Lecture 5 Stress Components and stress states

operational stress system

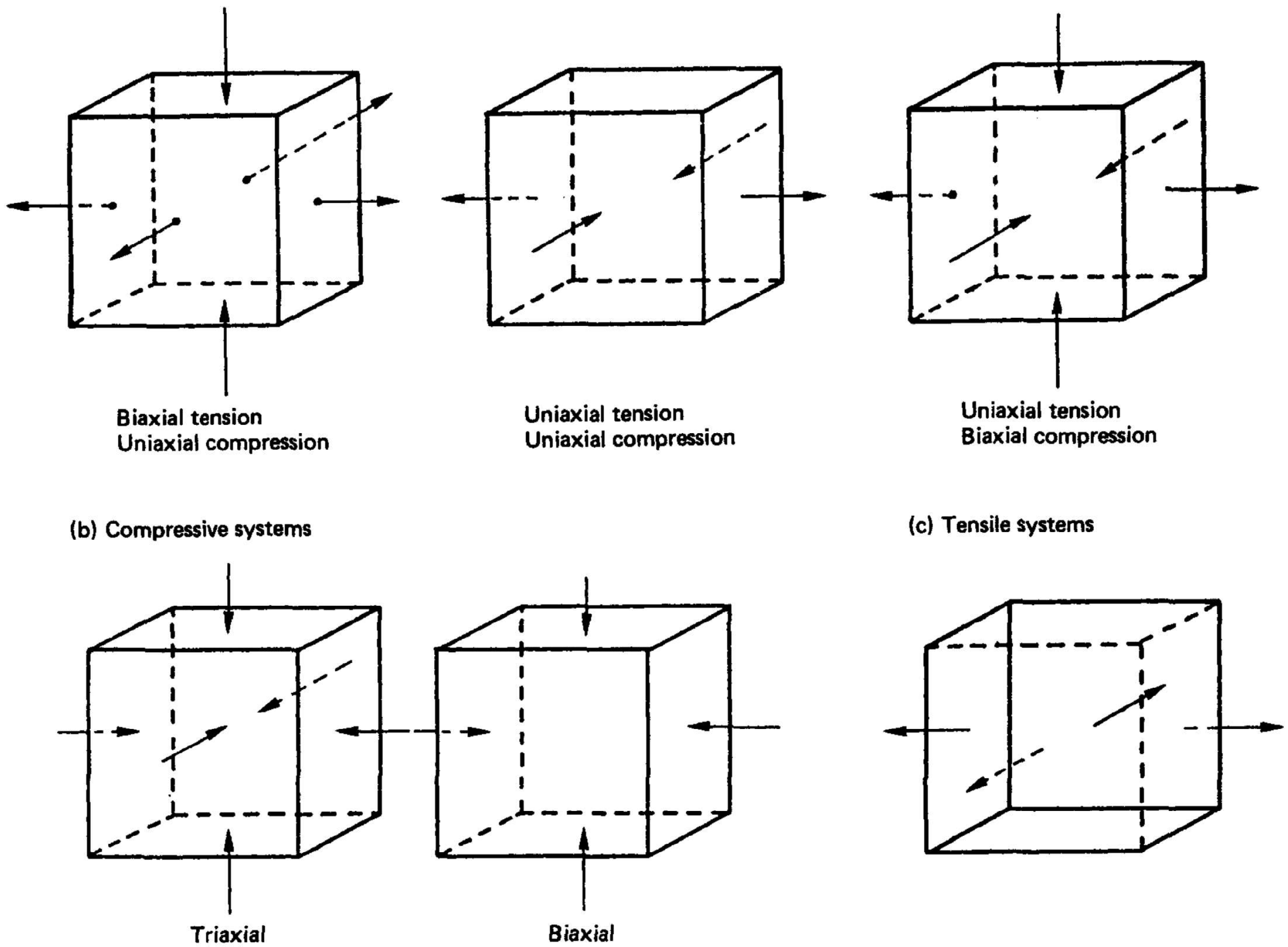
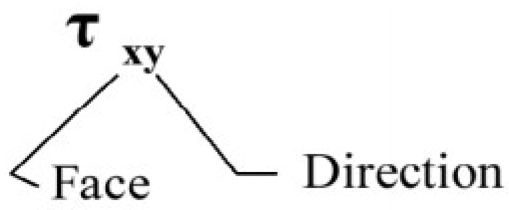


Figure 4.2 Process stress classification system

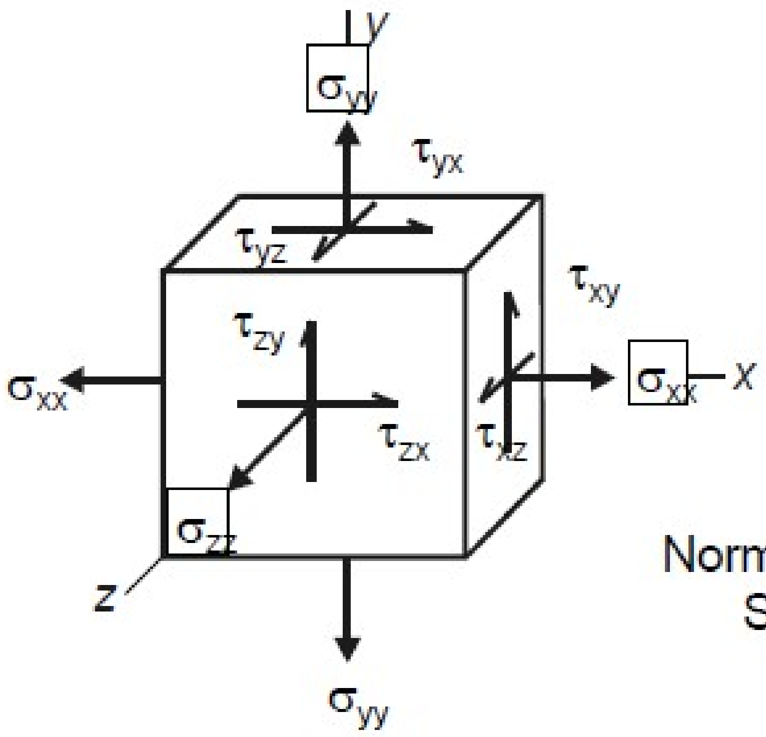
Representing stress as a tensor

The normal stress and shear stress are described in terms of cartesian coordinate by second order tensor

rank 2 tensor is a matrix



The normal and shear stresses on a stress element in 3D can be assembled into a 3x3 matrix known as the **stress tensor**.

