ELECTRICAL

Engineering

Fundamentals of Electrical Engineering Circuits

2.1 Charge, current, Kirchhoff's current law

The fundamental electric quantity is charge, the smallest amount of charge that exists is the charge carried by an electron

$q_e = -1.602 \times 10^{-19} C$ $q_p = +1.602 \times 10^{-19} C$

Electric current: the time rate of change of charge passing through a predetermined area (usually a cross section of a metal wire)

$i = \frac{\Delta q}{\Delta t}$ $\Delta q \Rightarrow$ units of charge through cross-sectional Area
$\Delta t \Rightarrow$ units of time ; $i \Rightarrow$ current

Amperes: unit of current = 1 ampere = 1 coulomb / second

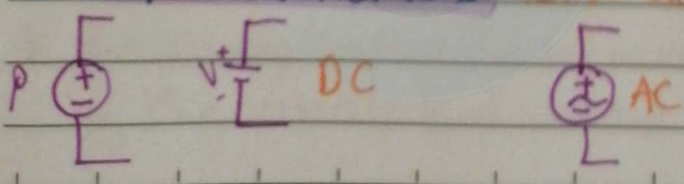
theoritical
* the positive direction of current flow is that of positive charges.

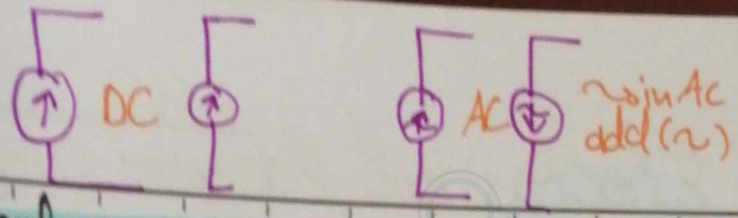
Kirchhoff's current law: because charge can't be created or destroyed must be conserved "the sum of the current at a node must equal zero" [node: junction of two or more conductors]

Voltage (potential difference): the total work per unit charge associated with the motion of charge btw 2 points V = joule / coulomb

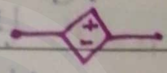
Source: elements that provide Energy   source
load: elements that dissipate Energy   + load -

ideal voltage source: provide a prescribed voltage across it's terminals irrespective of the current flowing through it. The amount of current supplied by the source is determined by the circuit connected to it "active element"





ideal current source: provides a prescribed current to any circuit connected to it, the voltage generated by the source is determined by the circuit connected to it

dependent or controlled sources: whose output (current or voltage) is a function of some other voltage or current 

power the rate at which energy is expended $P = IV$

Electric circuit has 3 components
1- source of power ex: battery ⇒ voltage or current source
2- load (appliance) ex: light bulb ⇒ Resistance, inductor, capicator.
3- inter connection ex: wires & cabels

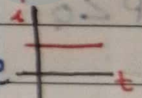
units desired to find in electric currents.
1- Voltage (derived) 2- current (Basic) 3- Power (derived)

SI Prefixes
T - Tera - 10^{12} G - Giga - 10^{9} M - mega - 10^{6} k - kilo - 10^{3}
m - milli - 10^{-3} μ - micro - 10^{-6} n - nano - 10^{9}

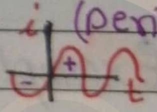
Columb = A x sec

2 ways to show a current (+ voltage) 1- quantity & direction

There are 2 types of current sources
1) Direct current (DC) ⇒ remains constant with time



2) Alternating current (AC) ⇒ varies sinusoidally with time



* Also in voltage you have 2 types DC[Battery] & AC[

$V_{ab} = V_a - V_b = (-V_{ba}) = V_b - V_a$

* my refernce is the +ve sign and that's where i begin.

* the assumption that the terminal + polarity is higher than - polarity by the amount of voltage drop; if -ve don't change your assumption

V ⇒ Volt
I ⇒ Amp
P ⇒ W

Vid#2
week1

Third Quantity: Power

Power: The rate of change of energy per unit time (Watts)

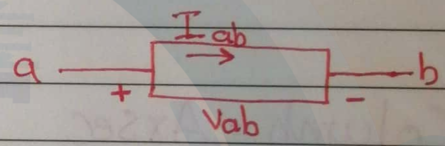
$$P = \frac{dw}{dt} = I\,V$$ ⇒ derived from voltage

Classification of circuit components

1) Passive components: absorb power, resistant, inductor, capacitor

2) Active components: deliver power. battery, source of power

Passive Sign convention

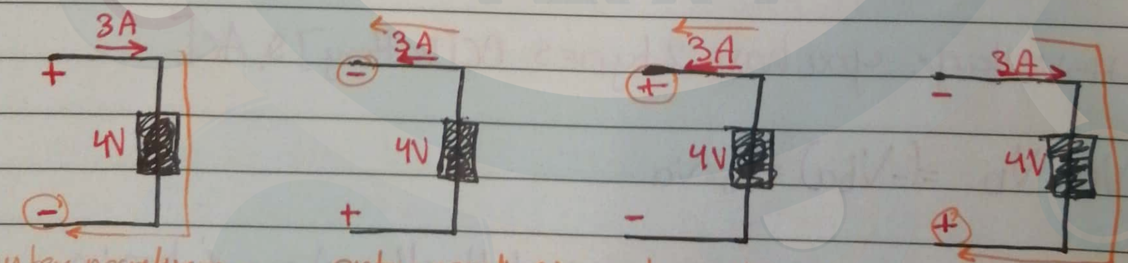


$$P = \pm V_{ab} \times I_{ab}$$

(+) → if current enters positive
(−) → if current enters negative

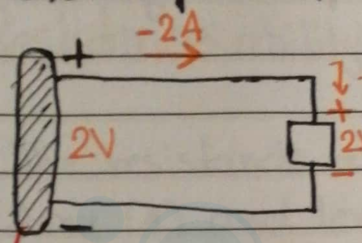
if $P \geq 0$ ⇒ passive ⇒ absorbs power
if $P < 0$ ⇒ Active ⇒ delivers power (absorbing negative power)

Example: find the power (Generated or absorbed) $P = \pm VI$



enters negative ⇒ −	enters negative (−)	enters positive (+)	enters positive (+)
$P = (-)4 \times 3 = -12W$	$P = (-)4 \times 3 = -12W$	$P = (+)\,4 \times 3 = 12W$	$P = (+)\,4 \times 3 = 12$
supplies 12W	supplies 12W	absorbs 12W	absorbs 12W

Example: find power and determine (supply/absorb)



current enters positive (+)

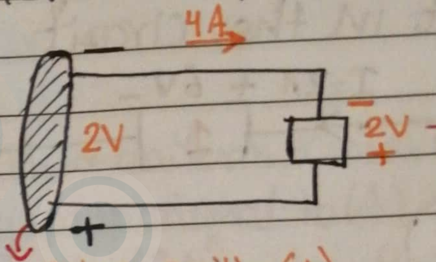
enters negative(-)

$P = (-)\,2 \times (-2) = 4W$

absorb power

$\Sigma P = 0$; element 2 $= -4W$

$P = (+)(-2) \times (2) = -4W$ ✓



enters positive (+)

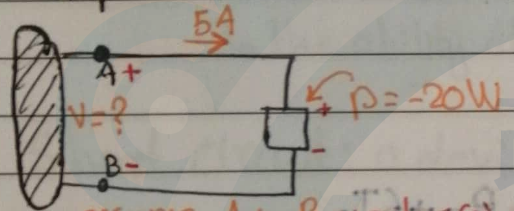
$P = (+)(2) \times (4) = 8W$

absorb Power

element 2 must be $= -8W$

$P = (-)(2) \times 4 = -8W$ ✓

Example: determine unknown voltage or current.



$P = -20W$

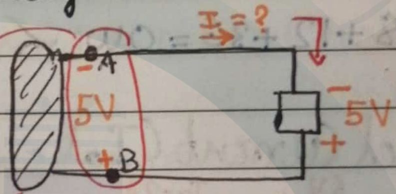
assume A+, B- ; enters(+)

$P = (+)\,V_{AB} \times 5 = -20$

$V_{AB} = -4V$

voltage in B is higher

you can assume whatever (+)/(-) you want; but stick to it.



$\Rightarrow P = 40W$

in here polarity is ready; don't assume

current enters negative

$P = (-)\,5 \times I = 40W$

$I = -8A$

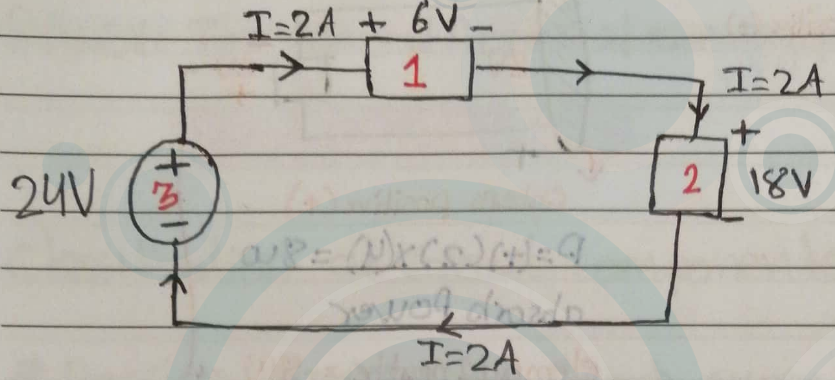
$V_{BA} \Rightarrow V_B - V_A = 5V$

Instantaneous power $P(t) = v(t)\,i(t)$

Energy absorbed or supplied by an element from t_0 to t

$$W = W(t_0 \to t) = \int_{t_0}^{t} p(\tau)\,d\tau = \int_{t_0}^{t} v(\tau)\,i(\tau)\,d\tau$$

Example: determine power absorbed or supplied by each element in the circuit



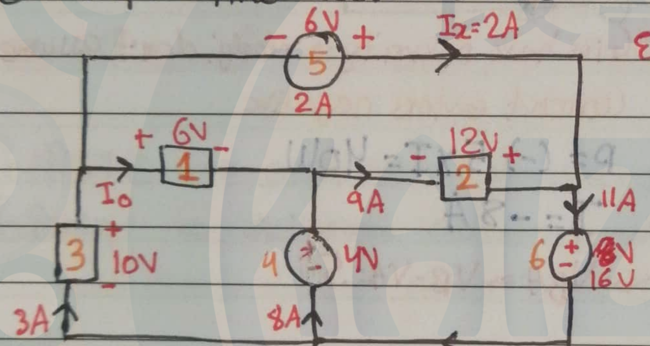
Element 3 ⇒ current enters negative (−) $P=(-)(24)(2)=-48W$ supply

Element 1 ⇒ current enters (+) $P=(+)(6)(2)=+12W$ ⇒ absorb

Element 2 ⇒ current enters (+) $P=(+)(18)(2)=+36W$ ⇒ absorb

$\therefore \sum P=0 \Rightarrow -48+12+36=0W$ ✓

Example find Current (I_o)



Element 1 ⇒ $P_1=+6I_o$

$P_2=-12\times9=-108W$

$P_3=-3\times10=-30W$

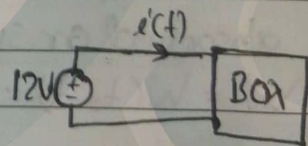
$P_4=-4\times8=\cancel{-36}-32W$

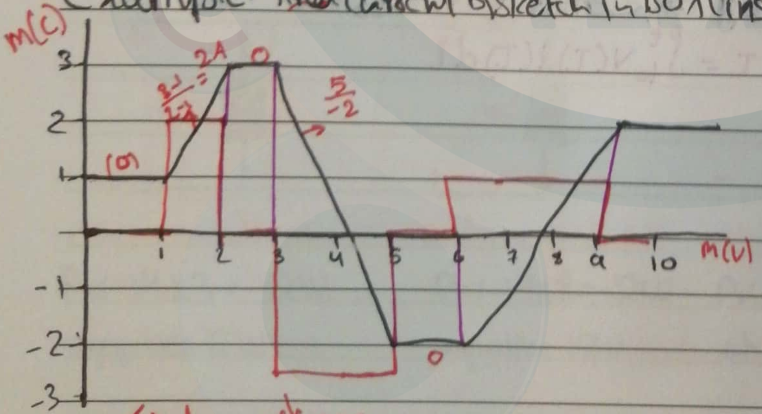
$P_5=-2\times6=-12W$

$P_6=11\times16=176$

$\sum P=0 \Rightarrow 6I_o-108-30-32-12+176=0 \Rightarrow 6I_o-6=0,\ I_o=-1A$

Example: find current & sketch in BOX (inst)



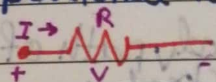


find slope in each point and that's I = P/V

Resistance

Vid#1
week 2.

Resistance: ability to resist current Ω OHM

"The resistance of any material with a uniform cross-sectional Area (A) and length (l) is inversly proportional to (A) and directly proportional to (l)" $R \propto l/A$  $R = \rho\, l/A$

Resistance always has absorbed power that turns to heat Energy

OHM's Law: $\boxed{V = IR}$ $\boxed{P = V^2/R}$ ↳ always positive

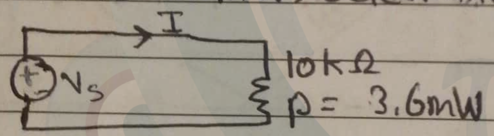
Conductivity $G = 1/R$ Siemens(S) or moho (℧) $\boxed{P = I^2 R}$

↳ the ability of an element to conduct current

Short circuit: a device with zero resistance

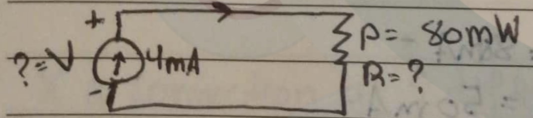
Open circuit: a device with zero conductance (∞ resistance)

Example: the power absorbed by the kΩ resistor in the following circuit is 3.6mW, Determine voltage & current in the circuit.



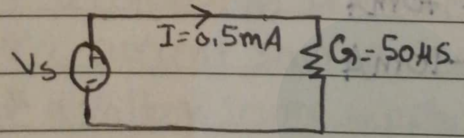
$P = I^2 R$; $3.6m = I^2 \times 10k$; $I = 0.6mA$

$V = IR$; $6\times10^{-4} \times 10\times10^{3} = 6V$



$P = I^2 R$;

$V_s = IR$



$G = 1/R \Rightarrow R = 1/50\mu\ \Omega$

$P = I^2 R$, $V = IR$

Nodes

Node(true node) is the point of connection of three or more circuit elements

binary node(b-node) has only 2 components connected to it

Branch: part of the circuit - collection of elements.

loop: repeated closed path of the current that does not cross any true nodes but once

- that means that i start and end at the same point

Mesh: a path that does not include a smaller path in it

Series & parallel connections

Series: when two or more elements are connected and they belong to the same branch — same I, different V

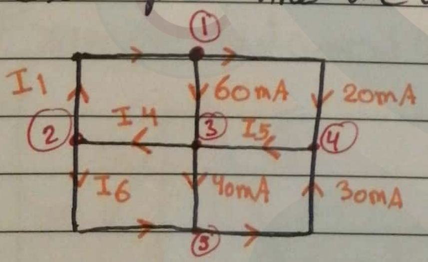
Parallel when two or more elements are connected btw the same 2 true nodes — same V, different I

Kirchhoff's current law (KCL) The Algebric sum of the currents entering a node (or a closed boundry) is zero

$I_1 + I_2 = I_3 + I_4$

$\boxed{\Sigma I_{in} = \Sigma I_{out}}$

Example: find the unknown currents



node 1 ⇒ $I_1 = 60 + 20 = 80\,mA$

node 4 ⇒ $30 + 20 = I_5 = 50\,mA$

node 3 ⇒ $50 + 60 = 40 + I_4 = 70\,mA$

node 2 ⇒ $70 = 80 + I_6 = -10\,mA$

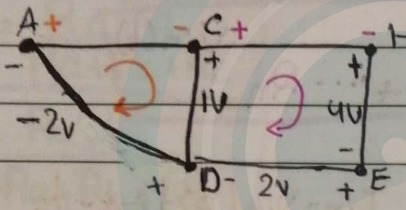
loop direction
Clockwise 2

Kirchhoff's Voltage Law (KVL) the Algebric sum of the voltage in any closed path or loop is zero

Sum of voltage drops = Sum of voltage rises

* i choose a point and go clockwise, if enters from (negative) the votage is negative and vise versa; the same applies to $(I \times R)$

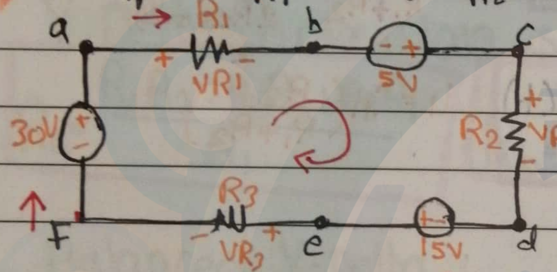
Example: find V_{AC} & V_{CH} in the following circuit



