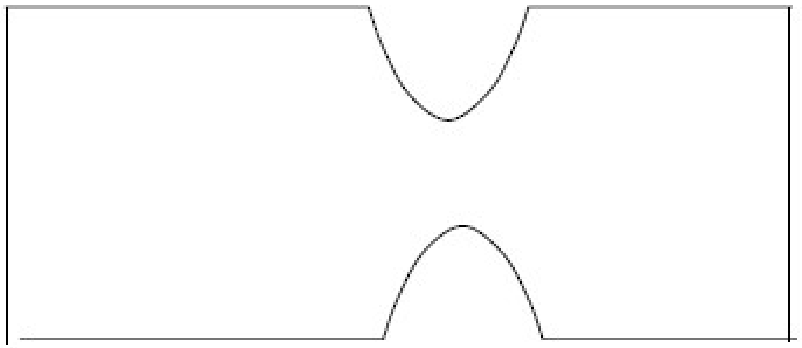
6-10 Stress Concentration and Notch

Fatigue Stress Concentration Factors



$$\sigma = K_f \frac{Mc}{I}$$

$$\sigma = K_t \frac{Mc}{I}$$

$$K_f = \frac{\text{Maximum stress in notched specimen}}{\text{Stress in notch - free specimen}}$$

or

$$K_f = \frac{\text{Endurance limit of a notched specimen.}}{\text{Endurance limit of a notch - free specimen.}}$$

- $\sigma = k_f \sigma_{nom+} = k_f \sigma_o$
- $\tau = k_{fs} \tau_{nom} = k_{fs} \tau_o$
- k_f is a reduced value of k_T and σ_o is the nominal stress.
- k_f called fatigue stress concentration factor

- $k_f = [1 + q(k_t – 1)]$
- $k_{fs} = [1 + q_{shear}(k_{ts} – 1)]$
  - k_t or k_{ts} and nominal stresses
    - Table A-15 & 16 (pages 1006-1013 in Appendix)
  - q and q_{shear}
    - Notch sensitivity factor
    - Find using figures 6-20 and 6-21 in book (Shigley) for steels and aluminums
    - Use q = 0.20 for cast iron
      - Brittle materials have low sensitivity to notches
    - As k_f approaches k_t, q increasing (sensitivity to notches, SC's)
    - If k_f ~ 1, insensitive (q = 0)
      - Property of the material

Notch Sensitivity Factor

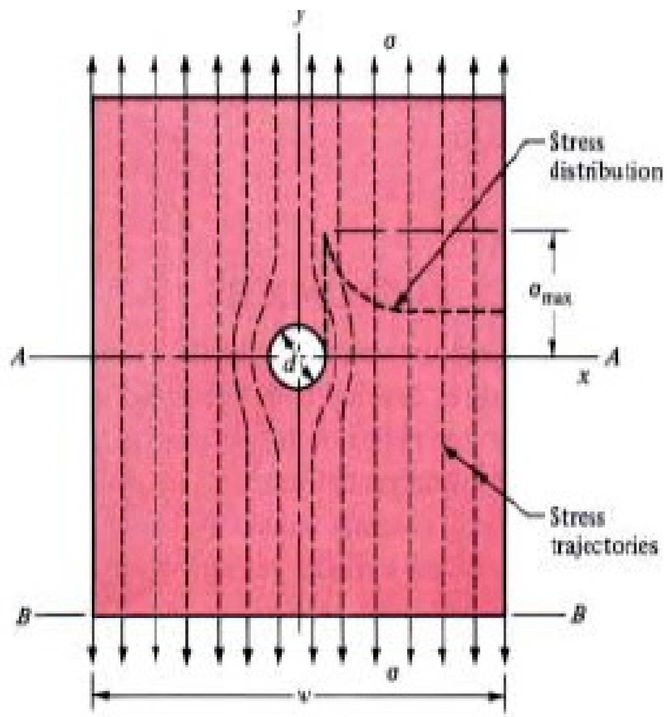
The notch sensitivity of a material is a measure of how sensitive a material is to notches or geometric discontinuities

$$q = \frac{K_f - 1}{K_t - 1} \qquad 0 \le q \le 1$$

$$K_f = 1 + q(K_t - 1) \qquad 1 \le K_f \le K_t$$

- Calculate **Fatigue Stress Concentration Factor K_f using K_t and q**:

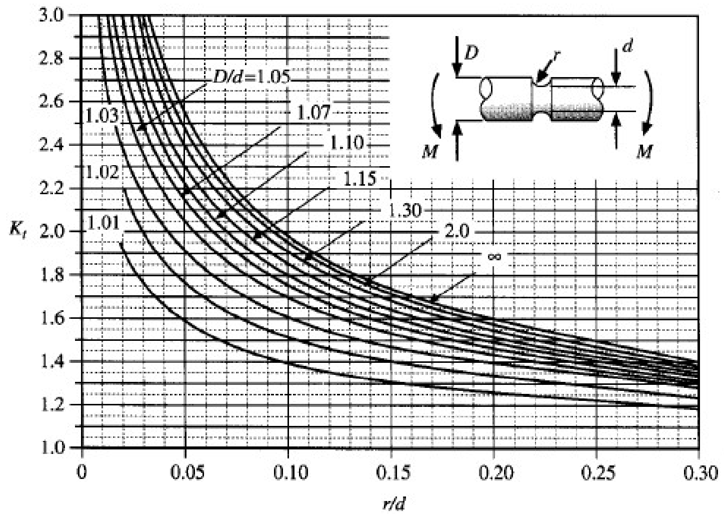
Geometric Stress Concentration Factors



$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

$$\sigma_{nom} = \frac{F}{A_0}$$

$$A_0 = (w - d)t$$



K_t is used to relate the maximum stress at the discontinuity to the nominal stress.

K_t is used for normal stresses

K_t is based on the geometry of the discontinuity

σ_{nom} is usually computed using the minimum cross section

$$K_f = 1 + q(K_t - 1) \quad or \quad K_{fs} = 1 + q_{shear}(K_{ts} - 1)$$

- For Steels and Aluminum (*2024*) the *notch sensitivity* for *Bending and Axial* loading can be found from *Figure 6-20* and for *Torsion* is found from *Figure 6-21*.

