Introduction

The load types :-

① tensile load (elongation)
② compressive load (contraction)
③ Shear load (relting)

→ constant with time → (Static load) $\xrightarrow[\text{failure}]{}$ Static fail
→ time Vaying load → (flunctuating load) → fatige failu

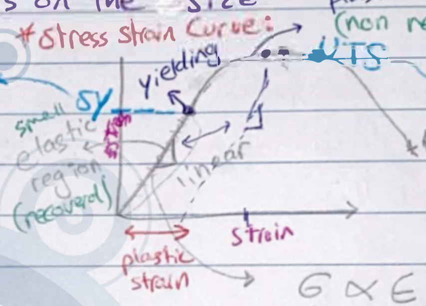
volume stays constant

load -Defomation chareteristics depends on the size

$$\sigma_{\text{stress}} = \frac{F \text{ (force)}}{A_o}$$
A_o original cross sectional area

$$\epsilon_{\text{strain}} = \frac{\Delta L \text{ (deformation elongation)}}{L_o}$$
L_o original length

\# stress strain Curve:



when it is Linear

$$\sigma_{\text{stress}} \propto \epsilon_{\text{strain}}$$
$$\sigma = \boxed{E}\,\epsilon$$
E → young's modulus, modulus of elasticity
$$E = \frac{\Delta\sigma}{\Delta\epsilon}$$

تغيرات بشكل كبير → plastic deformation

* if severe changes in the shape of the material occurs
↳ loss of function

قوة المادة ← *Strength : magnitude of strength at which something occurs
↳ yeilding
↳ necking
↳ Fracture

Sy (Yielding strength) : level of stress at which yielding occurs , Plastic deformation begins

* measured material properties are not exact quantities
↳ Description of stregth is statistical in nature
↳ strength → random variable

مثال
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Mean Strength

* Factor of safety :-

$\underset{stress}{6} = \frac{F}{A}$

الوحدة $[6] = \frac{N_{ewton}}{m^2} = \underset{Pa.}{pascal}$

$= \frac{N}{(mm)^2} = M\,pa.$

10^{-3} ملي ← (mm) ; ↑ mega $= 10^6$

* if force $\xrightarrow{\text{وحدتها}}$ pound

$[6] = \frac{pound}{in^2_{inch}} = psi \quad = Kpsi$

↑ pound per square inch

$\underset{strain}{\varepsilon} = \frac{\Delta L}{L_o}$

$[\varepsilon] = \frac{\cancel{mm}}{\cancel{mm}}$

strain has no unit

$\varepsilon = \frac{\Delta L}{L_o} = \frac{L_f - L_o}{L_o}$

$\boxed{\text{tension}}$ ⊕ $\longrightarrow L_f > L_o$

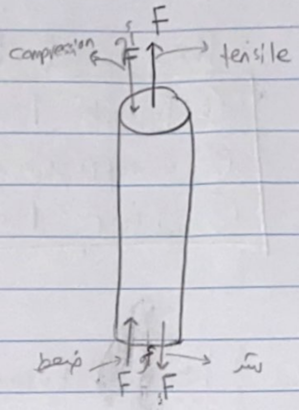
$\Delta L \longrightarrow +ve$

$\varepsilon \longrightarrow +ve \;/\; 6 \longrightarrow +ve$

$\boxed{\text{compression}}$ ⊖ $\longrightarrow L_f < L_o$

$\Delta L \longrightarrow -ve$

$\varepsilon \longrightarrow -ve \;/\; 6 \longrightarrow -ve$



Ratio ← * factor of safety (n) → كلما زاد كلما كان أفضل

it doesn't have a unit ← $\text{(n)} = \frac{Strength}{stress_{max}} = \frac{Sy\,(\text{yeild strength})}{6_{max}\ \text{stress}}$ yield FOS → factor of safety

$Sy > 6 \longrightarrow$ No failure

$6 > Sy \longrightarrow$ failure

$n = \frac{Sy}{6}$

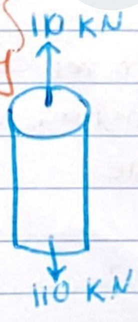
$n > 1 \longrightarrow$ No failure

$n < 1 \longrightarrow$ failure

البعض يعكسون المقام

$\boxed{1.2 - 4}$ غالبا n تكون في هذا ال Range

1) * Deterministic factor of safety



(Diameter) D = 20 mm

Sy = 480 Mpa

find FOS ?

Area الدائرة → $\pi\,(\text{نق})^2$

$6 = \frac{F}{A} = \frac{110 \times 10^3\ N}{\frac{\pi}{4}\,(20)^2\ mm^2}$

$= 350.14\ Mpa$

$n = \frac{480}{350.14} = 1.37 > 1$

No failure → (Survival)

2) * stochastic factor of safety

probability failure يصير أو ما يصير

$S_y \rightarrow$ random variable
$\sigma \rightarrow$ random variable
سوف نعتبرهم Normally Distributed

$P(\sigma > S_y) \rightarrow$ failure
$P(\sigma < S_y) \rightarrow$ Survival
مجموعهم 100 %

Probability of Survival (Reliability)

S_y: mean $= \dots$ (μ)
Standard deviation $= \dots$ ($\hat{\sigma}$)

$\mu_s = \dots$
$\hat{\sigma}_s = \dots$

σ : Mean $= \dots$ (μ)
Standard deviation $= \dots$ ($\hat{\sigma}$)

$\mu_\sigma = \dots$
$\hat{\sigma}_\sigma = \dots$

Reliability $P(S_y > \sigma)$
$P(\underbrace{S_y - \sigma}_{\text{margin}(m)} > 0)$
$m = S_y - \sigma$
$P(m > 0)$ survival
$P(m < 0)$ failure

* example :- يشبه سؤال 1.20

Strength:
$\mu_s = 553$ Mpa , $\hat{\sigma}_s = 42.7$ Mpa

Stress:
$\mu_\sigma = 473$ Mpa , $\hat{\sigma}_\sigma = 23.8$ Mpa

* $m = S_y - \sigma$
margin ← $m \rightarrow$ normally distributed
Mean ← $\mu_m = \mu_s - \mu_\sigma$
Variance ← $V_m = V_{sy} + V_\sigma$ (variability is cumulative)
$V_{sy} = \hat{\sigma}_s^2$
$\hat{\sigma} = \sqrt{\text{variance}}$
$\hat{\sigma}_m = \sqrt{\hat{\sigma}_s^2 + \hat{\sigma}_\sigma^2}$

$P(m > 0) \rightarrow$ Reliability
↓ Standarization
Z

① $\boxed{n = \frac{\mu_s}{\mu_\sigma}} = \frac{553}{473} = 1.17$ (Mean)

② $m = S_y - \sigma$
$\mu_m = 553 - 473 = 80$ Mpa
$\hat{\sigma}_m = \sqrt{42.7^2 + 23.8^2} = 48.9$ Mpa

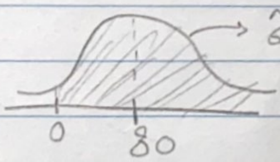
③ Reliability → $P(m > 0)$

$P\left(\frac{m - \mu_m}{\hat{\sigma}_m} > \frac{0 - 80}{48.9}\right)$ Standarization

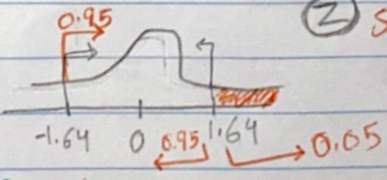
$P(Z > -1.64)$

$R = 0.95$
$P_{failure} = 0.05$

Final answer
$n = 1.17$
$R = 0.95$



0.05.05
Lolipop
0.05



$P(Z > -1.64) = P(Z < 1.64) = 1 - P(Z > 1.64)$

* Types of Loadings → ① ② ③ ④

stretching or compression for the material ← ① Normal Force (P) → force عمودي على ال cross section

Displacement for material ← ② Shear Force (V) → Force parralel for the cross section

Bending for material ← ③ Bending Moment (M) → Moment حوالين axis Parrallel for cross section

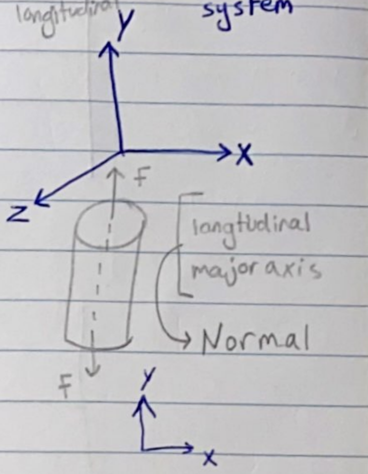
twisting for material ← ④ Torque (T) → Moment حوالين axis عمودي على ال cross section

Chapter (3)

[Types of loading and Types of Stress]

حول around longitudinal

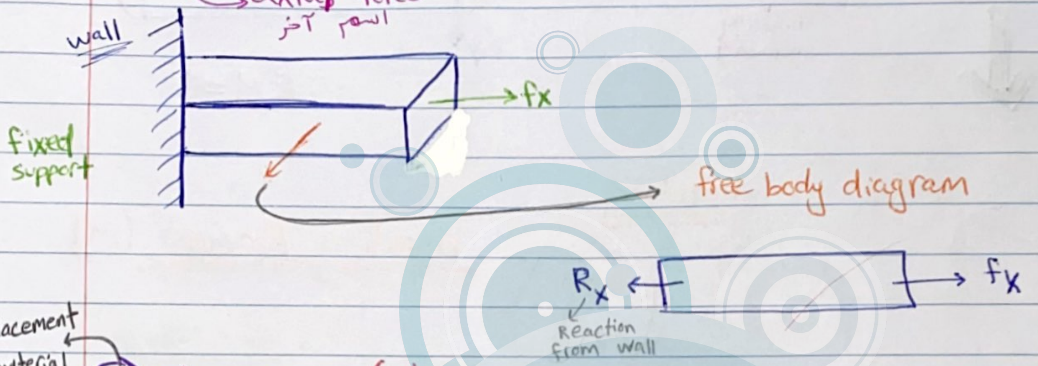
coordinate system



① Normal Force (tension or compression) (P)

⊕ve (tension), ⊖ve (compression)

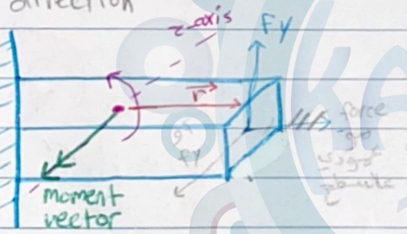
↳ axial force اسم آخر

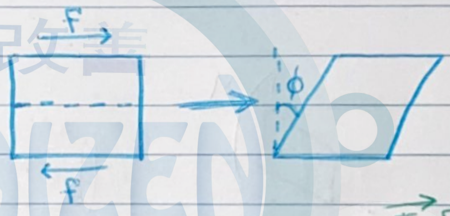


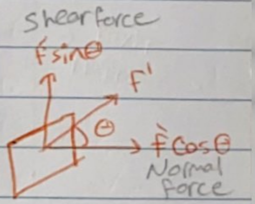
Displacement for material

② Shear Force (V)

↳ two equal forces in different direction



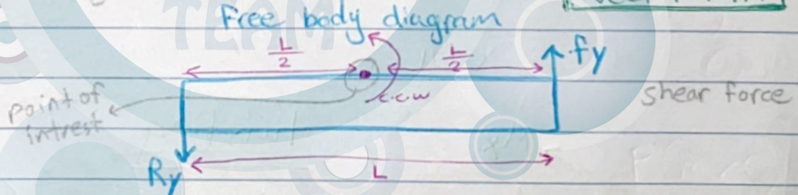




$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

vector form

Free body diagram



①Sense of rotation]→ Right hand Rule RHR

②moment vector]

2D → Moment حوالين نقطة

3D → Moment حوالين axis

scalar form ← [$M = Fd$]

↑ perpendecular distance

تكرار → Rotational motion → Moment → $M = Fd$ → c.w / c.c.w

$[M] = N.m$

vector form ← [$\vec{M} = \vec{r} \times \vec{f}$]

$\vec{r}$ اتجاهه from point of interest to line of action for force

$\vec{F} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

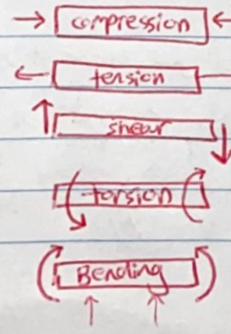
$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$

*cross product

$|\vec{M}| = |\vec{r}||\vec{F}| \sin\phi$

$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

مراجعة وتلخيص



compression, tension] Axial Force / Normal Force (axially loaded member)

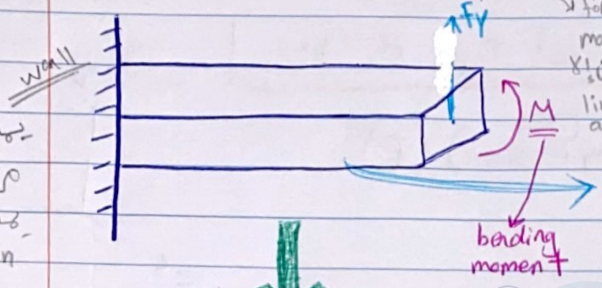
shear] Shear Force

torsion] Torque

Bending] Bending Moment

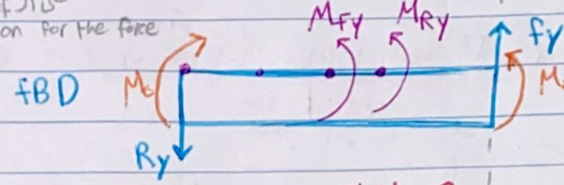
$$\overset{\uparrow}{M} = F\,\overset{\uparrow}{d}$$

