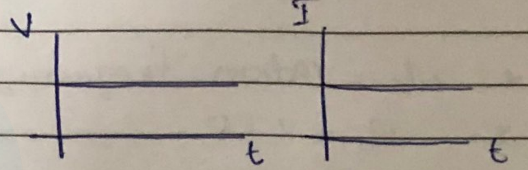
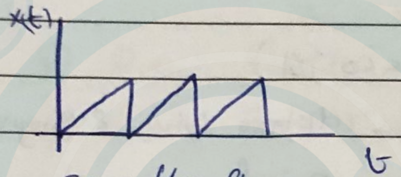
20/7/2020

Time dependent signal sources

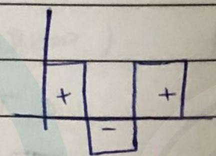
DC sources → fixed value



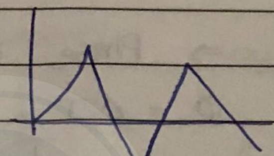
AC-sources → generates voltages & currents varrying with time



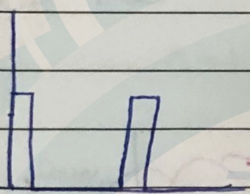
saw thooth



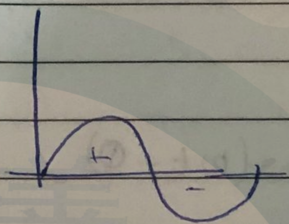
square signal



triangle signal



Pulse train signal



Sine wave signal

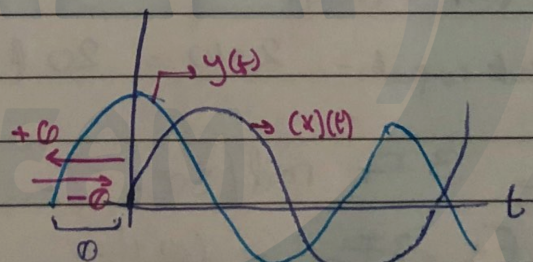
* Periodic signal: its a signal which repeats itself after one periode (T)

* sinusodial signals

$X(t) = A\cos(\omega t + \theta)$

$y(t) = B\cos\omega(t)$

* كل شي sin نحولها cos



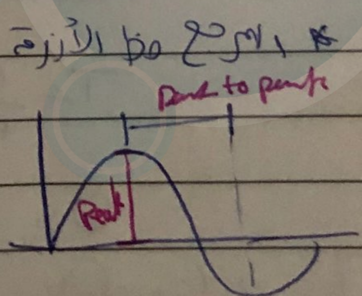
* المرجع هو الأزرق ، الهادئ

Amplitude of signal → peak value

Peak → قمة للصفر (P)

Peak to peak → ~~[illegible]~~

2P = قمة وقاع



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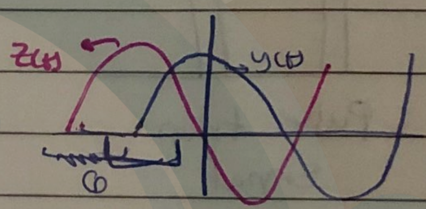
* ω = radian frequency ⇒ unit: rad/sec
* $\omega = 2\pi F$

* f: Natural Freq → unit: HZ
* Period ⇒ $T = \frac{1}{f}$ sec   كل شو بتعيد نفسها

* θ ⇒ Phase shift   (cos θ بس مش الزاوية)
* $\theta = \frac{\Delta T}{T} * 360°$ ΔT → distance btw 2 signals in second
  T → period

$\boxed{* \sin \omega t = \cos(\omega t - 90°)}$ دايماً نحولها cos

$Z(t) = \cos(\omega t + \theta)$



ex $i(t) = 10 \cos(377 t) + 30°)$

* moved to left ×30 بالزاوية

$I_{peak} = 10$ A

$I_{peak\ to\ peak} = 2 * 10 = 20$ A

$\omega = 377$ rad/sec $= 2\pi f$

$f = \frac{377}{2\pi} = 60$ Hz

$T = \frac{1}{f} = .016$ sec

$\theta = 30°$

$i(.1\ sec) = 10 \cos(377 * .1 + 30) = 7.8$A

Complex

20/7/2022

* complex number

$i / j = \sqrt{-1}$

$j^2 = -1$

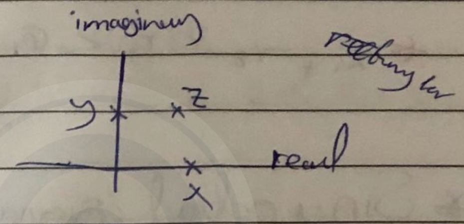
complex number has 2 form

① rectangular form

$Z = x + jy$

② Polar form

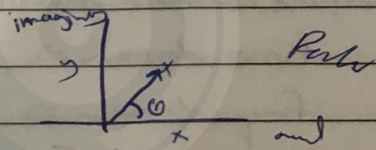
$Z = r \angle \theta$



* conversion from rect to polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1} \frac{y}{x}$ → ٤+١ ربع نقطة

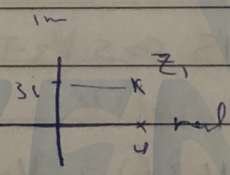


* conversion from polar to rect

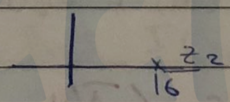
$x = r \cos \theta$

$y = r \sin \theta$

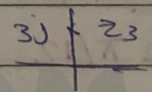
$\boxed{ex}$ $Z_1 = 4 + 3j$



$Z_2 = 16$



$Z_3 = 3j$



* Mathmatical op.

1 addition $X_1 = a + jb$ $X_2 = c + jd$

$X_1 + X_2 = a + c + j(b + d)$

2 subtraction

$X_1 - X_2 = a - c + j(b - d)$

3 Multiplication

$X_1 * X_2 = (a + jb) * (c + jd) = ac + jad + jcb - bd$

4 division

$\frac{X_1}{X_2} = \frac{a + jb}{c + jd} * \left(\frac{c - jd}{c - jd}\right) = \frac{ac + bd}{c^2 - d^2} + j \frac{bc - ad}{c^2 - d^2}$

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* Polar

$X_1 = r_1 \angle \theta_1$

$X_2 = r_2 \angle \theta_2$

* $X_1 * X_2 = r_1 \angle \theta_1 * r_2 \angle \theta_2 = r_1 r_2 \angle \theta_1 + \theta_2$

* $\frac{X_1}{X_2} = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$

* $X_1 \mp X_2 = r_1 \angle \theta_1 \mp r_2 \angle \theta_2$ → use calculator

* Sinuodial signals in freq domain

time domain ⇒ $X(t) = A \cos(\omega t + \theta)$

freq domain ⇒ $X(j\omega) = A \angle \theta$ → phase shift refrenced to cosine signal

↳ peak value

* الزاوية لازم تكون بدلالة cos

ex $v_1(t) = 15 \cos(377t + \frac{\pi}{4})$ v

$v_2(t) = 15 \cos(377t + \frac{\pi}{12})$ v

find $V_s(t) = V_1(t) + V_2(t)$

Soln:

$V_1(j\omega) = 15 \angle \pi/4 = 10.61 + j * 10.61$ v

$V_2(j\omega) = 15 \angle \pi/12 = 14.49 + j\ 3.88$ v

$V_{(s)}(j\omega) = V_1(jw) + V_2(jw)$

$= 25.1 + j\ 14.49$

$= 28.98 \angle \pi/6$

$V_s(t) = 28.98 \cos(377t + \frac{\pi}{6})$

* نفس 377 للإثنين *

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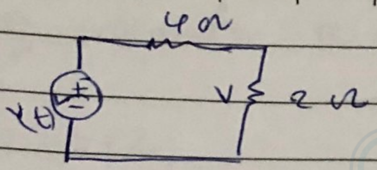
ex



$v(t) = 12 \cos(377t + 40^\circ)$

find $i(t)$ + $V_2(t)$

$\bar{V}_{J\omega} = 12 \angle 40$

$V_2(J\omega) = \dfrac{(12\angle 40) * 2}{2+4} = 4\angle 40$

$V_2(t) = 4 \cos(377t + 40)\,V$

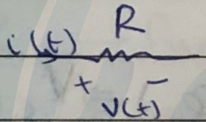
$I(J\omega) = \dfrac{V_2(J\omega)}{2} = \dfrac{4}{2}\angle 40 = 2\angle 40\ A$

$i(t) = 2\cos(377t + 40^\circ)\ A$

* Impedence:

→ we will describe the circuit elements RLC in AC circuit ⇒ they will have impedence

① Resistor



if $V(t) = V_{amp} \cos(\omega t + \theta)$

$\bar{V}(J\omega) = V_m \angle \theta$

$V = IR$

to find $I(J\omega) = \dfrac{V(J\omega)}{R} = \dfrac{V_m\angle\theta}{R} = \dfrac{V_m}{R}\angle\theta$

$i(t) = \dfrac{V_m}{R}\cos(\omega t + \theta)$

$Z(J\omega) = \dfrac{V(J\omega)}{I(J\omega)} = \dfrac{V_m\angle\theta}{\frac{V_m}{R}\angle\theta} = R\angle\theta$ zero

$\boxed{Z_R(J\omega) = R}\ \Omega$

no Phase shift

* inductor

$$v(t) = L \frac{di(t)}{dt}$$

if $i_L(t) = I_m \cos(\omega t + \theta)$

$$I_L(j\omega) = I_m \angle \theta$$

$$V(t) = I_m(\omega L)(-\sin(\omega t + \theta))$$

$$= I_m(\omega L)\cos(\omega t + \theta + 90)$$

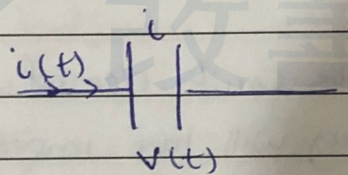
$$V(j\omega) = I_m(\omega L)\angle \theta + 90^\circ$$

$$Z_L(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{I_m(\omega L)\angle \theta + 90}{I_m \angle \theta}$$

$$Z_L(j\omega) = \omega L \angle(\theta + 90 - \theta)$$

$$Z_L(j\omega) = (\omega L \angle 90) = (j\omega L)\,\Omega$$

* capacitor



$$i_c(t) = C \frac{dv_c(t)}{dt}$$

if $V_c(t) = V_m \cos(\omega t + \theta) \Rightarrow V_c(j\omega) = V_m \angle \theta$

so →

$$i_c(t) = V_m(C\omega)(-\sin(\omega t + \theta))$$

$$= (C\omega)V_m \cos(\omega t + \theta + 90)$$

$$I_c(j\omega) = C\omega\, V_m \angle \theta + 90^\circ$$

$$Z_c(j\omega) = \frac{V_c(j\omega)}{I_c(j\omega)} = \frac{V_m \angle \theta}{C\omega(V_m)\angle \theta + 90}$$

$$\frac{1}{C\omega}\angle \theta - \theta - 90$$

$$Z_c(j\omega) = \frac{1}{C\Omega}\angle -90 = \frac{-j}{\omega C}\ \Omega$$

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* Using calculator to solve complex num.

