The use of Mohr circles to solve two-dimensional stress problems

the transformation of plane stress can be represented in graphical form, known as Mohr's circle the equation of Mohr's circle can be derived from the transformation equations for plane stress

$$\sigma_{x1x1} - \frac{\sigma_{xx} + \sigma_{y}}{2} = \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x1y1} = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

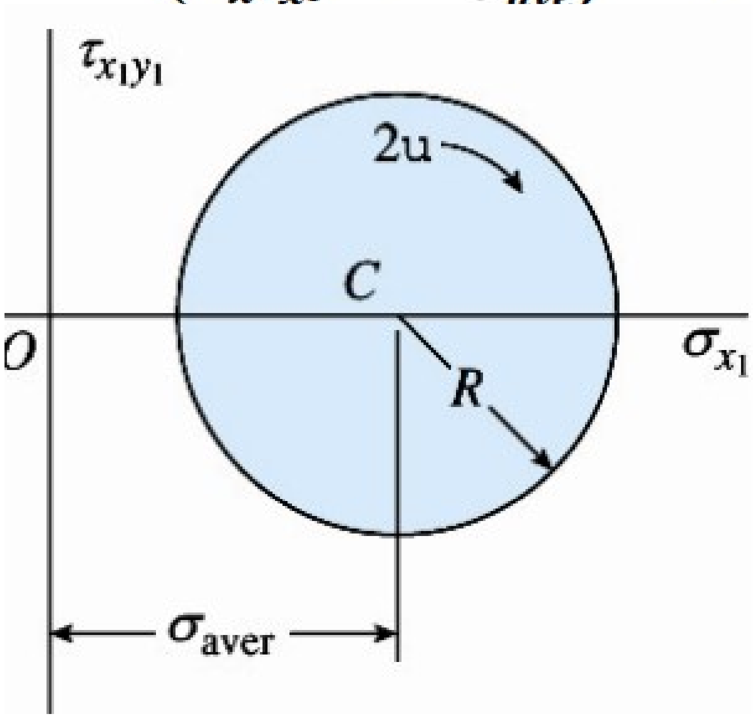
to eliminate the parameter 2θ, we square both sides of each equation and then add them together,

$$(\sigma_{x1x1} - \frac{\sigma_{xx} + \sigma_{yy}}{2})^2 + \tau_{x1y1}^2 = (\frac{\sigma_{xx} - \sigma_{yy}}{2})^2 + \tau_{xy}^2$$

let $\sigma_{ave} = \dfrac{\sigma_{xx} + \sigma_{yy}}{2}$ $\quad R^2 = (\dfrac{\sigma_{xx} - \sigma_{yy}}{2})^2 + \tau_{xy}^2$

The equation can be written

$$(\sigma_{x1\,x1} - \sigma_{ave})^2 + \tau_{x1y1}{}^2 = R^2$$

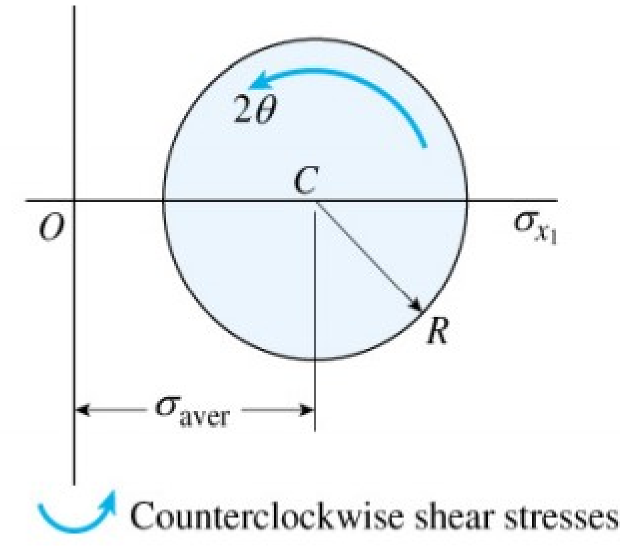


The Shear Stress will be considered positive when a pair of shear stress acting on opposite sides of the element produce a clockwise

Note that clockwise shear stresses are plotted upward and counterclockwise shear stresses are plotted downward.

We can plot the normal stress σx1 positive to the right and the shear stress τ x1y1 positive upwards, i.e. the angle 2 θ will be positive when clockwise counterclockwise shear stresses are plotted downward