كلما زادت المسافة
بتزيد ال moment



يمنع ال bean
من أن تتحرك
يمنعها من ال
rotation

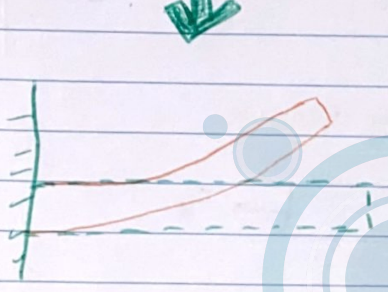
هاي ال force
بتعمل moment
بكل مكان إلا
على ال line of
action for the force



line of action
تبع ال force

$M = F\cancel{d}$

$\Sigma M = 0$

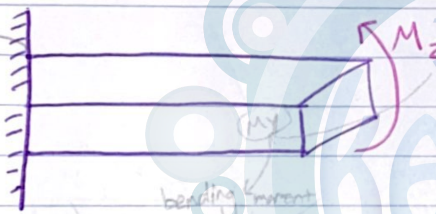


bending

انحناء
في ال
body

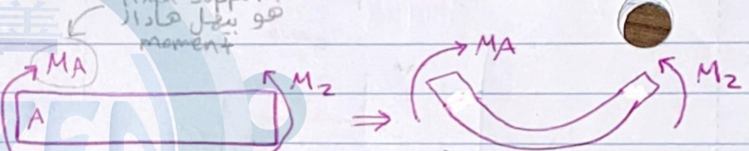
(يكون
بسبب
ال
moment)

Bending Moment (M)



Fixed support

bending moment
بجهة مختلفة

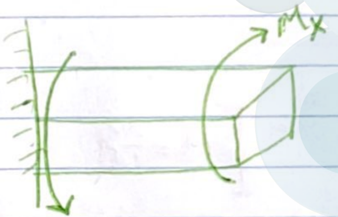


fixed support
هو بيعمل هادا ال
moment

راح يصير فيه curve
إذا صار bending
moment

في هذه الحالة

ال X هو العامودي على ال cross section



twisting

Torsional Moment (Torque) (T)

على ال
axis
العامودي
لل
Cross section
مثل ال Normal

هنا ال moment
بحاول يعمل
twisting
ل يسمى
tershinal moment
(Torque)

* Moment
حوالين ال axis العمودي على ال cross section
بكون ⟵ Torque

مثل
Normal
force

* إذا كان ال Moment مش حوالين ال axis العمودي
على ال cross section بكون ⟵ Bending Moment
(حوالين ال axis ال
Parrallel for the cross section)

مثل shear
force

Torque
ينتج عنا stress

Types of stress → ① Normal stress (σ)
→ ② Shear stress (τ)

① Normal Stress (σ → sigma) $\sigma = \frac{F_{Normal}}{A}$ → P

② Shear Stress (τ → tau) $\tau = \frac{F_{shear}}{A}$ → V

Hooke's Law :-

in Normal → $\sigma = E\,\epsilon$

Normal stress ↗ σ; ↑ Modulas of Elacticity; ↖ Normal strain

in shear → $\tau = G\,\gamma$

shear stress ↗ τ; ↑ Modulas of rigidity; ↖ shear strain

Types of Loading and Types of Stress → ① Normal Stress (σ)
→ ② Shear Stress (τ)

Types of Loading:

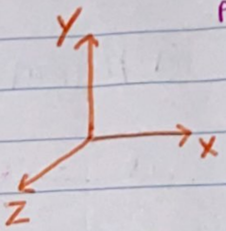
→ ① Axial force (P) → Normal stress $\sigma = \frac{P}{A}$ → Axial force normal force
→ ② Shear force (V) → Shear Stress
→ ③ Bending moment (M) → Normal stress
→ ④ Torque (T) → Shear stress
T

$$\uparrow \sigma = \frac{P}{A} \uparrow$$

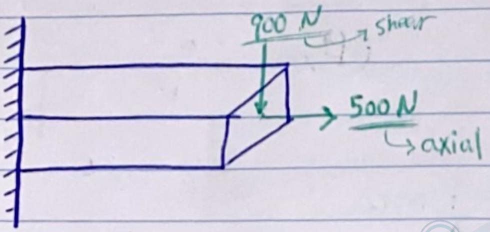
كلما زاد ال load بزيد ال stress

Uniformly Distributed Stresses

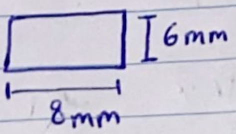
page 102
103



*Example :-



Cross Section
C.S

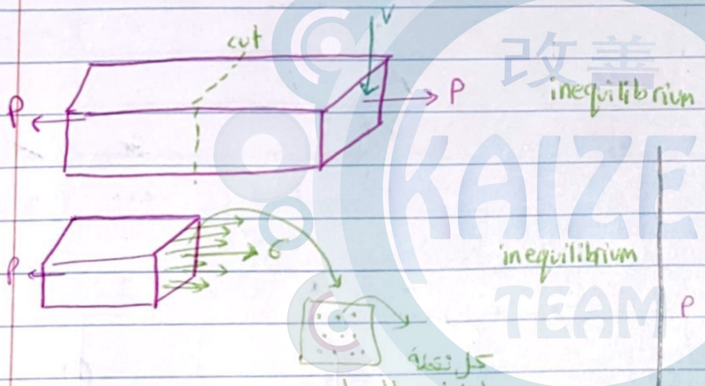


① Determine the Normal stress caused by Fx=500N

$$\sigma = \frac{F}{A} \longrightarrow \sigma = \frac{500\,N}{6mm \times 8mm} = 10.4 \text{ Mpa}$$

② Determine the average shear stress caused by Fy=900 N

$$\tau = \frac{V}{A} \longrightarrow \tau = \frac{900\,N}{6mm \times 8mm} = 18.75 \text{ Mpa}$$



Normal stress is uniformally distributed

((كل النقاط لهم نفس قيمة (σ)))

((ليس كل النقاط لهم نفس قيمة (τ)))

Shear stress is not uniformally distributed

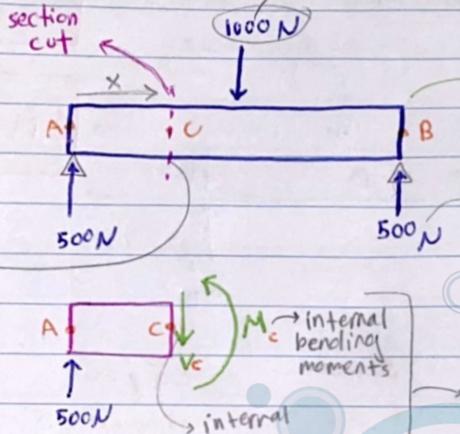
Average $\tau = \frac{V}{A}$

in equilibrium
$\Sigma F=0$
$\Sigma M=0$

(3.2) Shear Force and Bending Moments in Beams

Internal loadings

* كل ما يزيد قيمة الـ load بتزيد قيمة الـ stress



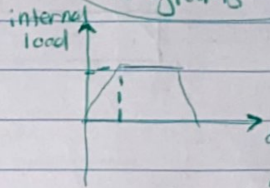
in equilibrium (متزن)
$\Sigma F=0$
$\Sigma M=0$

Reaction بيجي من الـ Support عشان الـ beam ما تتحرك

بعد ما نعمل section cut الـ Loadings بصيرو external

هدول يلي بيعملو الـ stress and they vary from point to point

Shear and moment diagrams



الهدف منه نحدد أعلى Load و أين موقعه

خطوات الحل

① FBD for the beam
② shear diagram
③ moment diagram ← بطلعه من الـ shear diagram

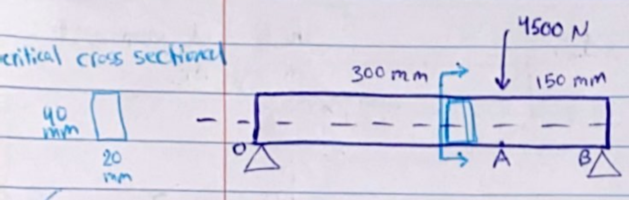
$\boxed{M = F d}$

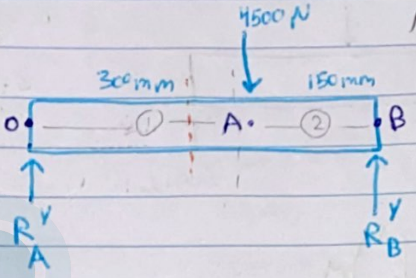
slope for the curve ← $\boxed{V = \frac{dM}{dx}}$

Area under the curve ← $\boxed{\Delta M = \int_{x_1}^{x_2} V\,dx}$

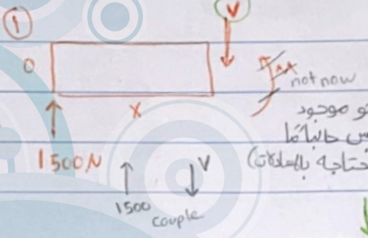
C.W ⊕
C.C.W ⊖ ← Shear force diagram :
+ moment diagram :

***Problem (3-35) page (145)**





يوجد مجهولين ← $\Sigma F = 0 \rightarrow R_A^y + R_B^y - 4500 = 0$
لازا نبدأ بمعادلة
الـ Moment → $\Sigma M = 0$



$\Sigma F_y = 0$

$1500 - V = 0$

$V = 1500 N$ → C.W +ve

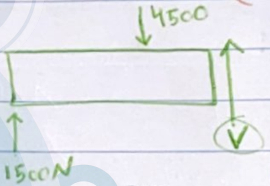
$(0 < x < 300 mm)$

$\Sigma M = 0$

$-(4500)(300) + (R_B^y)(450) = 0$

clockwise, counterclockwise

$R_B^y = 3000 N$



3000 couple V

$V = 4500 - 1500$

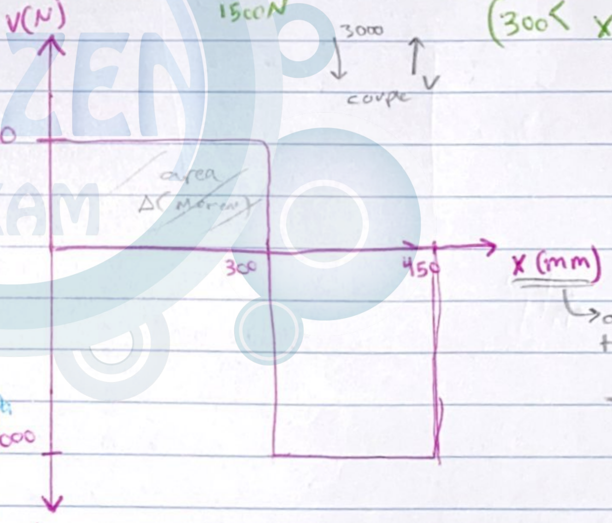
$V = 3000 N$ → C.C.W -ve

$(300 < x < 450)$

$\Sigma F_y = 0$

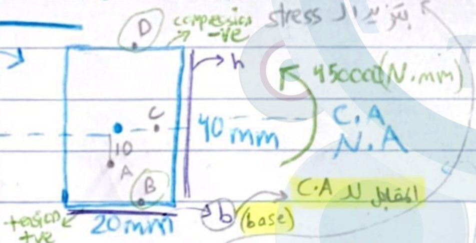
$300 - 4500 + R_o^y$

$R_o^y = 1500 N$



x (mm) → distance from the left side of the beam x →

كل ما زادت المسافة بتزيد الـ stress



$\sigma = \frac{Mc}{I}$

$\boxed{I = \frac{1}{12} b h^3}$

inhercia for rectangular form

$I = \frac{1}{12}(20)(40)^3$

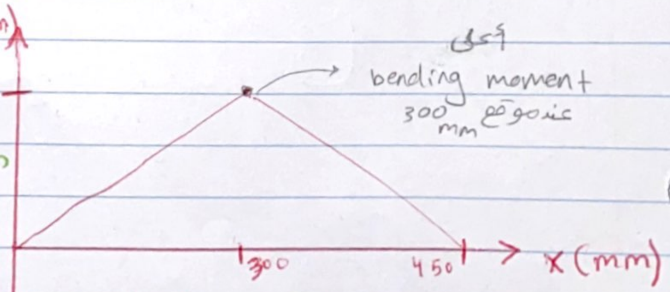
Ⓑ 8 Ⓓ اسمهم Critical points لأنه ما بتتبعد عن الـ C.A أكثر اشي

at ⓐ $\sigma = \frac{Mc}{I} = \frac{(450\,000)(10)}{\frac{1}{12}(20)(40)^3} = 42.19$ MPa

at ⓑ $\sigma = \frac{Mc}{I} = \frac{(450000)(20)}{\frac{1}{12}(20)(40)^3} \oplus = 84.38$ MPa tension

at ⓒ $\sigma = \frac{Mc}{I} = \frac{(450000)(0)}{\frac{1}{12}(20)(40)^3} = 0$ MPa

at ⓓ $\sigma = \frac{Mc}{I} = \frac{(450000)(20)}{\frac{1}{12}(20)(40)^3} \ominus = 84.38$ MPa → compression



أعلى bending moment عند 300 mm

لأن مساحة الشكل في منحنى (V) تحت الـ axis يكون نازل لتحت ←

$M_{450} - M_{300} = \int_{300}^{450} V\,dx$

$= (150)(3000)$

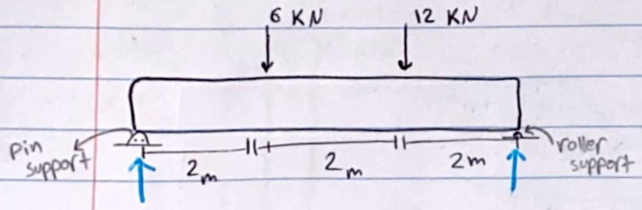
$= 450000$

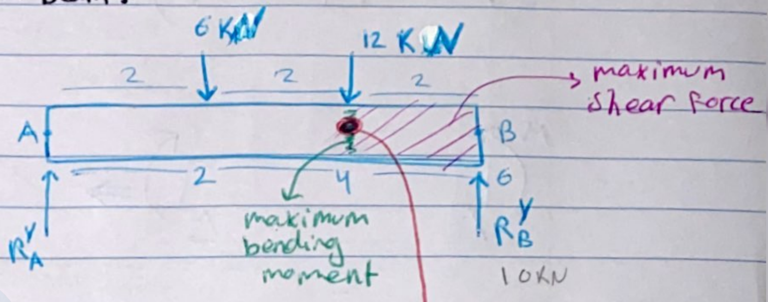
$\Delta M = (1500) \times (300) = 450\,000$

$-3000 \times 150 = -450\,000$

① FBD
② Shear Diagram
③ moment Diagram

*Draw the shear and moment diagrams for the beam.