By Calc

$x = 3+2j$

$y = 5+j$

$\Rightarrow x * y = (3+2j)(5+j) \Rightarrow$ mode $\to 2 \to$

$= (13+13j) \to$ rec

$= 18.38 \angle 45$ mode $\to 2 \to$ shift $\to 3$

$\Rightarrow x \div y = \frac{17}{26} + \frac{7}{26} j$

$= .707 \angle 22.4$

rec $\leftarrow$ Polar عشان أحول من

$\hookrightarrow$ shift $\to$ 4

$\Rightarrow 3 \angle 20 + 4 \angle -15 = 6.682 - 9.21 \times 10^{3} j$

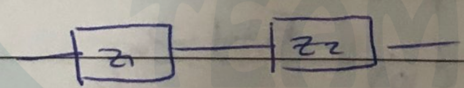
$= 6.68 \angle -0.08^\circ$

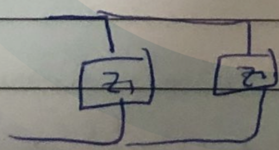
[* Review

$Z_R(j\omega) = R \to$ fixed with freq variation

$Z_L(j\omega) = j\omega L = \omega L \angle 90$

$Z_C(j\omega) = \frac{1}{C\omega} * -j = \frac{1}{\omega C} \angle -90$

* series  $= Z_1 + Z_2$

* Parallel  $= \frac{1}{Z_1} + \frac{1}{Z_2}$

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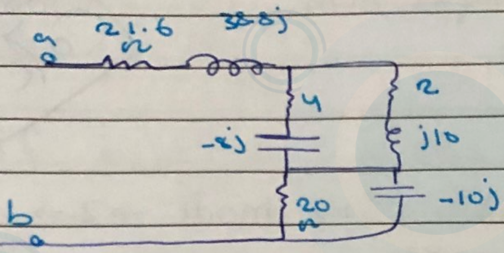
(ex)



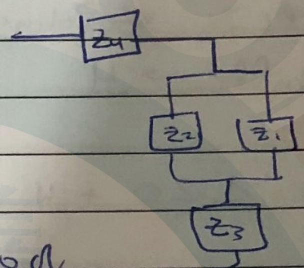
find Z_{eq} btw a & b

2 ∥ j10 → series $Z_1 = 2 + j10$

4 ∥ -8j → series $Z_2 = 4 - j8$

-10j ∥ 20 → Parallel $Z_3 = \frac{1}{20} + \frac{1}{-j8} = 4 - j8$

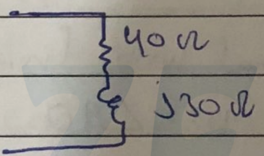
21.6 ∥ (38.8j) → series $Z_4 = 21.6 + 38.8j$



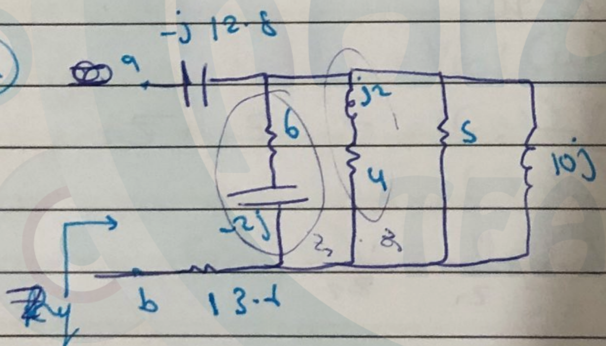
$Z_5 = \frac{1}{Z_2} + \frac{1}{Z_1}$ ⇒ Parallel

$Z_5 = 14.4 - .8j$

$Z_{eq} = Z_4 + Z_3 + Z_5 = 40 + j30\,\Omega$



(ex)

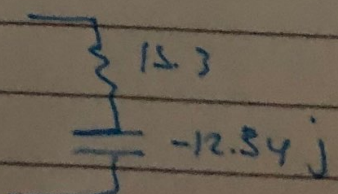


Find Z_{eq}

$Z_1 = 4 + 2j$ $Z_2 = 6 - 2j$

$\frac{1}{Z_q} = \frac{1}{4+2i} + \frac{1}{6-2j} + \frac{1}{5} + \frac{1}{10j} \longrightarrow Z_q = \frac{22}{13} + \frac{6}{13} i$

الجواب → $15.3 - 12.34j$ →



21/7/2020

AC - analysis steps

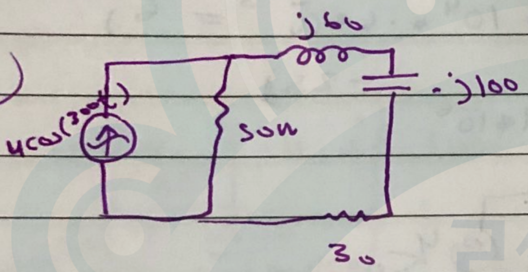
1 note the frequency of the Sinusodial excitation
2 convert all source to phasor form (freq domain)
3 convert each circuit elements to impedance form

$Z_R = R \qquad Z_L = \omega L j \qquad Z_C = \frac{-j}{C\omega}$

4 Solve the resulting phasor circuit using any method (mesh Nodal, superposition, S.T, S.P, Norton, thevenin, KCl, KVL ohm, divider)

(5) convert to time domain

(ex)



Find $V_c(t)$

Soln

(1) $\omega = 300$ rad/sec

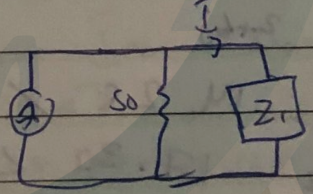
(2) j 60

-j100

(2) $4\cos(300t) = 4\angle 0^\circ \ A = I_s$

(3) $Z_1 = 30 + 60j - 100j$

$Z_1 = 30 - 40j \ \Omega$



using current division

$$I = \frac{4\angle 0 * \frac{1}{Z_1}}{\frac{1}{Z_1} + \frac{1}{50}} = 2 + j$$

$V_{c \text{ (phasor)}} = I * -j100 = 223.6 \angle -63.4^\circ$ volt

$V_c(t) = 223.6 \cos(300t - 63.4^\circ)$ volts

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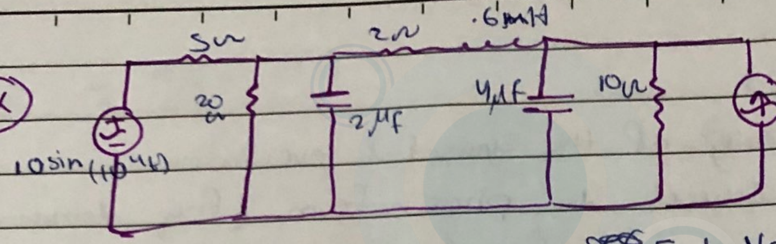
ex



Find Value of nodal voltages

① $10^4 = \omega$ rad/sec

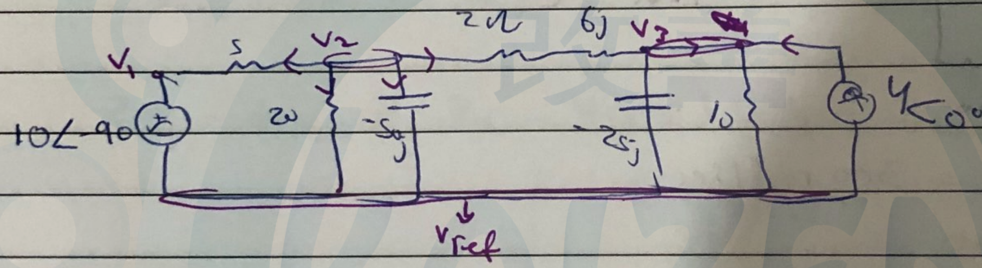
② $10\sin(10^4 t) = 10\cos(10^4 t - 90°) = 10\angle -90$

$4\cos 10^4 t = 4\angle 0°$

③ $2\mu f \to \frac{-j}{C\omega} \to \frac{-j}{2 * 10^4 * 10^{-6}} = -50j = Z_c$

$.6\ mH \Rightarrow Z_L = j * 10^4 * .6 * 10^{-3} = 6j$

$4\mu f = Z_c = \frac{-j}{10^4 * 4 * 10^{-6}} = -25j$



@ V_1 $V_1 = 10\angle -90$

@ V_2 $\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_2}{-50j} + \frac{V_2 - V_3}{2+6j} = 0$