Alternative sign conversion for shear stress

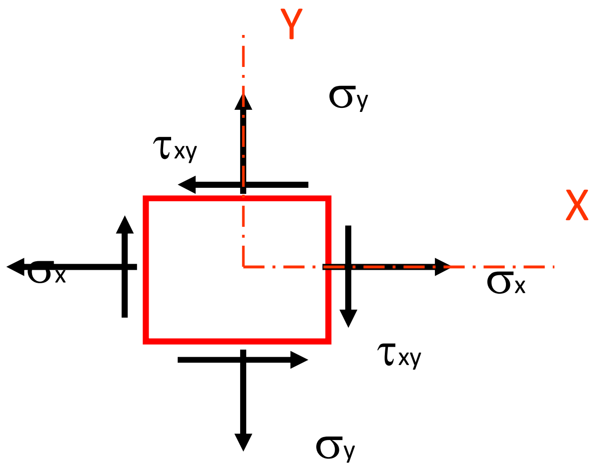


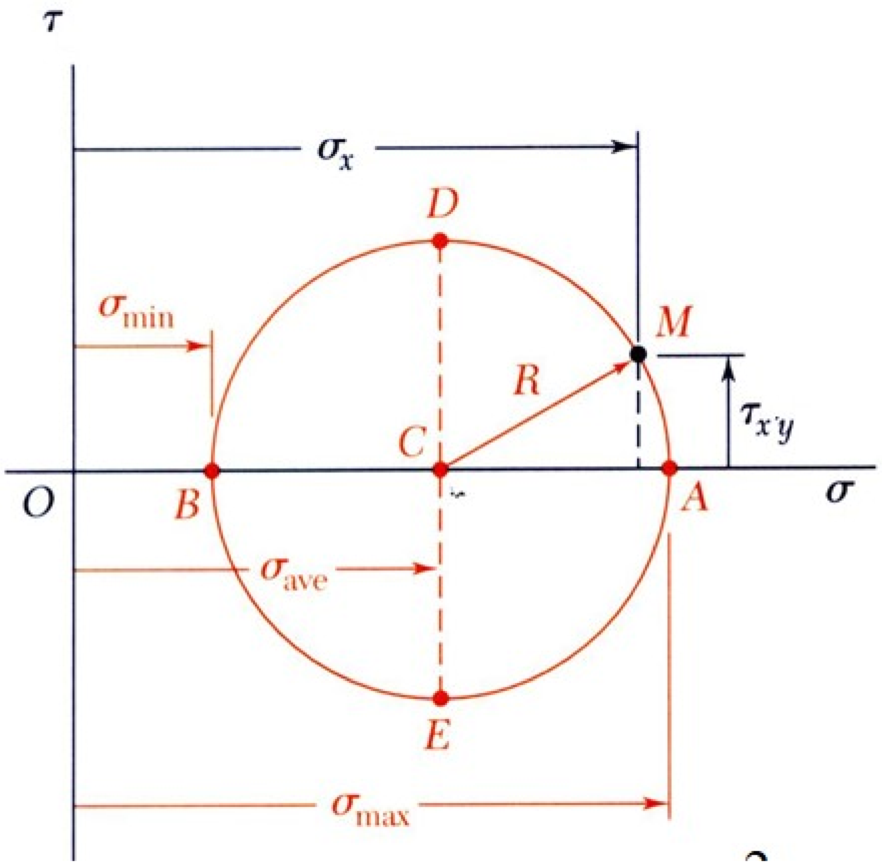
Construction of Mohr's Circle

- The x-axis represents normal stresses.
- The y-axis represents shear stresses.
-The circle is centered on the x-axis and intercepts it at the principal stresses (which are on the x-axis because there is no shear).
-The angle of rotation of the diameter (= 2θ) is twice the angle of rotation of the axes in real space (= θ).

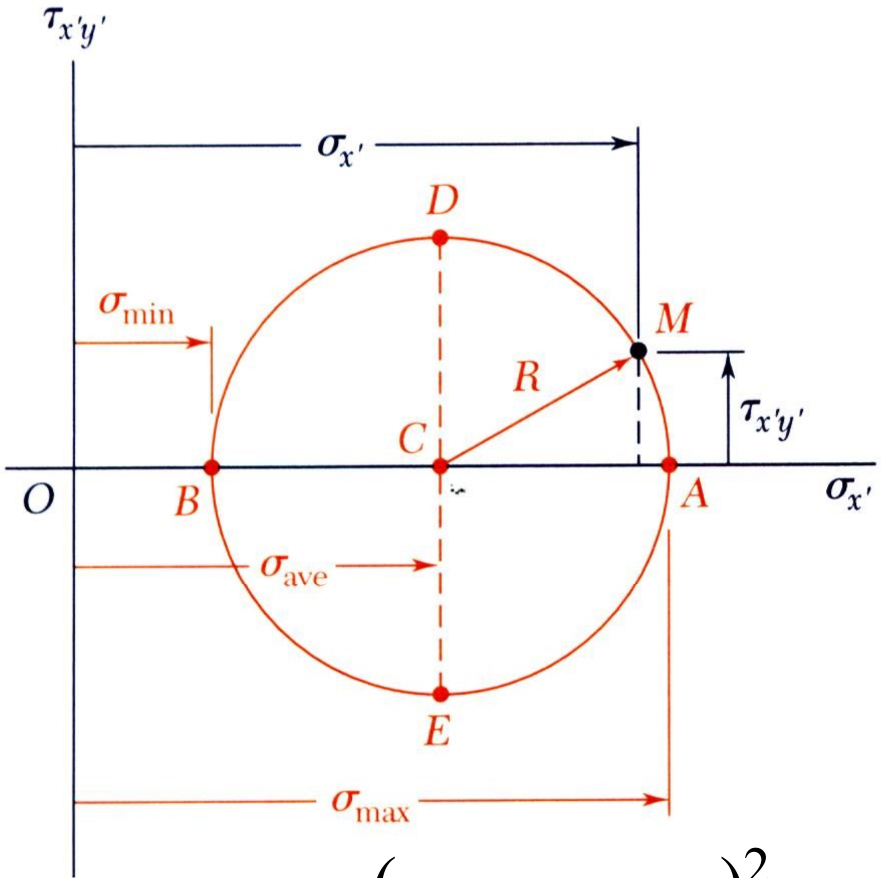
Uses of the Mohr circle

- To find principal axes and strains.
-To find the maximum shear stress.
-Knowing the stress tensor for one
-orientation, we can find the tensor for any othe





