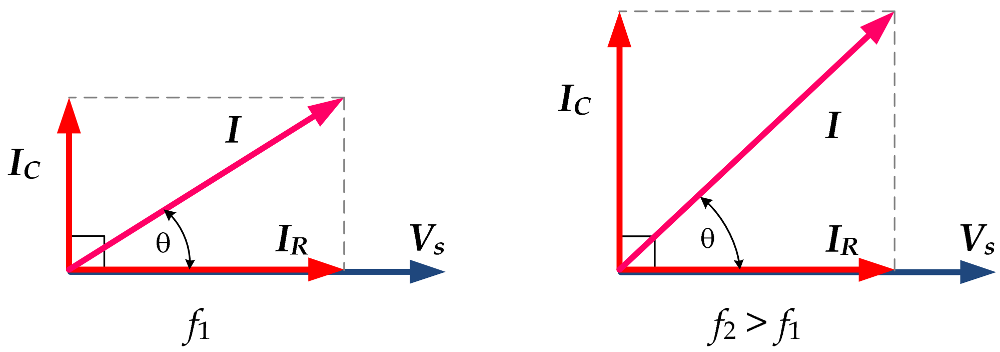
AC-excited parallel RC circuit

For the parallel RC circuit shown below, the total admittance seen by the AC source is given by:

$$\boldsymbol{Y} = \boldsymbol{Y_R} + \boldsymbol{Y_C} = \frac{1}{R} + j\omega C = \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \measuredangle \tan^{-1}(\omega RC)$$



The following phasor diagram shows how the capacitive admittance change with frequency affects the capacitor current $\boldsymbol{I_C}$, which increases with increasing frequency for a constant source voltage $\boldsymbol{V_S}$. The resistance current $\boldsymbol{I_R}$, however, stays constant, which means that the total current $\boldsymbol{I}$ changes (due to phasor addition) along with its phase shift compared to the source voltage $\boldsymbol{V_S}$. Of course, the current leads the source voltage due to the capacitive load.



Power and Power Factor

The average *complex power* $\boldsymbol{S}$ (units of VA) is given by the combination of the average *real power* P and the average *reactive power* Q as follows:

$$\boldsymbol{S} = P + jQ = V_{rms}I_{rms}\measuredangle(\measuredangle V - \measuredangle I) = \frac{1}{2}V_pI_p\measuredangle(\measuredangle V - \measuredangle I)$$

Hence, the real power P (units of W) is given by:

$$P = V_{rms}I_{rms}\cos(\measuredangle V - \measuredangle I) = \frac{1}{2}V_pI_p\cos(\measuredangle V - \measuredangle I)$$

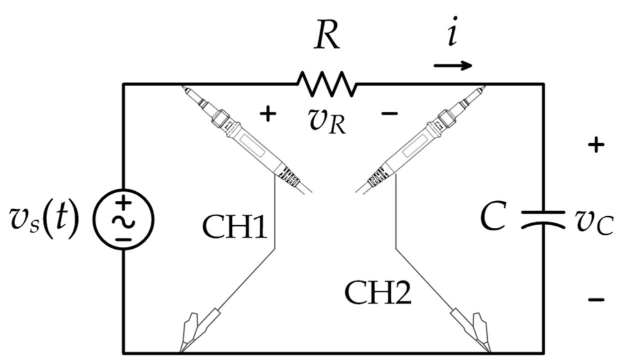
And the reactive power Q (units of VAR) is given by:

$$Q = V_{rms}I_{rms}\sin(\measuredangle V - \measuredangle I) = \frac{1}{2}V_pI_p\sin(\measuredangle V - \measuredangle I)$$

where the $\cos(\measuredangle V - \measuredangle I)$ quantity is known as the *power factor* (PF), which is lagging for inductive loads, and leading for capacitive loads. Finally the *apparent power* is given by $|\boldsymbol{S}| = V_{rms}I_{rms} = 0.5 \times V_p \times I_p$.