@ V_3 $\frac{V_2 - V_3}{2+6j} + 4\angle 0° = \frac{V_3}{-25j} + \frac{V_3}{10}$

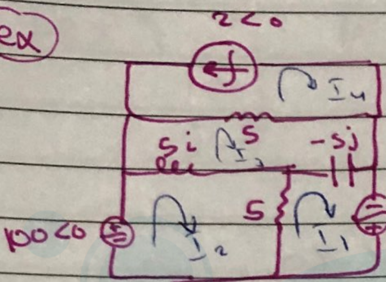
$V_1 = 14.28\angle -54.8$ volt

$V_2 = 17.33\angle 1.99$ volt

$V_1 = 14.28\cos(10^4 t - 54.8°)$ Ⓥ

$V_2 = 17.83\cos(10^4 t + 1.99°)$ Ⓥ

ex



write mesh equ

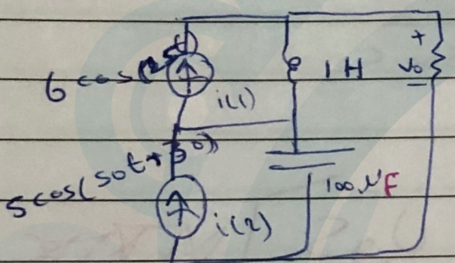
Sol $I_4 = -2\angle 0$ A

@ I_1 → $-50\angle 0 + 5I_1 - 5jI_1 - 5I_2 - -5jI_3 = 0$

@ I_2 → $5I_1 + j5I_1 - 100\angle 0 - 5I_1 - 5iI_3 = 0$

@ I_3 → $5I_3 + 5iI_3 - 5jI_3 + 5jI_1 + 5iI_2 - 5I_4 = 0$

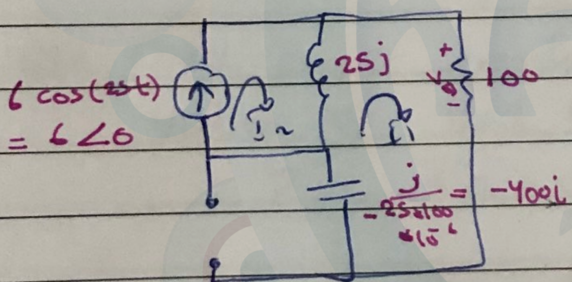
ex



find vo

* Note if the sources didn't have same radian freq (w) → use only S. Position *

(1) leave i_1



$I_2 = 6\angle 0$

@ I_1 → $100I_1 - 400jI_1 + 25jI_1 - 25j * I_2 = 0$

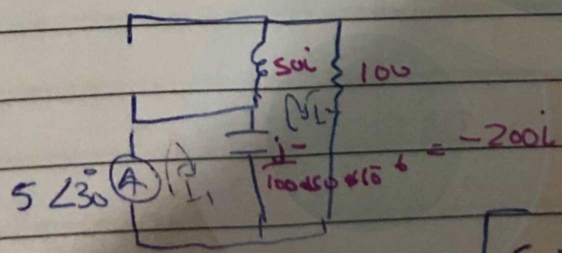
I_2 [illegible]

$V_o' = 100 * I_1$

$= 38.65 \angle 165$ volt

w = 25

leave i_2



$I_1 = 5\angle 30$

$(50i + 100 - 200i)I_1 + 200iI_2 = 0$

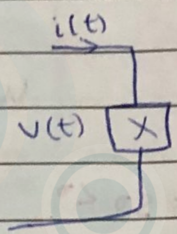
w = 50

$V_o'' = I_2 * 100 = 554.7\angle -3.7°$ V

[$\Sigma V_o \neq V_o' + V_o''$ → due to diff w

$V_o = 36.65\cos(25t + 165°) + 554.7\cos(50t - 3.7)$

Ex Given:



$v(t) = 12\sin(500t+30)$

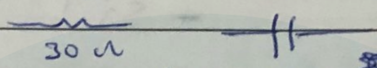
$i(t) = .2\cos(500t)$

Soln → $v(t) = 12\cos(500t-\cancel{60}) = 12\angle -60$

$I(t) = .2\angle 0$

$X = \dfrac{V}{I} = \dfrac{12\angle -60}{.2\angle 0} = 60\angle -60\ \Omega$

$X = 30 - 51.9j\ \Omega$



$X = 51.9$

$X_c = \dfrac{1}{\omega C} \rightarrow 51.9 = \dfrac{1}{500C}$

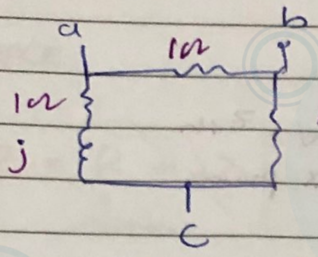
$\boxed{C = 38.5\ \mu F}$

[aj+b ـــــ حول كل شي الـ]

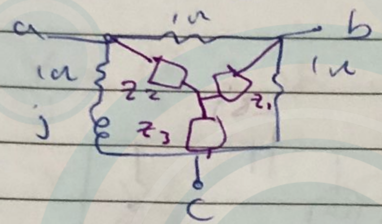
Y-Δ circuit

[$Z \rightarrow j \,/\, -j$

ex



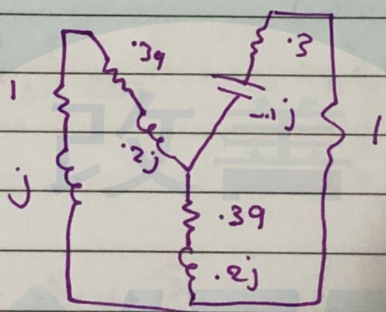
convert from Δ to Y



$$Z_1 = \frac{1 * 1}{1 + 1 + (1+j)} = \frac{1}{3+j} \cong .3 - .1j\,\Omega$$

$$Z_2 = \frac{1(1+j)}{3+j} \cong .39 + j.2\,\Omega$$

$$Z_3 = \frac{1 * (1+j)}{3+j} \cong .39 + .2j\,\Omega$$



max power transfer in AC circuit.

value of load to get max power transfer

$Z_L = Z^*_{th}$ → conjegate

ex: $Z_1 = 3 + 2j$

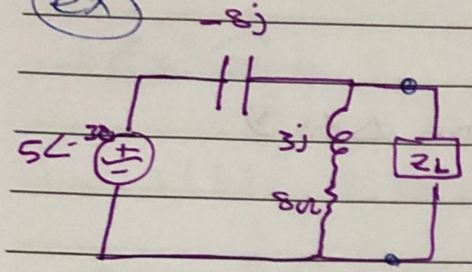
$Z_1^* = 3 - 2j$

* only change the sign of imaginary part

ex: $12\angle -60^\circ = Z_1$

$12\angle 60^\circ = Z_1^*$

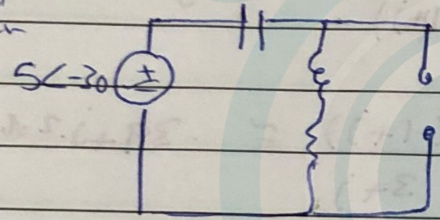
ex



Q
[Find V_{th}, I_N, Z_{th}
Z_L for max power

Solu →

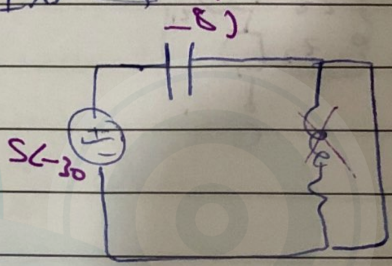
find V_{th}



$$V_{th} = \frac{(5\angle -30)(8+j8)}{8+8j-8j}$$

$$V_{th} = 7.07\angle 15^\circ$$

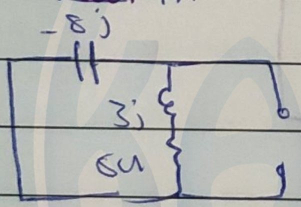
find I_N →



$$I_N = \frac{5\angle -30}{-8j}$$

$$I_N = .625\angle 60\ A$$

* find Z_{th}



$$Z_{th} = 8 - j8\ \Omega$$

* the value of $Z_{load} \Rightarrow P_{max}$

$$\boxed{Z_{load} = Z_{th}^* = 8 + j8\ \Omega}$$

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Admitance

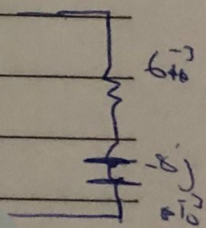
$$\underbrace{Z}_{\text{impedance}} = \underbrace{R}_{\text{resistors}} \pm jX \rightarrow \text{reactance } (X_L \text{ or } X_C)$$

$$\underbrace{Y}_{\text{admitance}} = \underbrace{G}_{\text{conductance}} \pm jB \rightarrow \text{suceptance}$$

$$Y = \frac{1}{Z} \qquad G \neq \frac{1}{R} \qquad \frac{1}{X} \neq B$$

$\boxed{\text{ex}}$ $Z = 100\angle 53^\circ$ Find Y

$$Y = \frac{1}{Z} = \frac{1}{100\angle 53} = \underbrace{6*10^{-3}}_{G} - \underbrace{j8*10^{-3}}_{B_L}$$



$$Z = 60.2 + j80$$

$$\frac{1}{60.2} = 16.6 \neq 6*10^{-3}$$

$$\frac{1}{80j} = 12.5 \neq 8*10^{-3}$$

$(1+i)^N i$

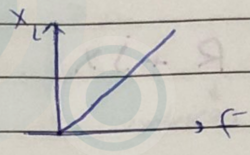
Notes

1. $Z_L(j\omega) = j\omega L = \omega L \angle 90$

$X_L = \omega L \rightarrow$ reactance

$\omega = 2\pi F$

$X_L = 2\pi F \cdot L$



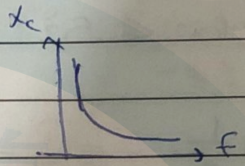
@ $F = 0 \rightarrow DC = 0$

$X_L = 0\ \Omega \rightarrow$ inductor acts as short circuit

@ $F = \infty$ $X_L = \infty\ \Omega \rightarrow$ open circuit

2. $Z_C(j\omega) = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90$

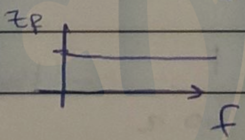
$X_C = \frac{1}{\omega C} \rightarrow$ reactance