$\Sigma M_A = 0$

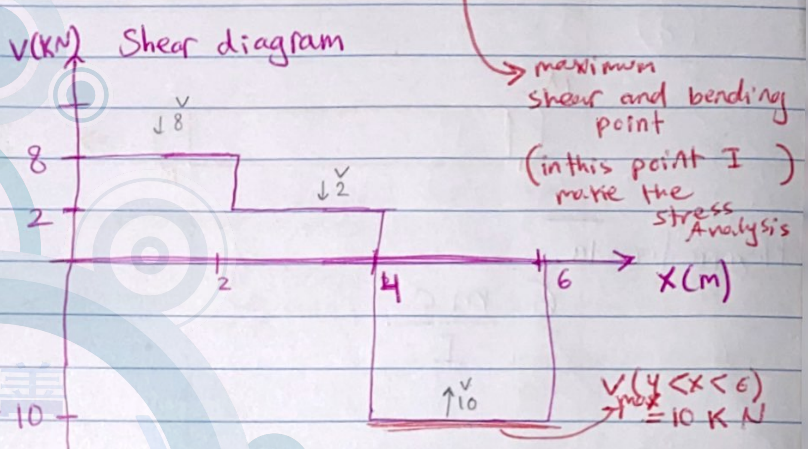
$-(6)(2) - (12)(4) + (R_B^y)(6) = 0$

$R_B^y = 10\ KN$

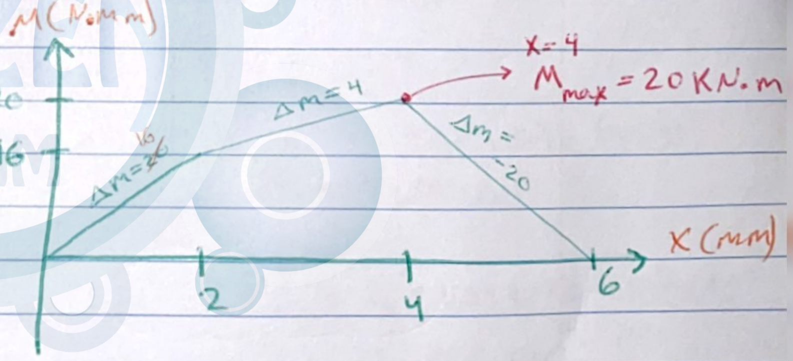
$\Sigma F_y = 0$

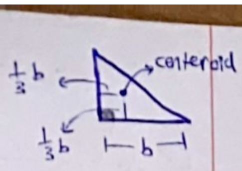
$10 - 6 - 12 + R_A^y = 0$

$R_A^y = 8$



$$\Delta M = \int_{x_1}^{x_2} V\,dx = \text{Area}$$

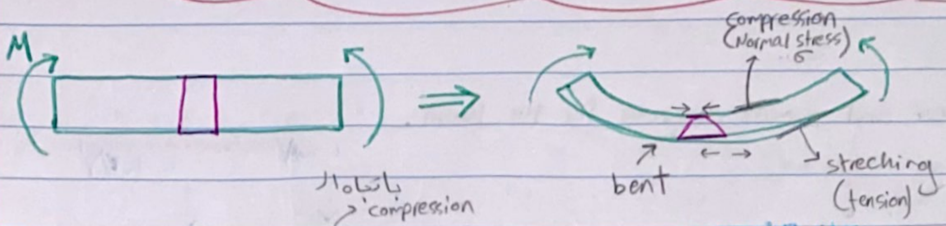


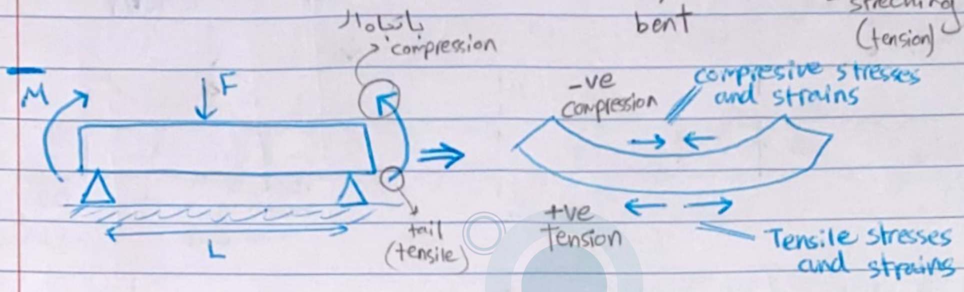


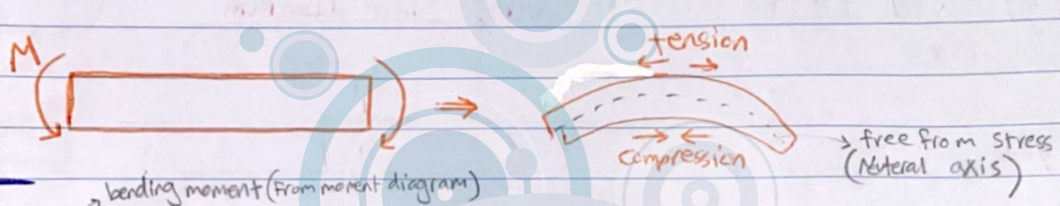
Part ①

Load → Stress

(3.10) Normal Stresses for Beams in Bending







Flexure formula

$$6 = \frac{m\,c}{I}$$

6 ← Normal stress

m → bending moment (From moment diagram)

c → distance from N.A Neautral axis (perpendicular distance)

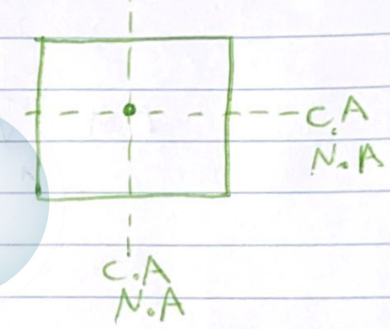
I → moment of inertia (غالباً تكون ثابتة)
قانون ثابت لكل شكل

N.A → Nuteral axis

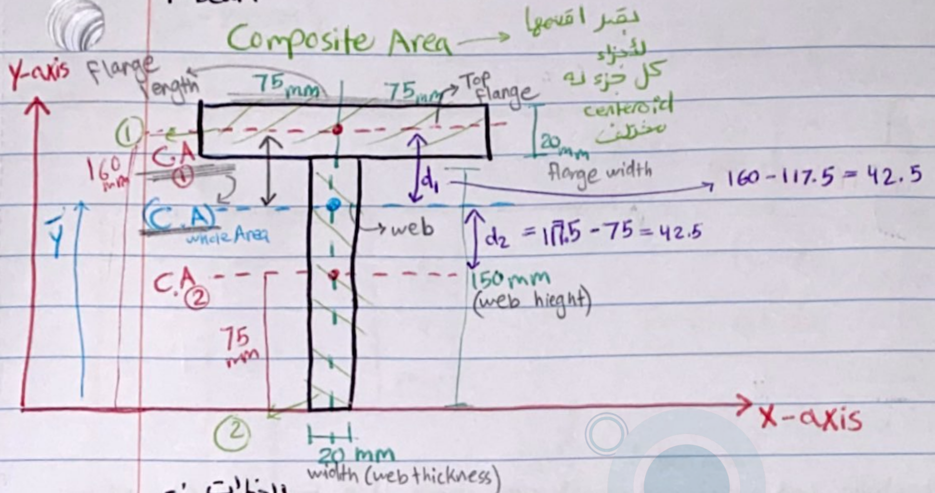
C.A → centeroidal axis

نفس بعض في ال straight beam

Straight beam



T-beam



الخطوات :-

① Centroidal and neutral axes ✓

② moment of inhertia about C.A

③ flexure formula

بكونو نفس بعض في حالة ال straight beams

①

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2}$$

coordinate For (C.A) whole Area

y telda coordinate For cintroide For second Area ②

$$\bar{y} = \frac{(75)(20 \times 150) + (160)(20 \times 150)}{(20 \times 150) + (20)(150)}$$

$$\bar{y} = 117.5 \text{ mm}$$

②

$$I = \frac{1}{12} b h^3$$

inhertia

الضلع المقابل لـ (C.A)

$$I_1 = \frac{1}{12}(150)(20)^3 = 100000 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(20)(150)^3 = 5625000 \text{ mm}^4$$

*Parallel axis theorem :-

$$\bar{I}_1 = I_1 + A_1 d^2$$

moment of enhertia about (C.A)$_1$

about (C.A)$_1$

perpendicular distance between (C.A) and (C.A)$_1$

Area of segment ①

$$\bar{I}_1 = I_1 + A_1 d^2$$

$$\bar{I}_1 = 100000 + (20 \times 150) \times (42.5)^2 = 5518750 \text{ mm}^4$$

$$\bar{I}_2 = I_2 + A_2 d^2$$

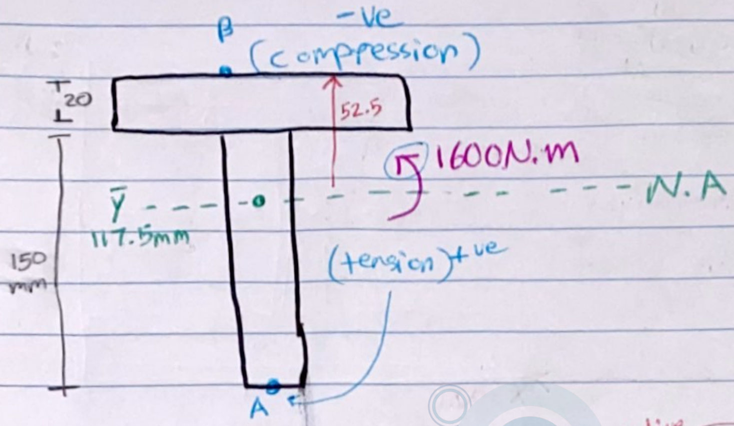
$$\bar{I}_2 = 5625000 + (20 \times 150) \times (42.5)^2$$

$$= 11043750 \text{ mm}^4$$

$$I = \bar{I}_1 + \bar{I}_2 = 11043750 + 5518750$$

$$I = 16562500 \text{ mm}^4$$

تابع !!



③ * find the maximum tensile stress and maximum compressive stress due to M = 1600 N.m

negative (-ve)

Maximum tensile stress

للتحويل إلى ملي متر

$$\sigma = \frac{MC}{I} = \frac{(1600 \times 10^3)(117.5)}{16\,562\,500}$$

نفس الإشي

$$\boxed{\begin{aligned} &1600(N.m) \\ &= 1600(N.10^3 mm) \end{aligned}}$$

$$[\sigma] = \frac{(N.\cancel{mm}) \times (\cancel{mm})}{mm^{\cancel{4}\,2}}$$

$$[\sigma] = \frac{N}{mm^2} = Mpa$$

$\sigma_{max} = 11.35$ Mpa

Maximum compressive stress

$$\sigma = \frac{MC}{I} = \frac{(1600 \times 10^3)(52.5)}{16\,562\,500}$$

$\sigma_{max} = \overset{\ominus}{}5.07$ MPa

there is online (SKyCiv Moment of Inhertia) Calculator to make sure from our answer

moment of enhertia

*Rectangle → $I = \frac{1}{12} b h^3$

table A-18

*Circular → $I = \frac{\pi D^4}{64}$

Part ②

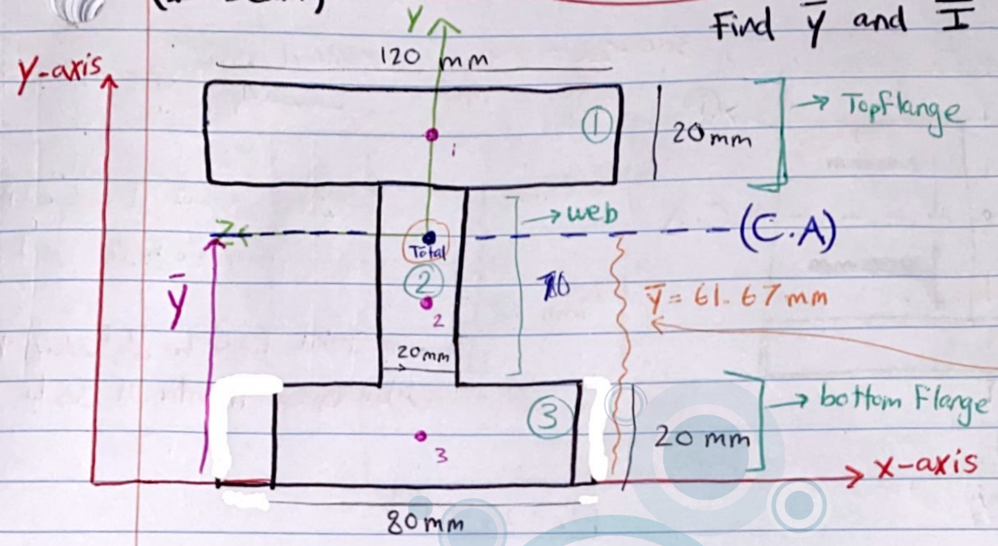
Normal Stresses for Beams in Bending

أهم شكل → (I - Beam)

