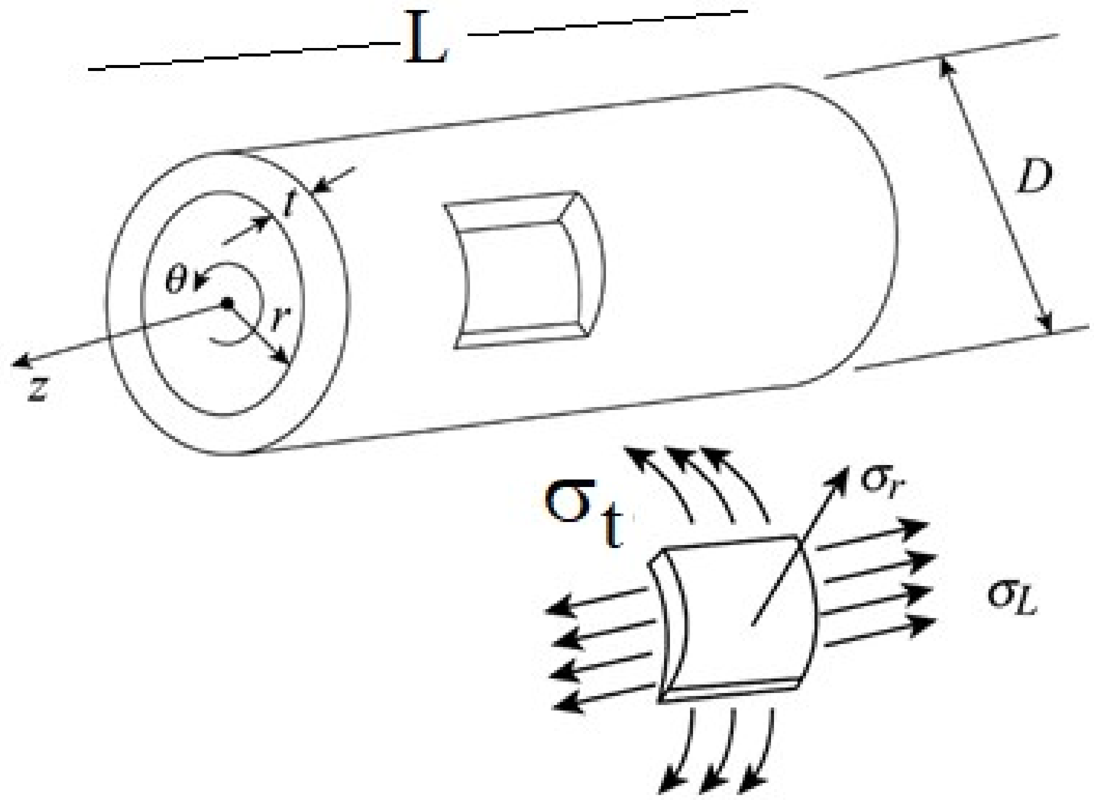
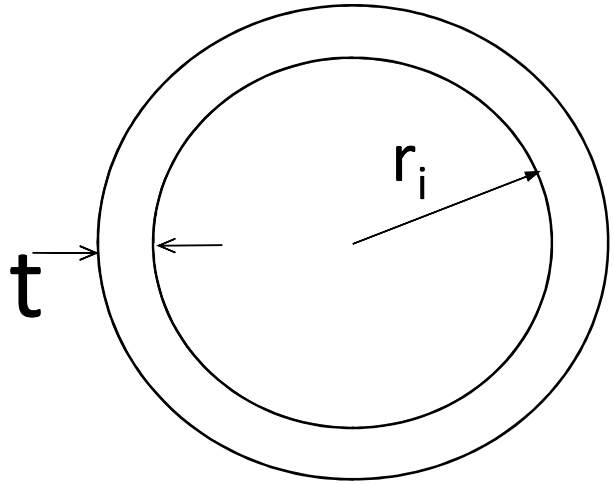
3-14 Stresses in Pressurized Cylinders