Mohr Circle for Plane Stress



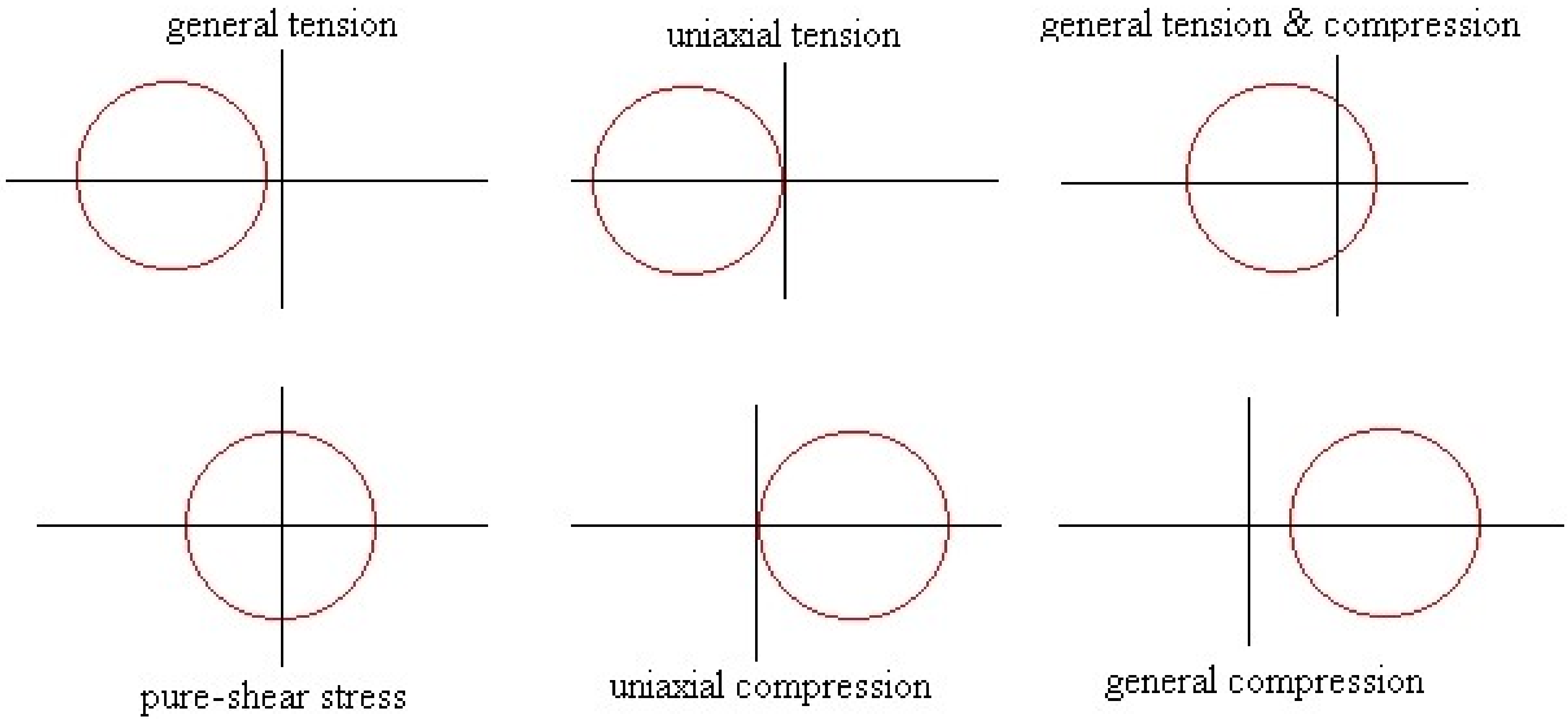
- The previous equations are combined to yield parametric equations for a circle,