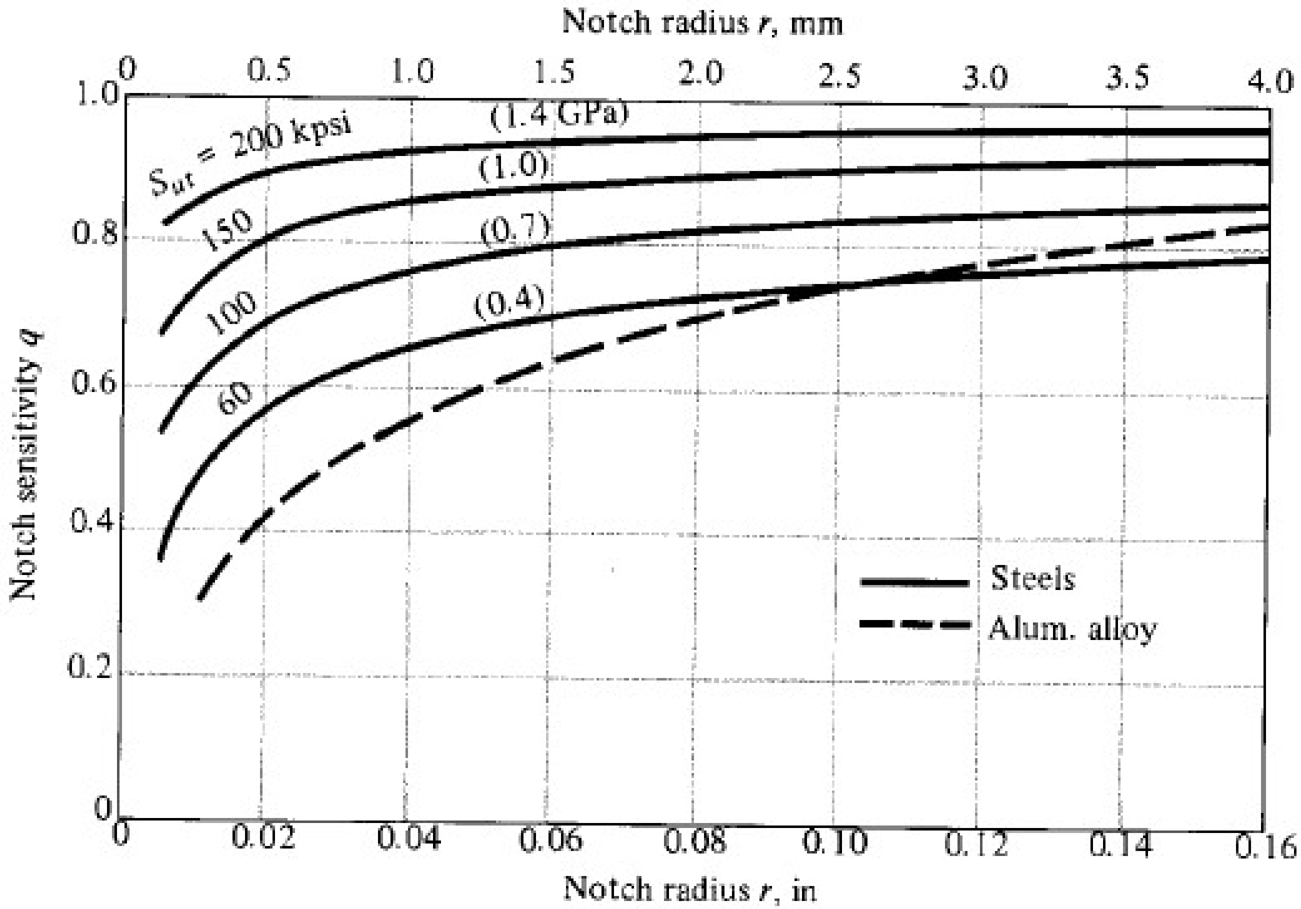


Fig 6-20 Reversed bending or reversed axial loading

- For cast iron, the notch sensitivity is very low from *0* to *0.2*, but to be conservative it is recommended to use $q = 0.2$

- *Heywood* distinguished between different types of notches (*hole, shoulder, groove*) and according to him, K_f is found as:

The modified Neuber equation

$$K_f = \frac{K_t}{1 + \frac{2(K_t - 1)}{K_t} \frac{\sqrt{a}}{\sqrt{r}}}$$

Where, r: radius

$\sqrt{a}$: is a constant that depends on the type of the notch.

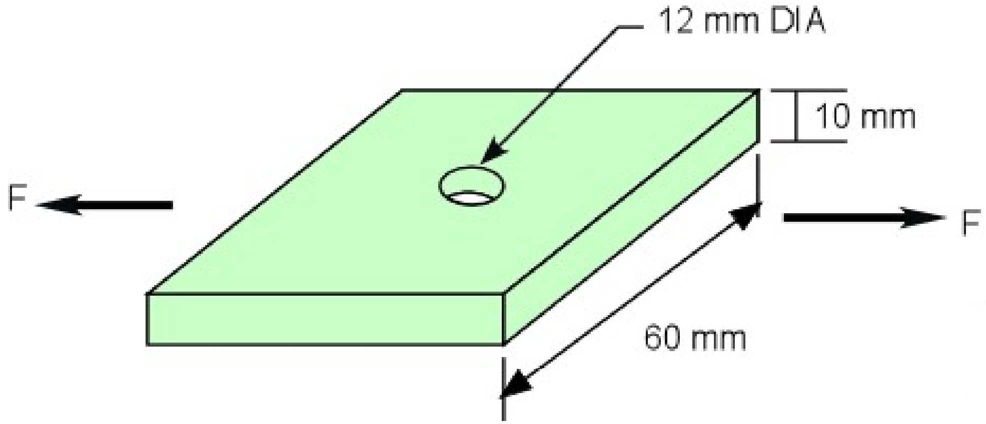
❖ For steels, $\sqrt{a}$ for different types of notches is given in *Table 6-15*.

Bending or axial: $$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{6–35a}$$

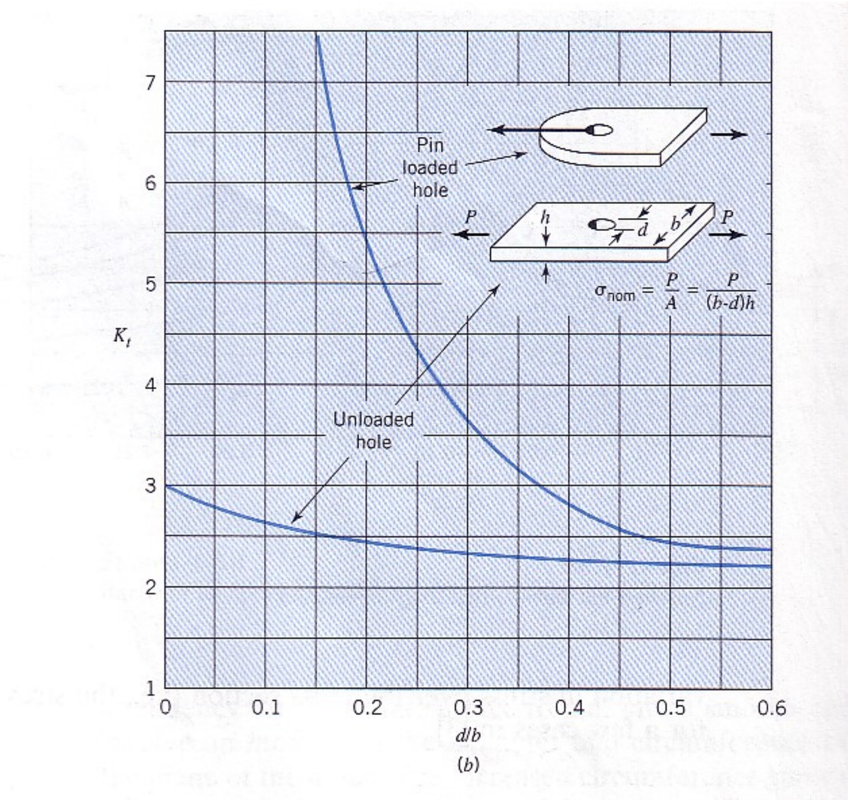
Torsion: $$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{6–35b}$$