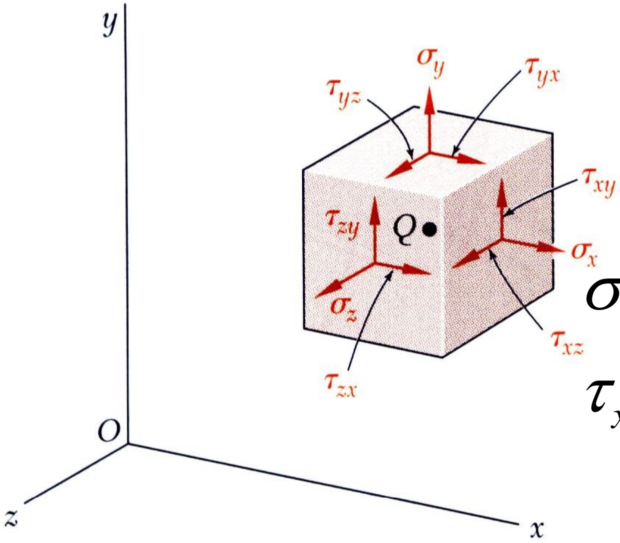


$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Normal stresses on the diagonal
Shear stresses off diagaonal

$\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$

Stress Components

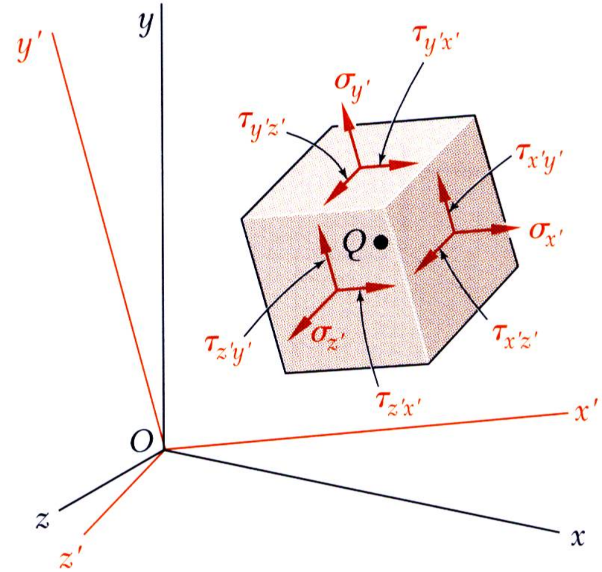


- General state of stress at a point is represented by 6 components,

$\sigma_x, \sigma_y, \sigma_z$ normal stresses

$\tau_{xy}, \tau_{yz}, \tau_{zx}$ shearing stresses

For equilibrium cross shears are equal



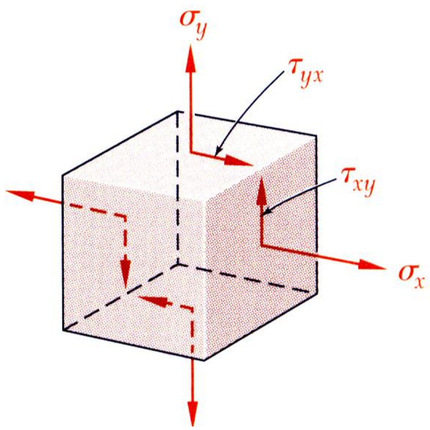
$$\tau_{xy} = \tau_{yx},\ \tau_{yz} = \tau_{zy},\ \tau_{zx} = \tau_{xz}$$

$$6\ components,$$

- If axes are rotated by ϕ how the components of stress are transformed

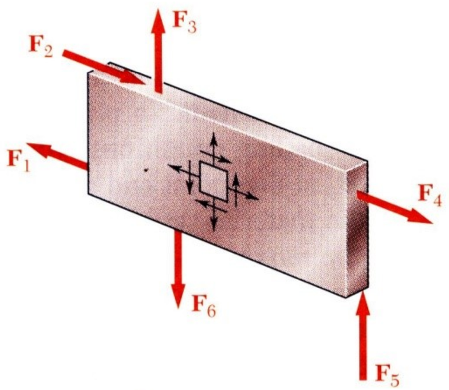
- New axes x', y', z'

2-D stresses, are called plane stress

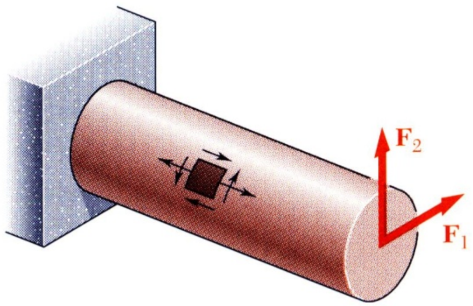


- *Plane Stress* - state of stress in which two faces of the cubic element are free of stress. For the illustrated example, the state of stress is defined by

$$\sigma_x, \sigma_y, \tau_{xy} \quad \text{and} \quad \sigma_z = \tau_{zx} = \tau_{zy} = 0.$$