Find $\bar{y}$ and $\bar{I}$

when we have 3 segments and moreہ

*الأفضل نتبع طريقة ال Table
عشان ما يتخربط الحل



+ The Total (C.A)
بروح قريب من ال Flange
الأكبر

Parrallel axis theorem

$$\boxed{\bar{I} = I + A d^2}$$

Y-telda
Y coordinate for every area

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2 + \tilde{y}_3 A_3}{A_1 + A_2 + A_3}$$

* Table Method

step Ⓐ

step Ⓑ $\frac{1}{12} b h^3$

I → Inhertia for each segment

d → perpendicular distance between Axis and Centroidal axis الكبير ← الصغير

Ī → inhertia for every segment about the (C.A)

Ī → by parralel axis theorem

segment	$\tilde{y}$	Area	$\tilde{y}A$	I	d	Ad^2	$\bar{I}$
1	100	120x20 2400	240×10^3	80×10^3	38.33	3526053	3606053 $\bar{I}_1$
2	55	1400	77×10^3	571.67×10^3	6.67	62284	633984 $\bar{I}_2$
3	10	1600	16×10^3	53.33×10^3	51.67	4271662	4324992 $\bar{I}_3$
		$\sum A = 5400$	$\sum \tilde{y} A = 333 \times 10^3$				$\sum \bar{I} =$ 8565029 mm^4

$$\bar{Y} = \frac{\sum \tilde{Y} A}{\sum A} = \frac{333 \times 10^3}{5400} = 61.67 \text{ mm}$$

in this case the ↗ upper ↘ Lower Flange will be the same dimensions and the (C.A) will be in the middle

* Example : I-beam with symmetric flanges

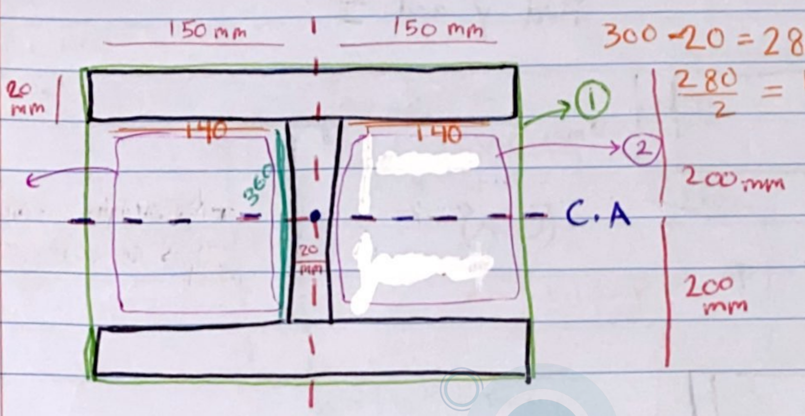
حسابًا وأبسط

①②③, Ⓘ كلهم لهم نفس ال (C.A)

300 - 20 = 280

$\frac{280}{2} = 140$

centroid axis بالمنتصف



400-20-20 = 360

*لحساب I بنسوي المربع① الكبير ناقص المربعين الصغار ②, ③

* المنطقة يلي ما فيها material باخذ ال inhertia تبعها سالب

Rectangular shape

حالة خاصة لأن الشكل symmetric

$$I = I_1 - I_2 - I_3$$

$$\boxed{I_2 = I_3 = I'}$$

$$I = I_1 - 2I'$$

$$I = \frac{1}{12}(300)(400)^3 - 2\left[\frac{1}{12}(140)(360)^3\right]$$

$$I = 511\ 360\ 000\ mm^4$$

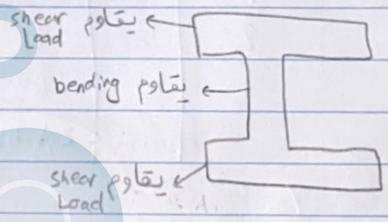
*This method is for I-beam with symmetrical flanges

I shape advantage → يقاوم stress

inhertia↑ عالية فيها نحسب stress for bending moment

$$6\downarrow = \frac{Mc}{I\uparrow}$$

اذا ال stress راح يقل

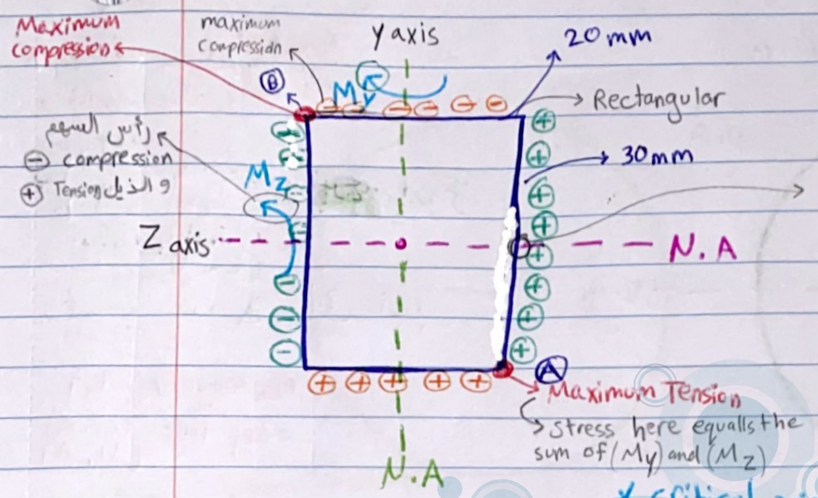


Part ③ Two Plane Bending

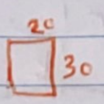
على نفس ال Cross Section

Rectangular Cross Section

Normal stresses for Beams in Bending



stress يوجد من My فقط



*راح ناخذ فقط

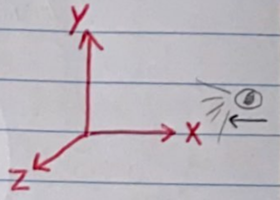
cross section → circle
→ rectangular

*for straight beams the Centroidal Axis is the same of Neutral Axis

Flexures formula

$$\boxed{\sigma = \frac{Mc}{I}}$$

في حالة one bending moment



Max corp.

Max ten.

*critical points in rectangular form are on the corners

*principle of Superposition

في حالة two (N.A) and two bending moment

$M_y = 180$ N.m

$M_z = 250$ N.m

*find Maximum Tensile stress and Maximum compressive stress

للتحويل إلى (mm)

المسافة من N.A إلى A

$$\sigma_{M_y} = \frac{(M_y)c}{I_y} = \frac{(180\times10^3)\times(10)}{\frac{1}{12}(30)(20)^3} = \boxed{90 \text{ MPa}}$$

$\frac{1}{12}(b)h^3$

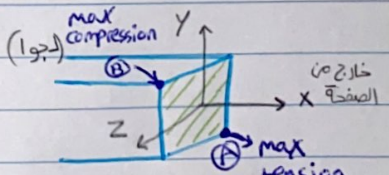
Dimension الموازي لل N.A

$$\sigma_{M_z} = \frac{M_z c}{I_z} = \frac{(250\times10^3)\times(15)}{\frac{1}{12}(20)\times(30)^3} = \boxed{83.33 \text{ MPa}}$$

Total stress (Net stress) → $\sigma_x = 90 + 83.33 = \boxed{173.33 \text{ Mpa}}$

باتجاه x

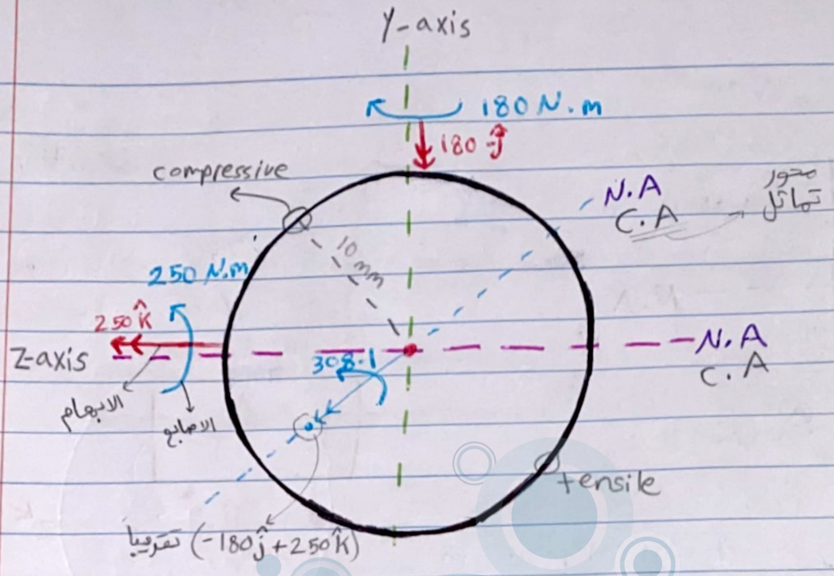
Mega



(لبرا السهم) في x-axis (عمودي)

Circular Cross Section

D = 20 mm



طريقة الحل لل circular cross section

* راح نحسب ال Resultant bending moment
و نشوف حوالين أي AXis

في الدائرة cross section
* أي محور يمر بالمركز
يكون (N.A) (C.A)
و يوجد عدد لا نهائي منهم

أما في Rectangular
only two (C.A) (N.A)

from Table A-18 Appendix

$$\boxed{I = \frac{\pi}{64} D^4}$$ → circular cross section

$\vec{M}_y = (-180\,\hat{j})\ N.m$

$\vec{M}_z = (250\,\hat{k})\ N.m$

vector → $\vec{M}_{net} = (-180\,\hat{j} + 250\,\hat{k})\ N.m$

magnitude → $|\vec{M}_{net}| = \sqrt{(180)^2 + (250)^2} = 308.1\ N.m$

maximum distance from (N.A)
(radias)

$$\sigma_x = \frac{(M_{net})\,c}{I} = \frac{(308.1)\times(10)}{\frac{\pi}{64}(20)^4} = 392.23\ Mpa$$

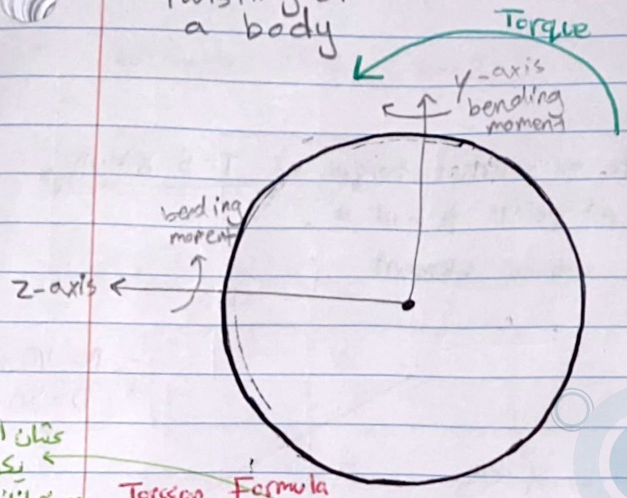
خلصنا ال Bending
Moment

Part (1)

Torque → Moment هو حوالين الـ longtudinal axis

Torsion

Twisting of a body



Torsion Formula

$$\tau = \frac{T\rho}{\boxed{J}}$$

internal Torque

Radial distance

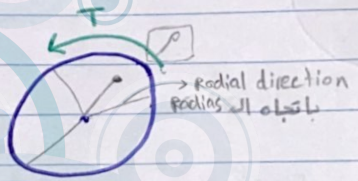
(taw) (shear stress)

Polar moment of inertia (twisting مقاومة الجسم لل)

Table A-18

عشان الجسم يكون inequilibrium بتولد فيه internal stress

Torque force



Flexures formula

*العلاقة السابقة

$$6 = \frac{MC}{I}$$

bending moment

perpendic distance from N,A

moment of inhertia

(مقاومة الجسم لل bending)

* Polar moment of inhertia

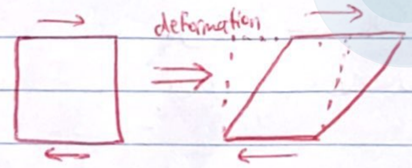
Circular

$$J = \frac{\pi D^4}{32}$$

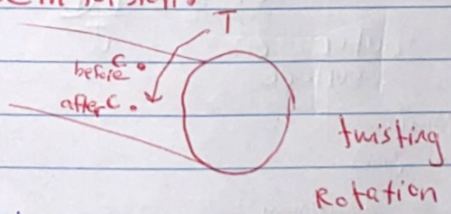
$$I = \frac{\pi D^4}{64}$$

moment of inhertia

* Torque : (Torsion) → any moment that is collinear with an axis of a mechanical part is called a Torque vector because the moment causes the part to be twisted about the axis, A bar that is subjected to such a moment is said to be in torsion.



displacement



$$\tau = \frac{T\rho}{J} \qquad (3\text{-}36)$$

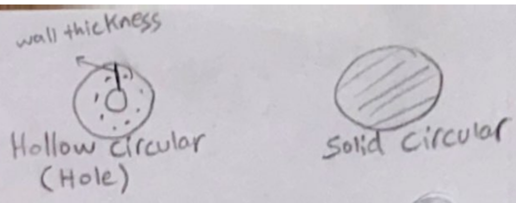
Sher stress

Torque

Torque (T) $\xrightarrow{\text{يولد}}$ Shear stress (τ)

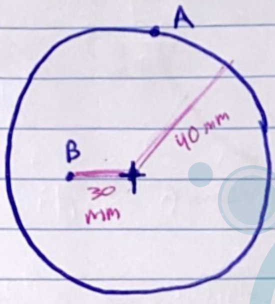
(tangential for the cross section)

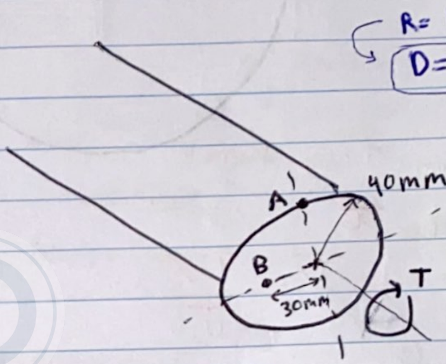
τ_{max}



* Example :-

F(5-1) The solid circular shaft is subjected to an internal torque of T= 5 KN.m, Determine the shear stress developed at points A and B. Represent each state of stress on a volume element