L
D
t
θ
r
z
σt
σr
σL

Thin-walled and thick-walled pressure vessels

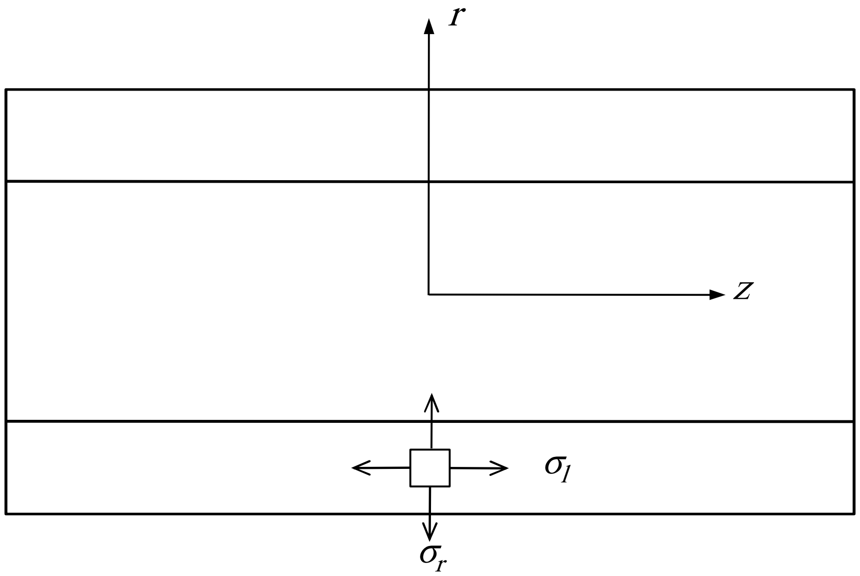


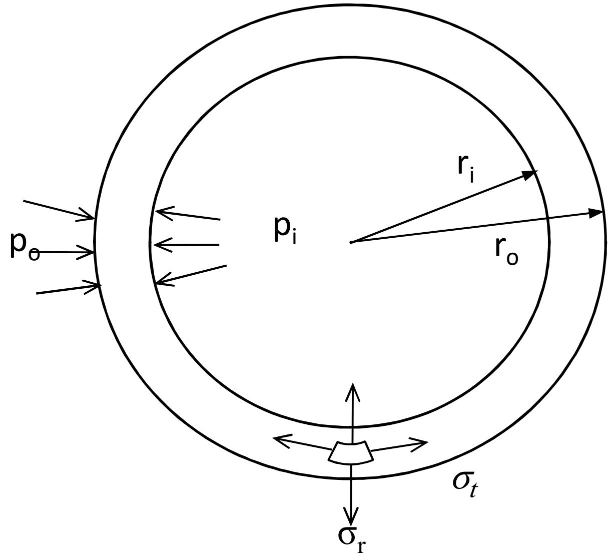
thin-walled cylinders

($r_i/t \geq 10$)

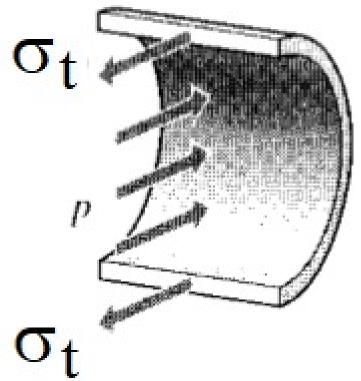
thick-walled cylinders

($r_i/t < 10$)





the circumferential or “hoop” stresses σt



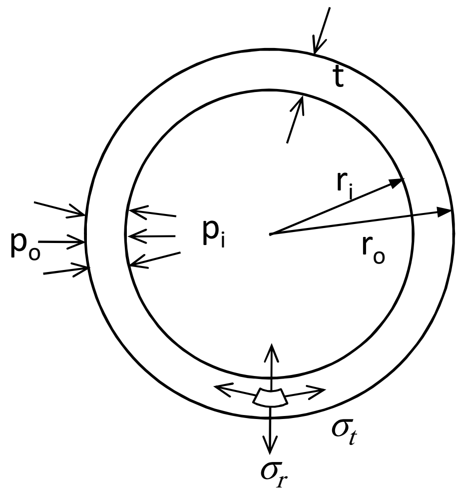
Thick-Walled Cylinders

- thick-walled pressure vessel, $r_i/t < 10$

  - For thick-walled pressure vessels

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$



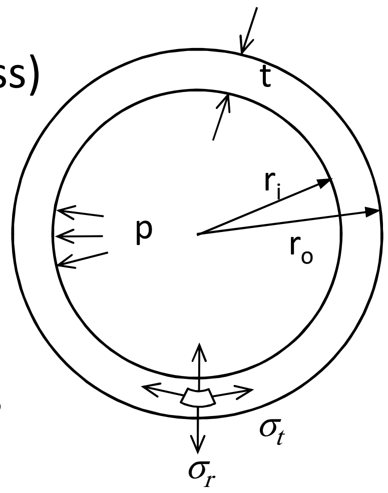
  - Maximum shear stress $\tau_{max} = \frac{1}{2}(\sigma_t - \sigma_r)$

  - If the ends of the cylinder are capped, must include longitudinal stress.

$$\sigma_l = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

Thin-Walled Pressure Vessels

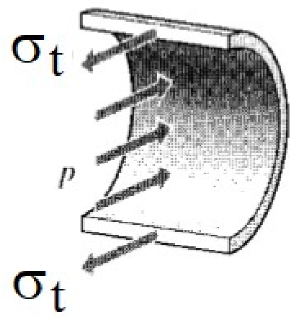
$$\sigma_t = \frac{pr_i}{t}$$ (hoop stress) $$\sigma_l = \frac{pr_i}{2t}$$ (longitudinal stress)



➢ In thin walled pressure vessels, the inner and outer radii are set equal to *r*, and the thickness is *t*.

- Radial stress (σ_r) is equal to *-p* on the inner surface, zero on the outer surface, and varies in between.