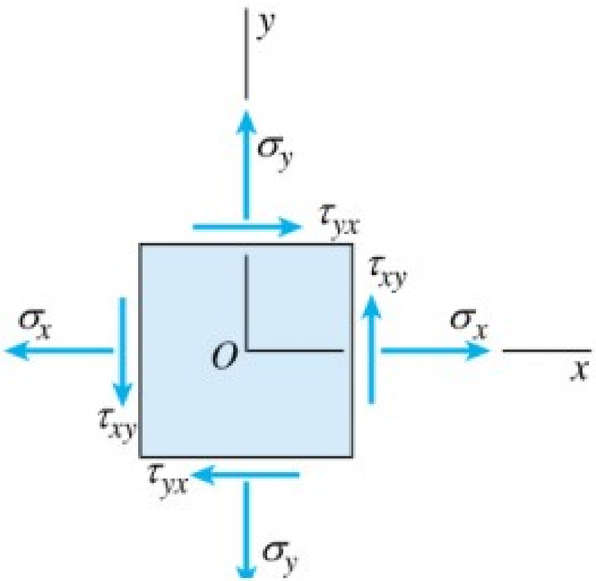


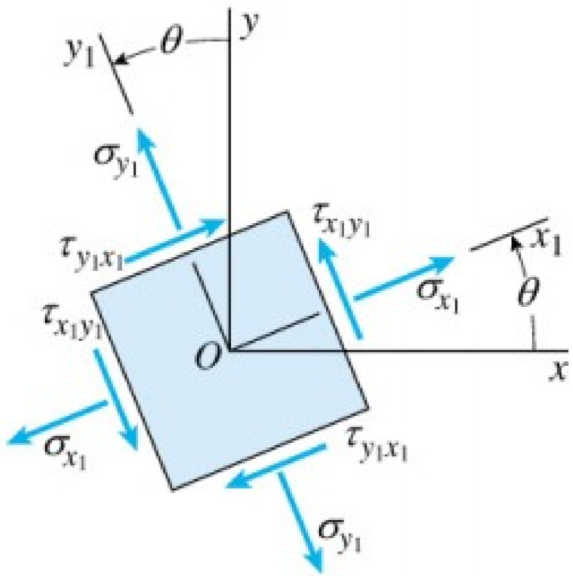
- State of plane stress occurs in a thin plate subjected to forces acting in the midplane of the plate.

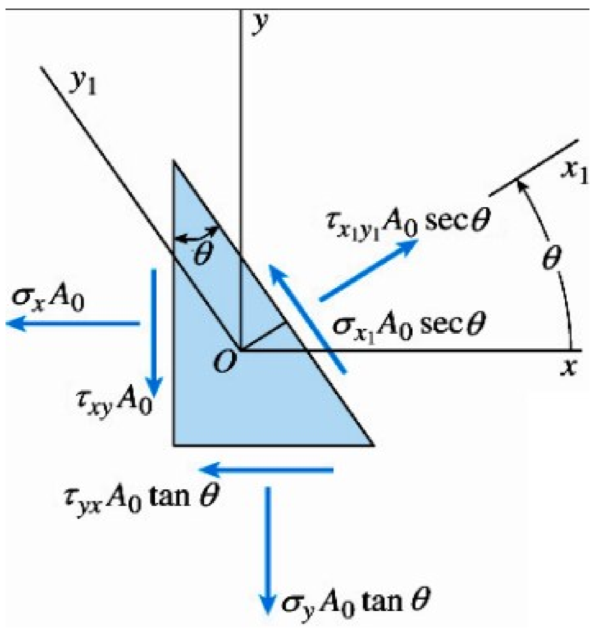


- State of plane stress also occurs on the free surface of a structural element or machine component, i.e., at any point of the surface not subjected to an external force.

Stresses on Inclined Sections

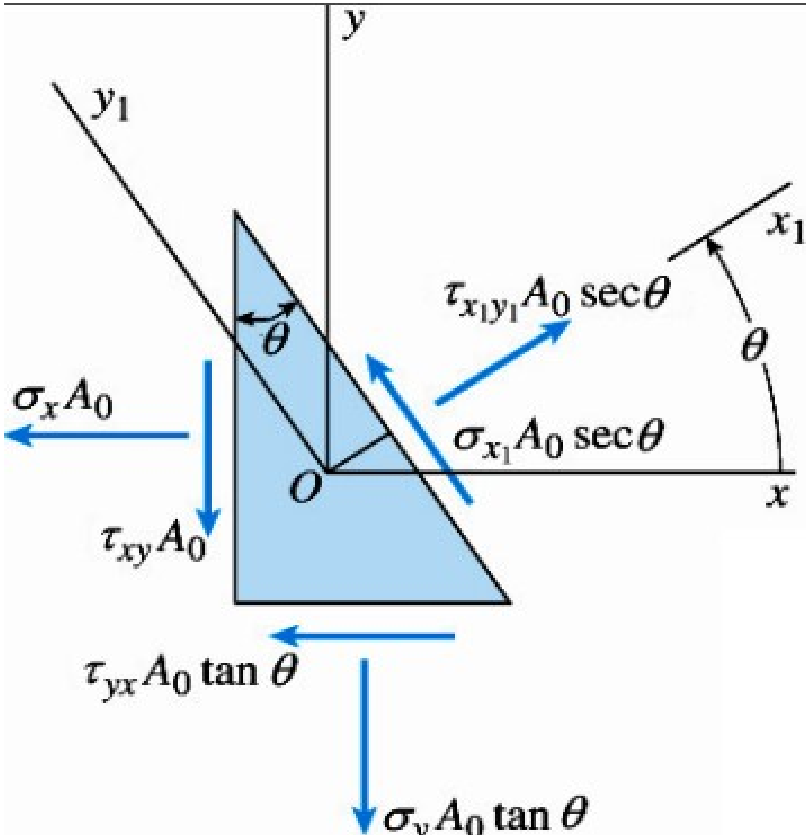






Transformation of Plane Stress

- Consider the conditions for equilibrium of a prismatic element with faces perpendicular to the *x*,1 and *y1*, axes.