Ex:AISI 1020 as-rolled steel

Ex: Find an expression for max. stress

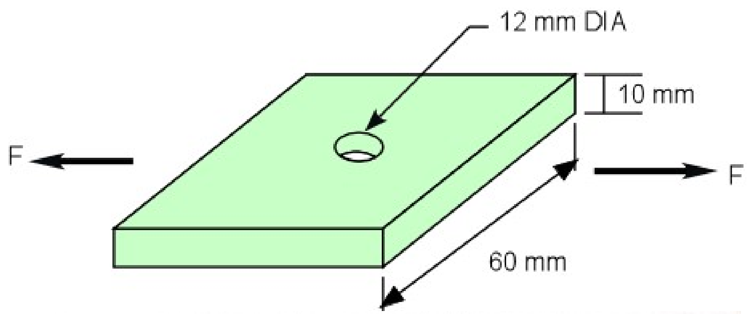


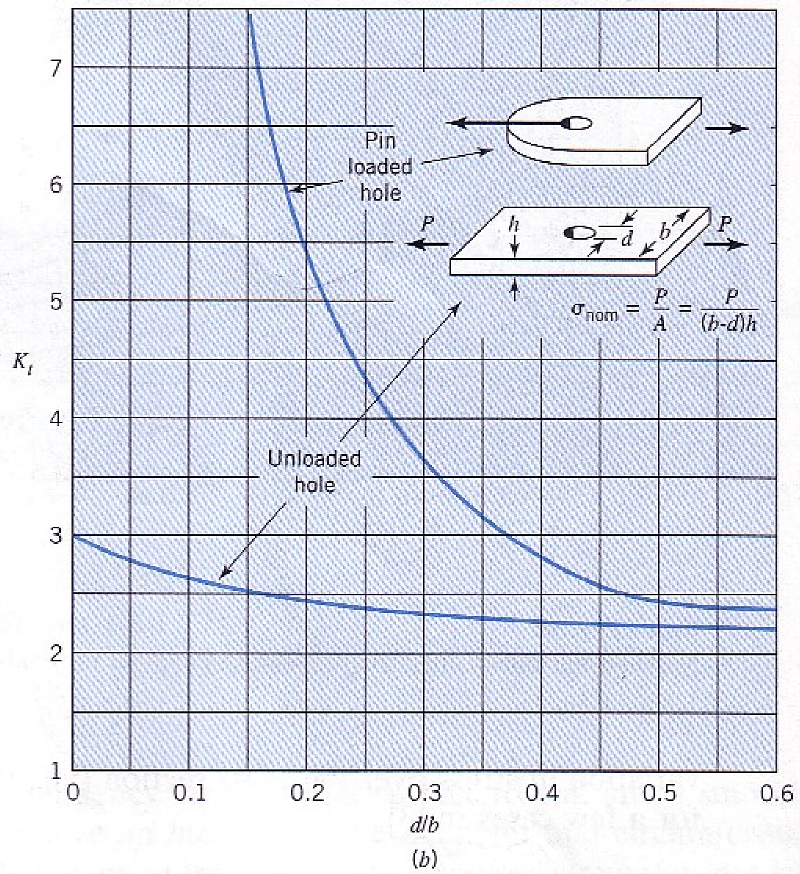
S_{ut} = 448 MPa = 65.0 ksi



Find σ'$_{nom}$ (without SC) •

$$\sigma'_{nom} = \frac{P}{A} = \frac{P}{(b-d)h} = \frac{F}{(60-12)10} = 2083F$$

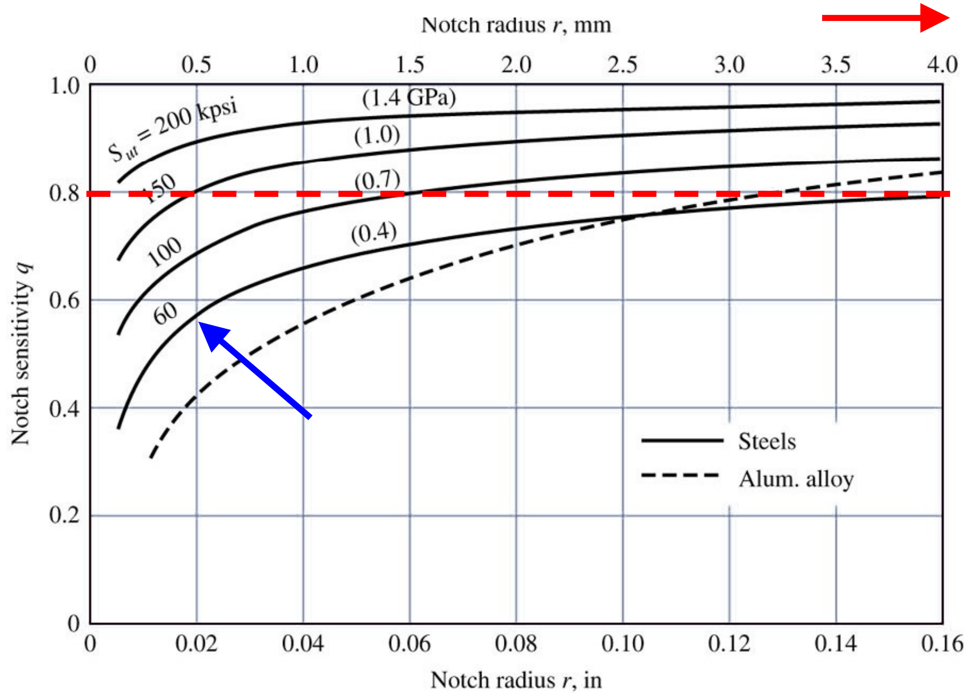




- Now add SC factor:

$$\sigma = k_f \sigma_{nom} = [1 + q(k_t - 1)]\sigma_{nom}$$

  - r = 6 mm
  - q ~ 0.8



- – Unloaded hole
  - – d/b = 12/60 = 0.2
  - – k_t ~ 2.5
- q = 0.8
- k_t = 2.5
- σ_{nom} = 2083 F

$$\sigma = [1 + q(k_t - 1)]\sigma_{nom}$$
$$\sigma = [1 + 0.8(2.5 - 1)]2083(F)$$
$$\sigma = 4583(F)$$

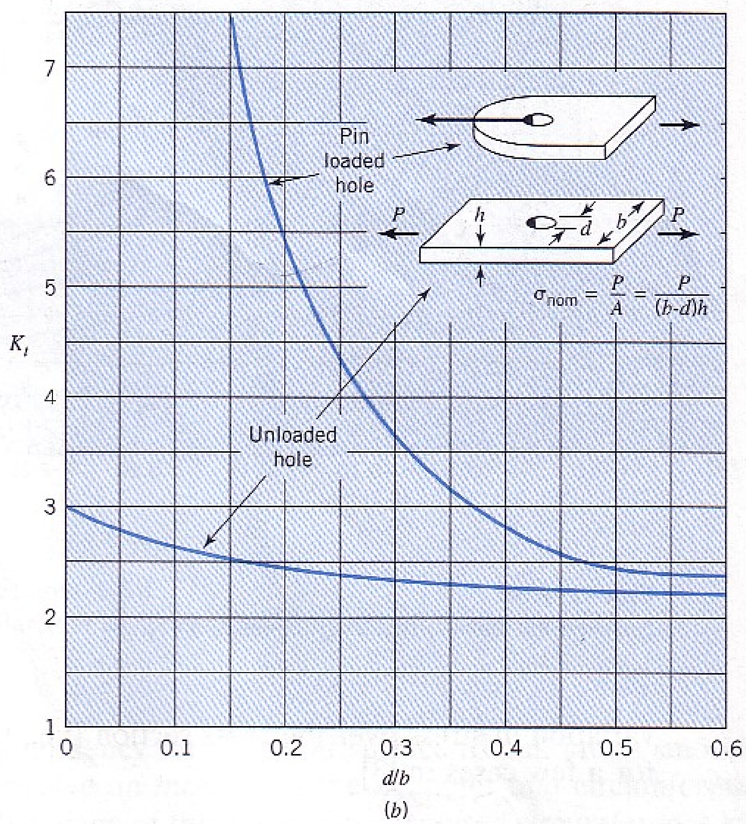


FIGURE 4.40
Plate with central hole (*a*) bending [7]; (*b*) axial hole [10].