@ $f = 0$ (DC) $\rightarrow X_C = \infty$ open circuit

@ $f = \infty$ (DC) $\rightarrow X_C = 0$ short circuit

3. $Z_R(j\omega) = R \rightarrow$ independent of freq



4. $Z = R \mp jX$

@ f $Z = R$ assume $V = V_m \angle \theta$ — Same angle

So $I = \frac{V_m \angle \theta}{R} = \frac{V_m}{R} \angle \theta$

So no phase shift btw voltage & current across the resistor

[Phase shift btw V & I = 0

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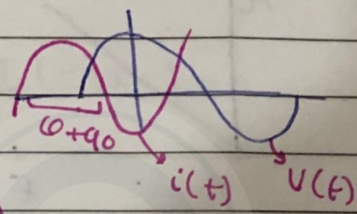
* if $Z = -jX_C \rightarrow$ pure capacitive.

assume $V = V_m \angle \theta$

so $I = \dfrac{V_m \angle \theta}{X_C \angle -90} = \dfrac{V_m}{X_C} \angle \theta + 90$

أول حالة رح تشوفها والعين هي i(t)

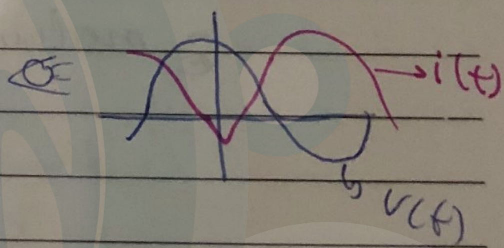


$\therefore$ current I leads voltage by 90 + voltage V lag the current by 90

* if $Z = jX_L \rightarrow$ pure inductive

assume $V = V_m \angle \theta$

$I = \dfrac{V_m \angle \theta}{X_L \angle 90} = \dfrac{V_m}{X_L} \angle \theta - 90$



$\therefore$ voltage V leads current by 90 + current I lag the voltage.

* if $Z = R + jX_L \rightarrow$ behaves like inductive

$0 < \theta < 90$

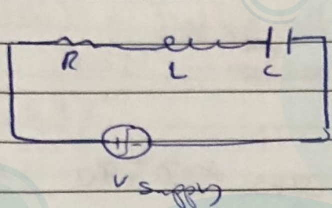
* if $Z = R - jX_C \rightarrow$ behaves like conductive

$0 > \theta > -90$

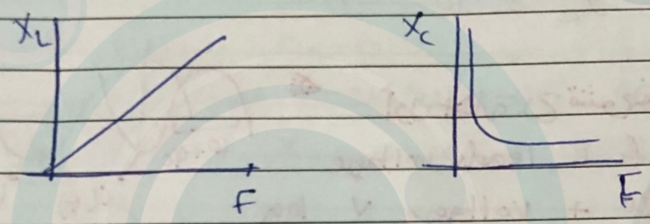
$(1+i)^N i$

26/7/2020 :)

Series RLC-Circuits



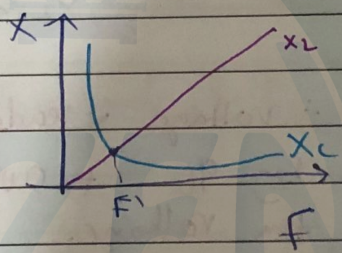
$Z_{eq} = R + j(X_L - X_C)$



@ low freq $X_C > X_L$ ⇒ assume circuit acts like capacitor (I leads) زي كأنها (L)

@ high freq $X_C < X_L$ ⇒ assume circuit acts like inductance

@ medium freq



@ f` $X_L = X_C$ → resonance frequency

@ resonant freq

$\frac{1}{WC} = WL \rightarrow \omega^2 = \frac{1}{LC} \rightarrow \omega_r = \frac{1}{\sqrt{LC}}$

resonant omega (resonant radian freq)

$W_r = \frac{1}{\sqrt{LC}} = 2\pi F_r$

$F_r = \frac{1}{\sqrt{LC} * 2\pi}$ ⇒ $Z_{eq} = R$ (Pure resistor)

$X_L = X_C$ → capacitor & inductor cancels each other

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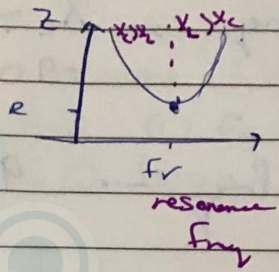
$Z_{eq} = R$ bec $X_L = X_C$



@ f_r by KVL

$$-V_{supply} + IR + I\,j\,X_L + I(-jX_C) = 0$$

@ f_r $\quad X_L = X_C$

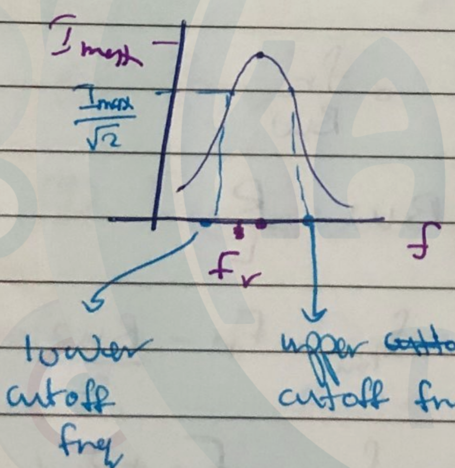
$$-V_{supply} + IR + 0 = 0$$

$$I_{max} = V_{supply} / R$$ Resonant خاصية

$$\vec{V}_C = -\vec{V}_L \rightarrow$$ متساوية عكس

out of phase

لما يلغوا بعض، فكل مصدر supply يروح على (R)

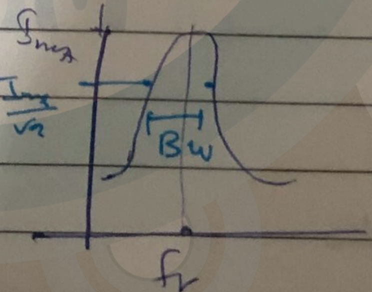


لما نشغل كالنص ال Power

$\hookrightarrow I_{max}/\sqrt{2}$

$$P = I^2 R$$

$$\frac{P}{2} = \frac{1}{2} I^2 R$$



$$\text{Band width} = f_H - f_L \quad \text{Hz}$$

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$\frac{(1+i)^N - 1}{(1+i)^N i}$

@ low freq → $X_C > X_L$ → -90 @ pure C

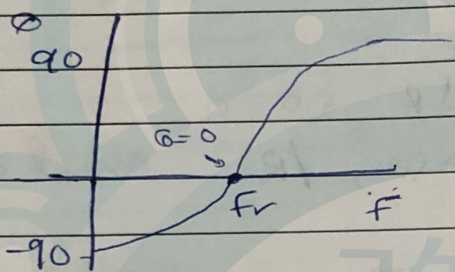
$-90 < \phi < 0$

$X_L > X_C$ → $90 = \phi$ → pure L

@ high freq → $90 > \phi > 0$

@ f_r ⇒ $X_L = X_C$ $\boxed{\phi = 0}$

$Z_{eq} = R$



* variation of φ @ Z_{eq} with freq

* Quality factor: relation btw max ~~pe~~ energy stored to the energy dissipated

$$Q = \frac{X}{R} = \frac{X_C}{R} = \frac{X_L}{R} = \frac{f_r}{BW}$$

* special case: if $Q > 10$ $BW = \frac{R}{L}$

$$f_{low} = f_r - \frac{BW}{2}$$

$$f_{high} = f_r + \frac{BW}{2}$$

$$I_{max} = \frac{V_{supply}}{R}$$

$$P_{max} = I_{max}^2 * R$$

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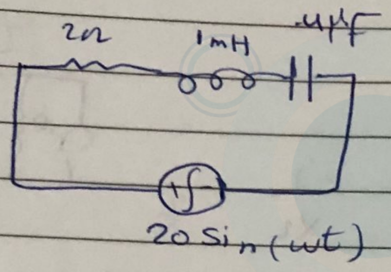
ex 8



assume $Q > 10$

find ① f_r ② ω_r ③ BW ④ Q ⑤ ω_L

Solu

$20 \sin(\omega t) \rightarrow 20 \cos(\omega t - 90)$

① $f_r = \dfrac{1}{\sqrt{2\pi LC}} = 7.9 \text{ kHZ}$

② $\omega_r = 2\pi f_r = 50 \text{ k rad/sec}$

③ $BW = \dfrac{R}{L}$ (Q>10 اذا فقط) $\Rightarrow 2000$ rad/sec

④ $Q = \dfrac{\omega_r}{BW} = 25$

$Q = \dfrac{f_r}{BW} \rightarrow Q < 10$

⑤ $\omega_L = \omega_r + \dfrac{BW}{2} = 2\pi f_L$

⑥ $\omega_{High} = \omega_r - \dfrac{BW}{2} = 2\pi f_h$

⑦ $I_{max} = \dfrac{V_{supply}}{R} = \dfrac{20\angle -90}{2} = 10\angle -90 \text{ A}$

⑧ $Z_{max} = R = 2\Omega$

⑨ $I_{(@\omega_L \& \omega_h)} = \dfrac{I_{max}}{\sqrt{2}}$

Note: if $Q < 10$

$BW = f_H - f_L = H_z$

$Q = \dfrac{f_r}{BW}$

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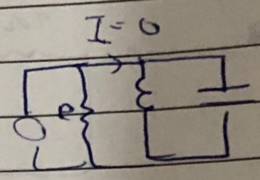
* Parallel RLC circuits



