Ch6 Fatigue

- Fatigue strength and endurance limit
- Estimating Fatique strength S_f and Endurance limit S_e
- Modifying factors

- The first major class of machine elements failure is due to static failure
- The second major class of component failure is due to dynamic loading
  - Variable stresses
  - Repeated stresses
  - Alternating stresses
  - Fluctuating stresses
- The ultimate strength of a material (S_u) is the maximum stress a material can sustain before failure assuming the load is applied only once and held
- A material can also fail by being loaded repeatedly to a stress level that is LESS than S_u
  - Fatigue failure

Fatigue Examples

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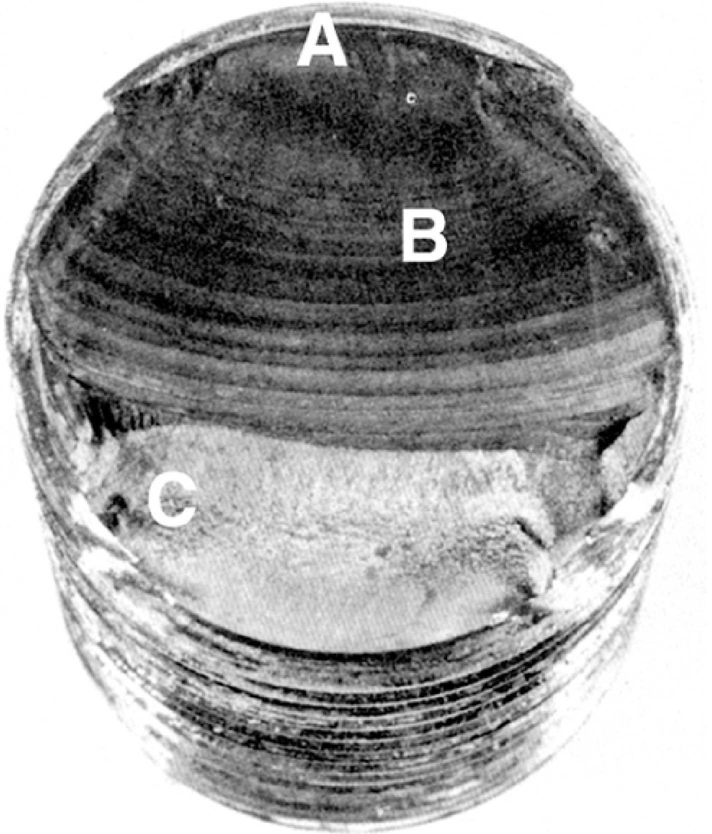

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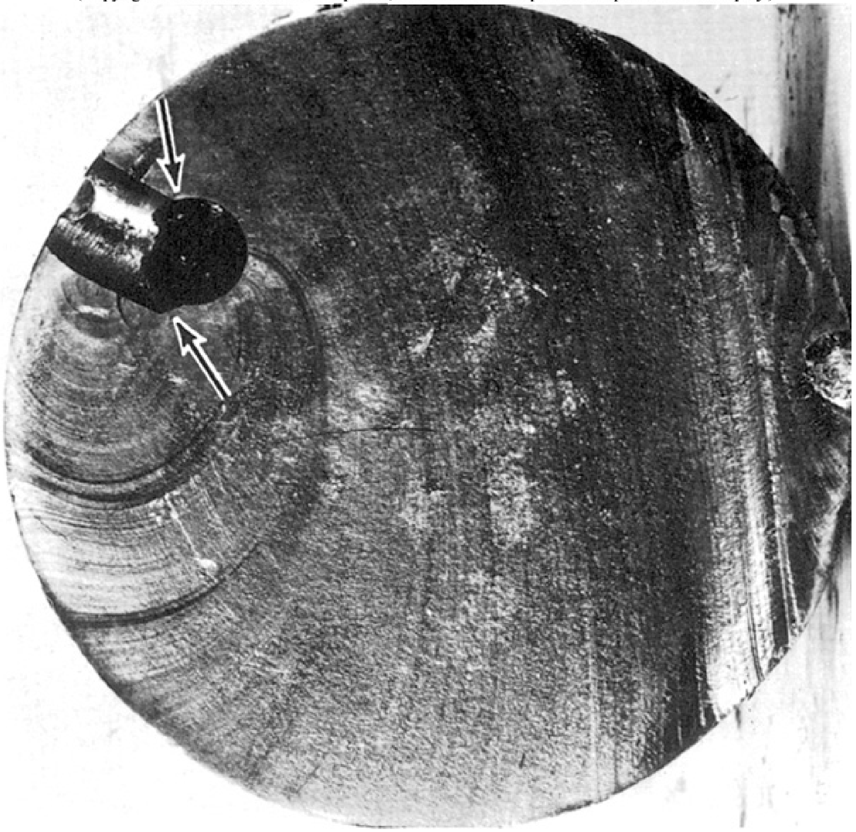