6–10 Estimate the endurance strength of a 1.5-in-diameter rod of AISI 1040 steel having a machined finish and heat-treated to a tensile strength of 110 kpsi.

$S_{ut} = 110$ kpsi $\qquad S_e' = 0.5(110) = 55$ kpsi

$k_a = aS_{ut}^{\;b}$: $\qquad a = 2.70,\ b = -0.265$

$= 2.70(110)^{-0.265} = 0.777$

assume the worst case rotating bending or torsion

$$k_b = 0.879d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$$

$$S_e = k_a k_b S_e' = 0.777(0.842)(55) = 36.0 \text{ kpsi}$$

6–14 A rectangular bar is cut from an AISI 1020 cold-drawn steel flat. The bar is 2.5 in wide by $\frac{3}{8}$ in thick and has a 0.5-in-dia. hole drilled through the center as depicted in Table A–15–1. The bar is concentrically loaded in push-pull fatigue by axial forces F_a, uniformly distributed across the width. Using a design factor of $n_d = 2$, estimate the largest force F_a that can be applied ignoring column action.

$$S_{ut} = 68 \text{ kpsi}$$

$$k_c = 0.85$$

$$S'_e = 0.5(68) = 34$$

$$k_b = 1$$

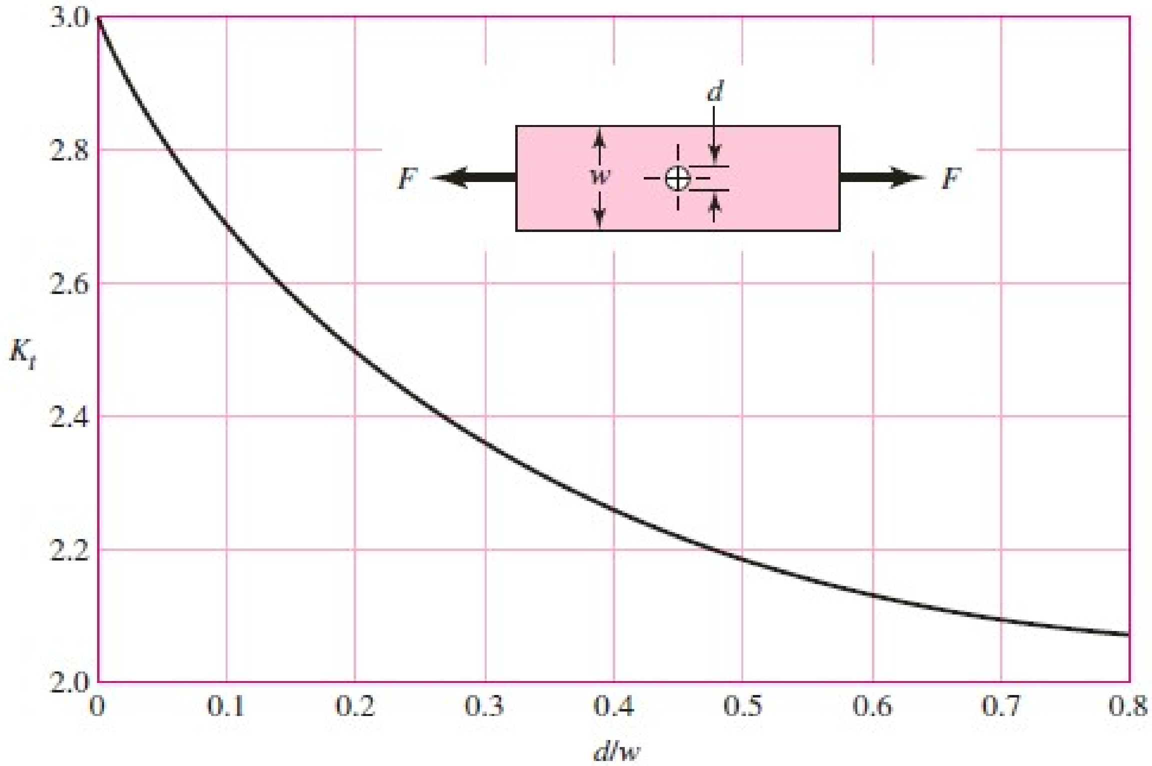
$$k_a = 2.70(68)^{-0.265} = 0.88$$

$$S_e = 0.88(1)(0.85)(34) = 25.4$$

$$d / w = 0.5 / 2.5 = 0.2, \; K_t = 2.5$$

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



$$d / w = 0.5 / 2.5 = 0.2$$

$$K_t = 2.5$$

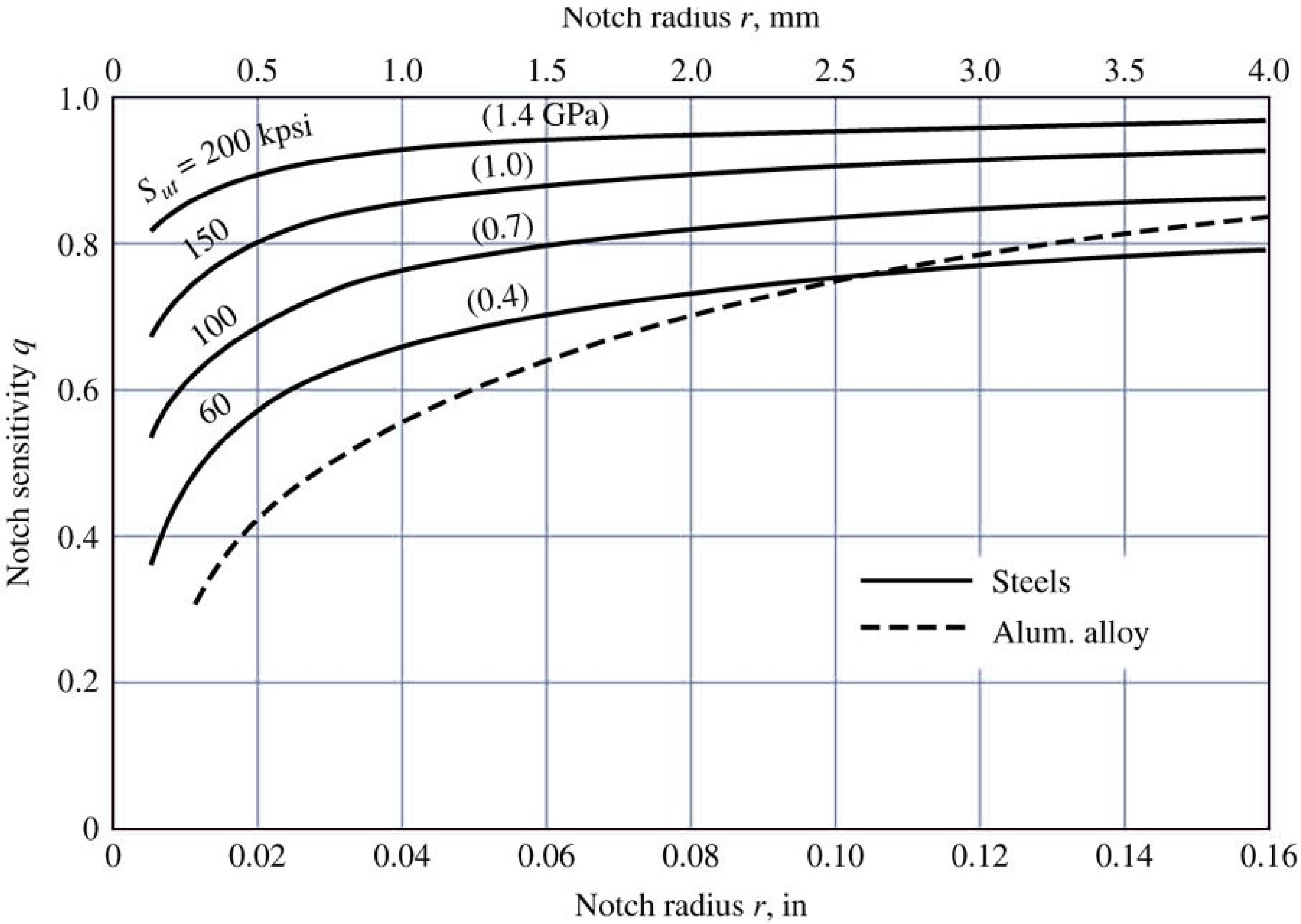
$$K_f = 1 + q(K_t - 1) \qquad q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35b)

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(68) + 1.51\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right)$$
$$= 0.09799$$

$$q = \frac{1}{1 + \frac{0.09799}{\sqrt{0.25}}} = 0.836$$



Notch radius r, mm
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1.0
0.8
0.6
0.4
0.2
0
Notch sensitivity q
S_ut = 200 kpsi
(1.4 GPa)
150
(1.0)
100
(0.7)
60
(0.4)
Steels
Alum. alloy
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
Notch radius r, in

$$K_f = 1 + 0.836(2.5 - 1) = 2.25$$

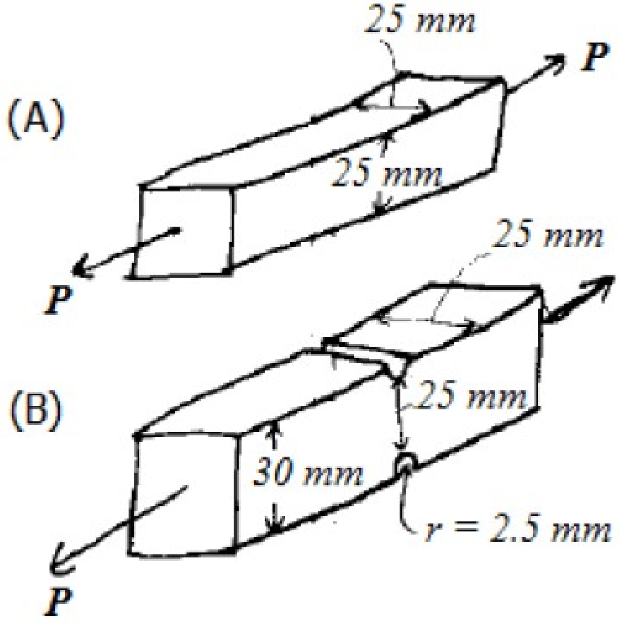
$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5 - 0.5)} = 3F_a$$

$$n_f = \frac{S_e}{\sigma_a} = \frac{25.4}{3F_a} = 2$$

$$F_a = 4.23 \text{ kips}$$

Example: The two axially loaded bars shown are made of *1050 HR* steel and have machined surfaces. The two bars are subjected to a completely reversed load P.