$+V_{AC}+1+2 \Rightarrow V_{AC}=-3V$

$+V_{CH}+4+2-1=0 \quad V_{CH}=-5V$

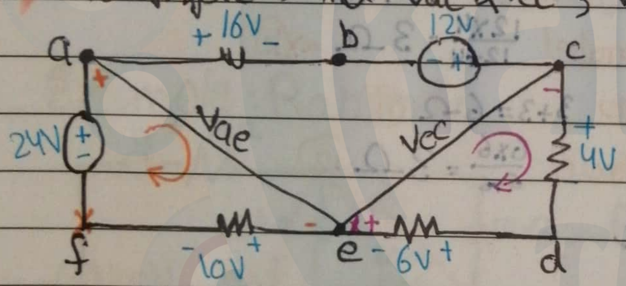
Example: $V_{R_1}=26V$, $V_{R_2}=14V$; Find V_{R_3}



$-30+26-5+14-15+V_{R_3}=0$

$V_{R_3}=-10V$

Example: find V_{ae} & V_{ec}; $V_{ae}=V_a-V_e$



$-24+V_{ae}+10=0$

$V_{ae}=14V$

$V_{ec}+4+6=0$

$V_{ec}=-10V$

* a connection of 2 different current sources in series is impossible
* a connection of 2 different voltage sources in parallel is impossible
* a current source supplying zero current is equivalent to an open circuit
* a voltage source supplying zero volt is equivalent to a short circuit

Five Apple

Vid#3
week 2

Voltage, Current Division

Series Resistors

"The sum of resistors in series is the sum of Resistors"

same current

$$\boxed{R_{eq} = R_1 + R_2 + \dots R_n}$$

Voltage Division → must be series

"In a series combination of n resistors, the voltage drop across the resistor R_j, $j = 1, 2, \dots n$ is

$$\boxed{V_j(t) = \frac{R_i}{R_1 + R_2 + \cdots R_n} V_{in}(t)}$$

Parallel Resistors

same voltage

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

$I_{in} = V_{in}\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)$

$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$ ← 2 Res only

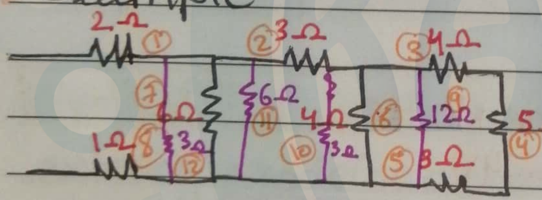
Current Division

must be parallel

$$\boxed{i_j(t) = \frac{G_j}{G_1 + G_2 + \dots G_n} i_{(in)}(t)}$$

$i_1 = i_n\left(\frac{R_2}{R_1 + R_2}\right)$

Example:



3 + 4 + 5 ⇒ series $4 + 5 + 3 = 12\,\Omega$ ⇒ 9

9 + 6 ⇒ parallel $\frac{12 \times 4}{12 + 4} = 3\,\Omega$ ⇒ 10

2 + 10 ⇒ series $3 + 3 = 6\,\Omega$ 11

11 + 7 ⇒ parallel $\frac{6 \times 6}{6 + 6} = 3\,\Omega$ 12

1 + 12 + 8 ⇒ series $2 + 3 + 1 = 6\,\Omega = R_{eq}$

T⊥ Y λ

→ Δ∇ → π ⫫

Y & Delta Networks

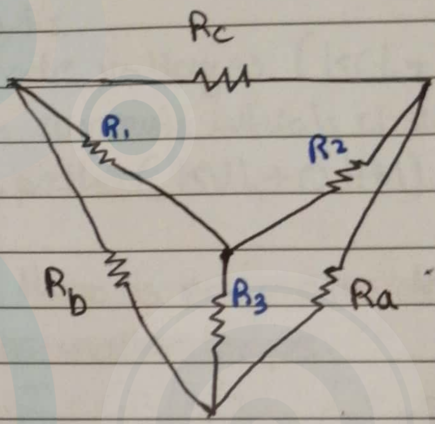
↳ a three terminal connection

Delta-Y Conversion

Solving for R_1, R_2, R_3

$\Delta \to T$

$\Pi \to Y$



$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

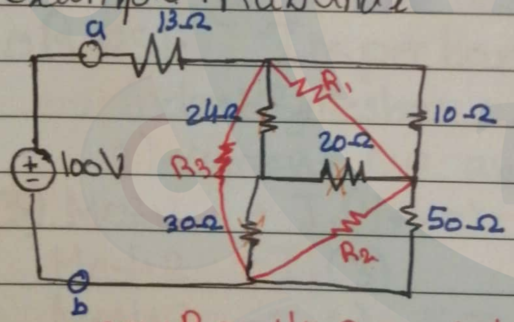
Y-Delta Conversion, solving for $R_a R_b R_c$

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \quad \boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \quad \boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$$

Balanced Y-Delta

when $R_1 = R_2 = R_3 = R_Y$ & $R_a = R_b = R_c = R_\Delta$ then $R_Y = \frac{R_\Delta}{3}$ or $R_\Delta = 3R_Y$

Example: Rabandi



*changing Δ→Y won't do me any good

* Y→Δ will simplify to series & parallel

$R_1 = \frac{(24\times20)+(20\times30)+(30\times24)}{30} = 60\,\Omega$ $\quad \to \frac{1800}{30}$

$R_2 = \frac{1800}{24} = 75\,\Omega$

$R_3 = 1800/20 = 90\,\Omega$

now R_1 and $10\,\Omega$ parallel ⇒ $\frac{60\times10}{70} = 8.571\,\Omega$

R_2 and $50\,\Omega$ parallel ⇒ $\frac{75\times50}{75+50} = 30\,\Omega$

$30 + 8.571 = 38.571$ // with R_3 $\quad \frac{90 * 38.57}{90+38.57} = 26.99$ series with 13 = $\boxed{39.99\,\Omega}$

$i = \frac{V}{R} = \frac{100V}{39.99\,\Omega} = 2.5\,A$

ch3 Mesh Analysis

2 Methods for circuit analysis

1) Nodal analysis to find all node voltages (KCL + OHM)
2) Mesh analysis: to find mesh currents which circulate around closed path (KVL + OHM)

* Mesh is a mesh if & only if there is no loop inside it
* a loop is a loop even if it contains smaller loops.

Mesh analysis procedure

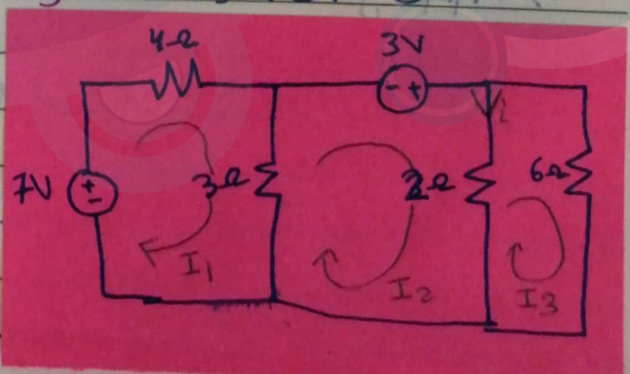
1) identify the meshes
2) suppse each mesh contains a different current "CW"
3) apply KVL to each mesh $V_1 + V_2 + RI + \ldots$
   * keep in mind that i will have an element with 2 mesh current $(I_1 - I_2) \Rightarrow I_1$ = my current mesh current
   $I_2 \Rightarrow$ the other mesh current moving in the opposite direction
4) You will then have j number of equations with j no. of meshes and j no. of unknowns (currents)
5) use simultaneous equations to solve. or matrices

must all be voltage sources. or at least no shared current source btwn meshes.

OR USE **Inspection Method**

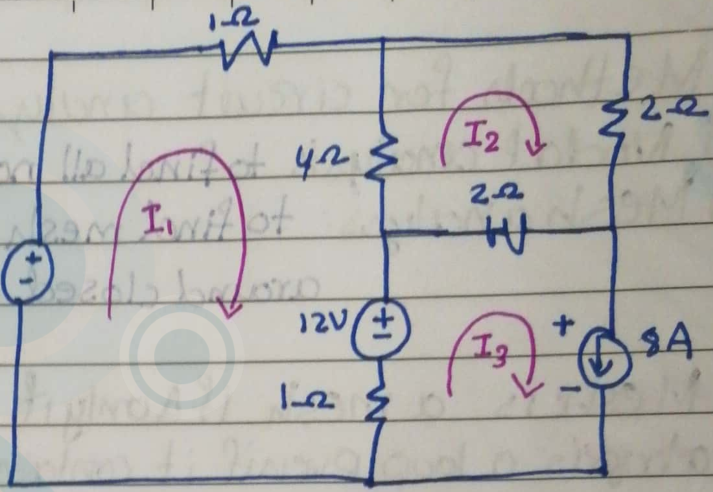
1) identify meshes
2) current CW and you move CW
3) Mesh1 $+ I_1(4+3\Omega) - I_2(3\Omega) - I_3(0) - 7V = 0$
   Mesh2 $-I_1(3) + I_2(3\Omega+2) - I_3(2\Omega) - 3V = 0$
   Mesh3 $-I_1(0) - I_2(2\Omega) + I_3(6+2\Omega) + 0V = 0$
4) solve 3 eq 3 unknowns

ex) find $i \Rightarrow i = (I_2 - I_3)$



Example

* Study note: the 8A current source is already I_3 (mesh current) no need to get it because it's not shared current and has (±) on it



mesh 1 → $+I_1(1+4+1\Omega) - I_2(4\Omega) - I_3(1\Omega) + 12V - 28V = 0$

$6I_1 - 4I_2 - 8 + 12 - 28 = 0 \Rightarrow 6I_1 - 4I_2 = 24$

mesh 2 → $-I_1(4\Omega) + I_2(4+2+2\Omega) - I_3(8) + 0 = 0$

$-I_1 4 + 8I_2 = 16$

find value of v in current source (8A)

use KVL ⇒ $+V_s + 1(8 - I_1) - 12 + 2(8 - I_2) = 0$

Mesh Analysis when current sources are shared btwn two meshes

1) Assign Mesh currents to every mesh in the circuit

2) Define the supermeshes (when 2+ shared current source.

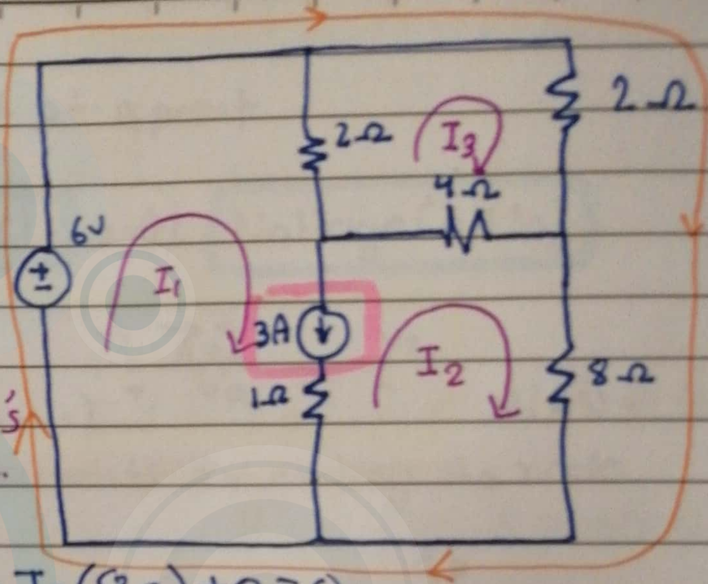
3) Write KVL for each regular mesh

4) Apply KVL & KCL to suppermeshes

5) Solve the system of equations

Example

in here the (3A) source is a shared current source so i need to use super mesh analysis



- Start with mesh 3 because it's not part of the shared current.

mesh3 ⇒ $-I_1(2\Omega) - I_2(4\Omega) + I_3(8\Omega) + 0 = 0$

now use super mesh (this huge orange mesh) [must NOT include the shared current source]

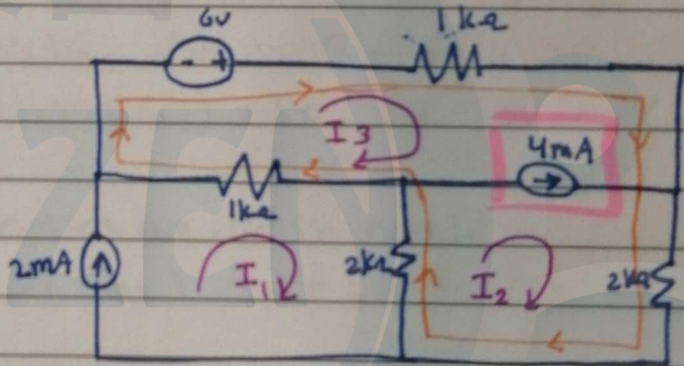
$+I_3(2\Omega) + I_2(8\Omega) - 6V = 0$ → i choose ±I depending on direction with/against

my third equation will come from the current source there

$3A = I_1 - I_2$

*solve

Example



$I_1 = +2mA$ ⇒ ready from current source

$4mA = I_2 - I_3$

super mesh

$-6V + 1k\Omega I_3 + 2k\Omega I_2 + 2k\Omega(\overset{-2mA}{I_2}) + 1k\Omega(I_3 - 2mA) = 0$

* solve.

study note: i tried to choose whichever loop/mesh that will avoid the shared current source.

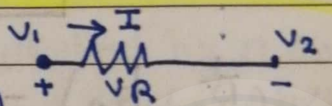
Nodal Analysis

week3 pt2.

Nodes: when 2+ elements meet at a point

Nodal Analysis ⇒ using $\boxed{KCL}$ and $\boxed{\text{Voltage (OHM)}}$

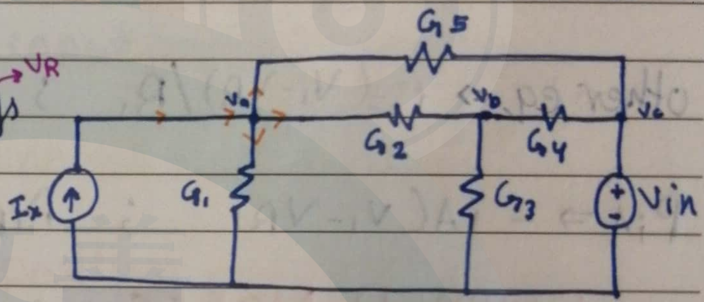
$$I = \frac{V_1 - V_2}{R} \quad \text{or} \quad I = G(V_1 - V_2)$$



$V_1 - V_2$

in node analysis i assume all currents are exiting the node unless it states outherwise.

in node analysis i must always assume a refrence where the $V = 0$, usally is the common one and is usually grounded ⏚



at node A ⇒ $(V_a - V_R)G_1 + (V_a - V_b)G_2 + (V_a - V_c)G_5 = I_s$

• $V_a(G_1 + G_2 + G_5) - V_b(G_2) - V_c(G_5) = I_s$

OR i could do it by inspection as we did in mesh

Node A ⇒ $+V_a(G_1 + G_2 + G_3) - V_b(G_2) - V_c(G_5) = I_s$

- name the nodes
- put (−)(+) based on the node you are working on
- put all the elements on that node that are btwn your node & other.

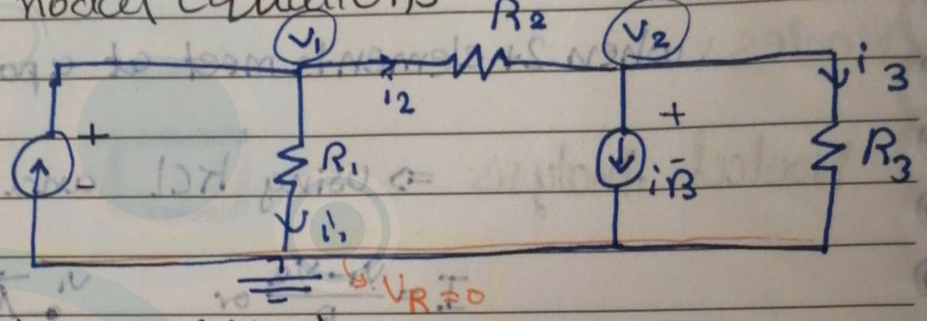
node B ⇒ $-V_a(G_2) + V_b(G_2 + G_3 + G_4) - V_c(G_4) = 0$ no current source

node c ⇒ i have a voltage source then ⇒ $V_c = V_{in} - V_A$ ⇒ $V_{in} = V_c$
and the voltage sourced isn't shared.