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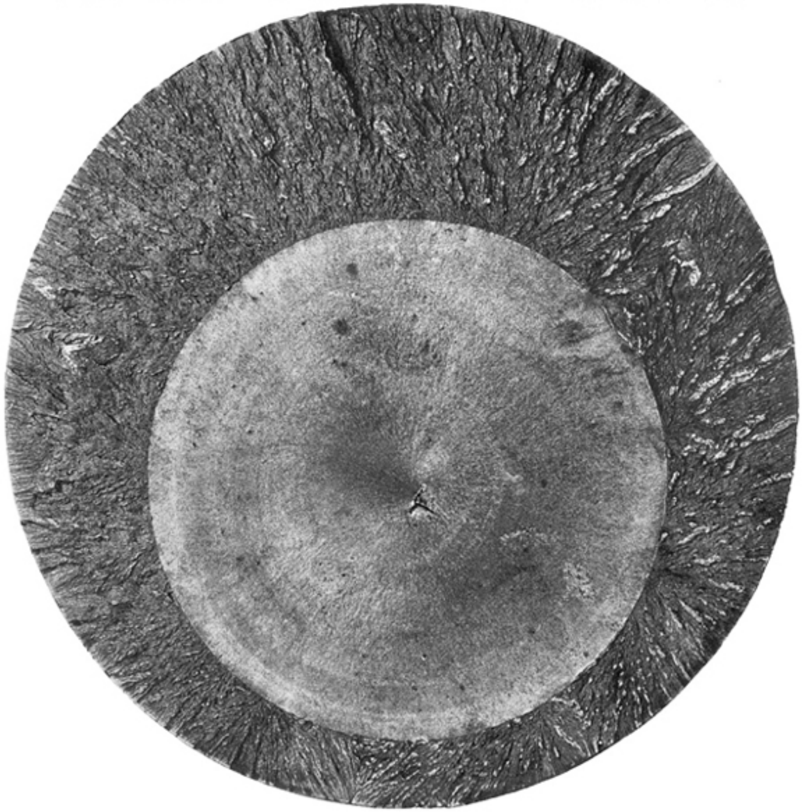
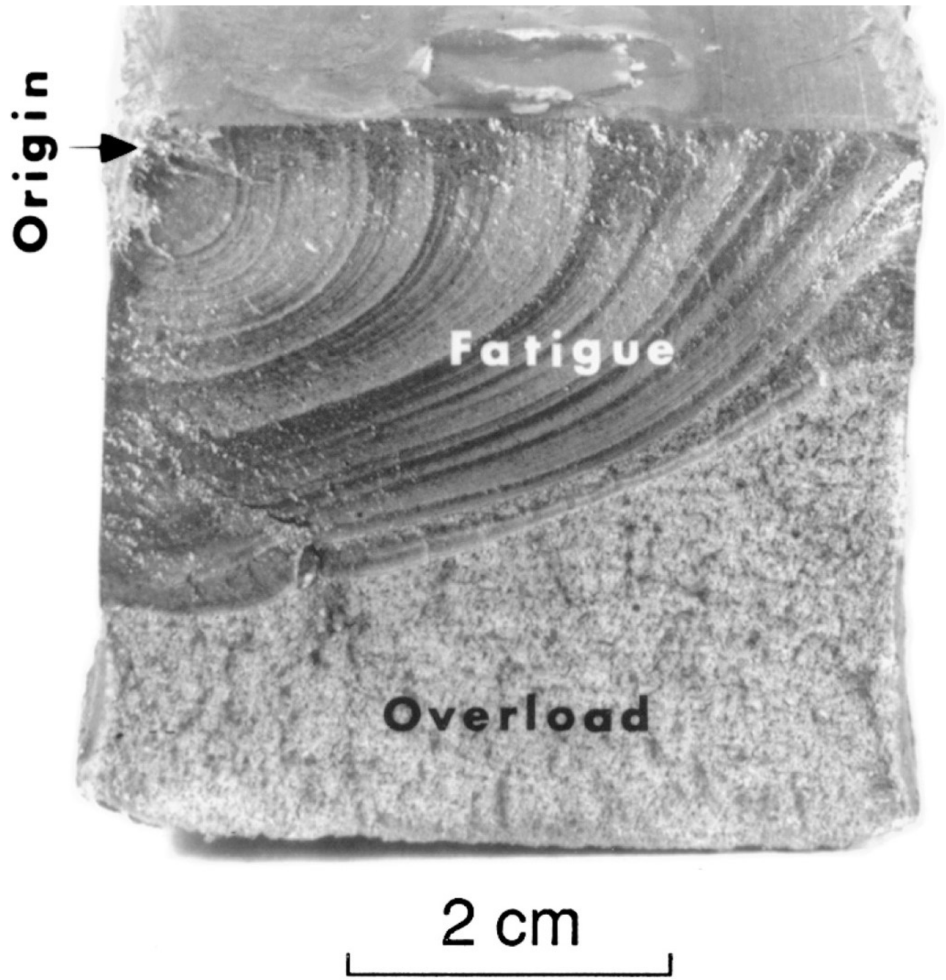