- σ_r is negligible compared to σ_t.



Ex: A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m2.

(a) Calculate the tangential and longitudinal stresses in the steel.

(b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m2?

ri/t =200/20=10 Thin walled vessel

$$\sigma_t = \frac{pr_i}{t}$$

$$\sigma_t = \frac{pr_i}{t} = \frac{4.5*200}{20}$$

$$\sigma_t = 45 \text{ MPa}$$

$$\sigma_l = \frac{pr_i}{2t}$$

$$\sigma_L = \frac{4.5*200}{2*20}$$

$$\sigma_L = 22.5\, MPa$$

Since

$$\sigma_t = \frac{pr_i}{t} \qquad \sigma_l = \frac{pr_i}{2t}$$

$\sigma_t = 2\sigma_l$

So the tangential stress is the critical stress

$$\sigma_t = \frac{pr_i}{t}$$

$$120 = \frac{p * 200}{20}$$

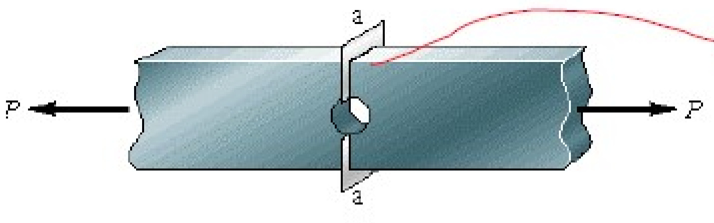
$p = 12$ MPa

5-2 Stress Concentration Factor, K_t

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{avg}}} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_{\text{avg}}}$$

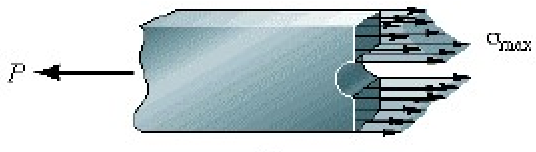
$$\sigma_{\text{avg}} = \frac{P}{A}, \text{ where } A \text{ is the smallest cross-sectional area.}$$

- Elementary stress equations don't apply in stress concentrations.



(a)

$$\sigma_{avg} = \frac{P}{A} = \frac{P}{(w\text{-}d)t}$$



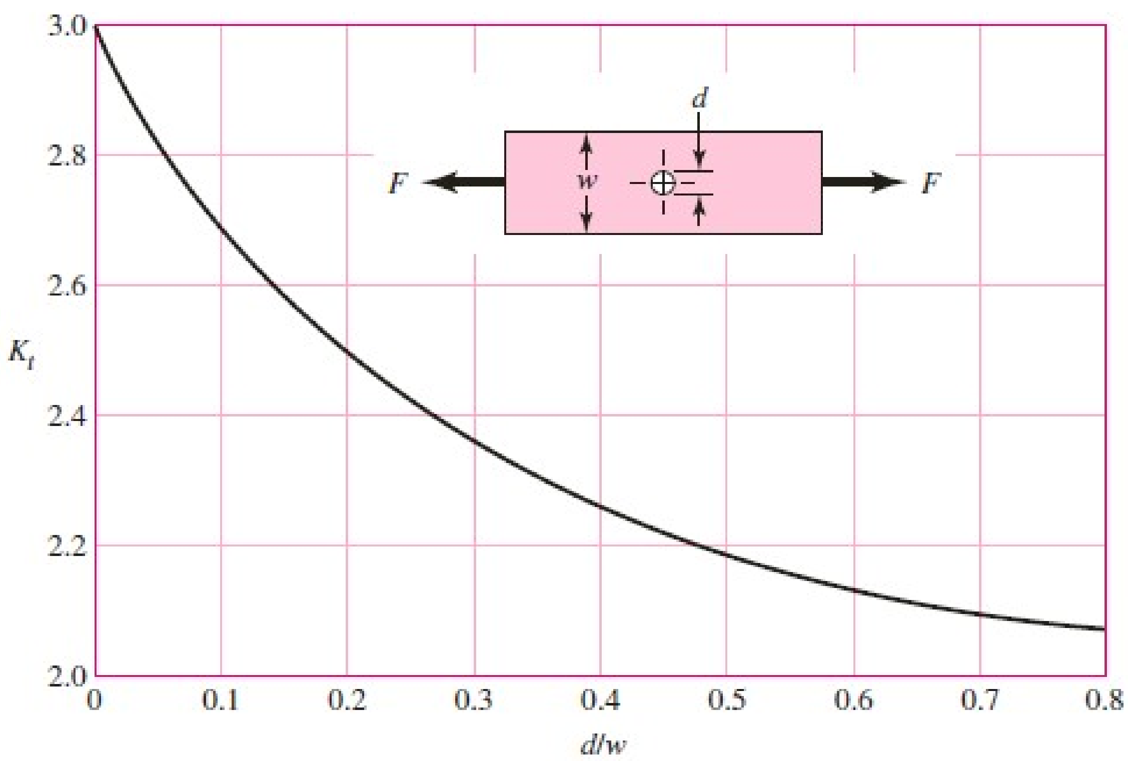
(b)