Five Apple

Scanned with CamScanner

Example: Write the nodal equations



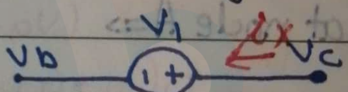
at node 1 $+V_1(\frac{1}{R_1}+\frac{1}{R_2}) - V_2(1/R_2) = i_A$ cuz it's entering

at node 2 $-V_1(1/R_2) + V_2(1/R_2 + 1/R_3) = -i_B$ cuz it's exiting

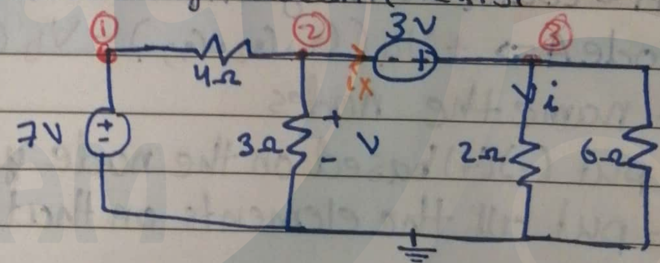
other eq $\Rightarrow$ $i_1 = (V_1 - V_R)/R_1$; $i_2 = (V_1 - V_2)/R_2$; $i_3 = \frac{V_2 - V_R}{R_3}$

$P_{iA} \Rightarrow -iA(V_1 - V_R)$; $P_{iB} \Rightarrow +i_B(V_2 - V_R)$

SUPPER NODES (when i have a shared voltage source btwn two node i use super nodes $V_1 = (V_C - V_D)$; then i assume there is a current (pick any direction you like) then continue solving on the nodes as if the voltage doesn't exist with respect to ix



example



$V_1 - 0 = 7$; $V_3 - V_2 = 3$

at node 2 $-V_1(1/4) + V_2(1/4 + 1/3) + ix = 0$

$ix = +V_3(1/2 + 1/6)$

$V = V_2 - V_R^{\,0}$ $\qquad i = (V_3 - 0)/2$

Superposition

Superposition:

Linearty: a system is linear if and only if it

1) If the input of the system is multiplied by a constant then the output is also multiplied by a same constant.

2) the output of the system to a sum of inputs is the sum of the outputs to each input applied seperately.

Example → $V = IR$ not linear ⇒ $P = I^2 R$

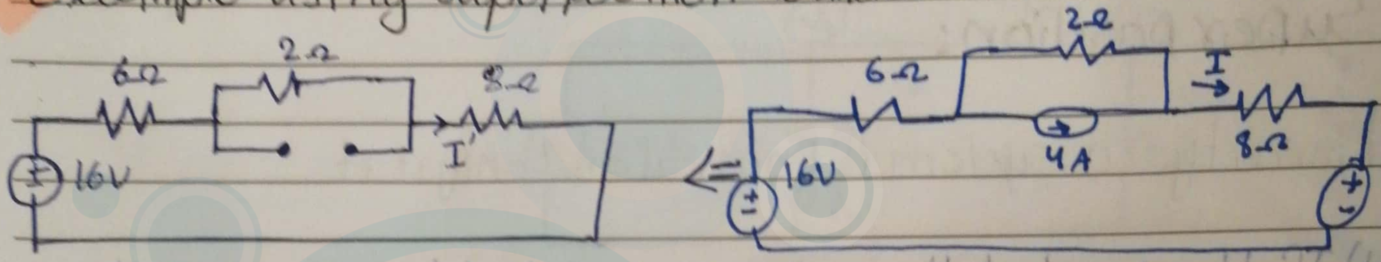
Steps for applying superposition principle

1) Kill all sources but one. find the desired current/voltage due to that source.
   How to kill Voltage source (replace with short circuit [line]) ——
   How to kill current Source (replace with open circuit [dutted line]) - - -
   keep in mind to keep the (+)(-) of V and direction of i

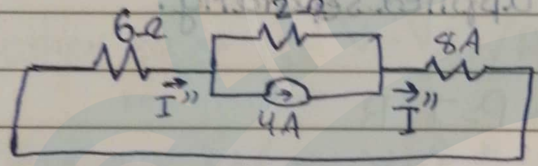
2) Repeat steps for each sources

3) find total desired voltage or current by algebricly adding the contribution due to each source.

Example: using superposition to find I

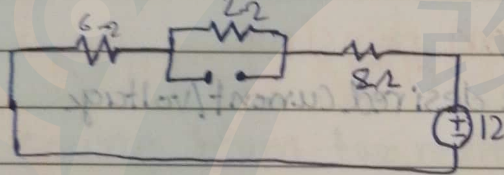


↳ in this way i replaced the current source with open circuit and voltage source with short circuit creating one simple circuit with $(8+2+6\,\Omega)=16\,\Omega$ and 16V in this way $I'=1A$.



current division

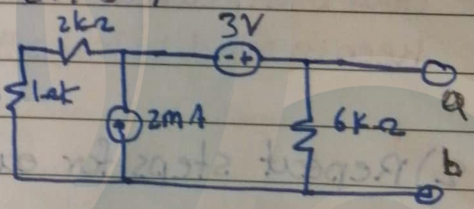
$I'' = 4\left(\frac{2}{16}\right) = 0.5A$



$I''' = -12/16 = -3/4\ A$

$I = I' + I'' + I''' \Rightarrow 1 + 0.5 - 0.75 = 0.75A$

Example



deleting the current source

$3/9 = 1/3 = 6 \times 1/3 = v_o' = 0.5v$

deleting the voltage source.

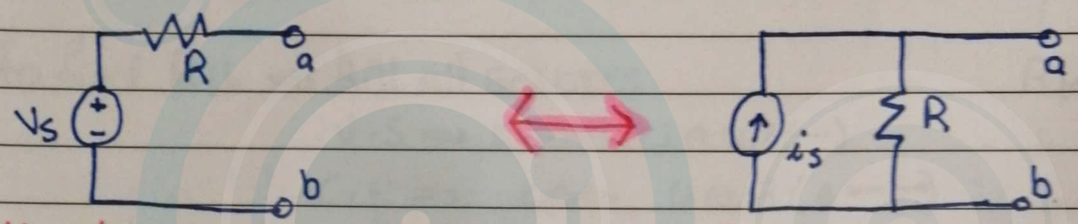
$2mA\left(\frac{3}{9}\right) = \frac{6m}{9} \times 6k = 4V = V_o''$

$V = V_o' + V_o'' = 0.5 + 4V = 4.5V$

Week4 Pt.1

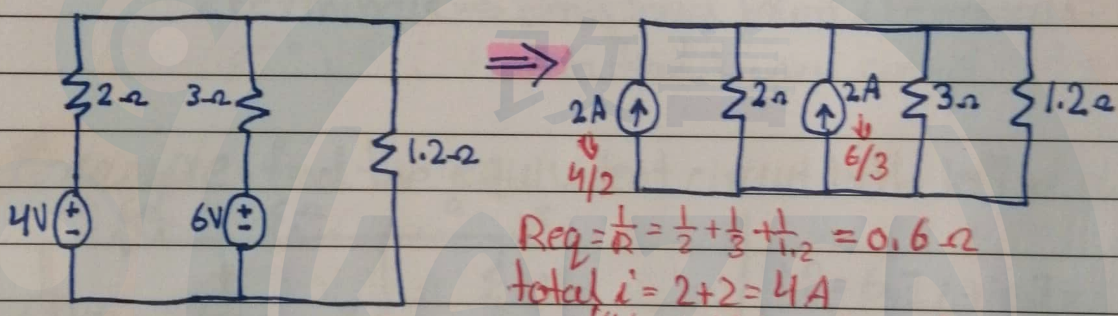
Source Transformation

Source transformation: process of replacing a voltage source in series with a ~~resistor~~ resistor R with a current source in parallel with a resistor (with the same resistance value as R) and vice versa $i_s = \frac{V_s}{R}$ $V_s = i_s R$



direction of current is important.

Example: find the current in the 1.2Ω resistor

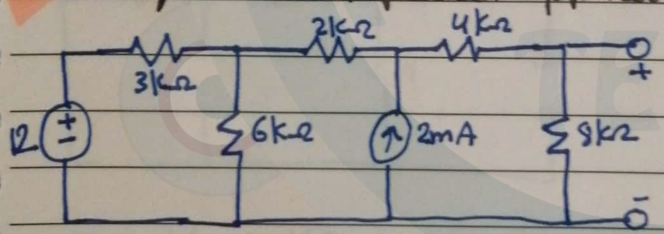


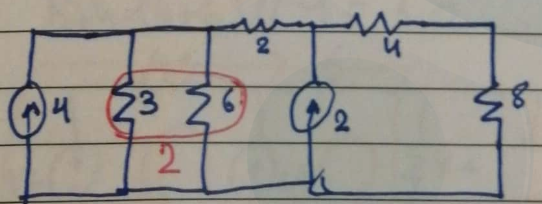
$R_{eq} = \frac{1}{R} = \frac{1}{2} + \frac{1}{3} + \frac{1}{1.2} = 0.6\,\Omega$

total $i = 2 + 2 = 4A$

using current division $\Rightarrow ((1/1.2)/(1/1.2 + 1/2 + 1/3)) \times 4 = 2A$

Example: use repeated application of ST to find V_o





* keep repeating till you get one voltage source and all the rest R are in ~~se~~ series then use voltage division.

$V_o = 8$

Week 4 Part 2

Thevenin and norton equivalent circuits.

Thevenin theorem: any linear circuit containing several voltages and Resistances can be replaced by just one single voltage in series with a single resistance connected across the load voltage (V_{th}) & Resistance (R_{th}).

to find R_{th} ⇒ kill all sources

V.S ⇒ short circuit (—)

C.S ⇒ open circuit

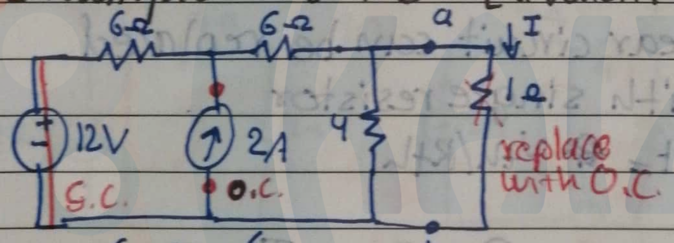
btwn 2 terminals ⇒ Req ⇒ R_{th}

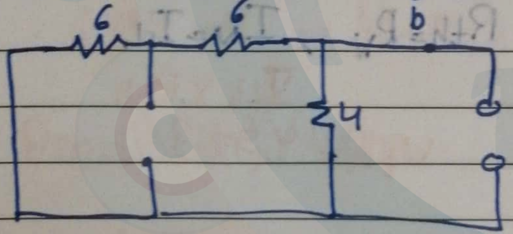
R_{load} = open circuit

to find V_{th} ⇒ V open circuit

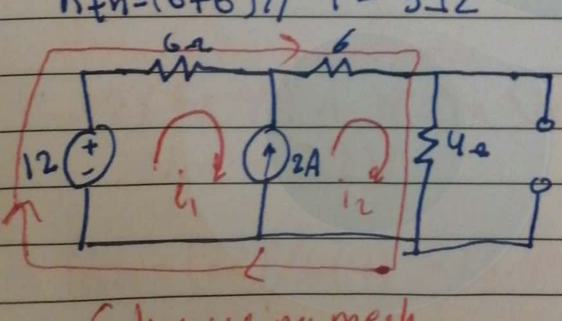
2 terminals ⇒ omit load btwn 2 terminals then find V open circuit

Example: find the equivalent circuit to the left at a, b then I





$R_{th} = (6+6) // 4 = 3\Omega$



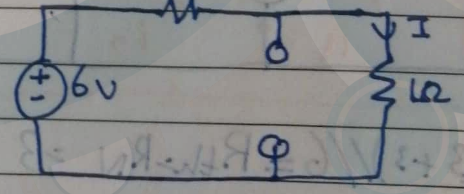
Solve using mesh

$12 = 6I_1 + 10I_2$

$2 = -I_1 + I_2$

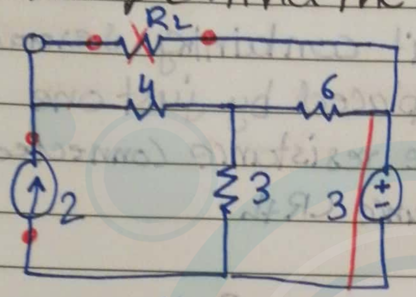
$I_2 = 1.5A$

$V_{oc} = V_{th} = 1.5 \times 4 = 6V$



$I = 6/4 = 1.5A$

Example: find thevenin equi circuit seen from R_L



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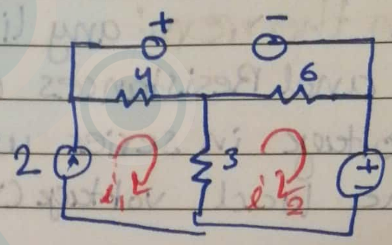
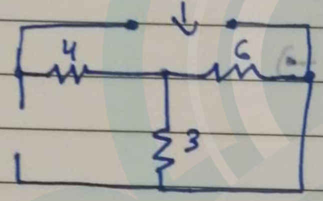


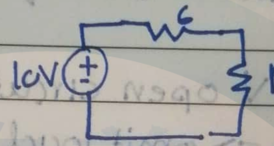
$i_1 = 2$ $\quad i_2 =$

⇓



$V_{oc} - 6k(\frac{1}{3}m - 4k(2m)) = 0$

$V_{oc} = 10V$



$6 // 3 = 2$

$2 + 4 = 6 = R_{th}$

$I = \frac{10}{R_L + 6}$; $P = I^2 R_L$

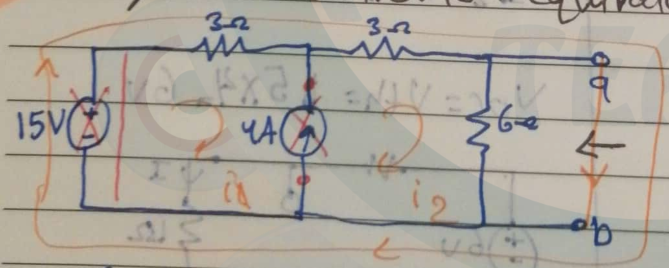
Norton theorem: any linear circuit can be replaced by a single current // with single resistor

$i_{norton} = i_{short\ circuit} = V_{th}/R_{th}$

Example find Norton equivalent $\quad R_{th} = R_N$, $I_N = I_{sh}$

$I_{th} \times R_{th}$

$V_{th} = 4{,}5 \times 3$



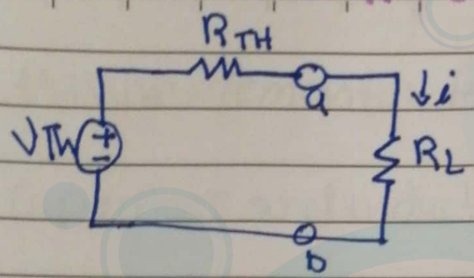
$(3+3) // 6 = R_{th} = R_N = 3$

$15 = 3i_1 + 3i_2 \quad I_2 = 4{,}5 = I_{th}$

$4 = -i_1 + I_2$

W4P4

Max Power transfer.



$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

maximum power to the Load.

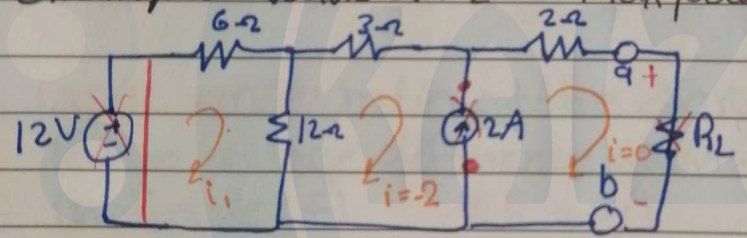
$$\boxed{P_L = (V_{th})^2 / (4R_{th})}$$

Efficency $= (P_{out}) / P(in)$
$= P_L / P_{source}$

$P_{soure} = (V_{th}^2)/2R_{sth}$

Efficency $= 0.5$ Always

Example: value of R_L for max power transfer; max power



$R_L = R_{th}$

$V_{oc} = V_{th}$

$6 // 12 = 4 \Rightarrow 4 + 3 + 2 = 9\Omega = R_L = R_{th}$

$P_m = (V_{th})^2 / 4R_{th}$

$i_1 = 18I_1 - 12(-2) = 12 \Rightarrow i_1 = -2/3 \text{ A}$

then take supermesh

$-12(6 \times -2/3) + 3(-2) + V_{oc} = 0$

$V_{oc} = 22V$

$P_{max} = (22)^2 / 4(9) = 13.44 \text{ W}$

week 5 pdf.