Fatigue Failure Feature

- Flat facture surface, normal to stress axis, no necking
- Stage one: initiation of microcracks
- Stage two: progress from microcracks to macrocracks, forming parallel plateau-like facture feature (beach marks) separated by longitudinal ridge
- Stage three: final cycle, sudden, fast fracture.



Bolt, unidirectional bending

Approach to fatigue failure in analysis and design

- Fatigue-life methods
  - Stress Life Method (Used in this course)
  - Strain Life Method
  - Linear Elastic Fracture Mechanics Method
- Stress-life method
  - Addresses high cycle Fatigue ($>10^3$) Well
  - Not Accurate for Low Cycle Fatigue ($<10^3$)

The 3 major methods

- Stress-life
  - Based on stress levels only
  - Least accurate for low-cycle fatigue
  - Most traditional
    - Easiest to implement
    - Ample supporting data
    - Represents high-cycle applications adequately
- Strain-life
  - More detailed analysis of plastic deformation at localized regions
  - Good for low-cycle fatigue applications
  - Some uncertainties exist in the results
- Linear-elastic fracture mechanics
  - Assumes crack is already present and detected
  - Predicts crack growth with respect to stress intensity
  - Practical when applied to large structures in conjunction with computer codes and periodic inspection

Stress-Life Method

- Specimen are subjected to repeated forces of specified magnitudes while the cycles are counted until fatigue failure

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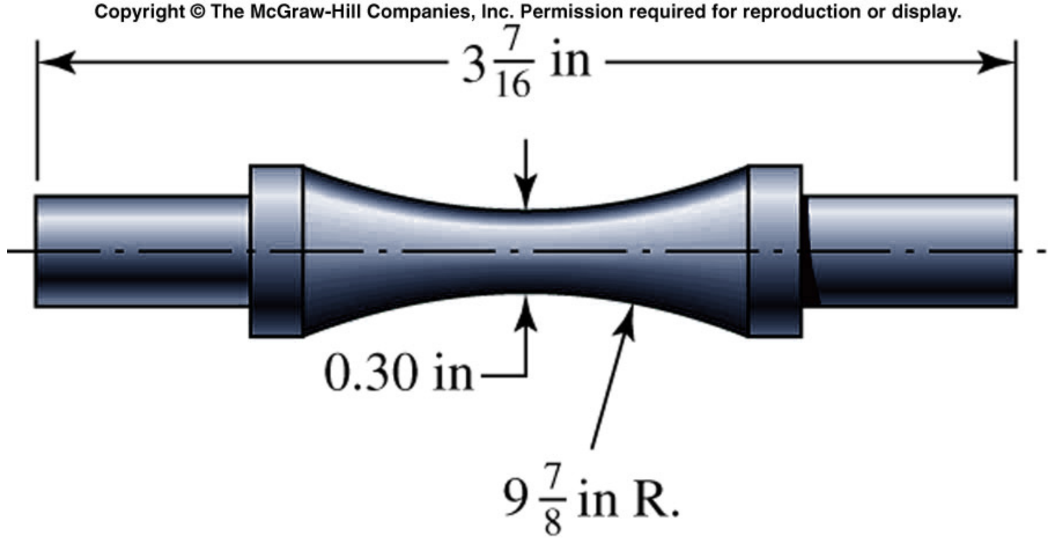
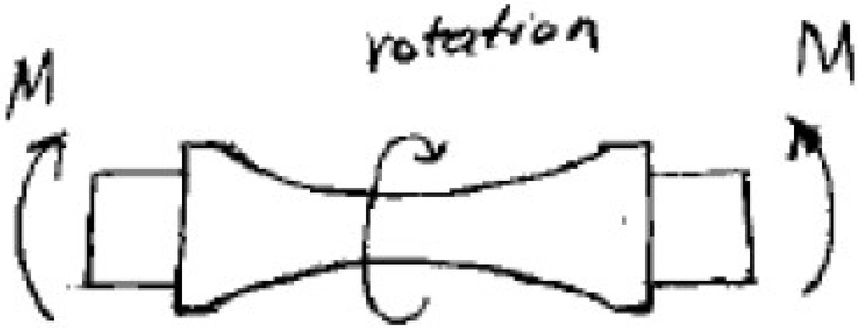