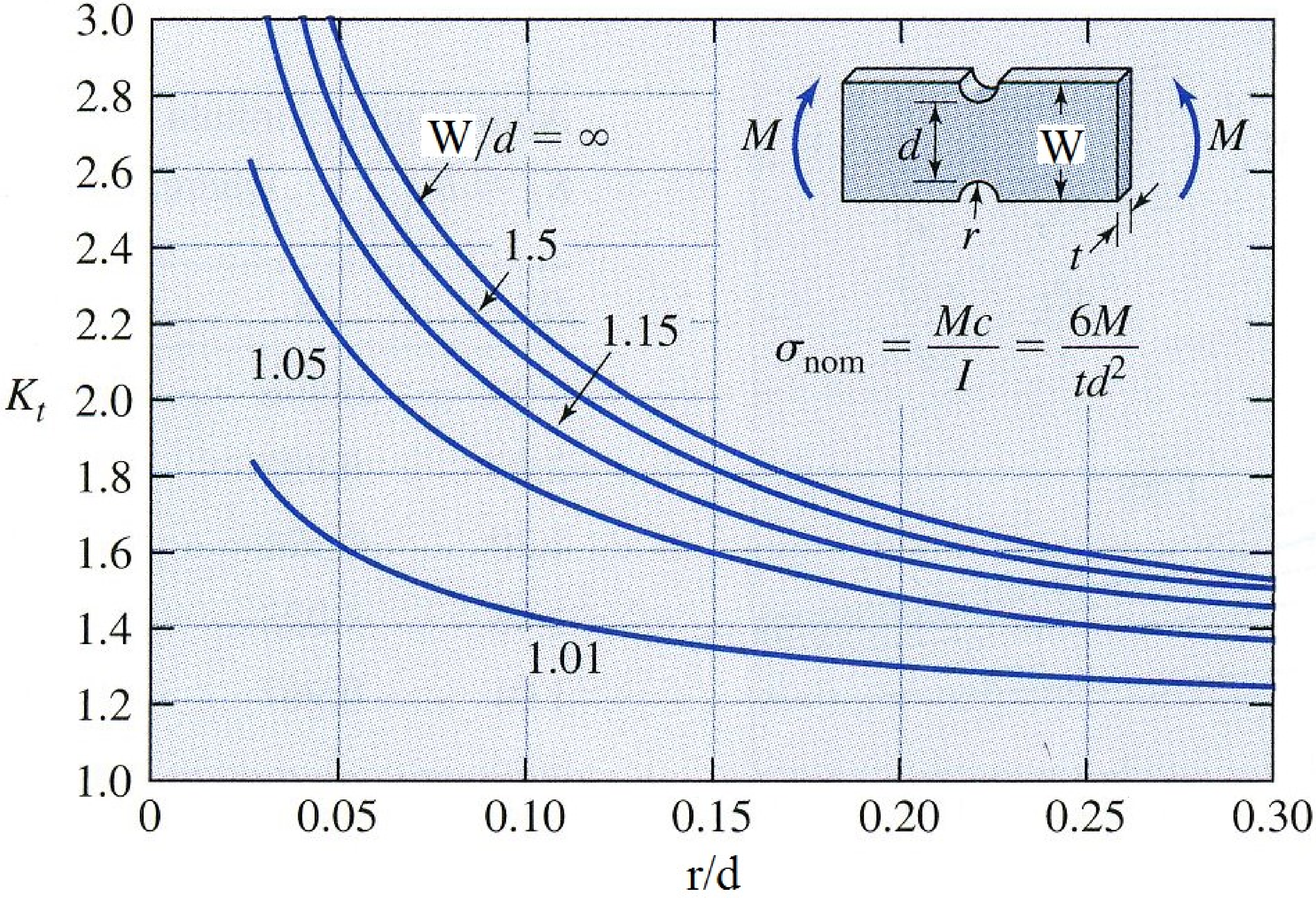
$$\boxed{\sigma_{\max} = K_t \sigma_{avg}}$$

Stress concentration factor *from charts*

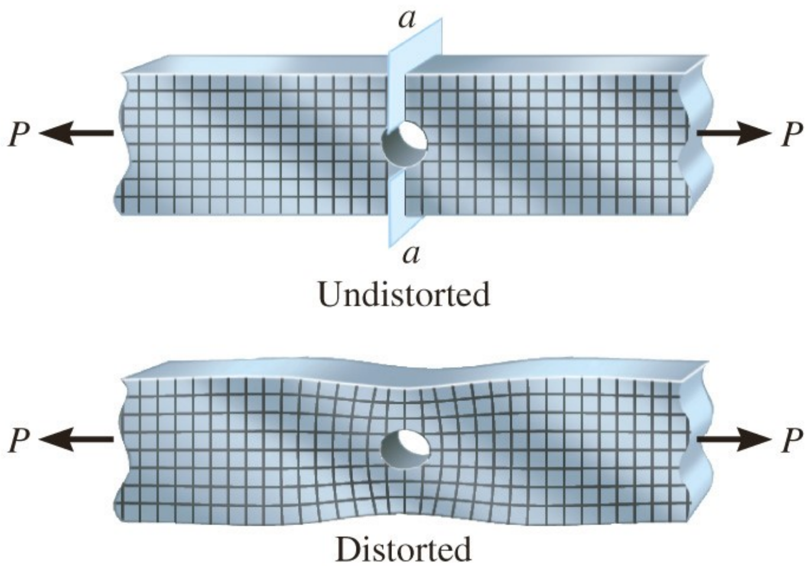
Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