Capacitors

Passive elements: Resistors, capacitors, inductors

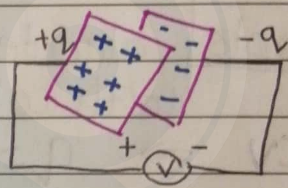
Capacitors and inductors can store energy - storage elements

DC ⇒ has Resistor, capacitor, inductor

Capacitor: consists of 2 conducting plates seperated by an insulator or dielectric

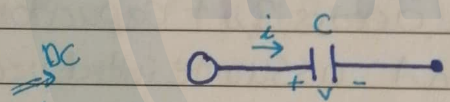
$$\boxed{C = \frac{\varepsilon A}{d}}$$

capacitor with applied voltage



factors Affecting the value of capacitance
1) Area: larger Area ⇒ larger capacitance (A)
2) spacing btwn the plates: smaller space ⇒ greater C (d)
3) material permitivity: higher permitivity ⇒ greater C (ε)

Capacitance: ratio of the charge on one plate of a capacitor to the voltage difference btwn the two plates measured in Farade (F) $F = C/V$



$$\boxed{i = C\frac{dv}{dt}}$$

* a constant voltage across a capacitor creates no current through the capacitor, = open circuit.

* i can change arbitrary but voltage won't (it take time to change)

* A capacitor has memory $V = \frac{1}{C}\int i\,dt + v(t_0)$

* capacitor voltage depends on the past history ($-\infty \rightarrow t_0$) of the capacitor current

Energy storing in capacitance

$$\boxed{P = vi = Cv\frac{dv}{dt}}$$

$$\boxed{w = \frac{1}{2}Cv^2} \qquad \boxed{w = q^2/2C}$$

under DC and steady state condition ⇒ Cap open circuit

Example| Calculate charge stored on a 3pF capacitor with 20V across it. Find Energy stored in capacitor

$q = CV \Rightarrow q = 3\times10^{-12} \times 20 = 60\ pC$

$w = \frac{1}{2}CV^2 = \frac{1}{2}\times3\times10^{-12}\times 400 = 600\ pJ$

example| the voltage across 5 μF capacitor is $v(t) = 10\cos 6000t\ V$, find current through it

$i = C\frac{dv}{dt} = 5\times10^{-6}\frac{d}{dt}(10\cos 6000t$
$= -5\times10^{-6}\times6000\times10\sin 6000t = -0.3\sin 6000t\ A$

example| determine the voltage across 2μF capacitor if the current is $i(t) = 6e^{-3000t}\ mA$ assumne initial $v=0$

$v = \frac{1}{C}\int_0^t i\,dt + v(0)$ and $v(0) = 0$

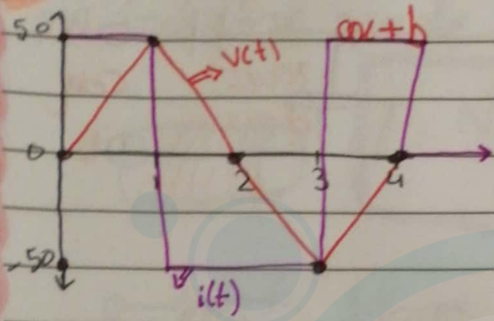
$v = \frac{1}{2\times10^{-6}}\int_0^t 6e^{-3000t}\,dt \cdot 10^{-3} = \frac{3\times10^{3}}{-3000}e^{-3000t}\Big|_0^t$

$= (1 - e^{-3000t})\ V$

(week 5 pt.2)

Capacitors pt2.

example Determine current through a $200\mu F$ when V is

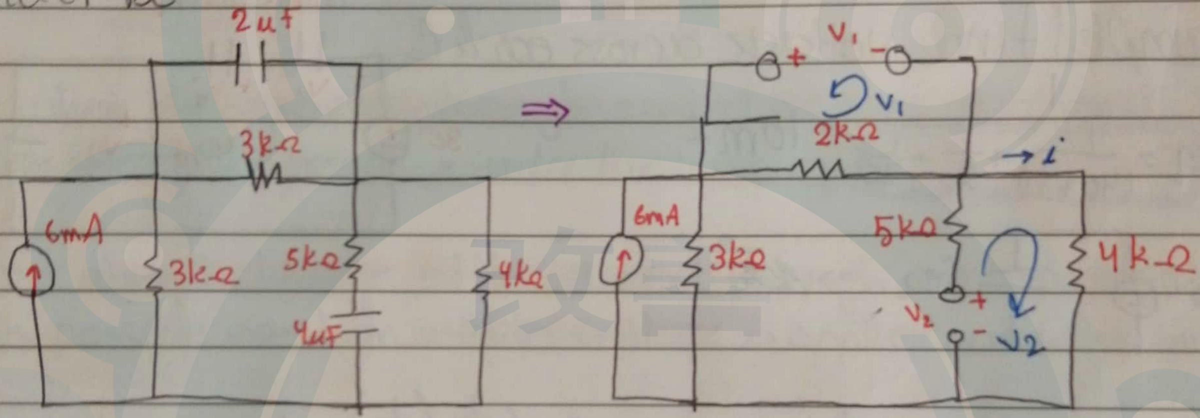


$$v(t) = \begin{cases} 50t\,V & 0<t<1 \\ 100-50t\,V & 0<t<3 \\ -200+50t\,V & 3<t<4 \\ 0 & \text{otherwise} \end{cases}$$

diff

$$i(t) = 200\mu \times \begin{cases} 50 \\ -50 \\ 50 \\ 0 \end{cases}$$

example find Energy in each capacitor under DC



under dc conditions, replace each by OC (no I, but has V)

$v_1 = 2000i = 4V$ $\qquad i = \frac{3}{3+2+4}(6mA) = 2mA$

$w_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2}(2\times10^{-3})(4)^2 = 16mj$

$v_2 = 4000i = 8V$

$w_2 = \frac{1}{2} C_2 v_2^2 = \frac{1}{2}(4\times10^{-3})(8)^2 = 128\,mj$

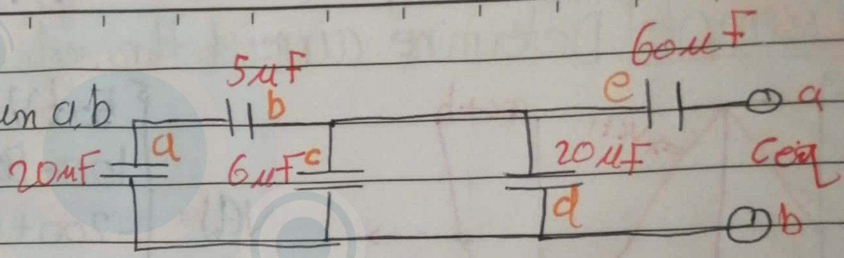
* i can treat capacitors ~~like~~ opposite resistor equ in series & parallel

parallel	$C_{eq} = C_1 + C_2 + C_3 + \dots C_n$	$i = i_1 + i_2 + i_3$
series	$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \dots 1/C_n$	$V = V_1 + V_2 + V_3$

Can also use Y-Δ but in opposite

$1/C = R$

example) Find eq C btwn a,b



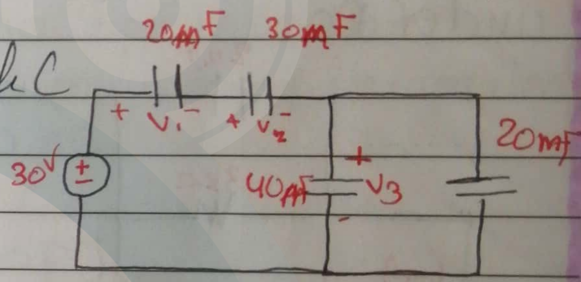
a, b series $\frac{1}{a} + \frac{1}{b} = \frac{5 \times 20}{5+20} = 4$

parallel with C => $4 + 6 = 10\mu$

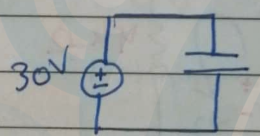
$10 + 20 = 30\ \mu$

$\frac{30 \times 60}{90} = 20\ \mu = eq$

example) Find voltage across each C



$Ceq = \frac{1}{\frac{1}{(40+20)} + \frac{1}{30} + \frac{1}{20}} = 10mF$



Using voltage division $V_1 = 30\left(\frac{1}{20} / \frac{1}{10}\right) = 15V$

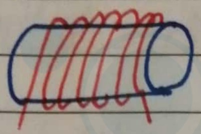
$V_3 = 30\left(\frac{1}{60} / \frac{1}{10}\right) = 5$ $\quad V_2 = 30\left(\frac{1}{30} / \frac{1}{10}\right) = 10V$

Five Apple

Inductors

Week 5 pt3

an inductor made of a coil of conducting wire



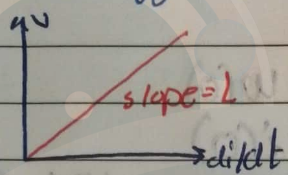
$$\boxed{L = \frac{N^2 \mu A}{l}}$$

N: # of turns
l: length
A: cross sectional Area
μ: permeability of the core

Stores Energy in seconds & restors in seconds

air-core

$$\boxed{V = L\frac{di}{dt}}$$



$$\boxed{i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)}$$

* when the current through an inductor is constant, the voltage across the inductor is zero $\boxed{\text{short circuit}}$ "DC"

* no abrupt change of the current through an inductor is possible except an infinite voltage is applied across the inductor

* can be used to generate a high voltage "igniting Element"

* the current through an inductor can't change abruptly

* ideal inductor doesn't dissipate energy

* Energy stored in inductor $\boxed{w = \frac{1}{2}Li^2}$ $\quad i(-\infty) = 0$

Ex $i = 10te^{-5t}$ $\quad L = 0.1H$ $\quad v = ?$ $\quad w = ?$

$V = L\frac{di}{dt}$

$V = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1-5t)V$

$w = \frac{1}{2}Li^2 \Rightarrow \frac{1}{2}(0.1)(100t^2)e^{-10t} = 5t^2e^{-10t}\ J$

ex] find current through 5-H if V is

$$v(t)\begin{cases}30t^2 & t>0 \\ 0 & t<0\end{cases}$$

find enery $0<t<5$
assume $i(0)=0$

$$i = 1/L \int_0^t v(t)\,dt + i(t_0)$$

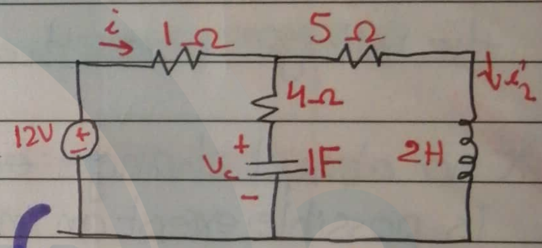
$$i = 1/5 \int_0^t 30t^2\,dt + 0 = 6 \times t^3/3 = 2t^3 A$$

Energy $0<t<5 \Rightarrow w(5) - w(0)$

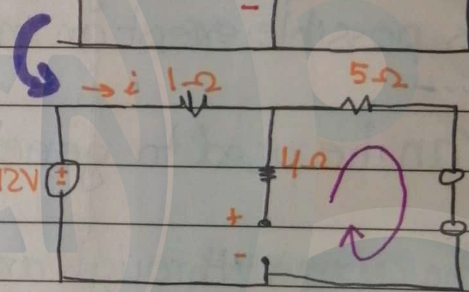
$$= 1/2\, L\, i(5) - 1/2\, L\, i(0)$$

$$= 1/2\; 5(2\times 5^3)^2 - 0 = 156.25 kJ$$

ex] under Dc conditions find
(a) i, V_c, i_L
(b) Energy stored in capacitor & inductor



on Dc, change Cap ⇒ OC, ind ⇒ S.C.
in that way there's no current through the 4-Ω Resistor so the circuit is of one loop $i = i_L$



$$i_L = i = \frac{V}{R} = \frac{12}{5+1} = 2A$$

$$-V_c + 5i = 0 \quad -V_c + 5(2) = 0 \quad , V_c = 10V$$

$$W_C = 1/2\, CV^2 = 1/2\,(1)(10)^2 = 50 J$$

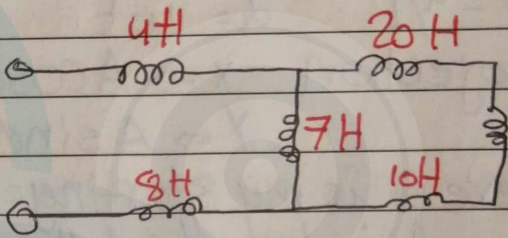
$$W_L = 1/2\, L i^2 = 1/2\,(2)(2)^2 = 4 J$$

Series inductors $L_{eq} = L_1 + L_2 + L_3 + \dots + L_n$

parallel inductors $1/L_{eq} = 1/L_1 + 1/L_2 + 1/L_3 + \dots + 1/L_n$

* just like a resistor, can use Y-Δ $L_Y = \frac{1}{3} L_\Delta$

ex] find Leq

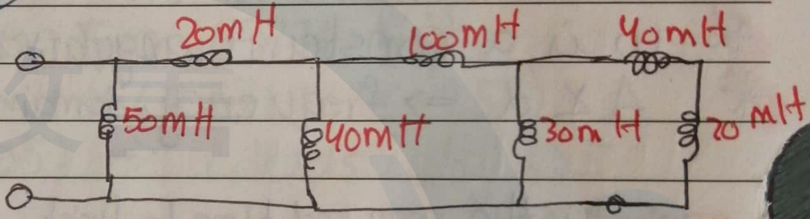


$20 + 12 + 10 = 42H$

$\frac{42 \times 7}{42+7} = 6H$

$6H + 4H + 8H = 18H = L_{eq}$

ex] Find leq



$40 + 20 = 60$

$\frac{60 \times 30}{60+30} = 20$

$20 + 100 = 120$

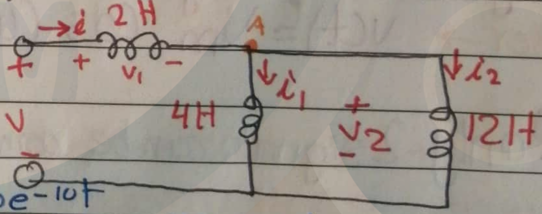
$\frac{120 \times 40}{120+40} = 30$

$30 + 20 = 50$

$\frac{50 \times 50}{50+50} = 25mH = L_{eq}$

ex] $i(t) = 4(2 - e^{-10t})$ mA if $i_2(0) = -1$mA

find $i_1(0)$; $v(t)$, $v_1(t)$, $v_2(t)$; $i_1(t)$, $i_2(t)$



- $i(0) = i_1(0) + i_2(0)$ KCL @(A)

  $4(2 - e^{-10(0)}) = i_1(0) - 1$

  $1 + 4 = i_1(0) = 5mA$
- $V_1 = L\frac{di}{dt} \Rightarrow V_1 = 2\frac{d}{dt}4(2 - e^{-10t}) = -80e^{-10t}$
- $L_{eq} = \frac{12 \times 4}{16} + 2 = 5H$, $V(t) = 5\frac{d}{dt}(4(2 - e^{-10t})) = 5/40 e^{10t}$ m
- $V_2 = V - V_1 \Rightarrow 5/40(e^{10t}) - (-80e^{-10t})$

$i_1 = \frac{1}{4}\int_0^t V_2 dt + i_1(0)$

$i_2 = 1/12 \int_0^t V_2 dt + i_2(0)$ $\Rightarrow$ OR $\Rightarrow$ $i_2(t) = i(t) - i_1(t)$

AC analysis

WG pt1

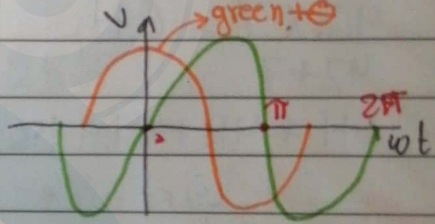
A sinusoidal voltage/current source produces a voltage that varies with time like a sinusoidal function (periodic), also cosine function

Phaser: $\alpha \Rightarrow$ angle , ω = angular velocity (ccw)

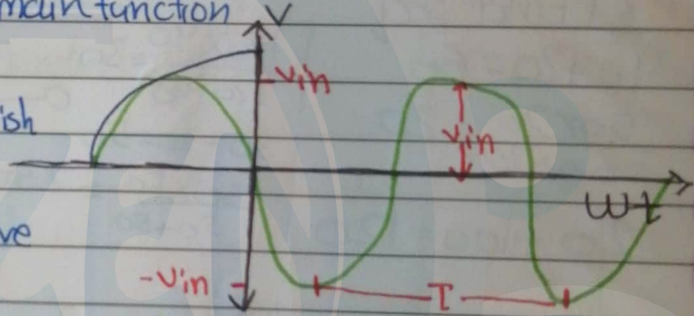
$\omega = \alpha / t \qquad \alpha = \omega t$

projection $x \Rightarrow A\cos\alpha = A\cos(\omega t) = A\cos(\omega t + \theta)$