a) Estimate the maximum value of the load P for each of the two bars such that they will have infinite life (*ignore buckling*).
b) Find the static and fatigue factors of safety n_s & n_f for bar (***B***) if it is to be subjected to a completely reversed load of $P = 50\ kN$.
c) Estimate the fatigue life of bar (***B***) under reversed load of $P = 150\ kN$ (*use* $f = 0.9$)



From *Table* $S_{ut} = 620\ MPa$ & $S_y = 340\ MPa$

a) $S_e' = 0.5(S_{ut}) = 310\ MPa$

Modifying factors:

- *Surface factor*: $k_a = a\, S_{ut}{}^{b}$, from Table 6-2: $a = 4.51, b = -0.265$

  ➔ $k_a = 4.51(620)^{-0.265} = 0.821$
- *Size factor*: $k_b = 1$ *since the loading is axial*
- *Loading factor*: $k_c = 0.85$ (*for axial loading*)
- *Other factors*: $k_d = k_e = k_f = 1$

Stress concentration (*for bar* ***B***):

From *Figure* A-15-3 with $w/d = 1.2$ & $r/d = 0.1$ ➔ $K_t \approx 2.13$

Using *Neuber* equation: $K_f = \frac{K_t - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} + 1$

From Table 6-15 for a groove: $\sqrt{a} = 104/S_{ut}$ ➔ $\sqrt{a} = 104/620$

Using the modified *Neuber* equation: $$K_f = \frac{K_t}{1+\frac{2(K_t-1)}{K_t}\frac{\sqrt{a}}{\sqrt{r}}}$$

From *Table 6-15* for a groove: $\sqrt{a} = 104/S_{ut}$ ➔ $\sqrt{a} = 104/620$

$$➔ \quad K_f = \frac{2.13}{1+\frac{2(2.13-1)}{2.13}\frac{(104/620)}{\sqrt{2.5}}} = 1.91$$

$$➔ \quad K_f = \quad 2$$

Thus,

For bar (**A**): $S_e = k_a k_c S'_e = (0.821)(0.85)(310) = \boxed{216.3\ MPa}$

For bar (**B**): $(S_e)_{mod} = \frac{S_e}{K_f} = \frac{216.3}{2} = \boxed{108.15 MPa}$

For bar (**A**): $S_e = k_a k_c S'_e = (0.821)(0.85)(310) = \boxed{216.3\ MPa}$

For bar (**B**): $(S_e)_{mod} = \frac{S_e}{K_f} = \frac{216.3}{1.91} = \boxed{113.3\ MPa}$

b) Static factor of safety n_s:

$$\sigma_o = \frac{P}{A_{net}} = \frac{50 \times 10^3}{25 \times 25} = 80\ MPa$$

$$\Rightarrow \quad n_s = \frac{S_y}{\sigma_o} = \frac{340}{80} = \boxed{4.25}$$

Fatigue factor of safety n_f:

$$n_f = \frac{(S_e)_{mod}}{\sigma_o} \qquad or \qquad n_f = \frac{S_e}{(K_f \sigma_o)} = \frac{216.3}{(\ \ 2\ \ \ )(80)} = \boxed{1.32}$$

$$n_f = \frac{S_e}{(K_f \sigma_o)} = \frac{216.3}{(1.91)(80)} = \boxed{1.42}$$

c) If we calculate the fatigue factor of safety with $P = 150\ kN$ we will find it to be less than one and thus the bar will not have infinite life.