$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

where

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

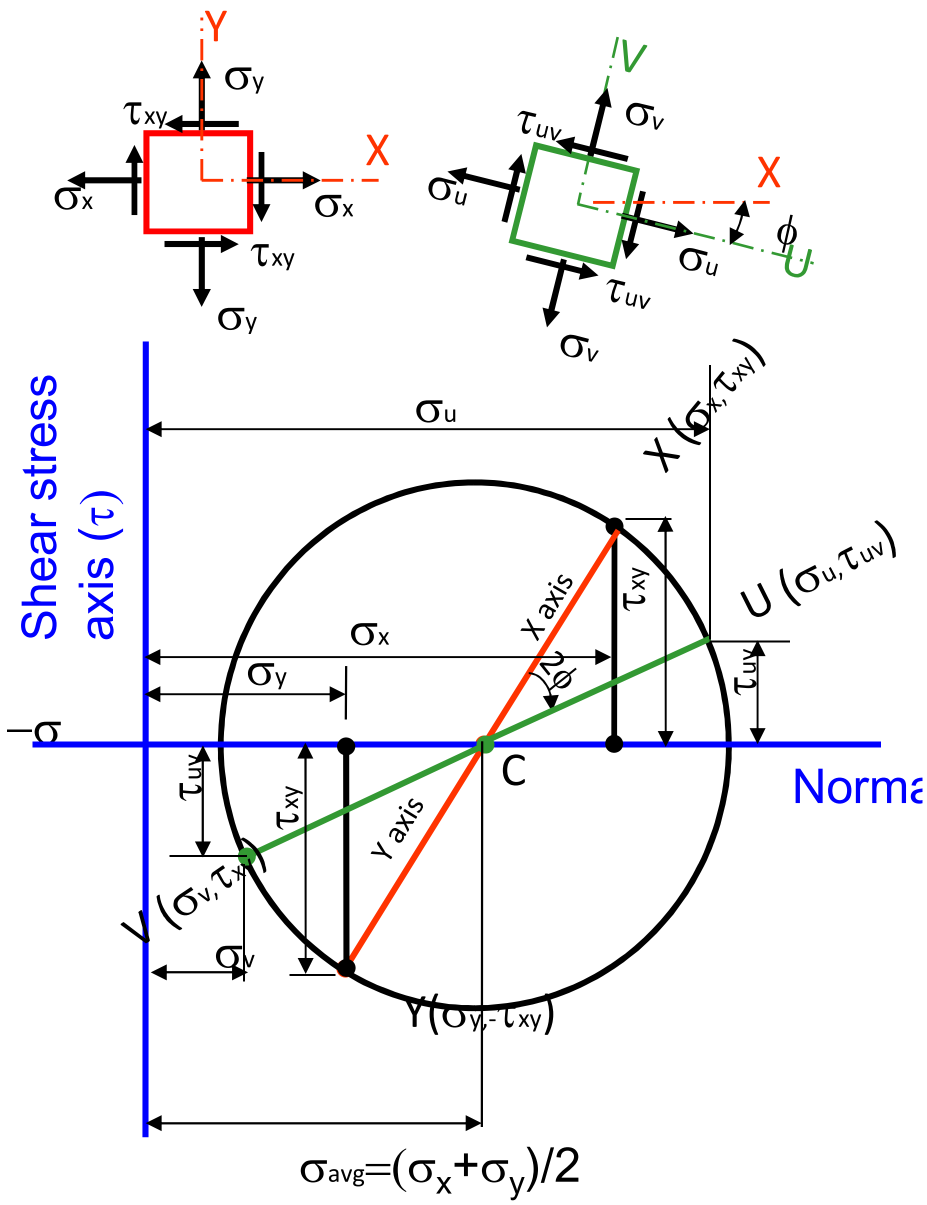
type of stress



CONSTRUCTION OF MOHR'S CIRCLE FOR A GIVEN STRESS ELEMENT

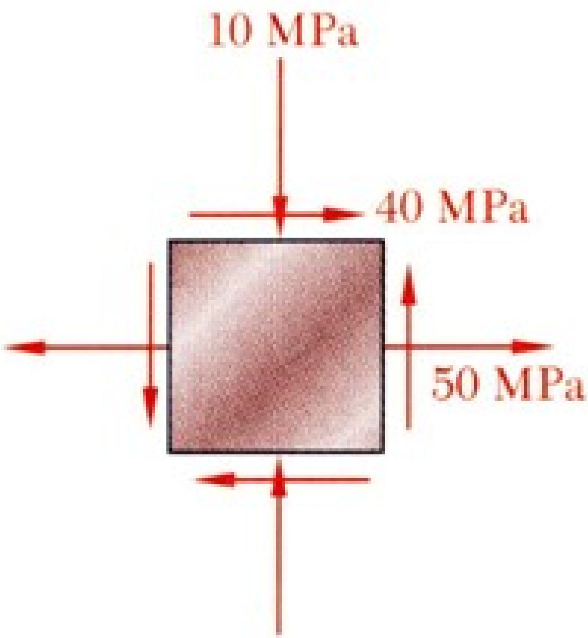
The axis system of Mohr circle is $\sigma-\tau$ axis .١

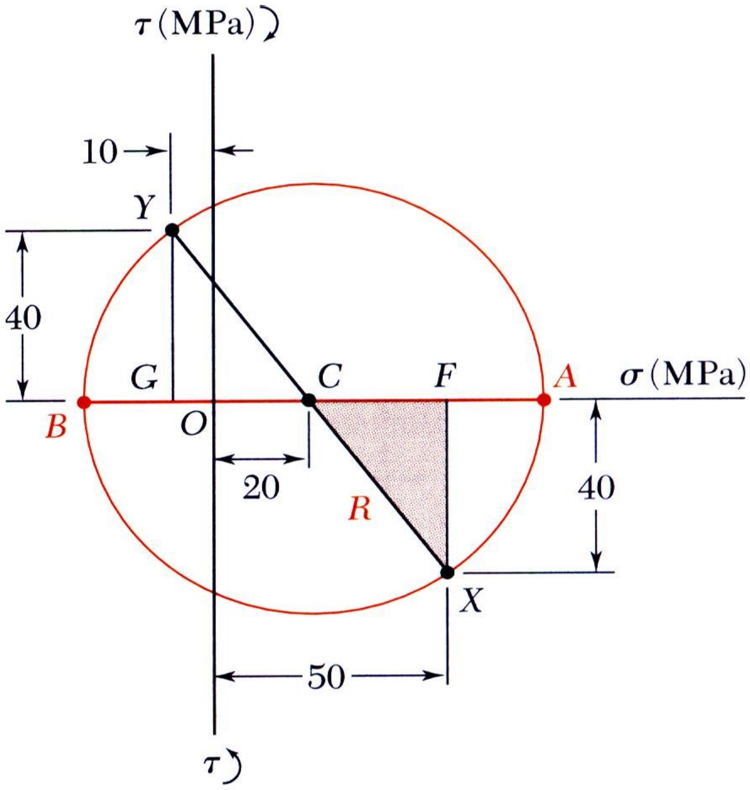
2. Mark the point X with a coordinate σ_x and τ_{xy} along the $\sigma-\tau$ coordinate system.
3. Mark another point Y with the coordinate σ_y and $-\tau_{xy}$
4. Join the XY line. Let at point C, XY intersects the horizontal axis. The point C denotes the average normal stress. The line CX denotes X axis and line CY denotes Y axis.
5. Note CX and CY are making 180^0 angle with each other, whereas in reality the X and Y axes are at an angle 90^0.
6. **RULE: ALL ANGLES IN MOHR'S CIRCLE IS TWICE THE REAL ANGLE.**
7. Use C as the center, and draw a circle with XY as the diameter.
8. To find stress in a direction U-V, which is ϕ angle CW from X-Y axis, draw a line UV through C at an angle 2ϕ CW from XY line.
9. The coordinate values of U & V denote the normal and shear stress in UV direction.



EX: For the stress matrix shown, (a) construct Mohr's circle, determine (b) the principal planes, (c) the principal stresses, (d) the maximum shearing stress and the corresponding normal stress.

$$\sigma = \begin{vmatrix} \sigma_{xx} & \tau xy & \tau xz \\ \tau yx & \sigma yy & \tau yz \\ \tau zx & \tau zy & \sigma zz \end{vmatrix} = \begin{vmatrix} 50 & -40 & 0 \\ -40 & -10 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$





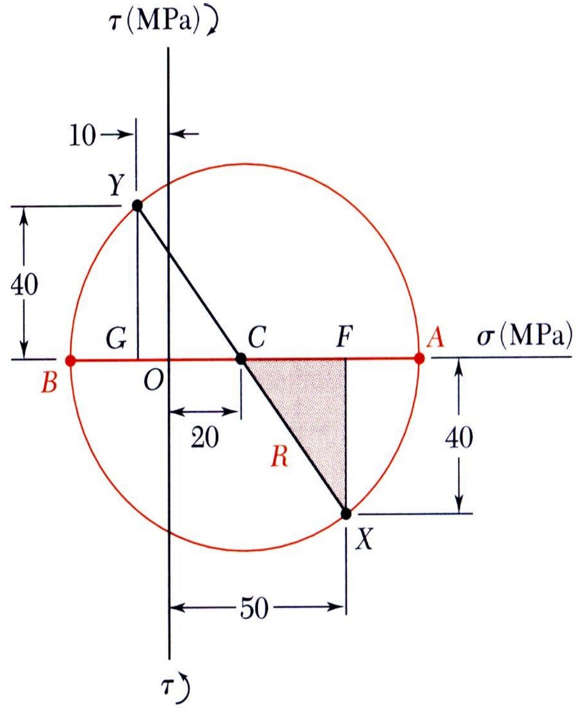
SOLUTION:

