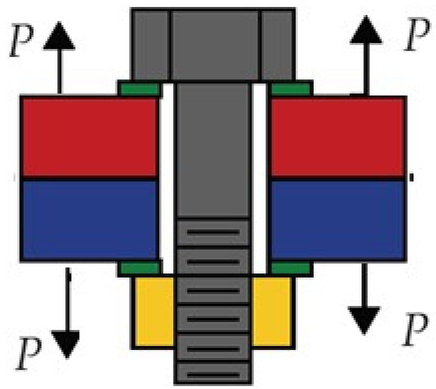
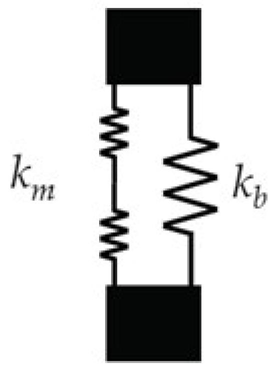
8–4 **Joints—Fastener Stiffness**

When an external tensile load " " is applied to the joint, the load will be divided between the bolt and the clamped member (*as long as the load " P" is not large enough to separate the clamped members),*





Most of the force is taken by the members

- Very little (<15%) of the force is taken by the bolt

P_b : portion of P taken by bolt.

P_m : portion of P taken by members.

$F_b = P_b + F_i$: resultant bolt load.

$F_m = P_m - F_i$: resultant members load.

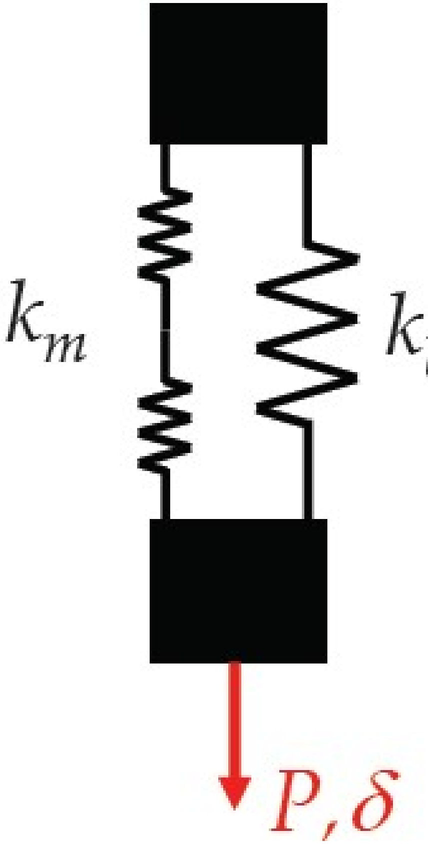
$C = P_b/P$: fraction of external load carried by bolt.

$(1 - C)$: fraction of external load carried by member.

- In order to find the portion of the external load carried by the bolt and the portion carried by the material, spring methodology is used where the bolt and the clamped material are represented as two springs in parallel.

- P_m = Portion of P taken by members
- P_b = Portion of P taken by bolt

$$P_b = \frac{k_b}{k_m + k_b} P = P\,C$$



$$P = P_m + P_b$$

$$P_m = P\,(1 - C)$$

$$\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m}$$

$$F_b = P_b + F_i$$

$$F_m = P_m - F_i$$

High preload = High load capacity

$$F_b = CP + F_i \qquad F_m = (1 - C)P - F_i$$

Since the bolt and members will have the same deflection:

$$\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m} \quad \rightarrow \quad P_b = P_m \frac{k_b}{k_m}$$

Knowing that,

$$C = \frac{P_b}{P} = \frac{P_b}{P_b + P_m} = \frac{P_m \frac{k_b}{k_m}}{P_m \frac{k_b}{k_m} + P_m}$$

$$\boxed{C = \frac{k_b}{k_b + k_m}}$$

The <u>Stiffness Constant</u>

of the joint

- According to the standard, the recommended value of preload is given as:

$$\boxed{F_i} = \begin{cases} 0.75\, F_p & for\ nonpermanent\ connections\ (reused\ fasteners) \\ 0.9\, F_p & for\ permanent\ connetions \end{cases}$$

Where F_p is the *Proof Load*: $\boxed{F_p = S_p A_t}$

Static load capacity

Typically the bolt fails first, why?

- It is the least expensive
- It is the most easily replaced

$$\sigma_b = \frac{CP}{A_t} + \frac{F_i}{A_t}$$

Proof load and stress

- S_p = proof stress = Limiting value of σ_b (~ 0.85 σ_y)

$$\frac{C\ n\ P}{A_t} + \frac{F_i}{A_t} = S_p$$

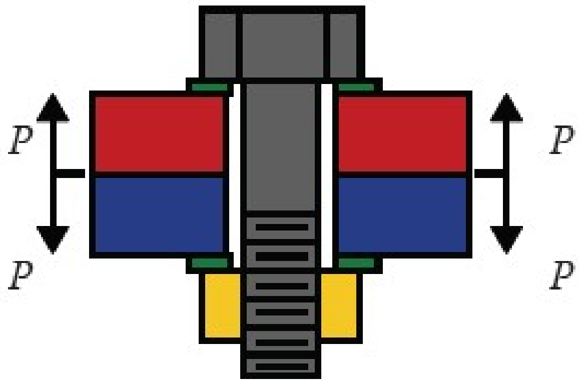
Load factor (like a factor of safety)

- $n > 1$ ensures $\sigma_b < S_p$

$$n = \frac{S_p\, A_t - F_i}{C\ P}$$

How high should the pre-load be?

