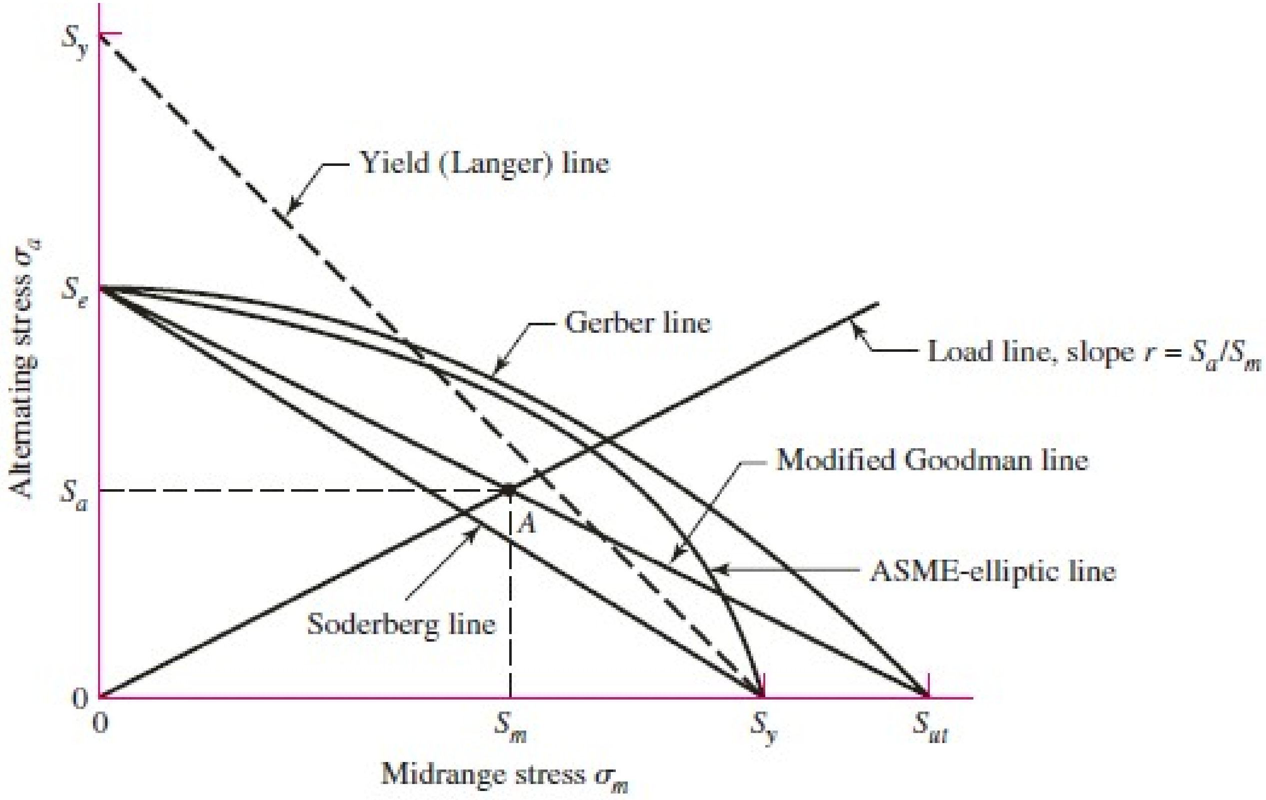
6-12 Fatique Failure Criteria for Flactuating Stress
Constant Life Fatigue Diagrams