$Y_R = \frac{1}{Z_R} = \frac{1}{R}$

$Y_L = \frac{1}{Z_L} = \frac{1}{j\omega L} = \frac{-j}{X_L}$

$Y_C = \frac{1}{Z_C} = \frac{1}{\frac{-j}{\omega C}} = j\omega C = X_C j$

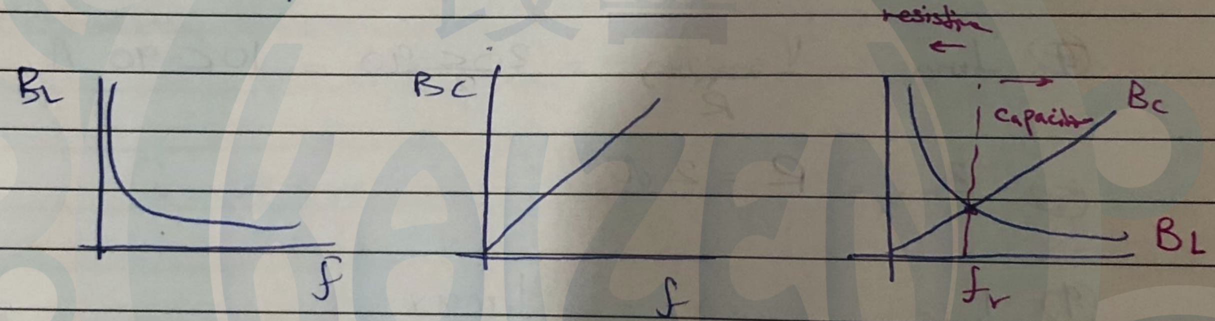
$Y_{total} =$

$\frac{1}{R} + \frac{-j}{X_L} + j X_C$

$Y = G + jB$

$B_L = \frac{-j}{X_L}$ $\quad B_C = jX_C$

$Y_{total} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$



@ resonance freq

(1) $B_L = B_C$

$\frac{1}{\omega L} = \omega C \rightarrow \omega_r = \frac{1}{\sqrt{LC}}$ $\quad f_r = \frac{1}{2\pi\sqrt{LC}}$

* @ resonance $|I_L| = |I_C|$ Because $B_L = B_C$ (same V on them)
But out of phase • circulating current btw L & C

$Y = \frac{1}{R} = G$

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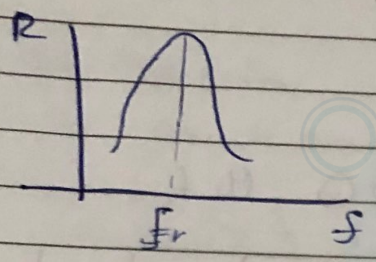
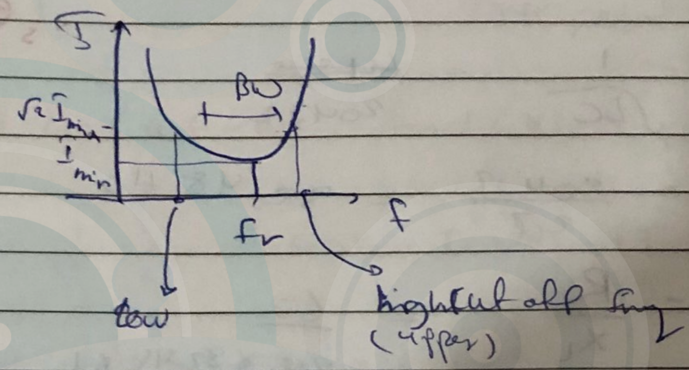


* $I_{min} = \frac{V_{sup}}{R}$



* $BW = f_H - f_L$ (HZ)
* Q = max energy stored / max energy lost

$$Q = \frac{R}{\omega L} = \omega RC = R\sqrt{\frac{C}{L}} = \frac{R}{X_L} = \frac{R}{X_C}$$

* $Q = \frac{\omega_r}{BW} = \frac{f_r}{BW}$ (لازم الوحدات يكونوا نفسها)

* $f_L = f_r - \frac{BW}{2}$
* $f_h = f_r + \frac{BW}{2}$

عند ال reson—— ← الزاوية بين V/I = 0

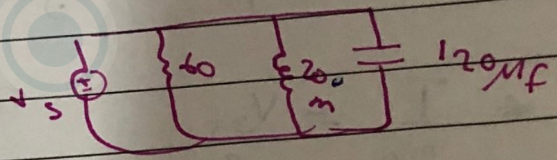
$((1+i)^N i$

ex:

$v_s = 100\cos(\omega t)$ (3) Q (4) f_L

find (1) f_r (2) BW

(5) f_H (6) I_{min}



$\omega_r = \frac{1}{\sqrt{LC}} = \frac{\cancel{645.5}}{204.12}$

(1) $f_r = \frac{204.12}{2\pi} = 32.48\ Hz$

(2) $Q = \frac{R}{X_L} = \frac{60}{2\pi * 32.48 * L} = \frac{60}{40.8} = 1.47$

(3) $BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22\ Hz$

(4) $f_H = f_r + \frac{1}{2}BW = 43.5\ Hz$

$f_L = f_r - \frac{1}{2}BW = 21.5\ Hz$

(5) $I_{min} = \frac{V}{R} = \frac{100\cos(204.12)t}{60} = \frac{100\angle 0}{60} = 1.67\ A$

Y at $f_r = \frac{1}{R} = \frac{1}{60}\ \mho$

Z at $f_r = R = 60\ \Omega$

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* the avg value : measuring the mean of the voltage or current over a period time.

$$\langle x(t)\rangle = \frac{1}{T}\int_0^T x(\tau)\,d\tau$$ → integral of the signal over period of time

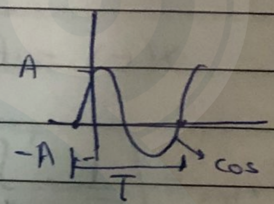
ex $x(t) = A\cos(\omega t - \theta)$ find $\langle x(t)\rangle$

$$\langle x(t)\rangle = \frac{1}{T}\int_0^T A\cos(\omega\tau + \theta)\,d\tau$$

$$\left(\frac{1}{T\omega} * A\right)\sin(\omega\tau+\theta)\Big]_0^T =$$

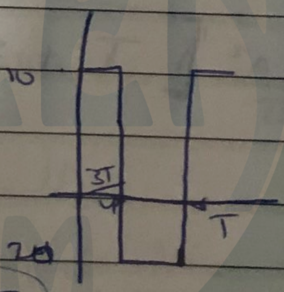
$\sin T = 0$

$\boxed{= 0 \text{ zero}}$



*Note: the avg of sinusoidal signal is zero independent of amplitude or frequency

ex find $\langle xt\rangle$



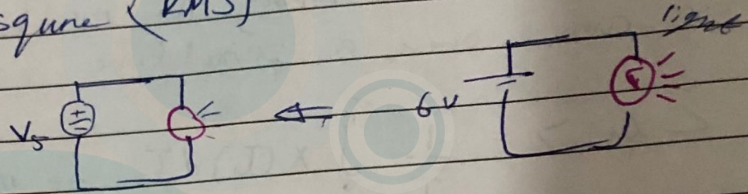
$$\langle xt\rangle = \frac{1}{T}\int_0^T x(\tau)\,d\tau$$

$$= \frac{1}{T}\left(\int_0^{3/4\,T} 10\,dT + \int_{3T/4}^{T} -20\,dT\right)$$

$$\frac{1}{T}\left[\frac{30}{4}T - \frac{20}{4}T\right] = \frac{10}{4} = x\langle(\tau)\rangle$$

$(1+i)^N i$

* Root mean square (RMS)



RMS = effective value of varying voltage or current

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T (x^2)(t)\,dt} = X_{effective}$$

ex $\quad x(t) = A\cos(\omega t)$

$$X_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T A^2\cos^2(\omega t)\,dt}$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sqrt{\frac{1}{T}\int_0^T A^2\left(\frac{1}{2}(1+\cos 2\omega t)\right)dt}$$

$$\sqrt{\frac{A^2}{2T}\Big[T + \sin 2\omega t\Big]_0^T} = \sqrt{(T+0)\frac{A^2}{2T}}$$

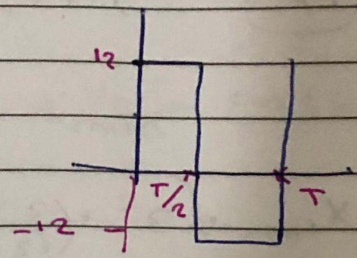
$$= \frac{A^2}{2}$$

* for sinusoidal signal

* $X_{rms} = \frac{\text{Peak value}}{\sqrt{2}}$

ex

$$x(t) = \begin{cases} 12 & 0 \le t \le \frac{T}{2} \\ -12 & \frac{T}{2} < t < T \end{cases}$$



$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T/2} 12^2\,dt + \int_{T/2}^{T} (-12)^2\,dt\right.}$$

$$\sqrt{\frac{1}{T}\left[\frac{144}{2}T + \frac{144}{2}T\right.} = \sqrt{144} = 12$$

Note: for square wave signal

① $x_{rms} = A \rightarrow$ max value

② for triangular signal

$$x_{rms} = \frac{A}{\sqrt{3}}$$

③ for DC - signal

$x_{rms} =$ DC value

ex: $V_{rms} = 220$ volt

$V_{peak} = ?$ $\quad V_{peak} = V_{rms} * \sqrt{2}$

$= 220 * \sqrt{2} = 311$ volt.