- Non-permanent: Some suggest 0.75 F_p
- Permanent: Some suggest 0.90 F_p



And the resultant bolt load is:

$$F_b = P_b + F_i = \boxed{CP + F_i}$$

And the resultant member load is:

$$F_m = P_m - F_i = \boxed{(1 - C)P - F_i}$$

- Note that these relations are valid only when ($F_m < 0$), meaning that the members are still under compressive load and did not get separated.

- If the external load is large enough to separate the members, then the entire load will be carried by the bolt: $\boxed{F_b = P}$ (*this should not happen*).

It's also necessary to ensure that separation will not occur (*if separation occurs, the bolt will carry the entire load).*

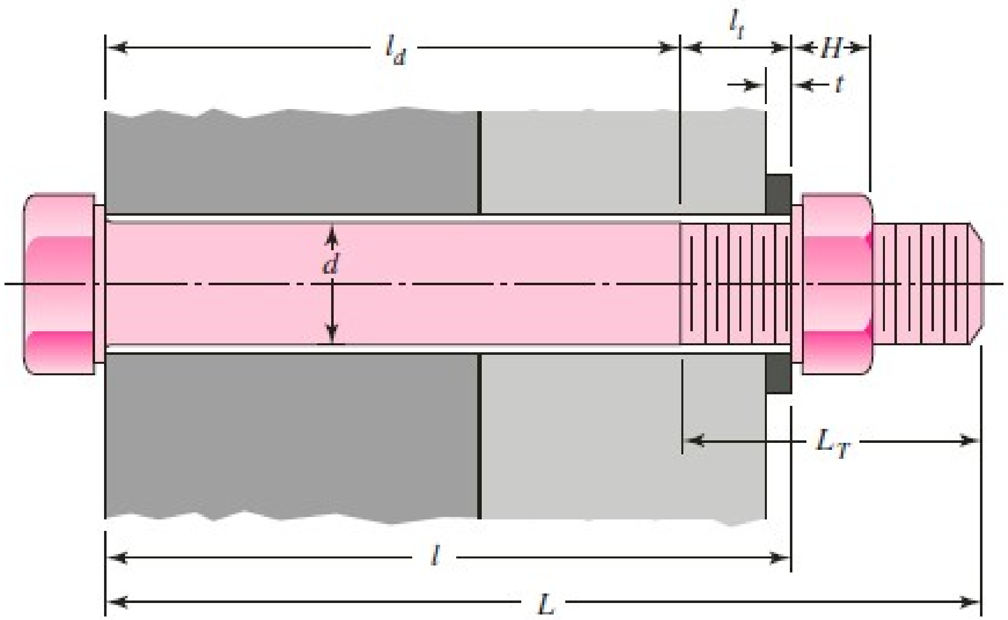
Separation occurs when:

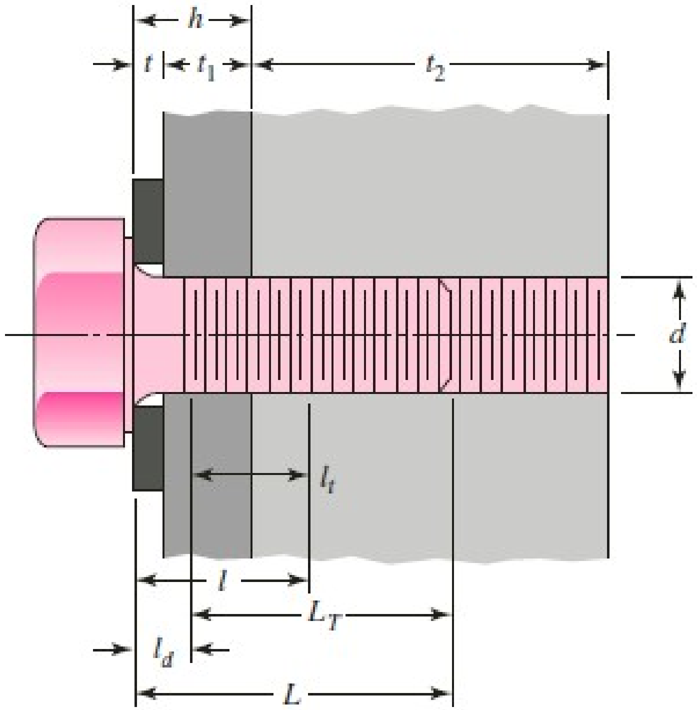
$$F_m = 0 = (1 - C)n_o P - F_i$$

$$\boxed{n_o = \frac{F_i}{P(1 - C)}}$$

Where is n_0 the *Load Factor guarding against joint separation*

- Both load factors n_L & n_o should be calculated, and the smaller of the two will be the load factor of safety for the joint.