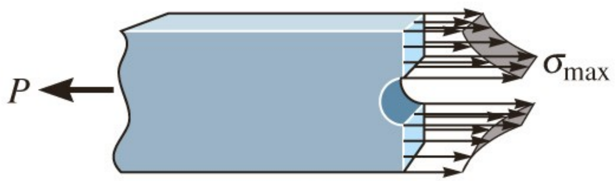




Stress concentration factors for a variety of geometries are found in Tables A-15 and A-16



(a)



Actual stress distribution
(b)

Average stress distribution
(c)

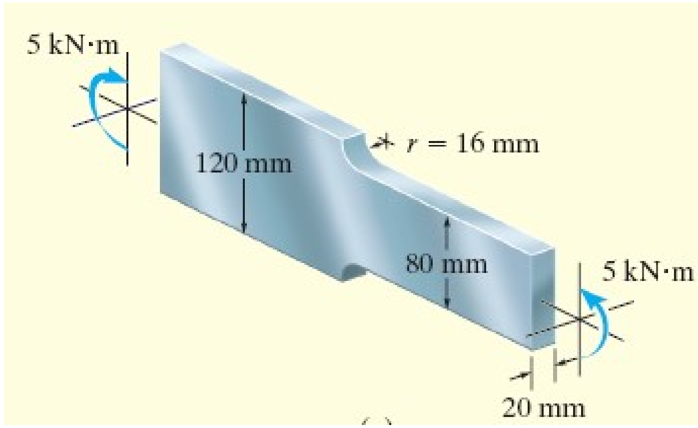
ExThe transition in the cross-sectional area of the steel bar is achieved using shoulder fillets. If the bar is subjected to a bending moment of 5 kNm, determine the maximum normal stress developed in the steel. The yield stress is σ_Y = 500 MPa.