* Note. $v(t) = V_1 \cos(\omega t) + V_2 \sin(\omega t) + \ldots + V_0$

$V_{rms} = ?$

$$V_{rms} = \sqrt{(V_{DC})^2 + \sum_{i=1}^{n} \frac{V_i^2}{2}}$$

$$\text{RMS total} = \sqrt{(RMS_1)^2 + (RMS_2)^2 + \ldots + (RMS_n)^2}$$

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ex : $x(t) = 3 + 2\cos(\omega t)$

$RMS_1 = 3$

$x_{rms} =$

$RMS_2 = \frac{Peak}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$$x_{rms} = \sqrt{3^2 + \left(\frac{2}{\sqrt{2}}\right)^2}$$

$$= \sqrt{9 + 2} = \sqrt{11}$$

ex :

$v(t) = 4\cos(100t) + 12\sin(100t) - 6\cos(300t) + 5$

$$V_{rms} = \sqrt{\left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{-6}{\sqrt{2}}\right)^2 + 5^2} = \sqrt{123} = 11.1$$

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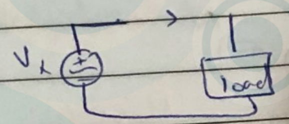
27/7/2020

* Power in AC-Circuit

(1) instantinous power (2) complex power (3) avg power

(1) instantinous power:



if $v(t) = \underset{\text{Peak}}{V_m} \cos(\omega t + \underset{\text{Phase}}{\theta_v})$

$i(t) = I_m \cos(\omega t + \theta_i)$

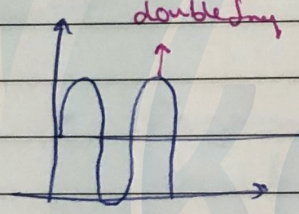
$P(t) = v(t)\, i(t)$

$= V_m \cos(\omega t + \theta_v) * I_m \cos(\omega t + \theta_i)$

مضاعف $= \frac{V_m I_m}{2} \cos(2\omega t + \theta_i + \theta_v) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

oscillating signal with double freq

constant value (avg power) (DC component)



(2) Avg power

$$P_{avg} = \frac{1}{T} \int_0^T P(t)\, dt$$

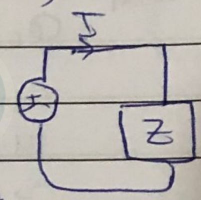
$$P_{avg} = \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)\, dt$$

$$= \frac{V_m I_m}{2} \left[\int_0^T \cos(2\omega t + \theta_v + \theta_i)\, dt + \int \cos(\theta_v - \theta_i)\, dt \right.$$

$$= \frac{V_m I_m}{2T} * T \cos(\theta_v - \theta_i)$$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

ex)



$$I = I_m \angle \theta_i$$

$$V = V_m \angle \theta_v$$

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$|Z| = \frac{V_m}{I_m}, \quad \theta_z = \theta_v - \theta_i$$

$$P_{avg} = \frac{1}{2} I_m^2 |Z| \cos(\theta_v - \theta_i)$$

$$= .5 \frac{V_m^2}{|Z|} \cos(\theta_v - \theta_i)$$

→ اذا جابة أربطوب |Z|

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$= I_{rms}^2 |Z| \cos(\theta_v - \theta_i)$$

$$= \frac{V_{rms}^2}{|Z|} \cos(\theta_v - \theta_i)$$

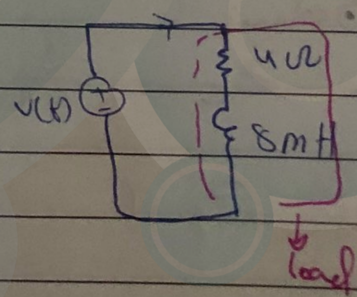
→ arms ب

* حسب المعطى بشتغل عليه

ex: → $V(t) = 14.14 \sin(377t)$

find ① P_{avg} for load RL

② $P(t)$



Soln:

$$v(t) = 14.14 \cos(377t - 90)$$

$$v(t) = 14.14 \angle -90$$

$$\omega = 377 \text{ rad/sec}$$

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$X_L = 377 * 8 * 10^{-3} = 3\Omega$

$Z_{load} = R + jX_L = 4 + 3j = 5\angle 36.9\,\Omega$

$|Z_{load}| = 5$

$\theta_{load} = 36.9 = \theta_v - \theta_i$

$I = \frac{V}{Z} = \frac{14.14\angle -90}{5\angle 36.9} = 2.83\angle -126.9\ A$

$I_{max} = 2.83$

$\theta_i = -126.9$

* $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= .5 * 14.14 * 2.83 \cos 36.9$

$P_{avg} = 16\ W$

or $P_{avg} = \frac{1}{2} * I_m^2\, |Z| \cos(\theta_v - \theta_i) \Rightarrow$

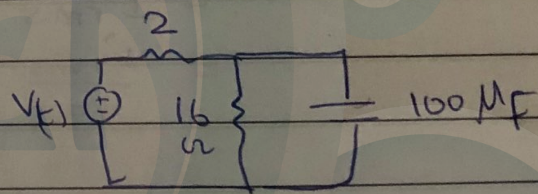
or $P_{avg} = \frac{1}{2} \frac{V_m^2}{|Z|} \cos(\theta_v - \theta_i) \Rightarrow$

(2) $P(t)_{inst} = V(t) * i(t)$

$= 14.14 \cos(377t - 90) * 2.83 \cos(377t - 126.9)$

$= 40 \cos(377t - 90)\ \cos(377t - 126.9)$ W

ex 2



$V = 110\angle 0$ volt

$\omega = 377$ rad/sec

find P_{avg} for the load

Solu:

$Z_C = \frac{-j}{\omega C} = \frac{-j}{377 * 100 * 10^{-6}} = -26.5j\,\Omega$

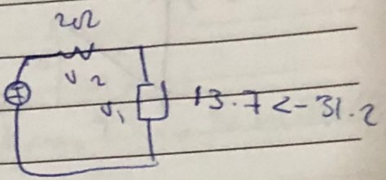
$Z_{load} = \frac{16 * -26.5j}{16 - 26.5j} = 13.7\angle -31.2\,\Omega$

$|Z|_{load} = 13.7$

$\theta_{load} = -31.2$

Five Apple

$$P_{avg} = \frac{1}{2} \frac{V_m}{|Z|_{load}} \cos(\theta_v - \theta_i)$$



$$V_1 = \frac{110\angle 0\,(13.7\angle -31.2)}{2 + 13.7\angle -31.2}$$

$$V_1 = 97.5 \angle -3.85 \text{ v}$$

$$V_m = 97.5$$

$$\theta = -3.85$$

$$P_{avg} = \frac{1}{2} \frac{(97.5)^2}{13.7} \cos(-31.2) = 296.8\,\omega$$

or $P_{avg} = \frac{1}{2} I_m^2 |Z| \cos(\theta_v - \theta_i)$

I_m → current @ load

$$I = \frac{V_1}{Z_{load}} = \frac{97.5 \angle -3.85}{13.7 \angle -31.2} = 7.116 \angle 27.35$$

$$P_{avg} = \frac{1}{2} (7.116)^2 (13.7) \cos(-31.2) = \cancel{96.6}\ 296.8\omega$$

* Complex power: product of rms voltage & the complex conjegate of the rms current

if voltage = $V_{rms} \angle \theta_v$

Current $I_{rms} \angle \theta i$

$$P_s = V_{rms} \angle \theta_v * (I_{rms}^*)$$

$$= (V_{rm} \angle \theta_v)(I_{rms} \angle -\theta_i)$$

$$= V_{rms} I_{rms} \angle \theta_v - \theta_i \quad \text{polar form}$$

$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{\text{real power}} + j\, \underbrace{V_{rms} I_{rm} \sin(\theta_v - \theta_i)}_{\text{reactive power}}$$

S → apparant Power (VA)

real power
avg Power
(W) dissipated pow

~~Real~~ reactive Power
(VAR)

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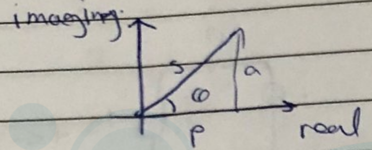
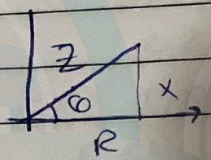


$$\theta = \theta_v - \theta_i$$
$$S = V_{rms} I^*_{rm}$$
$$= P + jQ$$
$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \ W$$
$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \ VAR$$



* $Z = R + jX$

$$Z = \frac{V_{rms}}{I_{rms}}$$

$$V_{rms} = I_{rms} * Z$$
$$S = (I_{rms} * Z) I^*_{rms}$$
$$S = |I_{rm}|^2 Z$$
$$S = |I_{rms}|^2 (R + jX)$$
$$= \underbrace{|I_{rms}|^2 R}_{P} + \underbrace{j |I_{rms}|^2 X}_{Q}$$

measure the energy exchanged between the source & the reactive impedence

* Summary:

$$S = V_{rms} I^*_{rms}$$
$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$
$$= |I_{rms}|^2 R + j |I_{rm}|^2 X$$

$$|I_{rms}| = I_{rms} \angle \theta$$

approaches ∞.