force equilibrium in x1-direction

$$\sum F_{x1x1} = 0$$

$$\sigma_{x1x1} A = (\sigma_x \cos\phi)(A\cos\phi) + (\sigma_y \sin\phi)(A\sin\phi) + (\tau_{xy} \sin\phi)(A\cos\phi) + (\tau_{yx} \cos\phi)(A\sin\phi)$$

$$\sigma_{x1x1} = (\sigma_x \cos^2\phi) + (\sigma_y \sin^2\phi) + (\tau_{xy} \sin\phi\cos\phi) + (\tau_{yx} \sin\phi\cos\phi)$$

$$\sigma_{x_1} A_0 \sec\theta - \sigma_x A_0 \cos\theta - \tau_{xy} A_0 \sin\theta - \sigma_y A_0 \tan\theta \sin\theta - \tau_{yx} A_0 \tan\theta \operatorname{con}\theta = 0$$

Transformation Equations for Plane Stress

$$\sigma_{x1x1} = \frac{\sigma_{xx} + \sigma_{y}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y1y1} = \frac{\sigma_{xx} + \sigma_{y}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x1y1} = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal Stresses and Maximum Shear Stresses

As we change the angle θ there will be maximum and minimum normal and shear stresses that are needed for design purposes

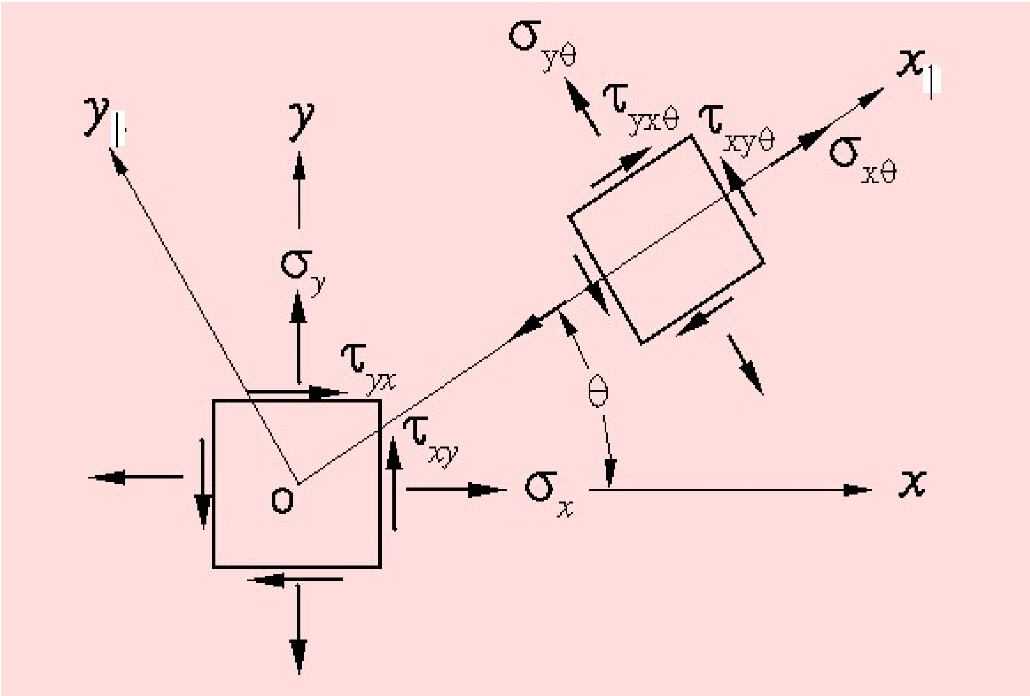
The maximum and minimum normal stresses are known as the principal stresses. These stresses are found by taking the derivative of $\sigma x1$ with respect to θ and setting equal to zero.

A property of a symmetric tensor is that there exists an orthogonal set of axes 1, 2 and 3 (called principal axes) with respect to which the tensor elements are all zero except for those in the diagonal differentiating the stress transformation formulae with respect to θ,

$$\frac{d\sigma_{x1x1}}{d\theta} = -\sin 2\theta(\sigma_{x1x1} - \sigma_{y1y1}) + 2\cos 2\theta \tau_{x1y1}$$

$$\frac{d\sigma_{y1y1}}{d\theta} = +\sin 2\theta(\sigma_{x1x1} - \sigma_{y1y1}) - 2\cos 2\theta \tau_{x1y1}$$

$$\frac{d\tau_{x1y1}}{d\theta} = -\cos 2\theta(\sigma_{x1x1} - \sigma_{y1y1}) - 2\sin 2\theta \tau_{x1y1}$$

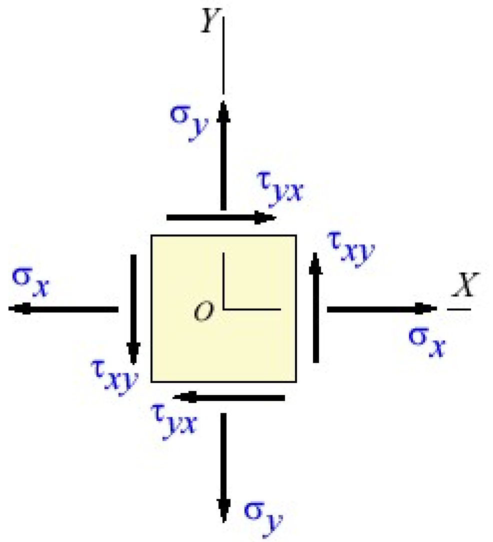


To maximize (or minimize) the stress, the derivative of $\sigma_{x'}$ with respective to the rotation angle θ is equated to zero. This gives