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.9 \times 620)^2}{216.3} = 1439.5\ MPa$$

$$b = -\frac{1}{3} log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3} log\left(\frac{0.9 \times 620}{216.3}\right) = -0.137$$

$$\sigma_o = \frac{P}{A_{net}} = \frac{150 \times 10^3}{25 \times 25} = 240\ MPa$$

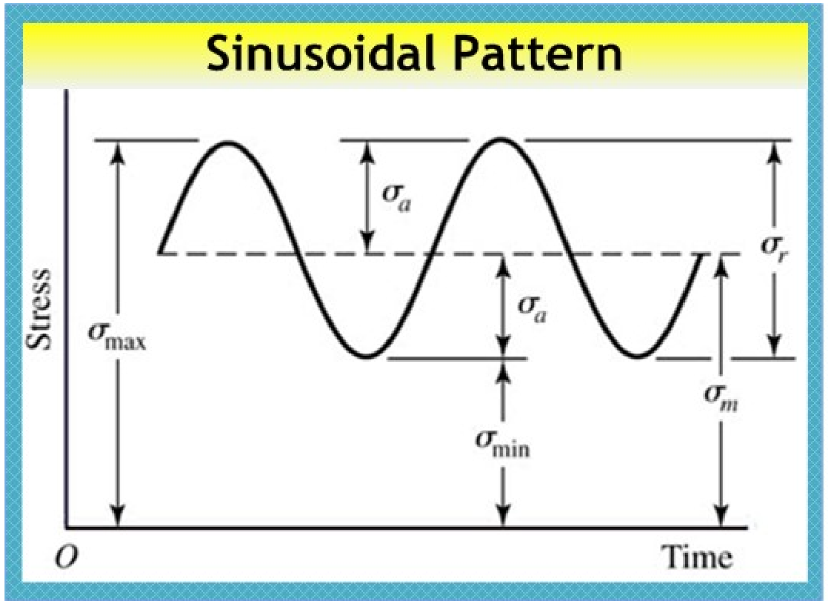
$$\sigma = K_f \sigma_o = 1.91 \times 240 = 458.4\ MPa$$

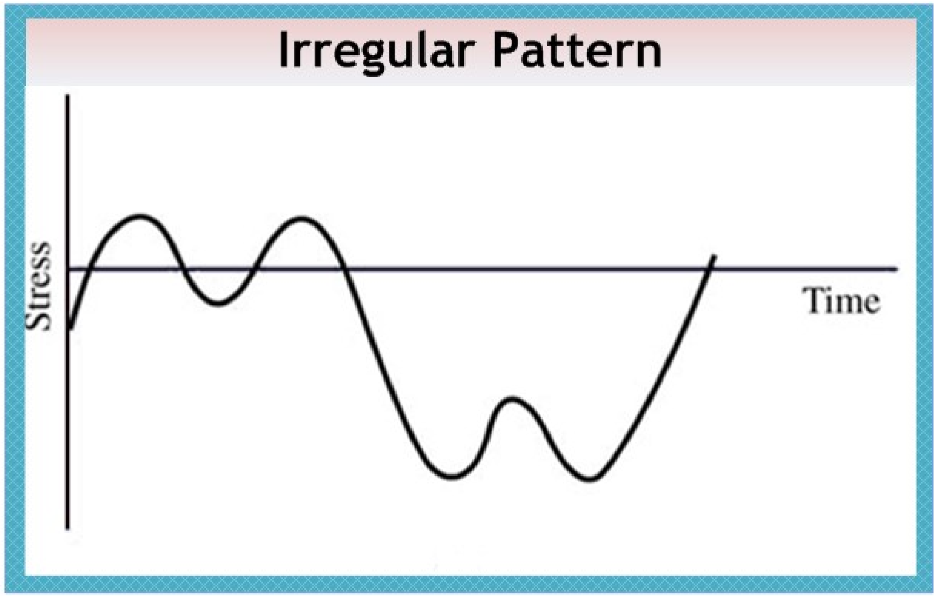
➔ $$N = \left(\frac{\sigma}{a}\right)^{1/b} = \left(\frac{458.4}{1439.5}\right)^{1/-0.137} = \boxed{4.24 \times 10^3\ cycles}$$

- The same result can be obtained if we divide both (S_e) and (fS_{ut}) by K_f,

6–11 Characterizing Fluctuating Stresses

- Fluctuating stresses in machinery often take the form of sinusoidal pattern because of the nature of some rotating machinery.
- Other patterns some quite irregular do occur.





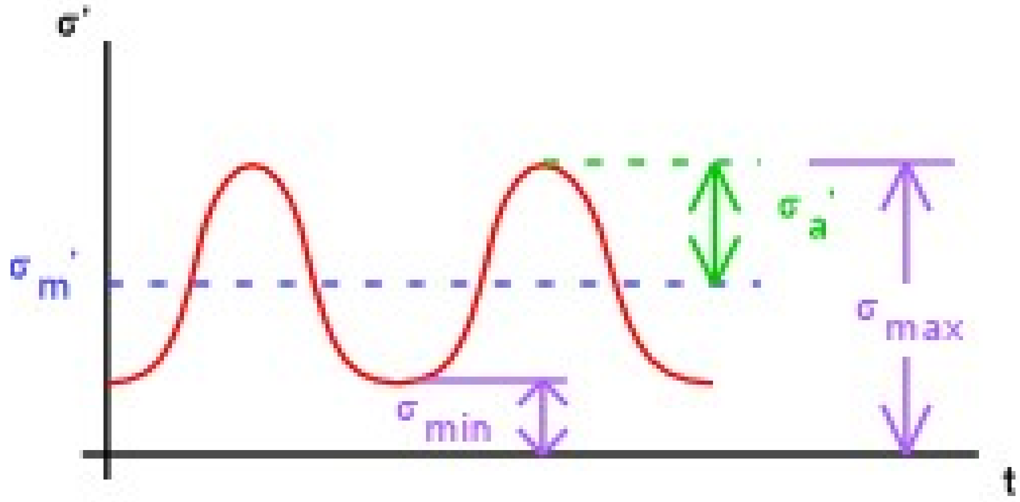
Fluctuating stresses

- Mean Stress

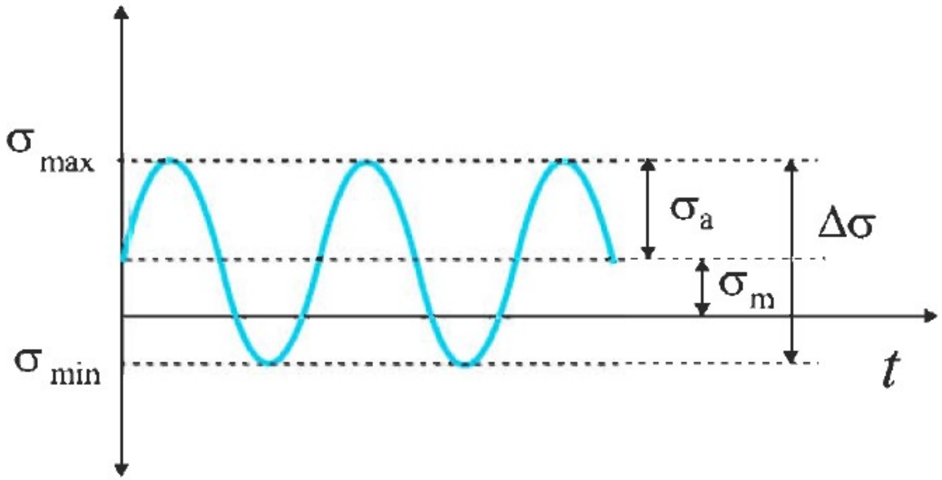
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