l Grip

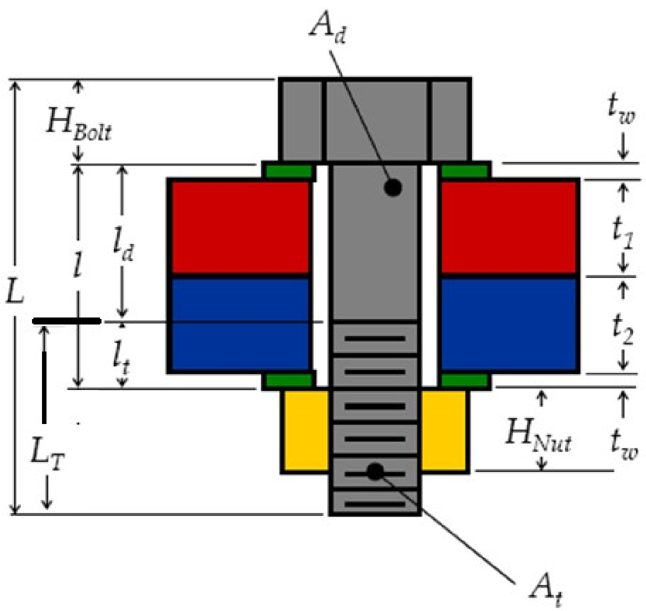
A_t Tensile stress area

A_d Major diameter area

l_t Threaded length in grip

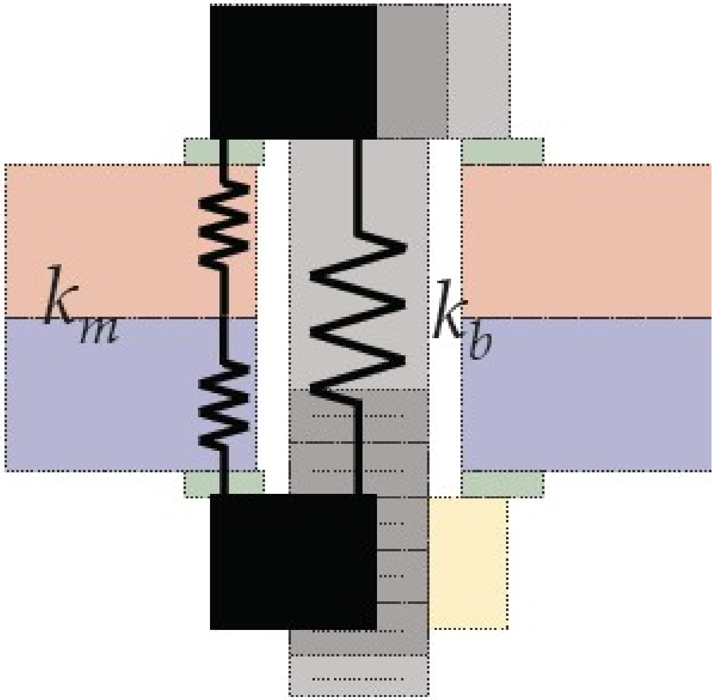
l_d Unthreaded length in grip

d Major diameter (unthreade



8-5 Joints – Member Stiffness

Preloaded joint modeled as series spring



Spring ***km, kb Need to find equivalent bolt and member stiffness***

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

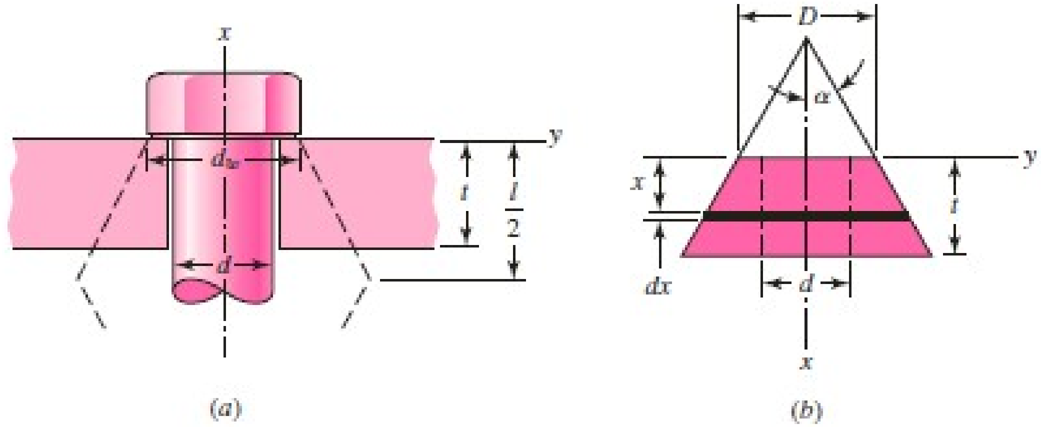
l_t: length of threaded portion of the grip

l_d: length of unthreaded portion.