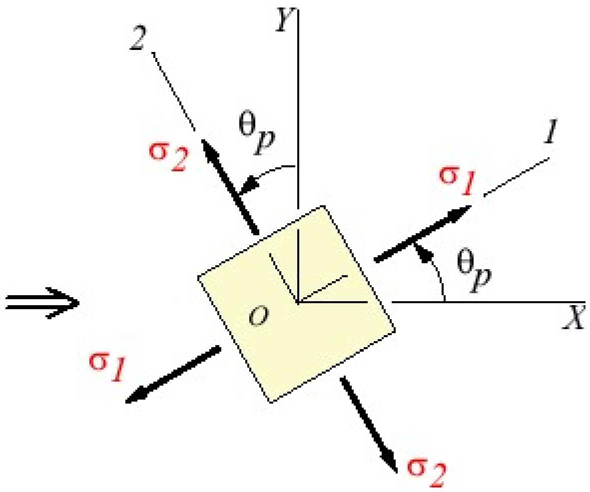
$$d\sigma_{x'} / d\theta = 0 - (\sigma_x - \sigma_x) \sin 2\theta_p + 2\tau_{xy} \cos 2\theta_p = 0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Location of Principal Planes

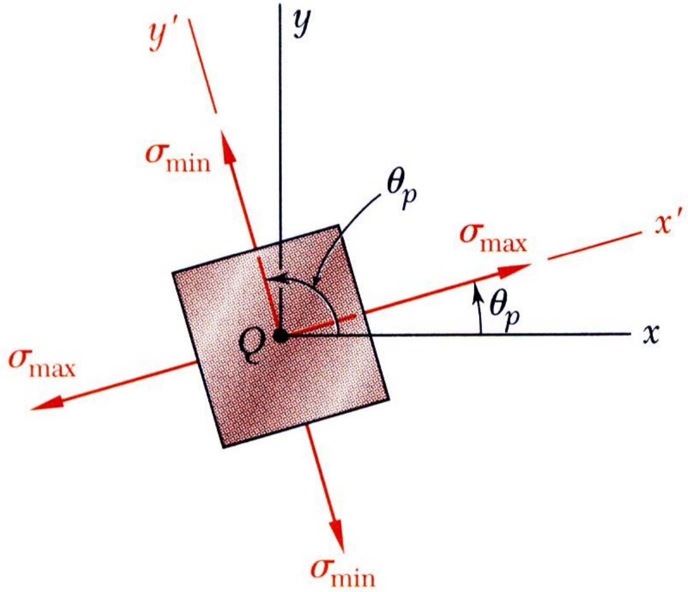


Stresses in given coordinate system



Principal stresses

The angle θ_p can be substituted back into the rotation stress equation to give the actual maximum and minimum stress values. These stresses are commonly referred to as σ_1 (maximum) and σ_2 (minimum),



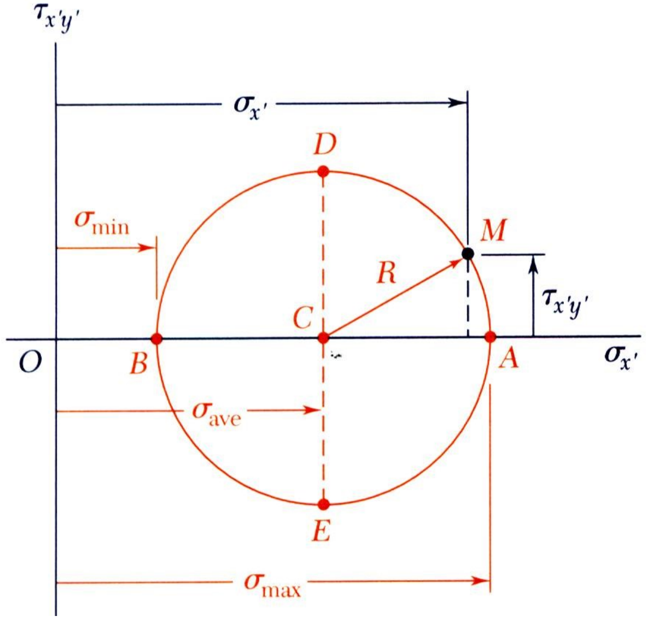
- *Principal stresses* occur on the *principal planes of stress* with zero shearing stresses.

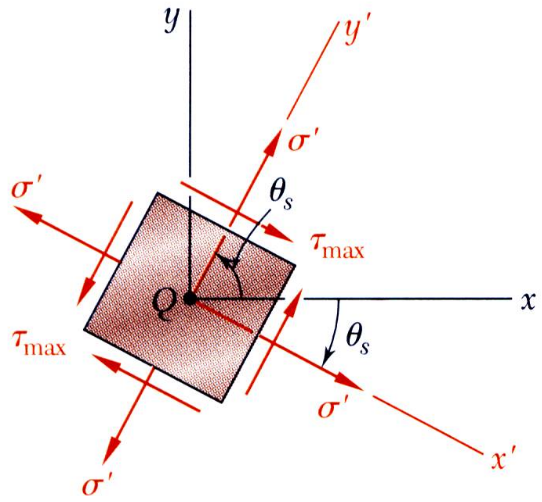
$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

two angles separated by 90°

Maximum Shearing Stress





Maximum shearing stress occurs for $\sigma_{x'} = \sigma_{ave}$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90^o and offset from θ_p by 45^o