Stress amplitude

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$



- Together, σ_m and σ_a characterize fluctuating stress



the important parameters to characterize a given cyclic loading l

- **Stress Range:** $$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

- **Stress amplitude:** $$\sigma_a = \frac{1}{2}\left(\sigma_{\max} - \sigma_{\min}\right)$$

- **Mean stress:** $$\sigma_m = \frac{1}{2}\left(\sigma_{\max} + \sigma_{\min}\right)$$

- **Load ratio:** $$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

Stress Range

$$\Delta\sigma = \sigma_r = \sigma_{max} - \sigma_{min}$$

Mean (Midrange Stress)

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress Amplitude (Alternating Stress)

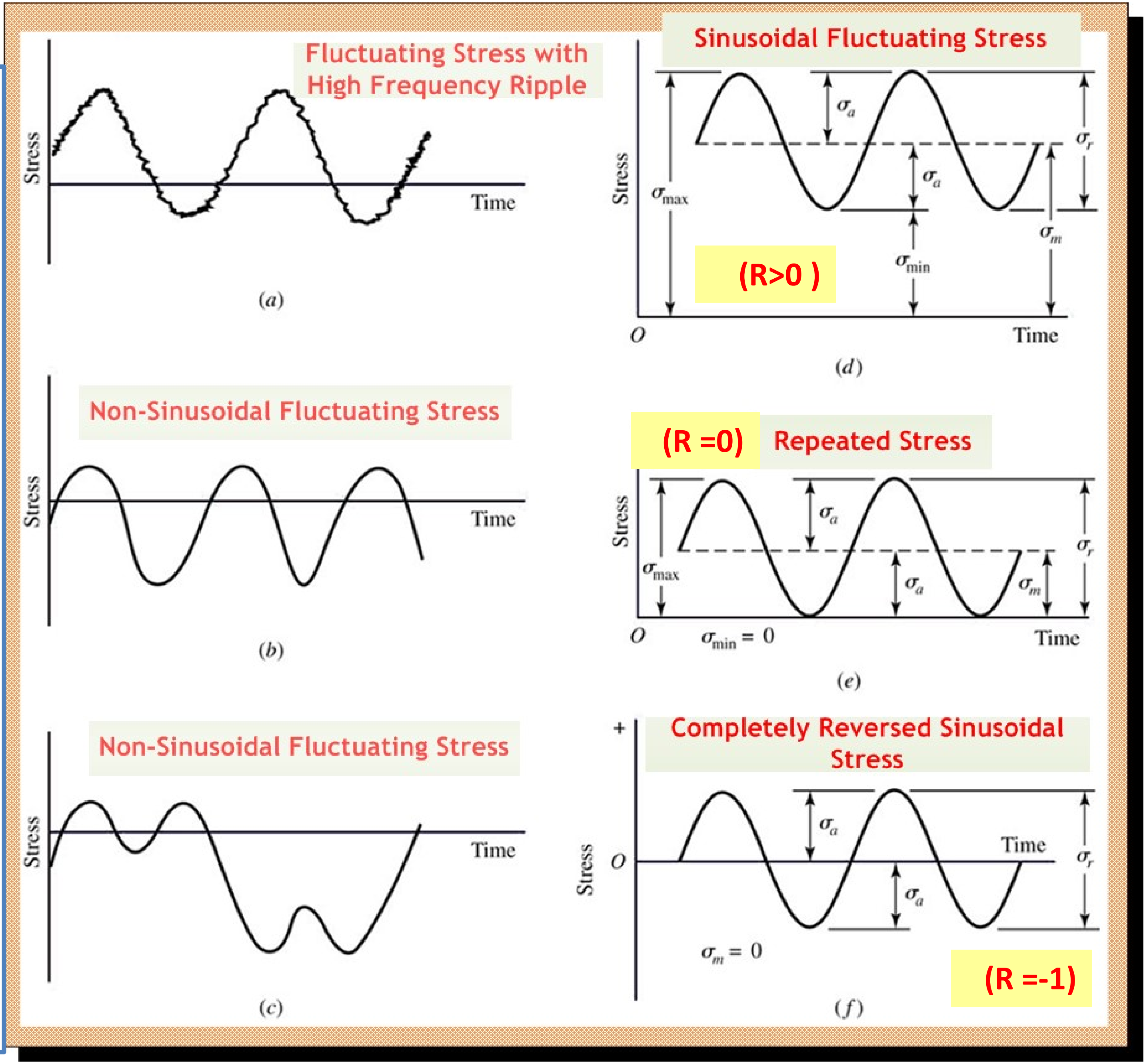
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

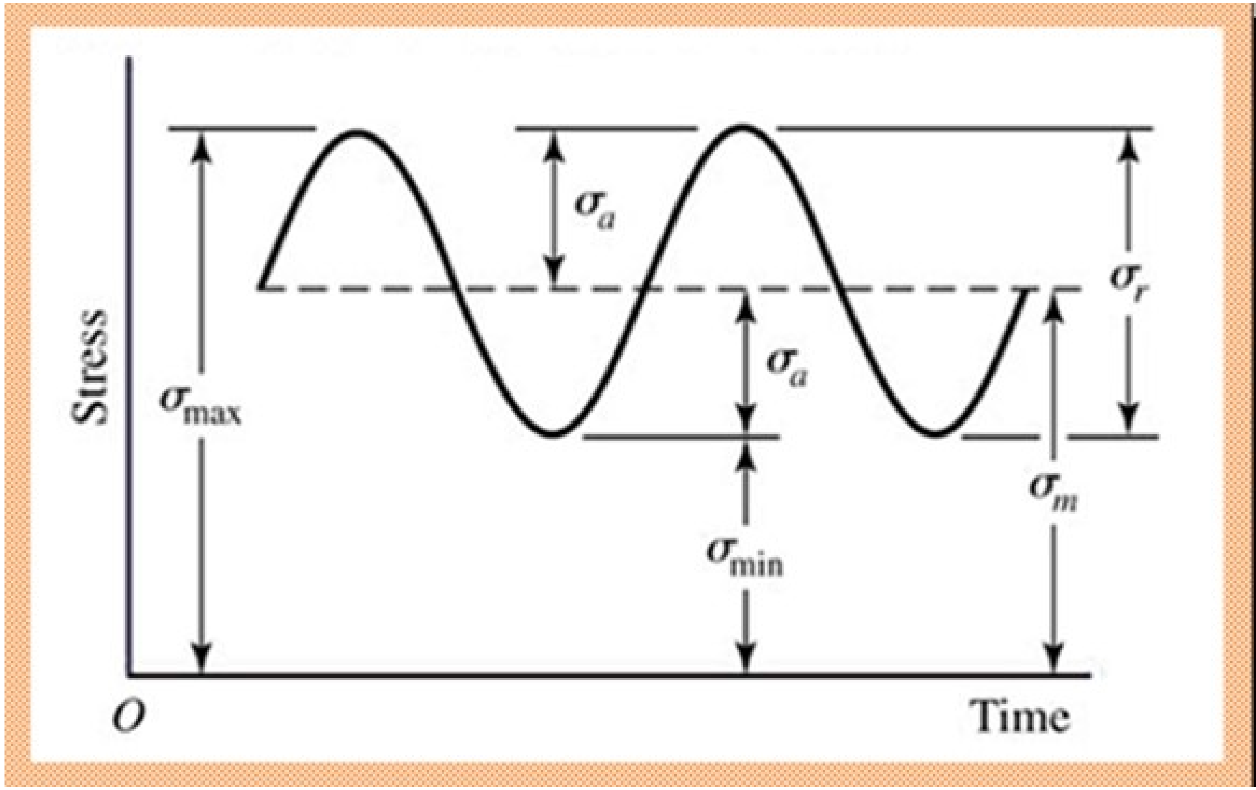
Stress Ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Stress Amplitude