A_d: "major-diameter area" of fastener.

A_t: tensile stress area

For members made of the same material (*same), the* effective stiffness is found ➢ to be:



$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} \qquad (8\text{–}22)$$

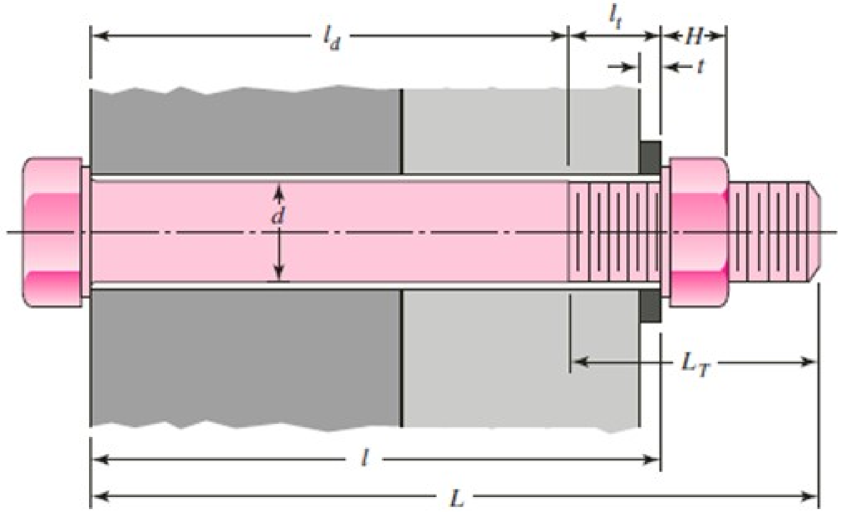
Prob. 8-11 : An M14 × 2 hex-head bolt with a nut is used to clamp together two 15-mm steel plates.

(*a) Determine a suitable length for the bolt, rounded up to the nearest 5 mm.*

(*b) Determine the bolt stiffness.*

(*c) Determine the stiffness of the members*

Table A-31, nut height $H = 12.8$ mm. $L \geq l + H = 2(15) + 12.8 = 42.8$ mm.



Add 2 theads after the nut

42.8+2×2=46.8 mm round up 47 mm

$A_d = \Pi\,(14^2)/4 = 153.9$ mm².

From Table 8-1, $A_t = 115$ mm². (table 8-1)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(19) + 115(11)} = 874.6 \text{ MN/m}$$

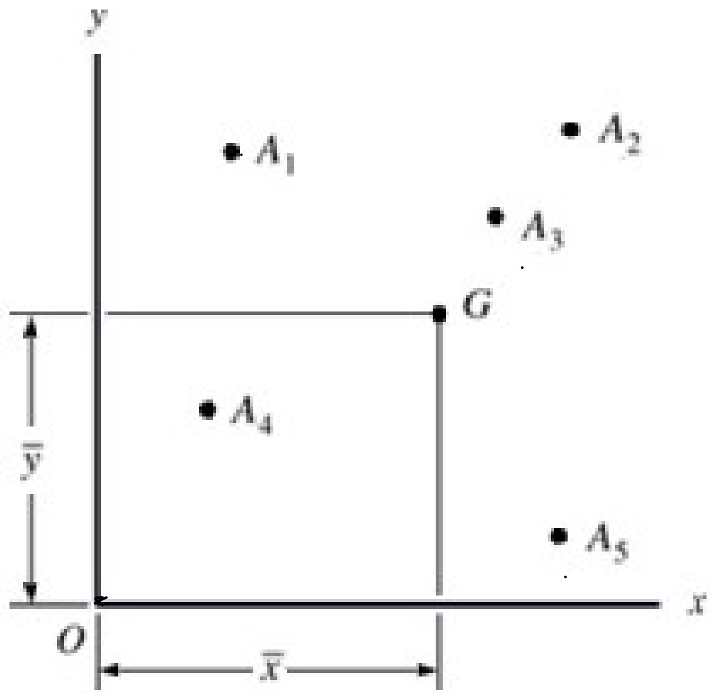
For a shear joint loaded by a shear force and a moment

In order to analyze the shear joints subjected to moment,

the *relative center of rotation between the two members* needs to be determined.

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$

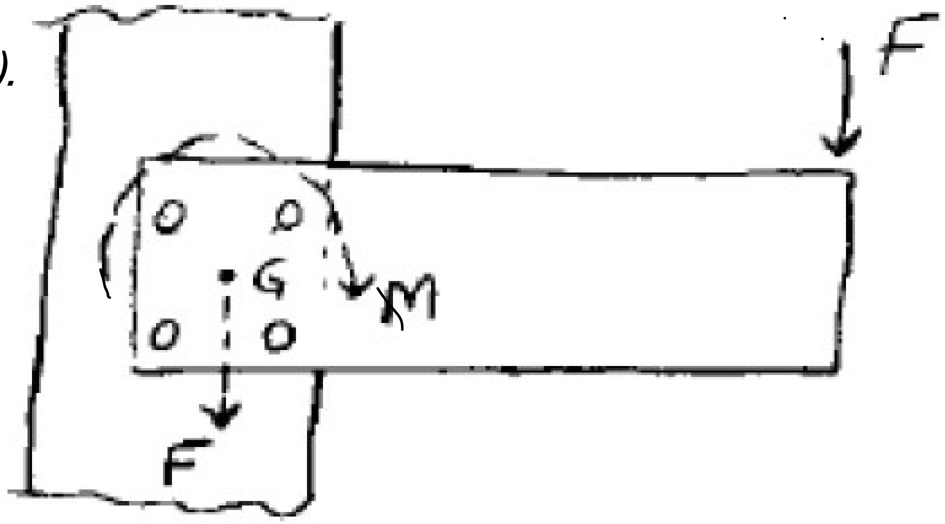
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$