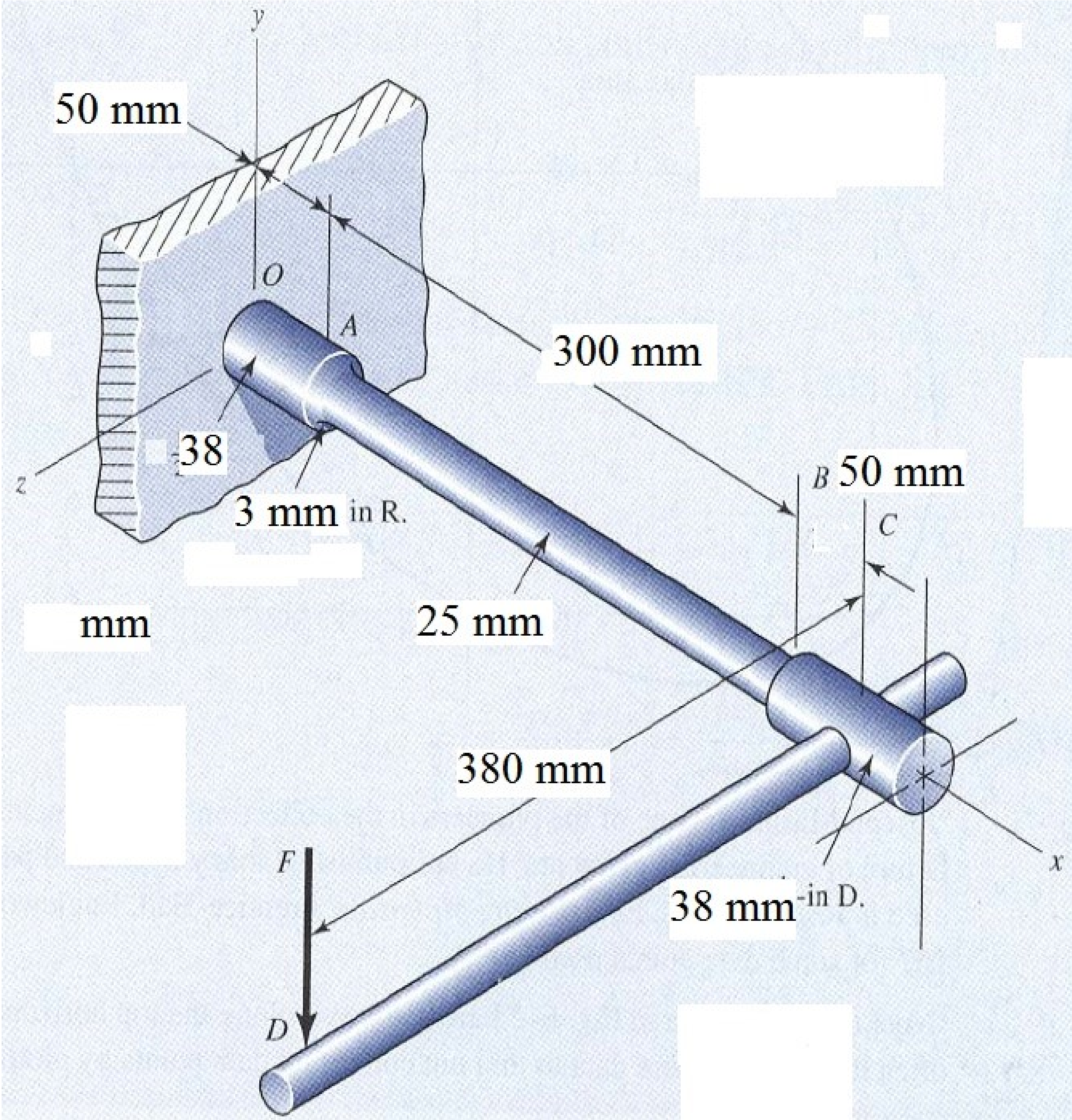
$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

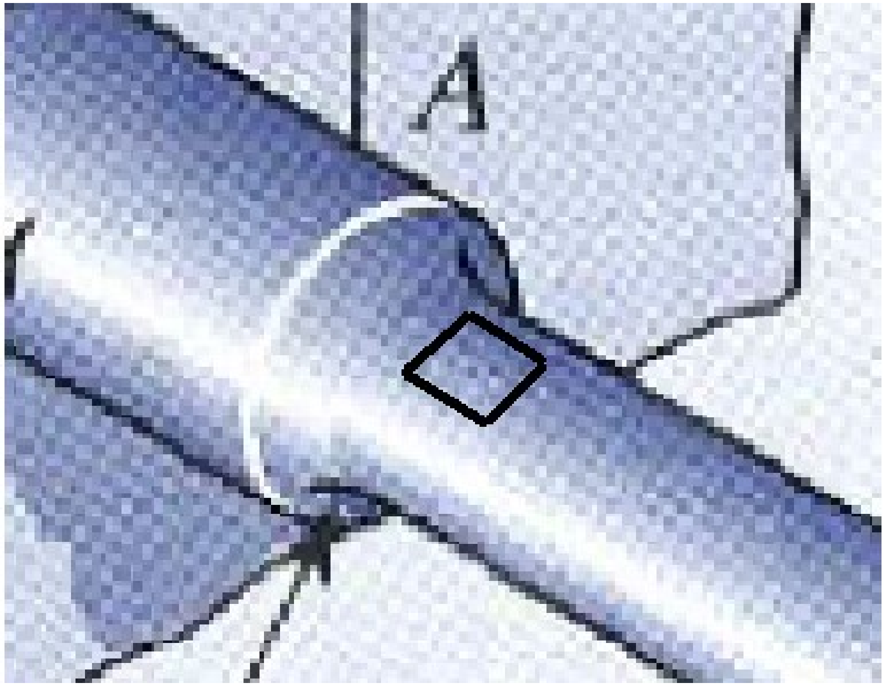
Ex :A force 100 N is applied at D near the end of the lever. The bar OABCis made of AISI 1035steel, forged and heat treated so that it has a minimum (ASTM) yield in strength of 55.8 Mpa . Find