From the geometry of the bar,

$$\frac{r}{d} = \frac{16}{80} = 0.2 \qquad \frac{D}{d} = \frac{120}{80} = 1.5$$

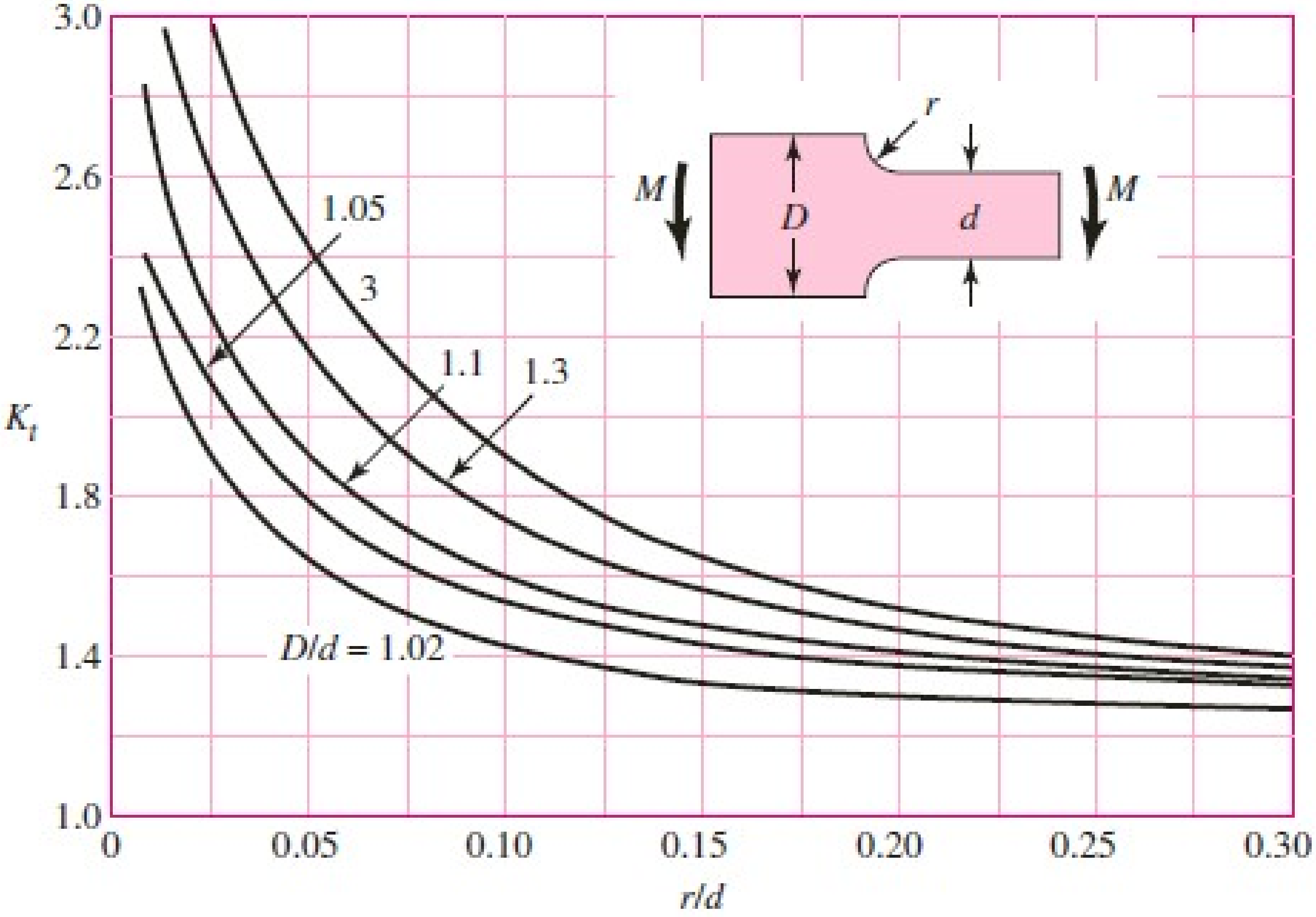
K is 1.45 and we have

$$\sigma_{\max} = K\frac{Mc}{I} = (1.45)\frac{(5)(0.04)}{\left[\frac{1}{12}(0.02)(0.08)^3\right]} = 340 \text{ MPa}$$



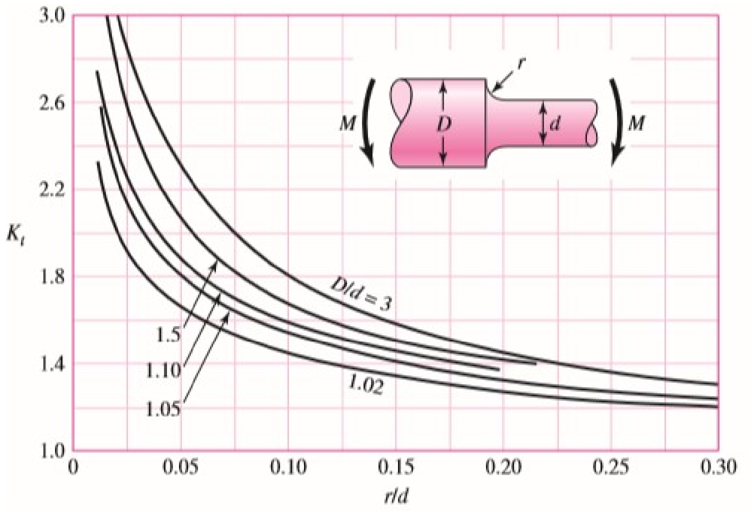
This result indicates that the steel remains elastic since the stress is below the yield stress (500 MPa).