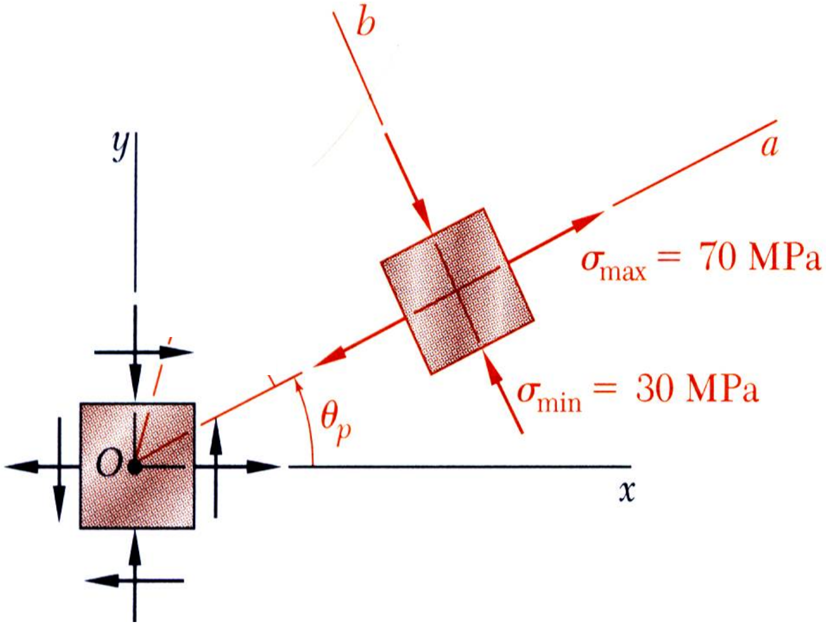
- Construction of Mohr's circle

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(50) + (-10)}{2} = 20\,\text{MPa}$$

$$CF = 50 - 20 = 30\,\text{MPa} \qquad FX = 40\,\text{MPa}$$

$$R = CX = \sqrt{(30)^2 + (40)^2} = 50\,\text{MPa}$$





- Principal planes and stresses

$$\sigma_{\max} = OA = OC + CA = 20 + 50$$

$$\boxed{\sigma_{\max} = 70\text{MPa}}$$

$$\sigma_{\min} = OB = OC - BC = 20 - 50$$

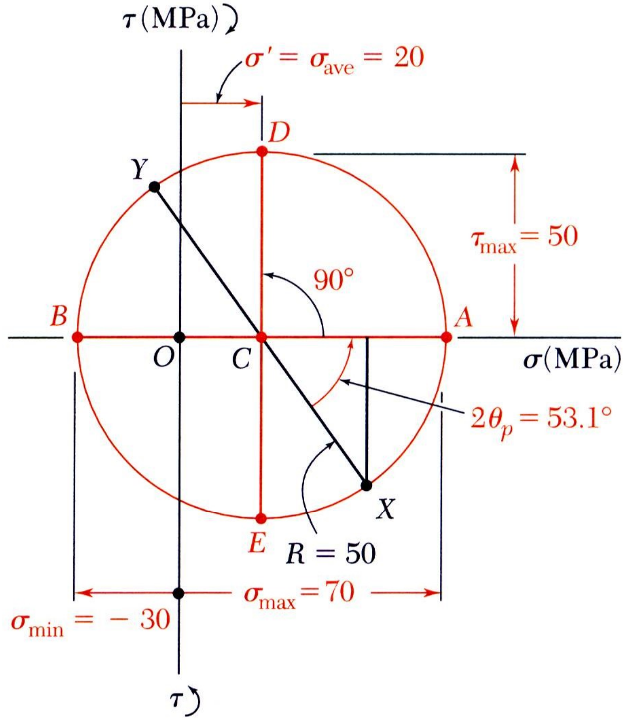
$$\boxed{\sigma_{\min} = -30\text{MPa}}$$

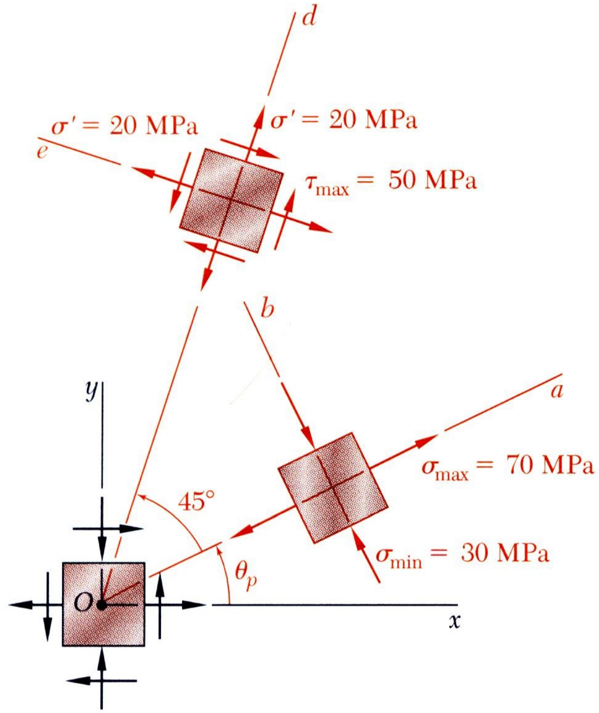
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{40}{30}$$

$$2\theta_p = 53.1°$$

$$\boxed{\theta_p = 26.6°}$$



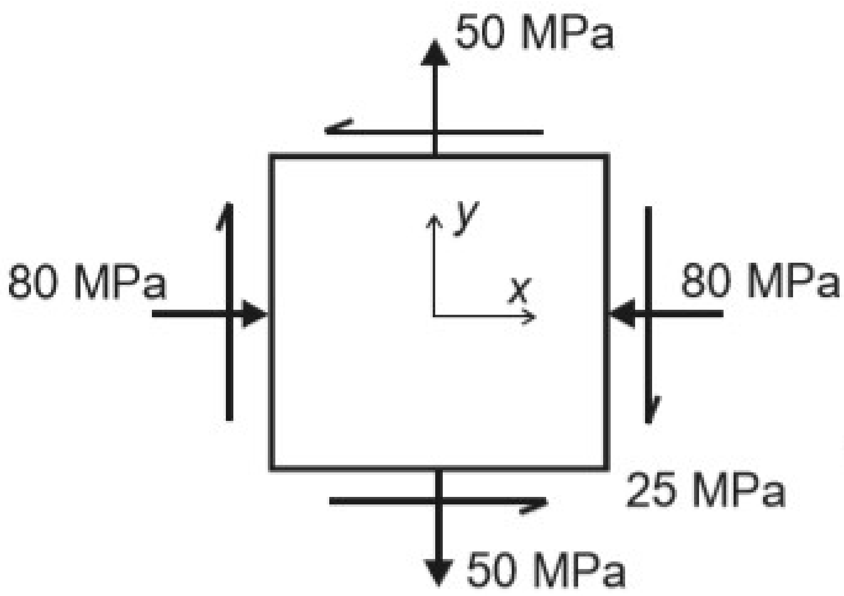


Maximum shear stress •

$$\theta_s = \theta_p + 45^\circ \qquad \tau_{\max} = R \qquad \sigma' = \sigma_{ave}$$

$$\boxed{\theta_s = 71.6^\circ} \qquad \boxed{\tau_{\max} = 50\ \text{MPa}} \qquad \boxed{\sigma' = 20\ \text{MPa}}$$