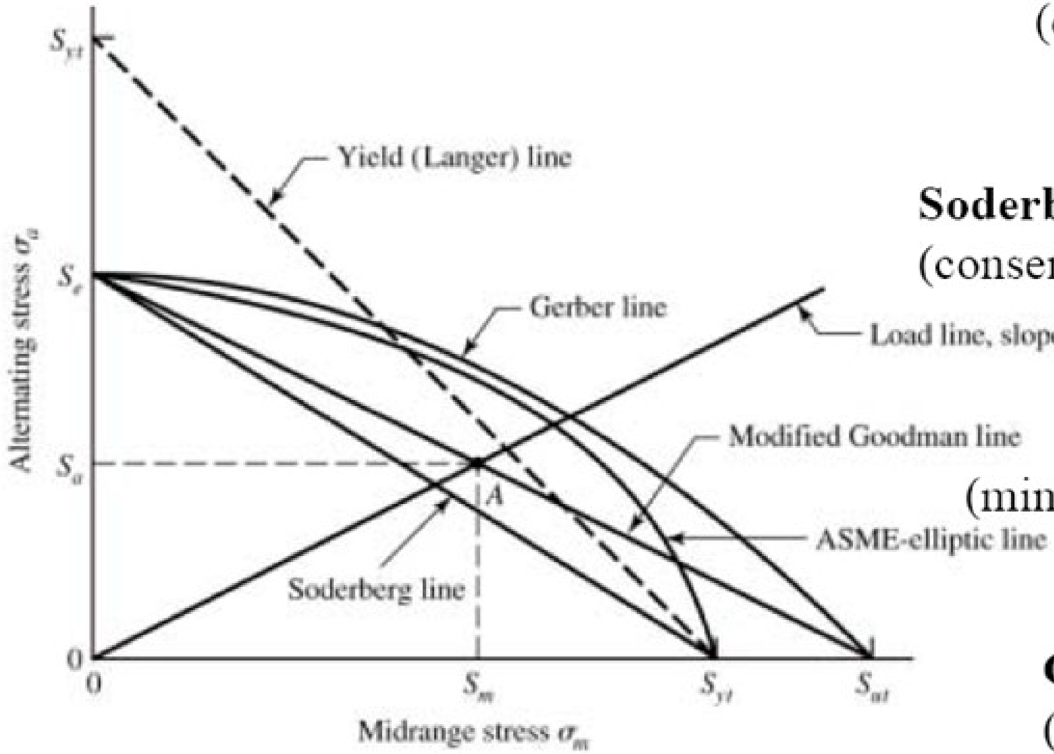
- There are 4 basic criteria of fatigue failure in the diagram σ_a - σ_m *:* Four straight lines relevant for infinite life:

– Soderberg, Modified Goodman, Langer (Yield-line), ASME/elliptic

Only the Sodeberg criterion ensures that plastic deformation will not appear



- Every point on Goodman (or Soderberg if that is preferred) line has the same lifetime



Langer
(elastic deformation)

$$S_a + S_m = S_y$$

Soderberg
(conservative assessment)

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$$

Goodman
(minimum assessment)

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Gerber
(the best fit)

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Modified Goodman and Langer

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

$$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$$

ASME- Elliptic and Langer

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

$\sigma_m \geq 0$

Soderburg	$\sigma_a/S_e + \sigma_m/S_y = 1/n$	(6–45)
mod-Goodman	$\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$	(6–46)
Gerber	$n\sigma_a/S_e + (n\sigma_m/S_{ut})^2 = 1$	(6–47)
ASME-elliptic	$(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2 = 1/n^2$	(6–48)

Check for localized yielding.

$$\sigma_a + \sigma_m = S_y/n$$

$$\tau_a + \tau_m = 0.577 S_y/n$$

6–25 The cold-drawn AISI 1040 steel bar shown in the figure is subjected to a completely reversed axial load fluctuating between 28 kN in compression to 28 kN in tension. Estimate the fatigue factor of safety based on achieving infinite life, and the yielding factor of safety. If infinite life is not predicted, estimate the number of cycles to failure.

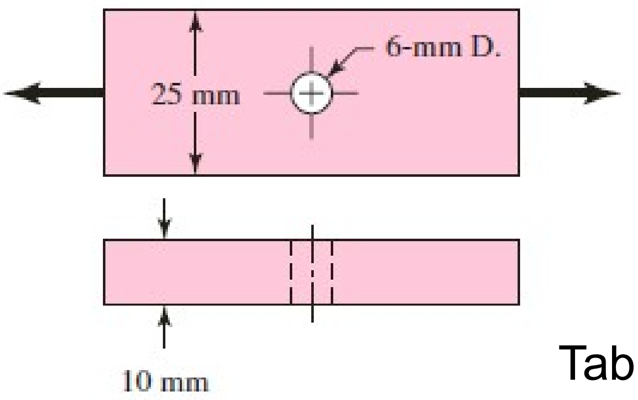


Table A-20

$$S_{ut} = 590 \text{ MPa}, S_y = 490 \text{ MPa}.$$

Surface Finish	Factor *a* S_{ut}, kpsi	Factor *a* S_{ut}, MPa	Exponent *b*
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York.