$Y \Rightarrow A\sin\alpha = A\sin(\omega t) = A\sin(\omega t + \theta)$



- * where θ is my starting angle
- * A is length of the vector
- * time domain function
- * ω is a constant angular velocity
- * A & α => frequency domain function



period: is the required time to finish one cycle T (seconds)

Vin: peak, highest value of wave

frequency: # of cycles in 1 second)

$f = 1/T$ (Hertz or cycles per second)

Angular frequency: $\omega = 2\pi f = 2\pi / T$ (radians per seconds)

phaser → $V(t) = V_m \cos(\omega t + \phi)$, phase shift $\overset{+}{\leftarrow}$ $\overset{-}{\rightarrow}$

* only 2 signals can be compared if and only if they have same frequency

orange leads green by ϕ; green lags orange by ϕ → phase angle

* Cosine leads sin by 90°; Sin lag cosine by 90°

↳ out of phase

$$\boxed{\theta = \frac{\text{Phase shift (no. of div)}}{T(\text{no. of div})} \times 360}$$

Peak 0 → max
Amplitude avg → max
if 0 = avg ⇒ peak = amplitude

- Change sinusoidal to phaser $\boxed{v = V_m \cos(\omega t - 90 + \phi)}$

ex| given a sinusoid $5\sin(4\pi t - 60^\circ)$ calculate amplitude, phase, angular frequency, period & frequency
amplitude = 5   angular frequency = 4π rad/s   period $1/f = 0.5$ sec.
phase = -60° $\omega = 4\pi = 2\pi f \Rightarrow f = 2Hz$
* if we add 10 then the only thing that's gonna change is the peak which is $(5+10) = 15$ = DC ⇒ average

ex| find the phase angle btwn $i_1 = -4\sin(377t + 25)^\circ$ and $i_2 = 5\cos(377t - 40^\circ)$, Does i_1 lead or lag i_2?
$\sin(\omega t + 90^\circ) = \cos\omega t$
$i_2 = 5 \cos(377t - 40^\circ + 90) = 5\sin(377t + 50^\circ)$
$i_1 = -4\sin(377t + 25^\circ) = 4\sin(377t + 180 + 25^\circ)$
$= 4\sin(377t + 205) \Rightarrow i_1$ leads i_2 by 155

$\cos(\omega t - 90^\circ) = \sin\omega t$ $\sin(\omega t + 180^\circ) = -\sin\omega t$

phasor   Rectangular ⇒ $Z = x + jy = r(\cos\phi + j\sin\phi)$
Polar ⇒ $Z = r\angle\phi$ $r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1} y/x$
Exponetial ⇒ $Z = re^{j\phi}$

ex| $(5+j2)(-1+j4) - 5\angle 60^\circ$
using symbolab change polar to cartesian Precalculus $(x+iy)$
then go to complex numbers Pre-calc and enter the equation.

$e^{\pm j\theta} = \underbrace{\cos\phi}_{\text{Real}} \pm \underbrace{j\sin\phi}_{\text{imaginary}}$ Euler's identity

time domain $\boxed{V_m\cos(\omega t + \phi) \Leftrightarrow V = V_m\angle\phi}$ phasor domain
must be cosine

Five Apple

AC Analysis pt.2

W6 pt 2.

ex| Transform the following sinusoids to phasors

a) $6\cos(50t-40^\circ) = 6\angle -40^\circ$

b) $4\sin(30t+50)$ change to cos "+90"

$= 4\cos(30t+50+90) = 4(\cos(30t+140) = 4\angle 140^\circ$

b leads a by $(140+40) = 180^\circ$

a lags b by $(140+40) = 180^\circ$

ex| Transform the phasors to sinusoids

a) $V = -10\angle 30^\circ = 10\angle 30+180 = 10\angle 210^\circ = 10\cos(\omega t + 210)$

b) $j(5-j12) = j5-(-1)(12) = 12+j5,\ r = \sqrt{12^2+5^2} = 13$

$\phi = \tan^{-1}(5/12) = 22.62^\circ \Rightarrow 13\cos(\omega t + 22.62^\circ)$

on calculator ⇒ mode → Cmplx ⇒ "write equ" ⇒ shift $\boxed{2}$ ⇒ $\boxed{3}$ ⇒ $\boxed{=}$

ex| $5\cos(10t\cancel{+30}) + 10\sin(10t+90^\circ) + 4$

i can add things up only if $\omega = \omega$

the 4 @ the end represents the DC AKA averge of signal $f=0$

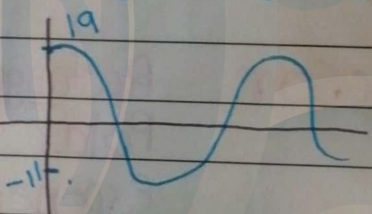
so i treat it like a real number.

$10\sin(10t+90) = 10(\cos(10t)$

$= 5\angle 0 + 10\angle 0 = 15\angle 0 + 4$

$= 15\cos(10t) + 4$ Amp = 15 peak = 15+4 = 19

min = 15 − 4 ⇒ 11



Diff btwn v(t) & V

- v(t) is instanteous or time domain representation
- V is the frequency or phaser domain representation
- v(t) is time-dependant v is not.
- v(t) is always real with no complx term. V is generally cmplx

$\omega = 2\pi f$

Time Domain		Phasor domain
$v(t) = V_m \cos(\omega t + \phi)$	$\Leftrightarrow$	$V = V_m \angle \phi$
$v(t) = V_m \sin(\omega t + \phi)$	$\Leftrightarrow$	$V = V_m \angle \phi - 90$
dv/dt	$\Leftrightarrow$	$J\omega V$
$\int v\,dt$	$\Leftrightarrow$	$V/J\omega$

$\boxed{ex}$ Use phasor approach, determine the current $i(t)$ in a circuit described by the integro-differential equ.

$$4i + 8\int i\,dt - 3\,di/dt = 50\cos(2t + 75°)$$

$$4i + 8i/J\omega - 3J\omega = 50\angle 75$$

$$i(4 + 8/J\omega) = 50\angle 75 + 3J\omega \qquad \omega = 2$$

$$i(4 + 8/J2) = 50\angle 75 + 3J2 \qquad I(4-10j) = 50\angle 75$$

$$i(4 + 4/J) = 50\angle 75 + 6J \qquad I = \frac{50\angle 75}{4-10j}$$

$$i(t) = 4.642\cos(2t + 143.2)$$

impedance $= Z\,(\Omega)$

$R \Rightarrow Z = R$

$L \Rightarrow Z = j\omega L$

$C \Rightarrow Z = -j/(\omega C) = 1/(j\omega C)$

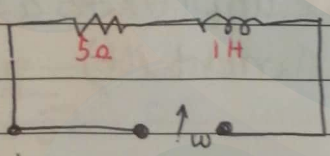
$V = IR \Rightarrow V = IZ$ (all in vectors)

$V\angle\phi = (I\angle\phi)(Z\angle\phi)$

Real $\pm$ J imag

Resistance — Reactant (Ω)

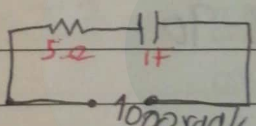
$\boxed{ex}$



$5\Omega \Rightarrow 5\Omega$; $1H \Rightarrow j10\omega$

$5 + j10 \qquad \omega = 10$

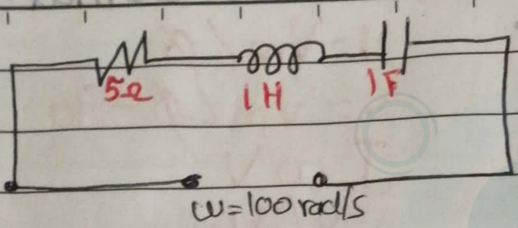
inductive load.



$5\Omega \Rightarrow 5\Omega \quad 1F$

$Z = 5\Omega\ J/100(1)$

capacitive load



$Z = 5 + J100 - J/100$

$= 5 + J\,99.99$

inductive load

Cases:

$Z \Rightarrow$ Resistive Pure (only real) rel $+ J0$ $\parallel \measuredangle 0$

inductive load real $+ JX$ $\parallel \measuredangle +ve$

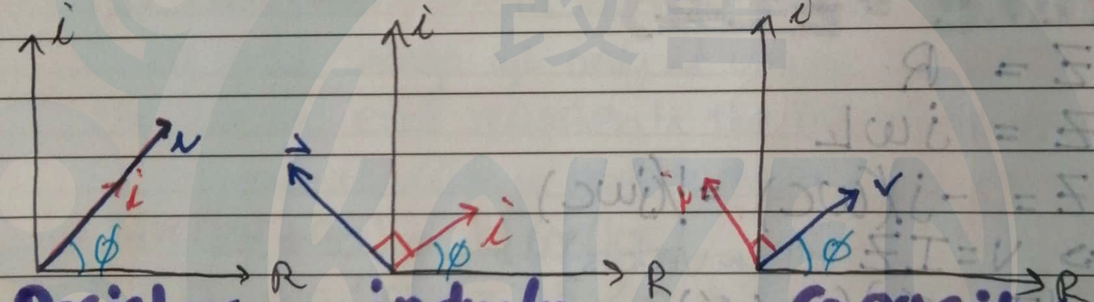
capacitive load $R - JX$ $\parallel \measuredangle -ve$

$1/Z \Rightarrow$ admitance $\Rightarrow y$ (S) $= \frac{1}{R+JX}$ $= \boxed{\frac{R-JX}{R^2+X^2}}$

$\boxed{Y = G \pm JB}$

conductance sospitance

$V = IZ \Rightarrow V = I/y \Rightarrow I = Vy$



Resistor in phase

inductor i lags v by 90

Capacitor v lags i by 90°

ex if voltage $v(t) = 6\cos(100t - 30°)$ is applied to a 50 µF capacitor, Calculate the current $i(t)$ through the capacitor

$6\cos(100t - 30) \Rightarrow 6\angle -30$

$50\,\mu F = Z_c = 1/J(100)\times 50\times 10^{-6}\,\Omega = 200\angle -90$

$I = V/Z = \frac{6\angle -30}{200\angle -90} = 30000\angle -60$

$= 30\cos(100t + 60)\,mA$

1.49FW

element impedance admittance

R	$Z = R$	$Y = 1/R$
L	$Z = j\omega L$	$Y = 1/j\omega L$
C	$Z = 1/j\omega C$	$Y = j\omega C$

$\frac{1}{Y} = Z$

L, $Z = j\omega L$

DC ⇒ short —o–o— $\omega = 0,\ Z = 0$

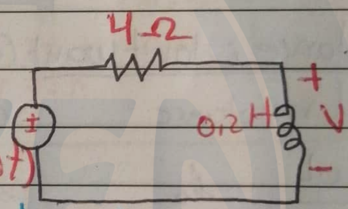
↑ frequency ⇒ open —o o— $\omega \to \infty,\ Z \to \infty$

C, $Z = 1/j\omega C$

DC ⇒ open —o o— $\omega = 0 \quad Z \to \infty$

high frequency ⇒ short —o–o— $\omega \to \infty \quad Z = 0$

ex) find $v(t)$ and $i(t)$



Change everything to phase (must always be cos)

$5\cos(10t) = 5\angle 0 \qquad 4\Omega = 4i \qquad 0.2H = j10 \times 0.2 = j2\Omega$

$KVL = -5\angle 0 + 4i + j2i = 0$

$\frac{5}{4+j2} =$ on calc ⇒ then shift 2 $r\angle\theta = 1.118\cos(10t - 26.56)A$

$V = IZ \Rightarrow j2 \times 1.118\angle -26.56$

on calc ⇒ shift $\boxed{2}$ $r\angle\theta$

Zin ⇒ find eq like Resistors.

W7 Pt.1

AC Analysis Pt3

Summary:- Source ⇒ $A\cos(\omega t + ph)$ phasor A ang=ph

impedance $Z =$ (Ω)
$$\left\{\begin{array}{l} R \Rightarrow R \\ L \Rightarrow J\omega L \\ C \Rightarrow 1/J\omega C = -J\omega C \end{array}\right\} \quad \begin{array}{l} \| \measuredangle \theta Z \\ R \pm Jx \end{array}$$

$$V = IZ \iff V = I/y$$

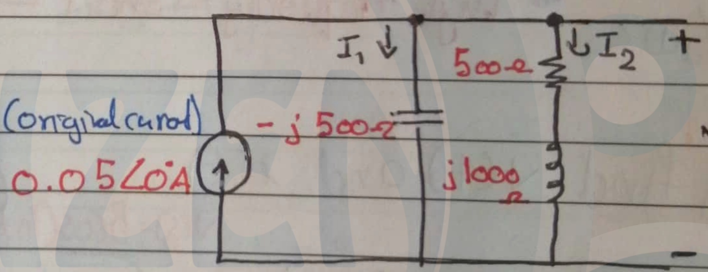
if Resistive load ⇒ V & I in phase "same angle"

if inductive load ⇒ i lags V $\measuredangle V - \measuredangle i = \measuredangle Z$

if capacitive load ⇒ i leads V $\theta_V - \theta_i = \theta_Z$

example: application of current Division for phasors

find I_1



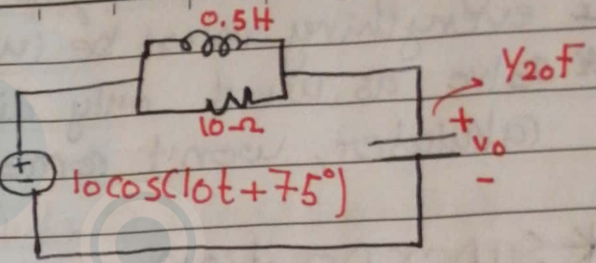
$$i = \frac{\text{resistance } i \text{ don't want (original current)}}{\text{total resistance}}$$

$$i_1 = \frac{R + j\omega L}{R + j\omega L + 1/j\omega C}\, i_0$$

$$= \frac{(500 + j1000)}{500 + j1000 - j500}(0.05\angle 0°) = i_1 = 0.079\angle 18.4° A$$

$$i_2 = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C}(i) = \frac{-j500}{500 + j1000 - j500}(0.05\angle 0) \Rightarrow i_2 = 0.03535\angle{-45°}A$$

Example:- Calculate $v_o(t)$



Change voltage => $10\angle 75$

0.5 H inductor => $j\omega L = j10 \times 0.5 = j5$

$1/20F \Rightarrow 1/j\omega c = 1/j10(1/20) = -j2$

let Z_1 = impedance of inductor // resistor

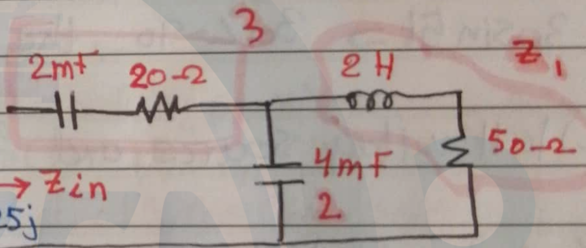
$Z_1 = 10 // j5 = (10 \times j5)/(10+j5) = 2+4j$

Z_2 = impedance of capacitor $= -j2$

V_0 = Voltage i want (original voltage)
total voltage

$V_0 = \frac{Z_2}{Z_1+Z_2}(V) = \frac{-j2}{-j2+2+4j} \times 10\angle 75 = 7.07\angle -60$
$= 7.07\ (\cos(10t-60))$

Exeemple: find Z_{in}, $\omega = 10$ rad/s



$Z_1 \Rightarrow$ Series $\Rightarrow 50 + j\omega L = 50 + j20$

$Z_1 // Z_2 \Rightarrow Z_2 \Rightarrow 1/j\omega C = 1/j(10)(4\times10^{-3}) = -25j$

$\frac{(50+j20)\times(-25j)}{50+j20-25j} = 10.891 + 1.089j$

$Z_3 \Rightarrow 20 + 1/j\omega c = 20 + 1/j(10)(2\times10^{-3}) = 20-50i$ $\quad Z_{total} = 32.37 - 73.76j$

Driving point impedance => total Z
Driving Point admittance => 1/total Z
phasor current = phasor voltage / total Z
particular response (steady state-response) = current in Sinosudal form.
current leads voltage & vise versa by ∡Z
To find which is which on graph find point that they both move together (up or down) and see who hits point first (down => point 0)

* Using Nodal Analaysis and Mesh

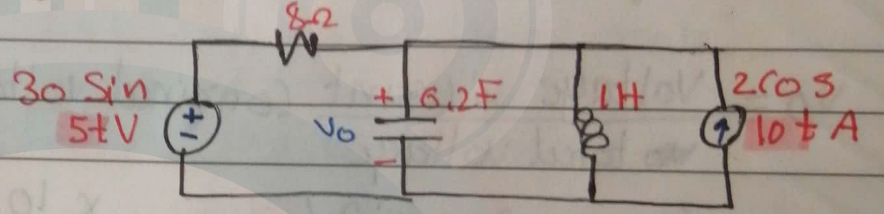
* everything must be in phasor form
* Solve as usual only differance can't solve using calculator, won't come compleated in test.