6.4 The Stress-Life Method

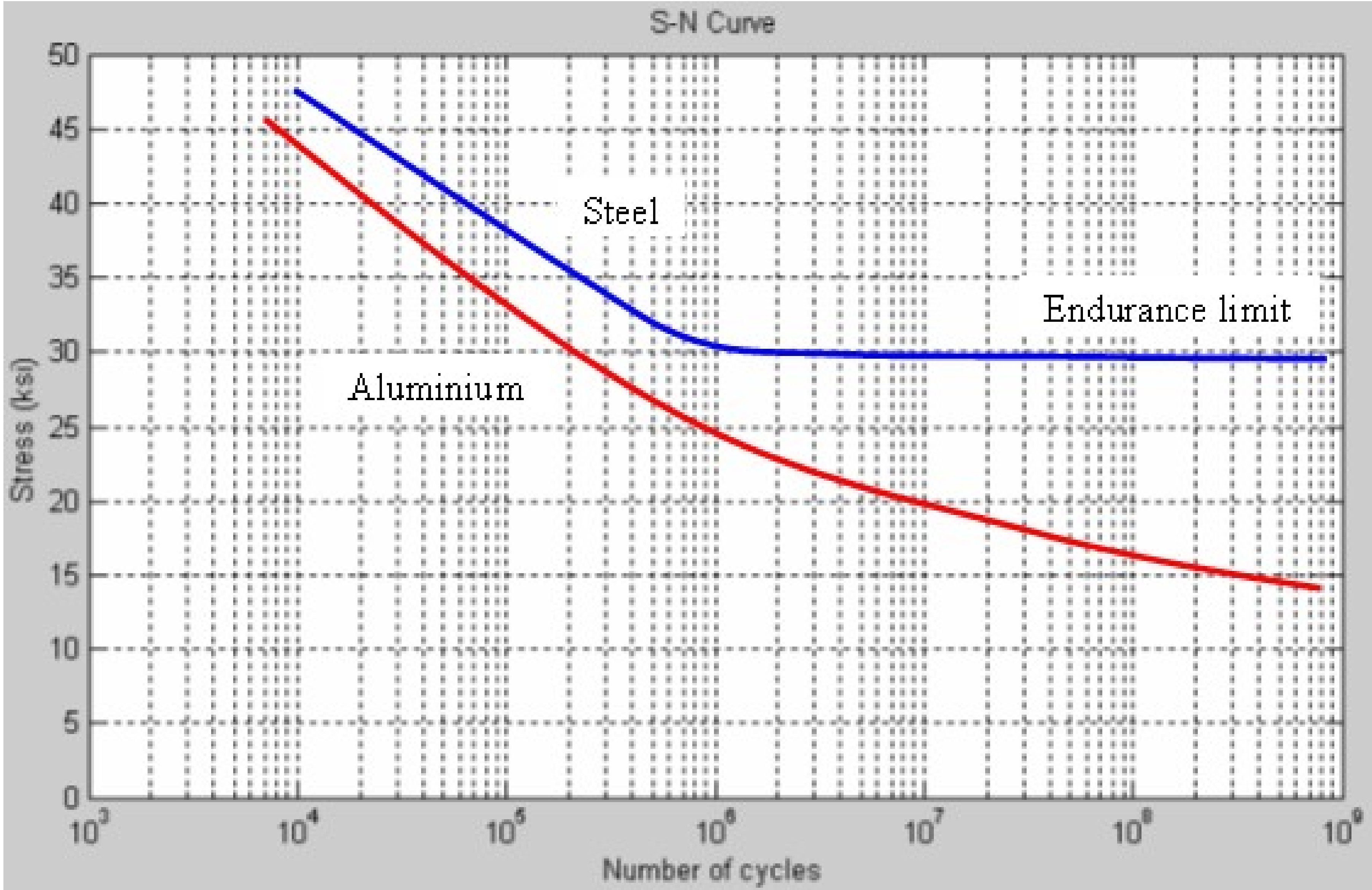
- This method relates the fatigue life to the alternating stress level causing failure



- The stress-life relation is obtained experimentally using *Moore high-speed rotating beam test*

  - The test is conducted by subjecting the rotating beam to a pure bending moment (of a fixed known magnitude) until failure occurs. (*Due to rotation, the specimen is subjected to an alternating bending stress)*

  - The data obtained from the tests is used to generate the fatigue strength *vs. fatigue life diagram which is known as the S-N diagram.*



Metals such as Aluminium and Copper, do not have a distinct. In these cases, a number of cycles (usually 10^7) is chosen to represent the fatigue life of the material.