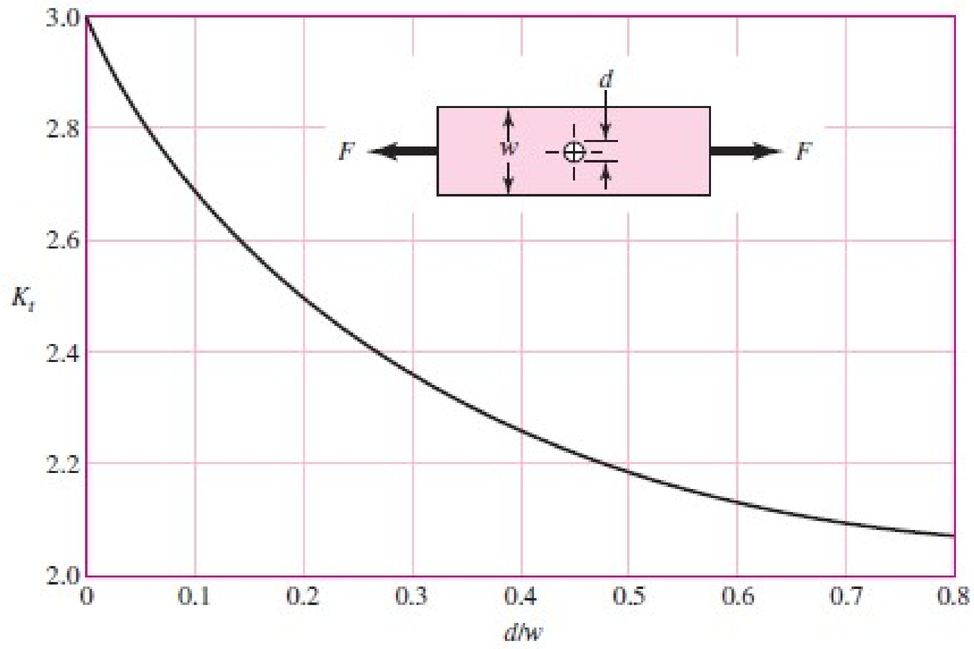
For axial loading there is no size effect,

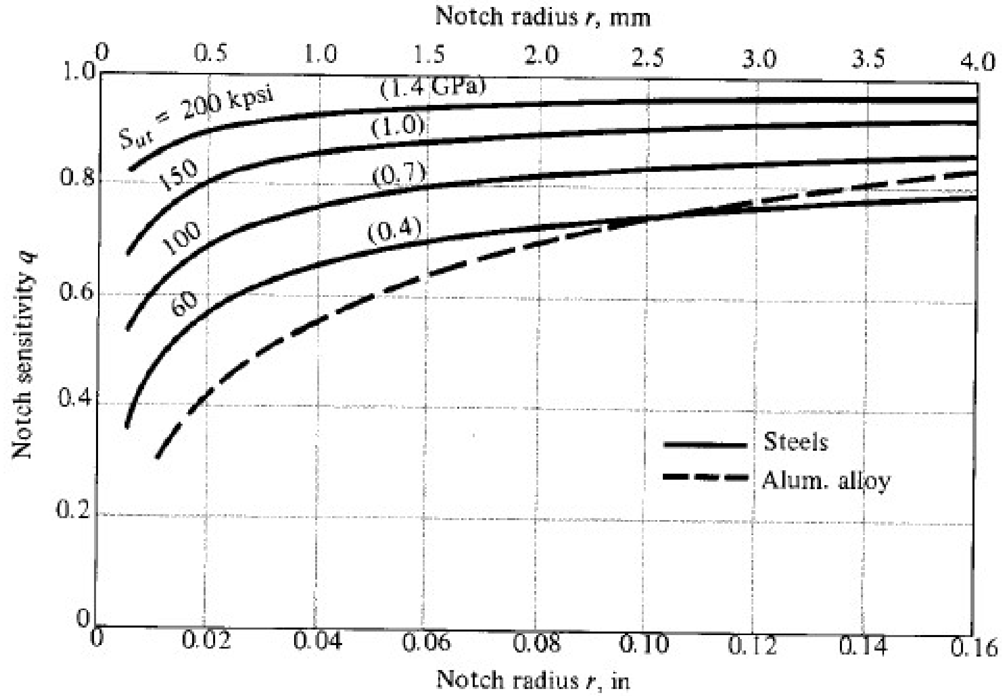
$$k_b = 1$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.