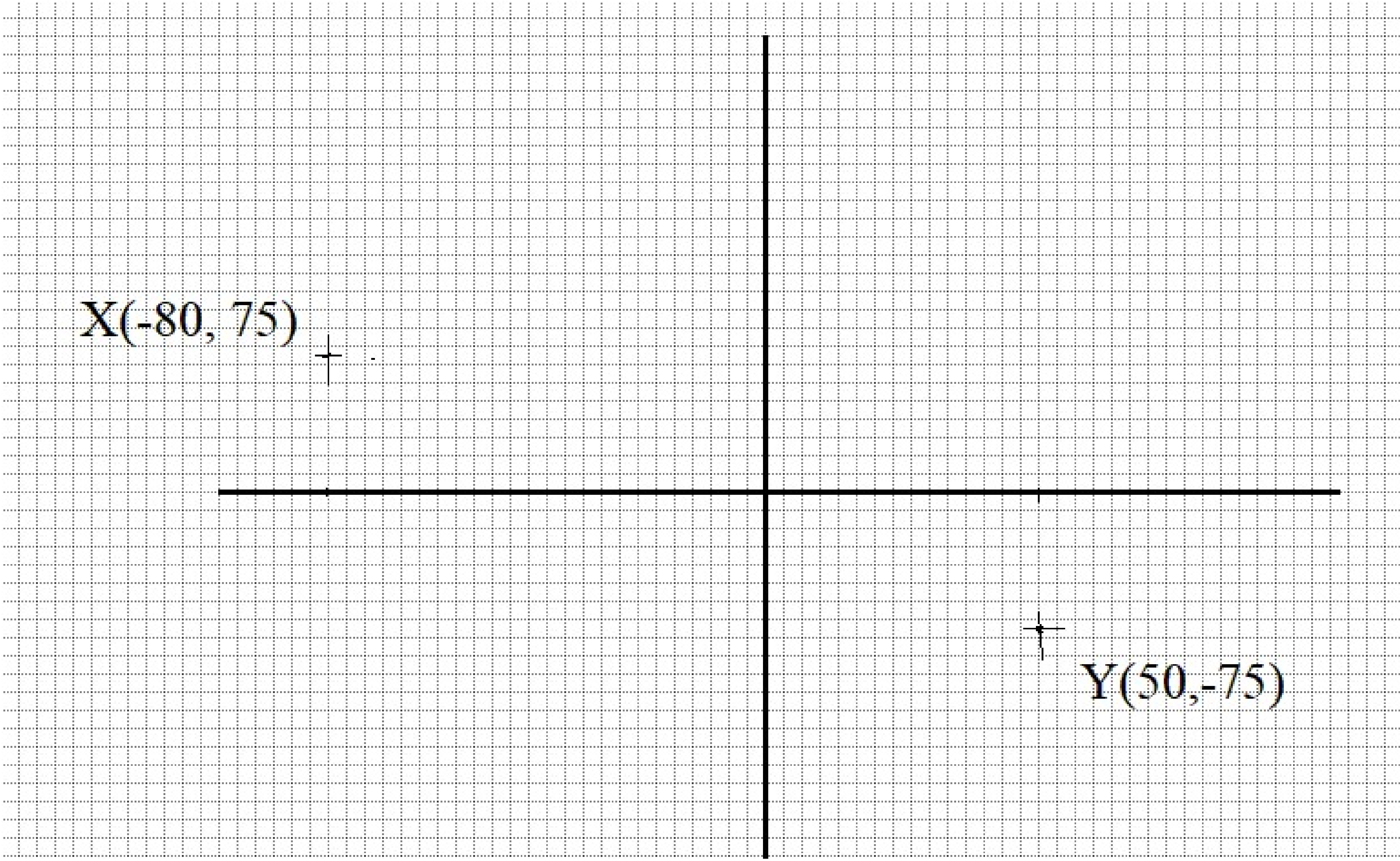
Example: The state of plane stress at a point is represented by the stress element below. Find the stresses on an element inclined at 30°clockwise