6-7 ENDURANCE LIMIT

- The symbol for endurance limit is S'_e for the rotating beam specimen
- The symbol for endurance limit Se for an actual machine element
- endurance limit can be related to the tensile strength through

$$S'_e = \begin{cases} 0.5\,S_{ut} & S_{ut} \leq 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

A method to estimate the endurance limit for steel

6–2 Estimate S'_e in kpsi for the following materials:

(*a*) AISI 1035 CD steel.

(*b*) AISI 1050 HR steel.

(*c*) 2024 T4 aluminum.

(*d*) AISI 4130 steel heat-treated to a tensile strength of 235 kpsi.

From A-20 in kpsi：

(*a*) AISI 1035 CD steel. $S_{ut} = 80$ kpsi $S'_e = 0.5(80) = 40$ kpsi

(*b*) AISI 1050 HR steel. $S_{ut} = 90$ kpsi $S'_e = 0.5(90) = 45$ kpsi

(*c*) 2024 T4 aluminum. Aluminum has no endurance limit.

(*d*) AISI 4130 steel heat-treated to a tensile strength of 235 kpsi

$$S_{ut} > 200 \text{ kpsi}, \; S'_e = 100 \text{ kpsi}$$

6-8Fatigue Strength

- **Region of low cycle fatigue:**

  The fatigue strength S_f is only slightly smaller than the tensile strength S_{ut} .

- **Region of high Cycle Fatigue**

  The purpose of this section is to develop methods of approximation of the S-N diagram in the high-cycle region, when information may be as scarse as the results of a simple tension test. Experience has shown high-cycle fatigue data are rectified by a logarithmic transform to both stress and cycles-to-failure.

6-8 Fatigue Strength

In the region of high cycle fatigue, the equation relating the fatigue strength S_f to the number of cycles to failure N may be given by the empirical curve fit equation:

$$S_f = a\,N^b \qquad (6\text{-}13)$$

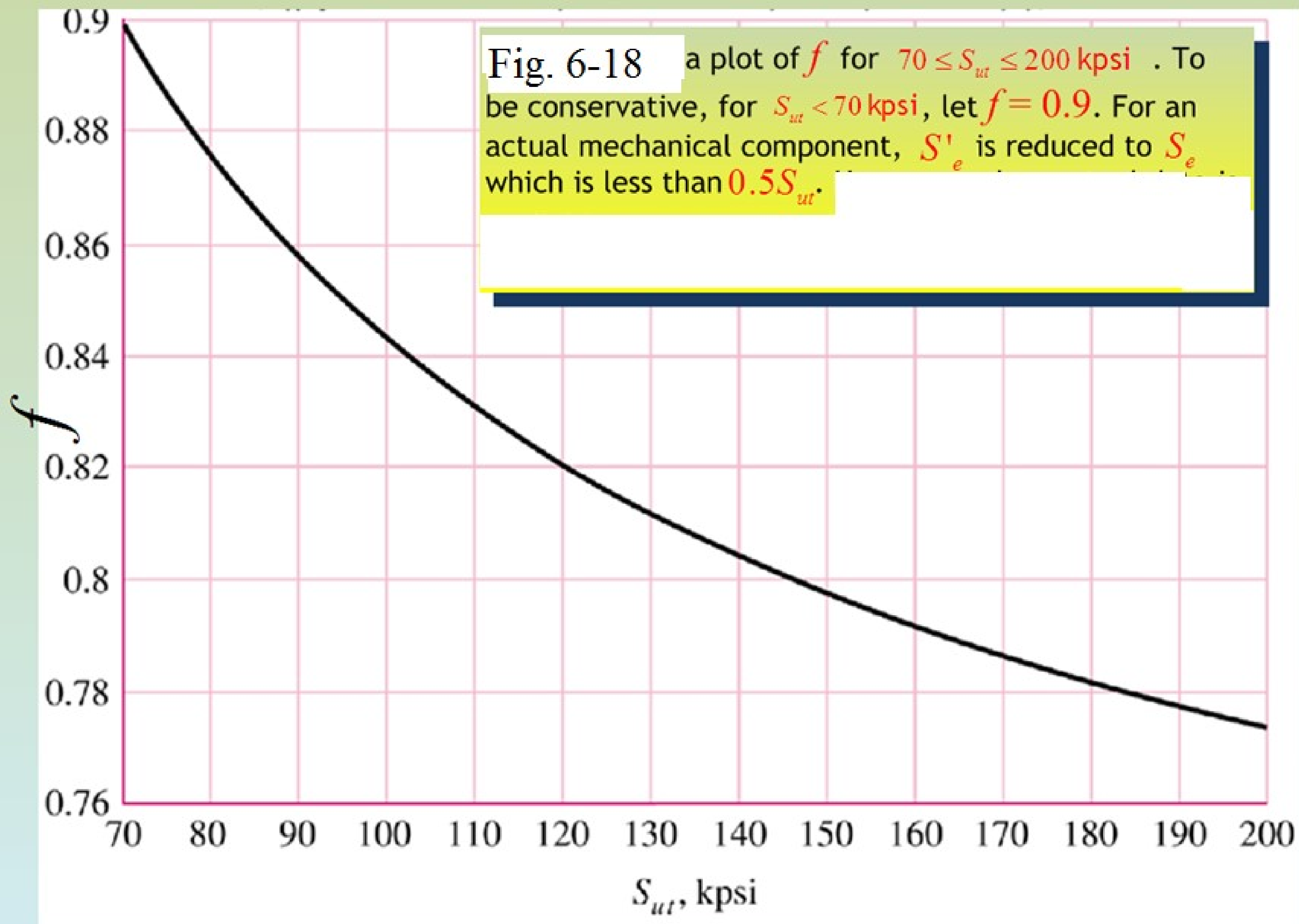
where N is the number of cycles to failure and a and b are given by