<u>PROCEDURE A – AC-EXCITED SERIES RC CIRCUIT</u>

1. Construct the circuit shown below. Assume that R = 3300 Ω, C = 0.1 μF.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 50 Hz, and *peak voltage* of $V_p = 5$ V.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift. Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the voltages $\boldsymbol{V_C}$ and $\boldsymbol{V_R}$ and current $\boldsymbol{I}$ in the circuit at the different frequencies in Tables 1 and 2. Make sure to evaluate both magnitude and phase for each complex quantity, and record the expected period T of the signals in milliseconds. Record the answers in the two tables? **Note**: Using MATLAB can quickly give you the theoretical answers if you define a vector of frequencies and then use array arithmetic.

4. Use the oscilloscope to measure the peak values of the voltages $\boldsymbol{V_S}$ and $\boldsymbol{V_C}$ and the phase shift of $\boldsymbol{V_C}$ compared to $\boldsymbol{V_S}$**.** Remember that you can change the horizontal sweep setting of the oscilloscope to make more accurate measurements of the phase. Also measure the period T in milliseconds of the source voltage $\boldsymbol{V_S}$. Record the measurements in Table 1?

CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. Use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_R}$ and its phase shift compared to $\boldsymbol{V_S}$**.** Also measure the period T in milliseconds of $\boldsymbol{V_R}$. Record the measurements in Table 2.

CAUTION: Do *not* put CH2 of the oscilloscope across the resistor R while simultaneously measuring $\boldsymbol{V_S}$ using CH1 since this will short circuit the capacitor. Rather, swap the locations of C and R in the circuit while keeping the oscilloscope connections unchanged, allowing you to measure $\boldsymbol{V_R}$.

Table 1

AC Source Frequency (Hz)	V_S (peak) (V)		V_S period T (ms)		V_C (peak) (V)		$\measuredangle V_C$ with V_s (Lag = negative)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
50	5							
100	5							
200	5							
300	5							
500	5							
700	5							
1100	5							
2000	5							

Table 2

AC Source Frequency (Hz)	V_R (peak) (V)		$\measuredangle V_R$ with V_s (Lead = positive)		V_R period T (ms)		I and $\measuredangle I$ (mA) $= V_R/R$	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
50								
100								
200								
300								
500								
700								
1100								
2000								

6. Can we just subtract the magnitudes of $|\boldsymbol{V_S}| - |\boldsymbol{V_C}|$ to obtain the magnitude $|\boldsymbol{V_R}|$? Why or why not?

. . . .

7. What is the relationship between the periods T of the two signals $\boldsymbol{V_S}$ and $\boldsymbol{V_R}$?

. . . .

8. Now evaluate the *measured* current $\boldsymbol{I}$ and its phase shift compared to $\boldsymbol{V_S}$ by applying Ohm's law to the resistor using the measured value of $\boldsymbol{V_R}$ (i.e., $\boldsymbol{I} = \boldsymbol{V_R}/R$). Record the answer in Table 2.

9. Using the values in Tables 1 and 2, evaluate the reactance of the capacitor $\boldsymbol{X_C}$ and the total impedance $\boldsymbol{Z}$ of the series R and C components, and record them in Table 3. Remember that the total impedance $\boldsymbol{Z}$ is a complex number, so you need to find both its magnitude and phase.

Table 3

AC Source Frequency (Hz)	$X_C = \lvert V_C\rvert/\lvert I\rvert$ (peak/peak) (kΩ)		$\lvert Z\rvert = \lvert V_S\rvert/\lvert I\rvert$ (peak/peak) (kΩ)		$\measuredangle Z = \measuredangle V_S - \measuredangle I$ (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.
50						
100						
200						
300						
500						
700						
1100						
2000						

10. Using the *measured* values in Table 3, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $\boldsymbol{X_C}$ and $|\boldsymbol{Z}|$ on the same plot versus source frequency; (2) $\measuredangle\boldsymbol{Z}$ versus source frequency; (3) $\boldsymbol{V_C}$ and $\boldsymbol{V_R}$ on the same plot versus source frequency.