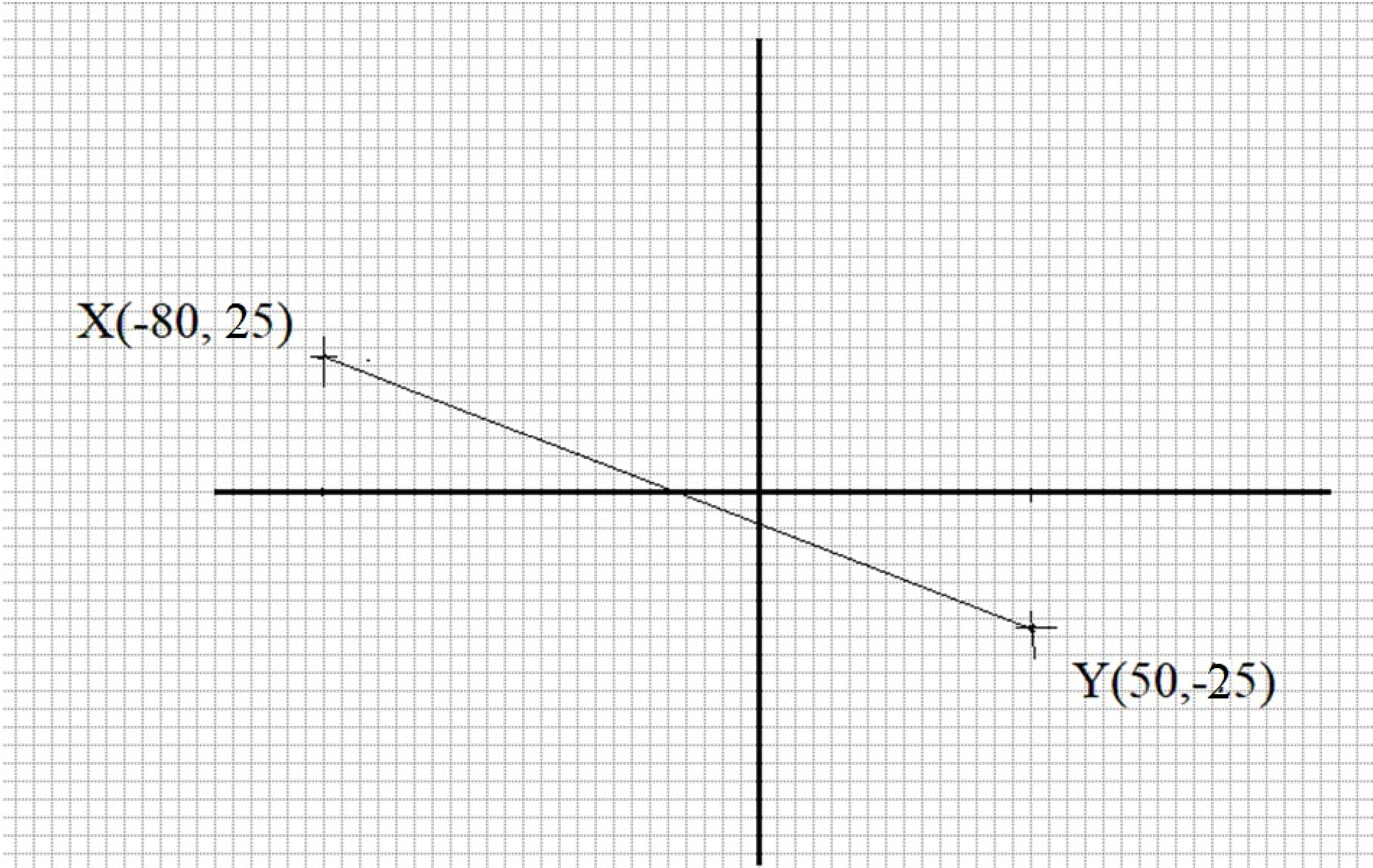


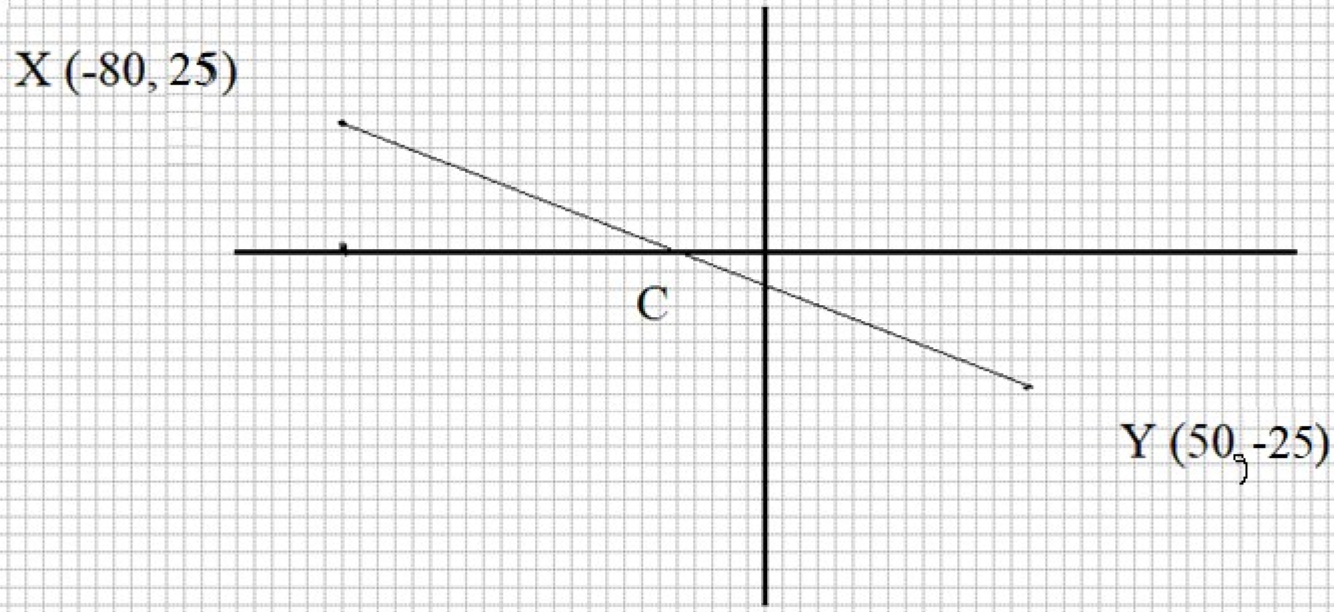
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\phi - \tau_{xy} \sin 2\phi$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi$$


X(-80, 75)
Y(50,-75)


X(-80, 25)
Y(50,-25)