For a shear joint loaded by a shear force and a moment,each fastener will carry two shear components:

- Primary shear (*due to the shear load).*

- Secondary shear (*due to the moment).*



The shear load will be divided evenly between the

fasteners (*assuming all fatteners have the same area) and each will have:*

Primary shear

$$F_n' = {}^{V}/_{n}$$

Where n is the number of fasteners

$$M_1 = F_A'' r_A + F_B'' r_B + F_C'' r_C + \cdots$$

r_A, r_B, r_C, etc., are the radial distances from the centroid

F'' are the moment loads

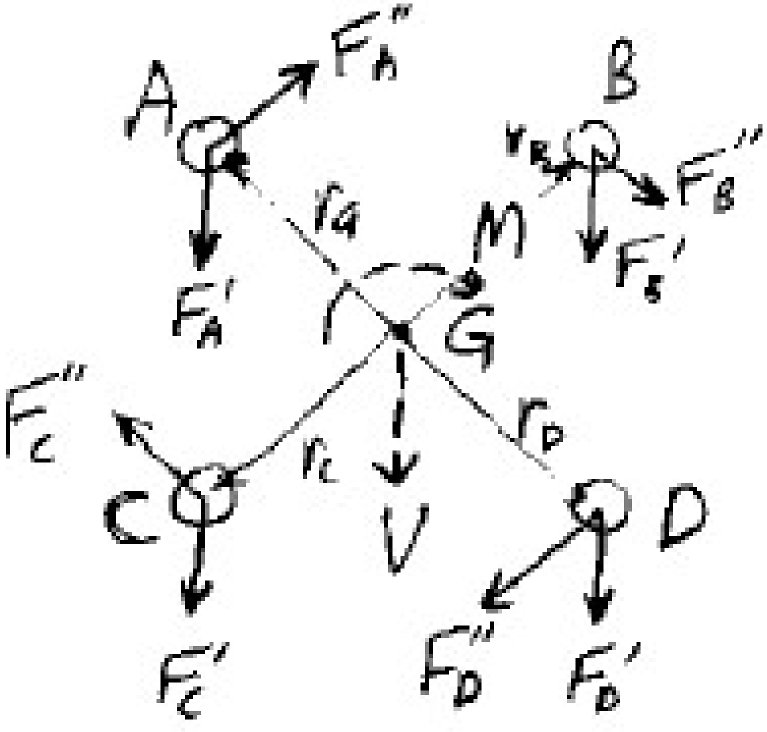
$$\frac{F_A''}{r_A} = \frac{F_B''}{r_B} = \frac{F_C''}{r_C}$$

$$F_n'' = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \cdots}$$

The moment introduces secondary shear in the fasteners and the value of the secondary shear (*assuming all fatteners have the same area and same)* *depends on the* distance of the fastener from the center of rotation "*G"* where the closer fastener to "*G", the less load it carries:*