27/7

* PF

* Power factor

$$PF = \cos(\theta_v - \theta_i)$$

$$0 < PF < 1$$

$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\,(V_{rms}) I_{rms} \sin(\theta_v - \theta_i)$$

$$P = V_{rms} I_{rms} \underbrace{\cos(\theta_v - \theta_i)}_{PF}$$

$$PF = \frac{P}{V_{rms} I_{rms}} = \frac{P}{|S|}$$

$$|S| = \sqrt{P^2 + Q^2}$$

Cases for PF

① when $PF = 1$ → pure resistive load

$\theta_v = \theta_i \qquad \cos(\theta_v - \theta_i) = \cos 0 = 1$

$\sin(\theta_v - \theta_i) = 0$

↳ $Q = 0$

② when $PF = 0$ → Pure ~~resistive~~ inductive load

$Q > 0 \qquad PF = \text{lag}$ → الزاوية موجب

$PF = 0 \qquad \cos(\theta_v - \theta_i) = \cos 90 = 0$

↳ special case for pure inductive

→ pure capacitive load

$Q < 0$

$PF = \cos(\theta_v - \theta_i) = \cos(90) = 0$

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③ inductive load PF → lag
capacitive load PF ⇒ leading

(ex) $PF = .8$ lag → inductive load
$|S| = 1000$ VA $Q > 0$

$S = |S| \angle + \cos^{-1}(PF)$
$PF = \cos(\theta_v - \theta_i)$
$\cos^{-1}(PF) = (\theta_v - \theta_i)$
lagging ⇒ θ_s is +ve

$S = |S| \angle(\theta_v - \theta_i)$

(ex) $Pf = .8$ lead
$|S| = 1000$ VA → capacitive
$Q < 0$

$PF = \cos(\theta_v - \theta_i)$
$\Rightarrow \theta_v - \theta_i = \cos^{-1}(PF)$
$S = |S| \angle - \cos^{-1}(PF)$
leading = θ_s = -ve

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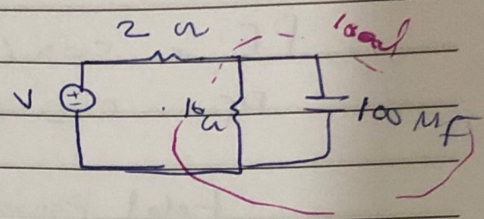
ex $V = 110\angle 0$

$\omega = 377$ rad/sec



find S P Q PF

Soln:

$Z_C = \frac{-j}{\omega C} = -j26.5\ \Omega$

$Z_1 = 16 \parallel -j26.5\ \Omega = \frac{16 \times -26.5j}{16 - j26.5} = 13.7\angle -31.2\ \Omega$

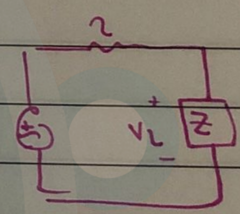
$I = \frac{V}{2+Z_1} = \frac{110\angle 0}{2 + 13.7\angle -31.2}$

$I = \underbrace{7.116}_{\text{peak}}\angle 27.35\ A$

$S = V_{rms} I^*_{rms}$

$= \left(\frac{110}{\sqrt{2}}\angle 0\right)\left(\frac{7.116}{\sqrt{2}}\angle -27.35\right) = 391.4\angle -27.35\ VA$

$= \underbrace{\cancel{349.4}\ 347.6}_{\text{total power}} - j\underbrace{179.8}_{\text{reactive power}} \rightarrow$ complex Power



$P = 347.6\ W \qquad Q = -179.8\ VAR$

$V_L = \frac{V_{supply} * Z_1}{2 + Z_1} = 97.5\angle -3.8\ V$

(سكت)

$Q = \frac{|V_L|^2}{X_C} = \frac{\left(\frac{97.5}{\sqrt{2}}\right)^2}{26.5} = 179.4\ VAR$

$P_{total} = P_{(2\Omega)} + P_{load}$

$P(2\Omega) = I_{rms}^2 * 2$

$= \left(\frac{7.116}{\sqrt{2}}\right)^2 * 2 = 50.64$

$P_{(16\Omega)} = \frac{(V_{L\,rms})^2}{16} = \frac{\left(\frac{97.5}{\sqrt{2}}\right)^2}{16} = 297.1\ W$

$P_{total} = 297.1 + 50.64 = 347.7\ W$

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$$PF = \cos(\theta_v - \theta_i)$$

$$PF = \cos(0 - 27.35) = .89 \text{ leading}$$

& capacitive لود

total power factor

بما انو Q بالسالب - يعني المرسومة

→ capacitive load

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Chp 11

[Power

Power inst: $I(t) * v(t)$

$$= \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

Power avg: $\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

Power complex: $\cancel{V_m}_{rms} \cancel{I_m}_{rms} (\angle \theta_r - \theta_i) = V_{rms} I^*_{rms}$

$\cancel{=}$ $V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$$= \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{\text{avg power}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{\text{Reactive power}}$$

(disipated power)

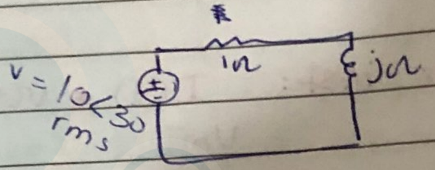
$= |I_{rms}|^2 (R + jX)$

* مع اكمال الاشارة.

28/7/2020

complex power ex:

ex) ass rms values:
find S P Q PF



Solu:

$Z = 1 + j\ \Omega = 1.41\angle 45$

$I_{rms} = \frac{V}{Z} = \frac{10\angle 30}{1.41\angle 45} = 7.07\angle -15\ A$

$S = I^*_{rms} * V^*_{rms} = (10\angle 30)(7.07\angle 15)$

$= 50 + 50j$

$P = 50\ W$

$Q = +50\ VAR$

$P = |I_{rms}|^2 R = 7.07^2 * 1 = 50\ W$

$Q = |I_{rms}|^2 |X| = (7.07)^2 * 1$

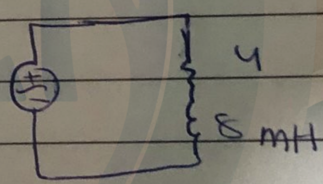
$= 50\ var$

$PF = \cos(\theta_v - \theta_i)$

$= \cos 30 - -15 = \cos 45 = .707$ $\boxed{lag}$

load is inductive

ex) $v(t) = 14.14 \sin(377t)$
find S, P, Q, Pf



Soln → $v(t) = 14.14 \cos(377t - 90)$

$\omega = 377\ rad/sec$ $8 * 10^{-3} * 377j = 3j$

$Z = 8*10^{-3} * 377j + 4 = 4 + 3j$

$S = I^*_{rms} * V_{rms}$

$\frac{V}{Z} = I = \frac{14.14 \cos(377t - 90)}{4+3j} = \frac{14.14\angle -90}{4+3j}$

من: رطا المين

→ $I = 2.828 \angle -127^\circ$ A

$\frac{I}{\sqrt{2}} = I_{rms} =$

$S = \frac{2.828 \angle 127}{\sqrt{2}} * \frac{14.14 \angle -90}{\sqrt{2}} = 20 \angle 37$ VA

$S = 16 + j12$

$P = 16W$ $\quad Q = 12 VAR$

inductive load (lag)

$P = I_{rms}^2 * R = \left(\frac{2.828}{\sqrt{2}}\right)^2 * 4 = 16W$

$Q = |I_{rms}|^2 \, |X| = \left(\frac{2.828}{\sqrt{2}}\right)^2 * 3 = 12 VAR$

$PF = \cos(\theta_v - \theta_i) = \cos(-90 - -127) = \cos 37$

$PF = .8$ lag

ex

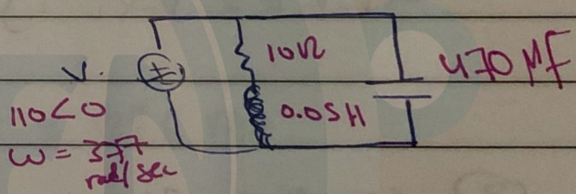
assume RMS values

find 1 dissipated power

2 total reactive value

3 complex power

4 PF



Soln → $C \rightarrow \frac{-j}{470 * 10^{-6} * 377} = 5.6\Omega$

$L \rightarrow j * .05 * 377 = 18.85$

① dissipated power → R فقط تتكون

$(1+i)^N i$

$$P = |I_{rms_R}|^2 R = \frac{|V_{rms}|^2}{2}$$

$$V_1 = \frac{(110\angle 0)(10)}{10 + j*18.85} = 51.55 \angle -62.05$$

$$P_R = \frac{|V_1|^2}{R} = \frac{(51.55)^2}{10} = 265.7\ W$$

② $Q_L = \frac{|V_L|^2}{X_L} = |I_L|^2 X_L$

$$V_L = \frac{(110\angle 0)(18.85\angle 90)}{10 + 18.85j} = 97.2 \angle 27.95\ (V)$$

$$Q_L = \frac{(97.2)^2}{18.85} = 501.2\ VAR$$

$$Q_C = \frac{(110)^2}{5.6} = 2160.7\ VAR$$

$$Q_{total} = jQ_L - jQ_C = 501.2j - 2160.7j = -j\,1659\ \text{volt AR}$$

③ complex power $= P_{total} + Q_{total}$

$$\vec{S} = 265.7 - j\,1659\ VA$$

عشان أتأكد لازم أحلها

$S = V_{rms} I^*_{rms} \Rightarrow$

$$I_{rms} = \frac{110\angle 0}{Z_{total}} = \frac{110\angle 0}{(10 + j\,18.851 - 5.6j)}$$

بتكون بكل → S

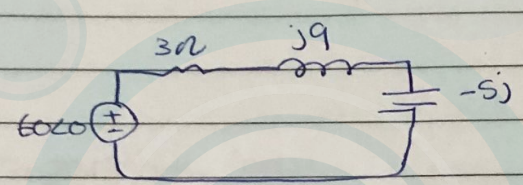
الدالة

28/7/2020

$S = 110\angle 0 * 15.3\angle -80$
$= 265.4 - j\,1659$ VA

(4) $Pf = \cos(\theta_v - \theta_i) = \cos(0-80) = .173$ lead capacitive

ex



assume RMS values
find S for the load

Soln:

$Z_{total}: 3+4j = 5\angle 53.13\ \Omega$

تريقة ①

$I_{rms} = \frac{60\angle 0}{5\angle 53.13} = 12\angle -53.13$ A

$S = V_{rms} I^*_{rms} = 60\angle 0\,(12\angle 53.13) = 720\angle 53.13$

$S = (432 + 576j)$ VA

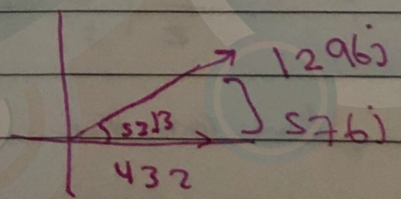
تريقة 2

$P = |I_{rms}|^2 * R = 12^2 * 3 = 432$ W

$Q_L = |I_{rms}|^2 X_L = 12^2 * 9 = 1296$ VAR

$Q_C = |I_{rms}|^2 X_C = 12^2 * 5 = 720$ VAR

$S = P + jQ = 432 + j1296 - 720j$
$S = 432 + 576j$ VA



$Pf = \cos(\theta_v - \theta_i)$
$Pf = \cos(0 + 53.13)$
$Pf = .6$ lagging

important facts related to complex power.