* Superposition

-used when i have different frequncies.

example:- find V_o using superposition

Since voltage & current source have different ω



1) kill current then solve
$30\sin 5t \Rightarrow 30\angle -90$, the $\omega = 5$ then use voltage division

2) kill voltage source and find impedance when $\omega = 10$

* can't add both because i have different frequncies write in time domain (frequence in $A\cos\omega t + \theta$

* using source transformation & Thevenin & Norton

Max power $Z_L = Z_{th}^*$ → pure resistance

W7Pt2.

AC Power Calc Pt.1

instanteous Power

$P(t) = V(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_I)$

$$\boxed{P(t) = \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{constant Power, Average Power}} + \underbrace{\tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{sinusoidal Power, dependant on } 2\omega}}$$

Average Power: $\theta_v = \theta_i \Rightarrow$ resistive load case

$\theta_v - \theta_i = \pm 90$ reactive load case $P = 0 \Rightarrow \cos 90 = 0$

$P = 0 \Rightarrow$ absorbs no average power

Example:- find inst & Avg Power absorbed by passive

$v(t) = 80\cos(10t + 20^\circ)$ $\quad i(t) = 15\sin(10t + 60^\circ)$ $\quad -90^\circ$

$= 15\cos(10t - 30)$

Average $= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \cdot 80 \cdot 15 \cdot \cos(20 - (-30)) = 385.67\,W$

inst $=$ Average $+ \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) = 385.6 + 600\cos(20t - 10^\circ)$

Example; current $I = 10\angle 30^\circ$ flows through impedance find Average P if $Z = 20\angle -22^\circ\,\Omega$

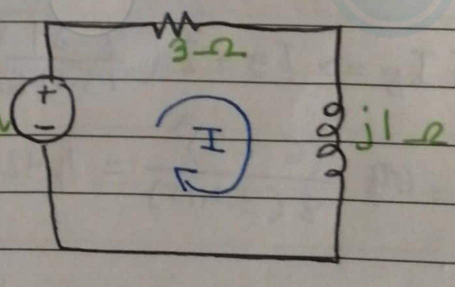
$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ $\quad \theta_v - \theta_i = \theta_Z$ $\quad V = IZ = 20 \times 10 = 200$

$\therefore = \frac{1}{2}\, 200 \times 10 \cos(-22) = 927.18\,W$

Example Avg Powers Absorbed by Resistor & inductor, Psupplied by the source

total $Z = 3 + j\,\Omega$

$I = \dfrac{8\angle 45}{3 + j} = 2.53\angle 26.56^\circ$



Avg Power in Resistor => $V_R = IR = 7.589\angle 26.56$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(2.53)(7.589)\cos(0) = 9.6\,W$

Avg Power in inductor $V_L = IL = 2.53\angle 116.57$

$P = \frac{1}{2}\; 2.53 \times 2.53\; \cos(90) = 0\,W$

Avg Power Applied $= \frac{1}{2}(8)(2.53)\cos(45 - 26.57) = 9.6\,W$

* watch out for the signs and where the current enters the voltage

Max Power transfer

if $x_L = -x_{th}$ and $R_L = R_{th}$

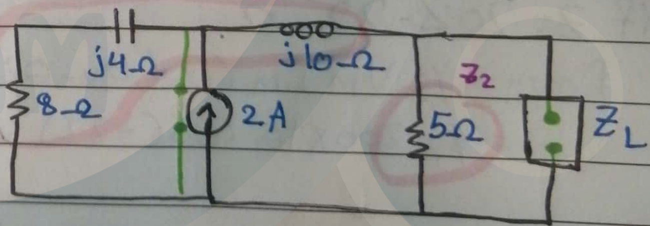
$$\boxed{P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}}$$

if the load is purely real $\boxed{R_L = \sqrt{R_{th}^2 + x_{th}^2} = |Z_{th}|}$

$\boxed{Z_L = R_{th} - jX_{th} = Z_{th}^*}$ => at max power transfer

Example: find Z_L that absorbs max Avg Power, max Avg Power

$Z_1 = $ series => $8 + j4 + j10 = 8 + 6i$



$Z_1 // Z_2 = \frac{(8+16j) \times 5}{8 + 6j + 5} = 3.415 + j0.73\Omega$

$Z_{th}^* = 3.415 - j0.73\Omega = Z_L$

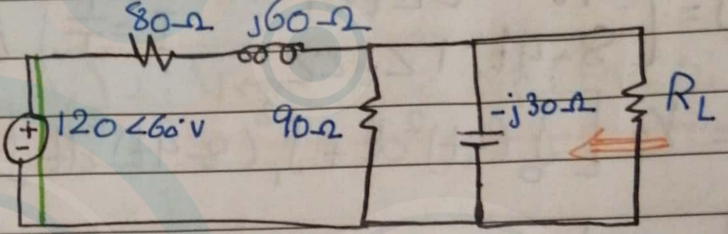
$V_{oc} = 5I_5 \Rightarrow I_5 = 2\left(\frac{8-j4}{13+j6}\right) = 6.25\angle -51.34° = V_{th}$

$P_{max} = \frac{(6.25)^2}{8(3.415)} = 1.429\,W$

Maximum Average Power for Resistive load

When load is purely resitive $\boxed{R_L = \sqrt{R_{th}^2 + X_{th}^2} = |Z_{th}|}$

*first find Z_{th} and V_{th} across R_L



$80\Omega + j60\Omega = 80 + j60$

$90 // -j30 = \dfrac{90 \times -j30}{90 - j30}$

$= 9 - 27j \; // \; 80 + j60 \Rightarrow 17.181 - j\,24.57\,\Omega = Z_{th}$

$R_L = |Z_{th}| = \sqrt{17.181^2 + 24.57^2} = 29.99\,\Omega \approx 30\,\Omega$

$\boxed{V_{th} = \overset{V_s}{120\angle 60}\left(\dfrac{9-27j}{(9-27j) + 80 + j60}\right)} = 35.98\angle -31.91$

Current through load $I = \dfrac{V_{th}}{Z_{th} + R_L} = \dfrac{35.98\angle -31.91}{30 + (17.181 - j\,24.57)} = 0.67\angle -4.4$

Max Avg Power absorbed by R_L is

$\boxed{P_{max} = \tfrac{1}{2} |I_L|^2 R_L} = \tfrac{1}{2} \times (0.67)^2 \times 30 = 6.863\ \text{W}$

Effective or RMS value

if $AC \Rightarrow P = \tfrac{1}{2} I_m^2 R$ if $DC \Rightarrow P = I_{ef}^2 R$

only if pure sin/cos

$\boxed{I_{eff} = I_m / \sqrt{2}}$

the effective value or root mean square (RMS) value of a perodic current is the DC value that delivers the same P_R As perodic current

$\boxed{I_{eff} = \sqrt{\dfrac{1}{T}\int_0^T i\,dt}}$

$P = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

$P_R = I_{Rms}^2 R = \dfrac{V_{Rms}^2}{R}$

Example

find RMS Value of the current waveform, calculate Avg Power if the current is applied to a 9Ω resistor

$$i(t) = \begin{cases} 4t & 0<t<1 \\ 8-4t & 1<t<2 \end{cases} \qquad I^2 = \frac{1}{T}\int_0^T i^2\,dt$$

$$I^2 = \tfrac{1}{2}\left[\int_0^1 (4t)^2 dt + \int_1^2 (8-4t)^2 dt\right] = 16/3$$

$$I_{rms} = \sqrt{16/3} = 2.309A$$

$$P = I_{rms}^2 R = (16/3)(9) = 48W$$

Ex2) $T=\pi$, $V(t) = 8\sin(t)$, $0<t<\pi$, find RMS, Avg P, if R=6

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt = 1/\pi \int_0^\pi (8\sin(t))^2 dt = 32$$

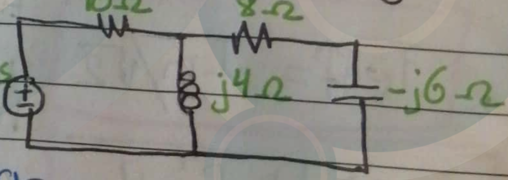
$$V_{rms} = \sqrt{32} = 5.65V \qquad P = \frac{V_{rms}^2}{R} = 32/6 = 5.333W$$

Apparent Power = $V_{rms} \times I_{rms}$ = S (VA)

Power factor = $\cos(\theta_v - \theta_i)$ $= \cos(\theta_Z)$ (0→1) (unitless)

P = S Power factor

Example:- calculate Pf seen by the source and the Avg power supplied by the source.



total $Z = 10 + j4 \,||\, (8 - j6) = \cancel{12.69}$

$= 12.69\angle 20.62°$

$pf = \cos(\theta_Z) = \cos(20.62) = 0.936$ lagging

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40\angle 0}{12.69\angle 20.62} = 3.152\angle -20.62°$$

Avg P supplied = Avg P absorbed

$P = I_{rms}^2 R = (3.152)^2 (11.88) = 118W$ OR

$P = V_{rms} I_{rms}\, pf = (40)(3.152)(0.936) = 118W$

Complex Power

$V = V_m \angle \theta_v \qquad I = I_m \angle \theta_i$

$$S = \tfrac{1}{2} V I^* = |V_{rms}||I_{rms}| \angle \theta_v - \theta_i$$

$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

P: is the average Power in Watts (Active) delivered to a load and it's the only useful power (W)

Q: is the reactive power exchange btwn the source and the reactive part of the load (VAR)

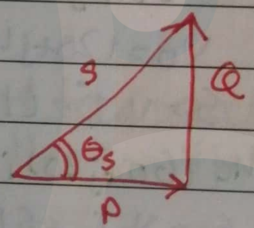
$Q = 0$	resistive loads	unity pf	$\therefore X = 0$
$Q < 0$	capacitive loads	leading pf	$-jX + R$
$Q > 0$	inductive loads	lagging pf	$R + jX$

$P = I^2_{RMS} R$

$Q = I^2_{rms} X$

$S = \sqrt{P^2 + Q^2}$

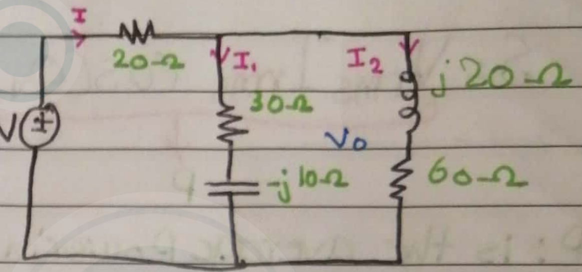
Power triangle ⇐



W8 P61

AC. Power Calc

Example: The 60 Ω resistor absorbs 240W Avg Power, Calc V & Cmplx ~~Avg~~ Power of each branch, total complex power?



$P = I_2^2 R \Rightarrow I_2 = \sqrt{\frac{P}{R}} = \sqrt{\frac{240}{60}} = 2$ rms.

$V_o = I_2 (60 + j20) = 120 + j40$

$I_1 = \frac{V_o}{30 - j30} = 3.2 + j2.4$

$I = I_1 + I_2 = 5.2 + j2.4$

$V = 20I + V_o = (104 + j48) + (120 + j40)$

$V = 224 + j88 = 240.67\angle 21.45$ rms

for the 20 Ω resistor

$V = 20I = 204 + j48 = 114.54\angle 24.8$, $I = 5.2 + j2.4 = 5.727\angle 24.8$

$S = VI^* = (114.54\angle 24.8)(5.727\angle -24.8) \Rightarrow S = 656$ VA

for the (30-j10) impedance

$V_o = 120 + j40 = 126.5\angle 18.43°$ $I_1 = 3.2 + j2.4 = 4\angle 36.87°$

$S_1 = V_o I_1^* = (126.5\angle 18.43°)(4\angle -36.87)$, $S_1 = 506\angle -18.44 = 480 - j160$ VA

for the (60+j20) impedance $I_2 = 2\angle 0°$

$S_2 = V_o I_2 = (126.5\angle 18.43°)(2\angle -0)$ $S_2 = 253\angle 18.43° = 240 + j80$ VA

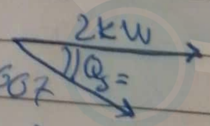
overall complex power supplied by the source

$S_T = VI^* = (240.67\angle 21.45°)(5.727\angle 24.8) = 1378\angle -3.35 = 1376 - j80$ VA

Example: loads parallel, load1 has 2kW, pf=0.75 leading, load2, 4kW pf=0.95 lagging, find pf of 2 loads, complex power supplied by source

load1 ⇒ leading ⇒ $-\cos(0.75) = \theta = -41.4$

$P_1 = S_1 \cos\theta_1 \Rightarrow S_1 = P_1/\cos\theta_1 = 2000/0.75 = 2000/\cos(-41.4) = 2666.667$



$Q_1 = S_1 \sin\theta_1 = -176.85$, $S_1 = P_1 + jQ_1 = 2000 - j1763.85$ (leading)

load2 ⇒ $P_2 = 4000$, $Pf = 0.95 = \cos\theta_2 \Rightarrow \theta_2 = 18.19$, $S_2 = P_2/\cos\theta_2 = 4210.53$

$Q_2 = S_2 \sin\theta_2 = 1314.4$, $S_2 = P_2 + jQ_2 = 4000 + j1314.4$ lagging

Total complex power ⇒ $S = S_1 + S_2 = 6 - j0.4495$ kVA

$pf = P/|S| = 6000/6016.18 = 0.9972$ leading

* Complex real, reactive powers of the sources equal the respective sums of the complex, real, reactive powers of the individual loads
* for parallel or series connection.

$$S = V_1 I_1^* + V_2 I_2^* + \dots V_N I_N^*$$
$$S = S_1 + S_2 + \dots + S_N$$

Power Factor Correction: W8Pt2.

is the process of increasing the power factor without altering the voltage or current to the original load
preferable to have high value, necessary for econimic reason.
put cappacitance in parallel with inductive load, vise versa

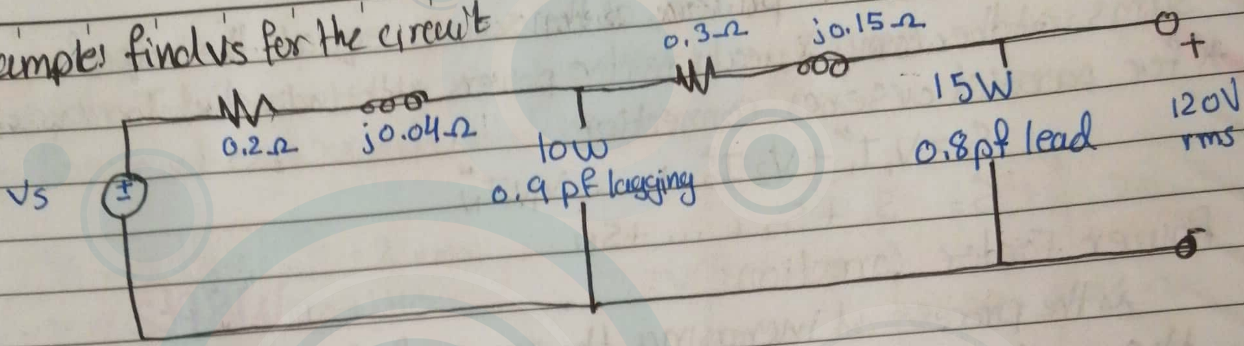
$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2) = \omega C V^2_{rms}$
$Q_1 = S \sin\theta_1 = P(\tan\theta_1)$
$C = Q_c / \omega V^2_{rms} = P(\tan\theta_1 - \tan\theta_2) = \omega V^2_{rms}$
$P = S_1 \cos\theta_1$, $Q_2 = P \tan\theta_2$
$L = V^2_{rms} / \omega Q_L$

Example: find the value of the capacitance needed to correct a load of 140k VAR at 0.85 lagging pf to unity pf. the load is supplied by a 110 volt(rms), 60Hz line.