Secondary shear

$$F_n'' = \frac{M r_n}{r_a^2 + r_b^2 + r_c^2 + \cdots}$$



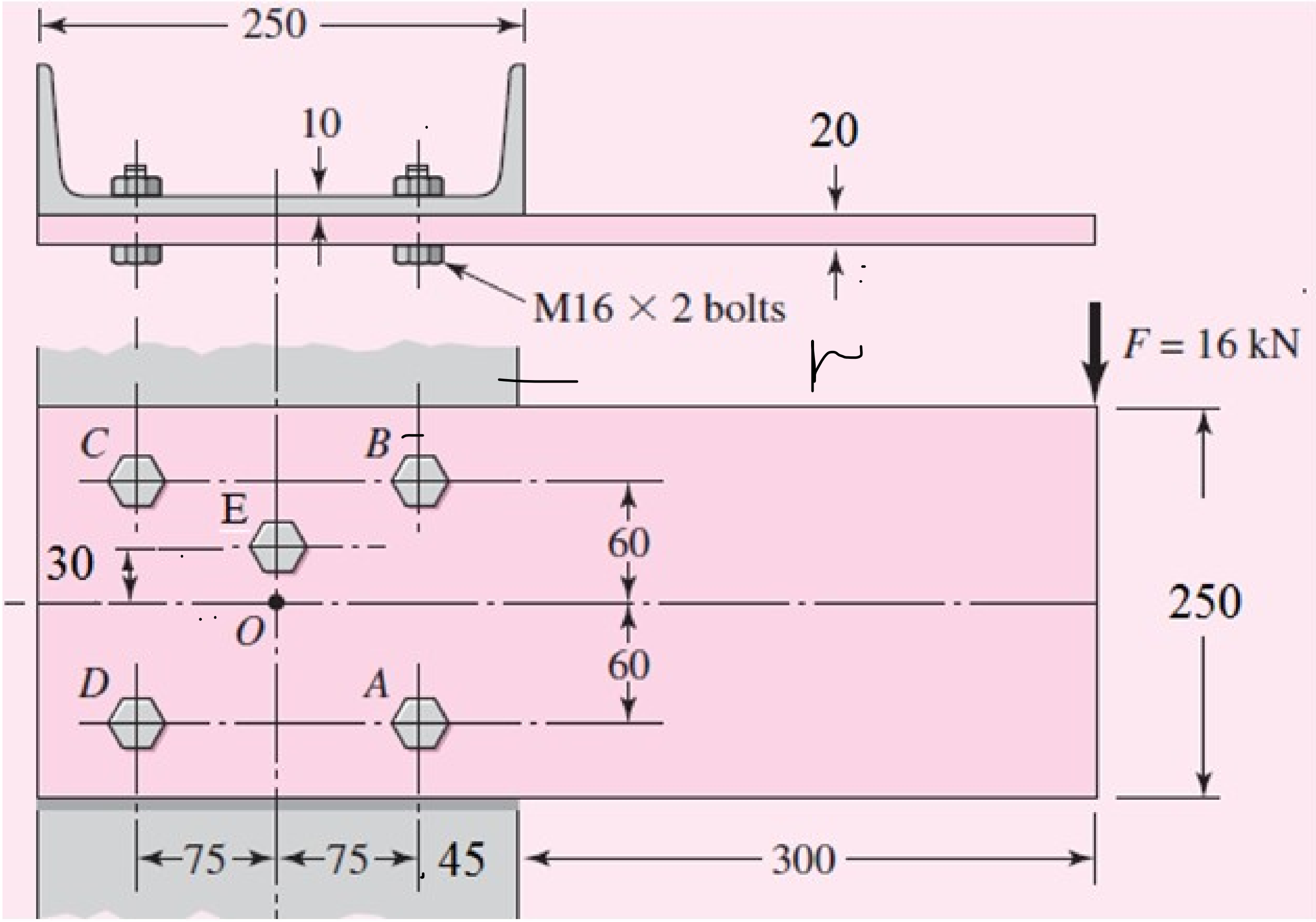
$$F_R = \left\|\vec{F}_n' + \vec{F}_n''\right\| = \sqrt{{F_n'}^2 + {F_n''}^2 + 2\, F_n'\, F_n'' \cos\theta}$$

Shown in the figure is a 20 by 250-mm rectangular steel bar cantilevered to a 250-mm steel channel using five tightly fitted bolts located at A, B, C, D and E. For F=16 kN load

find

(a) The resultant load on each bolt

(b) The maximum shear stress in each bolt



$$X = \frac{75 + 75 - 75 - 75 + 0}{5} = 0$$

$$\bar{Y} = \frac{-60 + 60 + 60 - 60 + 30}{5} = 6$$

G(x,y)=G(0,6)

$$F' = \frac{16}{5} = 3.2$$

M=(300+45+75)*16=6720 N.m

$$F''_n = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + r_D^2 + r_E^2}$$

$$r_A = \sqrt{75^2 + 66^2} = 99.9$$

$$r_C = \sqrt{75^2 + 54^2} = 92.4$$

$$r_D = \sqrt{75^2 + 66^2} = 99.9$$

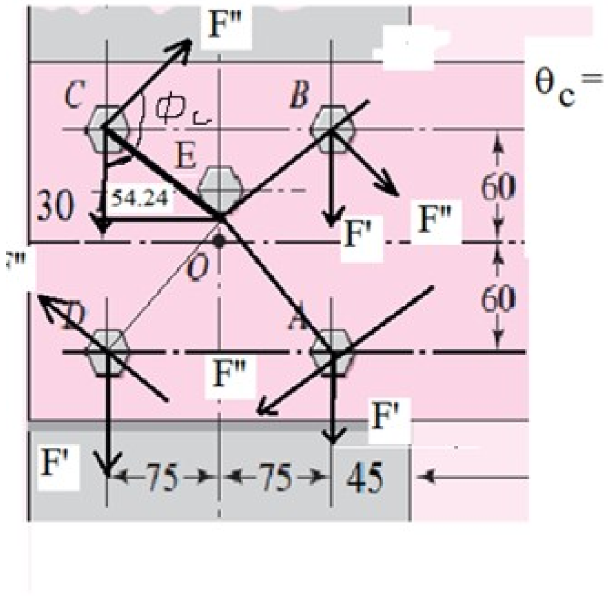
$$r_B = \sqrt{75^2 + 54^2} = 92.4$$

$$r_E = 24$$

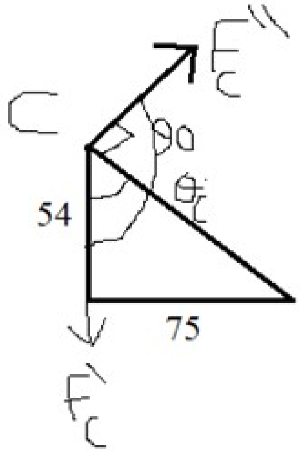
$$F''_A = F''_D = \frac{6720 * 99.9}{2 * 99.9^2 + 2 * 92.4^2 + 24^2} = 17.9 \text{ kN}$$