11. For the above plots, state your conclusions under the plot?

12. Using the *measured* values of $\boldsymbol{V_S}$ and $\boldsymbol{I}$ in Tables 1 and 2, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 4 below. Also find the power factor (PF) and state whether it is leading or lagging.

Table 4

AC Source Frequency (Hz)	$\|S\|$ (mVA)	$\measuredangle S$ (degrees)	P (mW)	Q (mVAR)	PF value	PF lead or lag
	Measured	**Measured**	**Measured**	**Measured**	**Measured**	**Measured**
50						
100						
200						
300						
500						
700						
1100						
2000						

13. Using the values in Table 4, plot (**by hand**) the following figure using the graph paper attached at the end of the report: $\boldsymbol{P}$ and $\boldsymbol{Q}$ on the same plot versus source frequency.

14. For the above plot, state your conclusions under the plot?

15. At what frequency the real power $\boldsymbol{P}$ is maximum? Why?

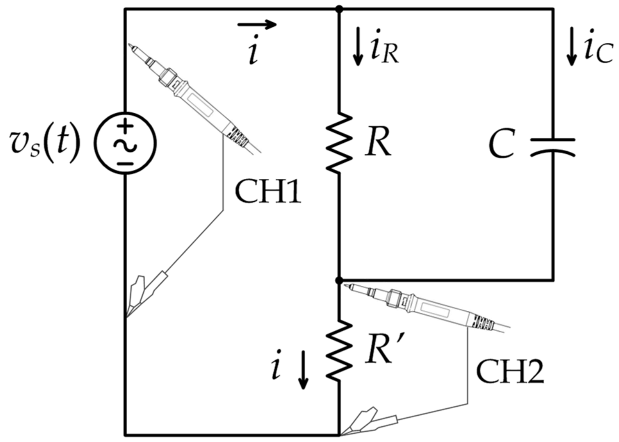
. . . .

16. At what frequency the magnitude of the reactive power $|\boldsymbol{Q}|$ is maximum? Why?

. . . .

PROCEDURE B – AC-EXCITED PARALLEL RC CIRCUIT

1. Construct the circuit shown below. Assume that R = 1000 Ω, C = 0.1 μF, and R' = 10 Ω. Make sure you use the correct resistor values.



2. Set the function generator to produce a sinusoidal waveform (AC) with frequency of 160 Hz, and *peak voltage* of $V_p = 5$ V.

CAUTION: Some older function generators have a defect and produce an AC signal with a slight DC shift. Hence, if you do not see a symmetric sinusoidal signal above and below zero volts, adjust the DC offset knob slightly to force a zero DC offset in the function generator output.

3. Use theoretical analysis to determine the currents $\boldsymbol{I_C}$, $\boldsymbol{I_R}$ and $\boldsymbol{I}$ in the circuit at the different frequencies in Tables 5, 6 and 7. Make sure to evaluate both magnitude and phase for each complex quantity. Record the answers in these tables? The small resistor R' was placed as a convenient way to measure the current $\boldsymbol{I}$ using the oscilloscope. It has a small resistance compared to other resistors in the circuit, which means you can neglect it in theoretical calculations.

4. Now use the oscilloscope to measure the peak value of the voltage $\boldsymbol{V_{R'}}$ and its phase shift compared to $\boldsymbol{V_S}$. If you want, change the horizontal sweep settings of the oscilloscope to make more accurate measurements of the phase. Record the measurements in Table 5?

CAUTION: Whenever you change the frequency of the function generator, verify the period of the signal from the oscilloscope to get accurate readings. Also re-check the peak-to-peak voltage as the function generator might change the amplitude when you change the frequency.

5. As you change the frequency of the source, use the oscilloscope to also measure the peak value of the voltage $\boldsymbol{V_S} \approx \boldsymbol{V_R}$ and record its phase shift as 0° compared to $\boldsymbol{V_S}$. Record the measurements in Table 5.

6. Use Ohm's law on R' to evaluate the current $\boldsymbol{I}$ from $\boldsymbol{V_{R'}}$, and Ohm's law on R to evaluate the current $\boldsymbol{I_R}$ from $\boldsymbol{V_R}$. Record the results in Table 6.