R= 40 mm
$\boxed{D= 80\text{ mm}}$

$$\underset{\text{tau}}{\tau}_B = \frac{T\rho}{J} = \frac{5\times10^{6}\times(30)}{\frac{\pi(80)^4}{32}} = 37.3\text{ Mpa}$$

ثابت لنفس ال (C.S)

$$\tau_A = \frac{T\rho}{J} = \frac{5\times10^{6}\times(40)}{\frac{\pi(80)^4}{32}} = 49.74\text{ Mpa}$$

ثابتة بسبب نفس الشكل لل (C.S)

*اختلف ال ρ (radial distance)

$$\uparrow \tau = \frac{T\overset{\text{ثابت}}{\rho}\uparrow}{\underset{\text{ثابت}}{J}}$$

τ تعتمد على ρ طردياً وخطياً

$\rho_{maximum}$ تكون على ال outer surface يعني المسافة (r) radias

* الأفضل نخلي ال Dimensions بال Newton / mm عشان يطلع الجواب النهائي MPa

T = 5 K N.m
= 5×10^3 N $\times 10^3$ mm
= 5×10^6 N.mm

$$\tau_{max} = \frac{T\rho_{max}}{J}$$

نفس الاشي

$$\tau_{max} = \frac{T r}{J}$$

outer surface

* عندما يطلب السؤال τ_{max} نحسبها عن ال outer surface لكن أولاً نحدد ال C.S (cross section) يلي عليها أعلى internal loading بعدين نحدد عليها ال Critical point

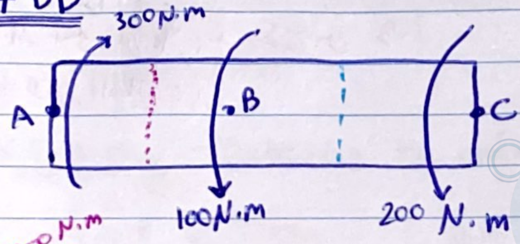
τ Maximum يكون عندما Radias = $\rho_{maximum}$

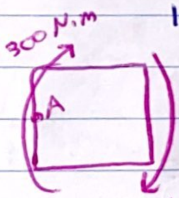
+Example (5-27)(5-28)

Determine maximum shear stress developed in the segments (AB) and (BC). The shaft has a diameter of 40 mm

$\tau = \frac{T\rho}{J}$ $\tau_{max} = \frac{Tr}{J}$

FBD





T̀ → عكس اتجاهها

internal torque to maintain in equilibrium

$\Sigma T = 0$

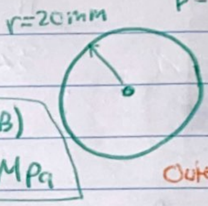
$\grave{T} = 300\ N.m$

AB

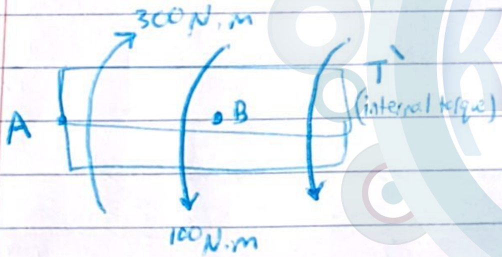
$\grave{T} = 300\ N.m$

للتحويل إلى ملي

$\tau = \frac{(300 \times 10^3)(20)}{\frac{\pi (40)^4}{32}} = \boxed{\tau_{max}(AB)\ 23.87\ MPa}$



+ أي نقطة على ال Outersurface بكون ال Radial distance Maximum الها



$\Sigma T = 0$

$300 - 100 - \grave{T} = 0$

$\grave{T} = 200\ N.m$

BC

$\tau = \frac{Tr}{J}$

$= \frac{(200 \times 10^3) \times (20)}{\frac{\pi (40)^4}{32}} = \boxed{\tau_{max}\ (BC)\ 15.92\ MPa}$

$\tau_{max}(AB) > \tau_{max}(BC)$

T لأنو ←
internal Torque
أكبر

*نادرا يجي هيك سؤال X
(يعطينا $\tau_{allowable}$ ويطلب D)

تابع
(الفرع الثاني)

***Example (5-28)**

Determine the required diameter of the shaft to the nearest mm if $\tau_{\text{allower stress}} = 60\ MPa$

Maximum allowable shear stress

هذه المعادلة تربط ثلاث اشياء ببعضها

$$\underset{\text{stress}}{\tau} = \frac{\overset{\text{load}}{T}\,r}{J} \quad \text{Size}$$

المنطقة لا أعلى torque Region for تكون بين AB
(حسبناه في الفرع السابق)

$$60 = \frac{(300\times10^3)\left(\frac{D}{2}\right)}{\frac{\pi D^4}{32}}$$

solve on calculator

$D_{minimum} = 29.42$ mm

*ال stress في ال material يعتمد عكسياً على ال size

$D = 30$ mm
(nearest mm)
Table (A-17) page (1066)

Gega → 10^9

Torsion

Part (2)

[angle of twist]

تعبّر عن مقدار ال deformation

θ → type of deformation

$$\theta = \frac{\textcircled{T}L}{GJ}$$

Torque

① FBD
② cut/section

length

جواب القانون
بال Radians

modulus of rigidity
(shear modulus of elasticity)

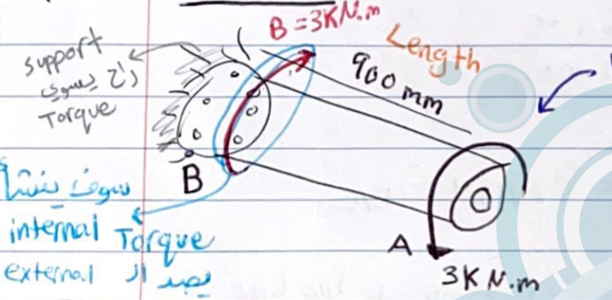
polar second moment of area

* اذا في applied torque ال support (الحيط) بيعمل Reaction Torque

* إذا في applied bending moment ال support بيعمل Reaction bending moment

F(5-9) *** Example** :- Determine the angle of twist of (A) with respect to (B)

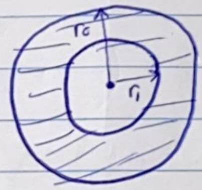
$G = 26$ Gpa



internal torque اذا بدي ادور
FBD بسوي

* قيمة internal Torque at any point is the same
لانو ال loading condition ما تغير على طول ال beam
كله عبارة عن قطعة وحدة وال internal load ما رح يتغير

Cross Section



$r_1 = 30$ mm
$r_o = 40$ mm

$D_1 = 60$ mm
$D_o = 80$ mm

$$\theta = \frac{TL}{GJ}$$

N.mm / N.m
mm / m
N/mm ← Mpa
N/m ← Pa
mm^4 / m^4

Hollow circular

$$\textcircled{J} = \frac{\pi(D_o^4 - D_i^4)}{32}$$

outer inner

$$\theta = \frac{(3\times10^3)(0.9)}{(26\times10^9)\times\left(\frac{\pi\times(0.08^4 - 0.06^4)}{32}\right)}$$

حولناها لل (m)

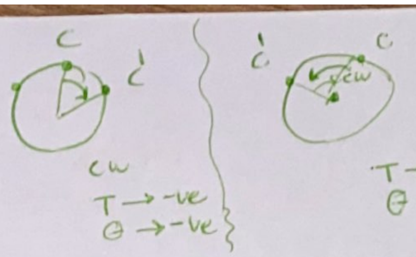
$\theta_{A/B} = 0.0378$ radians

angle of twist of A with respect to B

لما يحكي A و B
بعوض ال length يلي بينهم
وال internal torque يلي بينهم

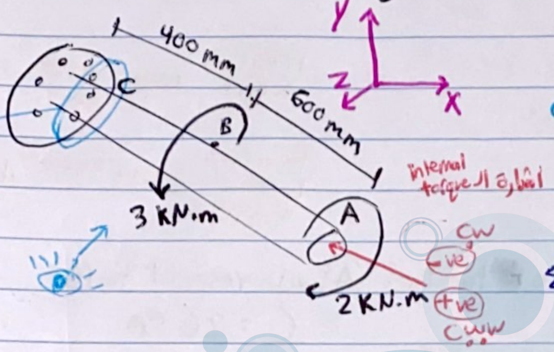
$$\text{radians} \xrightarrow{\times\frac{180}{\pi}} \text{degree}$$

$$\text{degree} \xrightarrow{\times\frac{\pi}{180}} \text{radians}$$

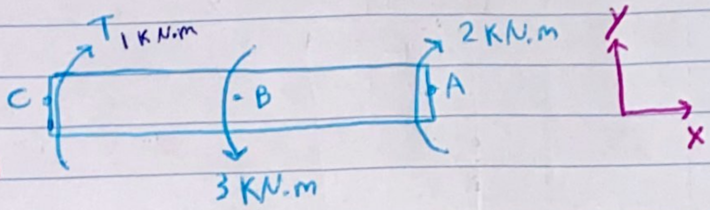


$\theta = \phi$
نفس الاشي

Example : The 60-mm diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of end (A) with respect to (C) [G = 75 Gpa]



في البداية نحسب قيمة ال Reaction Torque



$\Sigma T = 0$

$2 - 3 + T = 0$

$\boxed{T = 1\ KN.m}$

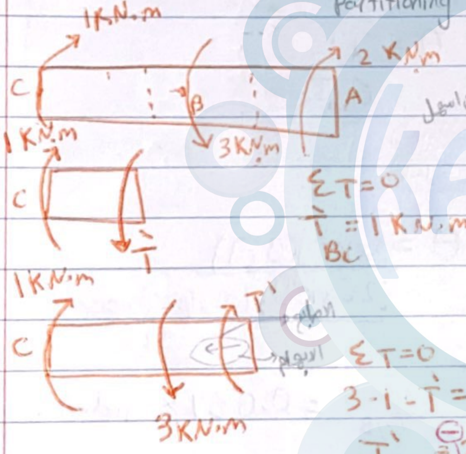
$$\theta_{A/C} = \theta_{A/B} + \theta_{B/C}$$

هنا ال loading conditions بيتغيرو لأنه في torque عالطريق بين (A) و (C)

بنسوي partitioning

$$\theta_{A/C} = \frac{T_{AB} L_{AB}}{GJ} + \frac{T_{BC} L_{BC}}{GJ}$$

بقدر اخذهم عامل مشترك عشان اسهل



$\Sigma T = 0$

$\overset{\rightharpoonup}{T}_{BC} = 1\ KN.m$

$\Sigma T = 0$

$3 - 1 - T' = 0$

$T'_{AB} = \ominus 2\ KN.m$

الاتجاه محور x minus

$$\theta_{A/C} = \frac{1}{(75 \times 10^9)\left(\frac{\pi \times (0.06)^4}{32}\right)} \times \left[(-2 \times 10^3)(0.6) + (1 \times 10^3)(0.4)\right]$$

طلعت c.w بالحل

$$\left[\theta_{A/C} = \ominus 0.00838 \text{ radians}\right.$$

* إذا طلب الزاوية بال degree بنسوي $\times \frac{180}{\pi}$ وبتطلع (-0.48)

لازم ال angle of twist تطلع رقم صغير جداً
(إذا طلعت رقم كبير لازم نشك بحلنا)

θ

* يجب تحديد الاتجاه للTorque لأنه بحدد اتجاه ال angle of twist

3-11 Shear Stresses for Beams in Bending

*Transverse Shear Formula

internal shear force (بسوي cut وبنقيمة) وفي الها diagrams

$$\tau = \frac{\boxed{V}Q}{Ib}$$

taw

Q → first moment of area $\int y\,dA$

I → second moment of area $\int y^2\,dA$

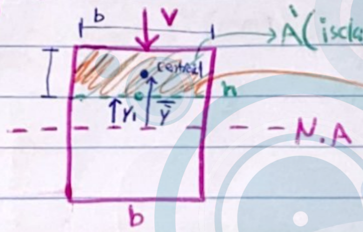
b → base (Dimension ال الموازي لل C.S)

+ قبل ما نستخدم ال transverse shear formula لازم نحدد ال location يلي فيو أعلى internal shear force عن طريق ال shear diagram

① نحدد ال critical cross sectional area يلي فيو أعلى internal shear force

② نحدد ال critical points يلي فيها أعلى shear stress due to this force

*Rectangular Cross section



A'(isolated area)

$Q = \int y\,dA$

$Q = \boxed{A'}\,\boxed{\bar{y}'}$

A' → Area for isolated area

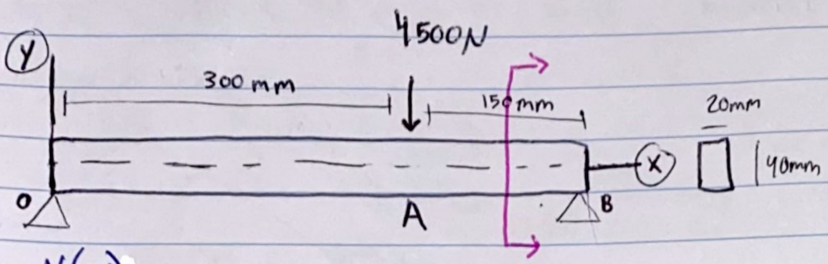
$\bar{y}'$ → location for centroid for isolated area

$$\tau = \frac{VQ}{\boxed{I}\,b}$$

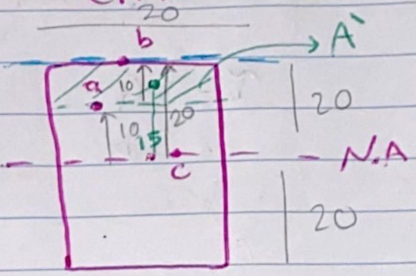
Q → for isolated area (area above the point of interest)