$$F''_C = F''_B = \frac{6720 * 92.4}{2 * 99.9^2 + 2 * 92.4^2 + 24^2} = 16.52\, kN$$

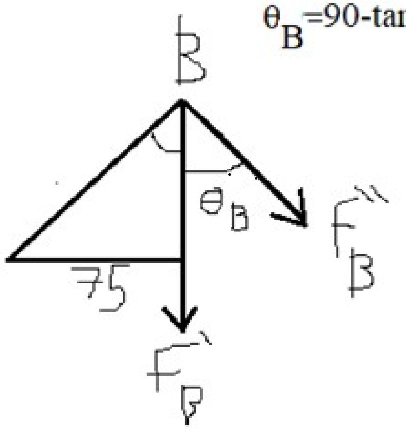
$$F''_E = \frac{6720 * 24}{2 * 99.9^2 + 2 * 92.4^2 + 24^2} = 4.3\ kN$$

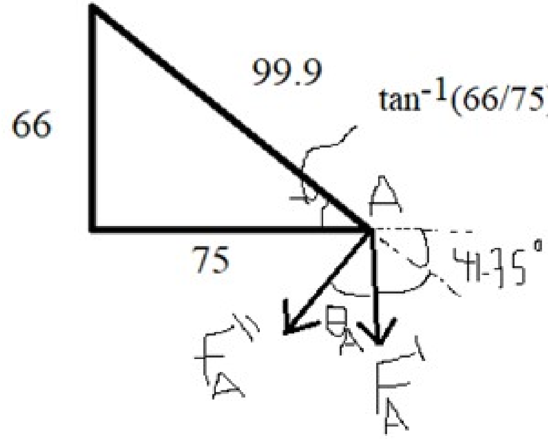


$\theta_c = \tan^{-1}(75/54) + 90 = 54.24 + 90 = 144.24$



$\theta_B = 90 - \tan^{-1}(75/54) = 35.76$

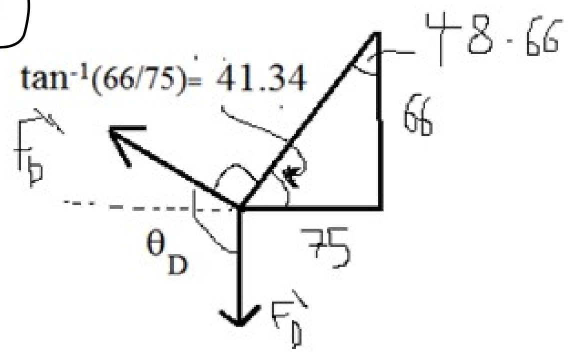




$\tan^{-1}(66/75) = 41.35°$

$\theta_A = 41.34$

$\theta_D = 48.66 + 90 - 90 + 9 = 138.66$



$\tan^{-1}(66/75) = 41.34$

$$F_A = \sqrt{(F')^2 + (F'')^2 + 2F'F''\cos 41.34} = 20.41$$

$$F_C = \sqrt{(F'_C)^2 + (F''_C)^2 + 2F'_C F''_C \cos(54.24+90)} = 14.05$$

$$F_B = \sqrt{(F'_B)^2 + (F''_B)^2 + 2F'_B F''_B \cos(35.76)} = 19.21$$

$$F_D = \sqrt{(3.2)^2 + (17.9)^2 + 2(3.2)(17.9)\cos 138.66} = 15.64$$

$$F_E = \sqrt{(3.2)^2 + 4.3^2} = 5.36$$

✓

FA=20.14KN

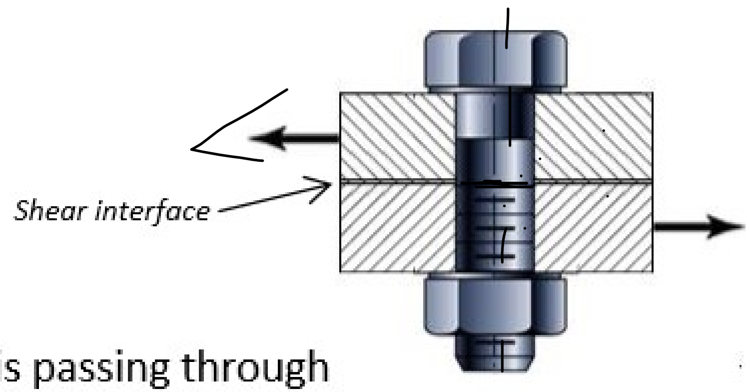
Bolts A and D are critical

Using standard bolts LT=38 mm ld=46-38=8 mm

Portion of the threaded part is in the joint

Thus shearing will be calc. using the root diameter

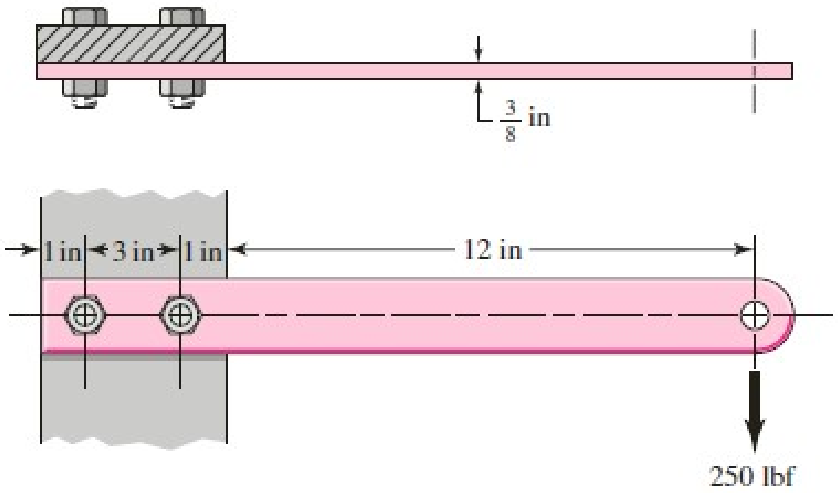
$$\tau = \frac{F}{A_s} = \frac{20.41 * 1000}{144}$$

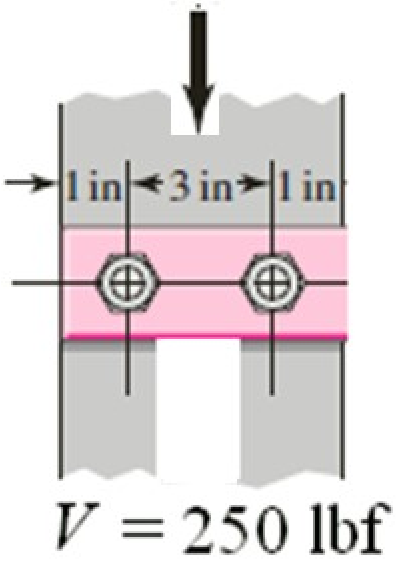


If the threaded portion of the fastener is passing through the shear interface,

Use the root diameter