$$S_e' = 0.5(590) = 295 \text{ MPa}$$

$$k_a = aS_{ut}^b = 4.51(590)^{-0.265} = 0.832 \qquad k_b = 1$$

$$k_c = 0.85$$

$$S_e = k_a k_b k_c S_e' = (0.832)(1)(0.85)(295) = 208.6 \text{ MPa}$$

A-15-1:

$$K_f = 1 + q(K_t - 1)$$

$$d / w = 0.24$$

$$q = 0.83 \qquad K_f = 2.20$$

$$K_t = 2.44$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

Check for localized yielding.

$$\sigma = \frac{28\,000}{10(25-6)} = 147.4 \text{ MPa}$$

Which one to use

$$\sigma_{\max} = K_f \sigma = 2.2 \times 147.4 = 324.2 < Sy$$

$$\sigma_{\max} = K_t \sigma = 2.4 \times 147.4 = 353.76 < Sy$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{324.2}{208.6} + \frac{0}{590}$$

n=0.64

$S_{ut} = 590$ Fig. 6-18: $f = 0.87$

$$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{[0.87(590)]^2}{208.6} = 1263$$

$$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{208.6}\right) = -0.1304$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{324.2}{1263}\right)^{\frac{1}{-0.1304}} = 33\,812 \text{ cycles}$$

$$N = 34\,000$$

6–26 Repeat Prob. 6–25 for a load that fluctuates from 12 kN to 28 kN. Use the Modified Goodman, Gerber, and ASME-elliptic criteria and compare their predictions.

$$S_e = 208.6 \text{ MPa}$$

$$K_f = 2.2$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (12\,000)}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Modified Goodman

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$$n_f = 1.20$$

Gerber

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{590}{231.6}\right)^2 \frac{92.63}{208.6}\left[-1+\sqrt{1+\left(\frac{2(231.6)(208.6)}{590(92.63)}\right)^2}\right] = 1.49$$

ASME-Elliptic

$$n_f = \sqrt{\frac{1}{(\sigma_a / S_e)^2 + (\sigma_m / S_y)^2}} = \sqrt{\frac{1}{(92.63 / 208.6)^2 + (231.6 / 490)^2}} = 1.54$$

the Modified Goodman line should predict failure significantly before the other two

6–14 Combinations of Loading Modes

$$\sigma_a' = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2} \tag{6–55}$$

$$\sigma_m' = \left\{ \left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}} \right]^2 \right\}^{1/2} \tag{6–56}$$

Conservative check for localized yielding using von Mises stresses.

$$\sigma_a' + \sigma_m' = S_y/n \tag{6–49}$$

$$\sigma' = \left[\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau^2{}_{xy} + \tau^2{}_{yz} + \tau^2{}_{zx})}{2} \right]^{\frac{1}{2}}$$

Combinations of Loading Modes

- In Fluctuating simple loads
  - Various criteria are used in relating the alternating stress (σ_a) to midrange stress (σ_m)
  - These include
    - Modified-Goodman, Gerber, ASME-elliptic, or Soderberg
  - Again only one type of load is allowed
- In Combinations of loading modes
  - More than one loading mode is allowed (bending, torsion and axial)

Combinations of Loading Modes

- When more than one type of loading (bending, axial, torsion) exists, use the Distortion Energy theory to combine them.

- Obtain von Mises stresses for both midrange and alternating components.

- Apply appropriate *Kf to each type of stress.*

- For load factor, use *kc = 1. The torsional load factor (kc = 0.59) is* inherently included in the von Mises equations.