Figure A–15–6

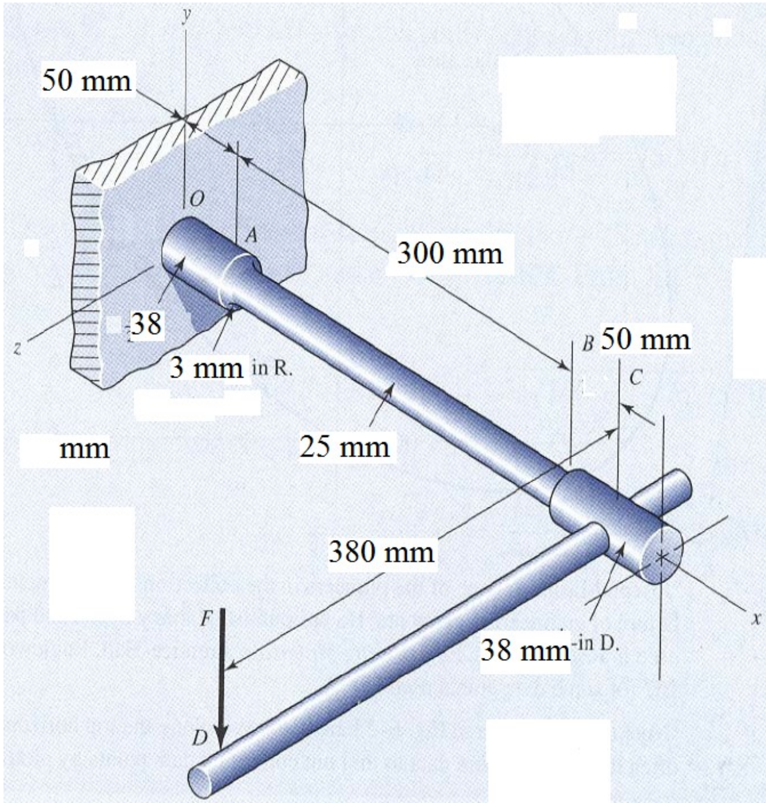


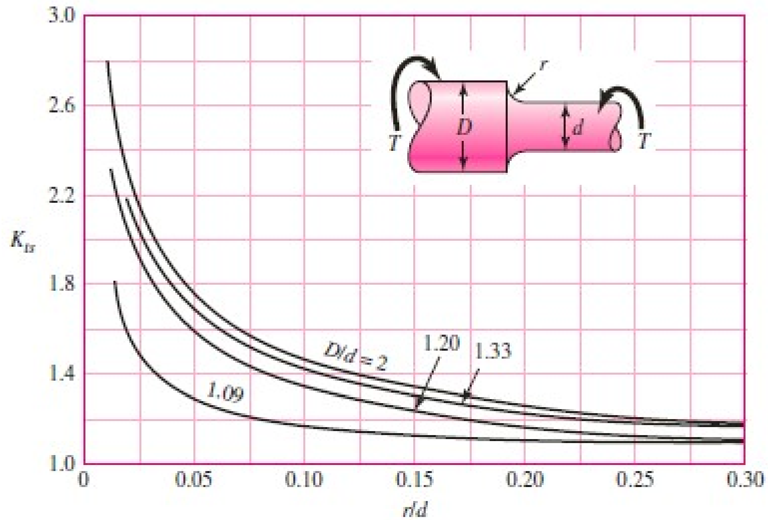
3.0
2.6
2.2
1.8
1.4
1.0
K_t
0
0.05
0.10
0.15
0.20
0.25
0.30
r/d
1.05
3
1.1
1.3
D/d = 1.02
r
M
D
d
M

$$\sigma_x = \frac{My}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi d^4}{}} = \frac{32 \times F \times 0.35}{\pi d^3} = 22.8\ \text{MPa}$$



r/d=3/25=0.12
D/d=38/25=1.52
Kt=1.7



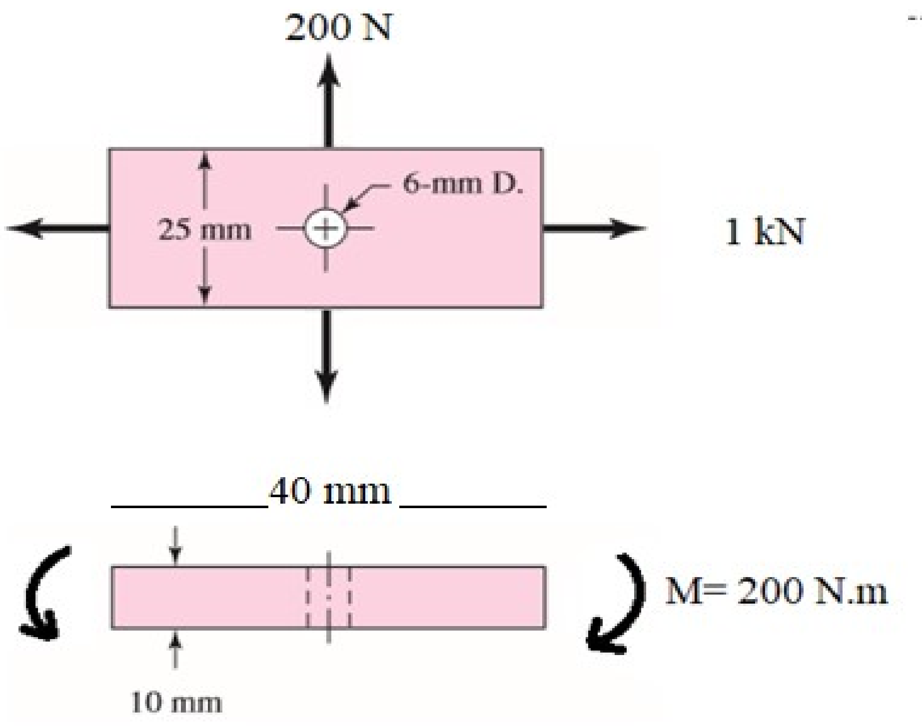


r/d=3/25=0.12
D/d=38/25=1.52
K_{ts}=1.35

$$\sigma_x = 1.7(22.8) = 38.76\ \text{MPa}$$

$$\tau_{zx_{max}} = K_{ts} \times \tau_{zx} = 1.35 \times 12.39 = 16.73\ \text{MPa}$$

Exam question . The steel bar shown in the figure is made of AISI 1006 cold- drawn steel and is loaded by a bending moment M= 200N.m, , and two axial loads of 1 kN and 200 N as shown in the figure.



a- For the critical stress element, determine the principal stresses and the maximum shear stress