$$A = \frac{\sigma_a}{\sigma_m}$$





$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

Stress Ratio $R = \frac{\sigma_{min}}{\sigma_{max}}$

Stress Ratio $A = \frac{\sigma_a}{\sigma_m}$

σ_{max} :maximum stress

σ_{min} :minimum stress

σ_a :amplitude (alternating) component

σ_m :midrange (mean) component

σ_r :range of stress

σ_s :static or steady stress

Effect of σ_m on 'S - N' curves

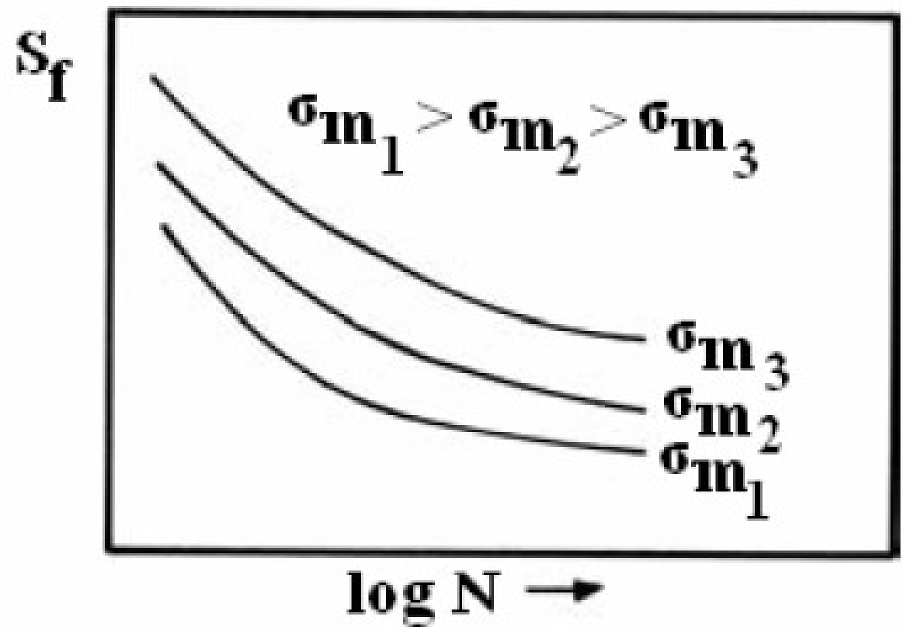
if σ_m not zero

- N_f decreases as σ_m increases

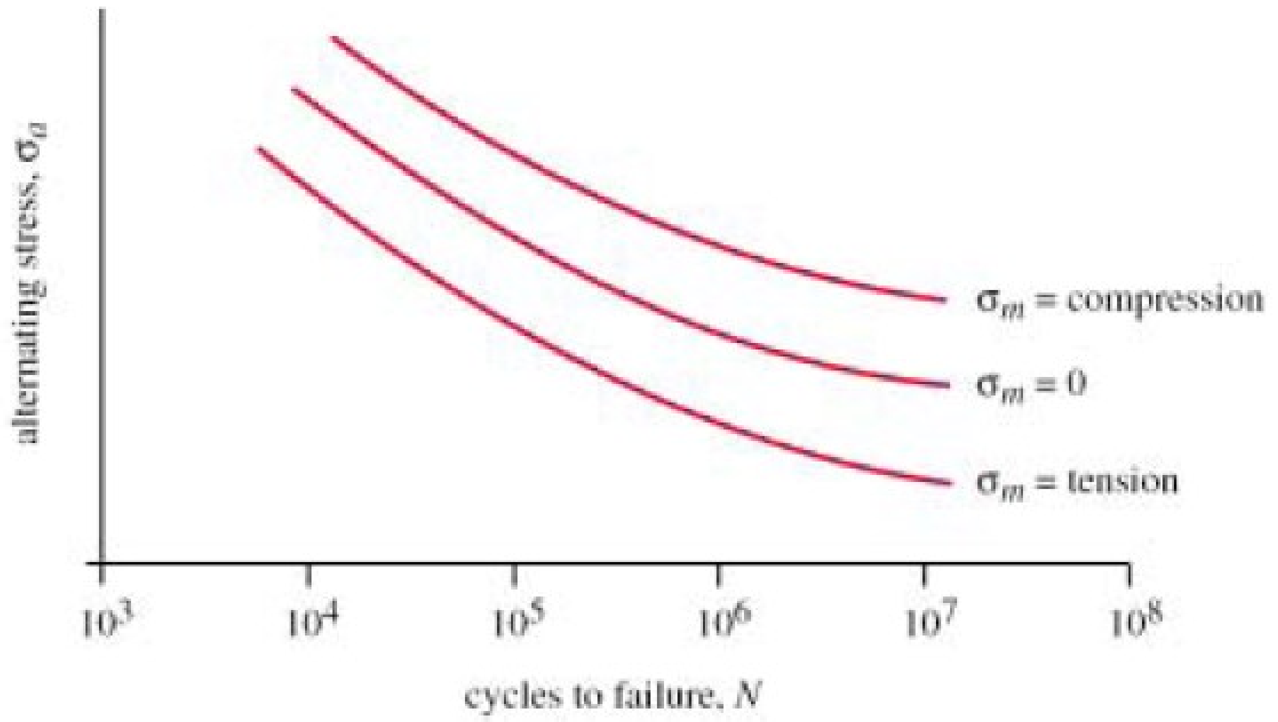
to maintain given N_f

- must decrease stress range, $\Delta\sigma$

- For a given stress amplitude σ_a, *as the* mean stress increases, the fatigue life decreases



S-N diagrams for different σ_m



There are several basic methods to obtain one diagram from these curves,
which enable to define intensity of ultimate stress amplitude for given midrange stress