- If needed, axial load factor can be divided into the axial stress

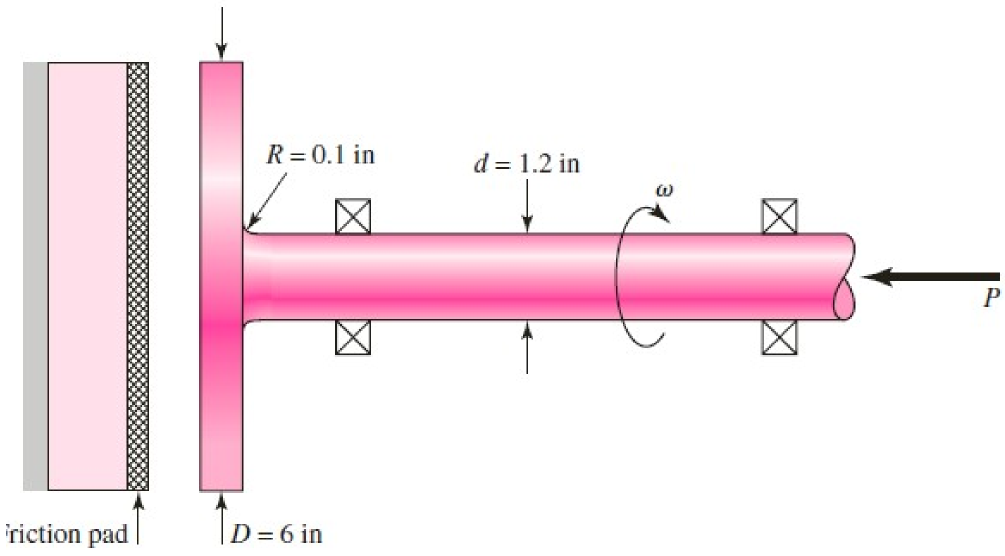
- Step 1
  - Compute alternating midrange stresses for each of the different loadings cases (normal and shear stresses) and apply appropriate stress conc. factor
  - Compute midrange stresses for each of the different loadings cases (normal and shear stresses) stress conc. Factor
- Step2
  - Compute an equivalent von Mises stress for the alternating and midrange stresses.
- Step3
  - Select failure criteria to use (mod-Goodman, Gerber etc)

6–57 A schematic of a clutch-testing machine is shown. The steel shaft rotates at a constant speed ω. An axial load is applied to the shaft and is cycled from zero to P. The torque T induced by the clutch face onto the shaft is given by

$$T = \frac{f P(D + d)}{4}$$

where D and d are defined in the figure and f is the coefficient of friction of the clutch face. The shaft is machined with $S_y = 120$ kpsi and $S_{ut} = 145$ kpsi. The theoretical stress-concentration factors for the fillet are 3.0 and 1.8 for the axial and torsional loading, respectively.

Assume the load variation P is synchronous with shaft rotation. With $f = 0.3$, find the maximum allowable load P such that the shaft will survive a minimum of 10^6 cycles with a factor of safety of 3. Use the modified Goodman criterion. Determine the corresponding factor of safety guarding against yielding.