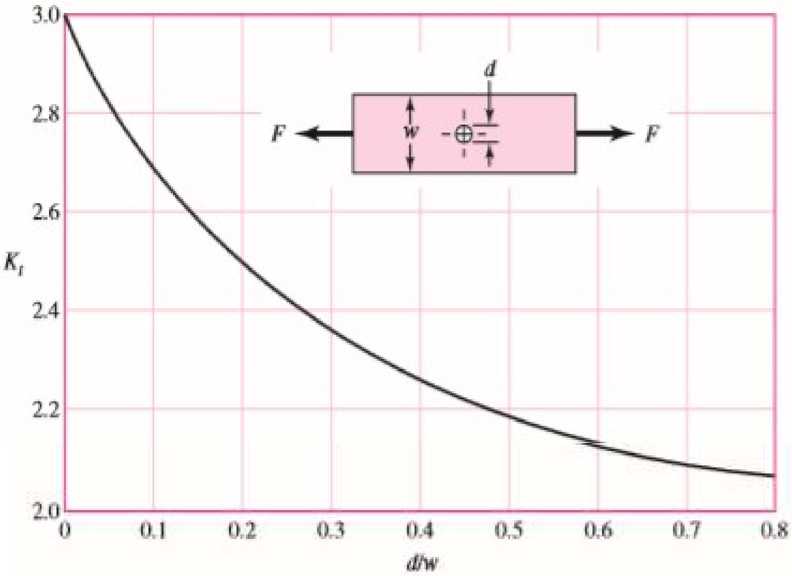
b- Compute the factor of safety, based upon the distortion energy theory, for the critical stress element of the member

AISI 1006 cold- drawn

UTS= 330 MPa, Sy=280 MPa

Figure A-15-1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



Normal stress due to Fx:

$$\sigma_x = \frac{F_x}{A} = \frac{1000}{(25-6)10} = 5.263\, MPa$$

d/w=6/25=0.24 , Kt=2.42

$$\sigma_{max} = K_t \sigma_x = 2.42\frac{F_x}{A} = 2.42 * 5.263 = 12.736 MPa$$

Normal stress due to Fy

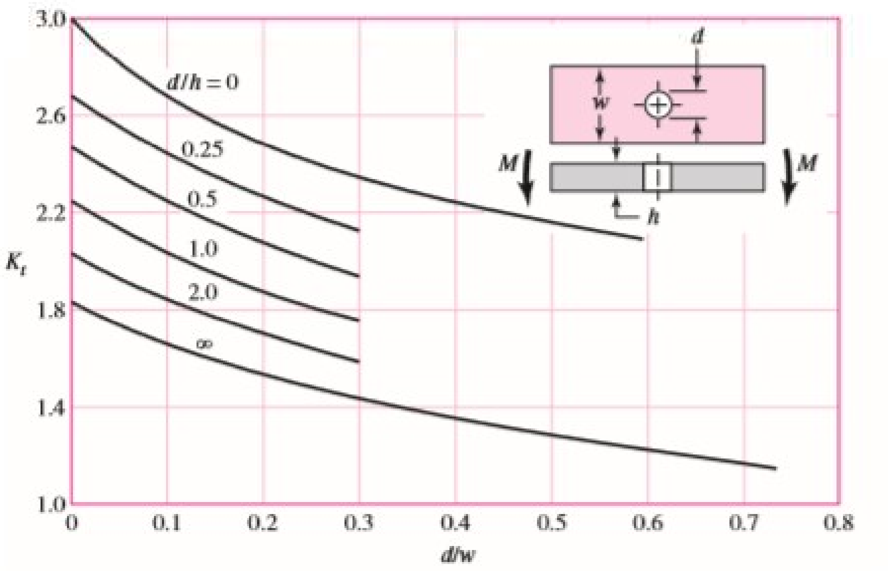
d/w=6/40=0.15 , Kt=2.58

$$\sigma_y = \frac{F_y}{A} = \frac{200}{(40-6)10} = 0.59MPa$$

$$\sigma_{max} = K_t \sigma_y = 2.58\frac{F_y}{A} = 2.58 * 0.59 = 1.52MPa$$

Figure A-15-2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w-d)h^3/12$.



$$\sigma_x = \frac{M_z \times \frac{d}{2}}{I} = \frac{200 \times 0.005}{(25-6)(10)^3} * 12 = 631.57 MPa$$

D/d=6/25=0.24 , d/h=6/10=0.6 Kt=2

$$\sigma_{max} = K_t \sigma_x = 2 \times 631.57 = 1263.14\, MPa$$

$$\sigma_{x\,total} = 12.736 + 1263.14 = 1275.88\, MPa$$

Principle stresses

$$\sigma_{1,2} = 1275.88, \quad 1.52$$

$$\tau_{max} = \frac{1275.88 - 1.52}{2} = 637.18$$

b-

$$Sy = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\sqrt{(1275.88 - 1.52)^2 + (1.52)^2 + (1275.88)^2} = 1275.12\, MPa$$

$$n = \frac{S_y}{\sigma_{eff}} = \frac{280}{1275.12} = 0.22$$