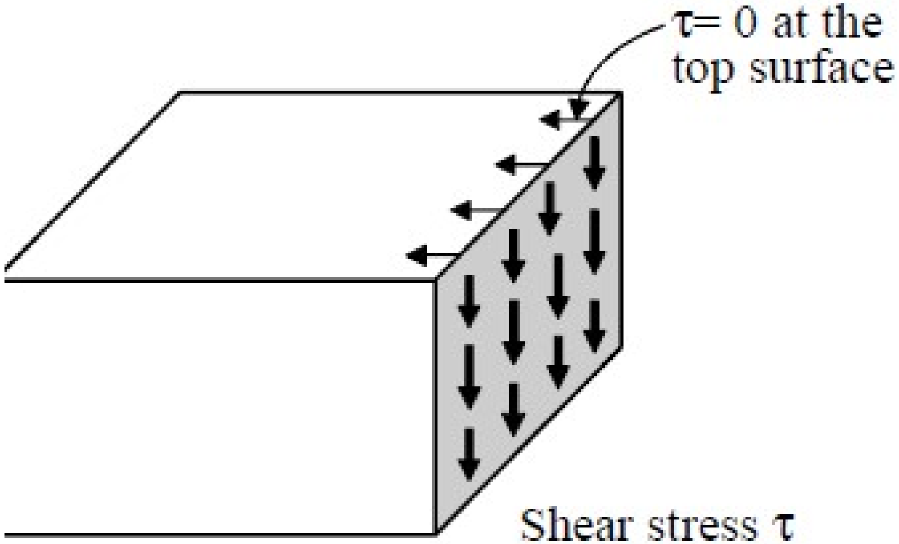
1-the maximum shear

2- the maximum shear required to initiate yielding According to Tresca.

<u>Assume that the lever DCwill not yield and that there is no stress concentration at A.</u>


y
50 mm
O
A
300 mm
38
z
3 mm in R.
B
50 mm
C
mm
25 mm
380 mm
F
38 mm-in D.
x
D


A


τ= 0 at the top surface
Shear stress τ

Find the critical section

The critical sections will be either point ***A or Point O. As the moment of inertia varies with r^4 then point A in the 25 mm diameter is the weakest section.***

The bending stress varies linearly with the distance from the neutral axis, *y, and is* given by

$$\sigma_x = -\frac{My}{I}$$

where *I is the second moment of area about the z axis.*

For a beam of diameter *d the maximum distance from* the neutral axis is *d/2,*

$I = \pi d^4/64$.

$$\sigma_x = \frac{My}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32 \times F \times 0.35}{\pi d^3} = 22.8 \text{ MPa}$$

$$\tau_{zx} = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16 \times F \times 0.38}{\pi (0.025)^3} = 12.39 \text{ MPa}$$

Check if CW or CCW

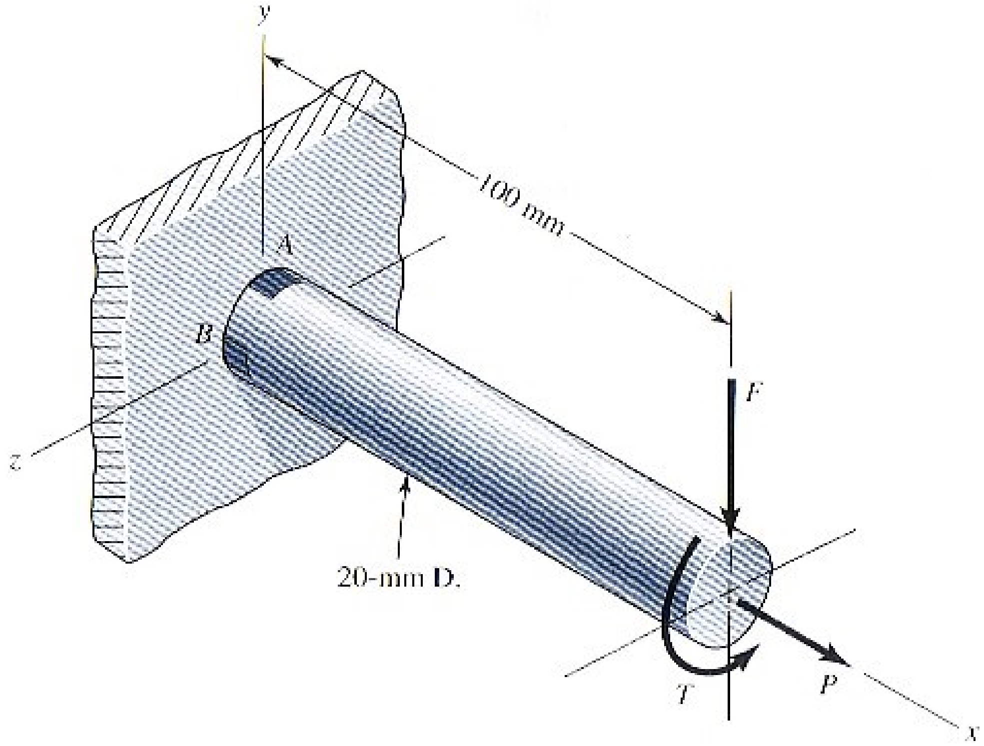
Apply the MSS theory. For a point undergoing plane stress with only one non-zero normal stress and one shear stress, the two non-zero principal stresses ***will have opposite signs***

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

$$\tau_{max} = \pm\sqrt{11.4^2 + 12.39^2} = 16.84 \text{ MPa}$$

16.84 MPa less than 55.8/2 Mpa no yielding

- Ex: A Bar is AISI 1020 hot-rolled steel $S_y = 331$ MPa
  - F = 0.55 kN
  - P = 8.0 kN
  - T = 30 Nm
- Find:
  - Principle stresses and max. shear stress
  - Factor of safety (n) at A