$$S_{ut} = 145 \text{ kpsi}$$
$$S_y = 120 \text{ kpsi}$$

use *kc = 1*

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
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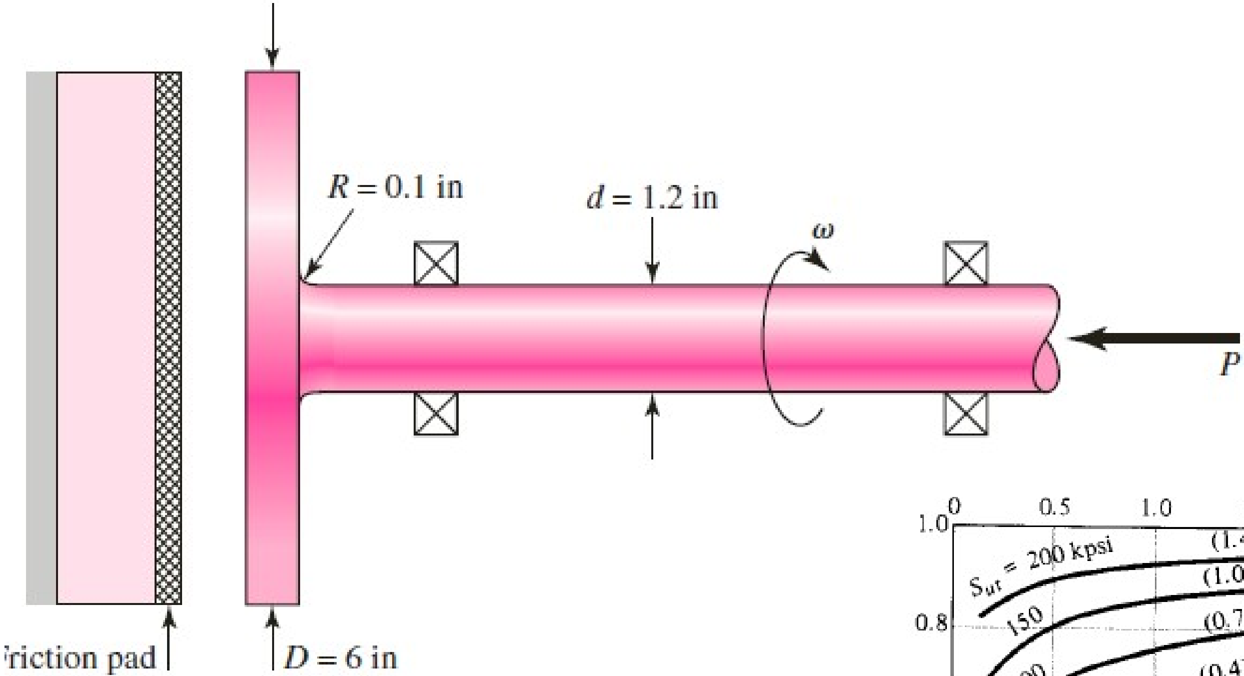
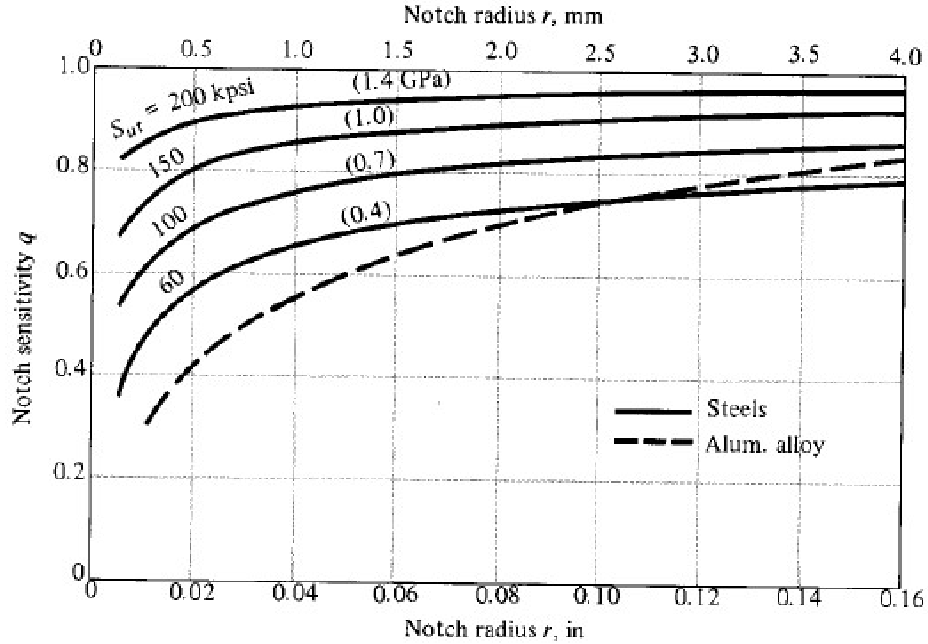
$$S'_e = 0.5(145) = 72.5 \text{ kpsi}$$
$$k_a = 2.70(145)^{-0.265} = 0.722$$
$$k_b = 0.879(1.2)^{-0.107} = 0.862$$
$$S_e = (0.722)(0.862)(72.5) = 45.12 \text{ kpsi}$$



$q = 0.9.$



$K_t = 3$

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(3-1) = 2.80$$

$q_s = 0.92.$ $\quad K_{ts} = 1.8$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8 - 1) = 1.74$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P \qquad \sigma_{\min} = 0$$

$$\sigma_m = \sigma_a = \frac{1}{2}(-2.476P) = -1.238P$$

$$T_{\max} = \frac{f\,P\left(D+d\right)}{4} = \frac{0.3P\left(6+1.2\right)}{4} = 0.54P$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

$$\sigma_a' = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(1.238P / 0.85)^2 + 3(1.385P)^2]^{1/2} = 2.81P$$

$$\sigma_m' = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Modified Goodman: $$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{2.81P}{45.12} + \frac{2.70P}{145} = \frac{1}{3}$$

$$P = 4.12 \text{ kips}$$

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{120}{(2.81)(4.12) + (2.70)(4.12)} = 5.29$$