$$a = \frac{(f\,S_{ut})^2}{S_e} \qquad (6\text{-}14)$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) \qquad (6\text{-}15)$$

where f is found from Figure 6-18.

Fig. 6-18 a plot of f for $70 \le S_{ut} \le 200$ kpsi . To be conservative, for $S_{ut} < 70$ kpsi, let $f = 0.9$. For an actual mechanical component, S'_e is reduced to S_e which is less than $0.5S_{ut}$.



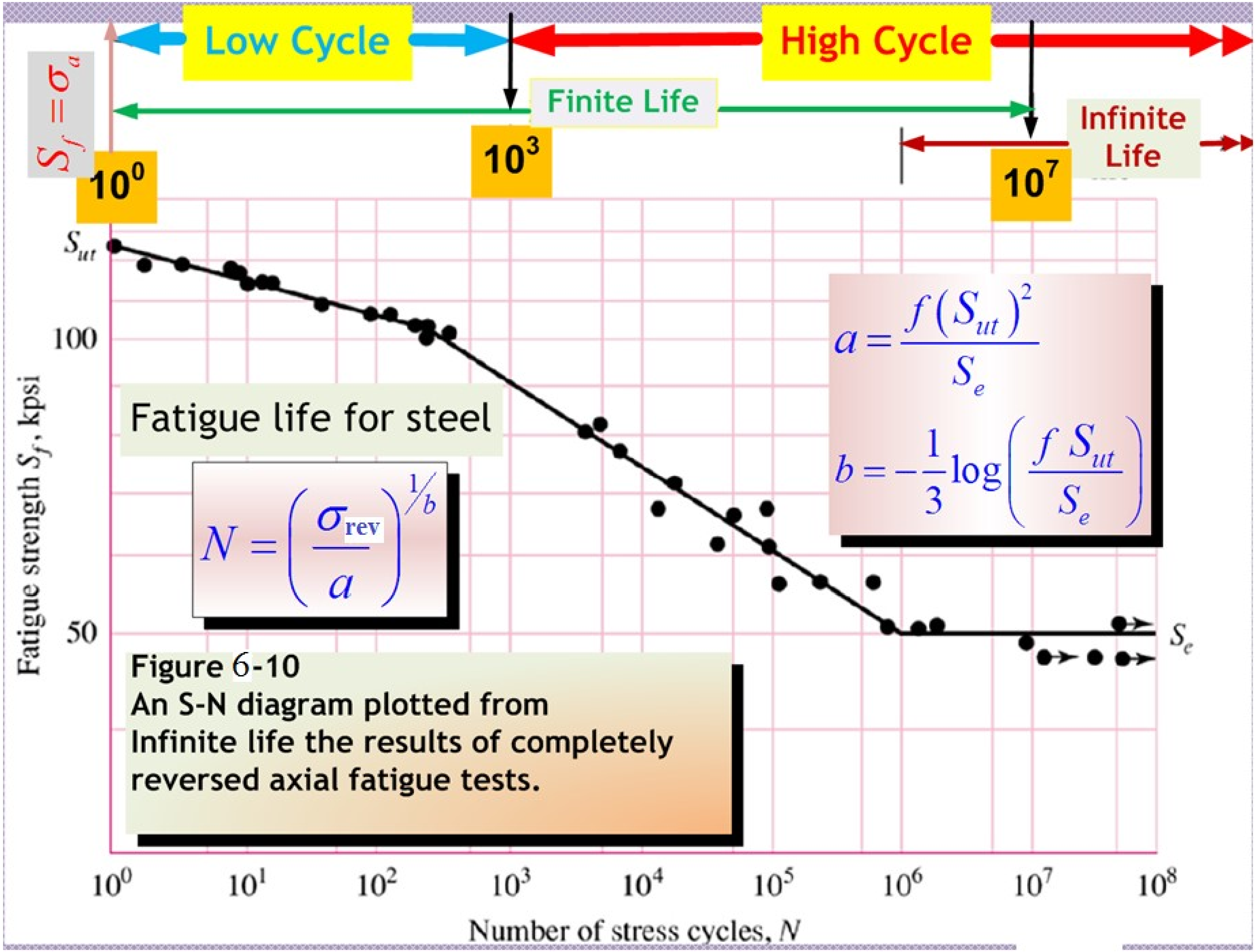


Figure 6-10
An S-N diagram plotted from
Infinite life the results of completely
reversed axial fatigue tests.

If a completely reversed stress σ_{rev} is given, setting $S_f = \sigma_a$ in Eq. (6-13), the number of cycles-to-failure can be expressed as

$$N = \left(\frac{\sigma_{rev}}{a} \right)^{1/b} \qquad (6\text{-}16)$$

Low-cycle fatigue is often defined (see Fig. 6-10) as failure that occurs in a range of cycles. $1 \le N \le 10^3$

6-4 A steel rotating-beam test specimen has an ultimate strength of 1600 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 900 MPa.

S_{ut} = 1600 MPa

from 6-8

$$S'_e = \begin{cases} 0.5 S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

S_e = 700 MPa

from Fig 6-18 we estimate f to be 0.77

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.77(1600)]^2}{700} = 2168.3 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400 \text{ cycles}$$

Given a 1050 HR steel, estimate

a) The endurance strength of a polished rotating beam specimen corresponding to 10^4 cycles to failure.

b) The expected life of a polished rotating-beam specimen under a completely reversed stress of 385MPa.

a) From Table A-20,

Sut=630 Mpa=91.373 kpsi

From Eq. (6-8)

a) From Fig. (6-18) for

S’e= (630)*0.5=315MPa= 45686.875 PSI

$f = 0.855$

From Eq. (6-14)

$$a = \frac{f\,(S_{ut})^2}{S_e}$$

and (6-15) ==➔

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

Thus Eq. (6-13) is:

$$S_f = a\,N^b$$

for $N = 10^4$ cycles to failure, the above equation becomes

$$S_f = a\,N^b = 1084\left(10^4\right)^{-0.0785} = 526\ \text{MPa}$$

b) From Eq. (6-16), with $\sigma_{\mathbf{rev}} = 385\ \text{MPa}$

$$N = \left(\frac{\sigma_{\mathbf{rev}}}{a}\right)^{1/b} = \left(\frac{385}{1084}\right)^{1/-0.0785} = 53.3\left(10^4\right)\text{cycles}$$