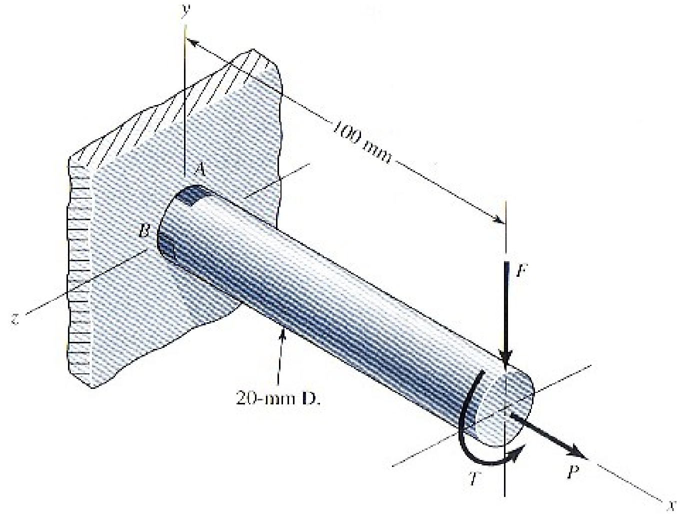


- Axial:

$$\sigma_x = \frac{P}{A} = \frac{P}{\left(\frac{\pi D^2}{4}\right)} = \frac{4P}{\pi D^2}$$

- Bending:

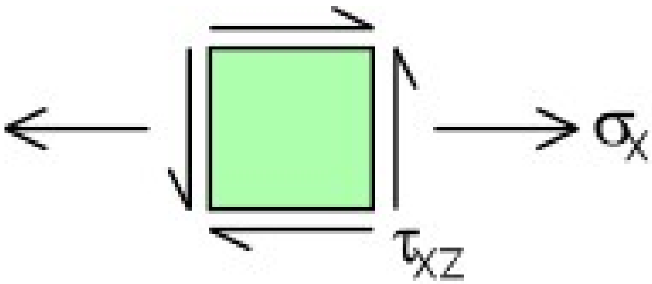
$$\sigma_x = \frac{My}{I} = \frac{(FL)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{64}\right)} = \frac{32FL}{\pi D^3}$$



- Shear: $\tau_{xy} = 0$

- Torsion:

$$\tau_{xz} = \frac{Tc}{J} = \frac{(T)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{32}\right)} = \frac{16T}{\pi D^3}$$



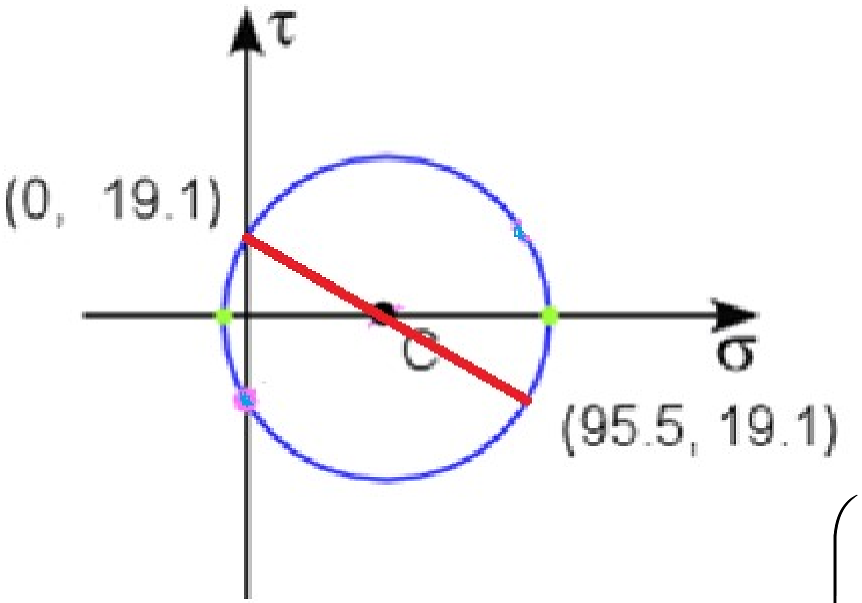
$$\sigma_x = \frac{4P}{\pi D^2} + \frac{32FL}{\pi D^3} = \frac{4PD + 32FL}{\pi D^3}$$

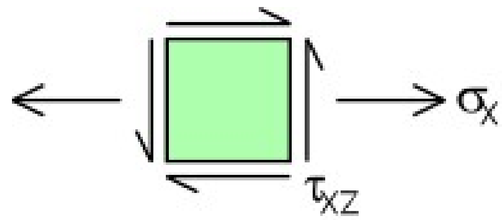
$$\tau_{xz} = \frac{16T}{\pi D^3}$$

- σ_x = 95.5 Mpa

τ_{xz} = 19.1 MPa

X (95.5, -19.1)
Y(0,19.1)





$$\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{95.5 - 0}{2}, 0\right) = (47.8, 0)$$

$$R = \sqrt{(\sigma_x - C_x)^2 + \tau_{xz}{}^2} = \sqrt{(95.5 - 47.8)^2 + 19.1^2} = 51.4$$

Find principal stresses

σ_1 = 99.2 MPa
σ_2 = -3.63 MPa

- Find the von Mises stress (σ_e)

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]}$$

$$\sigma_e = \sqrt{\frac{1}{2}\left[(99.2 - 0)^2 + (0 + 3.63)^2 + (99.2 + 3.63)^2\right]}$$

$$\sigma_e = 101 MPa$$

- S_y for our material = 331 MPa

$$n = \frac{S_y}{\sigma_e} = \frac{331}{101} = 3.28$$