I → for C.S area كاملة

problem (3-35) * **Example**



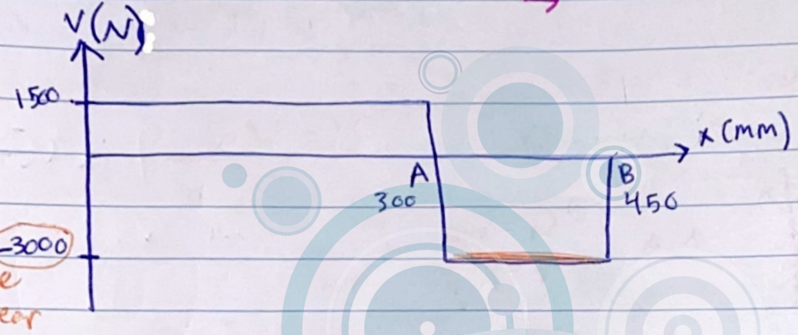
Critical section



$V = 3000\ N$

Maximum Value for internal shear force

Shear Diagram



Maximum value for internal shear بغض النظر عن الاشارة وبالمنطقة بين A و B

$$\tau = \frac{VQ}{Ib}$$

$$Q_a = A`\ \bar{y}`$$

$$Q_a = 10 \times 20 \times 15$$

$$Q_a = 3000\ mm^3$$

$$\tau_a = \frac{(3000)(3000)}{\frac{1}{12}(20)(40)^3 \times 20} = 4.22\ Mpa$$

$$\tau_b = \frac{(3000) \times 0}{\dots\dots} = 0$$

$$Q_b = A`\ \bar{y}$$

Area يلي فوق b لأنها outer

$= 0 \times \dots$

$= 0$ zero

ⓑ → outer surface the tranVerse

Second Part of Mid exam Material

3.5 Cartesian Stress Components

سوف نتكلم عن ٤ في part ②

* Stress element

* Stress transformation → نحول stresses من plane الى plane آخر داخل ال material

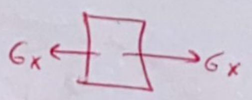
→ (principle stresses / maximum shear stress / principle angle)

في بعض المراجع يسمونها Maximum Normal Stresses

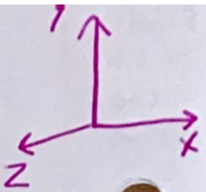
* mohr's circle → Graphical representation for stress state في نقطة معينة في ال material

(نفس principle stresses لكن بنمثلهم graphically)

* fallivre theries → Tressca / Vonmises



Uniaxial stress state



(3.5)

*** Stress element** — infinite ~~tesmile~~ esimal elements → has NO SIZE

نأخذه من ال material

To represent stress state at specific point from material.

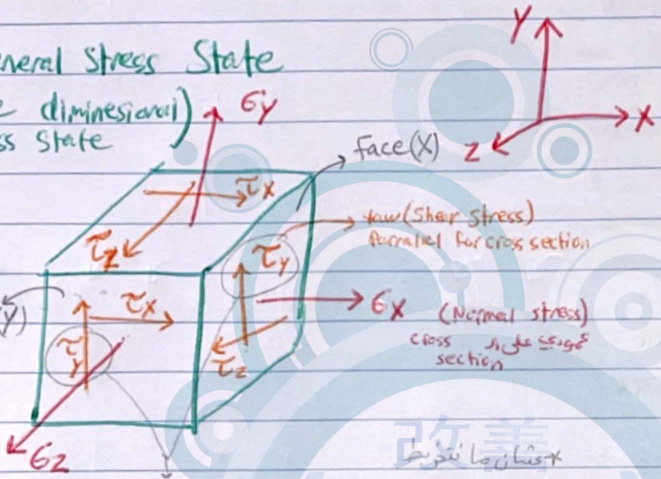
critical point :-

σ = ... Mpa

τ = ... Mpa

General Stress State
(Three dimensional) Stress state

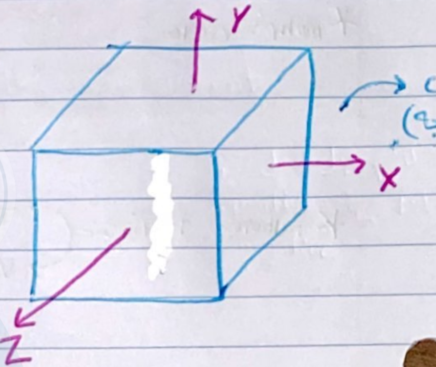
in Reality this cube have no size and no dimensions



الثنين على Y لكن كل واحد على surface مختلف

* عشان ما نتخربط بمسميات ال τ نلجأ إلى τ□□ surface Normal → direction

(Stress element) له عدة اشكال لكن غالباً يكون مكعب



6x , 6y , 6z

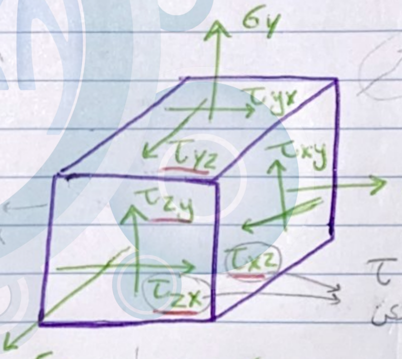
τxy ; τxz , τyx , τyz , τzx , τzy

Surface Normal — Direction

$\tau_{xy} = \tau_{yx}$

$\tau_{zy} = \tau_{yz}$

$\tau_{xz} = \tau_{zx}$



عالسطح الخلفي عشان ال eqillibrium

(cross Shears)

$\tau_{xz} = \tau_{zx}$

نفس المقدار لكن عكس الاتجاه

(9 Stress Components)

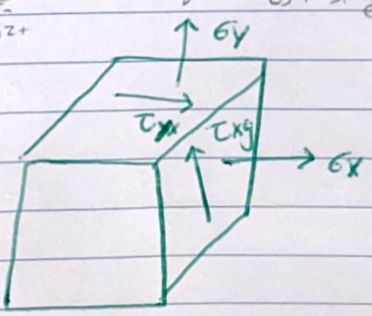
6 independent

3 dependent

* إذا كان material في in equillibrium واجيت قطعت هذا الجزء برضه باقي الأجزاء حيكونوا in equillibrium

فبرضه ال stress element بكون in equillibrium

في على ال فوق +Y وتحت -Y يمين +X و شمال -X قدام +Z و ورا -Z

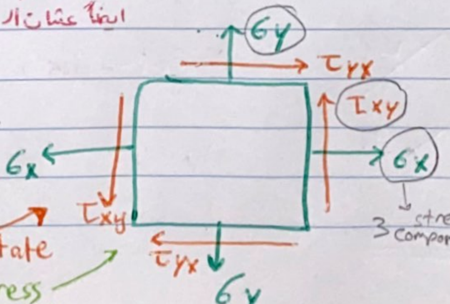


plane stress state

انه في Plane معين بطل عليه stresses
Σ Stresses = zero

6z

اعتبرنا 6z , τzy , τzx يساوو صفر و بالتالي τxz و τyz يساوو صفر ايضاً عشان ال equillibrium



plane stress state

2D - stress state

Square X

لانه مكعب بس انه عم نطلع عليه Side view

(3.6) Stress Transformation

ما بيجي عليه أسئلة
في الامتحان لكن مهم الفهم
عشان نفهم (Principle Stresses)

مراجعة سريعة

Stress element → is a tool that help us to represent the stresses at a specific point in the material

↓ ⓧⓨⓩ
in 3-dimension
we draw a cube
and put the stresses
on it

*إذا كان عندي Direction (axis) معين ما عليه stresses
بصير اتطلع على المكعب من جهة معينة فبس مربع
so we can represent the stresses in a single plane

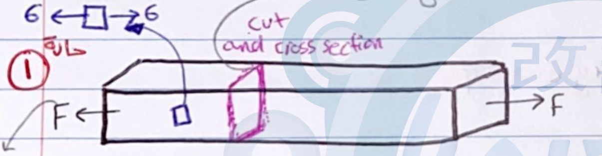
مثلاً إذا كان ال ⓩ ما عليه stresses كل ال stresses بحطهم على Plane (X-Y)
و بصير اسمها (plane stress state)
(2-D state)

*إذا كانت ال stresses بـ direction واحد فقط
بكون اسمها (axial stress state)

مثلاً Normal من اليمين و الشمال

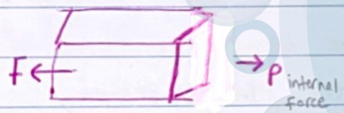
في هذا المثال
سوف نأخذ
نفس ال
material
ونفس ال
external
loadings

حالة ①



external
Forces
(ما بيعملو stress)

external forces
causes internal
forces to
develop and
internal causes
make the
stress



$\tau = 0$

→ P internal force يلي بتعمل ال stress

$$\boxed{\sigma = \frac{P}{A_o}}$$

stress state

A_o → Area for cross section

*we have single stress component and it's Normal

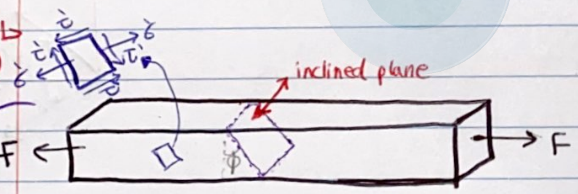
σ → transformed to → σ', τ'

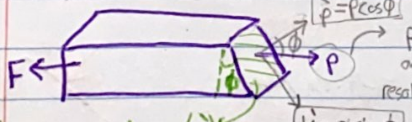
*if we rotate the stress element by an angle "ϕ", we will get a new stress state that is different from the original one.

"Stress transformation"

حالة ②

أخذنا
cross
section
بطريقة
مائلة





$P' = P\cos\phi$

P → Resultant force and can be resolved to two components

$V' = P\sin\phi$

stress الأصلية P ما بتعمل
لكن مركباتها جهة يلي
بيعملو σ، τ

Area (A') تختلف عن (A_o)

$A_o < A'$

$A' = \frac{A_o}{\cos\phi}$

Normal stress → $\sigma' = \frac{P'}{A} = \frac{P\cos\phi}{\frac{A_o}{\cos\phi}} = \boxed{\frac{P}{A_o}\cos^2\phi = \sigma'}$

shear stress → $\tau' = \frac{V'}{A'} = \frac{P\sin\phi}{\frac{A_o}{\cos\phi}} = \boxed{\frac{P}{A_o}\sin\phi\cos\phi = \tau'}$

Stress State
مختلف تماماً
لأن ال Plane مائل

باخذها من x إلى x'

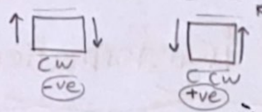
ϕ → +ve c.c.w, −ve c.w

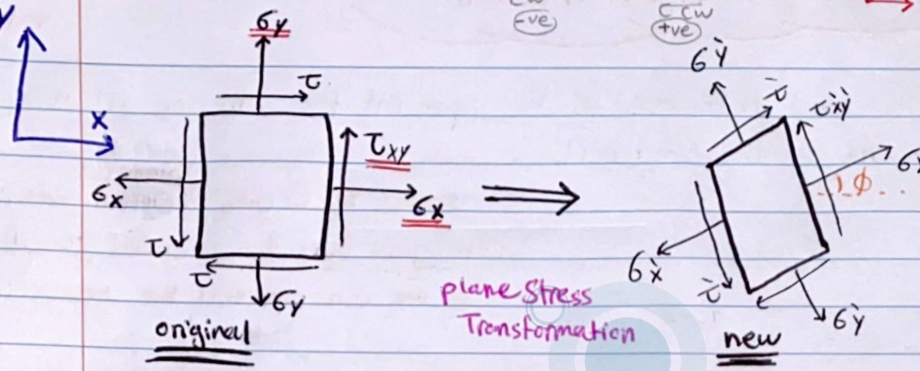
sign conventions regarding for stresses

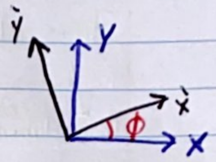
σ → +ve in tension, −ve in compression

τ → +ve tend to rotate c.c.w, −ve tend to rotate c.w

توضيح







*الزاوية φ دائماً measured from X-axis

المعطيات في السؤال باللون الأحمر

المطلوب

السؤال بيجي هيك → * Given the original stress state, Find the stresses on the new element

المعطيات
σ_x
σ_y
τ_{xy}
ϕ

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\phi) + \tau_{xy}\sin(2\phi)$$

لازم قبل الحل نحدد إذا هي positive or negative

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin(2\phi) + \tau_{xy}\cos(2\phi)$$

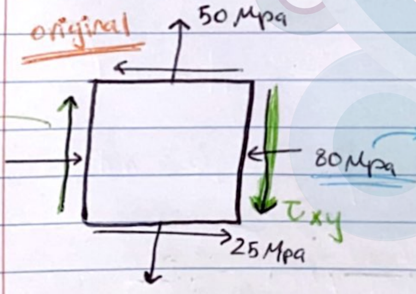
ما بيجي هيك أسئلة لكن عشان نفهم الموضوع →

***example:**

The State of plane stress at a point is represented by the element shown, Determine the state of stress at the point on another element oriented 30° clockwise from the position shown

لما stress element is rotated by an angle

original



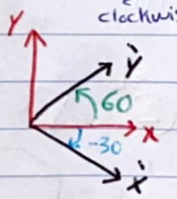
بحاولو يلفوه cw إذا τ ⊖

compression إذا سالب

$\sigma_x = -80$ Mpa (compression), $\sigma_y = +50$ Mpa (tension), $\tau_{xy} = -25$ Mpa (cw)

$\phi = \phi + 90$ إذا y' $= 60°$

$\phi = -30$ clockwise



$$\sigma_{x'} = \frac{-80+50}{2} + \frac{-80-50}{2}\cos(\overset{2\times -30°}{-60}) + (-25)\times\sin(-60) = -25.8 \text{ Mpa}$$

compression

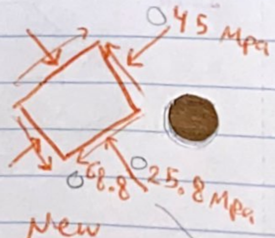
φ بعوض φ تساوي 90° + φ = 60°

$$\sigma_{y'} = \frac{-80+50}{2} + \frac{-80-50}{2}\cos(\overset{2\times 60°}{120}) + (-25)\sin(120) = -4.15 \text{ Mpa}$$

compression

$$\tau_{x'y'} = -\frac{-80-50}{2}\sin(-60) - 25\cos(-60) = -68.8 \text{ Mpa}$$

tend to rotate c.w



ما بصير احط المقدار سالب
لأنه خلص عبرت عنه عن طريق الاتجاه للسهم

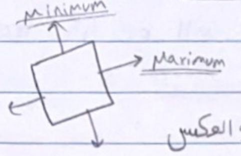
(3.6) Principle Stresses

*مفهوم **Principle Stresses**

"ϕ_p" principle angle [*في زاوية معينة إذا باخذ الـ stress element فيها بتكون $\sigma_{\dot{x}}$ maximum

"ϕ_s" Maximum shear stress angle [*في زاوية أخرى معينة إذا باخذ stress element فيها بتكون $\tau_{\dot{x}\dot{y}}$ Maximum

* Principle Stresses : is the $\underbrace{\text{maximum normal stress and minimum}}_{\text{يوجد بينهم علاقة}}$ normal stress.