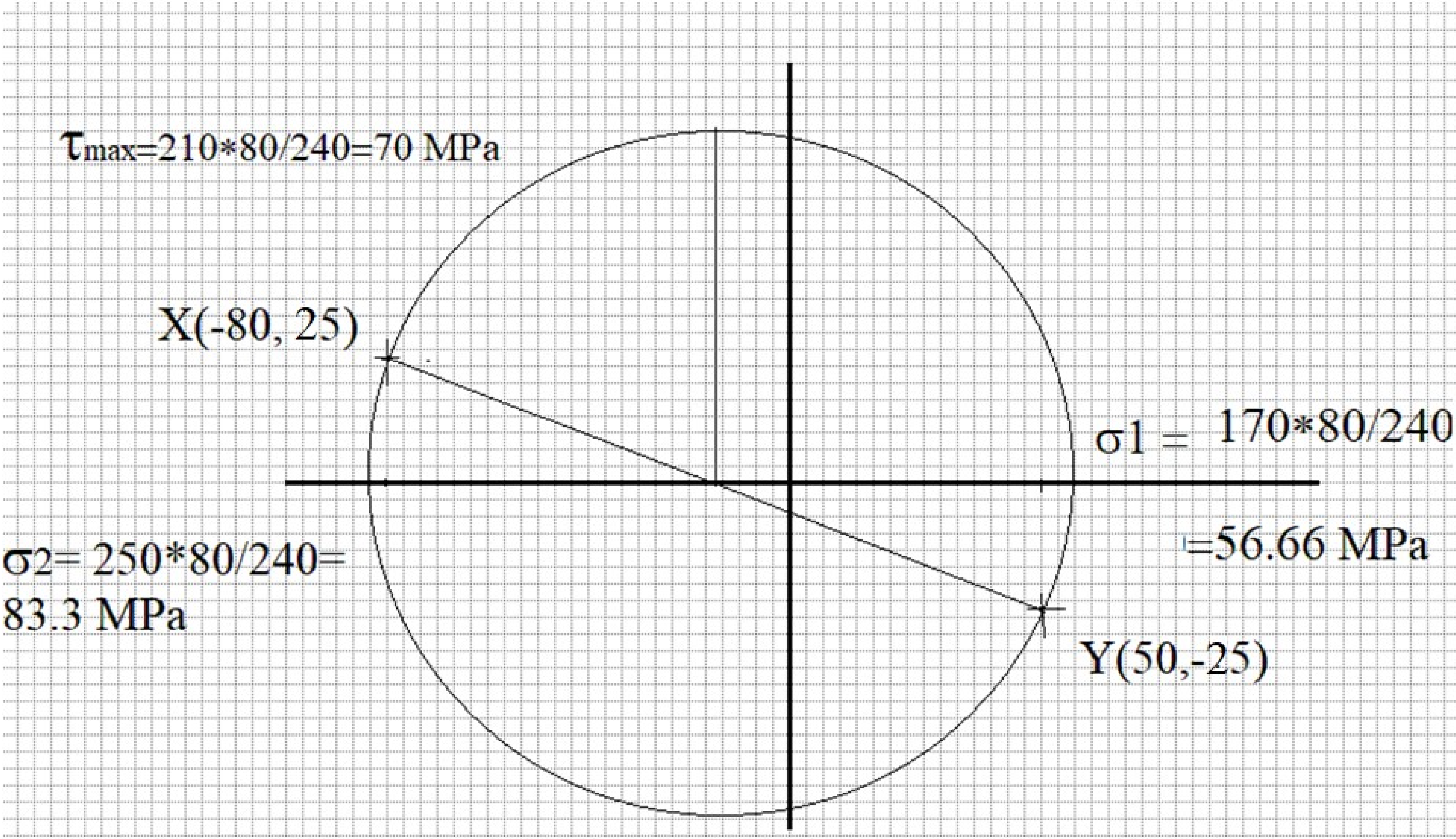
from MPa to length
Units 240/80

$\sigma = -80$ MPa = -240 units

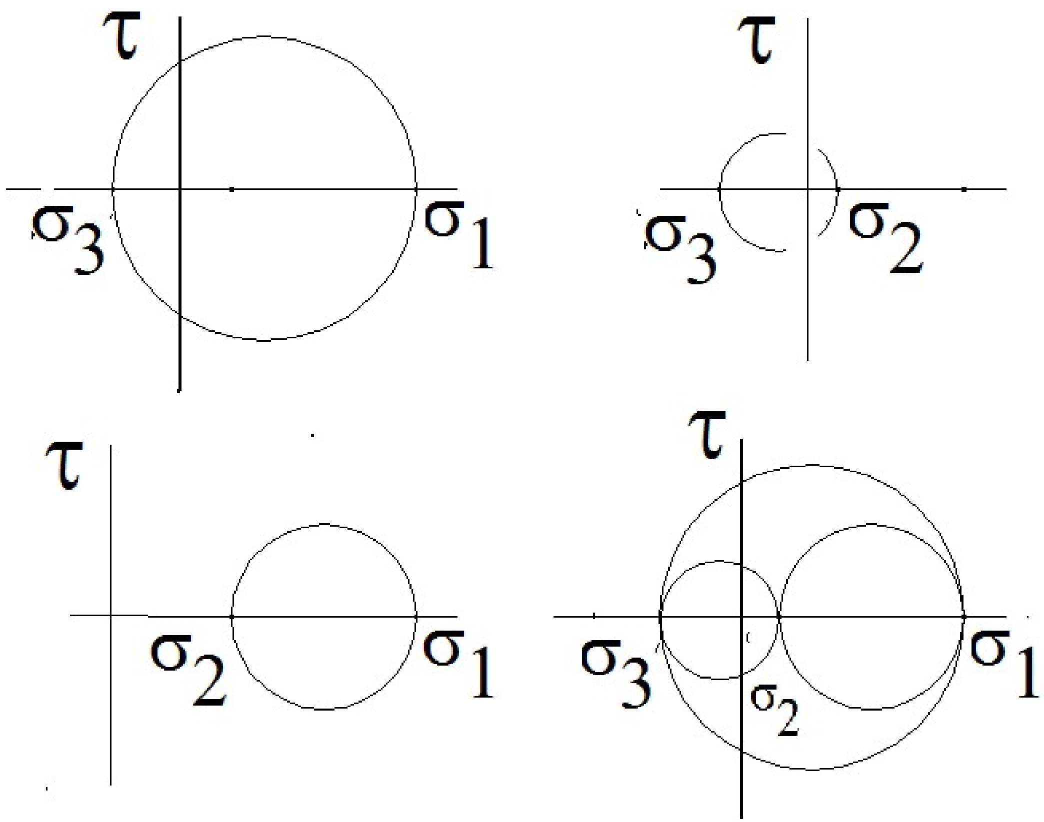
$\sigma = 50$ MPa = 150 units

$\tau = 25$ MPa =75 units

$\sigma = -80$ MPa = -240 units


τmax=210*80/240=70 MPa
X(-80, 25)
σ1 = 170*80/240
=56.66 MPa
σ2= 250*80/240=
83.3 MPa
Y(50,-25)

Mohr's Circle for Triaxial Stress



the principal normal stresses are ordered so

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$