Keep in mind that these are only estimates.

6-8 Endurance Limit Modifying Factors

The rotating-beam specimen used in the laboratory to determine endurance limits is prepared very carefully and tested under closely controlled conditions. It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory. Some differences include

- Material: composition, basis of failure, variability
- Manufacturing: method, heat treatment, fretting corrosion, surface condition, stress concentration
- Environment: corrosion, temperature, stress state, relaxation times
- Design: size, shape, life, stress state, stress concentration, speed, fretting, galling

Marin’s Equation

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. Marin’s Equations is therefore written as:

$$S_e = k_a \, k_b \, k_c \, k_d \, k_e \, k_f \, S'_e \qquad (6\text{-}18)$$

S_e : Endurance limit at the critical location of a machine part in geometry and condition of use

S'_e : rotary-beam test specimen endurance limit

$$S_e = k_a\, k_b\, k_c\, k_d\, k_e\, k_f\, S'_e$$

k_a = Surface condition modification factor

k_b = Size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor

k_f = miscellaneous-effects modification factor

When endurance tests of parts are not available, estimations are made by applying Marin factors to the endurance limit.

Surface factor k_a

$$k_a = a\, S_{ut}^{b} \tag{6-19}$$

where S_{ut} is the minimum tensile strength and a and b are to be found in Table 6-2.

Table 6-2 Parameters for Marin surface modification factor, Eq. (6-19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

Size factor k_b

The size factor k_b for bending and torsion may be given by:

$$k_b = \begin{cases} (d/3)^{-0.107} = 0.879\, d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91\, d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24\, d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51\, d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \quad (6\text{-}20)$$

For axial loading there is no size effect, so

$$\boxed{k_b = 1} \quad (6\text{-}21)$$

Size factor k_b

QUESTION:

What to do with Eq.(6-20) if a round bar in bending is not rotating or when a non-circular cross-section is used?