والعكس

خطوات الحل :- $\sigma_{\dot{x}\,max}$

① ايجاد الزاوية يلي بتعطيني Maximum normal stresses

② ايجاد Maximum Normal Stresses

ونفس الخطوات لـ $\tau_{\dot{x}\dot{y}\,max}$

* The magnitude of $\sigma_{\dot{x}}$ and $\tau_{\dot{x}\dot{y}}$ depend on the angle of inclination "ϕ".

$(\sigma_{\dot{x}}, \tau_{\dot{x}\dot{y}}) \xrightarrow{} \text{function of } "\phi"$

يعتمدوا على فاي

→ principle angle "ϕ_p"

→ Principle stresses (σ_1, σ_2)

max ← we are interested in this one

min

To find (ϕ_p) principle angle $\Rightarrow$ $\boxed{\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$

فاي الزاوية بتعطيني Maximum Normal Stress

To find (σ_1, σ_2) principle stresses $\Rightarrow$ $\boxed{\sigma_1, \sigma_2 = \frac{\sigma_x \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$

$(\sigma_1)_{max}$

$(\sigma_2)_{min}$

→ max shear stress angle "ϕ_s"

→ max in-plane Shear Stress "$\tau_{\substack{max \\ in\,plane}}$"

To find (ϕ_s) maximum shear stress angle $\Rightarrow$ $\boxed{\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\,\tau_{xy}}}$

تشبه معادلة (ϕ_s) لكنها مقلوبة ومضروبة بسالب

To find maximum in plane Shear Stress $(\tau_{\substack{max \\ in\text{-}plane}}) \Rightarrow \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

*أهم اشي في هذا الدرس

نحسب σ_1, σ_2, τ_{max} in plane ← هاد باي بهمونا

(الزوايا موكتير مهمة)

عند ال ϕ_p

① No ~~Shear~~ stress acts on the principle plane.

عند ال ϕ_s

② There is an average Normal Stress on the planes of max in-plane shear stress. $\rightarrow \frac{\sigma_x+\sigma_y}{2}$

③ An element subjected to maximum Shear stress will be 45° from the position of an element that is subjected to the principe stresses.

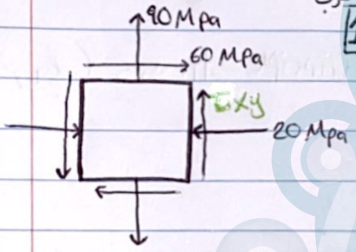
that means the difference between "ϕ_s" and "ϕ_p" = 45°

* يعني اذا اوجدت وحدة ϕ_p أو ϕ_s بطرح أو بجمع 45 و بلاقي الثانية ← عالحالتين صحيح

[1] find (principle stress angle) and (principe stresses) and (draw the stress element that have the principle stresses)

[2] find (maximum in-plane shear stress) and (maximum shear stress angle)

*example



$\sigma_x = -20$ Mpa, $\sigma_y = 90$ Mpa, $\tau_{xy} = 60$ Mpa

المطلوب [1]

$$(\sigma_1, \sigma_2) = \underbrace{\left(\frac{\sigma_x+\sigma_y}{2}\right)}_{\sigma_{average}} \pm \underbrace{\left(\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}\right)}_{\tau_{max}\text{ in plane (maximum shear)}}$$

$$(\sigma_1, \sigma_2) = \frac{-20+90}{2} \pm \sqrt{\left(\frac{20-90}{2}\right)^2 + (60)^2}$$

Principle Stresses ← $(\sigma_1, \sigma_2)\ 35 \pm 81.4$

$\sigma_1 = 116.4$ MPa → maximum

$\sigma_2 = -46.4$ Mpa → minimum

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\phi_p = \cancel{\tan}\left(\frac{2\times 60}{-20-90}\right) \rightarrow \frac{-12}{11}$$

$$2\phi_p = \tan^{-1}\left(\frac{-12}{11}\right)$$

$$\phi_p = \frac{-47.49^\circ}{2}$$

principle angle ← $\phi_p = -23.7^\circ$

(-) يعني clockwise سب

* إذا طلب السؤال Draw the stress Element that have the principle stresses

* عشان اعرف وين احط σ_1 σ_2 (مين على ال x و مين على ال y)

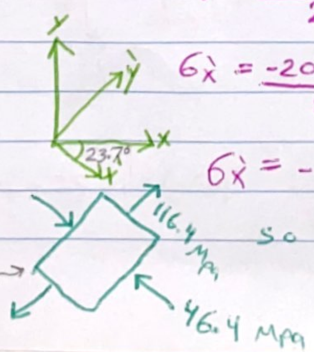
بعوض بهاد القانون و بعوض ($\phi \rightarrow \phi_p$) والرقم القريب منه هو على $\sigma_{\acute{x}}$

$$\sigma_{\acute{x}} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi_p + \tau_{xy}\sin 2\phi$$

$$\sigma_{\acute{x}} = \frac{-20+90}{2} + \frac{-20-90}{2}\cos(\overset{2\times -23.7^\circ}{-47.49^\circ}) + 60\sin(-47.49)$$

$$\sigma_{\acute{x}} = -46.4 \text{ Mpa} = \sigma_2$$

so now I can draw



*طبعاً ما في (τ) shear stress لأنه احنا على ال Principle plane

تابع

المطلوب $\boxed{2}$

$$* \left(\underset{\text{in-plane shear stress}}{\tau_{max}} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \right)$$

نلاحظ أن معادلة ال τ_{max} shear stress in-plane هي جزء من معادلة ال Normal principle stresses

* يعني فعليا احنا حاسبينها

$$\underset{\text{in plane}}{\tau_{max}} = \sqrt{\left(\frac{-20-90}{2}\right)^2 + 60^2}$$

$$\underset{\text{in plane}}{\tau_{max}} = 81.4 \text{ Mpa}$$

يا بحل يا دغري باخذها من part $\boxed{1}$

$$* \tan 2\phi_s = -\frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$

$$\tan 2\phi_s = \frac{-20-90}{2\times60} \rightarrow \frac{11}{12}$$

$$2\phi_s = \tan^{-1}\left(\frac{11}{12}\right)$$

$$2\phi_s = 42.5^\circ$$

$$\phi_s = 21.3^\circ$$

يا بحل يا دغري باخذها من part $\boxed{2}$ عن طريق

$$\phi_s = \phi_p + 45^\circ$$

$$\phi_s = -23.7^\circ + 45^\circ$$

$$\phi_s = \overset{\oplus}{}21.3^\circ \rightarrow \text{c.cw}$$

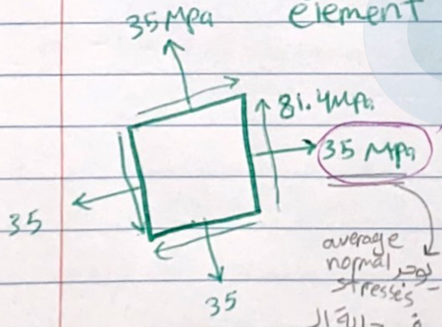
* والـ Maximum shear stress angle هي الـ

$$\phi_s = \phi_p + 45^\circ$$

أو

$$\phi_s = \phi_p - 45^\circ$$

* إذا كان مطلوب ارسم ال stress element



$$\sigma_{average} = \frac{\sigma_x+\sigma_y}{2} = \frac{-20+90}{2} = 35 \text{ Mpa}$$

بقدر اطلعها من part $\boxed{1}$ في قانون σ_x

average normal stresses بوجد في حالة ال Max in-plane shear stress

(3.6) Mohr Circle

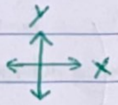
→ is a Graphical method to represt the relationship between σ and τ

سهل جداً وبسيط
وبيجي عليه أسئلة بالهيد

*** Revision**

$(x-a)^2 + (y-b)^2 = r^2$

المحور ← A   المحور   radias

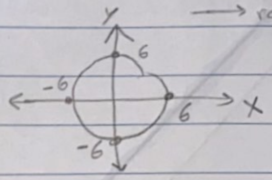


Center → (a,b)

Radias → (r)

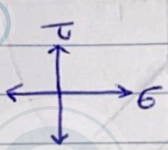
* Special case
$x^2+y^2=r^2$
center → $(0,0)$
radias → (r)

* example to understand
$x^2+y^2=36$
→ center $(0,0)$
→ radias (r)



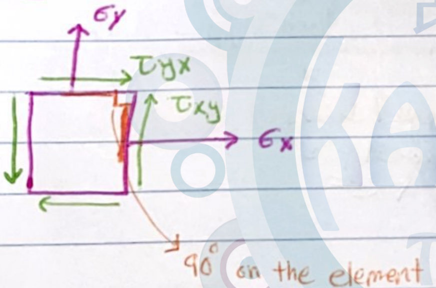
$(\sigma_x - \sigma_{avg})^2 + \tau_{xy}^2 = R^2$

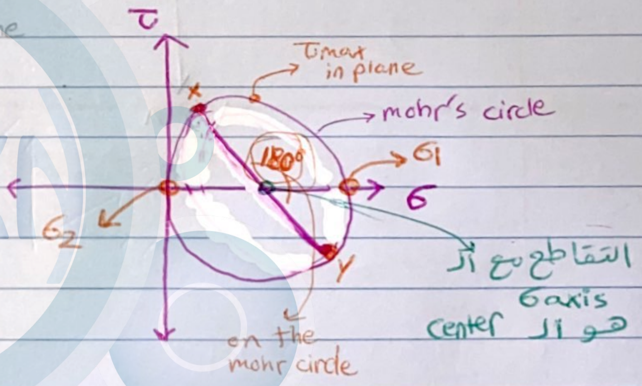
المحور   المحور   Radias



Center → $(\sigma_{avg}, 0)$

radias → $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$ → نفسه قانون τ_{max} in plane



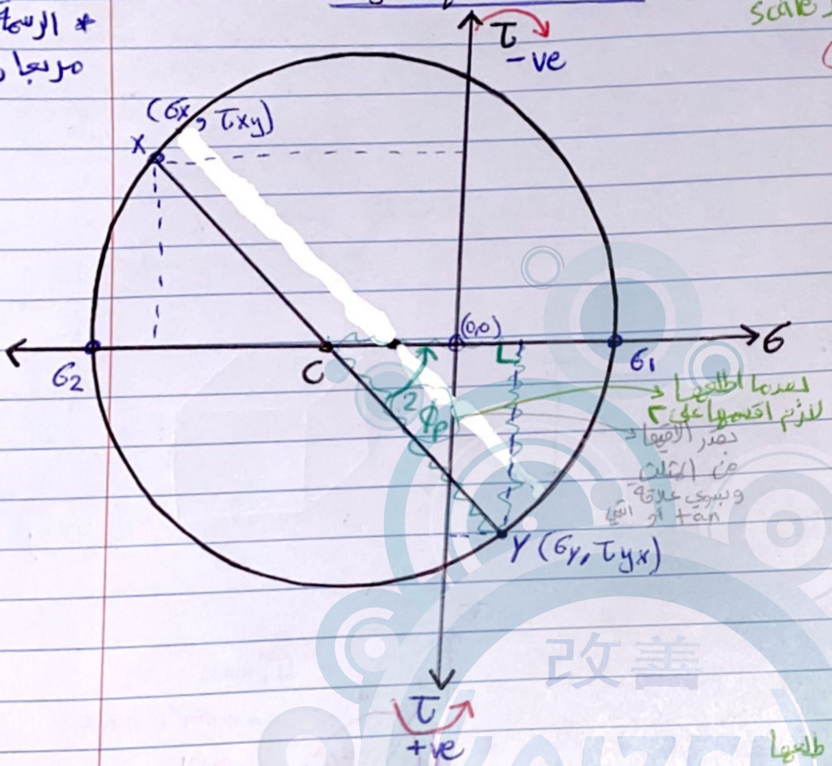


* الزاوية على الelement = $\frac{1}{٢}$ الزاوية على الcircle

* Rotation θ on the element Corresponds to Rotation 2θ on circle

***example:** A mohr's circle shown for a point in a physical object that is subjected to plane stress, If 1 grid square (scale) = 8 Mpa, determine the maximum in-plane shear stress τ_{max}