Table 5

AC Source Frequency (Hz)	$V_{R'}$ (peak) (V)		$\measuredangle V_{R'}$ with V_s (Lead = positive)		$V_R = V_C \approx V_S$ (peak) (V)		$\measuredangle V_R$ with V_s (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
160					5		0°	0°
320					5		0°	0°
800					5		0°	0°
1100					5		0°	0°
1600					5		0°	0°
2200					5		0°	0°
3500					5		0°	0°
6500					5		0°	0°

Table 6

AC Source Frequency (Hz)	I (peak) (mA) $= V_{R'}/R'$		$\measuredangle I$ with V_s (Lead = positive)		I_R (peak) (mA) $= V_R/R$		$\measuredangle I_R$ with V_s (degrees)	
	Theory	Meas.	Theory	Meas.	Theory	Meas.	Theory	Meas.
160							0°	0°
320							0°	0°
800							0°	0°
1100							0°	0°
1600							0°	0°
2200							0°	0°
3500							0°	0°
6500							0°	0°

Table 7

AC Source Frequency (Hz)	$I_C = I - I_R$ (mA) (magnitude (peak) and phase (degrees)) (phasor subtraction)	
	Theory	Measured
160		
320		
800		
1100		
1600		
2200		
3500		
6500		

7. In Table 7, use phasor subtraction to find $\boldsymbol{I_C} = \boldsymbol{I} - \boldsymbol{I_R}$ from the measured values in previous tables.

8. Using the values in Tables 5, 6 and 7, evaluate the susceptance of the capacitor $\boldsymbol{B_C}$ and the total admittance $\boldsymbol{Y}$ of the parallel R and C components, and record them in Table 8. Remember that the total admittance $\boldsymbol{Y}$ is a complex number, so you need to find both its magnitude and phase.

Table 8

AC Source Frequency (Hz)	$B_C = \lvert I_C \rvert / \lvert V_C \rvert$ (peak/peak) (mS)		$\lvert Y \rvert = \lvert I \rvert / \lvert V_S \rvert$ (peak/peak) (mS)		$\measuredangle Y = \measuredangle I - \measuredangle V_S$ (degrees)	
	Theory	**Meas.**	**Theory**	**Meas.**	**Theory**	**Meas.**
160						
320						
800						
1100						
1600						
2200						
3500						
6500						

9. Using the *measured* values in Table 8, plot (**by hand**) the following figures using the graph paper attached at the end of the report: (1) $\boldsymbol{B_C}$ and $|\boldsymbol{Y}|$ on the same plot versus source frequency; (2) $\measuredangle \boldsymbol{Y}$ versus source frequency; (3) $\boldsymbol{I_C}$ and $\boldsymbol{I_R}$ on the same plot versus source frequency.

10. For the above plots, state your conclusions under the plot?

11. Using the *measured* values of $\boldsymbol{V_S} = \boldsymbol{V_R}$ and $\boldsymbol{I}$ in Tables 5 and 6, evaluate the apparent power, real power, and reactive power generated by the source (function generator) and record them in Table 9 below. Also find the power factor (PF) and state whether it is leading or lagging.

Table 9

AC Source Frequency (Hz)	$\lvert S \rvert$ (mVA)	$\measuredangle S$ (degrees)	P (mW)	Q (mVAR)	PF value	PF lead or lag
	Measured	**Measured**	**Measured**	**Measured**	**Measured**	**Measured**
160						
320						
800						
1100						
1600						
2200						
3500						
6500						

12. Using the values in Table 9, plot (**by hand**) the following figure using the graph paper attached at the end of the report: $\boldsymbol{P}$ and $\boldsymbol{Q}$ on the same plot versus source frequency.

13. For the above plot, state your conclusions under the plot?

14. At what frequency the real power $\boldsymbol{P}$ is maximum? Why?

. . . .

15. At what frequency the magnitude of the reactive power $|\boldsymbol{Q}|$ is maximum? Why?

. . . .

**** End ****