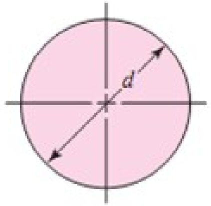
Use *effective dimension* d_e where

$$d_e = 0.370\, d \tag{6-24}$$

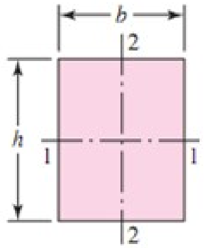
as the effective size of a round corresponding to a non-rotating solid or hollow round. Table 6-3 provides areas of common structural shapes undergoing non-rotating bending

Table 6-3

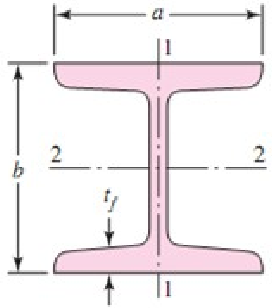
Areas of common non-rotating structural shapes



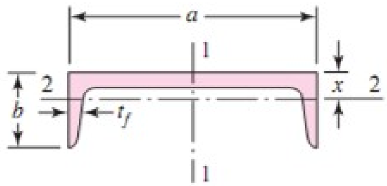
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



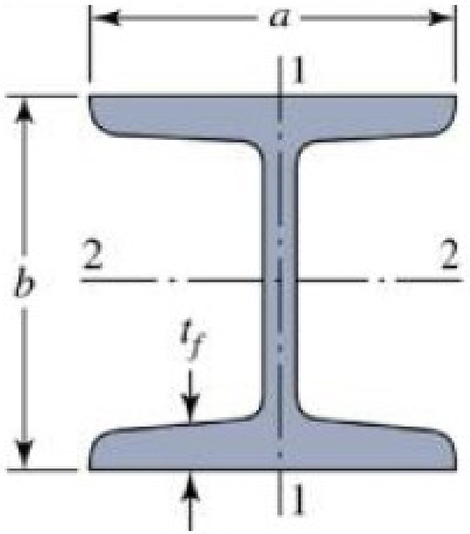
$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$$

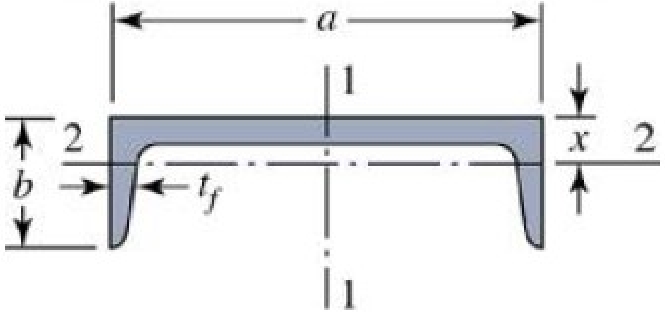


$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b-x) & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.1at_f & & axis1-1 \\ 0.05ab & t_f > 0.025a & axis2-2 \end{cases}$$

$$d_e = \left(\frac{A_{0.95\sigma}}{0.0766}\right)^{0.5}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & axis1-1 \\ 0.052xa + 0.1t_f(b-x) & axis2-2 \end{cases}$$

$$d_e = \left(\frac{A_{0.95\sigma}}{0.0766}\right)^{0.5}$$

Loading factor k_c

Average values for the load factor are given by

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{-}26)$$

Temperature factor Kd

For steels, the endurance limit slightly increases as temperature rises, then it starts to drop. Thus, the temperature factor is given as:

$$k_d = 0.9887 + 0.6507(10^{-3})T_c - 0.3414(10^{-5})T_c^2 + 0.5621(10^{-8})T_c^3 - 6.426(10^{-12})T_c^4$$

For $37 \leq T_c \leq 540\ °C$

The same values calculated by the equation are also presented in <u>*Table 6-4*</u>

where: $k_d = \left(\frac{S_T}{S_{RT}}\right)$

Reliability Factor (k_e)

The endurance limit obtained from testing is usually reported at mean value (*it has a normal distribution with* $\hat{\sigma} = 8\%$).

- For other values of reliability, k_e is found from *Table 6-5*.

Miscellaneous-Effects Factor (k_f)

It is used to account for the reduction of endurance limit due to all other effects (*such as Residual stress, corrosion, cyclic frequency, metal spraying, etc.*).

However, those effects are not fully characterized and usually not accounted for. Thus we use ($k_f = 1$).

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620