① Impedence triangle

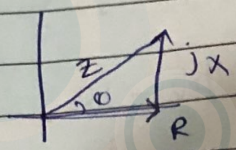
$Z = R + jX$

$R = |Z| \cos \theta$

$jX = |Z| \sin \theta$

$\theta = \tan^{-1}(X/R)$

$|Z| = \sqrt{R^2 + X^2}$



② Power triangle

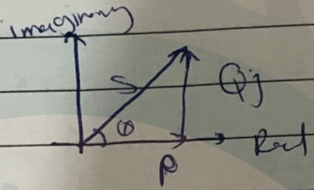
$S = P + Qj$

$P = |S| \cos \theta$

$Q = |S| \sin \theta$

$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$

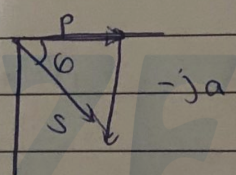
$|S| = \sqrt{P^2 + Q^2}$



$S = P + Qj$

$S = P - jQ$

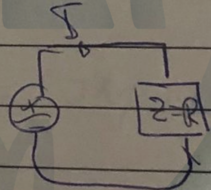
$s = P - jQ$



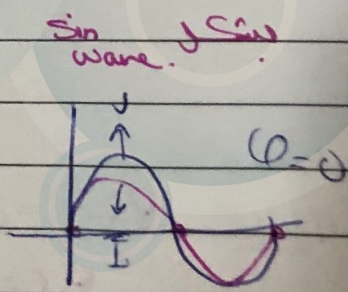
③ Resistive load

$Z = R$

assume $V_s = V\angle\theta_V$



$I = \frac{V_s}{Z} = \frac{V\angle\theta_V}{R} \rightarrow \frac{V}{R}\angle\theta_V$



You can see $\theta_I = \theta_V$

$\theta_V - \theta_I = 0$

∴ the voltage & the current across R are inphase

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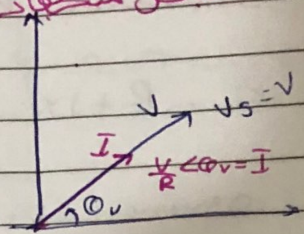
$PF = \cos(\theta_v - \theta_i)$

$PF = 1$

$Q = |S| \sin(\theta_v - \theta_i) = 0$

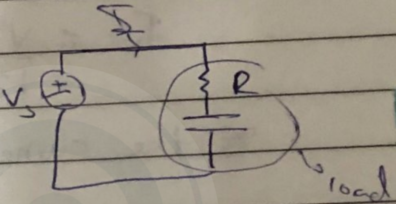
$P = |S| \cos(\theta_v - \theta_i) = |S|$

بشكل متجهات



* capacitive load

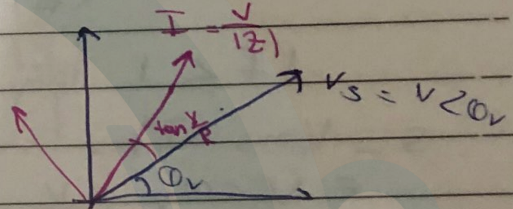
$Z = R - jX = |Z| \angle -\tan^{-1}\left(\frac{X}{R}\right)$



assume $V_s = V\angle\theta_v$

$I = \frac{V_s}{Z} = \frac{V\angle\theta_v}{|Z| \angle -\tan^{-1}\left(\frac{X}{R}\right)}$

$I = \frac{V}{|Z|} \angle\theta_v + \tan^{-1}\left(\frac{X}{R}\right)$ → positive value



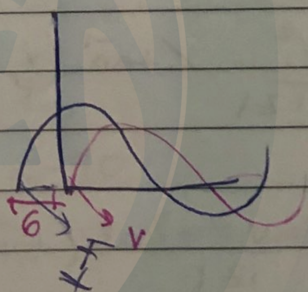
* the current leads the voltage

$Pf = \cos(\theta_v - \theta_i)$

$\cos\left(\theta_v - \left(\theta_v + \tan^{-1}\left(\frac{X}{R}\right)\right)\right)$

$\cos\left(-\tan^{-1}\left(\frac{X}{R}\right)\right)$

$PH < 1$ → leading



$S = V_{rms} I^{*}_{rms}$

$= V\angle\theta_v \left(\frac{V}{|Z|} \angle -\theta_v + \tan^{-1}\frac{X}{R}\right)$

$\frac{V^2}{|Z|} \angle -\tan^{-1}\left(\frac{X}{R}\right)$

$= \frac{V^2}{|Z|} \cos\left(-\tan^{-1}\frac{X}{R}\right) + j\frac{V^2}{|Z|} \sin\left(-\tan^{-1}\frac{X}{R}\right)$

$Q < 1$ → negative

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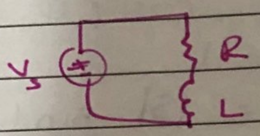
$((1+i)^N i$

* Inductive loads



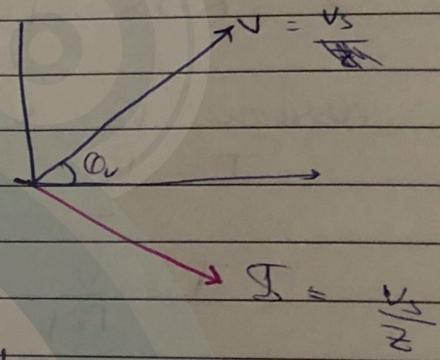
$Z = R + jX = |Z| \angle \tan^{-1}\left(\frac{X}{R}\right)$

assume $V_s = V \angle \theta_v$

$I = \frac{V_s}{Z} = \frac{V \angle \theta_v}{|Z| \angle \tan^{-1}\left(\frac{X}{R}\right)}$

$I = \frac{V}{|Z|} \angle \theta_v - \tan^{-1}\left(\frac{X}{R}\right)$

* the current lag voltage

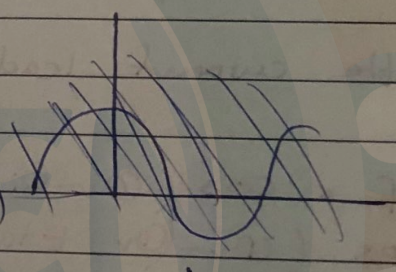


$PF = \cos(\theta_v - \theta_i)$

$= \cos(\theta_v - \theta_v - \tan^{-1}\left(\frac{X}{R}\right))$

$= \cos \tan^{-1}\left(\frac{X}{R}\right)$

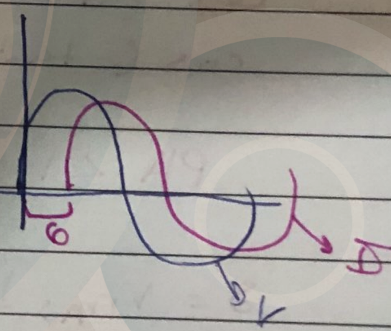
PF < 1 lag



$S = V_{rms} * I^*_{rms}$

$= V \angle \theta_v \left(\frac{V}{|Z|} \angle -\theta_v + \tan^{-1}\frac{X}{R}\right)$

$= \frac{V^2}{|Z|} \angle \tan^{-1}\left(\frac{X}{R}\right)$ → angle is +



$= \frac{V^2}{|Z|} \cos\left(\tan^{-1}\left(\frac{X}{R}\right)\right) +$

$j \frac{V^2}{|Z|} \sin\left(\tan^{-1}\left(\frac{X}{R}\right)\right)$ $Q > 1$

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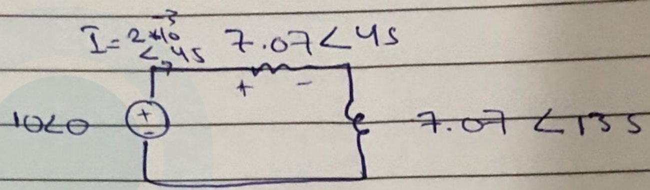
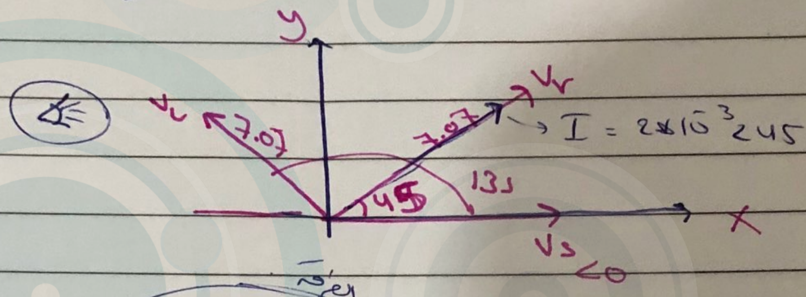
ex



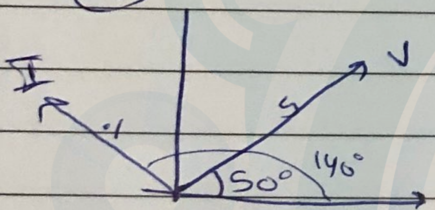
Draw phasor diagram

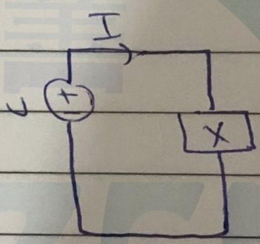


I lags V_L 135−45

V_L leads V_L

ex





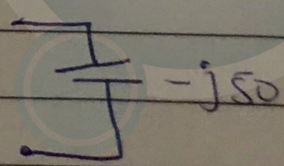
$V = 5\angle 50$

$I = .1\angle 140$

$$X = \frac{V}{I} = \frac{5\angle 50}{.1\angle 140}$$

$$X = 50\angle -90$$

$X = -j50 \rightarrow$ Pure capacitive



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