8–77 A $\frac{3}{8}$- × 2-in AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 250 lbf as illustrated. The bar is secured to the support using two $\frac{3}{8}$ in-16 UNC SAE grade 4 bolts. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.





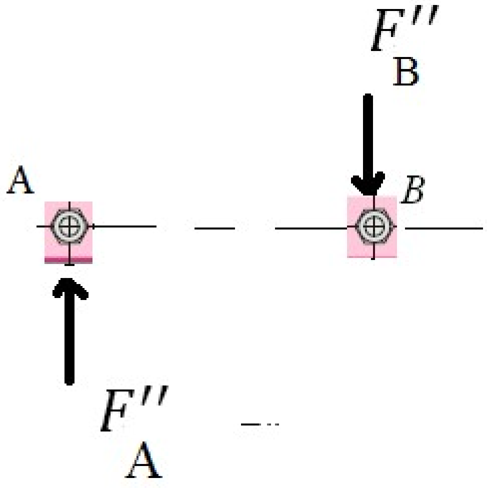
$M = 3625\ \text{lbf} \cdot \text{in}$
$V = 250\ \text{lbf}$

$\bar{x} = 1.5\ \text{in}$

$\bar{y} = 0$

Secondary shear

$$F_n'' = \frac{M r_n}{r_a^2 + r_b^2 + r_c^2 + \cdots}$$



$$F''_A = \frac{3625 \times 1.5}{1.5^2 + 1.5^2} = 1208.3$$

$$F''_B = \frac{3625 \times 1.5}{1.5^2 + 1.5^2} = 1208.3$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}$$

$$F_B = 1208 + 125 = 1333 \text{ lbf}$$

$$\tau_{max} = \frac{F_{max}}{A} = \frac{1333}{0.1104} = 12\ 070 \text{ psi}$$

$$A = (\pi / 4)(0.375^2) = 0.1104 \text{ in}^2$$

Table 8-9

$$S_y = 100 \text{ kpsi}$$

$$S_{shear} = 0.577(100) = 57.7 \text{ kpsi}$$

$$n = \frac{57.7}{12.07} = 4.78$$

Bearing area

$$A_B = td = 0.375\,(0.375) = 0.1406 \text{ in}^2$$

$$\sigma_B = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\,481 \text{ psi}$$

$$n = \frac{100}{9.481} = 10.6$$

on member:

$S_y = 54$ kpsi.

$$n = \frac{S_y}{\sigma_B} = \frac{54}{9.481} = 5.70$$

Bending of member

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\ 080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13$$