* الرسمة فيها مربعات



عدد المربعات X الـ scale

① σ_x, σ_y, τ_{xy}

$\sigma_x = \boxed{11} \times \boxed{8} = \ominus 88$ Mpa

موجودة بالسالب

$\sigma_y = 3 \times 8 = 24$ Mpa

$\tau_{xy} = 5 \times 8 \overset{\ominus}{=} 40$ Mpa

لأنه السالب فوق والموجب تحت

② σ_1, σ_2, τ_{max}

$\sigma_1 = 36$ Mpa

$\sigma_2 = \ominus$ _____

$\tau_{max} =$

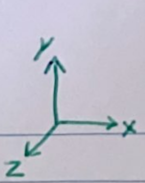
③ ϕ_p

$\tan 2\boxed{\phi_p} = \dfrac{2\,\tau_{xy}}{\sigma_x - \sigma_y}$

بعدما اطلعها لازم اقسمها على ٢

(I) Invariant → لا يتغير مجموعهم
stress

(3.7) General Three-Dimensional Stress

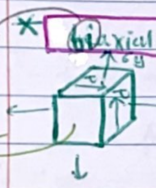


Revision

*** uniaxial Stress state**

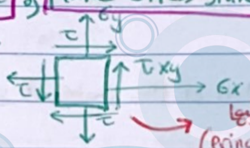
---

(2) *** biaxial stress State** , **plane Stress State** (I neal three stress component to specify this stress state) $[\sigma_x, \sigma_y, \tau]$



in this case there is a plane that have zero stresses

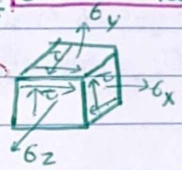
⟹



ناقشنا عليها
(principle stresses and mohr's circle) → σ_1, σ_2 ⟶ σ_x, σ_y ⇓ σ_1, σ_2

*** General three dimensional Stress state** , **triaxial stress state** (3)

درسنا هنا #



في هذا الدرس سوف نناقش كيف يمكننا حساب principle stresses mohr circle في هذه الحالة

we need (I need 6 Stress components) $[\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}]$

in this case we have all (σ) and (τ)
there is a total of 9 stress components
↙ 6 independent ↘ 3 dependent

$\sigma_x, \sigma_y, \sigma_z$
⇓
$\sigma_1, \sigma_2, \sigma_3$

---

* Go back to the book page (101) for the equations

* في العادة mohr circle ما بتدخل
في هذا ال section

* Example :

The figure shows a triaxial stress element having a critical three dimensional stress state where $\sigma_x = 50$ Mpa , $\sigma_y = -75$ Mpa , $\sigma_z = 25$ Mpa , $\tau_{xy} = 25$ Mpa , $\tau_{zx} = 100$ Mpa .

ⓐ calculate the first, second, and third stress invariants.

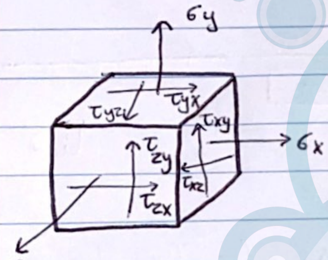
ⓑ Solve the Characteristic equation for the principle normal stresses.

ⓒ calculate the maximum shear stress

ⓓ draw the resulting Mohr circle representation of the state of stress. (25pts)

المعطيات:

$\sigma_x = 50$ Mpa	$\tau_{xy} = 10$ Mpa
$\sigma_y = -75$ Mpa	$\tau_{yz} = 25$ Mpa
$\sigma_z = 25$ Mpa	$\tau_{zx} = 100$ Mpa



* From equation (3-15) in the book

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \longrightarrow ⊛$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z \longrightarrow \boxed{I_1 = 0}$$

$$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_{zx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 \longrightarrow \boxed{I_2 = -15100}$$

$$I_3 = \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{yz} \tau_{zx} - \sigma_x \tau_{yz}^2 - \sigma_y \tau_{zx}^2 - \sigma_z \tau_{xy}^2 \longrightarrow \boxed{I_3 = 672500}$$

*الان برجع اعوض بالمعادلة ⊛

equation ←

معادلة تكعيبية

$$\sigma^3 - \cancel{0\sigma^2} + (-15100)\sigma - 672500 = 0$$

$$(\sigma^3) + 0 - 15100(\sigma) - 672500 = 0$$

max ← $\sigma_1 = 141$ Mpa

intermidiate ← $\sigma_2 = -56.4$ Mpa

min ← $\sigma_3 = -84.5$ Mpa

$\sigma_1 \geq \sigma_2 \geq \sigma_3$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 141.$$

page (233

(5.3) Failure Theories

طريقة لايجاد Factor of safety
عندما يكون عندي اكثر من
Stress component

Factor of safety ⟶ $n = \frac{S_y}{\sigma}$ → strength, → stress

$n > 1 \rightarrow$ survival
$n < 1 \rightarrow$ Failure

$\sigma_1, \sigma_2, \sigma_3, \tau_{max}$

⟶ we have more than one stress component

⟶ only one strength component

في هذا الدرس ⟶ ★representative stress

① Tresca : MSS

$$\boxed{n = \frac{S_y}{2\tau_{max}}}$$

strength ⟶ table (A-20)
calculated

② Vonmises : DE

$$\boxed{n = \frac{S_y}{\sigma'}}$$

strength
calculated

$$\sigma' = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

vonmisses stress
vonmisses equivalent

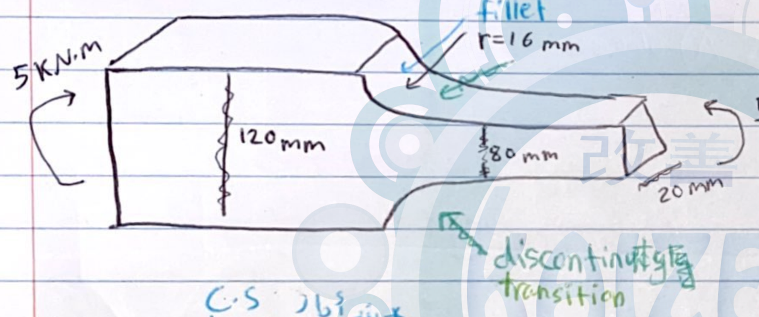
(3.13) Stress Concentration

$\sigma_o = \frac{MC}{I}$ $\qquad \tau_o = \frac{T\rho}{J}$

actual ← $\sigma_{max} = K_t \, \sigma_o$ $\qquad \tau_{max} = K_{ts} \, \tau_o$

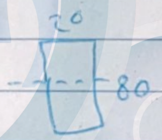
Nominal

*** Example:**



عند الـ C.S الصغير

$\sigma = \frac{MC}{I} = \frac{(5\times10^6)\times(40)}{\frac{1}{12}(20)(80)^3} = 234.4$ Mpa



$\sigma_o = 234.4$ Mpa

figure (A-15-6)

$\boxed{K_t}$

$\frac{r}{d} = \frac{16}{80} = 0.2$

$\frac{D}{d} = \frac{120}{80} = \underline{\underline{1.5}}$ → interpolation لأنها مو موجودة

→ $\boxed{K_t = 1.45}$ → أي قيمة قريبة عادي بيزبط

$\sigma_{max} = (1.45)\times(234.4) = 340$ Mpa

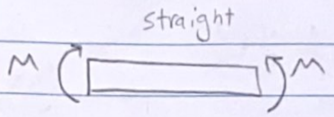
↓

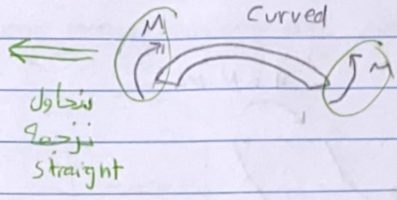
principle stresses بنطلع

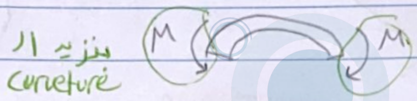
(3.18) Curved Beams in bending

Beams → straight
Beams → curved

ال beams هنا منحنين من الأول
curved beams و ايضاً تعرضنو ر
bending moment

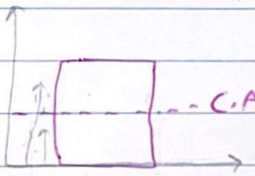




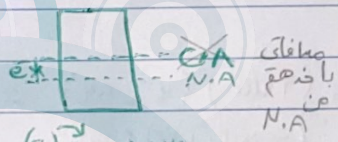


* Straight

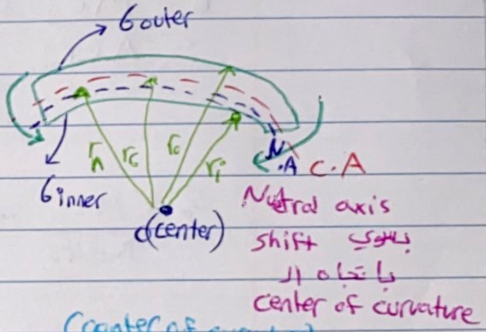
$\sigma = \frac{Mc}{I}$
c → distance from N.A
I → N.A

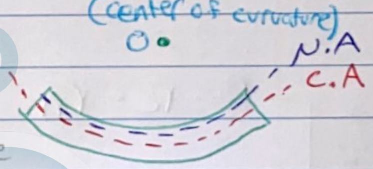


* Curve



(e) → eccentricity





$\sigma = \frac{Mc}{Aer}$
M → bending moment
c → distance from N.A
A → area for cross section
e → exentricity
r → radial distance from O to the point of interest

هون شغلنا بالحل
(المسافة بين C.A و N.A)

$* \; e = r_c - r_n$

$* \; r_c = \frac{\Sigma \tilde{r} A}{\Sigma A} = \frac{\tilde{r}_1 A_1 + \tilde{r}_2 A_2 \cdots}{A_1 + A_2 \cdots}$

$\tilde{r}$ → r telda
المسافة من ال C.A
center of curvature
لل C A_1 ثم C A_2
⋮

$r_n = \frac{\Sigma A}{\Sigma \int \frac{dA}{r}} = \frac{A_1 + A_2 + \cdots}{\int \frac{dA_1}{r_1} + \int \frac{dA_2}{r_2}}$ → موجود في الجداول

any point → $\sigma = \frac{Mc_o}{Aer}$

* critical stress element

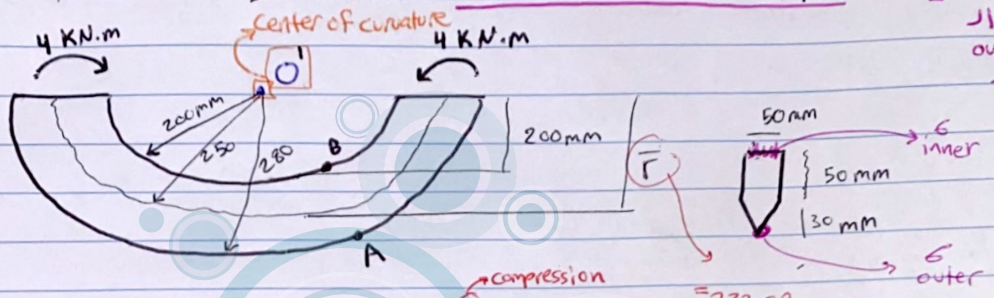
تكون ال critical يا على ال outer يا على ال inner

$$\boxed{\sigma_o = \frac{Mc_o}{Aer_o}}$$

$$\boxed{\sigma_i = \frac{Mc_i}{Aer_i}}$$

* Example

The curved bar has a cross sectional area shown , If it is subjected to bending moments ~~of~~ of 4 KN.m , determine the maximum normal stress developed. → (Critical) يا بكون على ال outer surfer يا على ال inner surface



=233.08

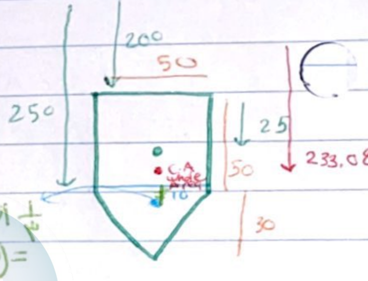
$$\sigma_i = \frac{M c_i}{A e r_i} = \frac{4\times10^{6} \times 31.42}{(3250)(1.66)(200)} = 116 \text{ Mpa}$$

⊖ compression

$$\sigma_o = \frac{M c_o}{A e r_o} = \frac{(4\times10^{6})(48.58)}{(3250)(1.66)(28)} = ⊕129 \text{ Mpa}$$

tension

∴ critical stress is on the outer



$\frac{1}{3}$ الارتفاع من القاعدة (10) =

نبدأ الحل من هنا

$$e = r_c - r_n$$

هذا ال cross section بقسمه إلى two compositie areas كل وحدة الها centeroidal

$$r_c = \frac{\Sigma \tilde{r} A}{\Sigma A} = \frac{225\times(50\times50) + (260)\times(\frac{1}{2}\times50\times30)}{50\times50 + \frac{1}{2}\times50\times30}$$

$\Sigma A = 3250\ m^2$

$\longrightarrow r_c = 233.08$ mm

$$r_n = \frac{\Sigma A}{\left(\Sigma \int \frac{dA}{r}\right)} = \frac{3250}{50\times\ln\left(\frac{250}{200}\right) + \frac{50\times280}{30}\ln\left(\frac{280}{250}\right) - 50}$$

Table 6-1 from strength book

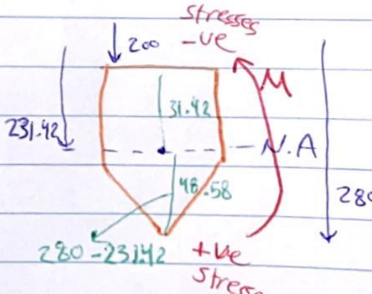
$\longrightarrow r = 231.42$ mm

$e = |r_c - r_n|$ → قيمة مطلقة

$r_c > r_n$ → بسوي shift باتجاه center of curvature

$= 233.08 - 231.42$

$\boxed{e = 1.66 \text{ mm}}$



الان نرجع لقوانين
σinner
σouter

Solved problems
3.14 stresses inPressurized Cylinders
3.16 Press and Shrink Fits

لم اكتب هذه المواضيع في الدفتر