$pf = 0.85 = \cos\theta$, $\theta = 31.79°$
$Q = S\sin\theta = S = Q/\sin\theta = 140/\sin(31.79°) = 265.8 \text{ kVA}$
$P = S\cos\theta = 225.93 \text{ kW}$
for $pf = 1 = \cos\theta \Rightarrow \theta_1 = 0$, since P remains same
$P = P_1 = S_1 \cos\theta_1 \Rightarrow S_1 = P_1/\cos\theta_1 = 225.93$
$Q_1 = S_1 \sin\theta_1 = 0 \Rightarrow Q_n = 0$
difference btwn the new Q_1 and the old Q is Q_c
$Q_c = 140 \text{ KVAR} = \omega C V^2_{rms}$

$$C = \frac{140 \times 10^3}{(2\pi)(60)(110^2)} = 30.69 \text{ mF.}$$

Example: find Vs for the circuit



$S_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$

$S_2 = V_2 I_2^*$, $I_2^* = S_2/V_2 = 15 - j11.25/120 = 0.125 + j0.09375$

$V_1 = V_2 + I_2(0.3 + j0.15)$

$V_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$, $V_1 = 120.02 + j0.0469$

$S_1 = 10 + j\,10/0.9\,\sin(\cos^{-1}(0.9)) = 10 + j4.843$

$S_1 = V_1 I_1$, $I_1^* = S_1/V_1 = \frac{11.111\angle 25.84^\circ}{120.02\angle 0.02} = I_1 = 0.093\angle -25.82^\circ$

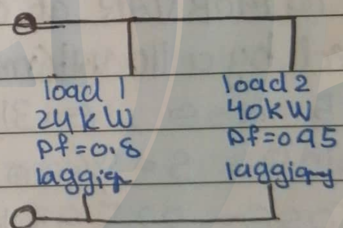
$I_1 = 0.0837 - j0.0405$, $I = I_1 + I_2 = 0.2087 + j0.053$

$V_s = V_1 + I(0.2 + j0.04)$

$V_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$

$V_s = 120.06 + j0.0658$ $V_s = 120.06\angle 0.03^\circ$ V

Example: A 120-Vrms, 60-Hz source, find pf, C



$\theta_1 = \cos^{-1}(0.8) = 36.87^\circ$, $S_1 = P_1/\cos\theta_1 = 24/0.8 = 30$ kVA

$Q_1 = S_1 \sin\theta_1 = 30(\sin(36.87)) = 18$ kVAR , $S_1 = 24 + j18$ kVA

$\theta_2 = \cos^{-1}(0.95) = 18.19$, $S_2 = P_2/\cos\theta_2 = 40/0.95 = 42.105$ kVA

$Q_2 = S_2 \sin\theta_2 = 13.144$ kVAR , $S_2 = 40 + j13.44$ kVA

$S = S_1 + S_2 = 64 + j31.144$ kVA

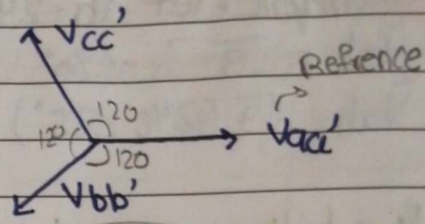
$\theta \tan^{-1}(31.144/64) = 25.95^\circ$ $pf = \cos\theta = 0.8992$ (lagging)

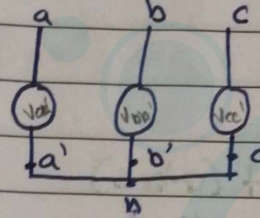
$\theta_2 = 25.95, \theta_1 = 0^\circ$, $Q_C = P(\tan\theta_2 - \tan\theta_1) = 64\tan(25.95) = 31.144$ kVAR

$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{31.144}{(2\pi)(60)(120)^2} = 5.74\,mf$

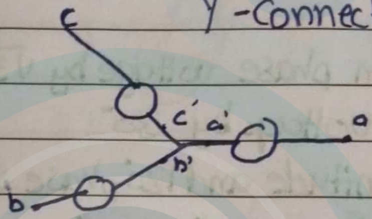
Three-Phase-System pt:1

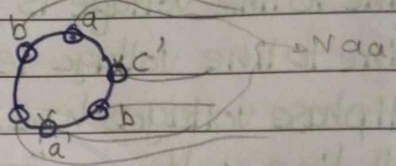
3 phase ⇒ produces constant power
⇒ has 6 terminal wires
connected as Y or Delta.



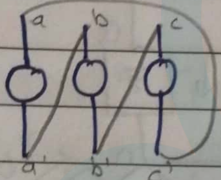


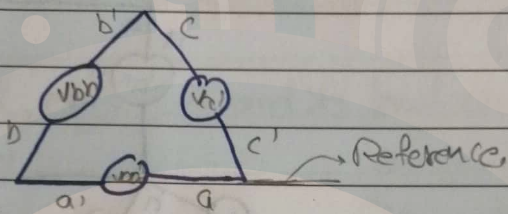
Y-connection



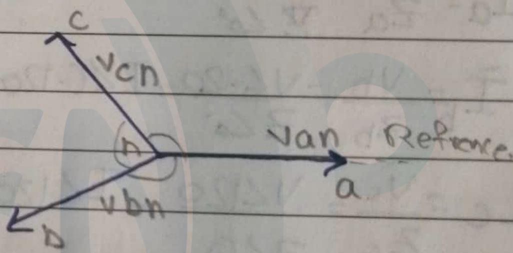


Delta connected





phase voltage ⇒ phase to ground voltage
line voltage ⇒ line to line voltage



$\bar{V}_{an} = \bar{V}_a = V\angle 0$
$\bar{V}_{bn} = \bar{V}_b = V\angle -120$
$V_{cn} = \bar{V}_c = V\angle 120$
} positive sequence

$\bar{V}_{ab} = V_{an} - V_{bn} = (V\angle 0) - (V\angle -120) = \sqrt{3}\, V\angle 30$ $-120°$
$\bar{V}_{bc} =$ $= \sqrt{3} V\angle -90$
$\bar{V}_{ca} =$ $\sqrt{3}\, V\angle 150$ $+120$

unless stated otherwise:
- all voltages are line-line
- all powers are for 3 phase
- positive sequence
- voltage sources are all balanced.

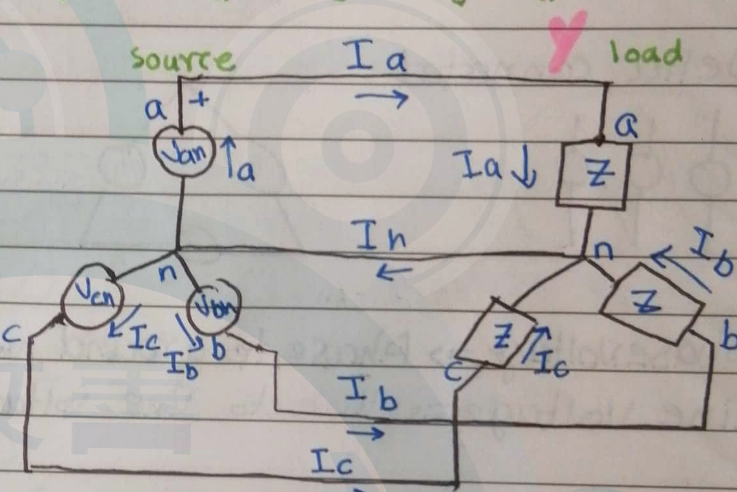
Example: let $V_a = 240\angle 25$, calculate line-line voltage

$\bar{V}_{ab} = \sqrt{3}\,\bar{V}_a \angle 30^\circ$

$\bar{V}_{ab} = \sqrt{3}(240\angle 25^\circ)\angle 30^\circ = 415.7\angle 55^\circ$

line to line voltage is greater than phase voltage by $\sqrt{3}$
line to line voltage leads phase voltage by 30°
All phase voltages have same magnitude and 120° phase shift (a-b-c)
All line voltages " " " " " " "

Y



$\bar{Z}_a = \bar{Z}_b = \bar{Z}_c = \bar{Z}$

$\bar{V}_{an} = \bar{V}_a = V\angle 0$

$\bar{V}_{bn} = \bar{V}_b = V\angle -120^\circ$

$\bar{V}_{cn} = \bar{V}_c = V\angle 120$

$$I_a = \frac{\bar{V}_a}{Z_a} = \frac{V\angle 0^\circ}{Z\angle \theta^\circ} = I\angle -\theta^\circ$$

$$\bar{I}_b = \frac{V_b}{Z_b} = \frac{V\angle -120}{Z\angle \theta^\circ} = I\angle(-120-\theta^\circ)$$

$$I_c = \frac{\bar{V}_c}{Z_c} = \frac{V\angle 120}{Z\angle \theta} = I\angle(120-\theta^\circ)$$

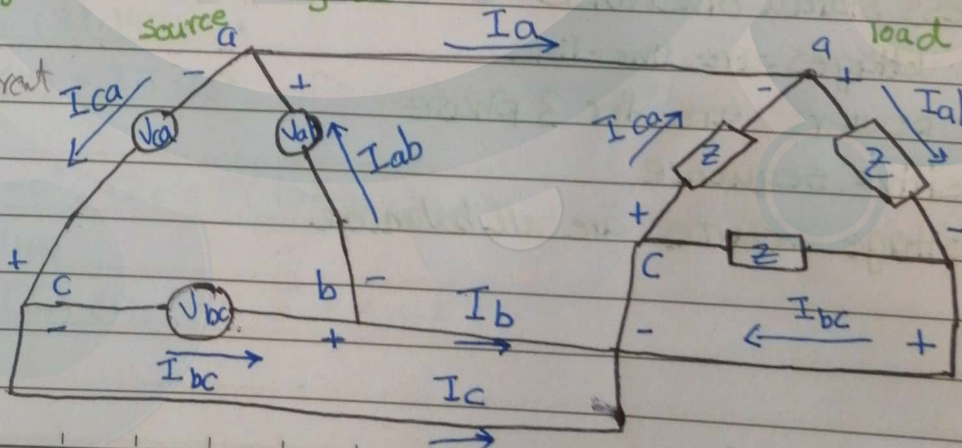
+120

$I_n = I_a + I_b + I_c = 0$

Δ

load voltage is equal to line-line voltage
line current is greater than load current $\bar{I}_a = \sqrt{3}\, I_{ab}\angle -(\theta + 30^\circ)$
line current lags load current by 30°

$I_a \Rightarrow$ line current
$I_{ab} \Rightarrow$ generator current



$\bar{I}_{ab} = \bar{I}_a + \bar{I}_{ca}$

$$I_{ab} = \frac{I_a}{\sqrt{3}}\angle 30^\circ$$

$I_{bc} = I_{ab}\angle \theta_{bc} - 120$

$I_{ca} = I_{ab}\angle \theta_{bc} + 120$

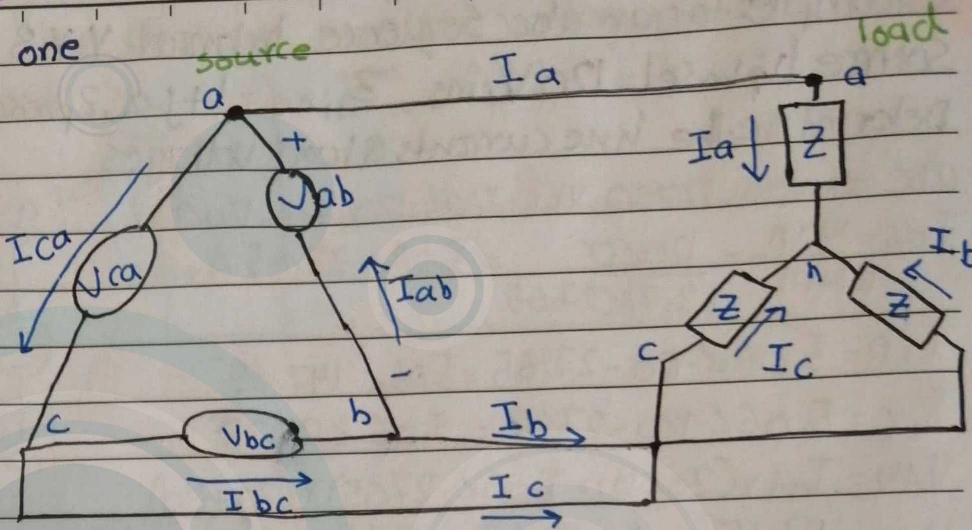
$I_{ac} = I_{ca}\angle \theta_{ca} - 180$

Mixed Connections

in this case, transform one

Y-Δ Transformation

$Z_Y = \frac{1}{3} Z_\Delta$

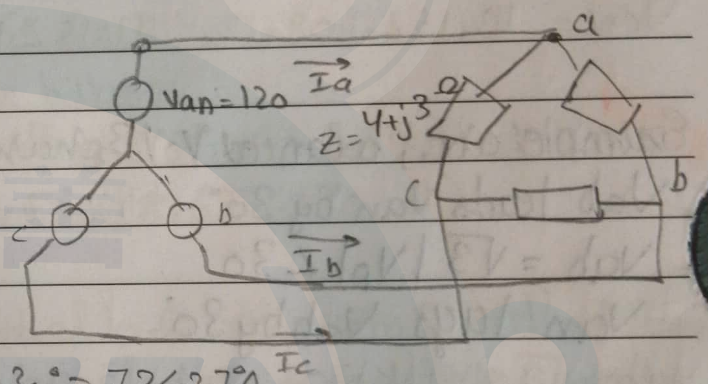


Example: Current of load, eq Y load and it's load current

Van ⇒ phase voltage = 120V

current of load $\bar{I}_{ab} = \frac{V_{ab}}{Z}$

$= \frac{\sqrt{3} \times 120\angle 30}{5\angle 37^\circ} = 41.57\angle -7^\circ A$



Part 2 ⇒ $Z_Y = \frac{Z_\Delta}{3} = \frac{5\angle 37^\circ}{3} = 1.67\angle 37$

Part 3 ⇒ load current in Δ load =

$\bar{I}_a = \sqrt{3}\, I_{ab}\angle -30 = \sqrt{3}(41.57\angle -7^\circ)\angle -30^\circ = 72\angle -37^\circ A$

$I_c = 72\angle -37+120$, $I_{ac} \Rightarrow I_{ca} = 41.57\angle -7+120$ $I_{ac} = I_{ca}\angle \theta - 180$

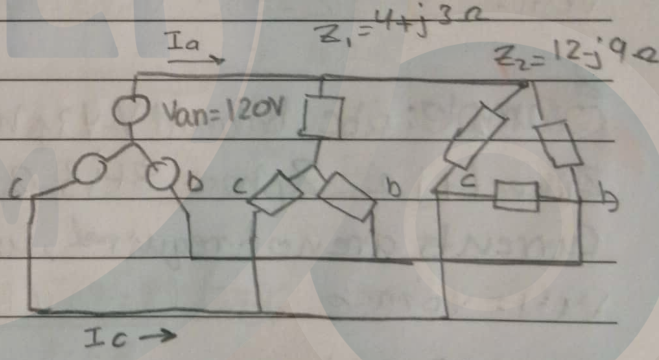
Examples: calculate line current

2 loads in parallel, transform 1

$Z_{2_Y} = \frac{1}{3} Z_2 = \frac{12-j9}{3} = 4-j3\,\Omega$

$Z_{2Y} // Z_1 = 25/8\,\Omega$

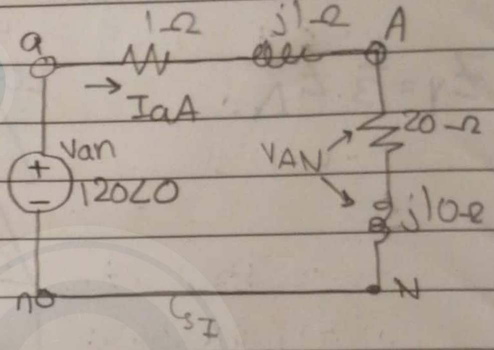
$I_a = \frac{120\angle 0}{25/8} = 38.4A$



Example: for an abc Sequence, balanced Y-Y 3 phase circuit
Source $|V_{phase}| = 120\,V_{rms}$, $Z_{line} = 1+j\,\Omega$, $Z_{phase} = 20+j10\,\Omega$
Determine the line currents & load voltages

abc ⇒ $V_{an} = 120\angle 0$, $V_{bn} = 120\angle -120$, $V_{cn} = 120\angle 120°$

$I_{aA} = \frac{V_{an}}{21+j11} = \frac{120\angle 0°}{23.71\angle 27.65} = 5.06\angle -27.65\ A_{rms}$



$I_{bB} = 5.06\angle -120 - 27.65 = 5.06\angle -147.65°$

$I_{cC} = 5.06\angle 120 - 27.65 = 5.06\angle 92.35°$

$V_{AN} = I_{aA} \times (20+j10) = (5.06\angle -27.65) \times (20+j10)$
$= 113.15\angle -1.08\ V_{rms}$

$V_{BN} = 113.15\angle -1.08 - 120 = 113.15\angle -121.08\ V_{rms}$

$V_{CN} = 113.15\angle -1.08 + 120 = 113.15\angle 118.92\ V_{rms}$

Example: abc, balanced Y-Y 3 phase $V_{an} = 120\angle 90\ V_{rms}$ find line Voltage

V_{ab} leads V_{an} by 30°
$V_{ab} = \sqrt{3}\,|V_p|\angle 30$
V_{an} lags V_{ab} by 30°

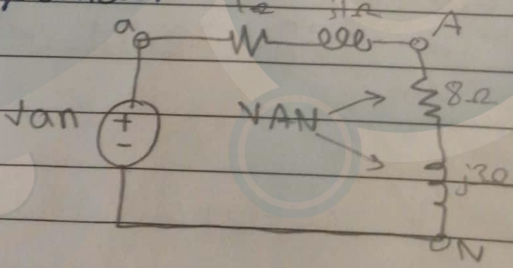
$V_{ab} = \sqrt{3} \times 120\angle 120$, $V_{bc} = \sqrt{3} \times 120\angle 0$, $V_{ca} = \sqrt{3} \times 120\angle 240$

$V_{ab} = 208\angle 0$, find phase voltage

$V_{an} = 208/\sqrt{3}\angle -30$, $V_{bn} = 208/\sqrt{3}\angle 150$, $V_{cn} = 208/\sqrt{3}\angle 90°$

Example: abc, balanced Y-Y 3 phase $V_{phase} = 104.02\angle 26$
$Z_{line} = 1+j\,\Omega$ Z_{phase} $8+j3\,\Omega$ determine Source phase voltage
Currents are not required, use inverse voltage divider

$V_{AN} = v_{phase}$ $V_{an} = \frac{(8+j3)+(1+j)}{8+j3} V_{AN}$



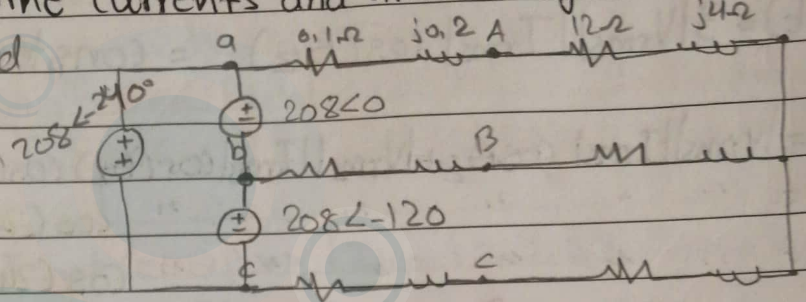
$V_{AN} = 1.15\angle 3.41$

$V_{an} = 120\angle 30°$
$V_{bn} = 120\angle -90°$
$V_{cn} = 120\angle 150°$

Example: Determine line currents and line voltages at the loads

Source is Delta connected

Convert to Y



$V_{ab} = V_L \angle 0°$

$V_{bc} = V_L \angle -120°$

$V_{ca} = V_L \angle 120°$

$V_{an} = V_L/\sqrt{3} \angle -30°$

$V_{bn} = V_L/\sqrt{3} \angle -150$

$V_{cn} = V_L/\sqrt{3} \angle 90°$

Analyse one phase $I_{aA} = \dfrac{208/\sqrt{3} \angle -30°}{12.1 + j4.2} = 9.38 \angle -49.14°$

$V_{AN} = (12 + j4) \times 9.38 \angle -49.19 = 118.65 \angle -30.71$

$V_{ab} = \sqrt{3}\,|V_p| \angle 30°$ $\quad V_{AB} = \sqrt{3} \times 118.65 \angle 0.71°$

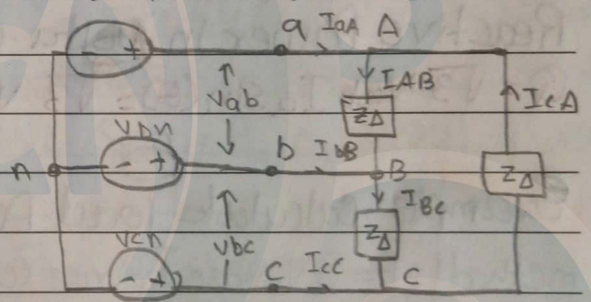
Determine the other phases using the balance.

$I_{bB} = 9.38 \angle -169.14°$ $\quad V_{BC} = \sqrt{3} \times 118.65 \angle -119.29°$

$I_{cC} = 9.38 \angle -71.86°$ $\quad V_{CA} = \sqrt{3} \times 118.65 \angle 120.71°$

Example: Delta connected load consists of 10Ω series with 20mH inductance source is Y connected, abc, 120-Vrms, 60Hz, find all line and phase currents

no need to convert, i have Van



$V_{an} = 120 \angle 30$, $V_{AB} = 120\sqrt{3} \angle 60°$

$Z_{inductance} = 2\pi \times 60 \times 0.02 = 7.54\,\Omega$

$Z_\Delta = 10 + j7.54\,\Omega = 12.52 \angle 37.02$, $Z_Y = 4.17 \angle 37.02°$

$I_{AB} = V_{AB}/Z_\Delta = 120\sqrt{3} \angle 60 / 10 + j7.54 = 16.6 \angle 22.98$

$I_{BC} = 16.6 \angle -97.02$ $\quad I_{CA} = 16.6 \angle 142.98$

$I_{aA} = 28.75 \angle -7.02$, $I_{bB} = 28.75 \angle -127.02$, $I_{cC} = 28.75 \angle 112.98$

$|V_\Delta| = \sqrt{3}\,|V_{phase}|$, $\theta_\Delta = \theta_{phase} + 30°$

line-phase voltage relationship

$|I_{line}| = \sqrt{3}\,|I_\Delta|$

$\theta_{line} = \theta_\Delta - 30°$ } ⇒ line-phase current relationship.

3 Phase Power

wlopt.

$P(t) = 3|V_{rms}||I_{rms}|\cos(\theta_z) = \text{constant}$

$P_a(t) = |V_{rms}||I_{rms}|\cos\theta_z + |V_{rms}||I_{rms}|\cos(\theta_z)\cos(2\omega t + 2\theta_v - \theta_z)$

$P_b(t) =$ " " " " " " $\cos(2\omega t + 2\theta_v - 120) - \theta_z)$

$P_c(t) =$ " " " " " " $\cos(2\omega t + 2(\theta_v + 120) - \theta_z)$

for single phase → $P = V_{phase} I_{phase} \cos(\theta)$ → $\theta_v - \theta_i = pf$
→ $Q = V_{phase} I_{phase} \sin(\theta)$

for 3-phase → $P = 3 V_{phase} I_{phase} \cos(\theta)$
→ $Q = 3 V_{phase} I_{phase} \sin\theta$

θ is the angle b/w phase voltage & phase current.

Real Power in Y circuit

$P = \sqrt{3} V_{ab} I_a \cos(\theta) = \sqrt{3} V_{line} I_{line} \cos(\theta)$ → θ_z

Reactive Power in Y circuit

$Q = \sqrt{3} V_{ab} I_a \sin(\theta) = \sqrt{3} V_{line} I_{line} \sin(\theta)$

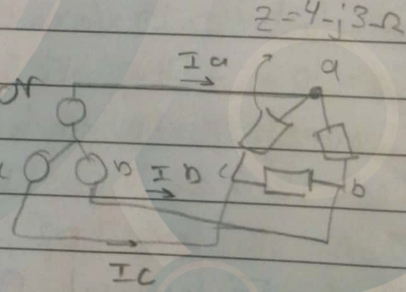
Real Power in Delta circuit → $\sqrt{3} V_{ab} I_a \cos(\theta) = \sqrt{3} V_{line} I_{line} \cos(\theta)$

$P = 3 V_{phase} I_{phase} \cos(\theta) = 3 V_{ab} I_{ab} \cos(\theta)$

Reactive Power in Delta circuit

$Q = \sqrt{3} V_{ab} I_a \sin(\theta) = \sqrt{3} V_{line} I_{line} \sin(\theta)$

Example: calculate load power



method 1: $P = 3 V_{phase} I_{phase} \cos\theta$

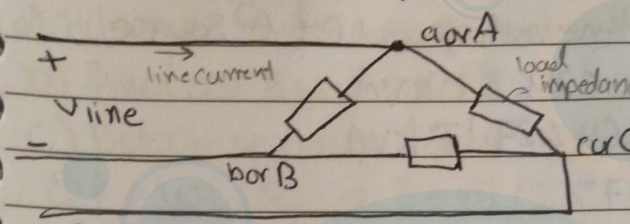
$P = 3 V_{ab} I_{ab} \cos\theta = 3 \times (\sqrt{3} \times 120) \times \frac{\sqrt{3} \times 120\angle 30}{4 - (i \times 3)}$

$\times \cos(\tan^{-1}(-3/4)) = 20.736\ kW$

method 2: $P = \sqrt{3}\ V_{line} I_{line} \cos\theta$

$P = \sqrt{3}\ V_{ab} I_a \cos\theta = \sqrt{3} \times 208 \times 72 \times \cos(\tan^{-1}(-3/4)) = 20.736\ kW$

Example: determine the magnitude of the line currents and the value of load impedance per phase in the delta.



$|V_{line}| = 208\ V_{rms}$
$P_{total} = 1200W$ inductive load
$pf = 20°$ lagging

$P_{total} = \sqrt{3}\,|V_{line}||I_{line}|\cos\overset{20°}{\theta_f} \Rightarrow |I_{line}| = 3.54\ A_{rms}$

$Q_{total} = \sqrt{3}\,|V_{line}||I_{line}|\sin\theta_f$

$|I_{line}| = \sqrt{3}\,|I_\Delta| \Rightarrow |I_\Delta| = 2.05\ A_{rms}$, $\theta_{line} = \theta_\Delta - 30°$

$|Z_\Delta| = |V_{line}| / |I_\Delta| = 101.46\angle{+20°}$

Example: abc, balanced Y-Y 3 phase, source $V_{phase} = 120V_{rms}$
$Z_{line} = 1+j\,\Omega$, $Z_{phase} = 20+j10\,\Omega$, determine real & reactive power per phase at the load, and total real, reactive and complex power @ the source

balanced circuit $\Rightarrow$ data on any phase is sufficient

$V_{an} = 120\angle0$, $V_{bn} = 120\angle-120°$, $V_{cn} = 120\angle120°$

Abc $\Rightarrow I_{aA} = V_{an} / 21+j11 = 120\angle0 / 23.7\angle27.65 = 5.06\angle-27.65$

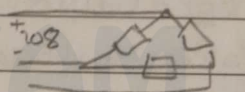
$V_{AN} = I_{aA} \times (20+j10) = I_{aA} \times 22.36\angle26.57 = 113.15\angle-1.08V$

$S_{phase} = V_{AN} I^*_{aA} = 113.15\angle-1.08 \times 5.06\angle27.65° = 572.54\angle26.57 = 512+j256.09$

$S_{source\ phase} = V_{an} \times I^*_{aA} = 120\angle0 \times 5.06\angle27.65 = 607.2\angle27.65 = 537.86+j281.78VA$

$p_{total\ source} = 3\times537.86W$, $Q_{total_s} = 3\times281.78VA$, $S_{total_s} = P_{ts}+Q_{ts} = |1821.6|\ VA$

Example total S, combined pf



$S = P+jQ$
$P = |S|\cos\theta_f$
$Q = |S|\sin\theta_f$
$P_f = \cos\theta_f$

Load 1: 24kW @ pf = 0.6 lagging
load 2: 10kW @ Pf = 1, load 3: 12kVA @ pf = 0.8 ~~lagging~~ leading

$P_1 = 24kW$ $pf = 0.6$ lag $\Rightarrow |S_1| = 40kVA$, $Q_1 = \sqrt{|S_1|^2 - |P_1|^2} = 32kVA$

lagging $\Rightarrow$ inductive $S_1 = 24 \overset{lag}{+} j32\ VA\times10^3$

$P_2 = 10kW$, $Pf = 1 \Rightarrow S_2 = 10 + j10\ kVA$

$P_3 = 9.6\ kW$, $|Q_3| = 7.2kVA$ leading pf $\Rightarrow$ capacitive $S_3 = 9.6 \overset{lead}{-} j7.2\ kVA$

$S_t = S_1+S_2+S_3 = 43.6+j24.8\ kVA = 50.160\angle29.63$ $pf = 0.869$ lagging

Continued:

if line current = 139.23 Arms and if the line impedance $Z_{line} = 0.05 + j0.02\,\Omega$ determine line voltage and pf @ source

$S_{load\ total} = 43.6 + j24.8\,kVA = 50.160\angle 29.63^\circ\,kVA$

$S_{source\ total} = 46.508 + j25.963 = 53.264\angle 29.17^\circ\,kVA$

$|S_{total}| = \sqrt{3}\,|V_{line}| \times |I_{line}|$ $\quad \theta_f = 29.17^\circ$

$|I_{line}| = 139.23$ Arms

$S_{line} = 3 \times Z_{line} I_{line} I^*_{line} = 3 \times Z_{line} |I_{line}|^2$

$S_{line} = 2908 + j1163\,(VA)$

$V_{line} = 53.264 / \sqrt{3} \times 139.13 = 220.87$ Vrms

$pf = \cos\theta_f = \cos(29.17) = 0.873$ lagging

Resonance

will pt I.

Must be pure resistive

$\omega_0 = 1/\sqrt{LC}$ rad/s or $f_0 = 1/2\pi\sqrt{LC}$ Hz

$Z = R + j(\omega L - 1/\omega C)$

$Z_{min} = R$ @ Resonance $\quad I = V_s/R \quad P = R I_{rms}^2$

$V_R = V_s \quad |V_L| = |V_C|$

$V_L = I(\omega L) = (V_s/R)(\omega L) = V_s(\omega L/R)$

if $\omega L/R > 1 \Rightarrow V_L > V_s$

$V_C = (V_s/R)/\omega C = V_s/R C\omega$

Quality factor $= V_L = V_s = \omega L/R = 1/\omega CR$

$1/(\omega CR) = Q_f$, Q_f = Reactive Power / Real power = Stored E / E dissipated

frequency responce at resonce circuit current

$I = |I| = V_m / \sqrt{R^2 + (\omega L - 1/\omega C)^2}$ series

$\omega_2 - \omega_1 = B_w$

Average power $P(\omega) = 1/2\, I^2 R$

highest Power $P(\omega_0) = 1/2\, V_m^2/R$

half Power $= P(\omega_1) = P(\omega_2) = 1/2\, \frac{(V_m/\sqrt{2})^2}{R} = V_m^2/4R$

$\omega_1 = R/2L + \sqrt{(R/2L)^2 + 1/LC} = \omega_2$ $\quad \omega_0 = \sqrt{\omega_1 \omega_2}$

Bandwidth $B = \omega_2 - \omega_1$

Example: a series connected circuit has $R=4\Omega$, $L=25mH$
a) calculate value of C that will produce a qf of 50.
b) find ω_1 & ω_2 and B
c) Determine avg Power dissipated at $\omega=\omega_0, \omega_1, \omega_2$ $V_m=100V$

$Q_f = WL/R \Rightarrow 50 \times 4/25 = \omega_0 = 8000$ rad/sec
$\Rightarrow (1/LC)^{0.5} = C$

b) $B = \omega_0/Q \Rightarrow 8000/50 = 160$ rad/s
$\omega_1 = \omega_0 - B\omega/2$, $\omega_2 = \omega_0 + B\omega/2$

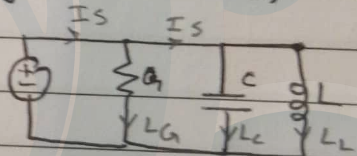
c) $P_{\omega_0} = (100^2/2(2\times4))$ $P(\omega_1) = 0.5P(\omega_0) = P\omega_2$

Parallel Resonance.

$Y = 1/R + j(\omega C - 1/\omega L) \rightarrow$ when $Y=0$

Resonance frequency $\omega_0 = 1/\sqrt{LC}$ rad/s or $f_0 = 1/2\pi\sqrt{LC}$ Hz
@ Resonance $Y = G = 1/R$
$B\omega = \omega_0/Q$

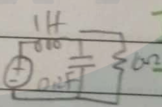
Example: if the source operates @ resonant frequency of the network, compute all branch currents



$V_S = 120\angle 0$, $G = 0.01S$, $C = 600\mu F$, $L = 120mH$
$I_G = 0.01 \times 120\angle 0 = 1.2\angle 0° = I_S$
$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{0.12 \times (6\times10^{-4})} = 117.85$ rad/s
$I_C = (1\angle 90°) \times (117.85) \times 600\times10^{-6} \times 120\angle 0° = 8.49\angle 90°$
$I_L = 8.49\angle -90$

$I_G = GV_S$
$I_C = j\omega CV_S = \frac{j\omega C}{Y} I_S$
$I_L = 1/j\omega L\, V_S = \frac{1/j\omega L}{Y} I_S$

Example: resonant frequency



$Z = 10 // \overset{c}{0.2} + j\omega l$
$c = 1/\omega(0.2) + j\omega L$

$\omega = \frac{\sqrt{19}}{2} = 2.179$ rad/s

total Z imag = 0

End of Electrical
2020/2021 Fall